Reflection relations and fermionic basis.

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1. Introduction

(Fateev-Fradkin-Lukyanov-Zamolodchikov-Zamolodchikov).

Consider the sine(h)-Gordon model defined by the Euclidian action

$$\mathcal{A} = \int \left\{ \left[\frac{1}{4\pi} \partial_z \varphi(z,\bar{z}) \partial_{\bar{z}} \varphi(z,\bar{z}) + \frac{2\boldsymbol{\mu}^2}{\sin \pi b^2} \cosh(b\varphi(z,\bar{z})) \right\} \frac{idz \wedge d\bar{z}}{2} \, .$$

Consider the one-point functions of descendants:

$$\frac{\langle P\big(\{\partial_z^k \varphi(0)\}, \{\partial_{\bar{z}}^k \varphi(0)\}\big) e^{a\varphi(0)} \rangle}{\langle e^{a\varphi(0)} \rangle} \, .$$

If P is even this must be invariant under

$$\sigma_1 : \; a \to -a \, .$$

Rewrite the action as

$$\mathcal{A} = \int \Big\{ \Big[\frac{1}{4\pi} \partial_z \varphi(z,\bar{z}) \partial_{\bar{z}} \varphi(z,\bar{z}) + \frac{\boldsymbol{\mu}^2}{\sin \pi b^2} e^{b\varphi(z,\bar{z})} \Big] + \frac{\boldsymbol{\mu}^2}{\sin \pi b^2} e^{-b\varphi(z,\bar{z})} \Big\} \frac{i dz \wedge d\bar{z}}{2} \, .$$

This describes perturbation of Liouville model with $c = 1 + 6Q^2$ ($Q = b + 1/b$).
Another basis of descendants and their one-point functions:

$$\frac{\langle L\big(\{\partial_z^k T_{z,z}(0)\}, \{\partial_{\bar{z}}^k T_{\bar{z},\bar{z}}(0)\}\big) e^{a\varphi(0)} \rangle}{\langle e^{a\varphi(0)} \rangle} \, .$$

It is natural to assume that these ones are invariant under

$$\sigma_2 : \; a \to Q - a \, .$$

Combine these EV into the vector $V_{N,\bar{N}}(a)$. Then in the UV limit we find the Riemann-Hilbert problem:

$$V(Q-a)=V(a), \quad V(a+Q)=(S(a)\otimes \bar{S}(a))V(a)\,,$$

with

$$S(a)=U(-a)U(a)^{-1}\,.$$

2. Reflection matrix.

Considering the domain $\phi_0 \to -\infty$ we have $\varphi(z,\bar{z})=\phi(z)+\phi(\bar{z})$ with

$$\phi(z)=\phi_0-2i\pi_0\log(z)+i\sum_{k\in\mathbb{Z}\setminus 0}^{\infty}\frac{a_k}{k}z^{-k}\,,$$

where the Heisenberg generators satisfy

$$[a_k,a_l]=2k\delta_{k,-l}\,,$$

and zero-mode is canonical: $\pi_0=\frac{\partial}{i\partial\phi_0}$.

It is convenient use the operator language:

$$e^{a\varphi(0)} \Longleftrightarrow \Phi_a = e^{a(\phi_0+\bar\phi_0)}|0\rangle \otimes \overline{|0\rangle}\,, \qquad a_k|0\rangle = 0, \quad k>0\,.$$

Then

$$P\big(\{\partial_z^k\varphi(0)\}\big)e^{a\varphi(0)} \Longleftrightarrow P\big(\{i(k-1)!a_{-k}\}\big)\Phi_a\,.$$

Virasoro algebra of central charge $c = 1+6Q^2$:

$$\mathbf{l}_k = \frac{1}{4}\sum_{j\neq 0,k} a_j a_{k-j} + (i(k+1)Q/2+\pi_0)a_k\,, \quad k\neq 0\,,$$

$$\mathbf{l}_0 = \frac{1}{2}\sum_{j=1}^{\infty} a_{-j}a_j + \pi_0(\pi_0+iQ)\,.$$

We have

$$\mathbf{l}_0\Phi_a = \Delta\Phi_a, \quad \Delta = a(Q-a)\,.$$

The correspondence with the local operators is

$$L\Big(\{\partial_z^k T_{z,z}(0)\}\Big)e^{a\varphi(0)} \Longleftrightarrow L\Big(\{k!\cdot \mathbf{l}_{-k-2}\}\Big)\Phi_a\ .$$

Zamolodchikov's local integrals $\mathbf{i}_1, \mathbf{i}_3, \mathbf{i}_5, \cdots$ act on $\mathcal{V}_a$. To be precise:

$$\mathbf{i}_1 = \mathbf{l}_{-1}\,, \qquad \mathbf{i}_3 = 2\sum_{k=-1}^{\infty}\mathbf{l}_{-3-k}\mathbf{l}_k\,,$$

$$\mathbf{i}_5 = 3\Big(\sum_{k=-1}^{\infty}\sum_{l=-1}^{\infty}\mathbf{l}_{-5-k-l}\mathbf{l}_l\mathbf{l}_k + \sum_{k=-\infty}^{-2}\sum_{l=-\infty}^{-2}\mathbf{l}_l\mathbf{l}_k\mathbf{l}_{-5-k-l}\Big)$$
$$+\frac{c+2}{6}\sum_{k=-1}^{\infty}(k+2)(k+3)\mathbf{l}_{-5-k}\mathbf{l}_k\ .$$

We consider the quotient space: $\mathcal{V}_a^{\mathrm{quo}} = \mathcal{V}_a \ / \ \sum_{k=1}^{\infty}\mathbf{i}_{2k-1}\mathcal{V}_a$. We shall use $\equiv$ for equality in this space.

$\mathcal{V}_a^{\mathrm{quo}}$ has subspaces of even degree only, $\dim\left(\mathcal{V}_{a,k}^{\mathrm{quo}}\right) = p(k/2)$.

Base on Virasoro side, $v_j^{(k)} = \mathbf{v}_j^{(k)}\Phi_a$, is generated by lexicographically ordered action of generators of Virasoro algebra with even indices $\mathbf{v}_1^{(k)}, \cdots, \mathbf{v}_{p(k/2)}^{(k)}$.

Base on Heisenberg side, $h_j^{(k)} = \mathbf{h}_j^{(k)}\Phi_a$, is generated by some polynomials of a_{-k}: $\mathbf{h}_1^{(k)}, \cdots, \mathbf{h}_{p(k/2)}^{(k)}$ of even degrees.

Level 2.

This case is rather trivial since the dimension equals one. Set

$$\mathbf{v}_1^{(2)} = \mathbf{l}_{-2}, \quad \mathbf{h}_1^{(2)} = (a_{-1})^2\,,$$

and obtain

$$v_1^{(2)} \equiv \frac{1}{4}(b+2a)(b^{-1}+2a)h_1^{(2)}\,.$$

Level 4.

Set

$$\mathbf{v}_1^{(4)} = (\mathbf{l}_{-2})^2, \quad \mathbf{v}_2^{(4)} = \mathbf{l}_{-4}\,; \qquad \mathbf{h}_1^{(4)} = a_{-1}^4, \qquad \mathbf{h}_2^{(4)} = a_{-2}^2\,.$$

We factorise out descendants of $\mathbf{i}_1$, this is enough

$$144 \cdot U^{(4)}(a)$$
$$= -\begin{pmatrix} -9 - 12a^2 + 16a^4 - 12aQ + 8a^3Q, & 12(3 + 16a^2 + 14aQ + 3Q^2) \\ 4a(2a^3 + 3a^2Q + 3a), & 12a(3Q + 2a) \end{pmatrix}$$

The determinant of this matrix is

$$\det(U^{(4)}) = C^{(4)} \cdot a(2a+b)(1/b+2a)(2a+3b)(3/b+2a)(1/b+2a+b)\,.$$

Explanation of multiplier a:

$$0 = \mathbf{l}_{-3}\mathbf{l}_{-1}|0\rangle = -2\mathbf{l}_{-4}|0\rangle + \mathbf{l}_{-1}\mathbf{l}_{-3}|0\rangle \equiv -2\mathbf{l}_{-4}|0\rangle\,.$$

Level 6.

The determinant:

$$\det(U^{(6)}) = \frac{N^{(6)}(a,b)(\Delta+2)}{3a^2 - 10Q^2 - 5}\,,$$

where the null-vector contribution

$$N^{(6)}(a,b) = C^{(6)} \cdot a(2a+b)^2(2a+3b)(2a+5b)(1/b+2a)^2(3/b+2a) \\ \times (5/b+2a)(1/b+2a+b)(2/b+2a+b)(1/b+2a+2b)\,,$$

Special vector

$$\mathbf{w}^{(6)} = \mathbf{l}_{-4}\mathbf{l}_{-2} + \frac{c-16}{2}\mathbf{l}_{-6}\,.$$

Level 8.

$$\det(U^{(8)}) = \frac{N^{(8)}(a,b)(\Delta+11)(\Delta+4)}{a^2(-21(76-19Q^2-30Q^4)-(991+1076Q^2)a^2+206a^4)}\,,$$

Two special vectors:

$$\begin{aligned}\mathbf{w}_4^{(8)} &= -28\,\mathbf{l}_{-4}(\mathbf{l}_{-2})^2 + 3(c-36)(\mathbf{l}_{-4})^2 - 2(5c-12)\mathbf{l}_{-6}\mathbf{l}_{-2} \\ &\quad + (4128-325c+5c^2)\mathbf{l}_{-8}, \\ \mathbf{w}_{11}^{(8)} &= 3(\mathbf{l}_{-4})^2 + 4\mathbf{l}_{-6}\mathbf{l}_{-2} + (5c-89)\mathbf{l}_{-8}\,.\end{aligned}$$

3. Fermions.

The space $\mathcal{V}_a^{\mathrm{quo}}$ is created by fermions $\boldsymbol{\beta}^*_{2m-1}$, $\boldsymbol{\gamma}^*_{2m-1}$.

The main property of our fermions.

Consider the generating functions

$$\boldsymbol{\beta}^*(\theta)=\sum_{m=1}^{\infty}\boldsymbol{\beta}^*_{2m-1}e^{-(2m-1)\theta}\,,\ \ \boldsymbol{\gamma}^*(\theta)=\sum_{m=1}^{\infty}\boldsymbol{\gamma}^*_{2m-1}e^{-(2m-1)\theta}\,,$$

then

$$\frac{\langle\boldsymbol{\beta}^*(\theta_1)\cdots\boldsymbol{\beta}^*(\theta_n)\boldsymbol{\gamma}^*(\eta_n)\cdots\boldsymbol{\gamma}^*(\eta_1)\Phi_\alpha(0)\rangle}{\langle\Phi_\alpha(0)\rangle}=\det\bigl(\omega(\theta_i,\eta_j|\alpha)\bigr)\,.$$

Experience teaches that

$$\sigma_1,\sigma_2\ :\ \boldsymbol{\beta}^*_{2m-1}\ \leftrightarrow\ \boldsymbol{\gamma}^*_{2m-1}\,.$$

This can be checked.

Introduce

$$\boldsymbol{\beta}^{*}_{2m-1} = D_{2m-1}(a)\boldsymbol{\beta}^{\mathrm{CFT}*}_{2m-1}, \quad \boldsymbol{\gamma}^{*}_{2m-1} = D_{2m-1}(Q-a)\boldsymbol{\gamma}^{\mathrm{CFT}*}_{2m-1}\,,$$

where

$$D_{2m-1}(a) = (-1)^m C^{2m-1} \frac{\Gamma\left(\frac{2a+(2m-1)b^{-1}}{2Q}\right)\Gamma\left(\frac{2(Q-a)+(2m-1)b}{2Q}\right)}{(m-1)!}\,.$$

For $I^{\pm} = \{2j_1^{\pm}-1, \cdots, 2j_n^{\pm}-1\}$ such that $\#(I^+) = \#(I^-)$ we have

$$\begin{aligned}&\boldsymbol{\beta}^{\mathrm{CFT}*}_{I^+}\boldsymbol{\gamma}^{\mathrm{CFT}*}_{I^-}\Phi_a \\ &\equiv C_{I^+,I^-}\Big(P^{\mathrm{even}}_{I^+,I^-}(\{\mathbf{l}_{-2k}\},\Delta,c) + d\cdot P^{\mathrm{odd}}_{I^+,I^-}(\{\mathbf{l}_{-2k}\},\Delta,c)\Big)\Phi_a\,,\end{aligned}$$

where

$$d = \tfrac{1}{6}\sqrt{(c-25)(24\Delta+1-c)} = (b-b^{-1})(Q-2a)\,,$$

C_{I^+,I^-} is the Cauchy determinant: cooked of $1/(j_p^+ + j_q^- - 1)$.

$$P^{\mathrm{even}}_{I^+,I^-}(\{\mathbf{l}_{-2k}\},\Delta,c) = (\mathbf{l}_{-2})^{\frac{1}{2}(|I^+|+|I^-|)} + \cdots\,.$$

Under σ_2 they transform as original ones:

$$\boldsymbol{\beta}^{\mathrm{CFT}*}_{2m-1} \longrightarrow \boldsymbol{\gamma}^{\mathrm{CFT}*}_{2m-1}, \quad \boldsymbol{\gamma}^{\mathrm{CFT}*}_{2m-1} \longrightarrow \boldsymbol{\beta}^{\mathrm{CFT}*}_{2m-1} \,.$$

Under σ_1:

$$\boldsymbol{\gamma}^{\mathrm{CFT}*}_{2m-1} \longrightarrow \left(\frac{2a-(2m-1)b}{2a+(2m-1)b^{-1}}\right) \boldsymbol{\beta}^{\mathrm{CFT}*}_{2m-1} \,,$$

$$\boldsymbol{\beta}^{\mathrm{CFT}*}_{2m-1} \longrightarrow \left(\frac{2a-(2m-1)b^{-1}}{2a+(2m-1)b}\right) \boldsymbol{\gamma}^{\mathrm{CFT}*}_{2m-1} \,.$$

The only way to be consistent is to require

$$\boldsymbol{\beta}^{\mathrm{CFT}*}_{I^+}\boldsymbol{\gamma}^{\mathrm{CFT}*}_{I^-}\Phi_a \equiv C_{I^+,I^-} \prod_{2j-1\in I^+}(2a+(2j-1)b^{-1}) \prod_{2j-1\in I^-}(2a+(2j-1)b)$$
$$\times \Big(Q^{\mathrm{even}}_{I^+,I^-}(\{a_{-k}\},a^2,Q^2) + g\cdot Q^{\mathrm{odd}}_{I^+,I^-}(\{a_{-k}\},a^2,Q^2)\Big)\Phi_a \,,$$

where $g = a(b-b^{-1})$.

4. Checks.

Level 2. We have $P^{\text{even}}_{\{1,1\}} = \mathbf{l}_{-2}$. Recalling

$$v_1^{(2)} \equiv \frac{1}{4}(b+2a)(b^{-1}+2a)h_1^{(2)}\,.$$

we get $Q^{\text{even}}_{\{1,1\}} = \frac{1}{4}(a_{-1})^2$.

Level 4. We have

$$P^{\text{even}}_{\{1\},\{3\}} = (\mathbf{l}_{-2})^2 + \frac{2c-32}{9}\,\mathbf{l}_{-4}, \quad P^{\text{odd}}_{\{1\},\{3\}} = \frac{2}{3}\,\mathbf{l}_{-4}\,.$$

Applying the matrix $U^{(4)}$ we see that in $P^{\text{even}}_{\{1\},\{3\}} + dP^{\text{odd}}_{\{1\},\{3\}}$ the multiplier $(2a+b^{-1})(2a+3b)$ factorises leaving

$$Q^{\text{even}}_{\{1,3\}} = -\frac{1}{144}\Big\{(4a^2(Q^2-2)-3)(a_{-1})^4 + 12(1+Q^2)a_{-2}^2\Big\}$$

$$Q^{\text{odd}}_{\{1,3\}} = \frac{1}{216}\Big\{(-3+4a^2)(a_{-1})^4 + 12a_{-2}^2)\Big\}\,.$$

Formulae for level 6 (BJMS) and level 8 (Boos) are more complicated. Denominators with $\Delta+2$; $\Delta+4$, $\Delta+11$ show up. We check our conjecture finding factorisation and formulae for $Q^{\rm even}_{\{2i-1,2j-1\}}$, $Q^{\rm odd}_{\{2i-1,2j-1\}}$.

5. Using reflection matrix for finding the fermionic basis.

General structure of determinants:

$$\det(U^{(k)}) = C^{(k)} \cdot N^{(k)}(a,b) \cdot \frac{D_V^{(k)}(\Delta,c)}{D_H^{(k)}(a^2,Q^2)}\,.$$

Let us look for $P^{\substack{\rm even\\ \rm odd}}_{I^+,I^-}$ in the form

$$P^{\rm even}_{I^+,I^-} = \mathbf{v}_1^{(k)} + \frac{1}{D_V^{(k)}(\Delta,c)} \sum_{i=2}^{p(k/2)} X_{I^+,I^-,i}(\Delta,c)\mathbf{v}_i^{(k)}\,,$$

$$P^{\rm odd}_{I^+,I^-} = \frac{1}{D_V^{(k)}(\Delta,c)} \sum_{i=2}^{p(k/2)} Y_{I^+,I^-,i}(\Delta,c)\mathbf{v}_i^{(k)}\,.$$

Polynomials $X_{I^+,I^-,i}(\Delta,c)$, $Y_{I^+,I^-,i}(\Delta,c)$ are of degree D in Δ. Introduce

$$T^+_{I^+,I^-}(a)=\frac{1}{2}\Big\{\prod_{2j-1\in I^+}(2a+(2j-1)b^{-1})\prod_{2j-1\in I^-}(2a+(2j-1)b)+(b\to 1/b)\Big\},$$

$$T^-_{I^+,I^-}(a)$$
$$=\frac{1}{2a(b-b^{-1})}\Big\{\prod_{2j-1\in I^+}(2a+(2j-1)b^{-1})\prod_{2j-1\in I^-}(2a+(2j-1)b)-(b\to 1/b)\Big\}.$$

These polynomials are invariant under $b\to b^{-1}$, hence they depend on b only through Q.

The main requirement is equivalent to two properties of polynomials:

$$D_V^{(k)}(\Delta(-a),c)D_H^{(k)}(a^2,Q^2)$$
$$\times\Big\{T^+_{I^+,I^-}(-a)\Big(D_V^{(k)}(\Delta,c)U_{1,j}^{(k)}(a)+\sum_{i=2}^{p(k/2)}X_{I^+,I^-,i}(\Delta,c)U_{i,j}^{(k)}(a)\Big)$$
$$-(Q^2-4)(Q-2a)T^-_{I^+,I^-}(-a)\sum_{i=2}^{p(k/2)}Y_{I^+,I^-,i}(\Delta,c)U_{i,j}^{(k)}(a)\Big\},$$

is even in a.

$$D_V^{(k)}(\Delta(-a),c)D_H^{(k)}(a^2,Q^2)$$
$$\times\Big\{-T^-_{I^+,I^-}(-a)\Big(D_V^{(k)}(\Delta,c)U_{1,j}^{(k)}(a)+\sum_{i=2}^{p(k/2)}X_{I^+,I^-,i}(\Delta,c)U_{i,j}^{(k)}(a)\Big)$$
$$+(Q-2a)T^+_{I^+,I^-}(-a)\sum_{i=2}^{p(k/2)}Y_{I^+,I^-,i}(\Delta,c)U_{i,j}^{(k)}(a)\Big\},$$

is odd in a.

These requirement provide a terribly overdetermined system of linear equations for our unknowns.
We check level 10. We compute

$$D_V^{(10)}(\Delta, c) = \Big(\Delta + 6\Big)\Big(-23794 + 2905c + (-2285 + 983c)\Delta$$
$$+ (1447 + 71c)\Delta^2 + (149 + c)\Delta^3 + 3\Delta^4\Big)\,,$$

Taking safe $D = 9$ we find all solutions! The actual degrees are

$$D = 7: \quad \{1\}, \{9\} \text{ even}, \; \{3\}, \{7\} \text{ even}, \; \{5\}, \{5\} \text{ even},$$
$$D = 6: \quad \{1\}, \{9\} \text{ odd}, \; \{3\}, \{7\} \text{ odd}, \; \{1, 3\}, \{1, 5\} \text{ even}, \; \text{odd}\,.$$

6. Duality. Duality is $b \to 1/b$. Like for $\sigma_{1,2}$, we have

$$\text{duality}: \quad \boldsymbol{\beta}^*_{2j-1} \; \leftrightarrow \; \boldsymbol{\gamma}^*_{2j-1}\,.$$

Let us have fun:

$$P^{\text{even}}_{\{5\},\{5\}}(\{\mathbf{l}_{-2k}\}) = (\mathbf{l}_{-2})^5 + \frac{20(22+2c+(c-16)\Delta)}{9(6+\Delta)}\mathbf{l}_{-4}(\mathbf{l}_{-2})^3$$

$$+ \frac{1}{6804\ D_V^{(10)}(\Delta,c)}$$

$$\times \Big\{6\Big(-2394125160 + 328307580c - 11439180c^2 + 2245740c^3 +$$

$$(4571783552 - 642113226c + 9291216c^2 + 1626898c^3)\Delta$$

$$+ (283889270 - 184441506c + 1485447c^2 + 487564c^3)\Delta^2$$

$$+ (-306733490 - 17698098c + 377931c^2 + 59192c^3)\Delta^3$$

$$+ (-32577650 - 3648594c + 199578c^2 + 1106c^3)\Delta^4$$

$$+ (-4856082 + 80724c + 4998c^2)\Delta^5 + (-126000 + 5040c)\Delta^6\Big)(\mathbf{l}_{-4})^2\mathbf{l}_{-2}$$

$$
\begin{aligned}
&+ 72\Big(306222000 - 173805840c + 10920960c^2 + 353640c^3 \\
&+ (381614464 - 23068800c - 1839477c^2 + 394058c^3)\Delta \\
&+ (-105570444 + 28836996c - 2363925c^2 + 120078c^3)\Delta^2 \\
&+ (-5062960 + 1902948c - 186516c^2 + 10948c^3)\Delta^3 \\
&+ (6142752 - 591276c + 16296c^2 + 168c^3)\Delta^4 \\
&+ (183204 - 17388c + 504c^2)\Delta^5\Big)\mathbf{l}_{-6}(\mathbf{l}_{-2})^2 \\
&+ 3\Big(-36240157632 + 6121778448c - 402247260c^2 + 15838734c^3 + 980700c^4 \\
&+ (61259894752 - 7807807432c + 120911226c^2 - 916009c^3 + 946078c^4)\Delta \\
&+ (-7496632304 + 562374632c - 138115254c^2 + 2579783c^3 + 269878c^4)\Delta^2 \\
&+ (-2902569880 + 343253716c - 42063144c^2 + 978190c^3 + 31388c^4)\Delta^3 \\
&+ (611052008 - 52433468c - 1301100c^2 + 80872c^3 + 1568c^4)\Delta^4 \\
&+ (38386992 - 1678896c - 154944c^2 + 7008c^3)\Delta^5 \\
&+ (3894912 - 324864c + 6912c^2)\Delta^6\Big)\mathbf{l}_{-6}\mathbf{l}_{-4}
\end{aligned}
$$

$$
\begin{aligned}
&+ 18\Big(84650153280 - 14906569500c + 601240950c^2 - 1997070c^3 + 598500c^4 \\
&+ (-63120449168 + 11108354394c - 726265569c^2 + 16981463c^3 + 370230c^4)\Delta \\
&+ (-4980065552 + 1173915830c - 54554649c^2 - 1312234c^3 + 171640c^4)\Delta^2 \\
&+ (2427198620 - 271665042c + 16272864c^2 - 804287c^3 + 26670c^4)\Delta^3 \\
&+ (11156180 + 22230214c - 2038338c^2 + 42124c^3 + 560c^4)\Delta^4 \\
&+ (9021768 + 97848c - 64800c^2 + 2064c^3)\Delta^5 \\
&+ (649152 - 54144c + 1152c^2)\Delta^6\Big)\mathbf{l}_{-8}\mathbf{l}_{-2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \Big(2402050721280 - 453732439584c - 1008508824c^2 + 736353804c^3 \\
&+ 10413480c^4 + 2116800c^5 + (-750420745088 + 104186820112c \\
&+ 5982020544c^2 - 1576485004c^3 + 55452740c^4 + 1381800c^5)\Delta \\
&+ (472993701600 - 34597963440c + 5014768290c^2 - 448750215c^3 \\
&+ 5613636c^4 + 585060c^5)\Delta^2 + (141065264032 - 14296085648c \\
&+ 1417241010c^2 - 80355379c^3 + 222908c^4 + 91140c^5)\Delta^3 \\
&+ (-36292325160 + 7660662252c - 400215072c^2 + 5783664c^3 \\
&- 80556c^4 \ + 5880c^5)\Delta^4 + (3111074008 - 286403588c + 23527848c^2 \\
&- 1241092c^3 + 23912c^4)\Delta^5 + (5295360 + 16262592c - 1153344c^2 \\
&+ 20352c^3)\Delta^6 + (4612608 - 297216c + 4608c^2)\Delta^7\Big)\mathbf{l}_{-10}\Big\} \, .
\end{aligned}
$$