Holographic Topological Insulators and Superconductors

Moshe Rozali

University of British Columbia

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Based on:

M.R., ArXiv/1210.0029, and PRL 109, 231601 (2012).

M.R. and Alex Vincart-Emard, ArXiv/1310.4510, to appear in JHEP.

Work in progress.

Outline:

Introduction and Motivation

Many interesting phenomena necessitate the introduction of spatial inhomogeneity:

- Momentum diffusion, e.g. in studies of conductivity.
- New probes of bulk phenomena: e.g. Josephson junctions.
- Phenomena confined to an interface.

This talk is an example of the latter, study of the gapless states living on the surface of topological materials. In this case the *only* interesting low energy physics is on the surface as the bulk is gapped.

The new tool required is solution of elliptic PDEs. This is done numerically, using pseudo-spectral methods and Newton iteration.

Topological Gapped Materials

When bulk is gapped low energy physics is trivial. Nevertheless, sometimes interesting things happen as consequence of topology in field space:

- Quantized transport coefficients (most famously Hall conductivity).
- Ground state degeneracy on topologically non-trivial manifolds.
- Interesting topological field theories describing low energy excitations.
- Most importantly for this talk: existence of gapless excitations on the edges of the bulk materials, and associated dissipationless transport.

Interesting connections to anomaly inflow, index theorems, topological QFTs and lattice fermions, among other issues.

Much of the treatment is at the level of free electrons (i.e. band theory). Can this be extended to interacting ones, and how does the structure of of topological phases get modified? To this end we use holography.

Topological Insulators

- New class of insulators (i.e. gapped states) characterized by topological quantum numbers (calculable e.g. from band theory).
- To be concrete, study the Z_2 three dimensional TI. In the continuum EFT, this topological insulator can be thought of as the theory of a free *massive* Dirac fermion ψ with mass M:

$$\mathcal{L}_\psi = \bar{\psi}(\gamma^\mu D_\mu - M)\psi$$

- Time reversal invariance implies that M is real, leaving the options of $M = \pm|M|$. The Z_2 topological invariant is encoded in the sign of the mass term. We define $M < 0$ to correspond to topologically non-trivial insulators.
- Locally indistinguishable (and boring) bulk physics for both signs of M, but this ambiguity has consequences in domain walls where $M(x)$ changes sign.

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Electromagnetic Response

Couple the massive fermion to external electromagnetic fields and integrate it out.

- The low energy theory of the electromagnetic fields is free:

$$\mathcal{L}_{\mathcal{A}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{16\pi} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- The θ term receives contribution from the chiral anomaly only, so it encodes the sign of the fermion mass term. $\theta = 0$ and $\theta = \pi$ are consistent with time reversal invariance, with our conventions $\theta = \pi$ is the topologically non-trivial insulator.
- As before, nothing interesting happens unless we consider an interface between a TI and topologically trivial insulator. In this level of description the interface is a domain wall where θ interpolates between 0 and π.

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Topologically Protected Edge States

Consider an interface between a (strong) TI and a trivial insulator:

- Index theorems predicts a single *gapless* Dirac fermion bound to the domain wall. This gapless state is stable to any time-reversal invariant perturbation.
- Each surface state gives a quantized contribution to the surface Hall conductivity in units of *half* of the usual quantized value of $\frac{e^2}{h}$.
- We can get Hall current without an external magnetic field, but we need an arbitrarily weak TR breaking perturbation to create a gap. The actual Hall response is then quantized in the usual units (and depends on how we generate the gap).
- Other exotic effects encoded in the Axion Electrodynamics action: magnetoelectric effect, image magnetic monopoles, Witten araday rotation,...
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Complicating Things

So far everything is well-described in terms of free fermions. Interesting to add complications such as noise or strong interactions, to see if there are new phenomena. Sadly this is purely of theoretical interest so far.
For example: Can one get *fractional* topological insulators? Parton construction J. Maciejko, X. -L. Qi, A. Karch and S. -C. Zhang, Phys. Rev. Lett. **105**, 246809 (2010)

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Holographic Realization

Realization of the parton construction using probe brane system. This is the D3-D7 system with the following identifications

C. Hoyos-Badajoz, K. Jensen and A. Karch, Phys. Rev. D **82**, 086001 (2010)

- The "statistical" gauge fields: large N SU(N) gauge theory, in the large N and large λ limit. Realized as the worldvolume gauge field of the D3 branes.
- Fermionic partons in the fundamental representation: 3-7 strings which are hypermultiplets of $N = 2$ SUSY. This multiplet contains both fermions and bosons, but only the fermions are expected to localize on the interface.
- Mass term M: the separation of the 3-7 branes in the two transverse directions. This is an a priori complex parameter; we take M to be real and depending on one of the spatial coordinates x, to realize an interface.

Now take the decoupling limit, do AdS/CFT with some probe branes.

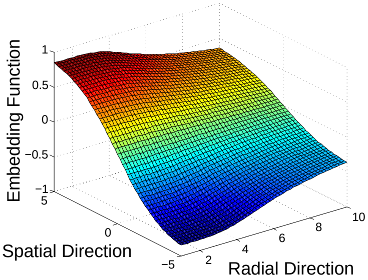
The Set Up

- D3 branes create the $AdS_5 \times S^5$ geometry, studied at finite temperature T

$$
\begin{aligned}
ds^2 &= \frac{1}{2}\pi^2 T^2 \left(-\frac{f^2(\rho)}{h(\rho)} dt^2 + h(\rho) d\vec{x}_3^{\,2}\right) + \frac{d\rho^2}{\rho^2} + d\Omega_5^2 \\
f(\rho) &= 1 - \frac{1}{\rho^4} \qquad\qquad h(\rho) = 1 + \frac{1}{\rho^4}
\end{aligned}
$$

- Probe D7 branes fill AdS_5, in the "black hole" type embedding, and wrap S^3 inside S^5 which is characterized by an angle θ.
- The asymptotic value of θ gives the mass of the hypermultiplets. We take this mass to vary over a spatial direction x. Later we'll introduce chemical potential which is spatially homogeneous. For now study the embedding function $\theta(\rho, x)$.

The Embedding



- The embedding function ($\cos\theta$ for an angle on S^5), displayed as function of radial and spatial coordinates.

- On the horizon the value of $\chi = \cos\theta$ tracks the profile of the asymptotic mass $m(x)$, as is typical of "black hole" embedding.
- We take that mass profile $m(x)$ to approximate a step function.
- Good exercise in application of Pseudo-Spectral methods. Solution shown done using Chebyshev grid with 41 points each direction.

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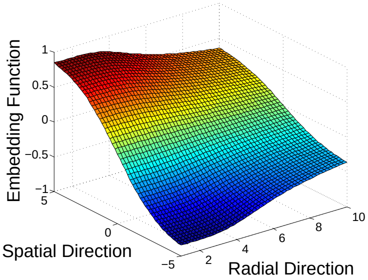

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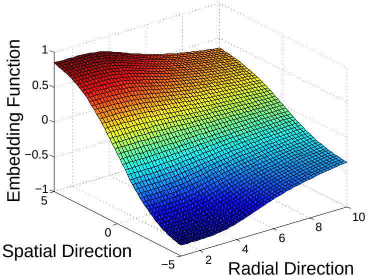

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Nature of the Surface Fluid

There are some indications the localized modes on the surface should have interesting physics:

- Only fermions localize. These fermions couple to other gapless modes (non-Abelian gauge field, or more generally quantum critical sector). Non-Fermi liquid?
- Fractional Quantum Hall response (once the system is gapped). Anyonic behaviour?
- Parent theories (probe brane metals or insulators) already have some unusual properties.

As a first step to probe this physics, put the system at a finite chemical potential and study the thermodynamics.

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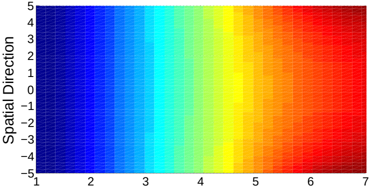
Finite Density



- The gauge field A_0 displayed as function of radial and spatial coordinates.
- Inhomogeneity only in the subleading (normalizable) mode.

- To study the nature of the edge fluid, turn on finite (spatially homogeneous) chemical potential.
- As expected, this turns on finite density sharply localized at the interface, where we have gapless modes.
- The density at the interface is approximately independent of details of the mass profile away from the interface. Other details, e.g the detailed density profile around the interface, do depend on the regulator.

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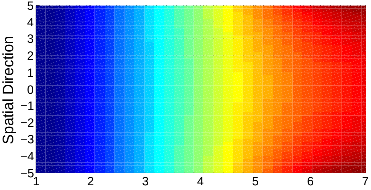

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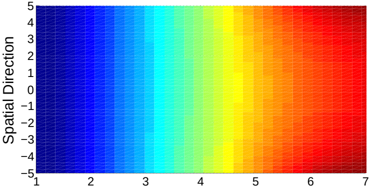

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Expansion of the Free Energy

To extract information from the thermodynamics, study the free energy as an expansion in $\tilde{\mu} = \frac{\mu}{T}$:

- Based on dimensional analysis alone $F = T^4 \mathcal{F}(\tilde{\mu})$.
- Contribution of a Fermi surface would scale as $\mathcal{F} \sim \tilde{\mu}^4$, leading to (3dim number) density scaling like μ^3.
- This is indeed the leading order contributions at large $\tilde{\mu}$. The exponent 3 is accurate to about 3-4 percent.
- The state on the interface is therefore *compressible*. Furthermore the state also does not break any symmetries (since the chiral condensate vanishes on the interface).
- Compressible states which break no continuous symmetries are sometimes taken to be an indication of a (potentially "hidden") Fermi surface. L. Huijse and S. Sachdev, "Fermi surfaces and gauge-gravity duality," Phys. Rev. D **84**, 026001 (2011)

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Hyperscaling Violation

To extract information about corrections, expand $F = \mu^4 \mathcal{F}(1/\tilde{\mu})$, a low temperature expansion.

- To leading order $\mathcal{F} = 1$. Subleading terms are sensitive to the excitations of the ground state.
- The dependence can be parametrized in terms of hyperscaling violation exponent θ, such that $S \sim T^{3-\theta}$ at low temperatures.
- For bosons $\theta = 0$, for Fermions $\theta = 2$. The fermionic contribution is larger, i.e. it comes at a lower order in the low temperature expansion.
- We find that with high confidence, the term in the expansion corresponding to $\theta = 2$ is non-vanishing. This is the value corresponding to excitations of a Fermi surface.

So, everything in the thermodynamics is consistent with the existence of a Fermi surface for the interface excitations. More probes of this Fermi surface in future work.

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Chiral Superconductors

Now let us turn to another set of gapped states, namely *superconductors*. Those also exhibit topological quantum numbers and gapless edge states. Most interestingly, under some circumstances those are *Majorana* modes, that is they are identical to their anti-particles.

Majorana modes are fascinating in that they are in some sense *fractionalized* and are related to non-Abelian statistics and topological quantum computations. Since they don't couple to any conserved global current (i.e. they are neutral) discovering them in experiment is a subtle and controversial issue.

For our purposes we need to know that superconductors with non-trivial order parameter, we'll focus on p+ip, may carry Majorana edge states, which are best thought of as two-fold degeneracy in the spectrum in the presence of an edge.

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Josephson Junctions

To realize holographic superconductor with non-trivial order parameter we use Gubser model, non-Abelian gauge field in asymptotically AdS_4 space, with $A = \Phi\tau^3 dt + w\tau^- d\zeta + w^*\tau^+ d\bar{\zeta}$.

Instead of constructing a single interface, we build an S-N-S Josephson junction, which has *two* such interfaces between our putative topological superconductor and the trivial material (here the metal sandwiched between the superconductors). The two superconductors have a phase difference we denote by Φ.

This may be a good way to probe the physics of Majorana fermions due to the following fact: if the superconductor is topological, such junctions are are doubly degenerate due to the Majorana modes. These two states are exchanged upon $\Phi \to \Phi + 2\pi$. Thus the physics is inherently 4π periodic (!!!).
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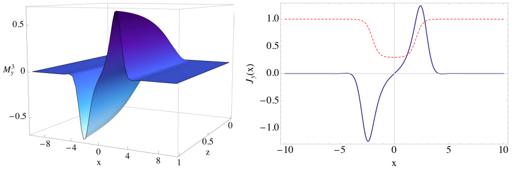
Chiral Edge Currents

First unusual feature of our holographic Josephson junction is the appearance of counter-propagating edge currents. We construct a Josephson junction by imposing a chemical potential modulated in the x-direction. This results in an unsourced Josephson current in the x-direction, encoded by the gauge field A_x^3.

However, in addition we have current in the y-direction, localized on the interfaces:



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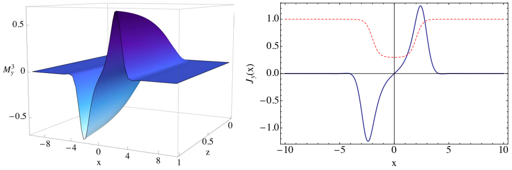

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Anomalous Josephson Effect?

The physics of the Josephson effect ought to be 4π periodic. There are conflicting experimental and theoretical report of seeing such unconventional periodicity in various systems including Josephson junction.

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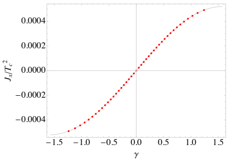

Other potentially anomalous effects (e.g. dependence of width of junctiuon) also turn out conventional.

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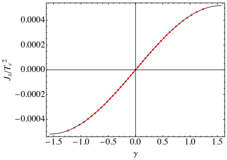

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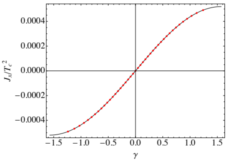

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Future Directions: Topological Superconductors

List of questions:

- Which holographic superconductors are topological?
- Which carry unpaired Majorana modes?
- How to discover such modes in holographic thought experiments?

To the last point

- Non-equilibrium Josephson junctions: AC effect or current noise.
- Fermionic probes: Andreev scattering and bound states (in progress).

More ambitiously: general understanding of topological quantum numbers and their consequences in strongly interacting systems.

Future Directions: Topological Insulators

It is interesting to probe more precisely the nature of the surface fluid:

- *In Progress*: Analytic structure in current correlators (spectral densities) at finite momentum, look for low energy spectral weight at finite momentum.

  R. J. Anantua, S. A. Hartnoll, V. L. Martin and D. M. Ramirez, arXiv:1210.1590 [hep-th]

Depending on results:

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