Known and unknown Zeldovich: Simplest Paths to Complexity — from Flames to Large Scale Structure

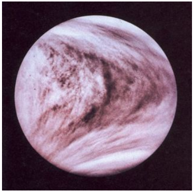
Blinnikov S.I.[1,2,3]

[1]ITEP, Moscow [2]MSU, NSU, Moscow, Novosibirsk [3]Kavli IPMU (WPI), Kashiwa

ACP Seminar, IPMU
22 May 2014

Yakov B. Zeldovich, 8 Mar 1914 - 2 Dec 1987

- From cosmology
- to circulation in Venerian atmosphere



Contribution of Ya.B. to physics and astrophysics

- hydrodynamics and chemistry of explosive phenomena
- nuclear chain reactions, picnonuclear reactions
- ultra-cold neutrons storage
- predictions of light isotopes
- muon catalysis in nuclear fusion
- conserved vector current (CVC) in weak interactions
- cosmological limit on neutrino mass
- Harrison-Zeldovich spectrum of primarily perturbations
- Zeldovich approximation for the growth of perturbations
- Sunyaev-Zeldovich effect
- particle creation in gravitational field

Less known contributions

Shakura, Shakura & Sunyaev on disk accretion

Bisnovatyi-Kogan & Blinnikov (1976) on accretion disk corona

Many other examples later in this talk

My work with Zeldovich: a book on Stellar Physics

Физические основы строения и эволюции звезд

Я. Б. Зельдович, С. И. Блинников, Н. И. Шакура

МГУ

1981

Ja. B. Zeldovics–Sz. I. Blinnyikov–Ny. I. Sakura

A csillagszerkezet és csillagfejlődés fizikai alapjai

Gondolat · Budapest, 1988

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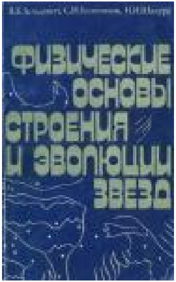
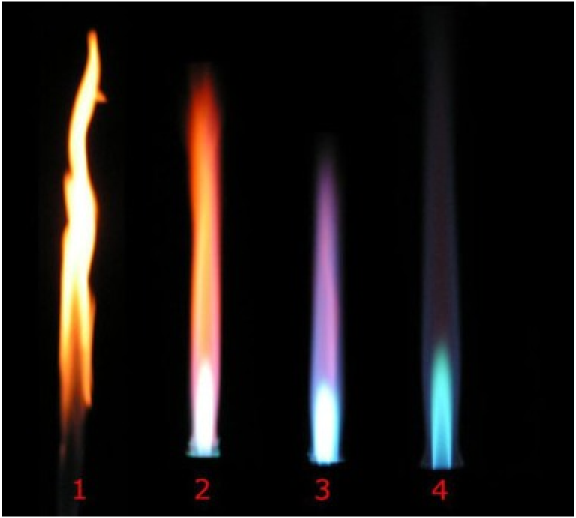


Gondolat · Budapest, 1988

His favourite problem: Flame’s different faces



Bunsenbrenner

Flame's different faces

A candle

Flame’s different faces

A candle in microgravity

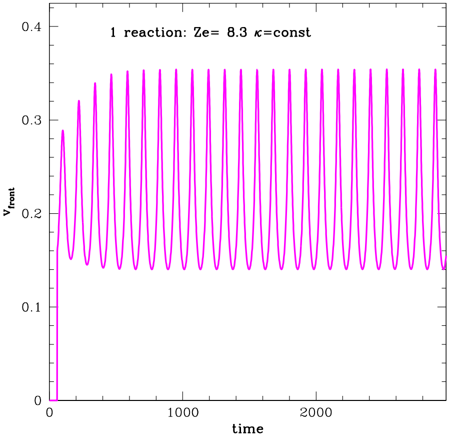
Buncefield, UK, 5 Dec 2005, oil storage depot

Ya.B. favourite problem – combustion theory

— Flame speed Zel'dovich-Frank-Kamenetsky
— ZND detonation
— Ze number
— Causal (thermal conductivity + diffusion) and spontaneous modes of flame propagation (+ Kolmogorov,Petrovsky,Piskunov KPP-1937)
— Nonlinear stabilization of Landau-Darrieus instability
— A paper about the nature of knocks in engines, on Spontaneous front, and with Aldushin,Hudyaev 1979 on KPP,
— Supervizing experiments on flame acceleration (Gostintsev et al. 1988)

Stable burning of powder: Katyusha

Pulsating front velocity in a solid propellant



Found by YaB Zeldovich (ZhETF 1942) for powder combustion.
A critical number for Arrhenius law $\mathrm{Ze_{cr}} \approx \mathbf{8.2}$, the instability sets in at $\mathrm{Ze} > \mathrm{Ze_{cr}}$

Zeldovich number

Dependence of heating ε on T:

$$\text{Ze} = \frac{\partial \log \varepsilon}{\partial \log T} \simeq \frac{E_a}{\mathcal{R}T} \quad \textbf{for Arrhenius law} \quad \varepsilon \propto \exp\left(\frac{-E_a}{\mathcal{R}T}\right)$$

– this is the Zeldovich number. Usually $\text{Ze} \gg 1$. In other words, the heating

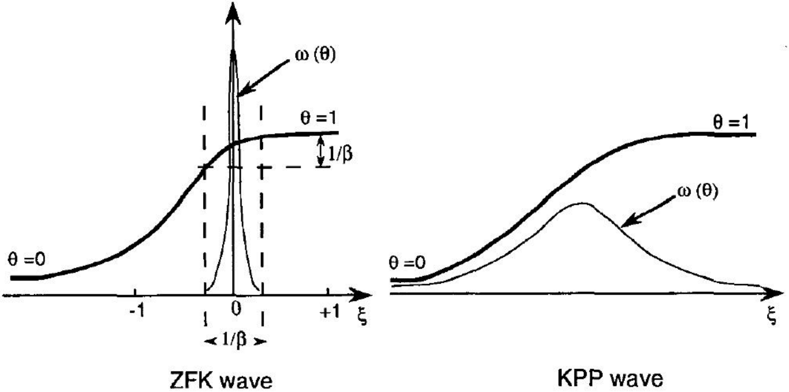
$$\varepsilon \propto T^{\text{Ze}}$$

in a narrow range of T.
For thermonuclear burning:

$$\text{Ze} = \frac{\partial \log \varepsilon}{\partial \log T} \simeq \frac{\alpha_G^{1/3}}{3T^{1/3}}.$$

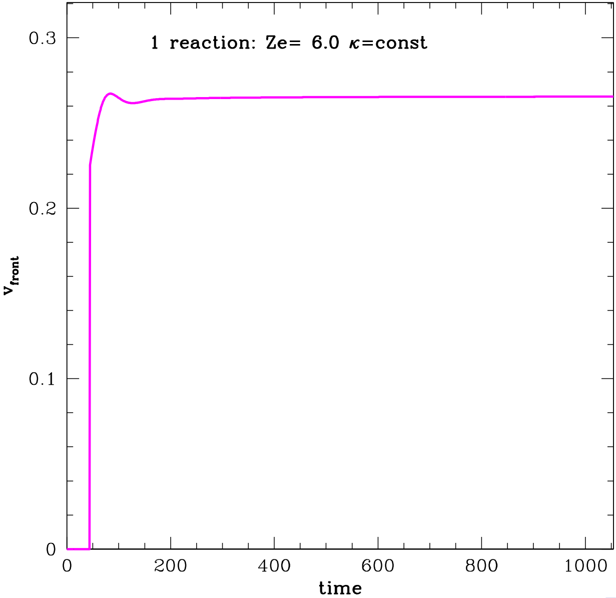
Many authors write β in place of Ze.

Zeldovich number (here β) in flame structure

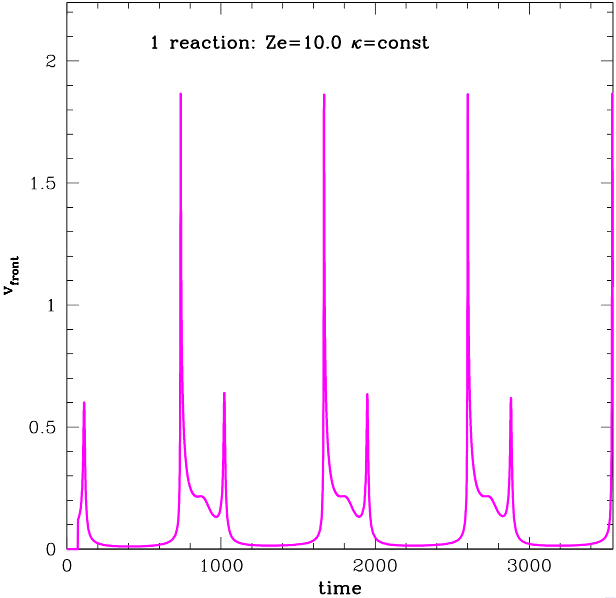


Effective Zeldovich number in a multi-reaction network is measured by the thickness of the reaction zone

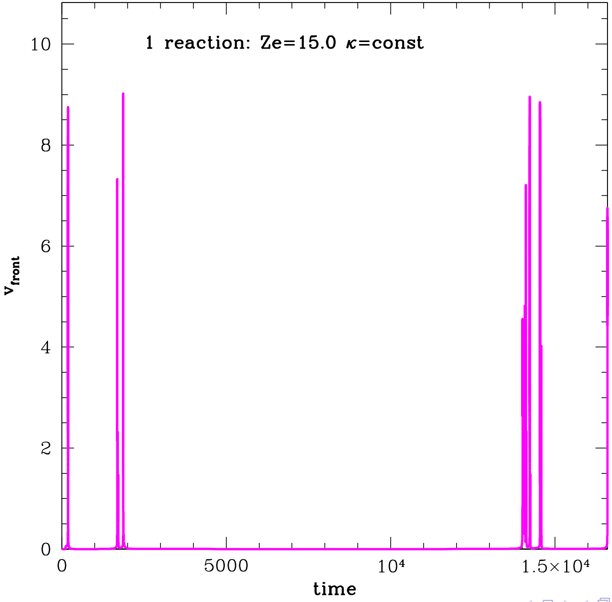
Front velocity in a solid propellant, stable, Ze=6



Unstable, periodic, Ze=10



Unstable, chaotic, Ze=15, cf Bayliss,Matkowsky 1990



Annu. Rev. Fluid Mech. 1994. 26 : 321–52

PREMIXED COMBUSTION AND GASDYNAMICS

P. Clavin

Combustion science is a fascinating field of nonequilibrium phenomena in complex systems. Although combustion processes are very diverse in nature, nevertheless, they have two fundamental characteristics: heat release by excess energy of chemical bonds and a strong temperature dependence of the chemical reaction rate. The basic laws of combustion may be obtained analytically by taking this thermal nonlinearity to its extreme limit. Yakov Borisovich Zeldovich is at the origin of this remarkable asymptotic method and one is readily convinced by reading Volume I of his selected works (Ostriker 1992), that nobody has contributed more than him to the understanding of combustion theory.

ZFK: Zel'dovich–Frank-Kamenetsky theory

Zel'dovich and Frank-Kamenetsky (1938) obtained flame speed in the chemical combustion - ignition due to heat conduction.
Laminar flame speed in the ZFK-wave:

$$S_\mathrm{L} \approx \mathrm{Ze}^{-1} \left(\frac{v_T l_T}{\tau_\mathrm{reac}(T_b)} \right)^{1/2} ,$$

where

$$\tau_\mathrm{reac}(T) \propto \exp \frac{E_a}{\mathcal{R}T} .$$

Thermonuclear burning in SNe Ia,

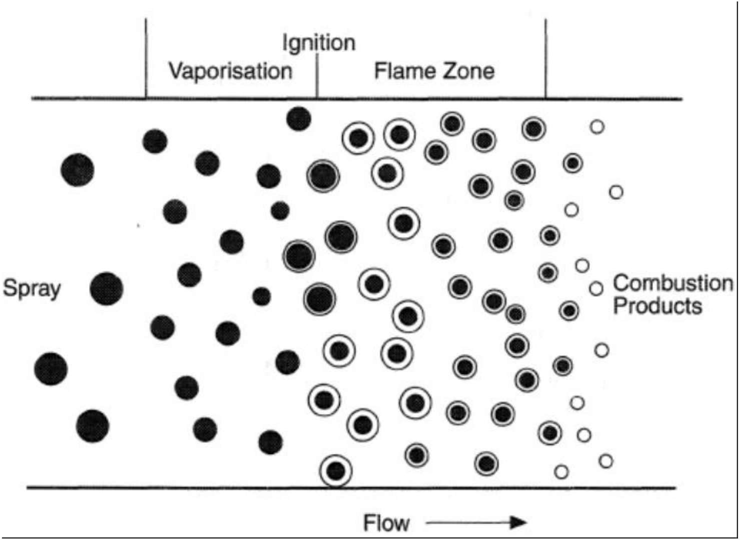
$$\tau_\mathrm{reac}(T) \propto \exp \frac{\alpha_G^{1/3}}{3T^{1/3}} .$$

Ya.B.Zeldovich, D.A.Frank-Kamenetsky, A.D.Sakharov

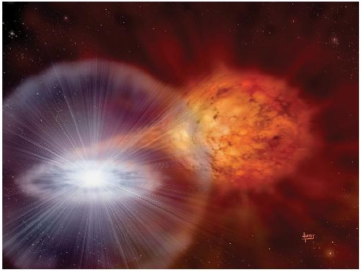
1946 or 1947

Sarov, 1950-ties

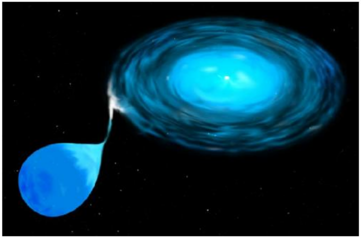
Flame structure



The three main models



Accretion model (Hydrogen)

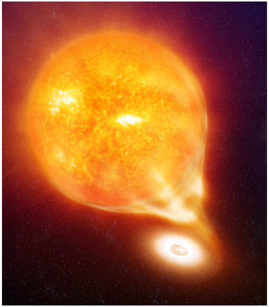


Accretion model (Helium)

2 WD merger model

Supernovae of type Ia

- Thermonuclear explosions of white dwarfs
- May be important for DE studies



Flame propagation

- In SD scenario the flame (deflagration) is born near the WD center
- While expanding the flame must accelerate and produce detonation HOW?

Supernovae of type Ia

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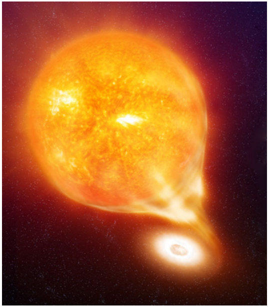

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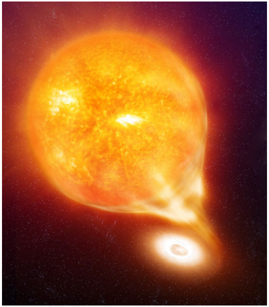

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Hydrodynamics of burning, solved by Ya.B. with a slide-rule

mass: $$\partial_t \rho + \partial_i (\rho \mathrm{v}_i) = 0,$$

chemistry: $$\partial_t (\rho X_j) + \partial_i (\rho \mathrm{v}_i X_j) = \rho R_j,$$

momentum: $$\partial_t (\rho \mathrm{v}_i) + \partial_j (\rho \mathrm{v}_i \mathrm{v}_j + p\delta_{ij}) = \partial_j \tau_{ij} + \rho f_i,$$

energy: $$\partial_t (\rho e) + \partial_i (\rho e \mathrm{v}_i) + p \partial_i \mathrm{v}_i = \partial_i (\kappa \partial_i T) + \rho \dot{S} + \tau_{ij} \partial_i \mathrm{v}_j.$$

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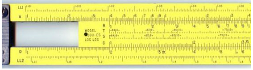

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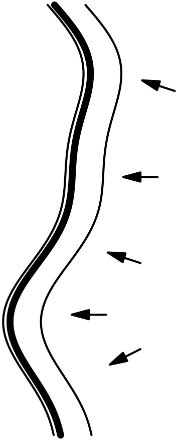
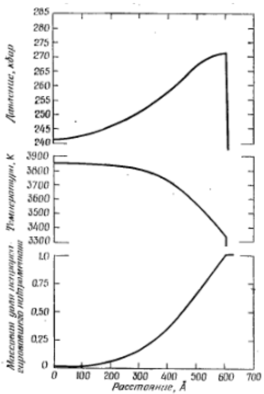
Steady modes of burning propagation

Deflagration



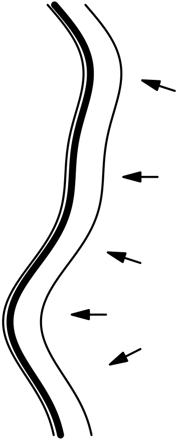
Detonation



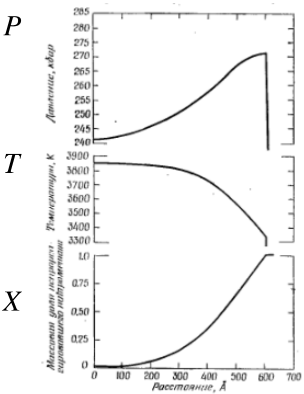
Mader (1985)

Steady modes of burning propagation

Deflagration

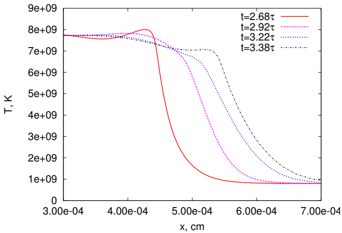


Detonation

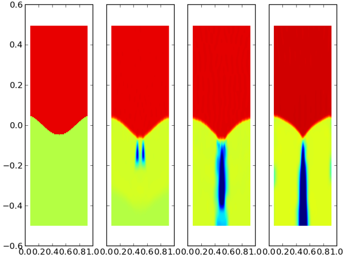


Mader (1985)

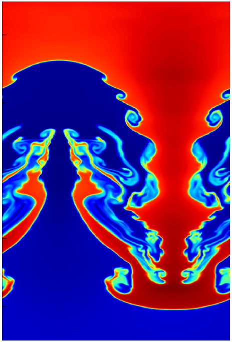
Flame instabilities



Thermopulsational



Landau–Darrieus



Rayleigh–Taylor–Landau

Flame instabilities

- Pulsational (1D)
  Bychkov, Liberman, 1995
  Blinnikov, Glazyrin, Dolgov, MNRAS 2013
- Landau–Darrieus (2D, 3D)
  Röpke et al. 2004, Bell et al. 2004
- Interaction with turbulence
  Schmidt et al. 2010
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Flame scale: δ_{flame} ↓ R_{WD}

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Flame scale:

δ_{flame}

$\downarrow$

R_{WD}

Flame structure and 1D pulsations of flames

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MNRAS **433**, 2840–2849 (2013)
doi:10.1093/mnras/stt909
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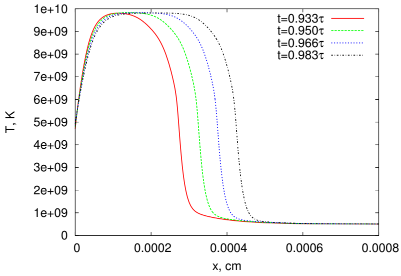
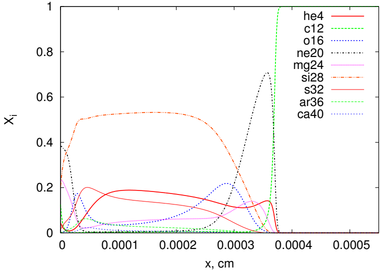
Flame fronts in Type Ia supernovae and their pulsational stability

S. I. Glazyrin,[1]★ S. I. Blinnikov[1,2,3]★ and A. Dolgov[1,2,4]★

[1] *Institute for Theoretical and Experimental Physics, Moscow 117218, Russia*
[2] *Novosibirsk State University, Novosibirsk 630090, Russia*
[3] *VNIIA, Moscow 127055, Russia*
[4] *University of Ferrara and INFN, I-44100 Ferrara, Italy*

Deflagration





Conditions at ignition:

$\rho \sim 10^9$ g/cm^3

$T \sim 10^8 - 10^{10}$ K

$\mathrm{v}_{\mathrm{flame}} \sim 100$ km/s

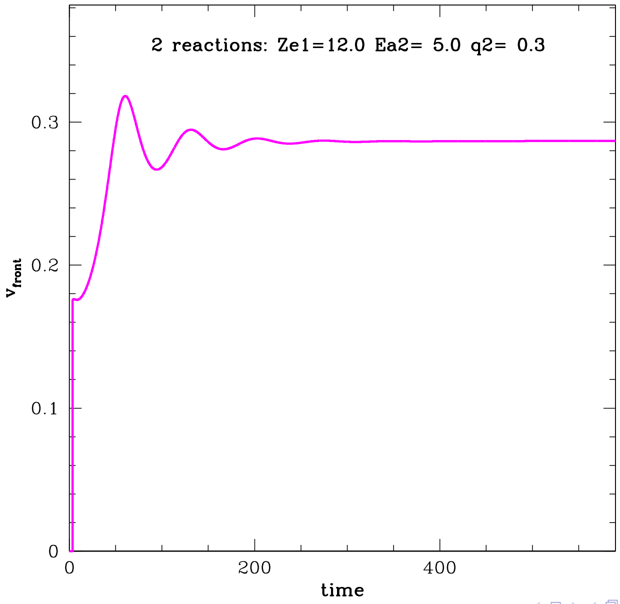
$\delta_{\mathrm{flame}} \sim 10^{-4}$ cm

$c_s \sim 10000$ km/s

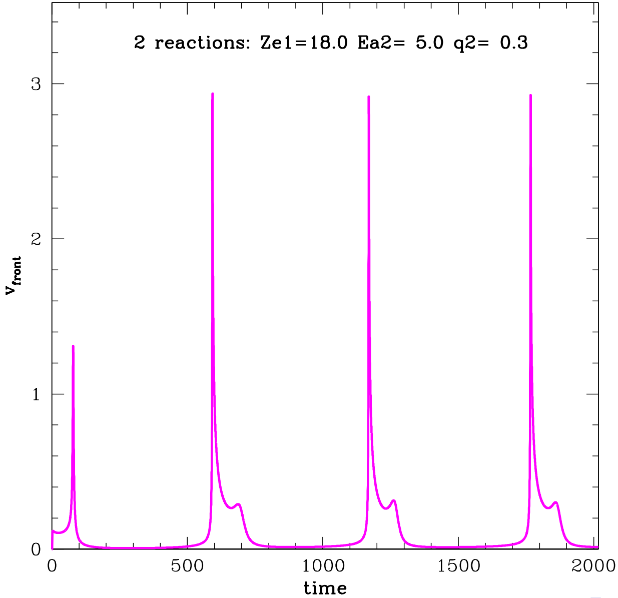
$R_{\mathrm{WD}} \sim 1000$ km

The flame is highly unstable in principle, but the pulsational 1D instability is not observed in simulations!

Stabilization by a second reaction with lower E_a



Front velocity in a solid propellant



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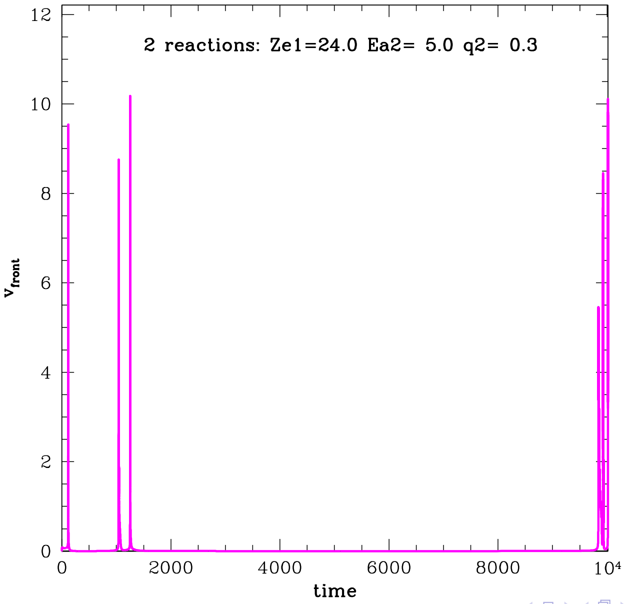


Thermopulsational instability

Bychkov, Liberman (1995) find $\mathrm{Ze}_{\mathrm{cr}} \approx 17$

Larger than in solids!

Cf. Nomoto et al. (1995)

Glazyrin, Blinnikov, Dolgov MNRAS (2013) explain why it is not observed in SNIa simulations.

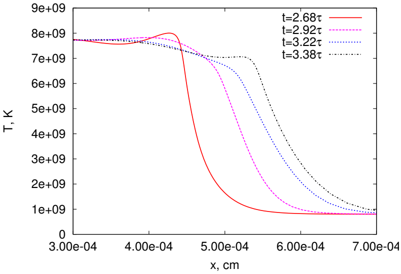
For Lewis number

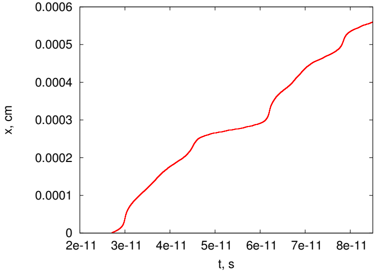
$$\mathrm{Le} \propto \varkappa / D \sim 10^4 \gg 1$$

the criterion for instability is Zeldovich number:

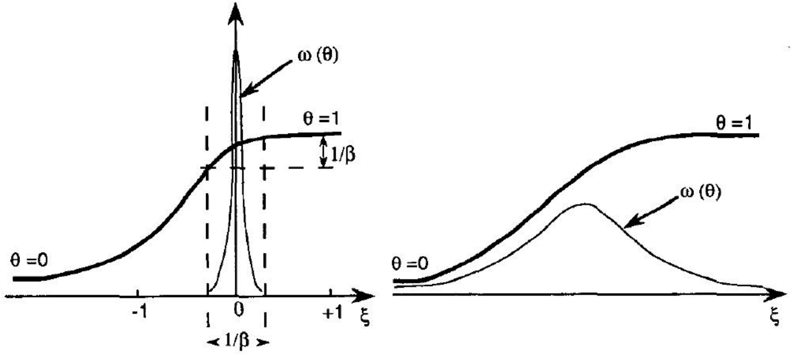
$$\mathsf{Ze} > \mathrm{Ze}_{\mathrm{cr}}$$

ρ, g/cm^3	$\mathrm{Ze}_{\mathrm{cr}}$
2×10^8	$18.4 < \mathsf{Ze} < 21.4$
7×10^8	$15.3 < \mathsf{Ze} < 16.7$
2×10^9	$13.7 < \mathsf{Ze} < 14.1$





Zeldovich number (here β) in flame structure



ZFK wave

KPP wave

Lower effective Zeldovich number in a relistic reaction network (measured by the thickness of the reaction zone) explains the TP-stability.

FRONT3D

FRONT3D is an open-source cartesian three-dimensional parallel code, under active development. It contains the following physics:

- Hydrodynamics (options: Kurganov–Tadmor, WENO, MUSCL–Hancock, MUSCL)
- MHD
- Premixed chemical (or nuclear) reactions
- Level–set method for burning flames
- Thermoconductivity
- Turbulence in k-epsilon model

and is designed primarily for the study of astrophysical phenomena, especially nuclear flames in SNIa explosions.
Download: **http://dau.itep.ru/sn/front3d**

Landau–Darrieus instability

- Hydrodynamic instability of a thin front
- Linear stage: Landau L. D., 1944; Darrieus unpublished

$$\omega = k u_n \frac{\mu}{1+\mu}\left(\sqrt{1+\mu-\frac{1}{\mu}}-1\right) \quad \text{where} \quad \mu = \frac{\rho_1}{\rho_2}$$

- Nonlinear stage: Zeldovich, 1966

$$\frac{dx}{dt} = \omega_{\mathrm{LD}} x - \frac{2}{\pi^2} k^2 u_n x^2$$

Stabilization in cusps

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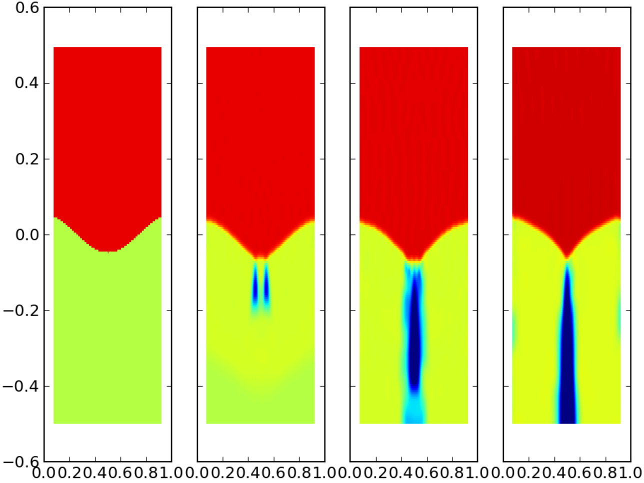
$$\frac{dx}{dt} = \omega_{\mathrm{LD}} x - \boxed{\frac{2}{\pi^2} k^2 u_n x^2}$$

Stabilization in cusps

Density: Flame goes up, red means larger values

Cf. Röpke et al. 2004. Here are FRONT3D results by S.Glazyrin.



Nonlinear stabilization of LD

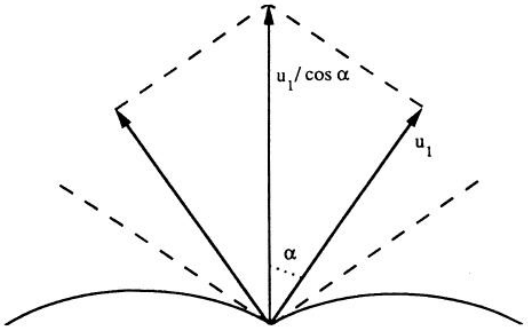


Fig. 1. The nonlinear stabilizing mechanism of the flame front instability

Zeldovich picture for a cusp velocity from 1D flame speed u_1
(source: Bychkov & Liberman, AA 1995)

ShZ: Reviews of Modern Physics, Vol. 61, No. 2, 1989

The large-scale structure of the universe: Turbulence, intermittency, structures in a self-gravitating medium

S. F. Shandarin

Institute for Physical Problems, Academy of Sciences of the USSR, 117 334 Moscow, Union of Soviet Socialist Republics
and Max-Planck-Institut für Physik und Astrophysik, Institut für Astrophysik, 8046 Garching bei München, Federal Republic of Germany

Ya. B. Zeldovich*

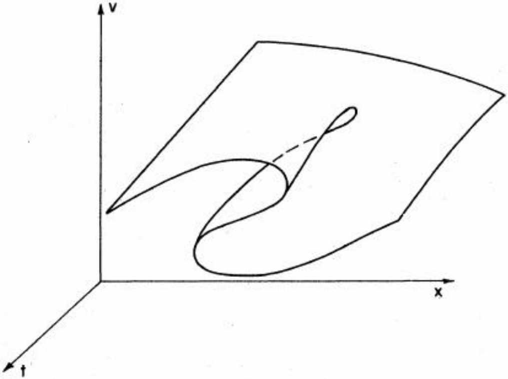
Institute for Physical Problems, Academy of Sciences of the USSR, 117 334 Moscow, Union of Soviet Socialist Republics

$$\mathbf{x}(t,\mathbf{q})=\mathbf{q}+t\cdot\mathbf{v}(\mathbf{q})\ , \tag{1.1}$$

ShZ: "Pancake" theory

It turns out that, at a rather late stage of the expansion of the universe (which will be defined later), the matter in the universe can be described as cold dust moving under the action of gravity alone. At this stage, the motion of each particle can be approximately described by a simple law (Zeldovich, 1970),

$$\mathbf{r}(t,\mathbf{q}) = a(t)[\mathbf{q} - b(t)\mathbf{s}(\mathbf{q})] \,, \tag{1.2}$$



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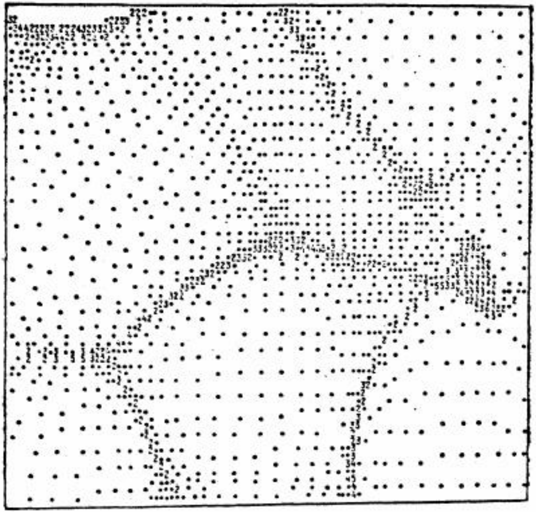


ShZ: Percolation

Our ideas about network structure have come from the impression that the dark regions (voids) are generally separated from one another, despite the fact that the total area occupied by them is greater than that of the bright regions (filaments). In contrast, the bright regions form a connected cellular structure.

...

Let us discuss this question in detail. It turns out that a good statistics characterizing the topological properties of random fields is **percolation** statistics (Zeldovich, 1982a, 1983; Shandarin, 1983b; Shandarin and Zeldovich, 1983, 1984).

ShZ: Analogy with light in refractive media

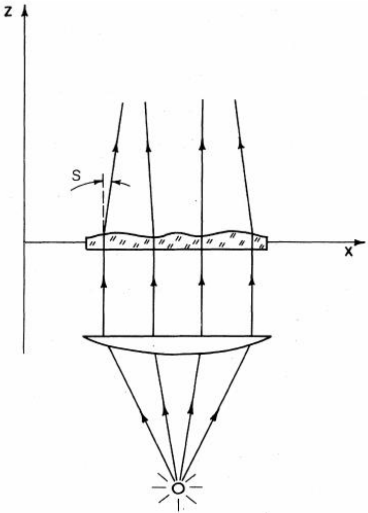


FIG. 10. The scheme of an optical experiment simulating the formation of the cellular structure in 2D.

ShZ: cellular structure of light caustics

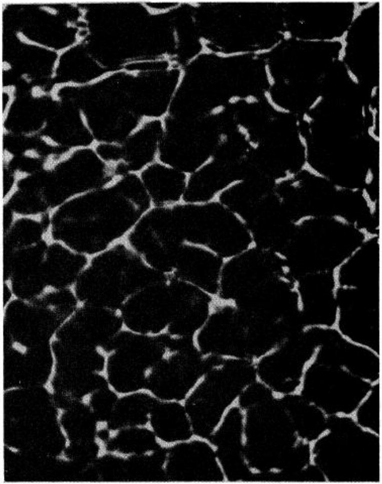
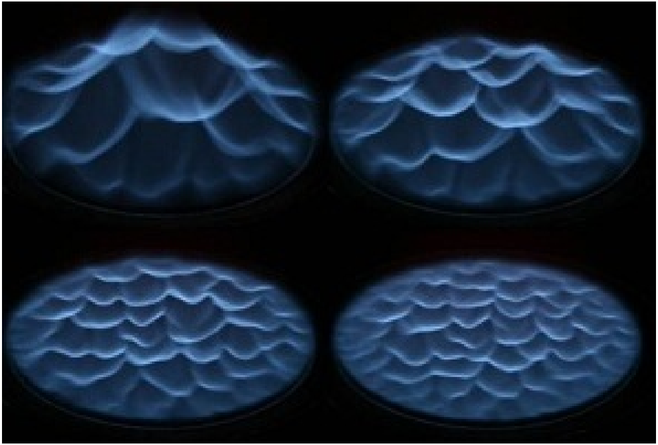


FIG. 11. Distribution of brightness on the screen in the optical experiment shown in Fig. 10.

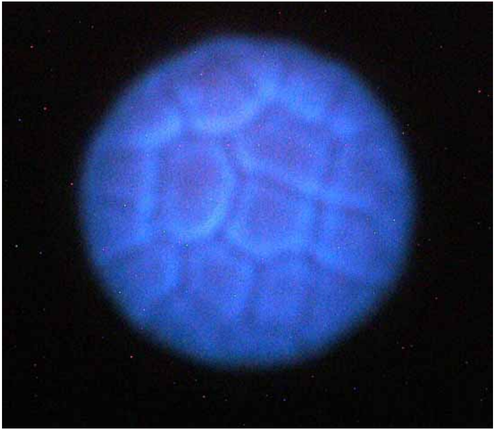
Cellular structure of flames – Ya.B. favourite



(Observation of Cellular Flames Formed on a Flat Burner)

Cellular flame observations

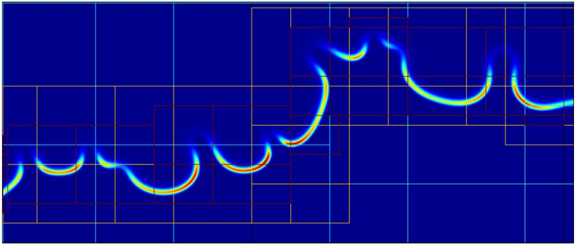
長岡技術科学大学・門脇/山崎研究室
Research Group of Kadowaki/Yamazaki, Nagaoka University of Technology, Japan



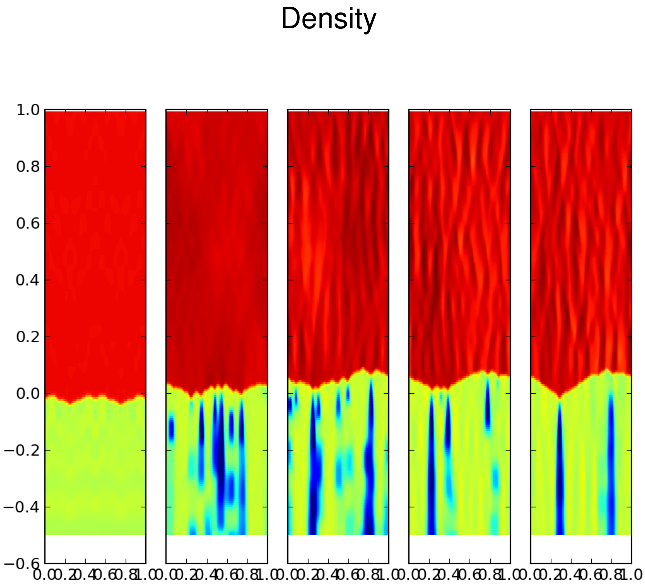
Cellular flame simulations

Berkeley Lab

Center for Computational Sciences and Engineering

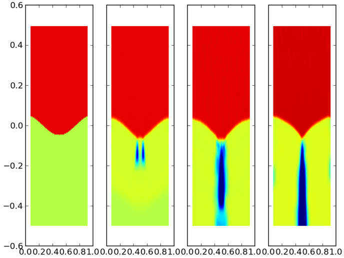


Back to LD. Vortical inflow: many cusps

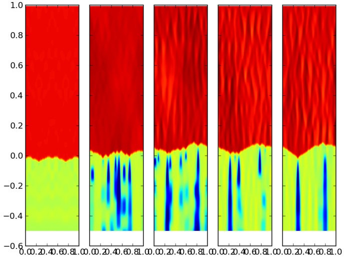


Landau–Darrieus Instability

- This is a hydrodynamical instability of a thin front
$$\omega = k u_n \frac{\mu}{1+\mu}\left(\sqrt{1+\mu-\frac{1}{\mu}}-1\right)$$
$$\mu \equiv \rho_u/\rho_b$$



- In a "channel" the perturbations merge into one cusp (cf Röpke'ea 2004) $\Rightarrow$ the flame is accelerated by only $3-4\%$

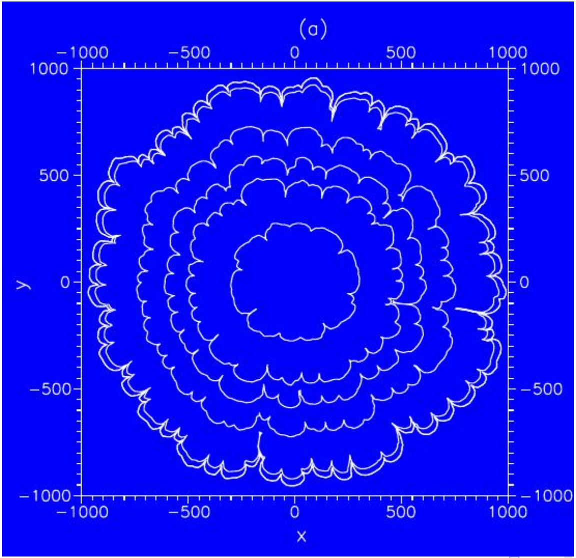


- But the flame in a supernova is expanding!

Spherical flame

- "Channel" bounds the size of perturbations.
- For a spherical flame the bound $\sim R_{\rm flame}$ grows with time.
- Fractalization of flames.

Fractalization of spherical flames, Blinnikov & Sasorov 1996



Deeper analogy of flames and cosmological perturbations

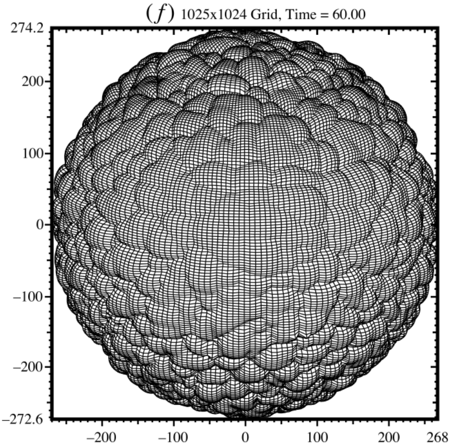
The above picture of fractalization is obtained with a so-called Frankel equation.

It is a potential approximation for the hydrodynamic flow.

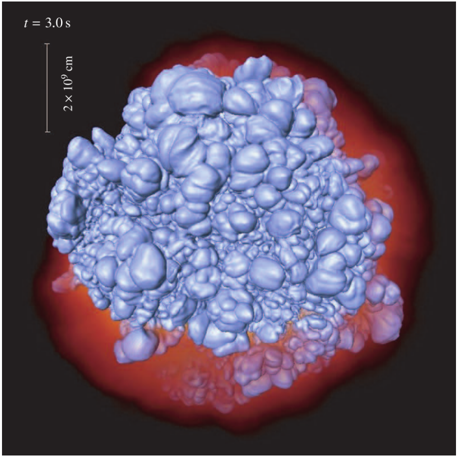
The simulations are done by a version of an N-body algorithm for gravitating particles!

Results of other groups

Röpke et al. 2007



D'Angelo et al. 2000



Experiments: Gostintsev et al. 1988

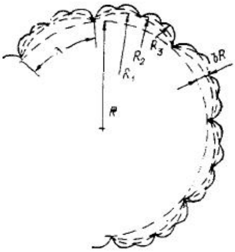


Fig. 3. Diagram of successive perturbations accompanying the development of an instability of the spherical flame front: R) radius of the unperturbed front; R_1, R_2, and R_3) radii of curvature for perturbations of the first, second, and third orders; bR) amplitude of the first-order perturbations; λ) wavelength.

Supervized by Ya.B.Zeldovich

Transition to detonation

E.g.,

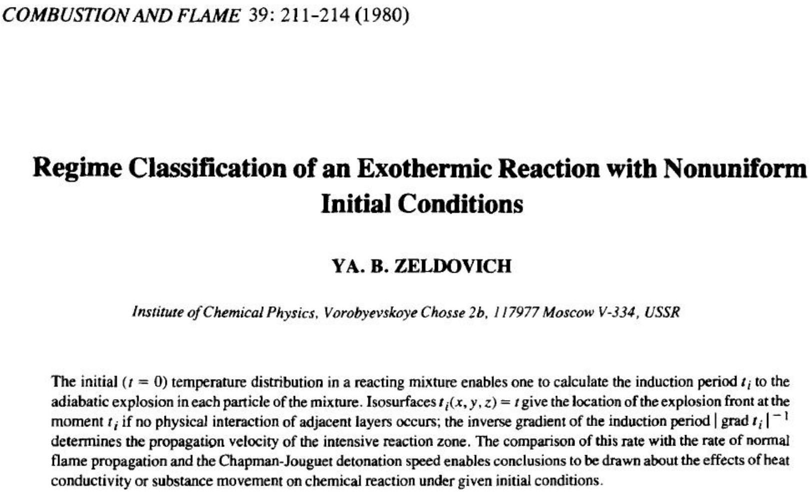
- A volume explosion ahead of the propagating flame

- Spontaneous regime transition (Zeldovich)

$$\mathrm{v}_{\mathrm{spont}} = \left(\frac{d\tau_b}{dx} \right)^{-1}$$

See e.g. Ciaraldi-Schoolmann, Seitenzahl, Röpke 2013

Spontaneous front


COMBUSTION AND FLAME 39: 211–214 (1980)

Regime Classification of an Exothermic Reaction with Nonuniform Initial Conditions

YA. B. ZELDOVICH

Institute of Chemical Physics, Vorobyevskoye Chosse 2b, 117977 Moscow V-334, USSR

The initial ($t = 0$) temperature distribution in a reacting mixture enables one to calculate the induction period t_i to the adiabatic explosion in each particle of the mixture. Isosurfaces $t_i(x, y, z) = t$ give the location of the explosion front at the moment t_i if no physical interaction of adjacent layers occurs; the inverse gradient of the induction period $|\mathrm{grad}\, t_i|^{-1}$ determines the propagation velocity of the intensive reaction zone. The comparison of this rate with the rate of normal flame propagation and the Chapman-Jouguet detonation speed enables conclusions to be drawn about the effects of heat conductivity or substance movement on chemical reaction under given initial conditions.

Zeldovich gradient mechanism

Stage of spontaneous flame propagation in supernovae

S. I. Blinnikov and A. M. Khokhlov

Institute of Theoretical and Experimental Physics, Moscow
Astronomy Council, Academy of Sciences, USSR, Moscow

(Submitted April 2, 1987)

Pis'ma Astron. Zh. **13**, 868–878 (October 1987)

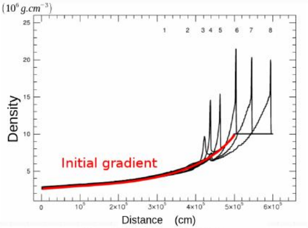
The stage of spontaneous flame propagation in a degenerate carbon–oxygen stellar core is considered. An estimate is obtained for the critical temperature of thermal instability development. The structure and evolution of the spontaneous combustion front are studied for an isentropic temperature profile. At the end of spontaneous combustion, the pressure, density, and velocity of matter behind the front are equal to the corresponding Chapman–Jouguet detonation values.

- Rely on an induction time (τ_i) gradient :
  - τ_i is the time needed to burn half of the carbon
  - A spontaneous combustion wave propagates from short τ_i to long τ_i

- If the gradient is sonic :

$$\nabla \tau_i = \frac{1}{Cs}$$

Overpressure accumulates at the wave front

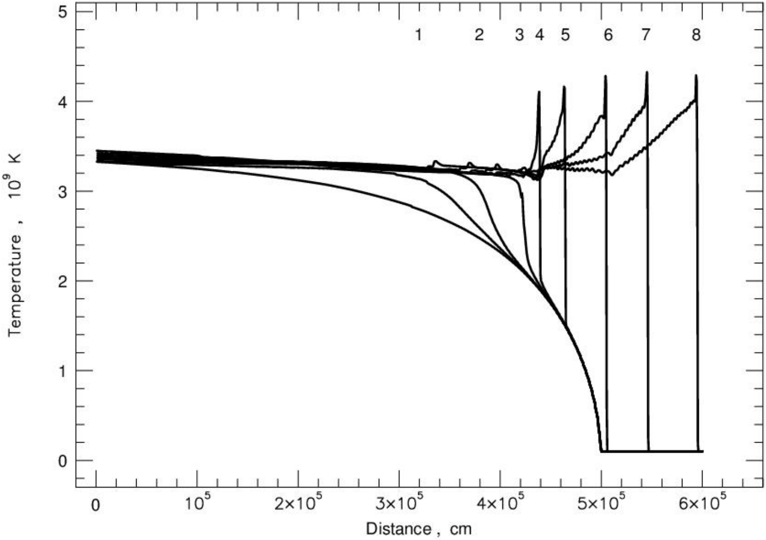


Khokhlov & al 1997

If the gradient is sonic and large enough

P_{CJ} can be reached $\Rightarrow$ self-sustained detonation

Gradient in temperature matters

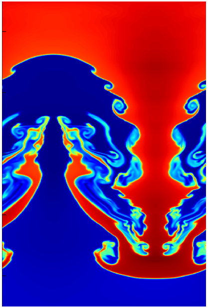


Rayleigh–Taylor–Landau Instability

- Necessary conidtion

$$\nabla \rho \cdot \boldsymbol{g} < 0$$

- Leads to turbulization of the flow



$v_n = 0$

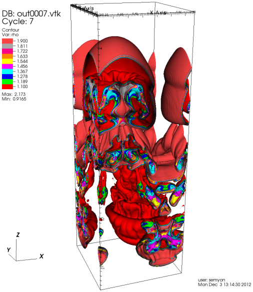
Dispersion relation at the linear stage

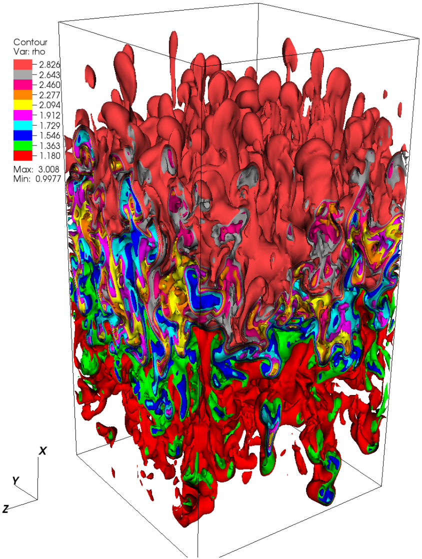
$$\omega = k u_n \frac{\mu}{1+\mu} \left(\sqrt{1 + \mu - \frac{1}{\mu} - \frac{\mu^2 - 1}{\mu^2} \frac{g}{k u_n^2}} - 1 \right)$$

– **see Landau-Lifshitz “Hydrodynamics”, it is NOT RT!**
Nonlinear stage: mixed zone of the width is formed

$$\Delta z \propto g t^2$$

RT mixing (without burning) FRONT3D results





Models for turbulence

- Reynolds averaged (RANS)
- Large eddy simulation (LES)
- ...

Reynolds averaging:

$$\overline{A}(\mathbf{x},t) = \frac{1}{T}\int\limits_{-T/2}^{T/2} A(\mathbf{x},t+t_1)dt_1,$$

$$A \equiv \overline{A} + A', \quad A \equiv \tilde{A} + A'', \quad \tilde{A} \equiv \frac{\overline{\rho A}}{\overline{\rho}}$$

One of the closure ways: k-ϵ model

New dynamical quantities

$$\bar{\rho}k \equiv \frac{\overline{\rho \mathrm{v}''^2}}{2}, \quad \bar{\rho}\epsilon \equiv \overline{\tau'_{ij}\partial_j \mathrm{v}'_i}$$

turbulent time scale, turbulent viscosity:

$$t_t = \frac{k}{\epsilon}, \qquad D = c_D \frac{k^2}{\epsilon}$$

“Gradient approximation” (Belen’kiy, Fradkin L(Lebedev)PI reports (1965)):

$$\overline{\mathrm{v}''_i A'} \sim -D\partial_i A$$

For Reynolds tensor

$$R_{ij} = \overline{\rho \mathrm{v}''_i \mathrm{v}''_j} = -\rho D \left(\partial_i \mathrm{v}_j + \partial_j \mathrm{v}_i - \frac{2}{3}\delta_{ij}\partial_k \mathrm{v}_k \right) + \frac{2}{3}\rho k \delta_{ij}$$

k-ϵ model

$$\partial_t \rho + \partial_i(\rho \mathrm{v}_i) = 0,$$
$$\partial_t(\rho \mathrm{v}_i) + \partial_j\left(\rho \mathrm{v}_i \mathrm{v}_j + p\delta_{ij}\right) = -\partial_j R_{ij},$$
$$\partial_t E + \partial_i(\mathrm{v}_i(E+p)) = -G_2 + \rho\epsilon + \partial_i(p a_i - Q_i^T),$$

$$\partial_t(\rho k) + \partial_i(\rho k \mathrm{v}_i) = G_1 + G_2 - \rho\epsilon + \partial_i(\rho c_k D \partial_i k),$$
$$\partial_t(\rho\epsilon) + \partial_i(\rho\epsilon \mathrm{v}_i) = \frac{\epsilon}{k}(c_{\epsilon 1} G_1 + c_{\epsilon 2} G_2 - c_{\epsilon 3}\rho\epsilon) + \partial_i(\rho c_\epsilon D \partial_i \epsilon),$$

$$R_{ij} = -\rho D\left(\partial_i \mathrm{v}_j + \partial_j \mathrm{v}_i - \frac{2}{3}\delta_{ij}\partial_k \mathrm{v}_k\right) + \frac{2}{3}\rho k \delta_{ij},$$
$$E = \rho e + \frac{\rho \mathrm{v}^2}{2}, \quad D = c_D \frac{k^2}{\epsilon}, \quad a_i = -c_\alpha D \frac{\partial_i \rho}{\rho},$$
$$G_1 = -R_{ij}\partial_i \mathrm{v}_j, \quad G_2 = a_i \partial_i p, \quad Q_i^T = -c_e \rho D \partial_i e.$$

k-ϵ model

$$\partial_t\rho + \partial_i(\rho \mathrm{v}_i) = 0,$$

$$\partial_t(\rho \mathrm{v}_i) + \partial_j\left(\rho \mathrm{v}_i \mathrm{v}_j + p\delta_{ij}\right) =$$

$$\partial_t E + \partial_i(\mathrm{v}_i(E+p)) = -G_2 + \rho\epsilon + \partial_i(pa_i - Q_i),$$

$G_2 = \rho^{-1}\overline{\rho' \mathrm{v}'_i}\partial_i p$

generation due to RTL

$$\partial_t(\rho k) + \partial_i(\rho k \mathrm{v}_i) = G_1 + \boxed{G_2} - \rho\epsilon + \partial_i(\rho c_k D \partial_i k),$$

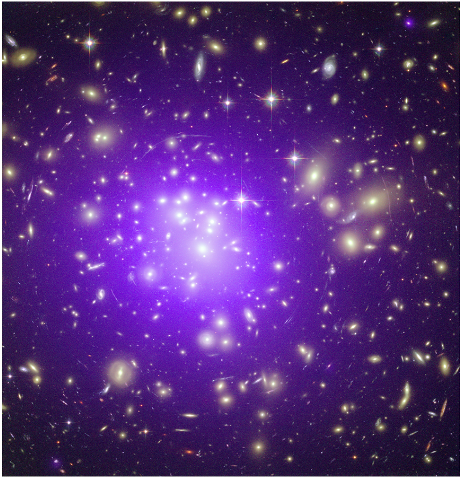
$$\partial_t(\rho\epsilon) + \partial_i(\rho\epsilon \mathrm{v}_i) = \frac{\epsilon}{k}\left(c_{\epsilon 1}G_1 + c_{\epsilon 2}G_2 - c_{\epsilon 3}\rho\epsilon\right) + \partial_i(\rho c_\epsilon D \partial_i \epsilon),$$

$$R_{ij} = -\rho D\left(\partial_i \mathrm{v}_j + \partial_j \mathrm{v}_i - \frac{2}{3}\delta_{ij}\partial_k \mathrm{v}_k\right) + \frac{2}{3}\rho k \delta_{ij},$$

$$E = \rho e + \frac{\rho \mathrm{v}^2}{2}, \quad D = c_D\frac{k^2}{\epsilon}, \quad a_i = -c_\alpha D\frac{\partial_i \rho}{\rho},$$

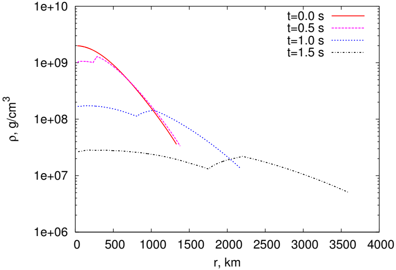
$$G_1 = -R_{ij}\partial_i \mathrm{v}_j, \quad G_2 = a_i \partial_i p, \quad Q_i^T = -c_e \rho D \partial_i e.$$

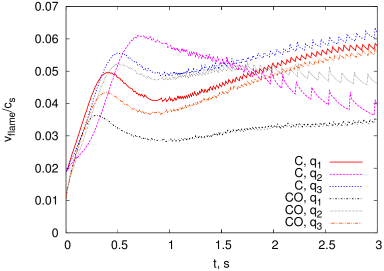
Turbulence models are important for understanding clusters of galaxies



The cluster Abell 1689. Credit: X-ray: NASA/CXC/MIT/E.-H Peng et al; Optical: NASA/STScI

1D burning of a WD with turbulence





- k-ϵ model

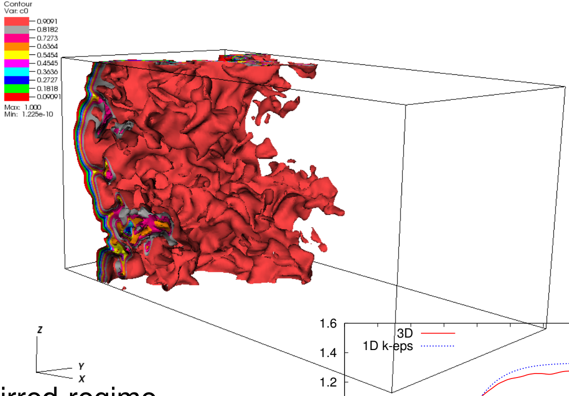
  (constants – Guzhova et al.VANT TPF (2005))

- flamelet regime velocity taken from Yakhot Comb. Sci. Tech (1988), should be checked with Pocheau (1994), as Schmidt'ea (2006) did

- $\rho_c = 2 \times 10^9$ g/cm^3, caloricity $q = (5.6 - 9.2) \times 10^{17}$ ergs/g

- "Helmholtz EOS"

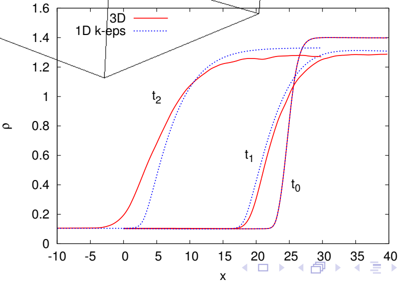
Recent FRONT3D results



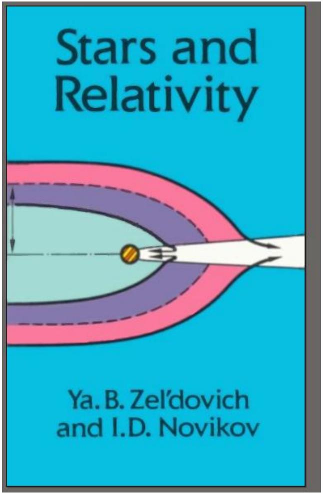
Stirred-regime,

Ka>1, the model works?

3D model at $t \approx t_2$



Zeldovich (1962)



Metastability of any star

10.13 THE METASTABILITY OF EVERY EQUILIBRIUM STATE

Let us digress somewhat from the collapse problem and make two remarks on problems of principle.

For stars composed of a cold Fermi gas and having a number of nucleons N less than the OV maximum, N_{max}, there always exist one or several static equilibrium configurations. Among these configurations is one with minimum total energy. It is stable with respect to small perturbations. Does that mean that the nucleons cannot be regrouped (without changing their number)! so that the resulting configuration (certainly nonstatic) would have a total energy (and consequently a mass M) smaller than the initial one? We will demonstrate below that such a conclusion would be incorrect; the minimum of energy corresponding to the stationary state is actually only a local minimum (Zel'dovich 1962*b*).

Cf. modern limits on boson Asymmetric Dark Matter (ADM) cross-sections.

Paradoksov (1967)

SOVIET PHYSICS USPEKHI VOLUME 9, NUMBER 4 JANUARY-FEBRUARY 1967

530.145

Methodological Notes

HOW QUANTUM MECHANICS HELPS US UNDERSTAND CLASSICAL MECHANICS

P. PARADOKSOV

Usp. Fiz. Nauk 89, 707-709 (August, 1966)

Paradoksov==Zeldovich: harmonic oscillator

But where is the promised nontrivial assistance rendered by quantum mechanics to classical mechanics? A simple example is found in adiabatic invariance. Let us consider the classical oscillator

$$m\ddot{x} = -k^2 x,$$

where the elastic constant k varies slowly with time [$k = k(t)$]. This can be the case of a heavy pendulum bob attached to a suspension of slowly varying length. For any given value of k the frequency is $\omega = k/\sqrt{m}$. In quantum theory the energy assumes the values $n\hbar\omega$, where the states are labeled with the integers n.*

Quantum mechanics helps to understand classical adiabatic invariants

Obviously, a slowly varying system in the n-th quantum state remains in the n-th state; the number n does not change. This is the statement of adiabatic invariance in quantum mechanics, and holds true for $n = 0$ or $n = 1$ in the case of a molecule, or for $n = 10^{30}$ (which is the order of magnitude of n for a clock pendulum). Clearly, for a slow adiabatic change of k, independently of the value of $\hbar$ or the specific value of n, we have

$$E = \text{const} \cdot \omega.$$

The proof of this equation is more complicated in classical mechanics. For any potential that is more

Non-adiabaticity means changing N, hence creation of particles! – Zeldovich's path to particle creation in gravitational fields.

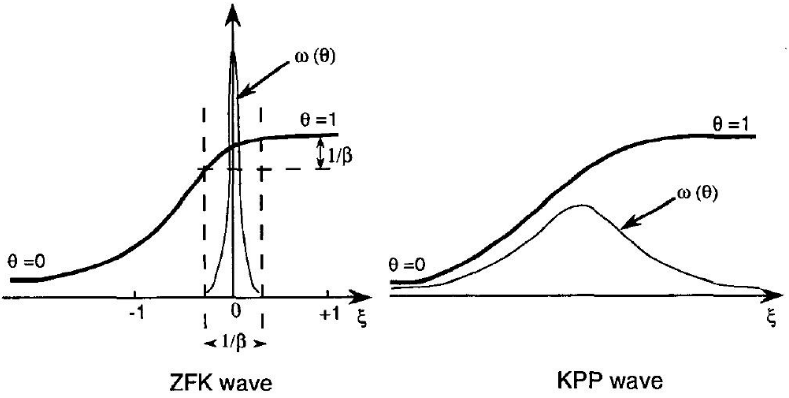
KPP: Kolmogorov, Petrovsky, Piskunov

ИССЛЕДОВАНИЕ УРАВНЕНИЯ ДИФФУЗИИ, СОЕДИНЕННОЙ С ВОЗРАСТАНИЕМ КОЛИЧЕСТВА ВЕЩЕСТВА, И ЕГО ПРИМЕНЕНИЕ К ОДНОЙ БИОЛОГИЧЕСКОЙ ПРОБЛЕМЕ [1]

А. Н. Колмогоров, И. Г. Петровский, Н. С. Пискунов

Бюл. МГУ (1937, сер. А, № 6)

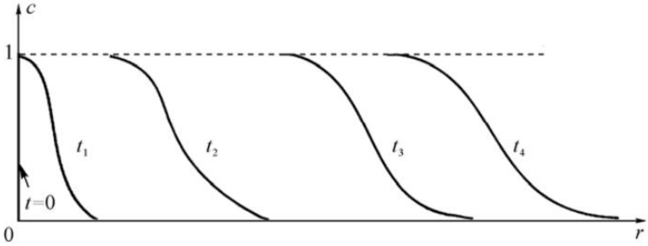
Comparing ZFK and KPP



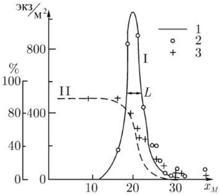
Clavin, Liñan 1984: ZFK at Ze $\gtrsim 3$.

KPP & Zeldovich equation in population kinetics

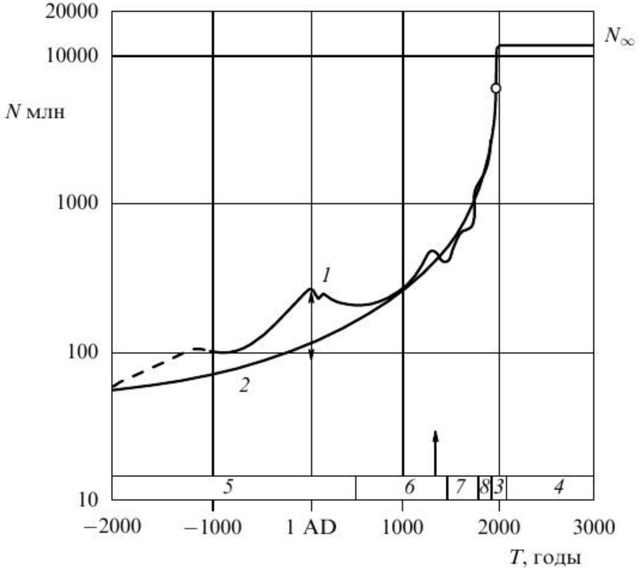
KPP wave and Zygogramma suturalis



Used to fight a weed “Ambrosia”



Planet Earth population (S.P.Kapitza 2010)



Conclusions

Ya.B. Zeldovich has found perhaps simplest paths to explanation of many complex phenomena.

All his work shows that the world on all scales is ruled by universal laws of physics.