

Formulation of effective theories for the dark matter direct detection

Natsumi Nagata

ACP seminar
09 April, 2014

Based on [J. Hisano, K. Ishiwata, N. N., [1004. 4090](#), [1007. 2601](#), and [1210. 5985](#)]
[J. Hisano, K. Ishiwata, N. N., M. Yamanaka, [1012. 5455](#)]
and [J. Hisano, K. Ishiwata, N. N., T. Takesako, [1104. 0228](#)]

Outline

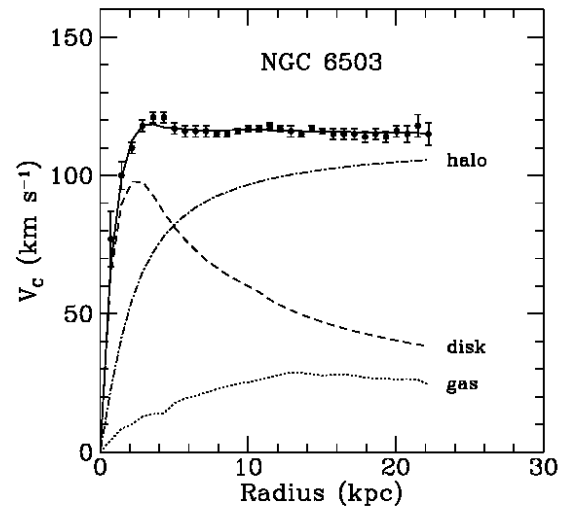
1. Introduction
2. The method of Effective theory
3. Some results
 - a) Pure bino DM
 - b) Pure Wino DM (high-scale SUSY scenario)
 - c) Minimal DM
 - d) KK photon DM in the MUED model
4. Conclusion and discussion

1. Introduction

Introduction

Evidence for dark matter (DM)

Galactic scale



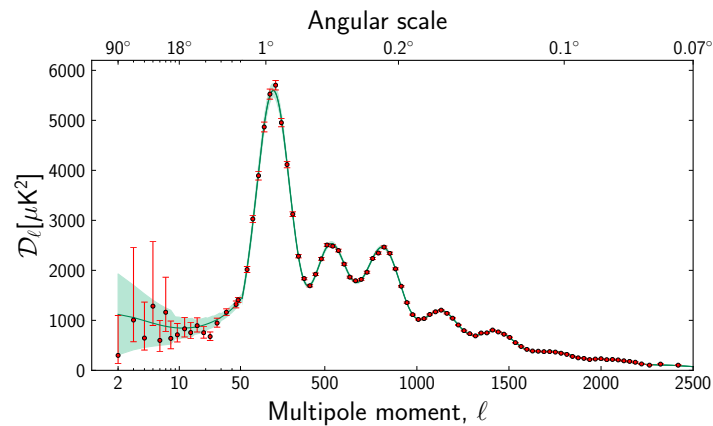
Begeman et. al. (1991)

Scale of galaxy clusters

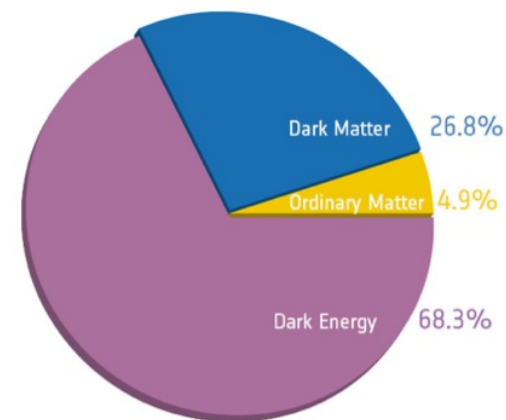


Clowe et. al. (2006)

Cosmological scale



Planck (2013)



Planck (2013)

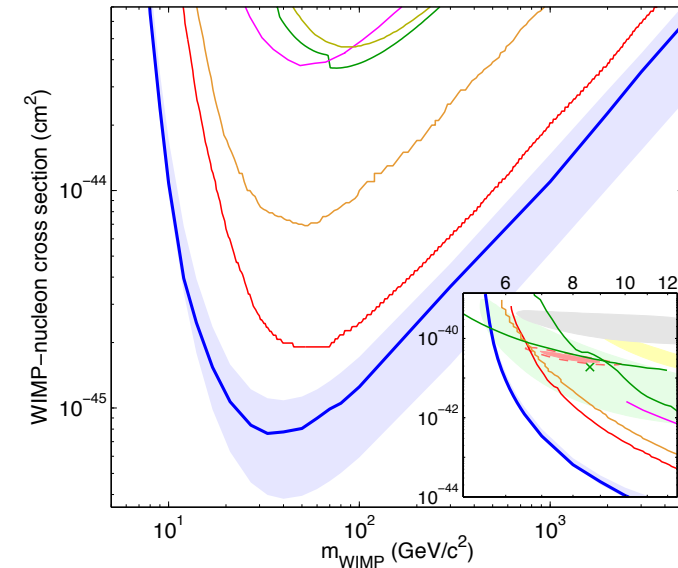
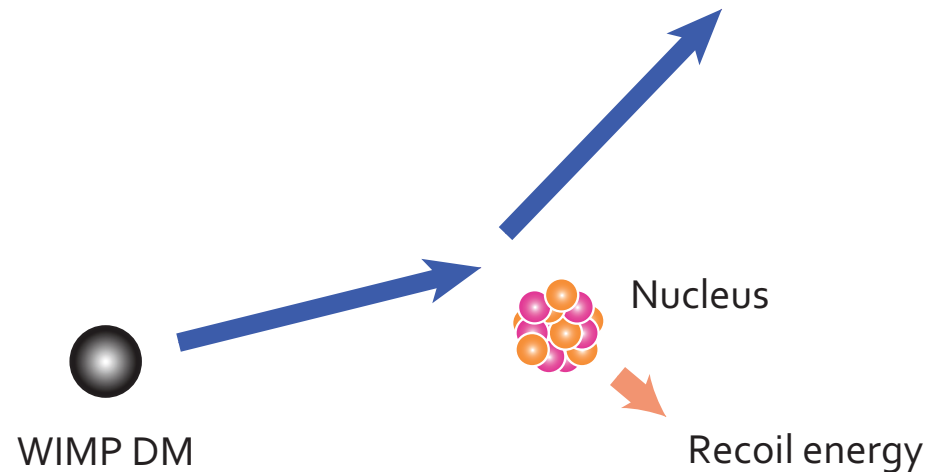
One of the most promising candidates for dark matter is

Weakly Interacting Massive Particles (WIMPs)

- have masses roughly between $10 \text{ GeV} \sim \text{a few TeV}$.
- interact only through weak and gravitational interactions.
- Their thermal relic abundance is naturally consistent with the cosmological observations [thermal relic scenario].
- appear in models beyond the Standard Model.

Introduction

Direct detection experiments



[Large Underground Xenon (LUX), arXiv: 1310.8214]

- LUX experiment gives a stringent constraint on spin-independent WIMP-nucleon scattering cross section.

$$\sigma_{\text{SI}} < 7.6 \times 10^{-46} \text{ cm}^2 \quad (\text{for WIMPs of mass } 33 \text{ GeV})$$

- Ton-scale detectors for direct detection experiments are expected to yield significantly improved sensitivities.

Motivation

To study the nature of dark matter based on direct detection experiments, the precise calculation of

the WIMP-nucleon scattering cross section

is required.

■ Previous works

- **For Majorana DM**
e.g.) M. Drees and M. Nojiri, Phys. Rev. D **48** (1993) 3483.
- **For vector DM**
H. C. P. Cheng, J. L. Feng and K. T. Matchev, Phys. Rev. Lett. **89**, 211301 (2002).
G. Servant and T. M. P. Tait, New J. Phys. **4**, 99 (2002).

- In these works, some of the leading contributions (especially those of gluon) to the scattering cross sections are not properly taken into account.
- We study the way of evaluating the cross section systematically by using the method of effective field theory

2. The method of effective theory

Method of effective theories

1. By integrating out heavy particles, we obtain the effective interactions of WIMP DM with quarks and gluons.

Operator Product Expansion (OPE)

$$\mathcal{L}_{\text{eff}} = \sum_i C_i(\mu) \mathcal{O}_i(\mu)$$

$C_i(\mu)$: Wilson coefficients

include short-distant effects

$\mathcal{O}_i(\mu)$: Effective operators

Higher-dimensional operators. Their nucleon matrix elements contain the effects of long-distance.

μ : factorization scale

A scale at which a high-energy theory is matched with the effective theory.

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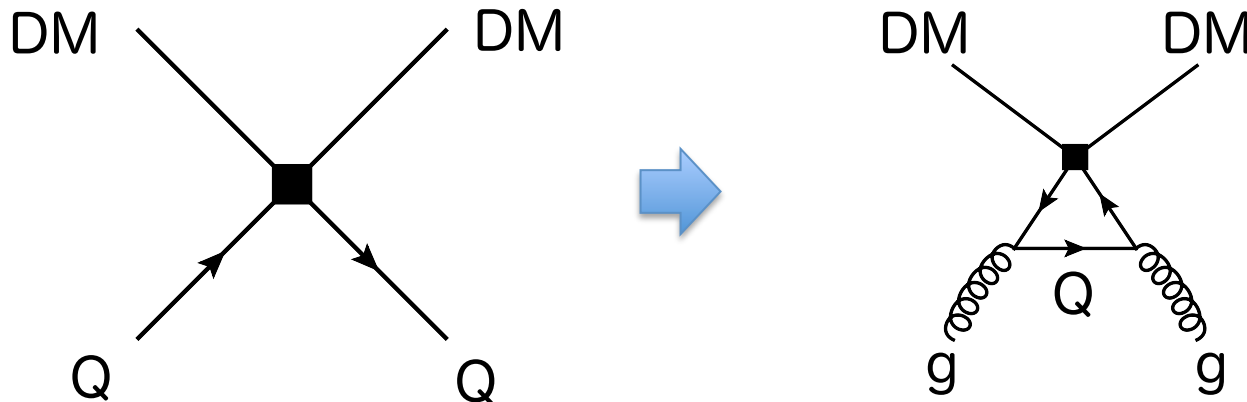
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A scale at which a high-energy theory is matched with the effective theory.

Method of effective theories

2. Evaluate the nucleon matrix elements of the effective operators (at a certain scale).

When evolving the operators down to the scale, we need to match the effective theories above/below each quark threshold.



3. By using the nucleon matrix elements, we evaluate the scattering cross section of DM with a nucleon

Effective Lagrangian for Majorana DM

Spin-independent

$$\mathcal{L}_{\text{eff}} = \sum_q C_S^q \mathcal{O}_S^q + C_S^g \mathcal{O}_S^g + \sum_{i=1,2} \sum_q C_{T_i}^q \mathcal{O}_{T_i}^q + \sum_{i=1,2} C_{T_i}^g \mathcal{O}_{T_i}^g ,$$

$\tilde{\chi}^0$: Majorana DM

Scalar-type

$$\mathcal{O}_S^q \equiv \frac{1}{2} \tilde{\chi}^0 \tilde{\chi}^0 m_q \bar{q} q ,$$

$$\mathcal{O}_S^g \equiv \frac{1}{2} \tilde{\chi}^0 \tilde{\chi}^0 G_{\mu\nu}^A G^{A\mu\nu} ,$$

Twist-2 type

$$\mathcal{O}_{T_1}^q \equiv \frac{1}{2} \tilde{\chi}^0 i \partial^\mu \gamma^\nu \tilde{\chi}^0 \mathcal{O}_{\mu\nu}^q ,$$

$$\mathcal{O}_{T_2}^q \equiv \frac{1}{2} \tilde{\chi}^0 i \partial^\mu i \partial^\nu \tilde{\chi}^0 \mathcal{O}_{\mu\nu}^q ,$$

$$\mathcal{O}_{T_1}^g \equiv \frac{1}{2} \tilde{\chi}^0 i \partial^\mu \gamma^\nu \tilde{\chi}^0 \mathcal{O}_{\mu\nu}^g ,$$

$$\mathcal{O}_{T_2}^g \equiv \frac{1}{2} \tilde{\chi}^0 i \partial^\mu i \partial^\nu \tilde{\chi}^0 \mathcal{O}_{\mu\nu}^g ,$$

Twist-2 operator

$$\mathcal{O}_{\mu\nu}^q \equiv \frac{1}{2} \bar{q} i (D_\mu \gamma_\nu + D_\nu \gamma_\mu - \frac{1}{2} g_{\mu\nu} \not{D}) q$$

$$\mathcal{O}_{\mu\nu}^g \equiv G_{\mu}^{a\rho} G_{\rho\nu}^a + \frac{1}{4} g_{\mu\nu} G_{\alpha\beta}^a G^{a\alpha\beta}$$

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Induces coupling of DM with
“mass of nucleon”

Twist-2 type

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$$\mathcal{O}_S^g \equiv \frac{1}{2} \tilde{\chi}^0 \tilde{\chi}^0 G_{\mu\nu}^A G^{A\mu\nu} ,$$

Induces coupling of DM with
“quark and gluon momenta”

Twist-2 type

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Nucleon matrix elements

Quark (scalar-type)

Nucleon matrix elements of scalar-type quark operators are evaluated by using the QCD lattice simulations.

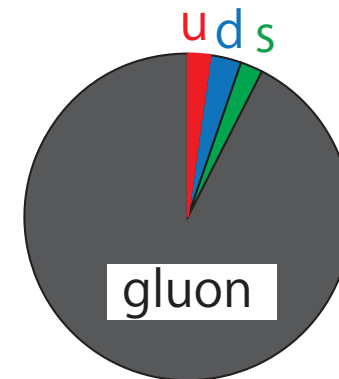
mass fractions

$$\langle N | m_q \bar{q}q | N \rangle / m_N \equiv f_{T_q} \quad (m_N : \text{Nucleon mass})$$

For proton	
f_{Tu}	0.023
f_{Td}	0.034
f_{Ts}	0.025
For neutron	
f_{Tu}	0.019
f_{Td}	0.041
f_{Ts}	0.025

Gluon contribution

$$1 - \sum_{q=u,d,s} f_{T_q} \equiv f_{TG}$$



Mass fractions for proton

Remarks.

Strange quark content is much smaller than those evaluated with the chiral perturbation theory.

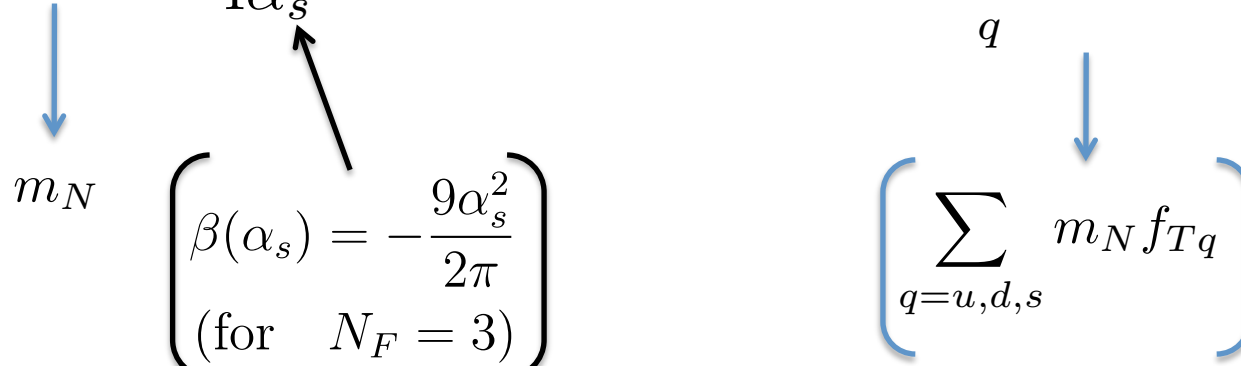
Nucleon matrix elements

Gluon (scalar-type)

Nucleon matrix element of scalar-type gluon operator is evaluated by using **the trace anomaly of the energy-momentum tensor**.

■ Trace anomaly of the energy-momentum tensor in QCD ($N_f = 3$)

$$\Theta_{\mu}^{\mu} = \frac{\beta(\alpha_s)}{4\alpha_s} G_{\mu\nu}^A G^{A\mu\nu} + (1 - \gamma_m) \sum_q m_q \bar{q}q$$



m_N

$\left(\begin{array}{l} \beta(\alpha_s) = -\frac{9\alpha_s^2}{2\pi} \\ \text{(for } N_F = 3) \end{array} \right)$

$\left(\sum_{q=u,d,s} m_N f_{Tq} \right)$

➡

$$\langle N | \alpha_s G_{\mu\nu}^A G^{A\mu\nu} | N \rangle = -\frac{8\pi}{9} m_N f_{TG}$$

Nucleon matrix elements

Twist-2 operators

Nucleon matrix elements of twist-2 operators are evaluated by using **the parton distribution functions (PDFs)**.

$$\langle N(p) | \mathcal{O}_{\mu\nu}^q | N(p) \rangle = \frac{1}{m_N} (p_\mu p_\nu - \frac{1}{4} m_N^2 \eta_{\mu\nu}) (q(2) + \bar{q}(2))$$

$$\langle N(p) | \mathcal{O}_{\mu\nu}^g | N(p) \rangle = \frac{1}{m_N} (p_\mu p_\nu - \frac{1}{4} m_N^2 \eta_{\mu\nu}) G(2)$$

Here, $q(2)$ and $G(2)$ are called **the second moments of PDFs**, which are defined by

$$q(2) + \bar{q}(2) = \int_0^1 dx \, x [q(x) + \bar{q}(x)]$$

$$G(2) = \int_0^1 dx \, x g(x)$$

Second moment at $\mu = m_Z$			
$G(2)$	0.48		
$u(2)$	0.22	$\bar{u}(2)$	0.034
$d(2)$	0.11	$\bar{d}(2)$	0.036
$s(2)$	0.026	$\bar{s}(2)$	0.026
$c(2)$	0.019	$\bar{c}(2)$	0.019
$b(2)$	0.012	$\bar{b}(2)$	0.012

Effective coupling of Majorana DM with nucleon

The SI coupling of Majorana DM with nucleon is given as

$$\mathcal{L}_{\text{eff}} = \frac{1}{2} f_N \bar{\chi}^0 \tilde{\chi}^0 \bar{N} N$$

$$f_N / m_N = \sum_{q=u,d,s} C_S^q f_{T_q} + \frac{3}{4} \sum_{i=1,2} \sum_{q=u,d,s,c,b} C_{T_i}^q (q(2) + \bar{q}(2)) \\ - \frac{8\pi}{9\alpha_s} C_s^g f_{TG} + \frac{3}{4} \sum_{i=1,2} C_{T_i}^g G(2)$$

The gluon contribution turns out to be comparable to the quark contributions even if it is induced by higher loop diagrams.

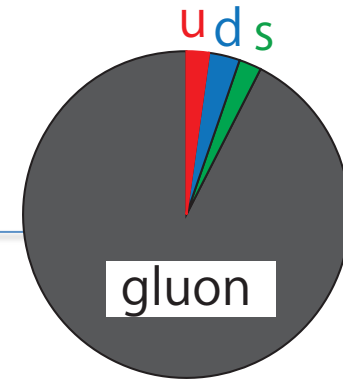
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suppressed by α_s

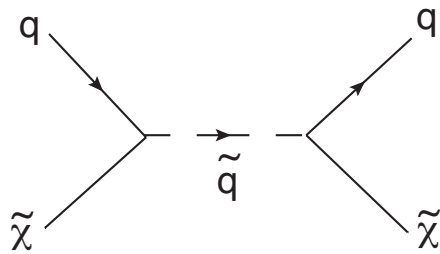
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3. Some results

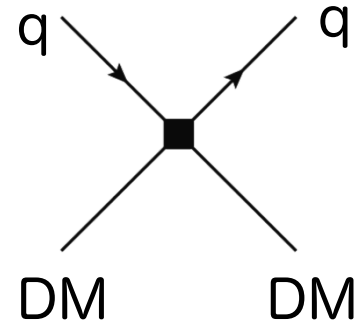
a) Pure bino DM

Pure Bino DM

The tree-level diagram:



Matching



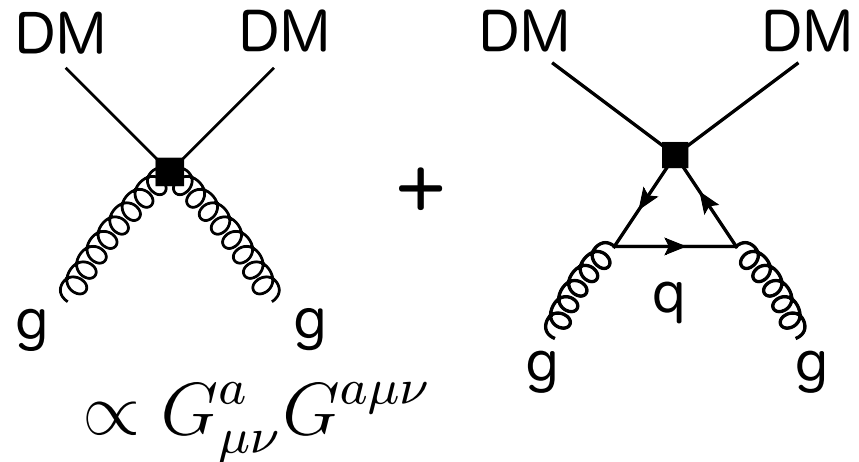
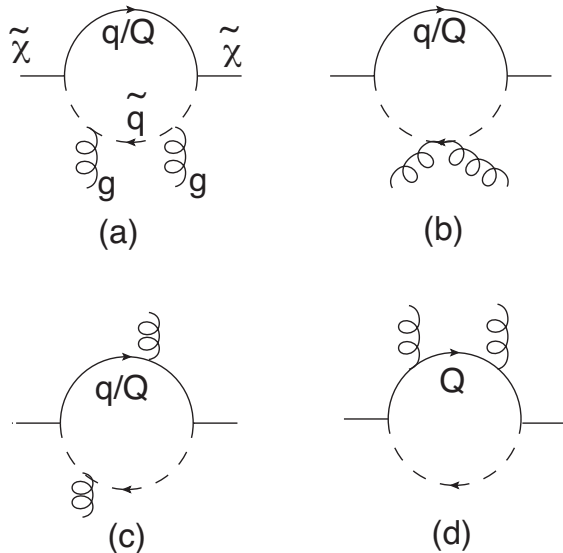
Scalar

$$\propto \bar{q}q$$

twist-2

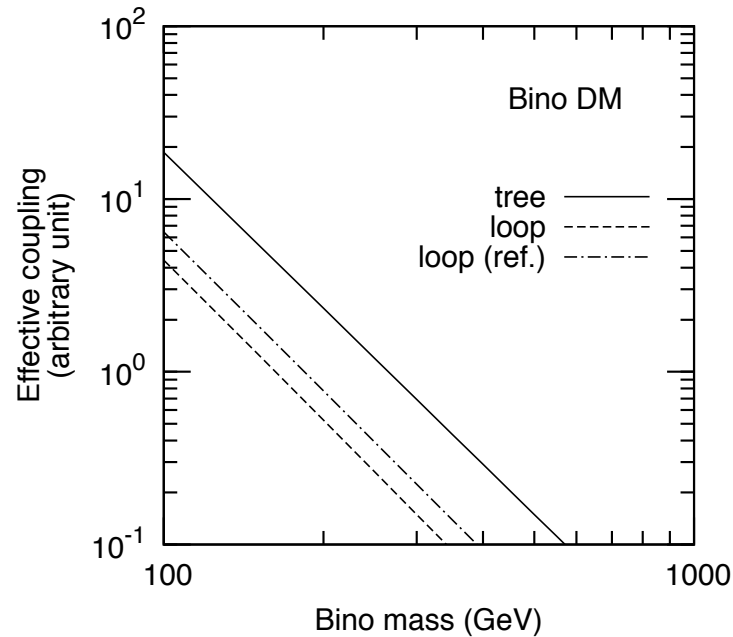
$$\propto \mathcal{O}_{\mu\nu}^q$$

1-loop diagrams:

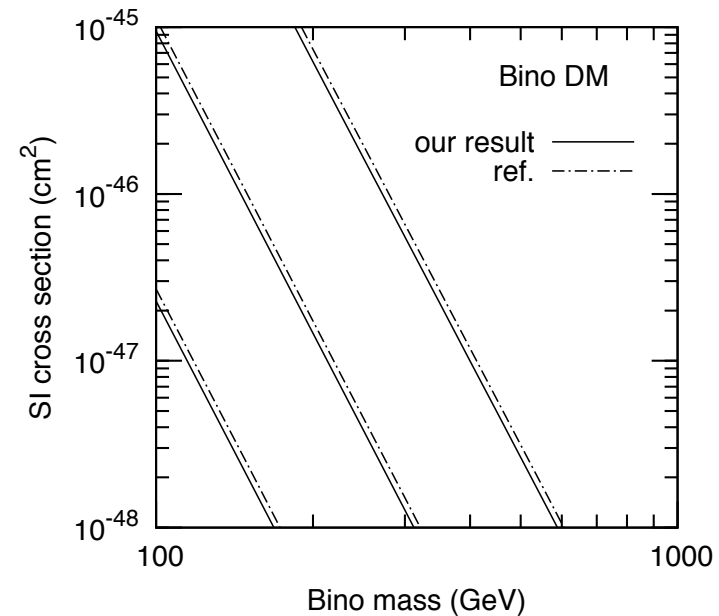


Only the short-distance contribution should be included into the Wilson coefficients.

Pure Bino DM



Each contribution to effective coupling f_p



Bino DM-proton SI cross sections

ref.) M. Drees and M. Nojiri, Phys. Rev. **D48** (1993) 3483.

We found O(10)% alternations in the SI cross sections

Due to a lack of matching in the previous calculation...

J. Hisano, K. Ishiwata, and N. Nagata, Phys. Rev. **D82** (2010) 115007.

3. Some results

- b) Wino DM (high-scale SUSY)
- c) Minimal DM

Mass spectrum

On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector

Scalar Particles



Gravitino



Higgsinos



Gauginos (Loop suppressed)



Gluino



Bino



Wino

Anomaly mediation

L. Randall and R. Sundrum (1998)

G.F. Giudice, M.A. Luty, H. Murayama, R. Rattazzi (1998)

Higgsinos can be light
(additional symmetries)

Mass spectrum

On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector

Scalar Particles



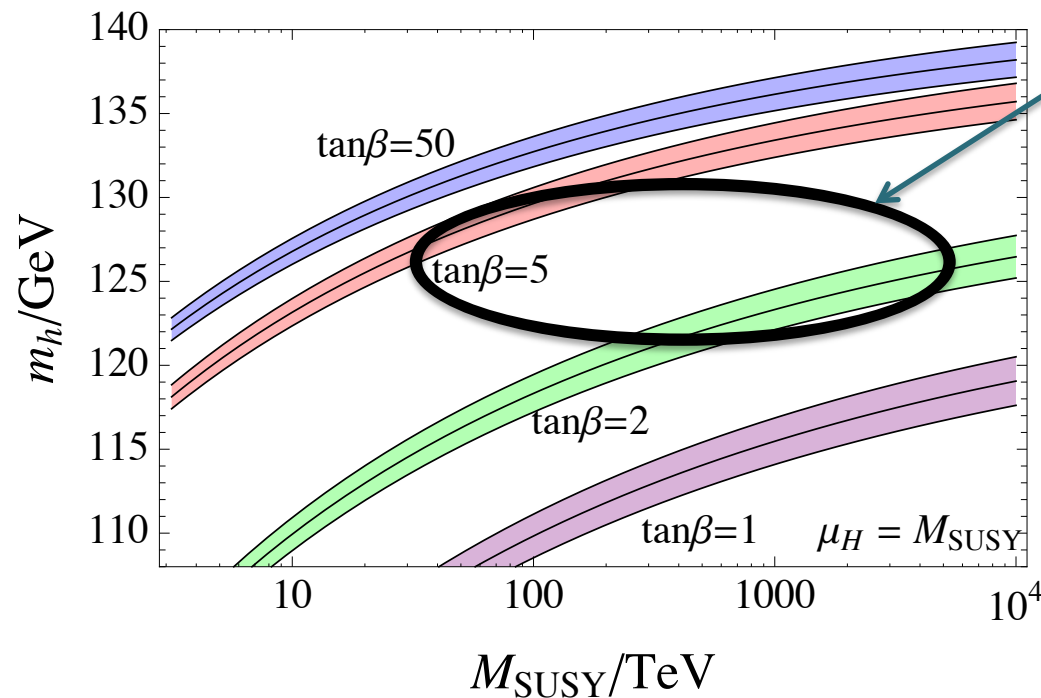
Gravitino



Higgsinos



$$M_S = 10^{(2-3)} \text{ TeV}$$



126 GeV Higgs boson

$\tilde{\nu}$ ino

M. Ibe, T.T. Yanagida (2012).

Mass spectrum

On the assumption of a generic Kahler potential and no singlet field in the SUSY breaking sector

Scalar Particles



Gravitino



Higgsinos



$$M_S = 10^{(2-3)} \text{ TeV}$$

Gauginos (Loop suppressed)



Gluino



Bino



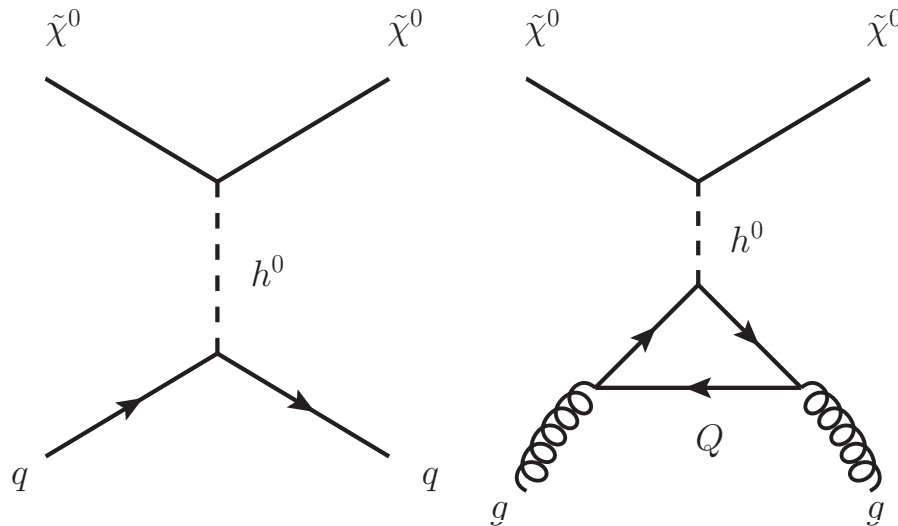
Wino

Higgsinos can be light
(additional symmetries)

$$M_S = O(1) \text{ TeV}$$

DM candidate

Diagrams Tree-level



“Higgs” contribution

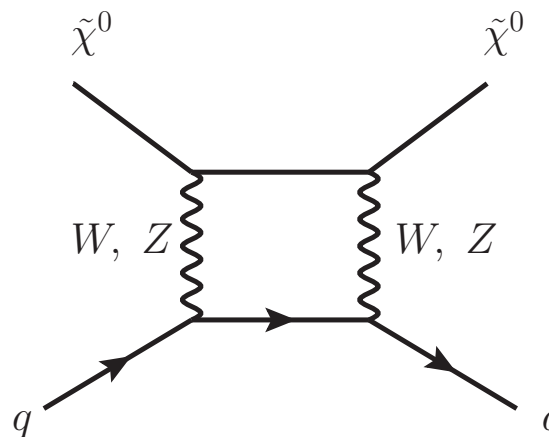
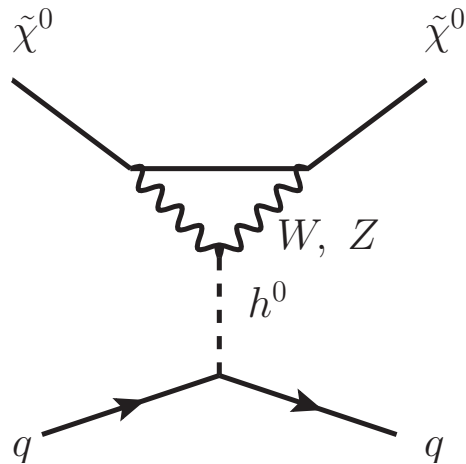
Effective coupling

$$C_q^H = \frac{g_2^2}{2m_W m_h^2} (Z_{12} - Z_{11} \tan \theta_W) (Z_{13} \cos \beta - Z_{14} \sin \beta)$$

(Z_{ij} : Neutralino mixing matrix)

$$\Rightarrow C_q^H \simeq \frac{g_2^2 (M_2 + \mu \sin 2\beta)}{2m_h^2 (M_2^2 - \mu^2)} \quad (|\mu \pm M_2| \gg m_Z)$$

Diagrams 1-loop



“Scalar”

$$\propto \bar{q}q$$

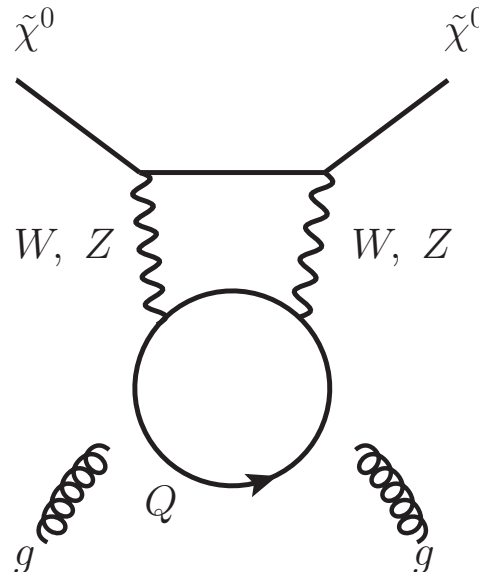
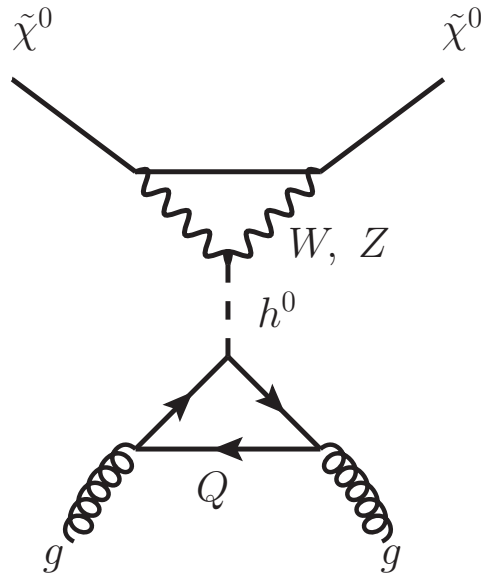
“twist-2”

$$\propto \mathcal{O}_{\mu\nu}^q$$

These interactions are not suppressed even if the DM mass is much larger than the W boson mass.

Non-decoupling effects

Diagrams 2-loop



Gluon contribution

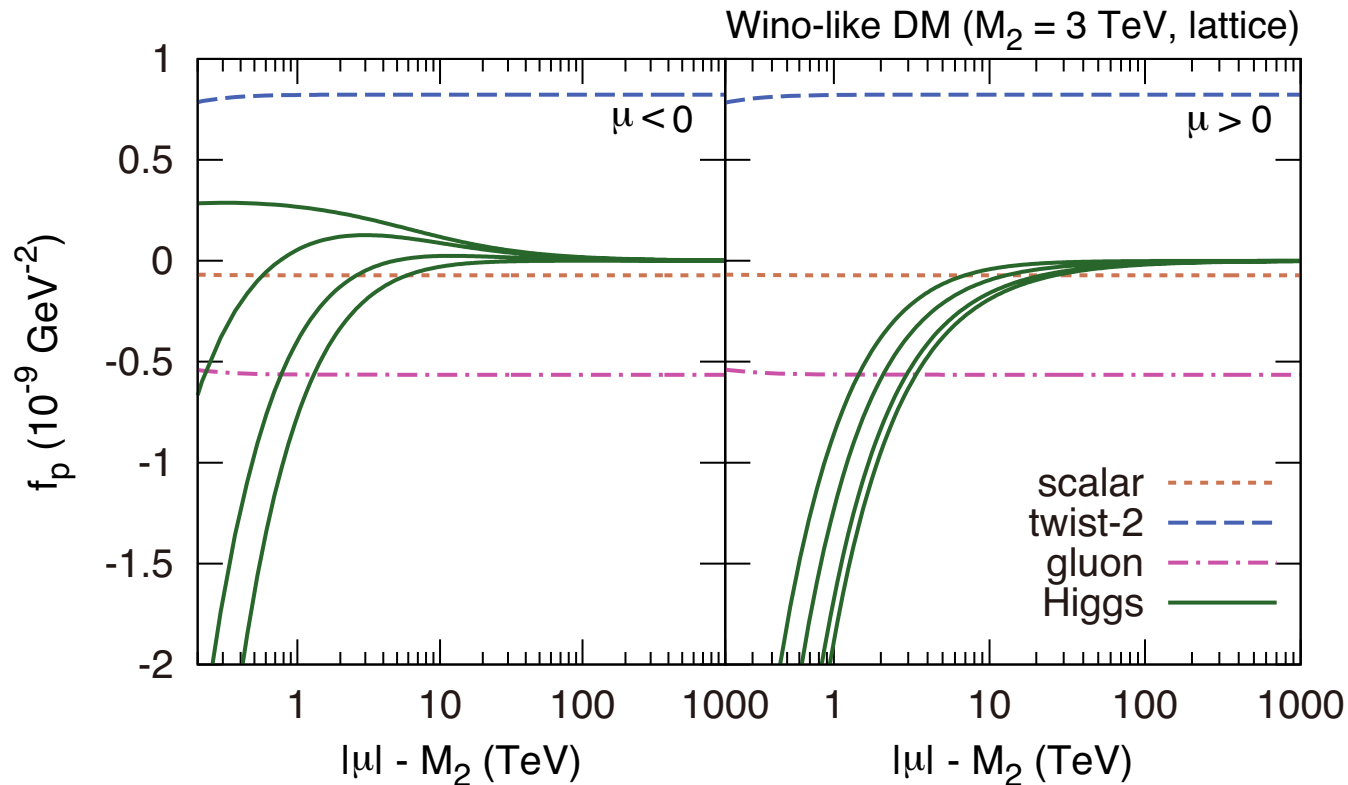
$$\propto G_{\mu\nu}^a G^{a\mu\nu}$$

- Neglected in previous calculations
- 2-loop gluon contribution can be comparable to 1-loop quark contribution
- non-decoupling

J. Hisano, K. Ishiwata, and N. Nagata, Phys. Lett. B **690** (2010) 311.

Wino-like DM

Effective coupling with a proton



$\tan \beta = 1, 2, 5, 50$

(from top to bottom)

$\tan \beta = 1, 2, 5, 50$

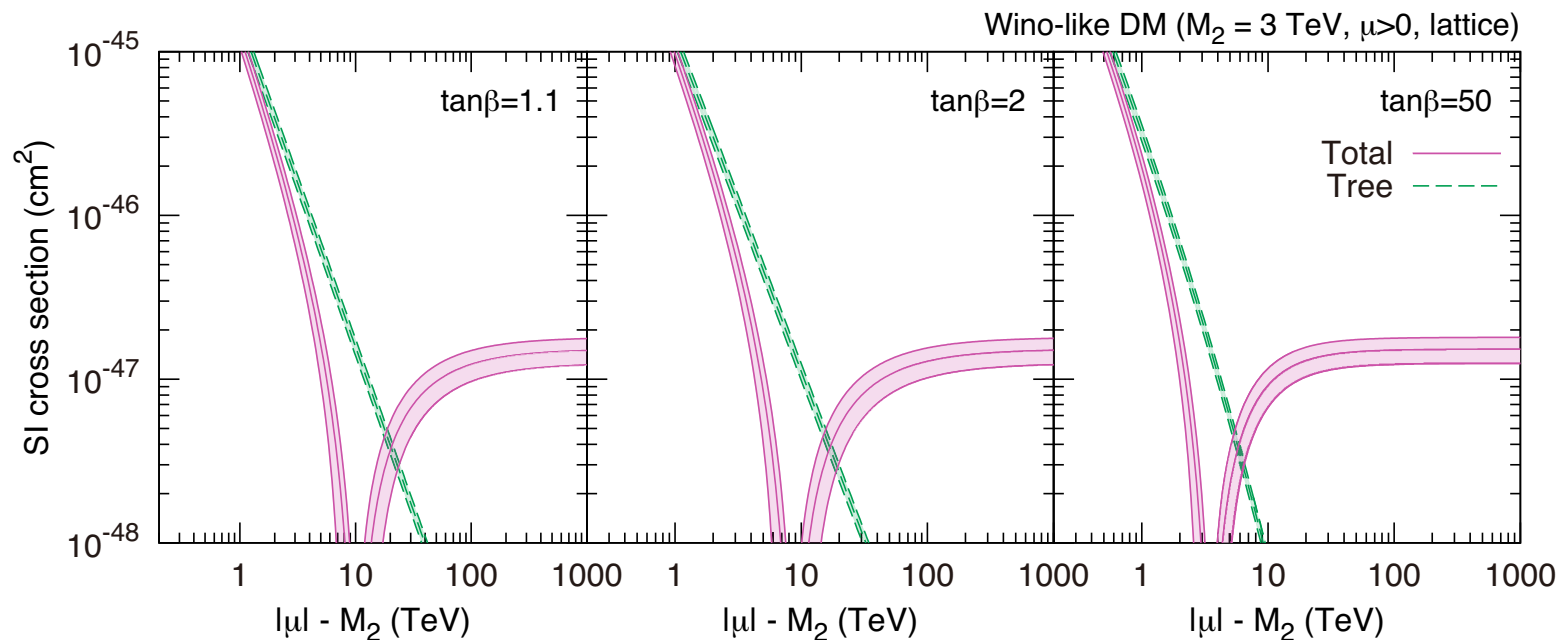
(from bottom to top)

There is a cancellation among these contributions

J. Hisano, K. Ishiwata, and N. Nagata, Phys. Rev. **D87** (2013) 035020.

Wino-like DM

Scattering cross sections with a proton

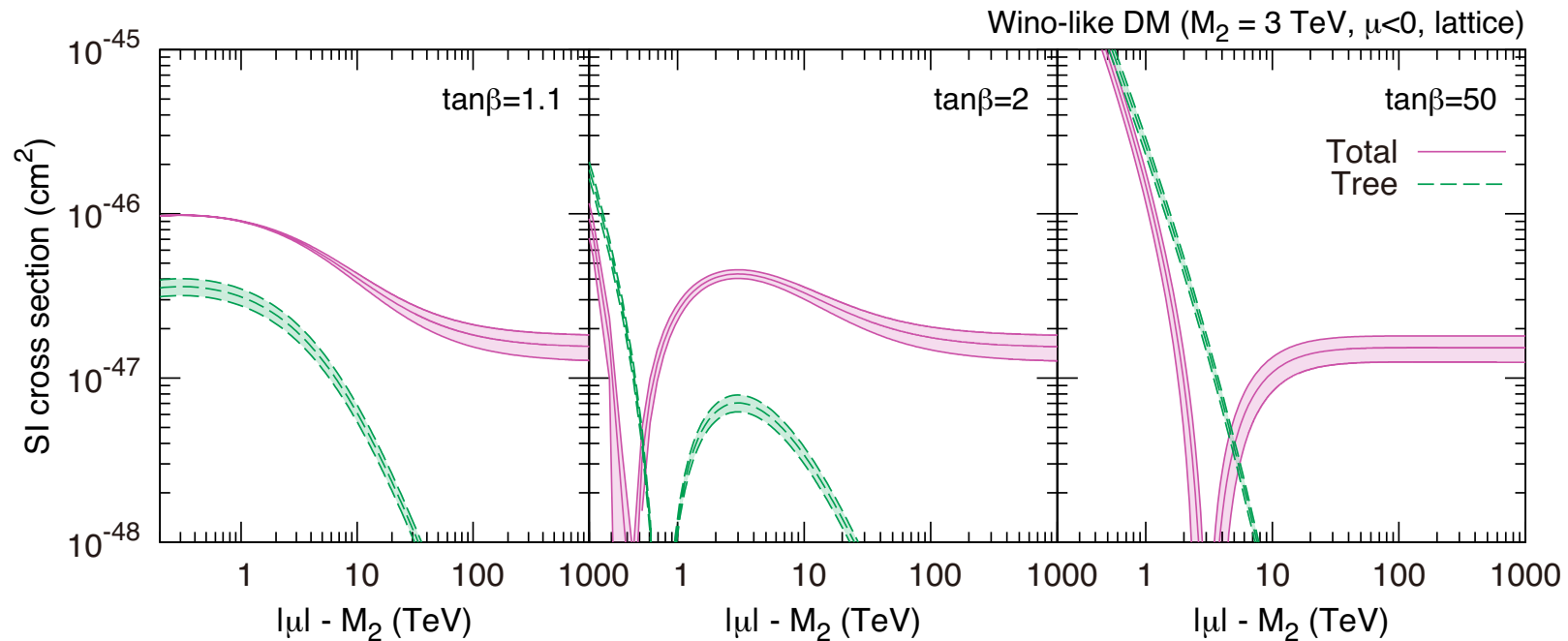


- Cancellations between tree- and loop-level contributions occur at a certain value of μ
- Loop contribution is dominant in a wide range of parameter region

J. Hisano, K. Ishiwata, and N. Nagata, Phys. Rev. **D87** (2013) 035020.

Wino-like DM

Scattering cross sections with a proton



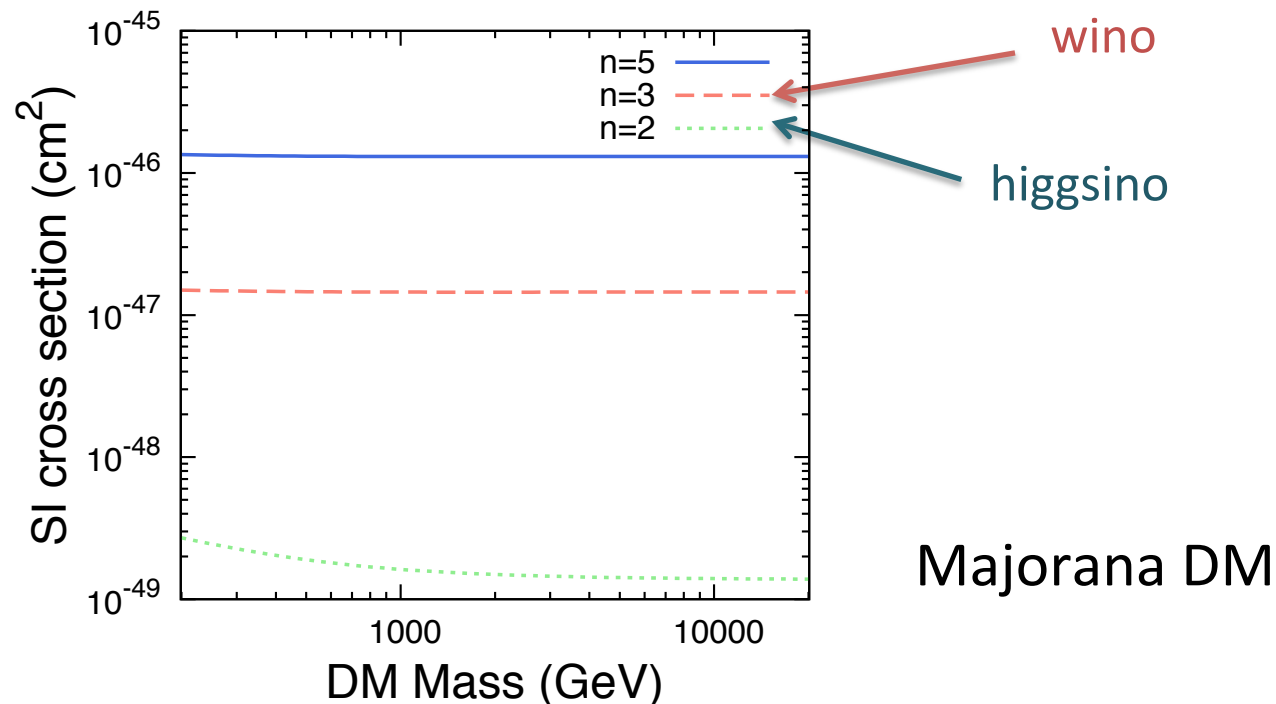
Tree-level contribution interferes constructively to the loop contribution in the case of low $\tan\beta$

Minimal DM

Minimal dark matter

M. Cirelli, N. Fornengo, A. Strumia (2005)

The neutral component of an $SU(2)_L$ n -tuplet is assumed to be DM in the Universe.



We again find cancellations among the contributions.

J. Hisano, K. Ishiwata, N. Nagata, and T. Takesako, JHEP **1107** (2011) 005.

3. Some results

d) KK-photon DM (Minimal UED)

Vector DM KK photon DM (MUED)

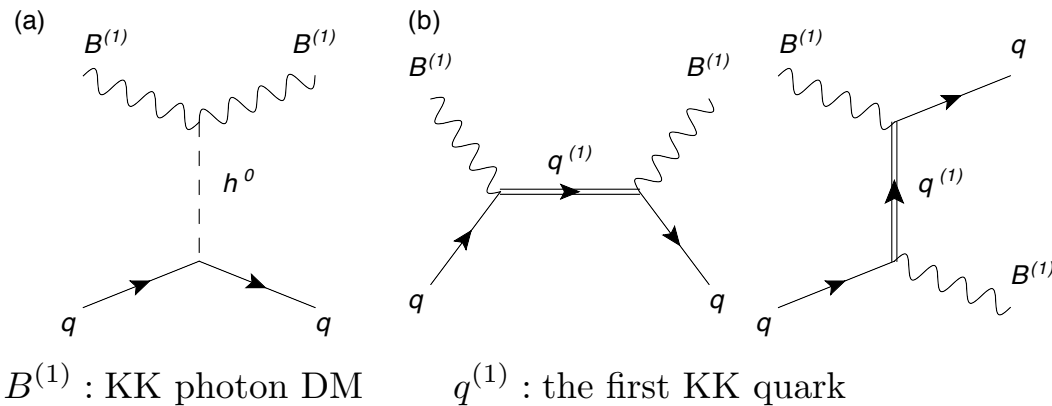
T. Appelquist, H. C. Cheng, B. A. Dobrescu (2001)

H. C. Cheng, K. T. Matchev, M. Schmaltz (2002)

Minimal Universal Extra Dimension (MUED) model

The first KK photon becomes the lightest Kaluza-Klein particle (LKP).

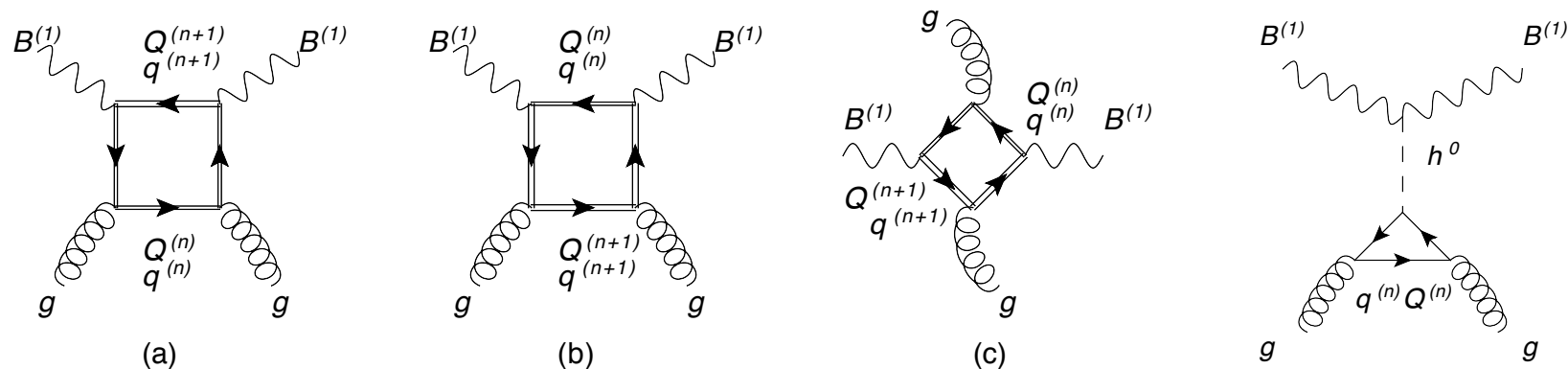
Tree-level diagrams:



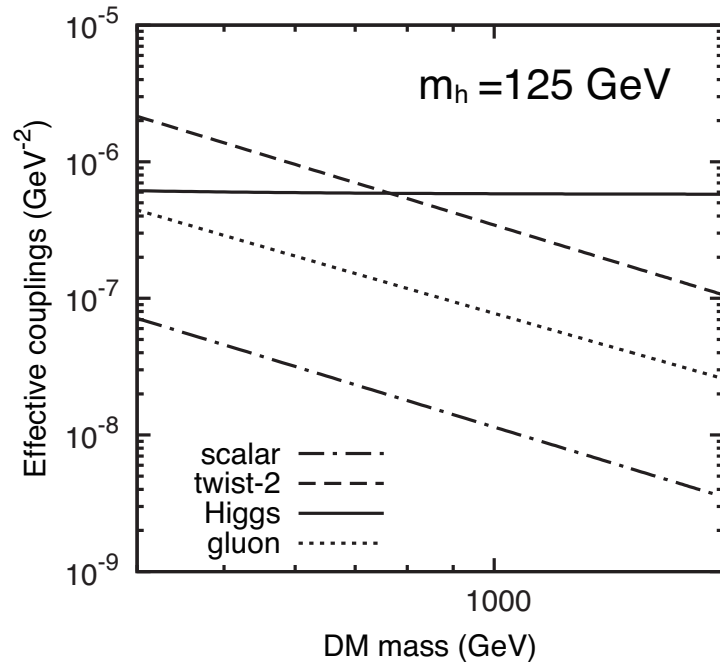
→ Vector DM

Neglected in previous calculations

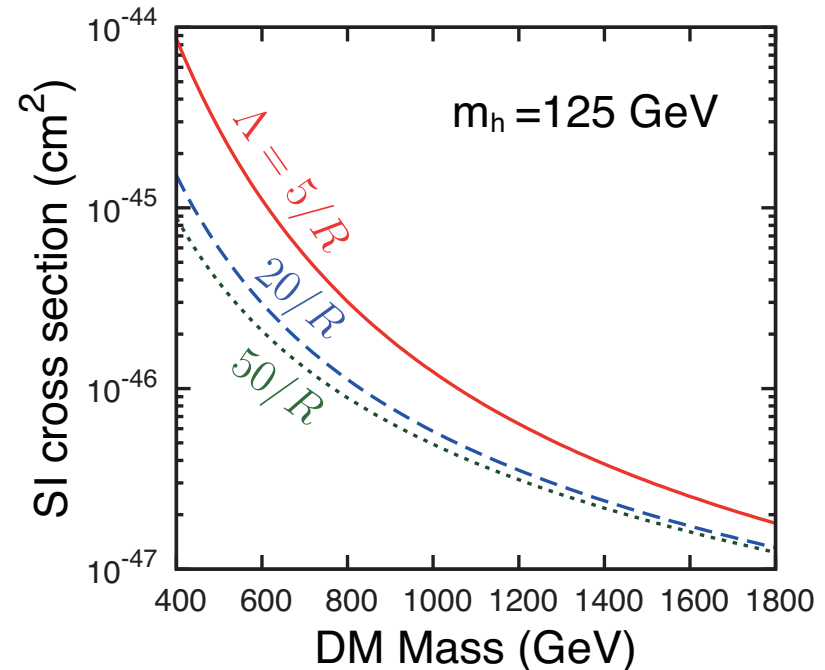
1-loop diagrams:



Vector DM KK photon DM (MUED)



Each contribution in the effective coupling



The SI scattering cross sections

- All of the contributions have the same sign (constructive).
- Resultant scattering cross sections are larger than those in previous work by about an order of magnitude in some parameter region.

4. Conclusion and discussion

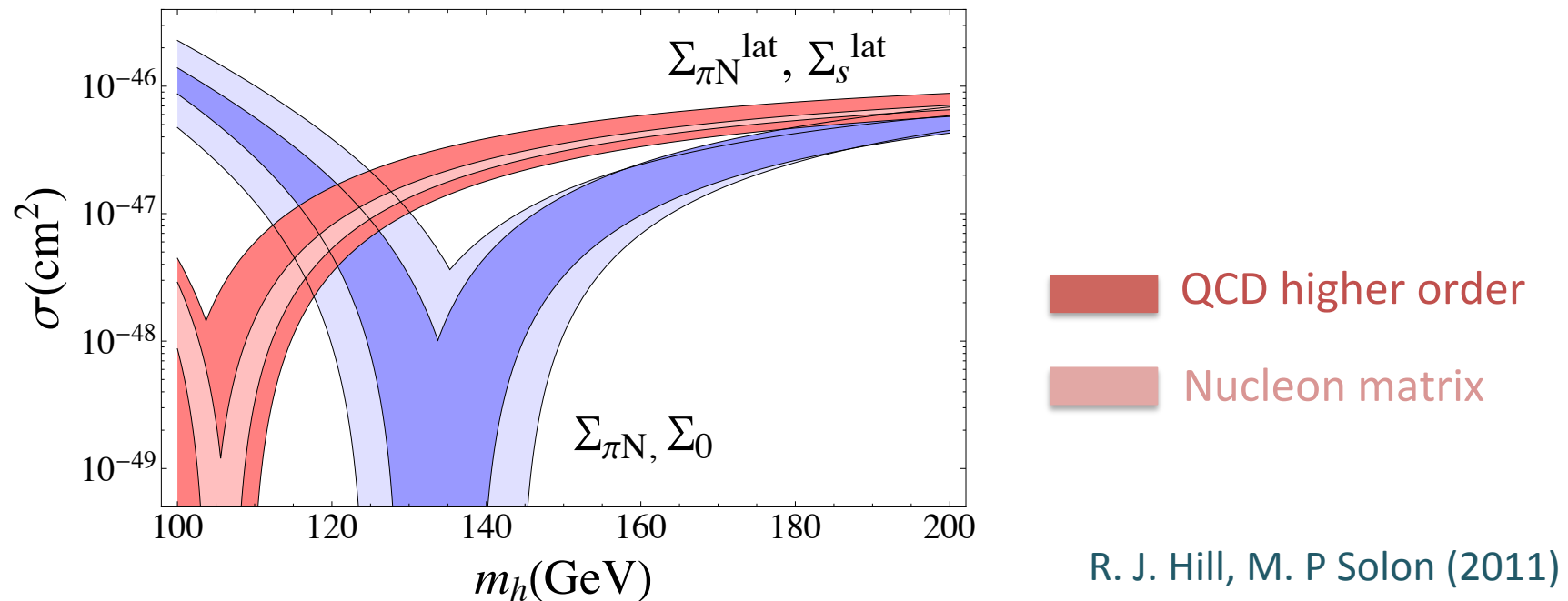
Summary

- We evaluate the elastic scattering cross sections of WIMP DM with nucleon based on the method of effective theory.
- The interaction of DM with gluon as well as quarks yields sizable contribution to the cross section, though the gluon contribution is induced at loop level.
- In the wino dark matter scenario we find the cross section is smaller than the previous results by more than an order of magnitude
- The cross section of the first Kaluza-Klein photon dark matter turns out to be larger.

Backup

Prospects

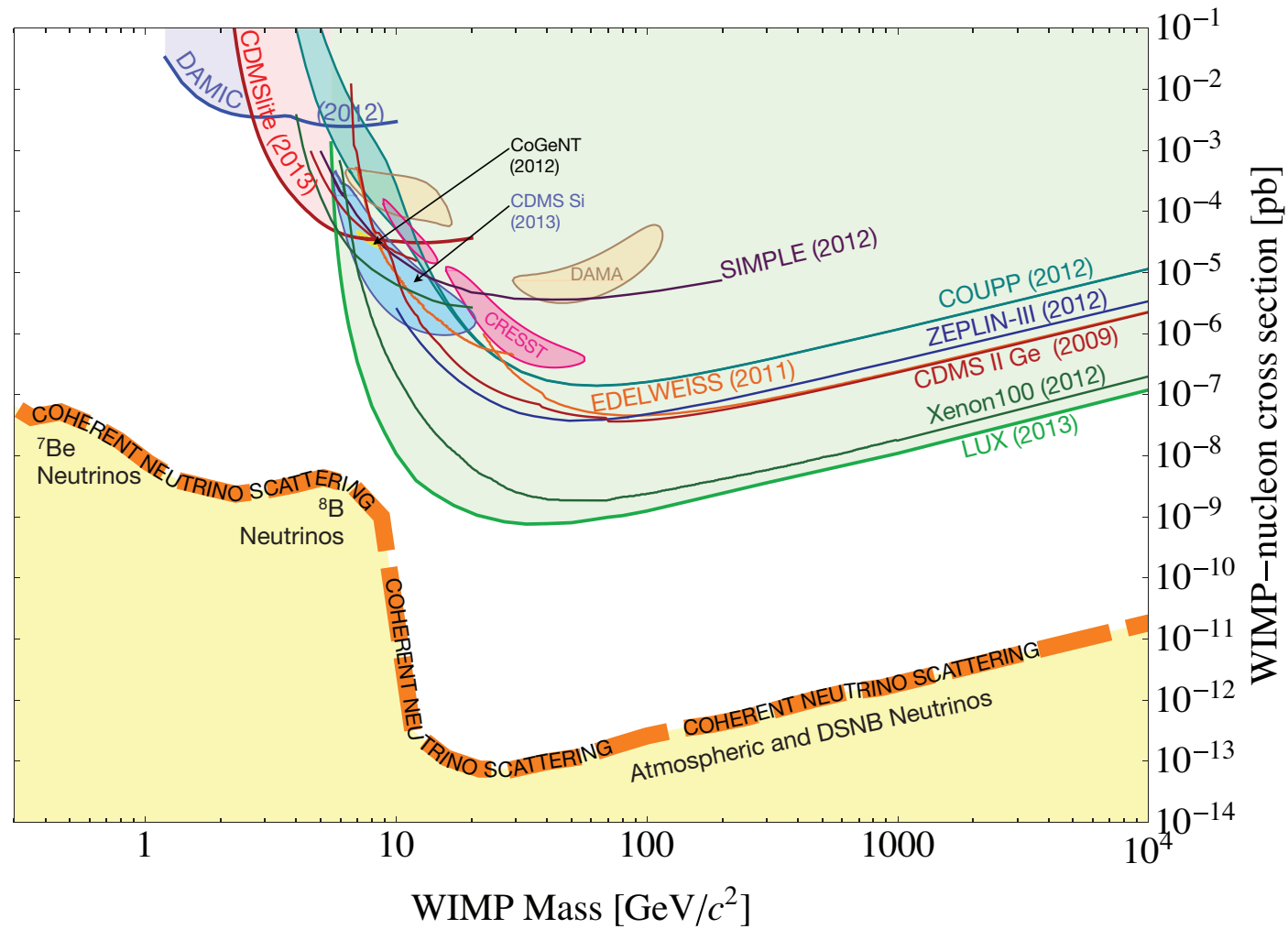
Wino-proton scattering cross sections



- Uncertainty mainly comes from the perturbative QCD.
- Need for higher order calculation.

J. Hisano, K. Ishiwata, N. Nagata, in progress.

Neutrino BG



J. Billard, E. Figueroa-Feliciano, L. Strigari [arXiv: 1307.5458].

Higgs-nucleon coupling

$$\mathcal{L}_{NNh} = -g_{NNh}\bar{N}Nh$$

$$\begin{aligned} g_{NNh} &= \frac{\sqrt{2}}{v} \sum_q \langle N | m_q \bar{q}q | N \rangle \\ &= \frac{\sqrt{2}}{v} \left[m_N (f_{Tu} + f_{Td} + f_{Ts}) - \frac{\alpha_s}{4\pi} \langle N | G_{\mu\nu}^a G^{a\mu\nu} | N \rangle \right] \\ &= \frac{\sqrt{2}}{v} m_N \left[\frac{2}{9} + \frac{7}{9} (f_{Tu} + f_{Td} + f_{Ts}) \right] \end{aligned}$$

Large mass fractions ($f_{Tq} \rightarrow \text{large}$)



Higgs-nucleon couplings are enhanced

Input parameters

$$\sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p | \bar{u}u + \bar{d}d | p \rangle$$

$$y = \frac{2 \langle p | \bar{s}s | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle}$$

$$\xi = \frac{\langle p | \bar{u}u - \bar{d}d | p \rangle}{\langle p | \bar{u}u + \bar{d}d | p \rangle}$$

$$\xi = 0.135 \pm 0.035$$

H. Y. Cheng (1989)

■ Lattice results (ours)

$$\sigma_{\pi N} = 53 \pm 2(\text{stat})_{-7}^{+21}(\text{syst}) \text{ MeV}$$

$$y = 0.030 \pm 0.016(\text{stat})_{-0.008}^{+0.006}(\text{syst})$$

H. Ohki et al. (2008)

■ Chiral perturbation (traditional)

$$\sigma_{\pi N} = 64 \pm 7 \text{ MeV}$$

M. M. Pavan et al. (2002)

$$y = 0.44 \pm 0.13$$

B. Borasoy and U. G. Meissner (1997)