Effective Temperature of Non-equilibrium Steady States in AdS/CFT

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Ref. S. N. and H. Ooguri (Caltech/KIPMU), PRD88 (2013) 126003 [arXiv:1309.4089].

We employ the natural unit: $k_B=c=\hbar=1$.

Fundamental question

What is temperature?

Fundamental question in statistical physics.

We have many answers.

Definitions of equilibrium temperature

$$P \propto e^{-E/T},\quad t_E \approx t_E + 1/T$$ Statistical distributions

$$dE = TdS$$ Thermodynamics

$$D = T\mu$$

Diffusion const. Mobility

Fluctuation-dissipation relation

Can we generalize the notion of temperature into non-equilibrium systems?

Fundamental question

Can we generalize the notion of temperature into non-equilibrium systems?

This question is too general.

What is the meaning of non-equilibrium?

Non-equilibrium systems

	Time independent	Time dependent
Linear response regime (vicinity of equilibrium)	Many success in linear response theory, hydrodynamics,…	
Beyond the linear response regime (non-linear regime, far from equilibrium)	Still frontier	

Non-equilibrium steady state (NESS)

Do we still have a notion of (generalized) temperature in NESS in non-linear regime?

Answer

- In the main part of my talk, I am going to show that the AdS/CFT correspondence suggests the presence of "effective temperature" in NESS.
- The effective temperature characterizes the relationship between the fluctuation and dissipation in NESS.
- I will show you an interesting (counter-intuitive) behavior of the effective temperature.

Main part

Non-equilibrium systems

	Time independent	Time dependent
Linear response regime (vicinity of equilibrium)	Many success in linear response theory, hydrodynamics,…	
Beyond the linear response regime (non-linear regime, far from equilibrium)	Still frontier	

Non-equilibrium steady state (NESS)

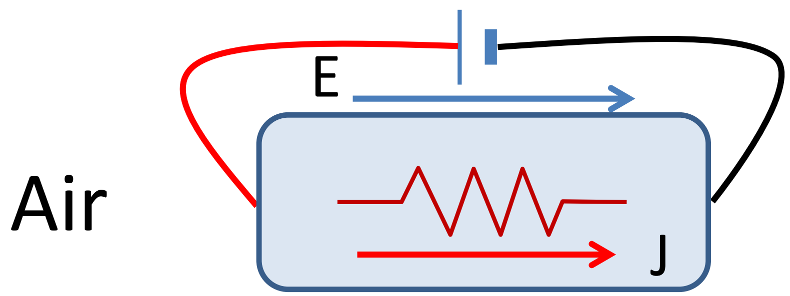
Do we still have a notion of (generalized) temperature in NESS in non-linear regime?

Non-equilibrium steady state (NESS)

Non-equilibrium, but time-independent.

A typical example:

A system with a constant current along the electric field.



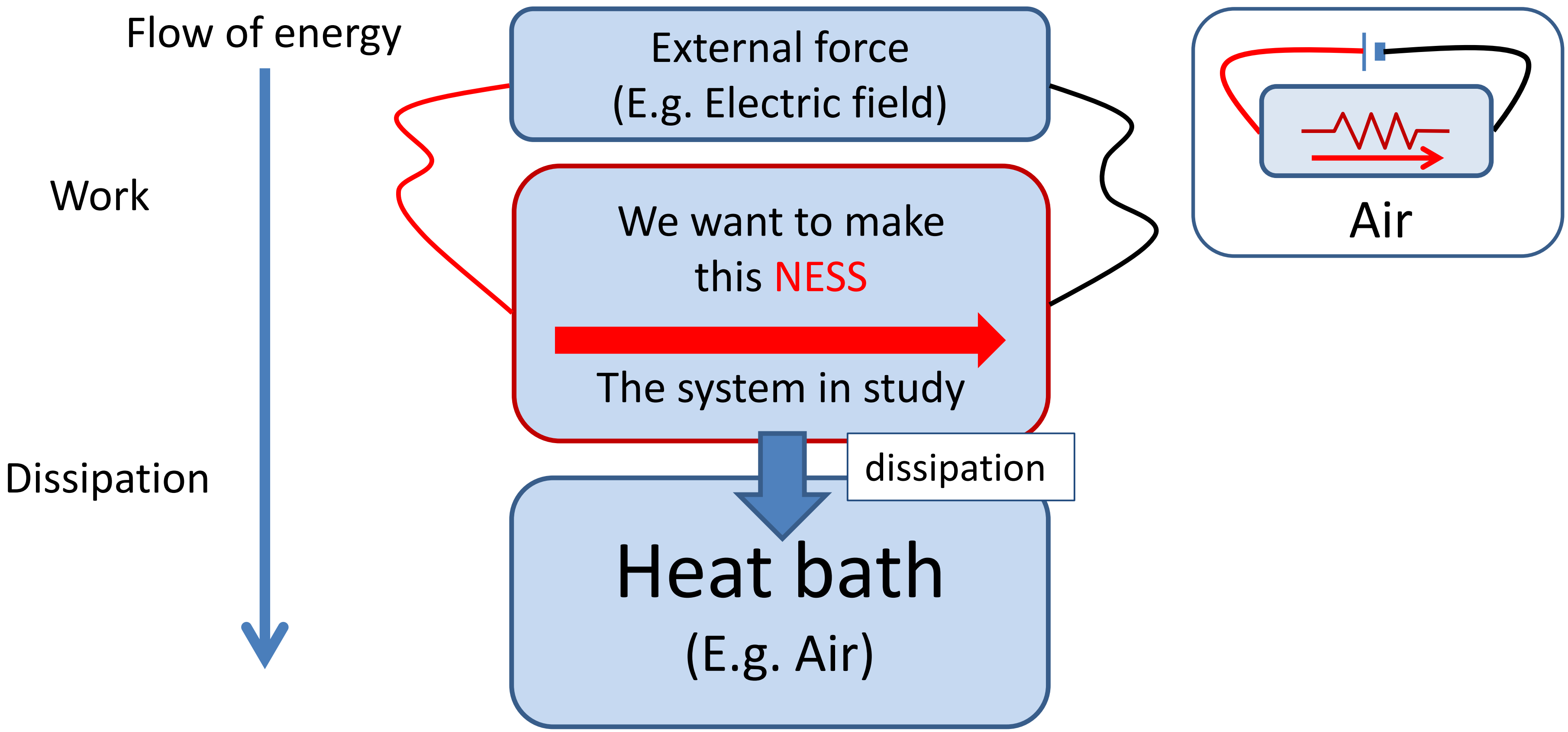
- It is non-equilibrium, because heat and entropy are produced.
- The macroscopic variables can be time independent.

In order to realize a NESS, we need an external force and a heat bath.

Setup for NESS

External force and heat bath are necessary.

Power supply drives the system our of equilibrium.



The subsystem can be NESS if the work of the source and the energy dissipated into the heat bath are in balance.

Our tool: AdS/CFT

Reasons to employ AdS/CFT

Analysis beyond the linear response regime:

- Still a challenge for the conventional approaches.
- We can deal with the systems in the non-linear regime by AdS/CFT at least for some cases.

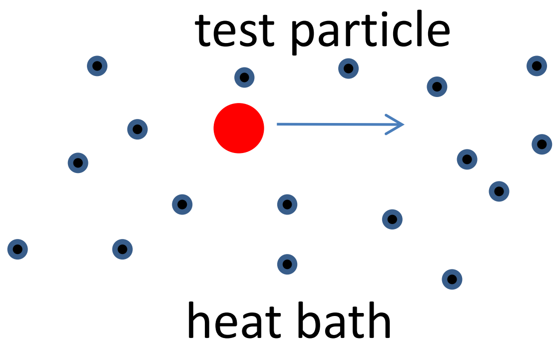
“Notion” of temperature:

- AdS/CFT may provide us a new picture on physics by virtue of the wisdom of general relativity.

NESS to consider

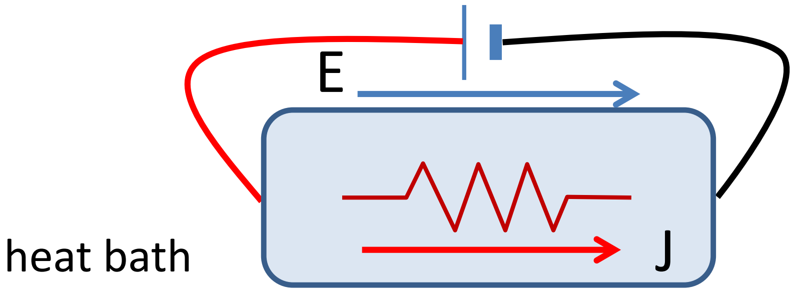
Langevin system

A test particle immersed in
a heat bath is driven by
a constant external force.



System with constant current

A system of charged particles
immersed in a heat bath is
driven by a constant external
electric field.



NESS in gauge theory

NESS: sector of quarks and antiquarks

Heat bath: sector of gluons

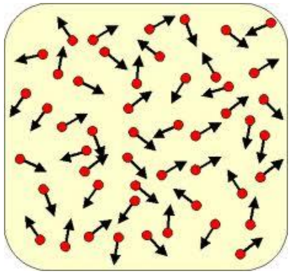
- The degrees of freedom of gluons are Nc times larger than that of quarks/amtiquarks.
- If we take the large-Nc limit, the gluon sector behaves as a good heat bath.

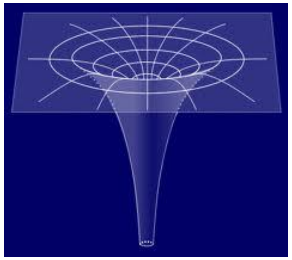
Strategy

A strongly-interacting quantum gauge theory

AdS/CFT ⟷ equivalent

A classical gravity (general relativity) on a curved spacetime in higher dimensions.





Heat bath (gluons) → Black Hole

Charged particle(s) (quarks) in the heat bath → Object immersed in the black hole geometry

Picture of many-body system is taken from internet.
Picture of black hole is taken from https://www.kahaku.go.jp/exhibitions/vm/resource/tenmon/space/theory/theory06.html

Objects in gravity dual

For the Langevin system

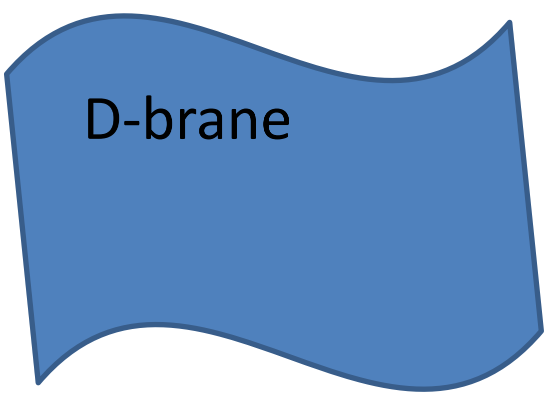
A single quark
as a test particle

A single string

For the system of conductor

A system of (many)
quarks (and anti-quarks)

A single D-brane



Strategy

Heat bath (gluons) → Black Hole

- A test particle
- Charged particles

→

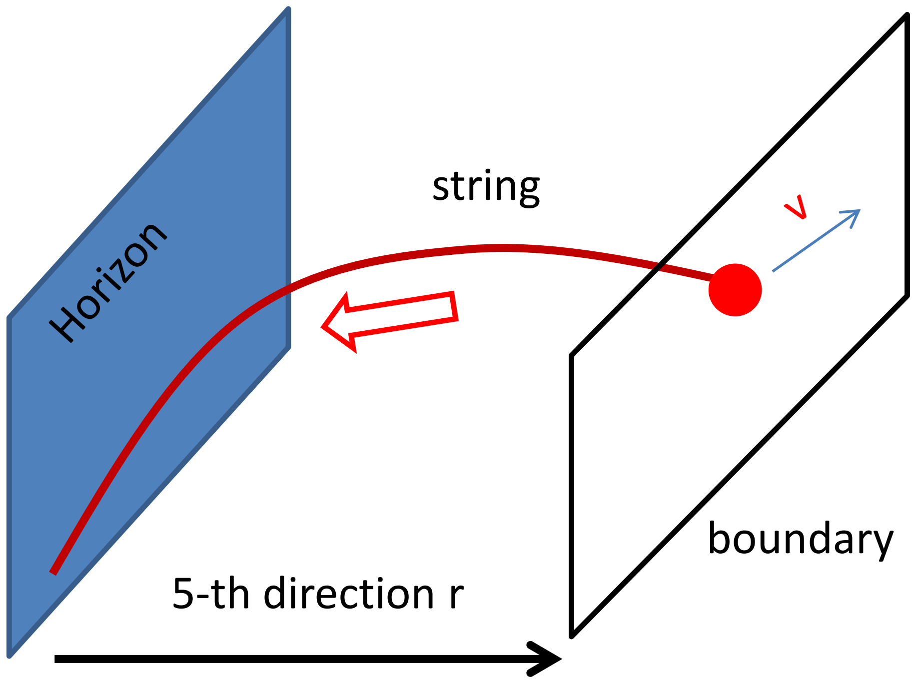
- F1
- Probe D-brane

The temperature of heat bath is always kept fixed (by definition).

Only need to solve the dynamics of the D-brane/F1 in the presence of external driving force.

This is just a problem of solving a non-linear differential equation, and we can do it!

Langevin system



[Gubser, 2006]
[Herzog et al., 2006]

Energy-momentum tensor of string

$T^0{}_r$ = energy flow into the black hole in unit time: dissipation

= Work in unit time by the force acting on the test particle

$$f = \left.\frac{\partial L}{\partial(\partial_r x)}\right|_{\text{boundary}} \neq 0 \quad \text{at} \quad v \neq 0.$$

[Gubser, 2006]
[Herzog et al., 2006]

Computation of drag force

[Gubser, 2006], [Herzog et al., 2006]

$$L_{\mathrm{string}} = -(\mathrm{tension})\sqrt{-\det\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}\right)}$$

$$X(t,r) = vt + x(r)$$

$$\partial_r \frac{\partial L}{\partial(\partial_r x)} = 0 \quad \Rightarrow \quad \frac{\partial L}{\partial(\partial_r x)} = f$$

$$(\partial_r x)^2 = f^2 \frac{g_{rr}}{-g_{tt}g_{rr}} \frac{(-g_{tt}) - g_{xx}v^2}{(-g_{tt})g_{xx} - f^2}$$

Right-hand-side can be negative.

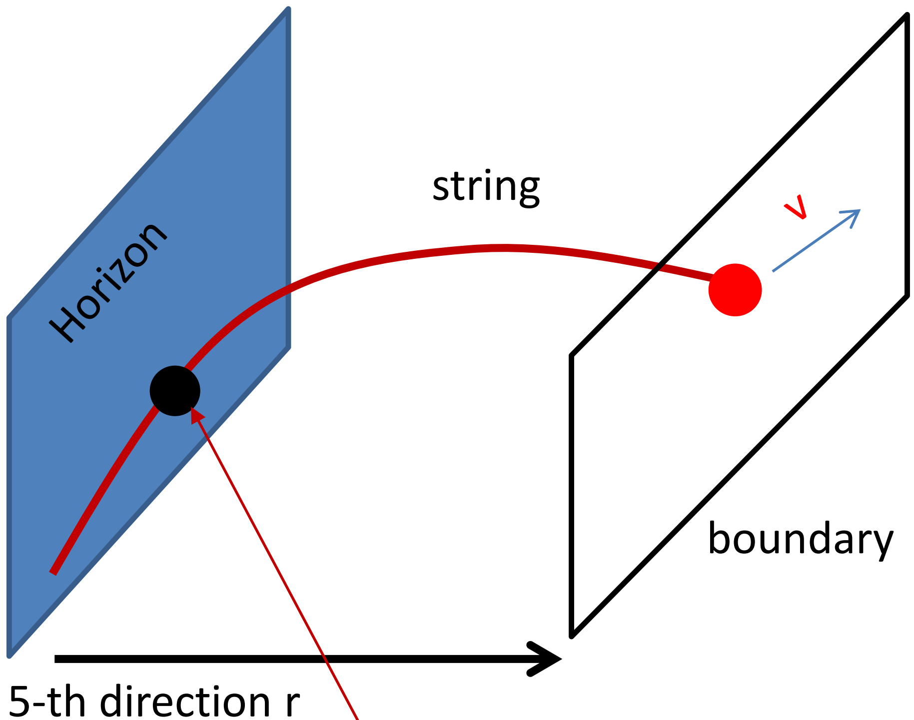
Let us define a point r=r_* by $(-g_{tt}) - g_{xx}v^2\Big|_{r_*} = 0$.

$$(-g_{tt})g_{xx} - f^2\Big|_{r_*} = 0$$

If f satisfies this, $\partial_r x$ can be real.

f is given as a function of v.

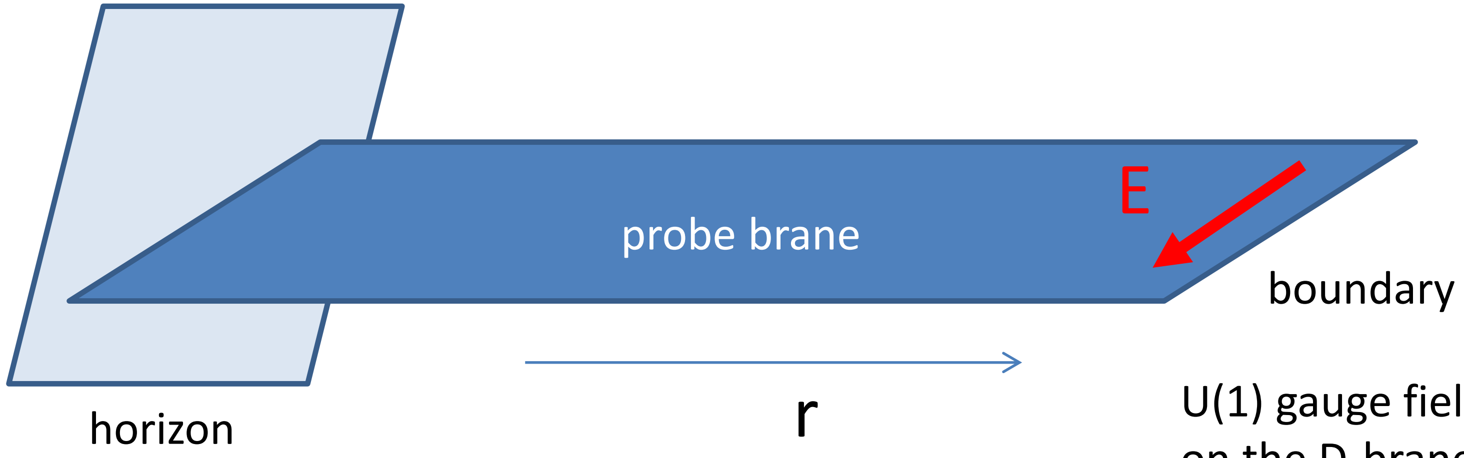
Langevin system



There is a special point ($r = r_*$).

For conudctors

[Karch and O'Bannon, 2007]



U(1) gauge field on the D-brane

$$L_{\mathrm{DBI}} = e^{-\phi}\sqrt{-\det\left(\partial_a X^\mu \partial_b X^\nu g_{\mu\nu} + F_{ab}\right)}$$

We apply an external electric field E.

$$A_1 = -Et + h(r) \quad \Rightarrow \quad J = \frac{\partial L}{\partial F_{r1}}$$

Relationship between E and J

[Karch and O'Bannon, 2007]

$$(F_{r1})^2 = J^2 \frac{g_{rr}}{|g_{tt}|} \frac{E^2 - |g_{tt}| g_{xx}}{J^2 - e^{-2\phi} |g_{tt}| g_{xx}^{q-1}}$$

q: number of spatial directions of the dual field theory.

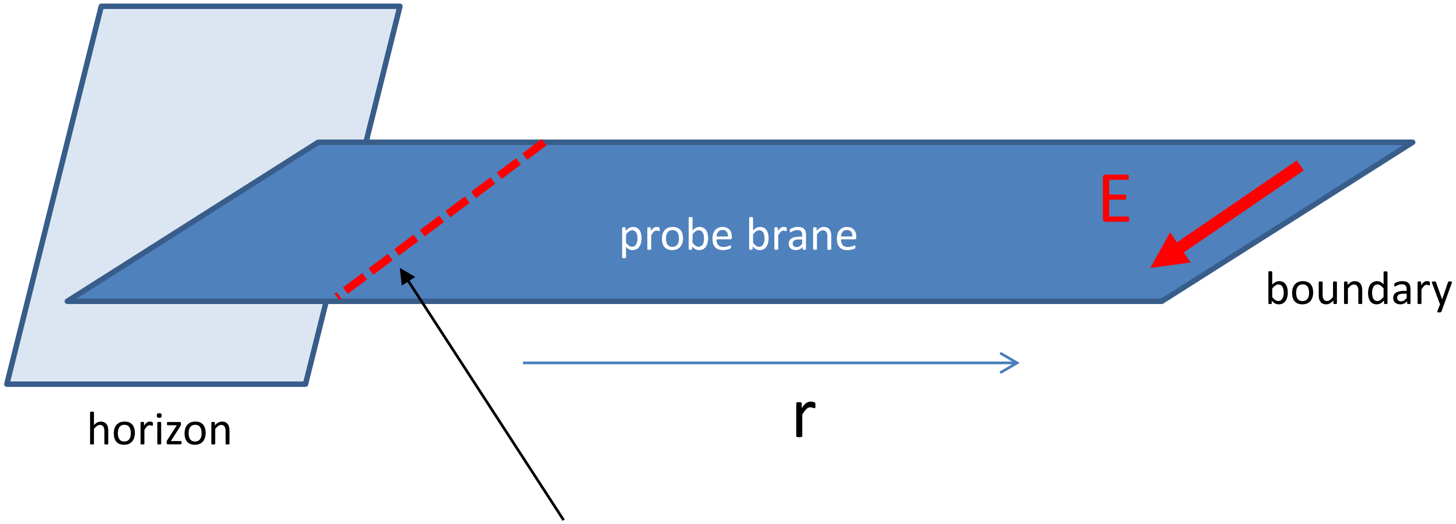
Again, we have a special point r_* given by

$$E^2 - |g_{tt}| g_{xx} = 0,$$

and J is given by $J^2 - e^{-2\phi} |g_{tt}| g_{xx}^{q-1} \Big|_{r_*} = 0$ in terms of E.

For conductors

[Karch and O'Bannon, 2007]



We have a special point $r=r_*$.

What is this special point?

“Special point” $r=r_*$

It is a “horizon” on the worldsheet/worldvolume seen by the small fluctuations.

(See also [Gubser 2008, Kim-Shock-Tarrio 2011, Sonner-Green 2012])

We call it “effective horizon.”

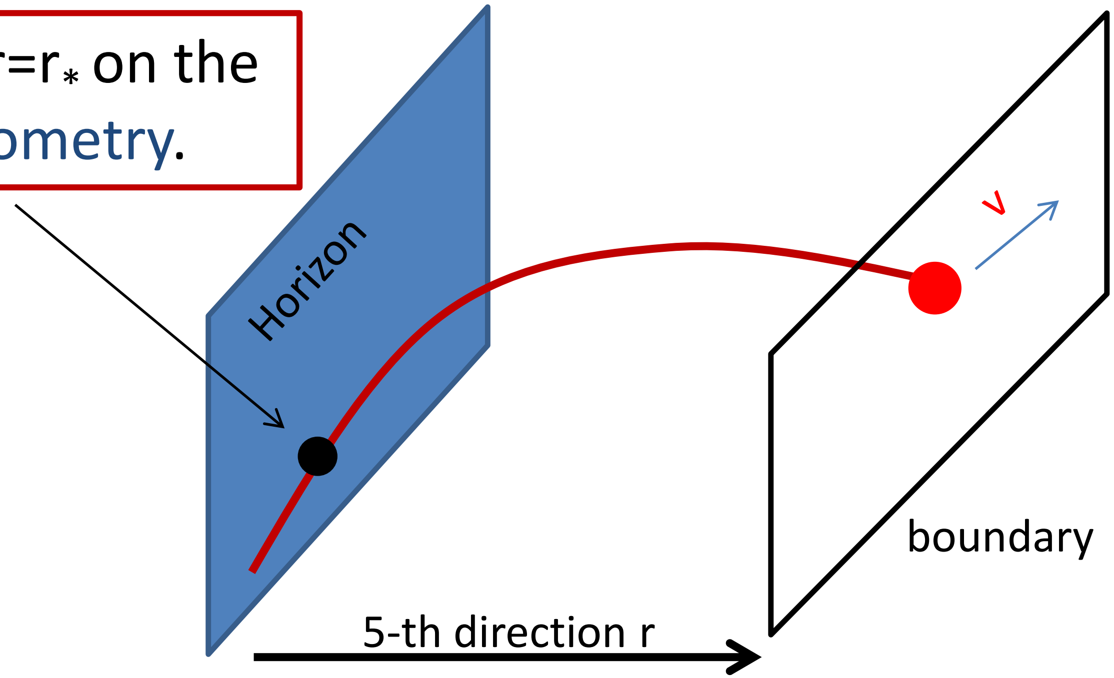
How to see it?

- We find a string/D-brane configuration in the presence of external driving force.
- Derive the equations of motion for small fluctuations of the worldvolume fields to read the effective metric.

Langevin system

See also, [Gubser, 2008]

Horizon at $r=r_*$ on the induced geometry.



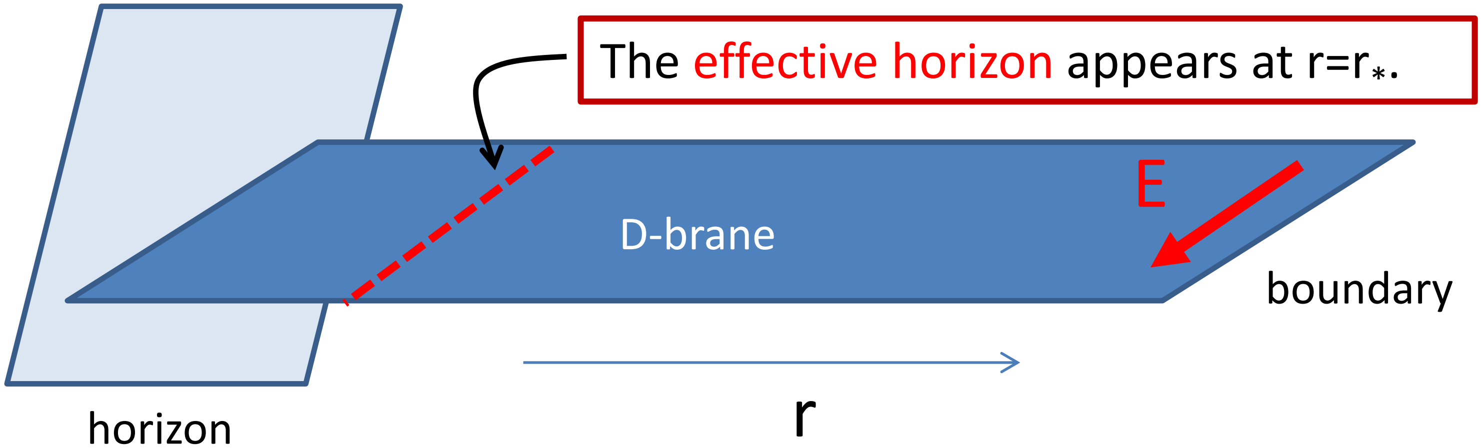
Equation of motion for small fluctuation δX of the string:

$$\partial_a\left(\sqrt{-\tilde{g}}\,\tilde{g}^{ab}\partial_b\,\delta X^{\mu}\right)=0,$$

$$\tilde{g}_{ab}=\partial_a X^{\mu}\partial_a X^{\nu} g_{\mu\nu}$$

Klein-Gordon equation on a curved spacetime given by the induced metric.

For conductors



Small fluctuation of electro-magnetic field on D-brane δA_b obeys to the Maxwell equation on a curved geometry:

$$\partial_a \left(\sqrt{-\bar{g}}\, \bar{g}^{ab}\, \delta f_{bc}\, \bar{g}^{cd} \right) = 0, \qquad \delta f_{bc} = \partial_b \delta A_a - \partial_a \delta A_b \, .$$

The metric is proportional to the open-string metric, but is different from the induced metric.

The geometry has a horizon at $r=r_*$.

(See also [Kim-Shock-Tarrio 2011, Sonner-Green 2012])

Now we have two temperatures

Black hole horizon gives the temperature of the heat bath.

The effective horizon on the string gives a different "Hawking temperature" that governs the fluctuations of the test particle.

We call this effective temperature T_{eff} of NESS.

> If the system is driven to NESS,
> $r_H < r_*$ at the order of v^2 (or E^2).

Two temperatures appear only in the non-linear regime.

Computations of effective temperature

[S. N. and H. Ooguri, PRD88 (2013) 126003]

We have computed T_{eff} for wide range of models.

- Heat bath:

  Near-horizon geometry of Dp-brane solution at T.

  [Itzhaki-Maldacena-Sonnenschein-Yankielowitcz, 1998]

$$ds^2 = r^{\frac{7-p}{2}}\left[-\left(1-{r_0^{7-p}}/{r^{7-p}}\right)dt^2 + d\vec{x}^2\right] + \frac{dr^2}{r^{\frac{7-p}{2}}\left(1-{r_0^{7-p}}/{r^{7-p}}\right)} + r^{\frac{p-3}{2}} d\Omega_{8-p}^2$$

- Test particle: probe D(q+1+n)-brane or F1 string

wrapped on n-sphere

- Charged particles: probe D(q+1+n)-brane

T_{eff} for Langevin system

Can never been understood as a Lorentz factor.

Beyond the linear-response regime

$$T_{\text{eff}} = \left(1 - v^2\right)^{\frac{1}{7-p}} \left(1 + Cv^2\right)^{\frac{1}{2}} = T + \frac{1}{2}\left(C - \frac{2}{7-p}\right) v^2 T + O(v^4)$$

$$c_0 = \frac{4\pi}{7-p}, \quad C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p} n\right)$$

This factor can be negative!

$T_{\text{eff}} < T$ can be realized.

For example, for the test quark in N=4 SYM: [Gubser, 2008]

$$T_{\text{eff}} = T \big/ \sqrt{\gamma} < T \qquad \gamma = 1 \big/ \sqrt{1 - v^2}$$

The temperature seen by the fluctuation can be made smaller by driving the system into out of equilibrium.

For conductors

Beyond the linear response regime

$$T_{\text{eff}} = c_0^{-1} \frac{\left((c_0 T)^{\frac{14-2p}{5-p}} + CE^2 \right)^{1/2}}{\left((c_0 T)^{\frac{14-2p}{5-p}} + E^2 \right)^{\frac{1}{7-p}}} = T + \frac{1}{2}\left(C - \frac{2}{7-p} \right) \frac{E^2}{(c_0 T)^{\frac{14-2p}{5-p}}} T + O(E^4)$$

$$c_0 = \frac{4\pi}{7-p}, \quad C = \frac{1}{2}\left(q + 3 - p + \frac{p-3}{7-p} n \right)$$

This can be negative.
E.g. : D4-D2 system (p=4, q=2, n=0)

$T_{\text{eff}} < T$ can happen.

The temperature seen by fluctuations can be made smaller by driving the system into NESS.

Is T_{eff}<T allowed?

It is not forbidden.

Some examples of smaller effective temperature:

[K. Sasaki and S. Amari, J. Phys. Soc. Jpn. 74, 2226 (2005)]

[Also, private communication with S. Sasa]

Is it OK with the second law?

No contradiction.

- NESS is an open system.
- The second law of thermodynamics applies to a closed system.
- The definition of entropy in NESS (beyond the linear response regime) is not clear.

What is the physical meaning of T_{eff}?

Fluctuation of string ⟺ Fluctuation of external force acting on the test particle

Fluctuation of electro-magnetic fields on the D-brane ⟺ Fluctuation of current density

Computations of correlation functions of fluctuations in the gravity dual is governed by the ingoing-wave boundary condition at the effective horizon.

$$\int dt \langle \delta f(t) \delta f(0) \rangle \Big|_{v \neq 0} = 2T_{\text{eff}} \frac{\operatorname{Im} G^R(\omega)}{-\omega} \Bigg|_{\substack{\omega \to 0, \\ v \neq 0}}$$

fluctuation | dissipation

See also, [Gursoy et al.,2010]

The fluctuation-dissipation relation at NESS is characterized by the effective temperature (at least for our systems).

What is temperature?

Definitions of equilibrium temperature:

$$P \propto e^{-E/T}, \ t_E \approx t_E + 1/T$$ Distributions

$$dE = TdS$$ Thermodynamics

$$D = T\mu$$ Fluctuation-dissipation relation

diffusion const. mobility

We have another definition of temperature:

$$\xi^a \nabla_a \xi^b \Big|_{\text{Horizon}} = 2\pi T \xi^b \Big|_{\text{Horizon}}$$ Hawking temperature

Killing vector

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T}, \; t_E \approx t_E + 1/T$$

Distributions

$$dE = TdS$$

Thermodynamics

$$D = T_{\text{eff}} \frac{\partial f}{\partial v}$$

diffusion const. differential mobility

Fluctuation-dissipation relation

They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\text{Eff. Horizon}} = 2\pi T_{\text{eff}} \, \xi^b \Big|_{\text{Eff. Horizon}}$$

Killing vector

Hawking temperature

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T}, \quad t_E \approx t_E + 1/T$$

Distributions

$$dE = TdS$$

Thermodynamics

$$D = T_{\mathrm{eff}} \frac{\partial f}{\partial v}$$

diffusion const. differential mobility

Fluctuation-dissipation relation

They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\mathrm{Eff.\,Horizon}} = 2\pi T_{\mathrm{eff}}\, \xi^b \Big|_{\mathrm{Eff.\,Horizon}}$$

Killing vector

Hawking temperature

What is temperature?

Definitions of effective temperature:

$$P \propto e^{-E/T_{\rm eff}}, \; t_E \approx t_E + 1/T$$

Distributions of small fluctuations

$$dE = TdS$$

Thermodynamics

$$D = T_{\rm eff} \frac{\partial f}{\partial v}$$

diffusion const. differential mobility

Fluctuation-dissipation relation

They give the same temperature.

$$\xi^a \nabla_a \xi^b \Big|_{\rm Eff.\,Horizon} = 2\pi T_{\rm eff}\, \xi^b \Big|_{\rm Eff.\,Horizon}$$

Killing vector

Hawking temperature

Thermodynamics in NESS?

$$dE \overset{??}{=} T_{\mathrm{eff}}\, dS$$

It is highly nontrivial.

Hawking radiation (Hawking temperature) is more general than the thermodynamics of black hole.

Hawking radiation:

It occurs as far as the “Klein-Gordon equation” of fluctuation has the same form as that in the black hole.

Thermodynamics of black hole:

We need the Einstein’s equation. It relies on the theory of gravity.

Example of "non-gravity"
Hawking radiation

Sonic black hole in liquid helium.

Fast

Sonic horizon where the flow velocity exceeds the velocity of sound.

Slow

- The sound cannot escape from inside the "horizon".
- It is expected that the sonic horizon radiates a "Hawking radiation" of sound at the "Hawking temperature".

[W. G. Unrhu, PRL51(1981)1351]

However, any "thermodynamics" associated with the Hawking temperature of sound has not been established so far.

[See for example, M. Visser, gr-qc/9712016]

Summary

At least for some examples of NESS:

- There exists two temperatures in the non-linear regime.
- The effective temperature appears in terms of the Hawking temperature at the effective horizon.
- It agrees with the coefficient in the generalized fluctuation-dissipation relation in NESS.
- $T_{eff} < T$ can happen for some cases.

Some more hint for non-equilibrium physics?