Stringy Differential Geometry and $\mathcal{N}=2\ D=10$ Supersymmetric Double Field Theory

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- It was the rotational $\mathbf{SO}(3)$ or Lorentz $\mathbf{SO}(1,3)$ symmetry that reorganized them into four or two compact equations.
- This talk aims to show that

Type IIA & IIB supergravities may undergo an analogous 'simplification' and 'unification',

restructured by **'Stringy Differential Geometry',**

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Talk based on works with Imtak Jeon & Kanghoon Lee

- Differential geometry with a projection: Application to double field theory
  JHEP 1104:014 arXiv:1011.1324
- Double field formulation of Yang-Mills theory PLB 701:260(2011) arXiv:1102.0419
- Stringy differential geometry, beyond Riemann PRD 84:044022(2011) arXiv:1105.6294
- Incorporation of fermions into double field theory JHEP 1111:025 arXiv:1109.2035
- Supersymmetric Double Field Theory: Stringy Reformulation of Supergravity
  PRD Rapid Comm. 85:081501 (2012) arXiv:1112.0069
- Ramond-Ramond Cohomology and O(D,D) T-duality JHEP 1209:079 arXiv:1206.3478
- **Stringy Unification of Type IIA and IIB Supergravities under $\mathcal{N} = 2$ $D = 10$ Supersymmetric Double Field Theory** PLB 723:245(2013) arXiv:1210.5078
- Comments on double field theory and diffeomorphisms JHEP 1306:098 arXiv:1304.5946
- Covariant action for a string in doubled yet gauged spacetime
  NPB 880:134(2014) arXiv:1307.8377

Related works on U-duality

- U-geometry : SL(5) with Yoonji Suh, JHEP 04 (2013) 147 arXiv:1302.1652
- M-theory and F-theory from a Duality Manifest Action
  with Chris Blair and Emanuel Malek, to appear in JHEP arXiv:1311.5109

- In Riemannian geometry, the fundamental object is the metric, $g_{\mu\nu}$.
  - Diffeomorphism: $\partial_\mu \longrightarrow \nabla_\mu = \partial_\mu + \Gamma_\mu$
  - $\nabla_\lambda g_{\mu\nu} = 0, \ \Gamma^\lambda_{[\mu\nu]} = 0 \longrightarrow \Gamma^\lambda_{\mu\nu} = \frac{1}{2} g^{\lambda\rho}(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu})$
  - Curvature: $[\nabla_\mu, \nabla_\nu] \longrightarrow R_{\kappa\lambda\mu\nu} \longrightarrow R$
- On the other hand, string theory puts $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ on an equal footing, as they form a multiplet of T-duality.
- This suggests the existence of a novel unifying geometric description of them, generalizing the above Riemannian formalism.
- Basically, Riemannian geometry is for *Particle* theory. *String* theory requires a novel differential geometry which geometrizes the whole NS-NS sector.

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Closed string

- The low energy effective action of $g_{\mu\nu}$, $B_{\mu\nu}$, ϕ is well known in terms of Riemannian geometry

$$S_{\text{eff.}} = \int_{\Sigma_D} \sqrt{-g}e^{-2\phi}\left(R_g + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu}\right).$$

- Diffeomorphism and B-field gauge symmetry are manifest,

$$x^\mu \rightarrow x^\mu + \delta x^\mu\,, \qquad B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu\Lambda_\nu - \partial_\nu\Lambda_\mu\,.$$

- Though not manifest, this enjoys T-duality which mixes $\{g_{\mu\nu}, B_{\mu\nu}, \phi\}$. Buscher

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T-duality

- Redefine the dilaton,

$$e^{-2d} = \sqrt{-g}e^{-2\phi}$$

- Set a $(D+D)\times(D+D)$ symmetric matrix, Duff

$$\mathcal{H}_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$$

- Hereafter, $A, B, ...$: 'doubled' $(D+D)$-dimensional vector indices, with $D = 10$ for SUSY.

- T-duality is realized by an $\mathbf{O}(D,D)$ rotation, Tseytlin, Siegel

$$\mathcal{H}_{AB} \longrightarrow M_A{}^C M_B{}^D \mathcal{H}_{CD}\,, \qquad\qquad d \longrightarrow d\,,$$

where

$$M \in \mathbf{O}(D,D)\,.$$

T-duality

- $\mathbf{O}(D,D)$ metric,

$$\mathcal{J}_{AB} := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

freely raises or lowers the $(D+D)$-dimensional vector indices.

Double Field Theory (DFT), 2009 - 2010

- Hull and Zwiebach , later with Hohm

$$S_{\rm DFT}=\int_{\Sigma_D} e^{-2d}\,L_{\rm DFT}(\mathcal{H},d)\,,$$

where

$$L_{\rm DFT}(\mathcal{H},d)=\quad \mathcal{H}^{AB}\Big(4\partial_A\partial_B d-4\partial_A d\,\partial_B d+\tfrac{1}{8}\partial_A\mathcal{H}^{CD}\partial_B\mathcal{H}_{CD}-\tfrac{1}{2}\partial_A\mathcal{H}^{CD}\partial_C\mathcal{H}_{BD}\Big)$$
$$+4\partial_A\mathcal{H}^{AB}\partial_B d-\partial_A\partial_B\mathcal{H}^{AB}\,.$$

- Spacetime is formally doubled, $y^A=(\tilde{x}_\mu,x^\nu)$, $A=1,2,\cdots,D{+}D$.
- Yet, Double Field Theory (for NS-NS sector) is a D-dimensional theory written in terms of $(D+D)$-dimensional language, i.e. tensors.
- All the fields MUST live on a D-dimensional null hyperplane or 'section', Σ_D.

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Section condition in Double Field Theory

- By stating DFT lives on a D-dimensional null hyperplane, we mean that, the $\mathbf{O}(D,D)$ d'Alembert operator is trivial, acting on arbitrary fields as well as their products:

$$\partial_A\partial^A\Phi = 2\frac{\partial^2}{\partial\tilde{x}_\mu\partial x^\mu}\Phi \equiv 0\,, \qquad \partial_A\Phi_1\partial^A\Phi_2 \equiv 0 \quad : \quad \text{section condition}$$

What does O(D,D) do in Double Field Theory ?

- O(D,D) rotates the D-dimensional null hyperplane where DFT lives.
- *A priori*, the O(D,D) structure in DFT is a 'meta-symmetry' or 'hidden symmetry' rather than a Noether symmetry.
- Only after dimensional reductions,

$$D = d + n \Longrightarrow d\,,$$

it can generate a Noether symmetry,

$$\mathrm{O}(n,n)$$

which is a subgroup of O(D,D) and 'enhanced' from O(n).

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String worldsheet origin of the section condition

- Closed string

$$X_L(\sigma^+) = \tfrac{1}{2}(x+\tilde{x}) + \tfrac{1}{2}(p+w)\sigma^+ + \cdots\,,$$

$$X_R(\sigma^-) = \tfrac{1}{2}(x-\tilde{x}) + \tfrac{1}{2}(p-w)\sigma^- + \cdots\,.$$

- Under T-duality,

$$X_L + X_R \;\longrightarrow\; X_L - X_R\,,$$

such that

$$(x,\tilde{x},p,w) \;\longrightarrow\; (\tilde{x},x,w,p)\,.$$

- Level matching condition for the massless sector,

$$p\cdot w \equiv 0 \quad\Longleftrightarrow\quad \partial_A\partial^A = 2\frac{\partial^2}{\partial x^\mu \partial\tilde{x}_\mu} \equiv 0\,.$$

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- Up to $\mathbf{O}(D,D)$ rotation, we may further choose to set

$$\frac{\partial}{\partial \tilde{x}_{\mu}} \equiv 0\,.$$

- Then DFT reduces to the effective action:

$$S_{\mathrm{DFT}} \Longrightarrow S_{\mathrm{eff.}} = \int_{\Sigma_D} \sqrt{-g}e^{-2\phi}\left(R_g + 4(\partial\phi)^2 - \tfrac{1}{12}H^2\right).$$

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- *What about the diffeomorphism and the B-field gauge symmetry?*

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Diffeomorphism & B-field gauge symmetry

- Introducing a unifying $(D+D)$-dimensional parameter,

$$X^A = (\Lambda_\mu, \delta x^\nu)$$

it is possible to spell a unifying transformation rule, up to the section condition,

$$\delta_X \mathcal{H}_{AB} \equiv X^C \partial_C \mathcal{H}_{AB} + 2\partial_{[A} X_{C]} \mathcal{H}^C{}_B + 2\partial_{[B} X_{C]} \mathcal{H}_A{}^C ,$$

$$\delta_X \left(e^{-2d}\right) \equiv \partial_A \left(X^A e^{-2d}\right) .$$

- In fact, these coincide with the generalized Lie derivative,

$$\delta_X \mathcal{H}_{AB} = \hat{\mathcal{L}}_X \mathcal{H}_{AB} , \qquad \delta_X (e^{-2d}) = \hat{\mathcal{L}}_X (e^{-2d}) = -2(\hat{\mathcal{L}}_X d) e^{-2d} .$$

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- **Definition** Siegel, Courant, Grana ...

$$\hat{\mathcal{L}}_X T_{A_1\cdots A_n} := X^B\partial_B T_{A_1\cdots A_n} + \omega\partial_B X^B T_{A_1\cdots A_n} + \sum_{i=1}^{n}(\partial_{A_i}X_B - \partial_B X_{A_i})T_{A_1\cdots A_{i-1}}{}^{B}{}_{A_{i+1}\cdots A_n}.$$

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$$[\hat{\mathcal{L}}_X, \hat{\mathcal{L}}_Y] \equiv \hat{\mathcal{L}}_{[X,Y]_\mathbf{C}},$$

where $[X,Y]_\mathbf{C}$ denotes the C-bracket,

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Diffeomorphism & B-field gauge symmetry

- Brute force computation can show that

$$\hat{\mathcal{L}}_X \mathcal{H}_{AB} \equiv X^C \partial_C \mathcal{H}_{AB} + 2\partial_{[A} X_{C]} \mathcal{H}^C{}_B + 2\partial_{[B} X_{C]} \mathcal{H}_A{}^C \,,$$

$$\hat{\mathcal{L}}_X \left(e^{-2d}\right) \equiv \partial_A \left(X^A e^{-2d}\right) \,,$$

  are symmetry of the action by Hull, Zwiebach & Hohm

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- This expression may be analogous to the case of writing the Riemannian scalar curvature, R, in terms of the metric and its derivative.

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- Proposal of a underlying stringy differential geometry for DFT
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  which 'unifies' IIA and IIB SUGRAs and 'contains' *more*.
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- **Gauge symmetries**

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  2. **A *pair* of local Lorentz symmetries, Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$**

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$$\partial_A \partial^A \Phi \equiv 0 \,, \qquad \partial_A \Phi_1 \partial^A \Phi_2 \equiv 0 \,,$$

which implies an invariance under a shift set by a 'derivative-index-valued' vector,

$$\Phi(x+\Delta) = \Phi(x) \qquad \text{if} \quad \Delta^A = \varphi \partial^A \varphi' \quad \text{for arbitrary functions } \varphi \text{ and } \varphi' \,.$$

- The section condition implies, and in fact can be shown to be equivalent to, an equivalence relation for the coordinates,

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- A 'physical point' is one-to-one identified with a 'gauge orbit' in the coordinate space.
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- **Bosons**
  - NS-NS sector
    - DFT-dilaton: d
    - DFT-vielbeins: $V_{Ap}\,,\quad \bar{V}_{A\bar{p}}$
  - R-R potential: $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$
- **Fermions**
  - DFT-dilatinos: $\rho^{\alpha}\,,\quad \rho'^{\bar{\alpha}}$
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Index	Representation	Metric (raising/lowering indices)
$A, B, \cdots$	**O**(D,D) **& DFT-diffeom. vector**	$\mathcal{J}_{AB}$
$p, q, \cdots$	**Spin**$(1, D{-}1)_L$ **vector**	$\eta_{pq} =$ **diag**$(-++\cdots+)$
$\alpha, \beta, \cdots$	**Spin**$(1, D{-}1)_L$ **spinor**	$C_{+\alpha\beta}\,,\quad (\gamma^p)^T = C_+\gamma^p C_+^{-1}$
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R-R potential and Fermions carry NOT $(D+D)$-dimensional BUT undoubled D-dimensional indices.

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***A priori*, O(D,D) rotates only the O(D,D) vector indices (capital Roman), and the R-R sector and all the fermions are O(D,D) T-duality singlet.**

The usual IIA $\Leftrightarrow$ IIB exchange will follow only after fixing a gauge.

- The DFT-dilaton gives rise to a scalar density with weight one,

$$e^{-2d}\,.$$

- The DFT-vielbeins satisfy the **four defining properties:**

$$V_{Ap}V^{A}{}_{q}=\eta_{pq}\,,\qquad \bar{V}_{A\bar{p}}\bar{V}^{A}{}_{\bar{q}}=\bar{\eta}_{\bar{p}\bar{q}}\,,\qquad V_{Ap}\bar{V}^{A}{}_{\bar{q}}=0\,,\qquad V_{Ap}V_{B}{}^{p}+\bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}}=\mathcal{J}_{AB}\,.$$

- For fermions, the gravitinos and the DFT-dilatinos are not twenty, but ten-dimensional Majorana-Weyl spinors,

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where $\mathbf{c}$ and $\mathbf{c}'$ are arbitrary independent two sign factors, $\mathbf{c}^2=\mathbf{c}'^2=1$.

- Lastly for the R-R sector, we set the R-R potential, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$, to be in the **bi-fundamental** spinorial representation of $\mathbf{Spin}(1,D{-}1)_L\times\mathbf{Spin}(D{-}1,1)_R$. It possesses the chirality,

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$$\gamma^{(D+1)}\mathcal{C}\bar{\gamma}^{(D+1)} = \mathbf{cc}'\,\mathcal{C}\,.$$

- *A priori* all the possible four different sign choices are equivalent up to $\mathbf{Pin}(1,D{-}1)_L \times \mathbf{Pin}(D{-}1,1)_R$ rotations.
- That is to say, $\mathcal{N}=2$ $D=10$ SDFT is chiral with respect to both $\mathbf{Pin}(1,D{-}1)_L$ and $\mathbf{Pin}(D{-}1,1)_R$, and the theory is unique, unlike IIA/IIB SUGRAs.
- Hence, without loss of generality, we may safely set

$$\mathbf{c} \equiv \mathbf{c}' \equiv +1\,.$$

- Later we shall see that while the theory is unique, it contains type IIA and IIB supergravity backgrounds as different kind of solutions.

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- The DFT-vielbeins generate **a pair of rank-two projectors,**

$$P_{AB} := V_A{}^p V_{Bp}\,, \qquad P_A{}^B P_B{}^C = P_A{}^C\,, \qquad \bar{P}_{AB} := \bar{V}_A{}^{\bar{p}} \bar{V}_{B\bar{p}}\,, \qquad \bar{P}_A{}^B \bar{P}_B{}^C = \bar{P}_A{}^C\,,$$

which are symmetric, orthogonal and complementary to each other,

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$$P_A{}^B V_{Bp} = V_{Ap}\,, \qquad \bar{P}_A{}^B \bar{V}_{B\bar{p}} = \bar{V}_{A\bar{p}}\,, \qquad \bar{P}_A{}^B V_{Bp} = 0\,, \qquad P_A{}^B \bar{V}_{B\bar{p}} = 0\,.$$

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- Further, we construct **a pair of rank-six projectors,**

$$\mathcal{P}_{CAB}{}^{DEF} := P_C{}^D P_{[A}{}^{[E} P_{B]}{}^{F]} + \tfrac{2}{D-1} P_{C[A} P_{B]}{}^{[E} P^{F]D} \,, \qquad \mathcal{P}_{CAB}{}^{DEF} \mathcal{P}_{DEF}{}^{GHI} = \mathcal{P}_{CAB}{}^{GHI} \,,$$

$$\bar{\mathcal{P}}_{CAB}{}^{DEF} := \bar{P}_C{}^D \bar{P}_{[A}{}^{[E} \bar{P}_{B]}{}^{F]} + \tfrac{2}{D-1} \bar{P}_{C[A} \bar{P}_{B]}{}^{[E} \bar{P}^{F]D} \,, \qquad \bar{\mathcal{P}}_{CAB}{}^{DEF} \bar{\mathcal{P}}_{DEF}{}^{GHI} = \bar{\mathcal{P}}_{CAB}{}^{GHI} \,,$$

which are symmetric and traceless,

$$\mathcal{P}_{CABDEF} = \mathcal{P}_{DEFCAB} = \mathcal{P}_{C[AB]D[EF]} \,, \qquad \bar{\mathcal{P}}_{CABDEF} = \bar{\mathcal{P}}_{DEFCAB} = \bar{\mathcal{P}}_{C[AB]D[EF]} \,,$$

$$\mathcal{P}^A{}_{ABDEF} = 0 \,, \quad P^{AB} \mathcal{P}_{ABCDEF} = 0 \,, \qquad \bar{\mathcal{P}}^A{}_{ABDEF} = 0 \,, \quad \bar{P}^{AB} \bar{\mathcal{P}}_{ABCDEF} = 0 \,.$$

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- For each gauge symmetry we assign a corresponding connection,
  - Γ_A for the DFT-diffeomorphism (generalized Lie derivative),
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$$\nabla_A = \partial_A + \Gamma_A\,, \qquad\qquad D_A = \partial_A + \Phi_A + \bar{\Phi}_A\,.$$

- The former is the 'semi-covariant' derivative for the DFT-diffeomorphism (set by the generalized Lie derivative),

$$\nabla_C T_{A_1A_2\cdots A_n} := \partial_C T_{A_1A_2\cdots A_n} - \omega\Gamma^B{}_{BC} T_{A_1A_2\cdots A_n} + \sum_{i=1}^{n} \Gamma_{CA_i}{}^{B} T_{A_1\cdots A_{i-1}BA_{i+1}\cdots A_n}\,.$$

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- ***By definition,*** **the master derivative annihilates all the 'constants',**

$$\mathcal{D}_A\mathcal{J}_{BC}=\nabla_A\mathcal{J}_{BC}=\Gamma_{AB}{}^{D}\mathcal{J}_{DC}+\Gamma_{AC}{}^{D}\mathcal{J}_{BD}=0\,,$$

$$\mathcal{D}_A\eta_{pq}=D_A\eta_{pq}=\Phi_{Ap}{}^{r}\eta_{rq}+\Phi_{Aq}{}^{r}\eta_{pr}=0\,,$$

$$\mathcal{D}_A\bar{\eta}_{\bar{p}\bar{q}}=D_A\bar{\eta}_{\bar{p}\bar{q}}=\bar{\Phi}_{A\bar{p}}{}^{\bar{r}}\bar{\eta}_{\bar{r}\bar{q}}+\bar{\Phi}_{A\bar{q}}{}^{\bar{r}}\bar{\eta}_{\bar{p}\bar{r}}=0\,,$$

$$\mathcal{D}_AC_{+\alpha\beta}=D_AC_{+\alpha\beta}=\Phi_{A\alpha}{}^{\delta}C_{+\delta\beta}+\Phi_{A\beta}{}^{\delta}C_{+\alpha\delta}=0\,,$$

$$\mathcal{D}_A\bar{C}_{+\bar{\alpha}\bar{\beta}}=D_A\bar{C}_{+\bar{\alpha}\bar{\beta}}=\bar{\Phi}_{A\bar{\alpha}}{}^{\bar{\delta}}\bar{C}_{+\bar{\delta}\bar{\beta}}+\bar{\Phi}_{A\bar{\beta}}{}^{\bar{\delta}}\bar{C}_{+\bar{\alpha}\bar{\delta}}=0\,,$$

including the gamma matrices,

$$\mathcal{D}_A(\gamma^{p})^{\alpha}{}_{\beta}=D_A(\gamma^{p})^{\alpha}{}_{\beta}=\Phi_A{}^{p}{}_{q}(\gamma^{q})^{\alpha}{}_{\beta}+\Phi_A{}^{\alpha}{}_{\delta}(\gamma^{p})^{\delta}{}_{\beta}-(\gamma^{p})^{\alpha}{}_{\delta}\Phi_A{}^{\delta}{}_{\beta}=0\,,$$

$$\mathcal{D}_A(\bar{\gamma}^{\bar{p}})^{\bar{\alpha}}{}_{\bar{\beta}}=D_A(\bar{\gamma}^{\bar{p}})^{\bar{\alpha}}{}_{\bar{\beta}}=\bar{\Phi}_A{}^{\bar{p}}{}_{\bar{q}}(\bar{\gamma}^{\bar{q}})^{\bar{\alpha}}{}_{\bar{\beta}}+\bar{\Phi}_A{}^{\bar{\alpha}}{}_{\bar{\delta}}(\bar{\gamma}^{\bar{p}})^{\bar{\delta}}{}_{\bar{\beta}}-(\bar{\gamma}^{\bar{p}})^{\bar{\alpha}}{}_{\bar{\delta}}\bar{\Phi}_A{}^{\bar{\delta}}{}_{\bar{\beta}}=0\,.$$

- It follows then that the connections are all anti-symmetric,

$$\Gamma_{ABC} = -\Gamma_{ACB} \,,$$

$$\Phi_{Apq} = -\Phi_{Aqp} \,, \qquad \Phi_{A\alpha\beta} = -\Phi_{A\beta\alpha} \,,$$

$$\bar{\Phi}_{A\bar{p}\bar{q}} = -\bar{\Phi}_{A\bar{q}\bar{p}} \,, \qquad \bar{\Phi}_{A\bar{\alpha}\bar{\beta}} = -\bar{\Phi}_{A\bar{\beta}\bar{\alpha}} \,,$$

and as usual,

$$\Phi_{A}{}^{\alpha}{}_{\beta} = \tfrac{1}{4}\Phi_{Apq}(\gamma^{pq})^{\alpha}{}_{\beta} \,, \qquad \bar{\Phi}_{A}{}^{\bar{\alpha}}{}_{\bar{\beta}} = \tfrac{1}{4}\bar{\Phi}_{A\bar{p}\bar{q}}(\bar{\gamma}^{\bar{p}\bar{q}})^{\bar{\alpha}}{}_{\bar{\beta}} \,.$$

- ***Further,*** **the master derivative is <u>compatible with the whole NS-NS sector,</u>**

$$\mathcal{D}_A d = \nabla_A d := -\tfrac{1}{2}e^{2d}\nabla_A(e^{-2d}) = \partial_A d + \tfrac{1}{2}\Gamma^B{}_{BA} = 0\,,$$

$$\mathcal{D}_A V_{Bp} = \partial_A V_{Bp} + \Gamma_{AB}{}^C V_{Cp} + \Phi_{Ap}{}^q V_{Bq} = 0\,,$$

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- **The connections assume the following most general forms:**

$$\Gamma_{CAB} = \Gamma^{0}_{CAB} + \Delta_{Cpq}V_{A}{}^{p}V_{B}{}^{q} + \bar{\Delta}_{C\bar{p}\bar{q}}\bar{V}_{A}{}^{\bar{p}}\bar{V}_{B}{}^{\bar{q}}\,,$$

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Here

$$\begin{aligned}\Gamma^{0}_{CAB} = \quad & 2\left(P\partial_{C}P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_{D}P_{EC} \\ & -\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_{D}d + (P\partial^{E}P\bar{P})_{[ED]}\right)\,,\end{aligned}$$

and, with the corresponding derivative, $\nabla^{0}_{A} = \partial_{A} + \Gamma^{0}_{A}$,

$$\Phi^{0}_{Apq} = V^{B}{}_{p}\nabla^{0}_{A}V_{Bq} = V^{B}{}_{p}\partial_{A}V_{Bq} + \Gamma^{0}_{ABC}V^{B}{}_{p}V^{C}{}_{q}\,,$$

$$\bar{\Phi}^{0}_{A\bar{p}\bar{q}} = \bar{V}^{B}{}_{\bar{p}}\nabla^{0}_{A}\bar{V}_{B\bar{q}} = \bar{V}^{B}{}_{\bar{p}}\partial_{A}\bar{V}_{B\bar{q}} + \Gamma^{0}_{ABC}\bar{V}^{B}{}_{\bar{p}}\bar{V}^{C}{}_{\bar{q}}\,.$$

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- Further, the extra pieces, Δ_{Apq} and $\bar{\Delta}_{A\bar{p}\bar{q}}$, correspond to the **torsion** of SDFT, which must be covariant and, in order to maintain $\mathcal{D}_A d = 0$, must satisfy

$$\Delta_{Apq}V^{Ap} = 0\,, \qquad \bar{\Delta}_{A\bar{p}\bar{q}}\bar{V}^{A\bar{p}} = 0\,.$$

  Otherwise they are arbitrary.

- As in SUGRA, the torsion can be constructed from the bi-spinorial objects, e.g.

$$\bar{\rho}\gamma_{pq}\psi_A\,, \qquad \bar{\psi}_{\bar{p}}\gamma_A\psi_{\bar{q}}\,, \qquad \bar{\rho}\gamma_{Apq}\rho\,, \qquad \bar{\psi}_{\bar{p}}\gamma_{Apq}\psi^{\bar{p}}\,,$$

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- The ‘torsionless’ connection,

$$\Gamma^{0}_{CAB} = 2\left(P\partial_{C}P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_{D}P_{EC}$$
$$-\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_{D}d + (P\partial^{E}P\bar{P})_{[ED]}\right),$$

further obeys

$$\Gamma^{0}_{ABC} + \Gamma^{0}_{BCA} + \Gamma^{0}_{CAB} = 0\,,$$

and

$$\mathcal{P}_{CAB}{}^{DEF}\Gamma^{0}_{DEF} = 0\,, \qquad \bar{\mathcal{P}}_{CAB}{}^{DEF}\Gamma^{0}_{DEF} = 0\,.$$

- **In fact, the torsionless connection,**

$$\Gamma^{0}_{CAB} = 2\left(P\partial_{C}P\bar{P}\right)_{[AB]} + 2\left(\bar{P}_{[A}{}^{D}\bar{P}_{B]}{}^{E} - P_{[A}{}^{D}P_{B]}{}^{E}\right)\partial_{D}P_{EC}$$
$$-\tfrac{4}{D-1}\left(\bar{P}_{C[A}\bar{P}_{B]}{}^{D} + P_{C[A}P_{B]}{}^{D}\right)\left(\partial_{D}d + (P\partial^{E}P\bar{P})_{[ED]}\right) ,$$

is uniquely determined by requiring

$$\nabla_{A}\mathcal{J}_{BC} = 0 \iff \Gamma_{CAB} + \Gamma_{CBA} = 0 \,,$$

$$\nabla_{A}P_{BC} = 0 \,,$$

$$\nabla_{A}d = 0 \,,$$

$$\Gamma_{ABC} + \Gamma_{CAB} + \Gamma_{BCA} = 0 \,,$$

$$(\mathcal{P} + \bar{\mathcal{P}})_{CAB}{}^{DEF}\,\Gamma_{DEF} = 0 \,.$$

- Having the two symmetric properties, $\Gamma_{A(BC)}=0$, $\Gamma_{[ABC]}=0$, we may safely replace ∂_A by $\nabla^0_A=\partial_A+\Gamma^0_A$ in $\hat{\mathcal{L}}_X$ and also in $[X,Y]^A_{\mathbf{C}}$,

$$\hat{\mathcal{L}}_X T_{A_1\cdots A_n}=X^B\nabla^0_B T_{A_1\cdots A_n}+\omega\nabla^0_B X^B T_{A_1\cdots A_n}+\textstyle\sum_{i=1}^n(\nabla^0_{A_i}X_B-\nabla^0_B X_{A_i})T_{A_1\cdots A_{i-1}}{}^B{}_{A_{i+1}\cdots A_n}\,,$$

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Semi-covariant curvature

- The usual curvatures for the three connections,

$$R_{CDAB} = \partial_A \Gamma_{BCD} - \partial_B \Gamma_{ACD} + \Gamma_{AC}{}^{E}\Gamma_{BED} - \Gamma_{BC}{}^{E}\Gamma_{AED} \,,$$

$$F_{ABpq} = \partial_A \Phi_{Bpq} - \partial_B \Phi_{Apq} + \Phi_{Apr}\Phi_B{}^r{}_q - \Phi_{Bpr}\Phi_A{}^r{}_q \,,$$

$$\bar{F}_{AB\bar{p}\bar{q}} = \partial_A \bar{\Phi}_{B\bar{p}\bar{q}} - \partial_B \bar{\Phi}_{A\bar{p}\bar{q}} + \bar{\Phi}_{A\bar{p}\bar{r}}\bar{\Phi}_B{}^{\bar{r}}{}_{\bar{q}} - \bar{\Phi}_{B\bar{p}\bar{r}}\bar{\Phi}_A{}^{\bar{r}}{}_{\bar{q}} \,,$$

are, from $[\mathcal{D}_A, \mathcal{D}_B]V_{Cp} = 0$ and $[\mathcal{D}_A, \mathcal{D}_B]\bar{V}_{C\bar{p}} = 0$, related to each other,

$$R_{ABCD} = F_{CDpq}V_A{}^p V_B{}^q + \bar{F}_{CD\bar{p}\bar{q}}\bar{V}_A{}^{\bar{p}}\bar{V}_B{}^{\bar{q}} \,.$$

- However, the crucial object in DFT turns out to be

$$S_{ABCD} := \tfrac{1}{2}\left(R_{ABCD} + R_{CDAB} - \Gamma^E{}_{AB}\Gamma_{ECD}\right) \,,$$

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Properties of the semi-covariant curvature

- Precisely the same symmetric property as the Riemann curvature,

$$S_{ABCD}=\tfrac{1}{2}\left(S_{[AB][CD]}+S_{[CD][AB]}\right)\,,$$

$$S^{0}_{[ABC]D}=0\,.$$

- Projection property,

$$P_{I}{}^{A}\bar{P}_{J}{}^{B}P_{K}{}^{C}\bar{P}_{L}{}^{D}S_{ABCD}\equiv 0\,.$$

- Under arbitrary variation of the connection, $\delta\Gamma_{ABC}$, it transforms as

$$\delta S_{ABCD}=\mathcal{D}_{[A}\delta\Gamma_{B]CD}+\mathcal{D}_{[C}\delta\Gamma_{D]AB}-\tfrac{3}{2}\Gamma_{[ABE]}\delta\Gamma^{E}{}_{CD}-\tfrac{3}{2}\Gamma_{[CDE]}\delta\Gamma^{E}{}_{AB}\,,$$

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- Generically, under $\delta_X P_{AB} = \hat{\mathcal{L}}_X P_{AB}$, $\delta_X d = \hat{\mathcal{L}}_X d$ (DFT-diffeomorphism), the variation of $\nabla_C T_{A_1\cdots A_n}$ contains an anomalous non-covariant part,

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- **For O(D,D) tensors:**

$$P_{C}{}^{D}\bar{P}_{A_1}{}^{B_1}\bar{P}_{A_2}{}^{B_2}\cdots\bar{P}_{A_n}{}^{B_n}\nabla_{D}T_{B_1B_2\cdots B_n}\,,$$

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$$\left.\begin{array}{l} P^{AB}\bar{P}_{C_1}{}^{D_1}\bar{P}_{C_2}{}^{D_2}\cdots\bar{P}_{C_n}{}^{D_n}\nabla_{A}T_{BD_1D_2\cdots D_n}\,, \\ \bar{P}^{AB}P_{C_1}{}^{D_1}P_{C_2}{}^{D_2}\cdots P_{C_n}{}^{D_n}\nabla_{A}T_{BD_1D_2\cdots D_n} \end{array}\right\}\quad \text{Divergences}\,,$$

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Projector-aided, fully covariant derivatives

- **For Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$ tensors:**

$$\mathcal{D}_p T_{\bar{q}_1\bar{q}_2\cdots\bar{q}_n}\,, \qquad \mathcal{D}_{\bar{p}} T_{q_1q_2\cdots q_n}\,,$$

$$\mathcal{D}^p T_{p\bar{q}_1\bar{q}_2\cdots\bar{q}_n}\,, \qquad \mathcal{D}^{\bar{p}} T_{\bar{p}q_1q_2\cdots q_n}\,,$$

$$\mathcal{D}_p\mathcal{D}^p T_{\bar{q}_1\bar{q}_2\cdots\bar{q}_n}\,, \qquad \mathcal{D}_{\bar{p}}\mathcal{D}^{\bar{p}} T_{q_1q_2\cdots q_n}\,,$$

where we set

$$\mathcal{D}_p := V^A{}_p\mathcal{D}_A\,, \qquad \mathcal{D}_{\bar{p}} := \bar{V}^A{}_{\bar{p}}\mathcal{D}_A\,.$$

These are the pull-back of the previous results using the DFT-vielbeins.

- **Dirac operators for fermions,** ρ^{α}, $\psi_{\bar{p}}^{\alpha}$, $\rho'^{\bar{\alpha}}$, $\psi_{p}'^{\bar{\alpha}}$:

$$\gamma^{p}\mathcal{D}_{p}\rho=\gamma^{A}\mathcal{D}_{A}\rho\,,\qquad\qquad \gamma^{p}\mathcal{D}_{p}\psi_{\bar{p}}=\gamma^{A}\mathcal{D}_{A}\psi_{\bar{p}}\,,$$

$$\mathcal{D}_{\bar{p}}\rho\,,\qquad\qquad \mathcal{D}_{\bar{p}}\psi^{\bar{p}}=\mathcal{D}_{A}\psi^{A}\,,$$

$$\bar{\psi}^{A}\gamma_{p}(\mathcal{D}_{A}\psi_{\bar{q}}-\tfrac{1}{2}\mathcal{D}_{\bar{q}}\psi_{A})\,,$$

$$\bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\rho'=\bar{\gamma}^{A}\mathcal{D}_{A}\rho'\,,\qquad\qquad \bar{\gamma}^{\bar{p}}\mathcal{D}_{\bar{p}}\psi'_{p}=\bar{\gamma}^{A}\mathcal{D}_{A}\psi'_{p}\,,$$

$$\mathcal{D}_{p}\rho'\,,\qquad\qquad \mathcal{D}_{p}\psi'^{p}=\mathcal{D}_{A}\psi'^{A}\,,$$

$$\bar{\psi}'^{A}\bar{\gamma}_{\bar{p}}(\mathcal{D}_{A}\psi'_{q}-\tfrac{1}{2}\mathcal{D}_{q}\psi'_{A})\,.$$

Incorporation of fermions into DFT 1109.2035

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- **For Spin$(1,D{-}1)_L \times$ Spin$(D{-}1,1)_R$ bi-fundamental spinorial fields,** $\mathcal{T}^{\alpha}{}_{\bar{\beta}}$ **:**

$$\mathcal{D}_{+}\mathcal{T} := \gamma^{A}\mathcal{D}_{A}\mathcal{T} + \gamma^{(D+1)}\mathcal{D}_{A}\mathcal{T}\bar{\gamma}^{A}\,,$$

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- Especially for the torsionless case, the corresponding operators are **nilpotent**

$$(\mathcal{D}^{0}_{+})^{2}\mathcal{T} \equiv 0\,, \qquad\qquad (\mathcal{D}^{0}_{-})^{2}\mathcal{T} \equiv 0\,,$$

and hence, they define **O(D,D) covariant cohomology.**

- **The field strength of the R-R potential, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$, is then defined by**

$$\mathcal{F} := \mathcal{D}^{0}_{+}\mathcal{C}\,.$$

- Thanks to the nilpotency, the **R-R gauge symmetry** is simply realized

$$\delta\mathcal{C} = \mathcal{D}^{0}_{+}\Delta \qquad \Longrightarrow \qquad \delta\mathcal{F} = \mathcal{D}^{0}_{+}(\delta\mathcal{C}) = (\mathcal{D}^{0}_{+})^{2}\Delta \equiv 0\,.$$

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- **Scalar curvature:**

$$(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD}\,.$$

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$$S_{p\bar{q}} + \tfrac{1}{2}\mathcal{D}_{\bar{r}}\bar{\Delta}_{p\bar{q}}{}^{\bar{r}} + \tfrac{1}{2}\mathcal{D}_{r}\Delta_{\bar{q}p}{}^{r}\,,$$

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Combining all the results above, we are now ready to spell

- **Type II *i.e.* $\mathcal{N}=2$ $D=10$ Supersymmetric Double Field Theory**

- **Lagrangian** :

$$\mathcal{L}_{\rm Type\,II} = e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD}-\bar{P}^{AB}\bar{P}^{CD})S_{ACBD}+\tfrac{1}{2}{\rm Tr}(\mathcal{F}\bar{\mathcal{F}})-i\bar{\rho}\mathcal{F}\rho'+i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$
$$+i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}^{\star}_{p}\rho-i\bar{\psi}^{\bar{p}}\mathcal{D}^{\star}_{\bar{p}}\rho-i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}^{\star}_{q}\psi_{\bar{p}}-i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}'^{\star}_{\bar{p}}\rho'+i\bar{\psi}'^{p}\mathcal{D}'^{\star}_{p}\rho'+i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}'^{\star}_{\bar{q}}\psi'_{p}\Big]\,.$$

where $\bar{\mathcal{F}}^{\bar{\alpha}}{}_{\alpha}$ denotes the charge conjugation, $\bar{\mathcal{F}}:=\bar{C}_{+}^{-1}\mathcal{F}^{T}C_{+}$.

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- **Torsions:** The semi-covariant curvature, S_{ABCD}, is given by the connection,

$$\Gamma_{ABC}=\Gamma^{0}_{ABC}+i\tfrac{1}{3}\bar{\rho}\gamma_{ABC}\rho-2i\bar{\rho}\gamma_{BC}\psi_{A}-i\tfrac{1}{3}\bar{\psi}^{\bar{p}}\gamma_{ABC}\psi_{\bar{p}}+4i\bar{\psi}_{B}\gamma_{A}\psi_{C}$$

$$+i\tfrac{1}{3}\bar{\rho}'\bar{\gamma}_{ABC}\rho'-2i\bar{\rho}'\bar{\gamma}_{BC}\psi'_{A}-i\tfrac{1}{3}\bar{\psi}'^{p}\bar{\gamma}_{ABC}\psi'_{p}+4i\bar{\psi}'_{B}\bar{\gamma}_{A}\psi'_{C}\,,$$

which corresponds to the solution for 1.5 formalism.

The master derivatives in the fermionic kinetic terms are twofold:
$\mathcal{D}^{\star}_{A}$ for the unprimed fermions and $\mathcal{D}'^{\star}_{A}$ for the primed fermions, set by

$$\Gamma^{\star}_{ABC}=\Gamma_{ABC}-i\tfrac{11}{96}\bar{\rho}\gamma_{ABC}\rho+i\tfrac{5}{4}\bar{\rho}\gamma_{BC}\psi_{A}+i\tfrac{5}{24}\bar{\psi}^{\bar{p}}\gamma_{ABC}\psi_{\bar{p}}-2i\bar{\psi}_{B}\gamma_{A}\psi_{C}+i\tfrac{5}{2}\bar{\rho}'\bar{\gamma}_{BC}\psi'_{A}\,,$$

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- **Lagrangian** :

$$\mathcal{L}_{\rm Type\,II} = e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD}-\bar{P}^{AB}\bar{P}^{CD})S_{ACBD}+\tfrac{1}{2}{\rm Tr}(\mathcal{F}\bar{\mathcal{F}})-i\bar{\rho}\mathcal{F}\rho'+i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$
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- **The $\mathcal{N}=2$ supersymmetry transformation rules are**

$$\delta_\varepsilon d = -i\tfrac{1}{2}(\bar\varepsilon\rho + \bar\varepsilon'\rho')\,,$$

$$\delta_\varepsilon V_{Ap} = i\bar V_A{}^{\bar q}(\bar\varepsilon'\bar\gamma_{\bar q}\psi'_p - \bar\varepsilon\gamma_p\psi_{\bar q})\,,$$

$$\delta_\varepsilon \bar V_{A\bar p} = iV_A{}^{q}(\bar\varepsilon\gamma_q\psi_{\bar p} - \bar\varepsilon'\bar\gamma_{\bar p}\psi'_q)\,,$$

$$\delta_\varepsilon\mathcal{C} = i\tfrac{1}{2}(\gamma^p\varepsilon\bar\psi'_p - \varepsilon\bar\rho' - \psi_{\bar p}\bar\varepsilon'\bar\gamma^{\bar p} + \rho\bar\varepsilon') + \mathcal{C}\delta_\varepsilon d - \tfrac{1}{2}(\bar V^A{}_{\bar q}\,\delta_\varepsilon V_{Ap})\gamma^{(d+1)}\gamma^p\mathcal{C}\bar\gamma^{\bar q}\,,$$

$$\delta_\varepsilon\rho = -\gamma^p\hat{\mathcal{D}}_p\varepsilon + i\tfrac{1}{2}\gamma^p\varepsilon\,\bar\psi'_p\rho' - i\gamma^p\psi^{\bar q}\bar\varepsilon'\bar\gamma_{\bar q}\psi'_p\,,$$

$$\delta_\varepsilon\rho' = -\bar\gamma^{\bar p}\hat{\mathcal{D}}'_{\bar p}\varepsilon' + i\tfrac{1}{2}\bar\gamma^{\bar p}\varepsilon'\,\bar\psi_{\bar p}\rho - i\bar\gamma^{\bar q}\psi'_p\bar\varepsilon\gamma^p\psi_{\bar q}\,,$$

$$\delta_\varepsilon\psi_{\bar p} = \hat{\mathcal{D}}_{\bar p}\varepsilon + (\mathcal{F} - i\tfrac{1}{2}\gamma^q\rho\,\bar\psi'_q + i\tfrac{1}{2}\psi^{\bar q}\,\bar\rho'\bar\gamma_{\bar q})\bar\gamma_{\bar p}\varepsilon' + i\tfrac{1}{4}\varepsilon\bar\psi_{\bar p}\rho + i\tfrac{1}{2}\psi_{\bar p}\bar\varepsilon\rho\,,$$

$$\delta_\varepsilon\psi'_p = \hat{\mathcal{D}}'_p\varepsilon' + (\bar{\mathcal{F}} - i\tfrac{1}{2}\bar\gamma^{\bar q}\rho'\bar\psi_{\bar q} + i\tfrac{1}{2}\psi'^q\bar\rho\gamma_q)\gamma_p\varepsilon + i\tfrac{1}{4}\varepsilon'\bar\psi'_p\rho' + i\tfrac{1}{2}\psi'_p\bar\varepsilon'\rho'\,,$$

where

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$$\hat\Gamma'_{ABC} = \Gamma_{ABC} - i\tfrac{17}{48}\bar\rho'\bar\gamma_{ABC}\rho' + i\tfrac{5}{2}\bar\rho'\bar\gamma_{BC}\psi'_A + i\tfrac{1}{4}\bar\psi'^p\bar\gamma_{ABC}\psi'_p - 3i\bar\psi_B\gamma_A\psi_C\,.$$

Type II $\mathcal{N}=2$ $D=10$ SDFT [1210.5078]

- **Lagrangian** :

$$\mathcal{L}_{\rm Type\,II} = e^{-2d}\Big[\tfrac{1}{8}(P^{AB}P^{CD} - \bar{P}^{AB}\bar{P}^{CD})S_{ACBD} + \tfrac{1}{2}\mathrm{Tr}(\mathcal{F}\bar{\mathcal{F}}) - i\bar{\rho}\mathcal{F}\rho' + i\bar{\psi}_{\bar{p}}\gamma_{q}\mathcal{F}\bar{\gamma}^{\bar{p}}\psi'^{q}$$
$$+ i\tfrac{1}{2}\bar{\rho}\gamma^{p}\mathcal{D}^{\star}_{p}\rho - i\bar{\psi}^{\bar{p}}\mathcal{D}^{\star}_{\bar{p}}\rho - i\tfrac{1}{2}\bar{\psi}^{\bar{p}}\gamma^{q}\mathcal{D}^{\star}_{q}\psi_{\bar{p}} - i\tfrac{1}{2}\bar{\rho}'\bar{\gamma}^{\bar{p}}\mathcal{D}'^{\star}_{\bar{p}}\rho' + i\bar{\psi}'^{p}\mathcal{D}'^{\star}_{p}\rho' + i\tfrac{1}{2}\bar{\psi}'^{p}\bar{\gamma}^{\bar{q}}\mathcal{D}'^{\star}_{\bar{q}}\psi'_{p}\Big]\,.$$

- The Lagrangian is **pseudo :** It is necessary to impose a **self-duality** of the R-R field strength by hand,

$$\tilde{\mathcal{F}}_{-} := \left(1-\gamma^{(D+1)}\right)\left(\mathcal{F} - i\tfrac{1}{2}\rho\bar{\rho}' + i\tfrac{1}{2}\gamma^{p}\psi_{\bar{q}}\bar{\psi}'_{p}\bar{\gamma}^{\bar{q}}\right) \equiv 0\,.$$

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- Under the $\mathcal{N}=2$ SUSY transformation rule, the Lagrangian transforms, disregarding total derivatives, as

$$\delta_{\varepsilon}\mathcal{L}_{\rm Type\,II}\simeq-\tfrac{1}{8}e^{-2d}\bar{V}^{A}{}_{\bar{q}}\delta_{\varepsilon}V_{Ap}\mathrm{Tr}\left(\gamma^{p}\tilde{\mathcal{F}}_{-}\bar{\gamma}^{\bar{q}}\overline{\tilde{\mathcal{F}}_{-}}\right)\,,$$

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This verifies, to the full order in fermions, **the supersymmetric invariance of the action, modulo the self-duality.**

- For a **nontrivial consistency check,** the supersymmetric variation of the self-duality relation is precisely closed by the equations of motion for the gravitinos,

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Equations of Motion for Bosons

- DFT-vielbein:

$$S_{p\bar{q}}+\mathrm{Tr}(\gamma_p\mathcal{F}\bar{\gamma}_{\bar{q}}\bar{\mathcal{F}})+i\bar{\rho}\gamma_p\tilde{\mathcal{D}}_{\bar{q}}\rho+2i\bar{\psi}_{\bar{q}}\tilde{\mathcal{D}}_p\rho-i\bar{\psi}^{\bar{p}}\gamma_p\tilde{\mathcal{D}}_{\bar{q}}\psi_{\bar{p}}+i\bar{\rho}'\bar{\gamma}_{\bar{q}}\tilde{\mathcal{D}}_p\rho'+2i\bar{\psi}'{}_p\tilde{\mathcal{D}}_{\bar{q}}\rho'-i\bar{\psi}'^{q}\bar{\gamma}_{\bar{q}}\tilde{\mathcal{D}}_p\psi'_q=0.$$

  This is DFT-generalization of Einstein equation.

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$$\mathcal{L}_{\mathrm{Type\,II}}=0\,.$$

  Namely, the on-shell Lagrangian vanishes!

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Equations of Motion for Fermions

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- Gravitinos,

$$\tilde{\mathcal{D}}_{\bar{p}}\rho+\gamma^{p}\tilde{\mathcal{D}}_{p}\psi_{\bar{p}}-\gamma^{p}\mathcal{F}\bar{\gamma}_{\bar{p}}\psi'_{p}=0\,,\qquad\qquad\tilde{\mathcal{D}}_{p}\rho'+\bar{\gamma}^{\bar{p}}\tilde{\mathcal{D}}_{\bar{p}}\psi'_{p}-\bar{\gamma}^{\bar{p}}\bar{\mathcal{F}}\gamma_{p}\psi_{\bar{p}}=0\,.$$

Equations of Motion for Fermions

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$$\gamma^{p}\tilde{\mathcal{D}}_{p}\rho - \tilde{\mathcal{D}}_{\bar{p}}\psi^{\bar{p}} - \mathcal{F}\rho' = 0\,, \qquad\qquad \bar{\gamma}^{\bar{p}}\tilde{\mathcal{D}}_{\bar{p}}\rho' - \tilde{\mathcal{D}}_{p}\psi'^{p} - \bar{\mathcal{F}}\rho = 0\,.$$

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- Turning off the primed fermions and the R-R sector truncates the $\mathcal{N}=2$ $D=10$ SDFT to $\mathcal{N}=1$ $D=10$ SDFT,

$$\mathcal{L}_{\mathcal{N}=1}=e^{-2d}\Big[\tfrac{1}{8}\left(P^{AB}P^{CD}-\bar{P}^{AB}\bar{P}^{CD}\right)S_{ACBD}+i\tfrac{1}{2}\bar{\rho}\gamma^{A}\mathcal{D}^{\star}_{A}\rho-i\bar{\psi}^{A}\mathcal{D}^{\star}_{A}\rho-i\tfrac{1}{2}\bar{\psi}^{B}\gamma^{A}\mathcal{D}^{\star}_{A}\psi_{B}\Big]\,.$$

- $\mathcal{N}=1$ **Local SUSY:**

$$\begin{aligned}
\delta_{\varepsilon}d&=-i\tfrac{1}{2}\bar{\varepsilon}\rho\,,\\
\delta_{\varepsilon}V_{Ap}&=-i\bar{\varepsilon}\gamma_{p}\psi_{A}\,,\\
\delta_{\varepsilon}\bar{V}_{A\bar{p}}&=i\bar{\varepsilon}\gamma_{A}\psi_{\bar{p}}\,,\\
\delta_{\varepsilon}\rho&=-\gamma^{A}\hat{\mathcal{D}}_{A}\varepsilon\,,\\
\delta_{\varepsilon}\psi_{\bar{p}}&=\bar{V}^{A}{}_{\bar{p}}\hat{\mathcal{D}}_{A}\varepsilon-i\tfrac{1}{4}(\bar{\rho}\psi_{\bar{p}})\varepsilon+i\tfrac{1}{2}(\bar{\varepsilon}\rho)\psi_{\bar{p}}\,.
\end{aligned}$$

$\mathcal{N} = 1$ SUSY Algebra [1112.0069]

- Commutator of supersymmetry reads

$$[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}] \equiv \hat{\mathcal{L}}_{X_3} + \delta_{\varepsilon_3} + \delta_{\mathbf{so}(1,9)_L} + \delta_{\mathbf{so}(9,1)_R} + \delta_{\rm trivial}\,.$$

where

$$X_3^A = i\bar{\varepsilon}_1\gamma^A\varepsilon_2\,, \qquad \varepsilon_3 = i\tfrac{1}{2}\left[(\bar{\varepsilon}_1\gamma^p\varepsilon_2)\gamma_p\rho + (\bar{\rho}\varepsilon_2)\varepsilon_1 - (\bar{\rho}\varepsilon_1)\varepsilon_2\right]\,, \qquad \text{etc.}$$

and $\delta_{\rm trivial}$ corresponds to the fermionic equations of motion.

Now we are going to

- parametrize the DFT-field-variables in terms of Riemannian variables,
- discuss the 'unification',
- choose a diagonal gauge of Spin$(1, D-1)_L \times$ Spin$(D-1, 1)_R$,
- and reduce SDFT to SUGRAs.

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- Assuming that the upper half blocks are non-degenerate, the DFT-vielbein takes the general form,

$$V_{Ap}=\frac{1}{\sqrt{2}}\begin{pmatrix}(e^{-1})_{p}{}^{\mu}\\(B+e)_{\nu p}\end{pmatrix},\qquad \bar{V}_{A\bar{p}}=\frac{1}{\sqrt{2}}\begin{pmatrix}(\bar{e}^{-1})_{\bar{p}}{}^{\mu}\\(B+\bar{e})_{\nu\bar{p}}\end{pmatrix}.$$

Here $e_{\mu}{}^{p}$ and $\bar{e}_{\nu}{}^{\bar{p}}$ are two copies of the D-dimensional vielbein corresponding to the same spacetime metric,

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$$\sqrt{2}\gamma^A\mathcal{D}_A\rho \equiv \gamma^m\left(\partial_m\rho + \tfrac{1}{4}\omega_{mnp}\gamma^{np}\rho + \tfrac{1}{24}H_{mnp}\gamma^{np}\rho - \partial_m\phi\rho\right),$$

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Unification of type IIA and IIB SUGRAs

- The $\mathcal{N}=2$ $D=10$ SDFT solutions are then classified into two groups,

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- That is to say, formulated in terms of the genuine DFT-field variables, i.e. V_{Ap}, $\bar{V}_{A\bar{p}}$, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$, etc. the $\mathcal{N}=2$ $D=10$ SDFT is a chiral theory with respect to the pair of local Lorentz groups. The possible four chirality choices are all equivalent and hence the theory is *unique*. We may safely put $\mathbf{c}\equiv\mathbf{c}'\equiv+1$ without loss of generality.
- However, the theory contains two 'types' of Riemannian solutions, as classified above.
- Conversely, any solution in type IIA and type IIB supergravities can be mapped to a solution of $\mathcal{N}=2$ $D=10$ SDFT of fixed chirality e.g. $\mathbf{c}\equiv\mathbf{c}'\equiv+1$.
- In conclusion, the single unique $\mathcal{N}=2$ $D=10$ SDFT unifies type IIA and IIB SUGRAs.

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with $\eta_{pq} = -\bar{\eta}_{\bar{p}\bar{q}}$, $\bar{\gamma}^{\bar{p}} = \gamma^{(D+1)}\gamma^{p}$, $\bar{\gamma}^{(D+1)} = -\gamma^{(D+1)}$, breaks the local Lorentz symmetry,

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- And it reduces SDFT to SUGRA:

$\mathcal{N} = 2$ $D = 10$ **SDFT** $\Longrightarrow$ $10D$ **Type II democratic SUGRA**

Bergshoeff, *et al.*; Coimbra, Strickland-Constable, Waldram

$\mathcal{N} = 1$ $D = 10$ **SDFT** $\Longrightarrow$ $10D$ **minimal SUGRA** Chamseddine; Bergshoeff *et al.*

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- **To the full order in fermions, $\mathcal{N}=1$ SDFT reduces to $10D$ minimal SUGRA:**

$$\begin{aligned}\mathcal{L}_{10D} = \det e \times e^{-2\phi}\Big[& R + 4\partial_\mu\phi\partial^\mu\phi - \tfrac{1}{12}H_{\lambda\mu\nu}H^{\lambda\mu\nu} \\ & + i2\sqrt{2}\bar{\rho}\gamma^m[\partial_m\rho + \tfrac{1}{4}(\omega + \tfrac{1}{6}H)_{mnp}\gamma^{np}\rho] - i4\sqrt{2}\bar{\psi}^p[\partial_p\rho + \tfrac{1}{4}(\omega + \tfrac{1}{2}H)_{pqr}\gamma^{qr}\rho] \\ & - i2\sqrt{2}\bar{\psi}^p\gamma^m[\partial_m\psi_p + \tfrac{1}{4}(\omega + \tfrac{1}{6}H)\gamma^{np}\psi_p + \omega_{mpq}\psi^q - \tfrac{1}{2}H_{mpq}\psi^q] \\ & + \tfrac{1}{24}(\bar{\psi}^q\gamma_{mnp}\psi_q)(\bar{\psi}^r\gamma^{mnp}\psi_r) - \tfrac{1}{48}(\bar{\psi}^q\gamma_{mnp}\psi_q)(\bar{\rho}\gamma^{mnp}\rho)\Big]\,.\end{aligned}$$

$$\delta_\varepsilon\phi = i\tfrac{1}{2}\bar{\varepsilon}(\rho + \gamma^a\psi_a)\,, \qquad \delta_\varepsilon e_\mu^a = i\bar{\varepsilon}\gamma^a\psi_\mu\,, \qquad \delta_\varepsilon B_{\mu\nu} = -2i\bar{\varepsilon}\gamma_{[\mu}\psi_{\nu]}\,,$$

$$\begin{aligned}\delta_\varepsilon\rho = & -\tfrac{1}{\sqrt{2}}\gamma^a[\partial_a\varepsilon + \tfrac{1}{4}(\omega + \tfrac{1}{6}H)_{abc}\gamma^{bc}\varepsilon - \partial_a\phi\varepsilon] \\ & + i\tfrac{1}{48}(\bar{\psi}^d\gamma_{abc}\psi_d)\gamma^{abc}\varepsilon + i\tfrac{1}{192}(\bar{\rho}\gamma_{abc}\rho)\gamma^{abc}\varepsilon + i\tfrac{1}{2}(\bar{\varepsilon}\gamma_{[a}\psi_{b]})\gamma^{ab}\rho\,,\end{aligned}$$

$$\begin{aligned}\delta_\varepsilon\psi_a = & \tfrac{1}{\sqrt{2}}[\partial_a\varepsilon + \tfrac{1}{4}(\omega + \tfrac{1}{2}H)_{abc}\gamma^{bc}\varepsilon] \\ & - i\tfrac{1}{2}(\bar{\rho}\varepsilon)\psi_a - i\tfrac{1}{4}(\bar{\rho}\psi_a)\varepsilon + i\tfrac{1}{8}(\bar{\rho}\gamma_{bc}\psi_a)\gamma^{bc}\varepsilon + i\tfrac{1}{2}(\bar{\varepsilon}\gamma_{[b}\psi_{c]})\gamma^{bc}\psi_a\,.\end{aligned}$$

Diagonal gauge fixing and Reduction to SUGRA

- After the diagonal gauge fixing, we may parameterize the R-R potential as

$$\mathcal{C}\equiv\left(\tfrac{1}{2}\right)^{\frac{D+2}{4}}\textstyle\sum_{p}^{\prime}\,\tfrac{1}{p!}\,\mathcal{C}_{a_1a_2\cdots a_p}\gamma^{a_1a_2\cdots a_p}$$

and obtain the field strength,

$$\mathcal{F}:=\mathcal{D}^{0}_{+}\mathcal{C}\equiv\left(\tfrac{1}{2}\right)^{\frac{D}{4}}\textstyle\sum_{p}^{\prime}\,\tfrac{1}{(p+1)!}\,\mathcal{F}_{a_1a_2\cdots a_{p+1}}\gamma^{a_1a_2\cdots a_{p+1}}$$

where $\sum_p^{\prime}$ denotes the odd p sum for Type IIA and even p sum for Type IIB, and

$$\mathcal{F}_{a_1a_2\cdots a_p}=p\left(D_{[a_1}\mathcal{C}_{a_2\cdots a_p]}-\partial_{[a_1}\phi\,\mathcal{C}_{a_2\cdots a_p]}\right)+\tfrac{p!}{3!(p-3)!}\,H_{[a_1a_2a_3}\mathcal{C}_{a_4\cdots a_p]}$$

- The pair of nilpotent differential operators, $\mathcal{D}^0_+$ and $\mathcal{D}^0_-$, reduce to a 'twisted K-theory' exterior derivative and its dual, after the diagonal gauge fixing,

$$\begin{array}{lll}\mathcal{D}^0_+ & \Longrightarrow & \mathrm{d}+(H-\mathrm{d}\phi)\wedge\\ \mathcal{D}^0_- & \Longrightarrow & *\left[\,\mathrm{d}+(H-\mathrm{d}\phi)\wedge\ \right]*\end{array}$$

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$$\mathcal{F} := \mathcal{D}_+^0 \mathcal{C} \equiv \left(\tfrac{1}{2}\right)^{\frac{D}{4}} \textstyle\sum_p' \tfrac{1}{(p+1)!}\, \mathcal{F}_{a_1 a_2 \cdots a_{p+1}} \gamma^{a_1 a_2 \cdots a_{p+1}}$$

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- **In this way,** **ordinary SUGRA $\equiv$ gauge-fixed SDFT,**

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Modifying $\mathbf{O}(D,D)$ transformation rule

- The diagonal gauge, $e_\mu{}^p \equiv \bar{e}_\mu{}^{\bar{p}}$, is incompatible with the vectorial $\mathbf{O}(D,D)$ transformation rule of the DFT-vielbein.

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- The $\mathbf{O}(D,D)$ rotation must accompany a compensating $\mathbf{Pin}(D{-}1,1)_R$ local Lorentz rotation, $\bar{L}_{\bar{q}}{}^{\bar{p}}$, $S_{\bar{L}}{}^{\bar{\alpha}}{}_{\bar{\beta}}$ which we can construct explicitly,

$$\bar{L}=\bar{e}^{-1}\left[\mathbf{a}^{t}-(g+B)\mathbf{b}^{t}\right]\left[\mathbf{a}^{t}+(g-B)\mathbf{b}^{t}\right]^{-1}\bar{e}\,,\qquad\qquad \bar{\gamma}^{\bar{q}}\bar{L}_{\bar{q}}{}^{\bar{p}}=S_{\bar{L}}^{-1}\bar{\gamma}^{\bar{p}}S_{\bar{L}}\,,$$

where $\mathbf{a}$ and $\mathbf{b}$ are parameters of a given $\mathbf{O}(D,D)$ group element,

$$M_{A}{}^{B}=\begin{pmatrix}\mathbf{a}^{\mu}{}_{\nu} & \mathbf{b}^{\mu\sigma}\\ \mathbf{c}_{\rho\nu} & \mathbf{d}_{\rho}{}^{\sigma}\end{pmatrix}.$$

Modified $\mathbf{O}(D,D)$ Transformation Rule After The Diagonal Gauge Fixing

$$
\begin{array}{ccc}
d & \longrightarrow & d \\
V_A{}^p & \longrightarrow & M_A{}^B\, V_B{}^p \\
\bar{V}_A{}^{\bar{p}} & \longrightarrow & M_A{}^B\, \bar{V}_B{}^{\bar{q}}\, \bar{L}_{\bar{q}}{}^{\bar{p}} \\
\mathcal{C}^\alpha{}_{\bar{\alpha}}\,, \quad \mathcal{F}^\alpha{}_{\bar{\alpha}} & \longrightarrow & \mathcal{C}^\alpha{}_{\bar{\beta}}(S_{\bar{L}}^{-1})^{\bar{\beta}}{}_{\bar{\alpha}}\,, \quad \mathcal{F}^\alpha{}_{\bar{\beta}}(S_{\bar{L}}^{-1})^{\bar{\beta}}{}_{\bar{\alpha}} \\
\rho^\alpha & \longrightarrow & \rho^\alpha \\
\rho'^{\bar{\alpha}} & \longrightarrow & (S_{\bar{L}})^{\bar{\alpha}}{}_{\bar{\beta}}\rho'^{\bar{\beta}} \\
\psi^\alpha_{\bar{p}} & \longrightarrow & (\bar{L}^{-1})_{\bar{p}}{}^{\bar{q}}\, \psi^\alpha_{\bar{q}} \\
\psi'^{\bar{\alpha}}_p & \longrightarrow & (S_{\bar{L}})^{\bar{\alpha}}{}_{\bar{\beta}}\psi'^{\bar{\beta}}_p
\end{array}
$$

- **All the barred indices are now to be rotated.** Consistent with Hassan
- **The R-R sector can be also mapped to $\mathbf{O}(D,D)$ spinors.**

Fukuma, Oota Tanaka; Hohm, Kwak, Zwiebach

Flipping the chirality: IIA $\Leftrightarrow$ IIB

- **If and only if $\det(\bar{L}) = -1$, the modified $\mathbf{O}(D, D)$ rotation flips the chirality of the theory, since**

$$\bar{\gamma}^{(D+1)} S_{\bar{L}} = \det(\bar{L})\, S_{\bar{L}} \bar{\gamma}^{(D+1)}\,.$$

- Thus, the mechanism above naturally realizes the exchange of Type IIA and IIB supergravities under $\mathbf{O}(D, D)$ T-duality.

- However, since $\bar{L}$ explicitly depends on the parametrization of V_{Ap} and $\bar{V}_{A\bar{p}}$ in terms of $g_{\mu\nu}$ and $B_{\mu\nu}$, it is impossible to impose the modified $\mathbf{O}(D, D)$ transformation rule from the beginning on the parametrization-independent covariant formalism.

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- With the semi-covariant derivative, we may construct YM-DFT :

$$\mathcal{F}_{AB} := \nabla_A \mathcal{V}_B - \nabla_B \mathcal{V}_A - i\,[\mathcal{V}_A, \mathcal{V}_B]\ , \qquad \mathcal{V}_A = \left(\begin{array}{c} \phi^{\lambda} \\ A_{\mu} + B_{\mu\nu}\phi^{\nu} \end{array}\right),$$

$$\begin{aligned} S_{\rm YM} &= \int_{\Sigma_D} e^{-2d}\,{\rm Tr}\Big(P^{AB}\bar{P}^{CD}\mathcal{F}_{AC}\mathcal{F}_{BD}\Big) \\ &\equiv \int {\rm d}x^D\sqrt{-g}e^{-2\phi}{\rm Tr}\Big(f_{\mu\nu}f^{\mu\nu} + 2D_{\mu}\phi_{\nu}D^{\mu}\phi^{\nu} + 2D_{\mu}\phi_{\nu}D^{\nu}\phi^{\mu} + 2i\,f_{\mu\nu}[\phi^{\mu},\phi^{\nu}] \\ &\qquad - [\phi_{\mu},\phi_{\nu}][\phi^{\mu},\phi^{\nu}] + 2\,(f^{\mu\nu} + i[\phi^{\mu},\phi^{\nu}])\,H_{\mu\nu\sigma}\phi^{\sigma} + H_{\mu\nu\sigma}H^{\mu\nu}{}_{\tau}\phi^{\sigma}\phi^{\tau}\ \Big)\,. \end{aligned}$$

- Similar to topologically twisted Yang-Mills, but differs in detail.
- Curved D-branes are known to convert adjoint scalars into one-form,

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Comment 1: Double field Yang-Mills theory 1102.0419

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$$\mathcal{S}=\frac{1}{4\pi\alpha'}\int\mathrm{d}^2\sigma\ \mathcal{L}\,,\qquad\qquad \mathcal{L}=-\tfrac{1}{2}\sqrt{-h}h^{ij}D_iX^MD_jX^N\mathcal{H}_{MN}(X)-\epsilon^{ij}D_iX^M\mathcal{A}_{jM}\,,$$

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  c.f. Hull; Tseytlin; Copland, Berman, Thompson; Nibbelink, Patalong; Blair, Malek, Routh

- Further, after parametrization and integrating out $\mathcal{A}_i^M$, it can produce either the standard string action for the 'non-degenerate' Riemannian case,

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or novel chiral actions for 'degenerate' non-Riemannian cases, e.g. for $\mathcal{H}_{AB}=\mathcal{J}_{AB}$,

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or novel chiral actions for 'degenerate' non-Riemannian cases, e.g. for $\mathcal{H}_{AB}=\mathcal{J}_{AB}$,

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- **String propagates in doubled yet gauged spacetime, 1307.8377**

$$\mathcal{S}=\frac{1}{4\pi\alpha'}\int \mathrm{d}^2\sigma\ \mathcal{L}\,,\qquad \mathcal{L}=-\tfrac{1}{2}\sqrt{-h}h^{ij}D_iX^MD_jX^N\mathcal{H}_{MN}(X)-\epsilon^{ij}D_iX^M\mathcal{A}_{jM}\,,$$

where

$$D_iX^M=\partial_iX^M-\mathcal{A}_i^M\,,\qquad \mathcal{A}_i^M\partial_M\equiv 0\,.$$

- The Lagrangian is symmetric with respect to the string worldsheet diffeomorphisms, Weyl symmetry, $\mathbf{O}(D,D)$ T-duality, target spacetime generalized diffeomorphisms and the coordinate gauge symmetry, thanks to the auxiliary gauge field, $\mathcal{A}_i^M$.

  c.f. Hull; Tseytlin; Copland, Berman, Thompson; Nibbelink, Patalong; Blair, Malek, Routh

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Summary

- Riemannian geometry is for *particle* theory. *String* theory requires a novel differential geometry which geometrizes the whole NS-NS sector and underlies DFT.

- The fundamental field-variables of $\mathcal{N}=2$ $D=10$ SDFT are, besides the fermions, the DFT-dilaton, d, DFT-vielbeins, V_{Ap}, $\bar{V}_{A\bar{p}}$, and the R-R potential, $\mathcal{C}^{\alpha}{}_{\bar{\alpha}}$.

- Novel differential geometic ingredients:
  - ▷ projectors, $P_{AB}=V_{Ap}V_{B}{}^{p}$, $\bar{P}_{AB}=\bar{V}_{A\bar{p}}\bar{V}_{B}{}^{\bar{p}}$, and semi-covariant derivative.
  - ▷ Spacetime being doubled yet gauged (section condition).

- $\mathcal{N}=2$ $D=10$ SDFT manifests simultaneously the symmetric structures:
  - ▷ $\mathbf{O}(10,10)$ *T-duality*
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$$P_{MN}-\bar{P}_{MN}=\mathcal{H}_{MN}=\left(\begin{array}{cc} 0 & N \\ N^{t} & 0 \end{array}\right), \qquad N^2=1\,.$$

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- The relaxation of the section condition: Geissbuhler; Graña, Marqués, Aldazabal; Berman, Musaev, Blair, Malek, Perry; Berman, Kanghoon Lee for Scherk-Schwarz and Blumenhagen, Fuchs, Lust, Sun for non-associativity
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