Black Hole dynamics at large D

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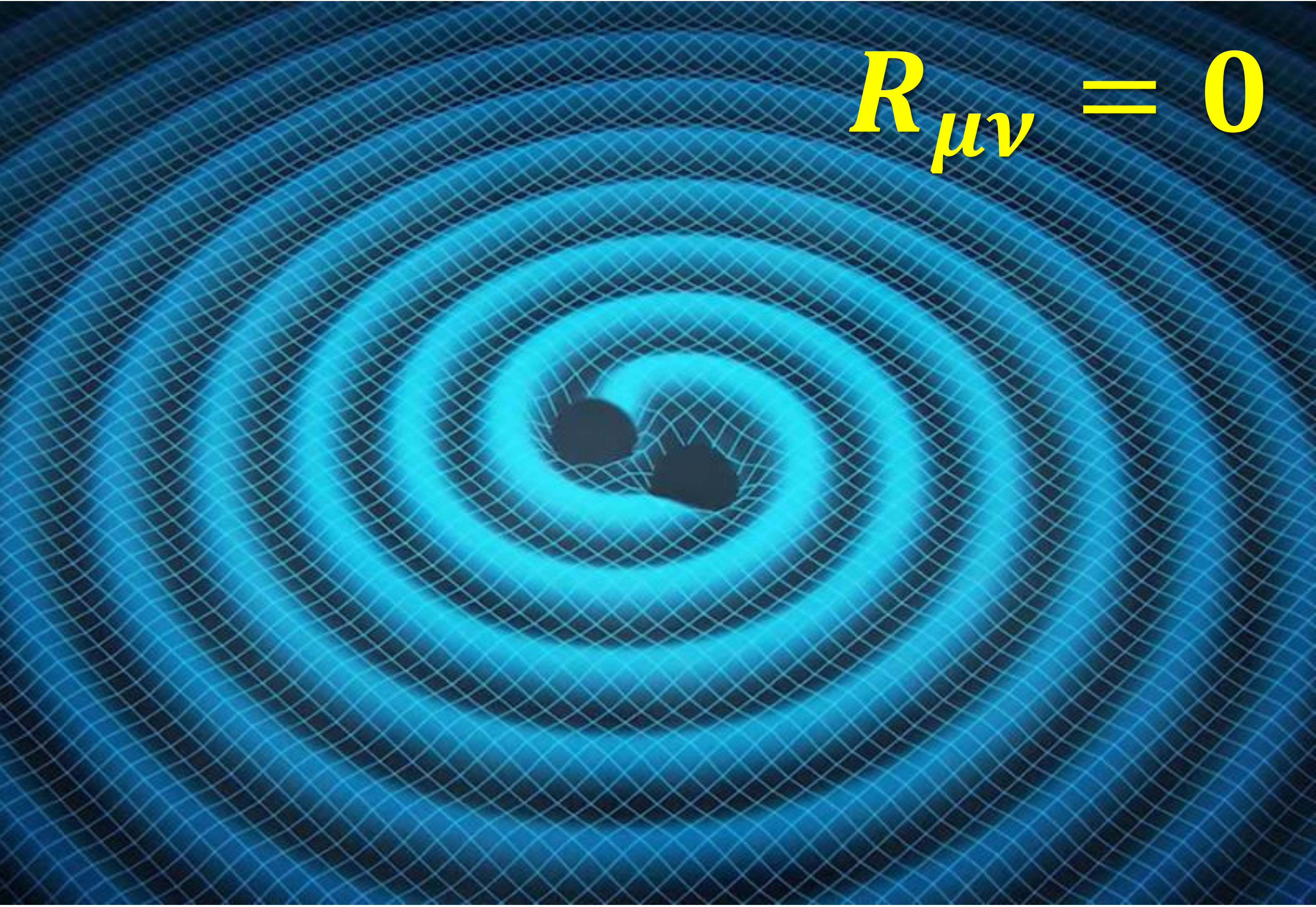
Daniel Grumiller
Takahiro Tanaka, Tetsuya Shiromizu

Nov 1915

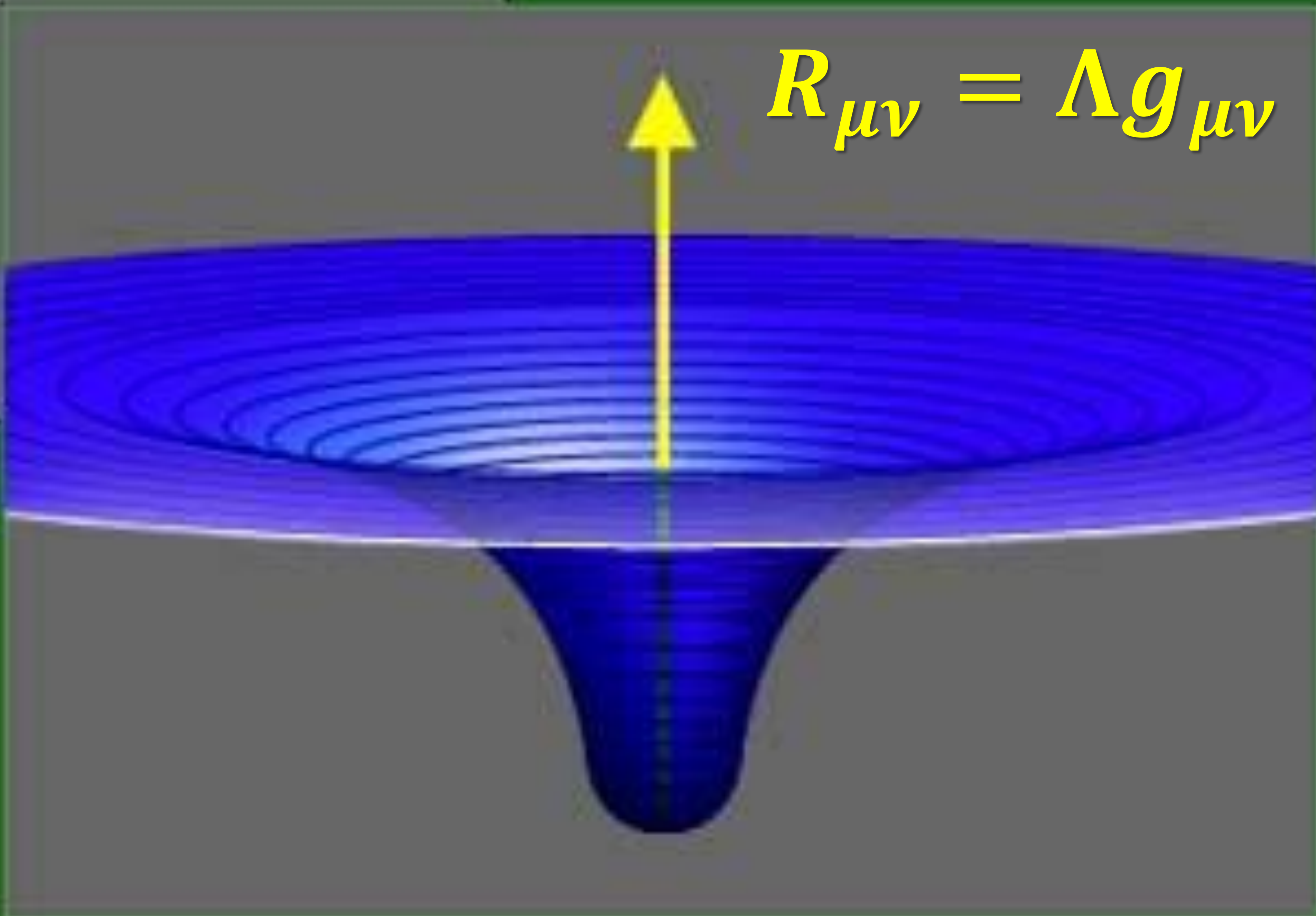
$$\boldsymbol{R_{\mu\nu} = 0}$$

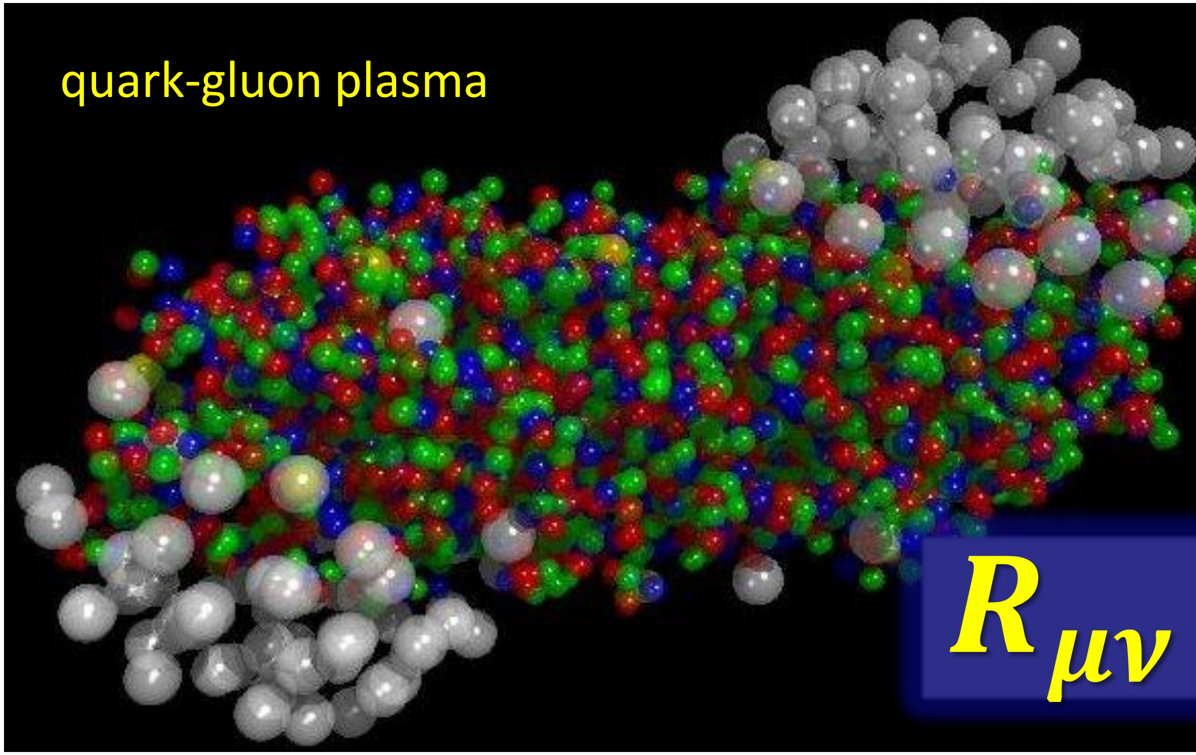
Feb 1917

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

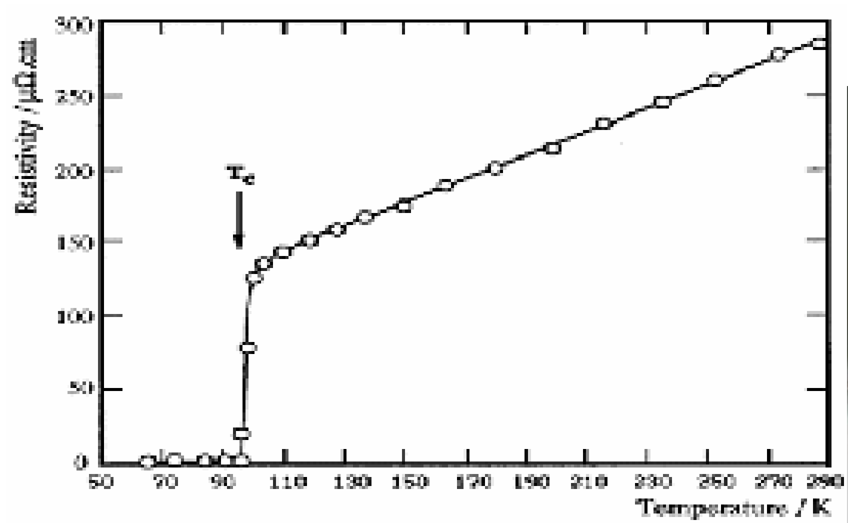

$R_{\mu\nu} = 0$

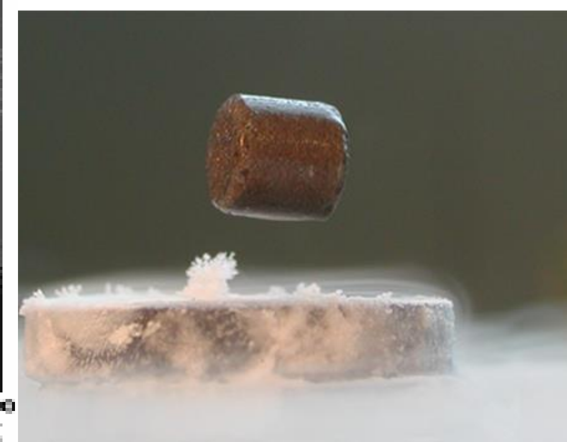
$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$





$$R_{\mu\nu} = -\Lambda g_{\mu\nu}$$





Even the simplest case

$$R_{\mu\nu} = 0$$

are very hard to solve

A small parameter can take you a long way

Quantum ElectroDynamics

Perturb around $e^2 = 0$

Quantum GluoDynamics

SU(3) Yang-Mills theory

No parameter?

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

parameter!

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

Well-defined for all N

Many problems can be formulated keeping N arbitrary

$\rightarrow$ N = continuous parameter

$\rightarrow$ expand in 1/N

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

Large N

keeps essential physics of N=3

- confinement
- asymptotic freedom

simplifies the theory

- reformulation in terms of string variables?

What parameter in

$R_{\mu\nu} = 0$?

What parameter in

$$R_{\mu\nu} = 0$$

$$\mu, \nu = 0, \ldots, 3$$?

$$R_{\mu\nu} = 0$$

$$\mu, \nu = 0, \ldots, \boldsymbol{D} - 1$$

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

Well-defined for all N

Many problems can be formulated keeping N arbitrary

$\rightarrow$ N = continuous parameter

$\rightarrow$ expand in 1/N

Classical General Relativity

D-diml Einstein's theory

Well-defined for all D

Many problems can be formulated keeping D arbitrary

→ D = continuous parameter

→ expand in 1/D

Quantum GluoDynamics

SU(**N**) Yang-Mills theory

Large N

- keeps essential physics of N=3
  - confinement
  - asymptotic freedom
- simplifies the theory
  - reformulation in terms of string variables?

Classical General Relativity

D-diml Einstein's theory

Large D

keeps essential physics of D=4

∃ black holes

∃ gravitational waves

simplifies the theory

reformulation in terms of string variables??

Shouldn't we take this analogy further?

YM: SU(N) local gauge group

GR: SO(D-1,1) local Lorentz group

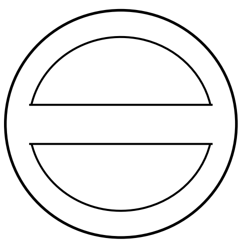
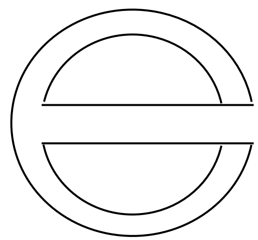
Strominger 1981

Bjerrum-Bohr 2004

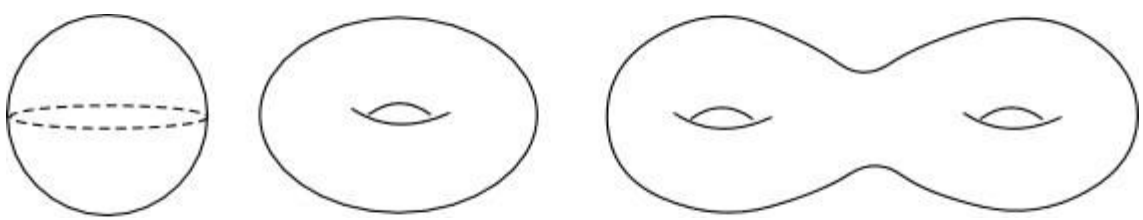
YM: SU(**N**) local gauge group

Large N: # gluon polarizations grows

Topological expansion of Feynman diagrams

 $\sim N^2$  $\sim N^0$ ...

Gluons arrange into worldsheets → **strings**!



Quantum GR: SO(**D**-1,1) local Lorentz group

Large D: # graviton polarizations grows

Topological expansion of Feynman diagrams?

Quantum GR: SO(**D**-1,1) local Lorentz group

Large D: # graviton polarizations grows

Topological expansion of Feynman diagrams?

Alas, no!

No arrangement into string worldsheets

Worse:

Large D $\rightarrow$ UV behavior infinitely bad

YM $\quad \leftrightsquigarrow \quad$ Quantum GR

$SU(N \to \infty)$ $\quad\quad$ $SO(D \to \infty, 1)$

Classical General Relativity

D-diml Einstein's theory

Well-defined for all D

Understand this theory first

Maybe later go back to quantum theory

Kol+Miyamoto et al

How do we take

$D \to \infty$

in

$R_{\mu\nu} = 0$?

Regard $R_{\mu\nu} = 0$ as a theory of

Black Holes

interacting with/via

gravitational waves

Black Hole
dynamics at large D

K Schwarzschild to A Einstein
(letter dated 22 December 1915)

"I made at once by good luck a search for a full solution. A not too difficult calculation gave the following result:"

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{r_0}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

In *D* dimensions

Tangherlini 1963

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right)dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

scale r_0

determines the length scale

of *all* bh dynamics

Large *D* black holes

r_0 not the only scale

Small *parameter* $1/D \Longrightarrow$ scale hierarchy

$$r_0/D \ll r_0$$

Localization of interactions

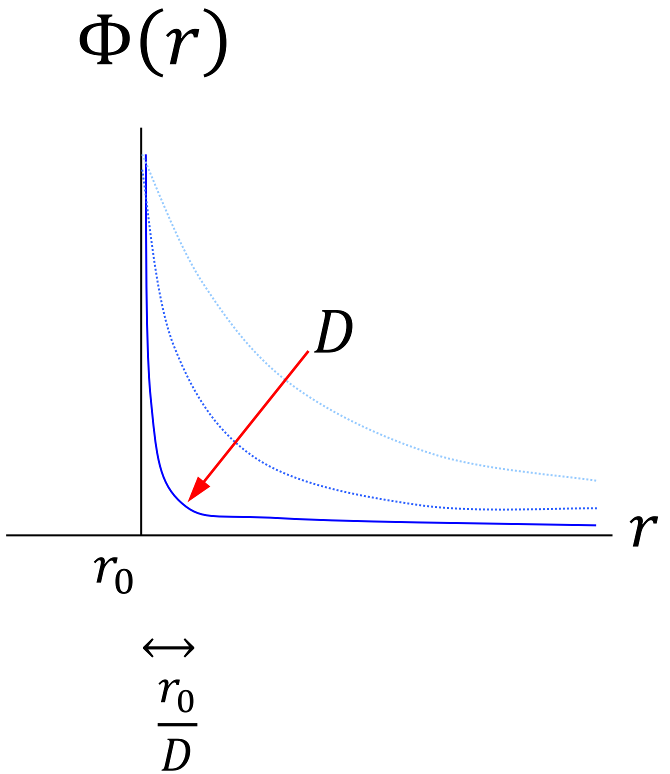
Large potential gradient:

$$\Phi(r) \sim \left(\frac{r_0}{r}\right)^{D-3}$$

$$\nabla\Phi\Big|_{r_0} \sim D/r_0$$

$\Longrightarrow$ Hierarchy of scales

$$\frac{r_0}{D} \ll r_0$$



Fixed $r > r_0$ $\quad D \to \infty$

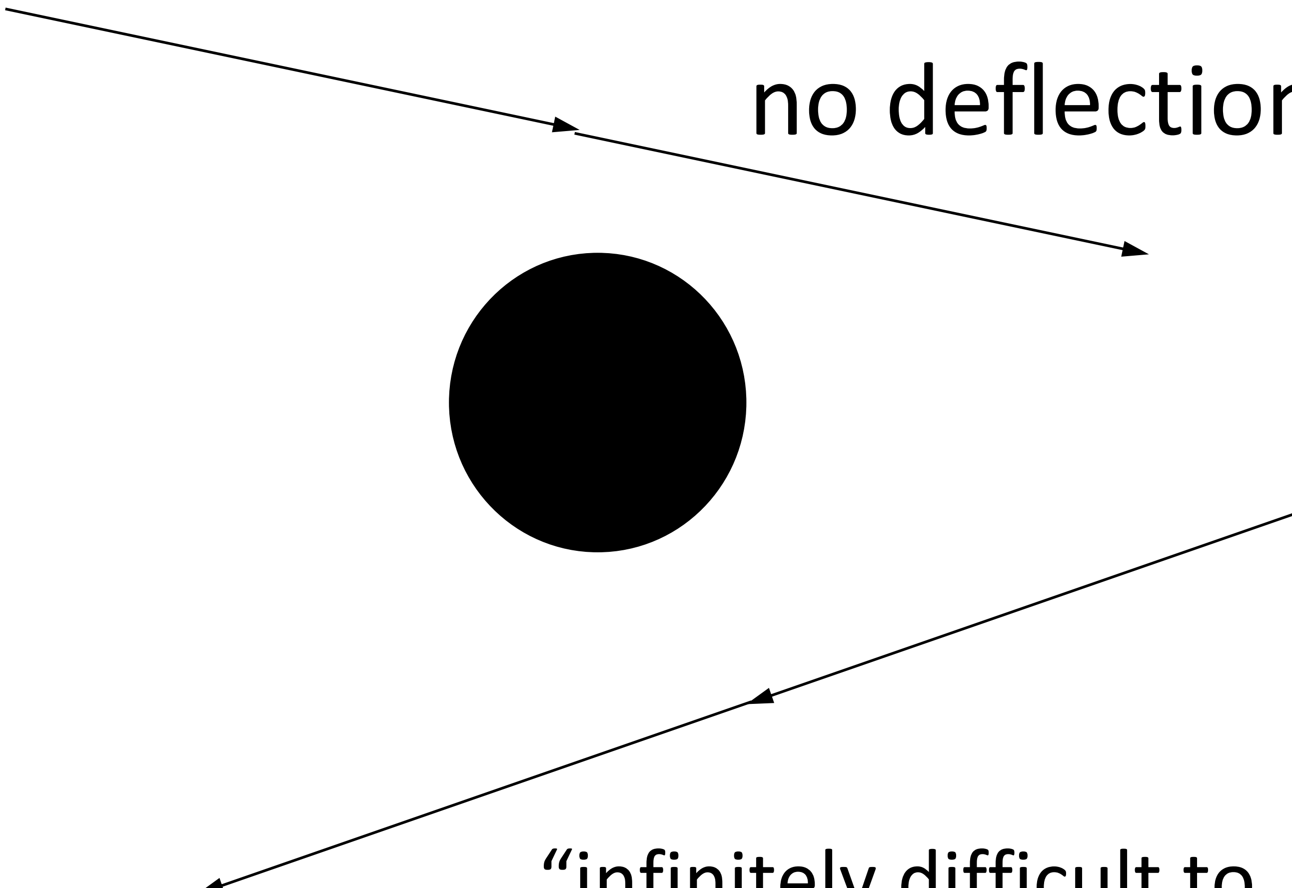
$$1 - \left(\frac{r_0}{r}\right)^{D-3} \to 1$$

$$ds^2 \to -dt^2 + dr^2 + r^2 d\Omega_{D-2}$$

Flat, empty space at $r > r_0$

no gravitational field

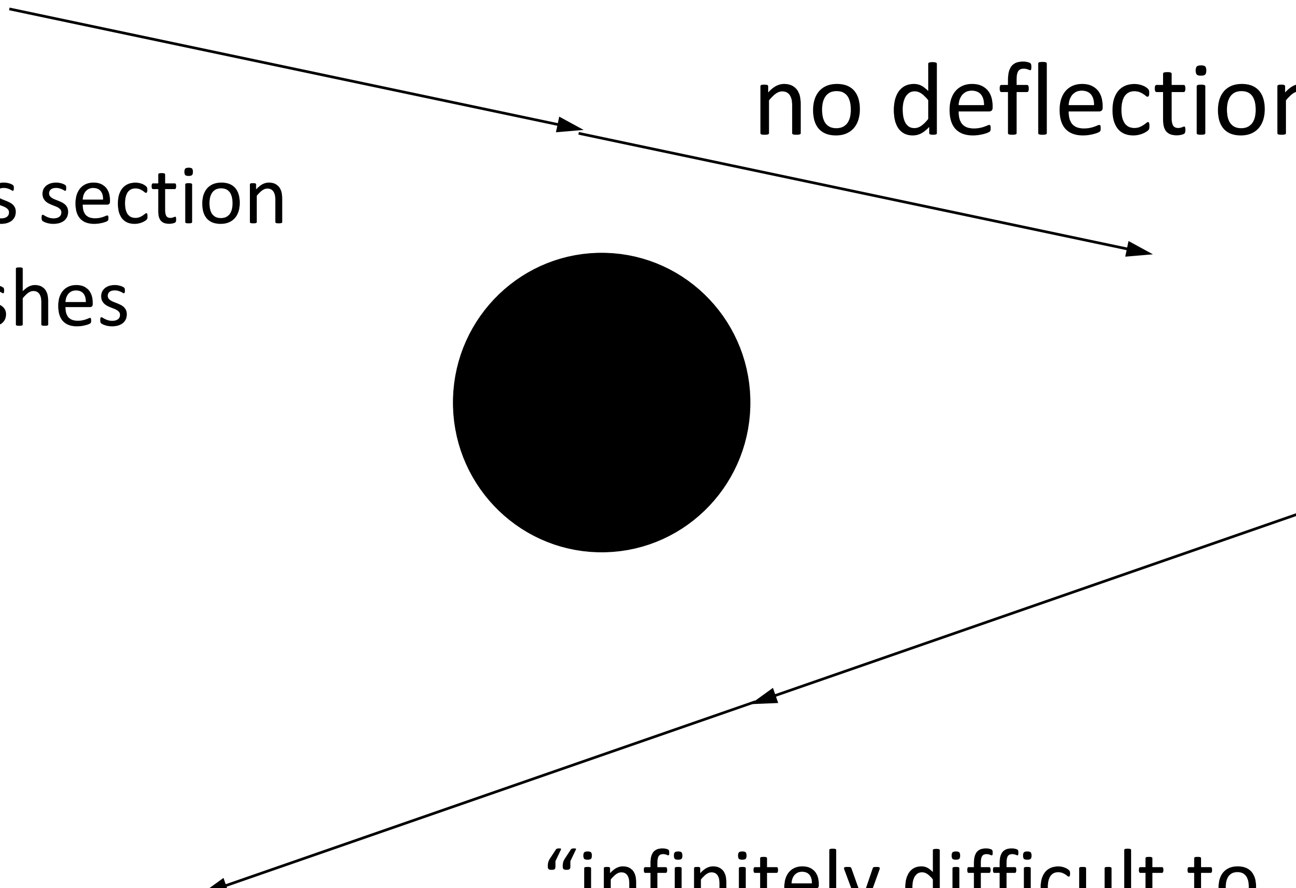
Black Hole scattering: no deflection



“infinitely difficult to catch a line of force”

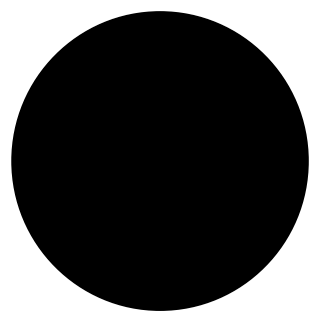
Black Hole scattering: no deflection

Cross section vanishes



“infinitely difficult to catch a line of force”

Black Hole scattering



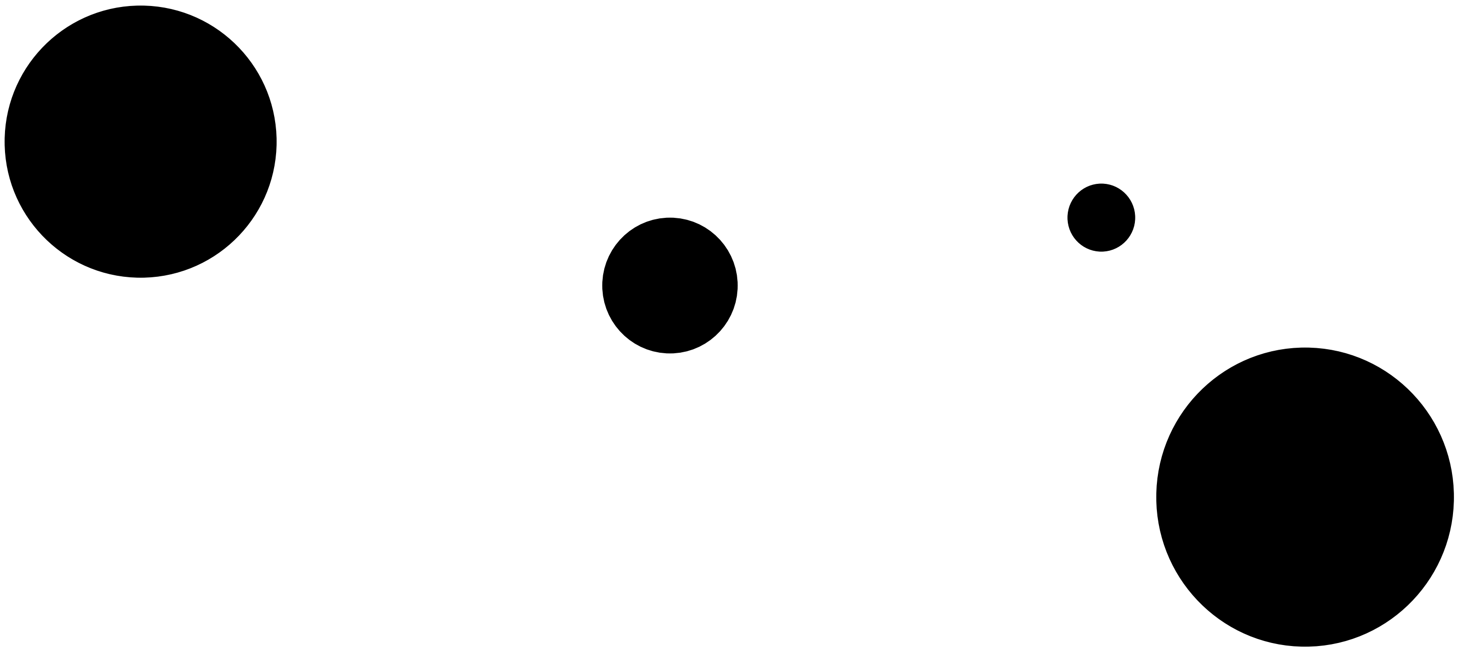
No absorption of waves

with wavelength

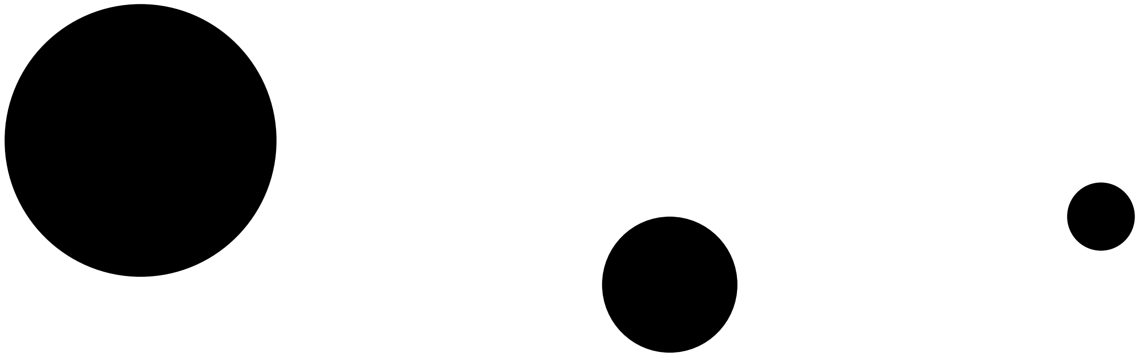
$\lambda \sim r_0$

Perfect reflection

No interaction



Holes cut out in Minkowski space

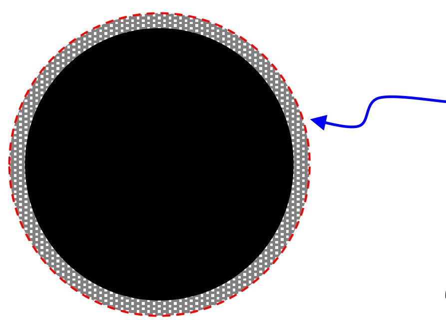


We are keeping length scales $\sim r_0$ finite as we send $D \to \infty$

"**Far-zone**" limit

Now take a limit that does *not* *trivialize* the gravitational field

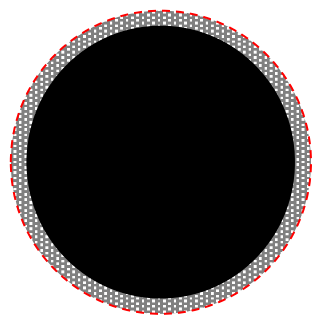
$$\left(\frac{r_0}{r}\right)^{D-3} = \mathcal{O}(1) \iff r - r_0 < \frac{r_0}{D}$$



$$r - r_0 \sim \frac{r_0}{D}$$

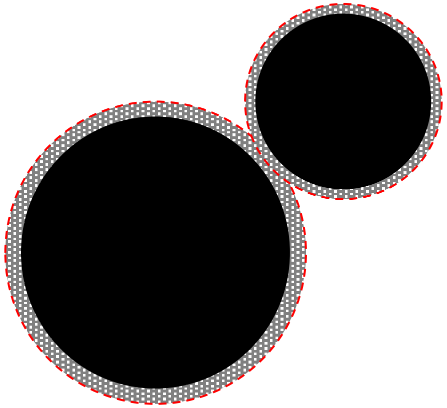
“**Near-horizon**” limit

Physics at $\sim r_0/D$ close to the horizon is *not* trivial



Perfect absorption
of waves with
$\lambda \sim r_0/D$
$\omega \sim D/r_0$

“**Near-horizon**” dynamics



Not an exact solution

Non-trivial interaction

“**Near-horizon**” dynamics

Large D ⇒ Two scales of BH physics

Far zone

$$\lambda \sim r_0$$

Dynamics in flat space with holes

Near-horizon

$$\lambda \sim r_0/D$$

Non-trivial curved space dynamics

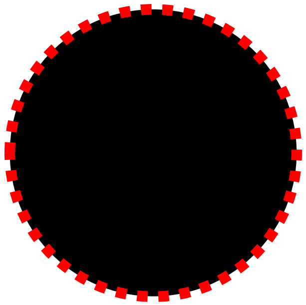
Two scales → ***Effective Theory*** thinking

Solve near-horizon equations

integrate-out short-distance dynamics

→ Boundary conds for far-zone fields

long-distance effective theory



Wave propagation in flat space
w/ bdry conds @ holes

Get practical

Solve BH problems by

Matched Asymptotic Expansion

(*a.k.a.* Classical Effective Field Theory)

1. Solve near-horizon w/ ingoing bdry conds
2. Solve far-zone w/ asymp bdry conds
3. Match where they overlap

Solve far-zone

Easy: flat spacetime

Solve near-horizon

Not trivial, but ∃ enhanced symmetry

$SL(2, \mathbb{R})$

Bonus: universality

✓ Analytic solution

Linear perturbations

Schw black hole scattering of waves

Schw(-AdS) black hole quasinormal modes

Instabilities of rapidly rotating black holes

Instabilities of black branes

Holographic superconductors

✓ Analytic + num'l ODE

Fully non-linear

(in progress)

Non-uniform black strings

"Black droplets" at AdS boundary

How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

How accurate?

Small expansion parameter: $\frac{1}{D-3}$

not quite good for $D = 4$ ...

But it seems to be $\frac{1}{\mathbf{2}(D-3)}$

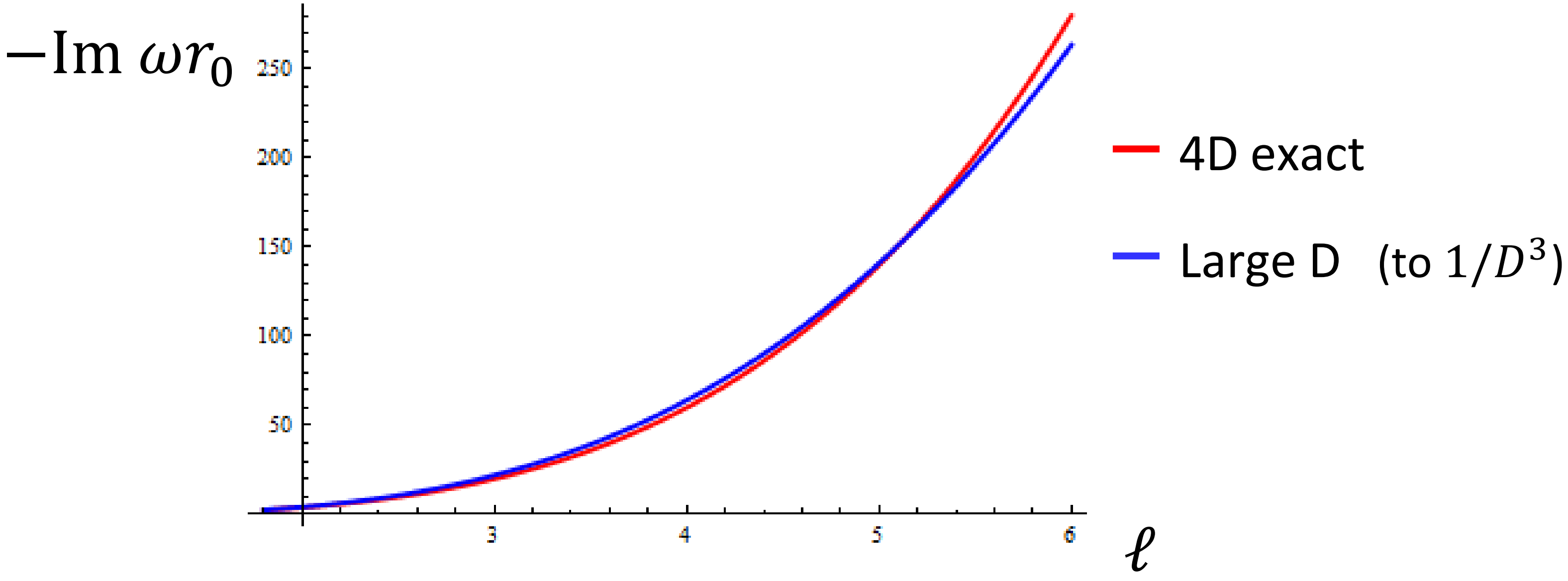
not *so* bad in $D = 4$, if we can compute higher orders

(in AdS: $\frac{1}{2(D-1)}$)

Quite accurate

Comparison with D=4

“algebraically special” quasi-normal mode



Conclusion so far

It works

(not obvious beforehand!)

Black Hole dynamics at large D (II)

Near-horizon geometry

$$ds^2 = -\left(1 - \left(\frac{r_0}{r}\right)^{D-3}\right) dt^2 + \frac{dr^2}{1 - \left(\frac{r_0}{r}\right)^{D-3}} + r^2 d\Omega_{D-2}$$

$$\left.\begin{aligned} \left(\frac{r}{r_0}\right)^{D-3} &= \cosh^2\rho \\ t_{near} &= \frac{D}{2r_0} t \end{aligned}\right\} \text{ finite as } D \to \infty$$

Near-horizon geometry

$$ds_{nh}^2 \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \;\; dt_{near}^2 + d\rho^2\right) + r_0^2 d\Omega_{D-2}^2$$

Near-horizon geometry

$$ds^2_{nh} \to \frac{4r_0^2}{D^2}\left(-\tanh^2\rho \; dt^2_{near} + d\rho^2\right) + r_0^2 d\Omega^2_{D-2}$$

2d string black hole

Elitzur et al
Mandal et al
Witten 1991

Soda 1993
Grumiller et al 2002

$$\ell_{string} \sim \frac{r_0}{D}, \qquad \alpha' \sim \left(\frac{r_0}{D}\right)^2$$

Near-horizon universality

2d string bh = near-horizon geometry of **all neutral non-extremal bhs**

rotation = local boost

(along horizon)

cosmo const = 2d bh mass-shift

Entropy

$$S \sim M^{1+\frac{1}{D-3}} \quad (D \text{ finite})$$

$$M = M_1 + M_2 \;\Rightarrow\; S > S_1 + S_2$$

Black hole merger → entropy gain

Cannot break up: entropy cost

Entropy

$$S \sim M^{1+\frac{1}{D-3}} \xrightarrow{D\to\infty} S \sim M$$

$$M = M_1 + M_2 \;\Rightarrow\; S = S_1 + S_2$$

Black hole merger: no entropy gain

Can break up at no entropy cost

Far-zone absence of interactions

Entropy, *near-horizon* view

$$S \sim M^{1+\frac{1}{D-3}} \rightarrow S \sim M$$

Hagedorn string entropy

$$S = T_{string} M$$

$$T_{string} = \frac{D}{2r_0}$$

Really strings?

What kind?

Or, is this just moonshine?

Near-horizon geometries

Well-defined limiting geometry

Requires small parameter/scale separation

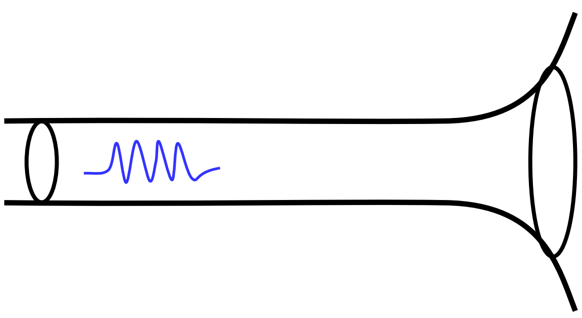
Well known: **(near-)extremal black holes**

small near-extremality parameter

$$\frac{\sqrt{M^2 - Q^2}}{M}, \qquad \frac{\sqrt{M^4 - J^2}}{M^2} \ll 1$$

(Near-)Extremal black holes

Throat geometries near-horizon

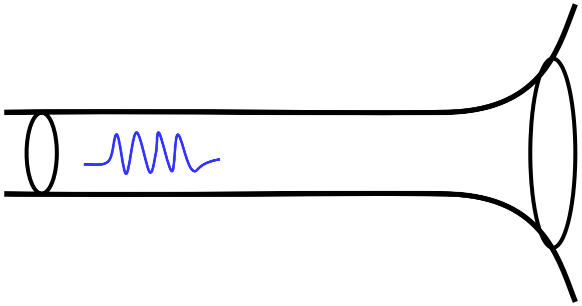


throat supports
“decoupled” dynamics

e.g. AdS/CFT decoupling limit

(Near-)Extremal black holes

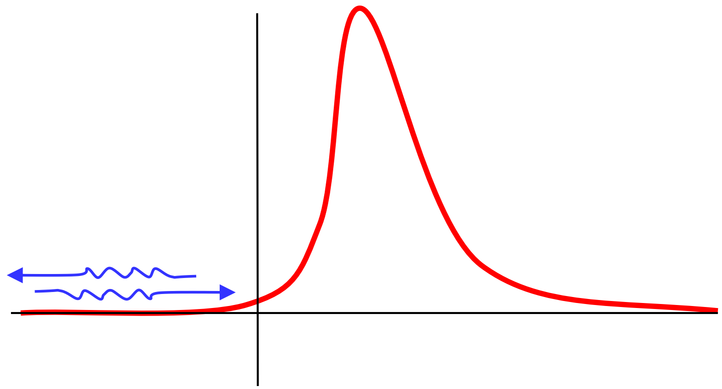
Decoupled dynamics:



finite-frequency excitations that are normalizable in n-h geometry

(Near-)Extremal black holes

Decoupled dynamics:



effective radial potential

finite-frequency excitations that are normalizable in n-h geometry

Is the large D limit a decoupling limit?

Is the large D limit
a decoupling limit?

No

Perturbative BH dynamics @ large D
is concentrated close to the horizon

States can be characterized in terms of their properties within N-H geometry

but N-H geometry is **not long** throat

$$ds^2_{nh} = \frac{4r_0^2}{D^2}\left(-\tanh^2\rho\ dt^2_{near} + d\rho^2\right) + r_0^2 d\Omega^2_{D-2}$$

small extent $\propto r_0/D$

crossed very quickly $t_{near} = \frac{D}{2r_0} t$

Can't expect to support excitations fully trapped within

Black Hole dynamics: Quasinormal modes

Quasinormal modes @ large D

Most QNMs are **not decoupled** states

not normalizable N-H states

But $\exists$ **a few decoupled** QNMs

normalizable N-H states

Non-decoupling and decoupling sectors are very different

Non-decoupling QNMs

High frequencies $\omega \sim D/r_0$

Small damping ratios $\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \to 0$

Control interaction between bh and environment

Little information about black hole

Universal spectrum

Decoupling QNMs

Low frequencies $\omega \sim D^0/r_0$

Damping ratio $\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} \sim 1$

Insulated from far-zone

Specific dynamics of each black hole

instabilities, hydrodynamic modes etc

Non-universal

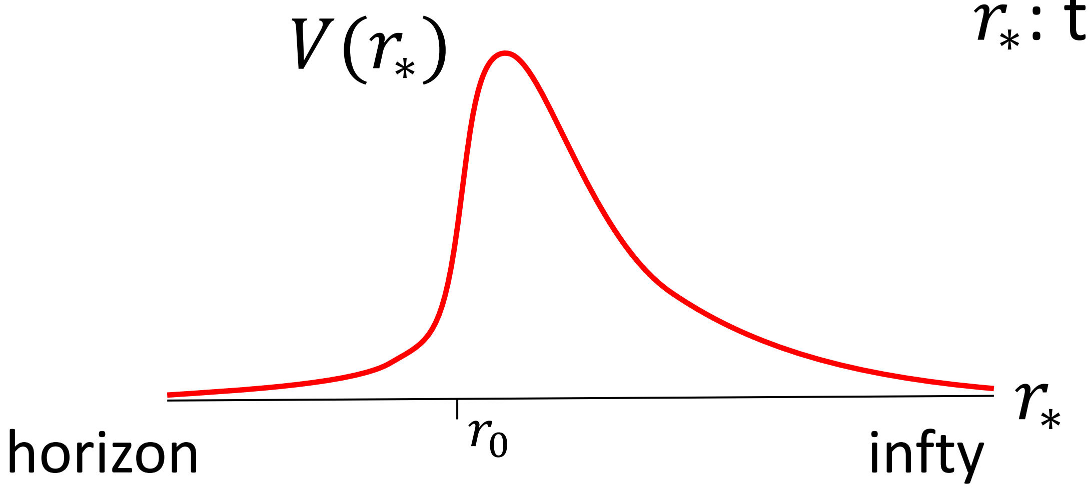
Massless scalar field dynamics

$\Box\Phi = 0$

$$\Phi = r^{-\frac{D-2}{2}}\,\phi(r)\,e^{-i\omega t}\,Y_\ell(\Omega)$$

$$\frac{d^2\phi}{dr_*^2} + (\omega^2 - V(r_*))\phi = 0$$

r_*: tortoise coord



Schwarzschild bh grav perturbations

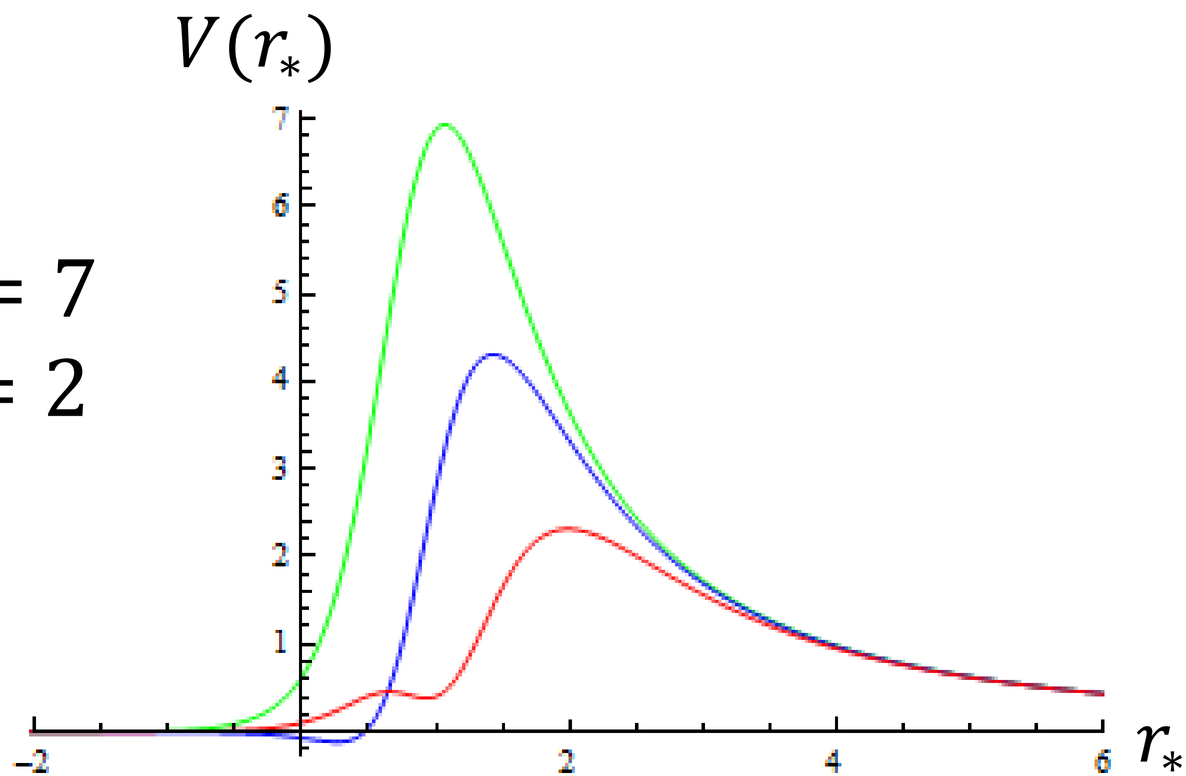
Kodama+Ishibashi

Gravitational scalar, vector, tensor modes

$SO(D-1)$ reps

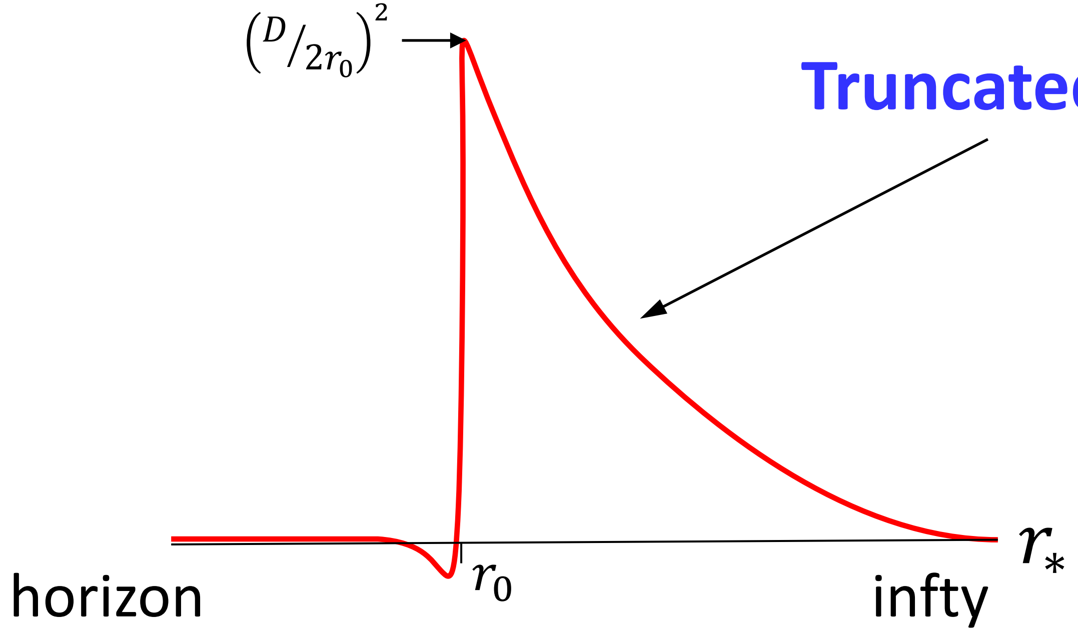
$D = 7$

$\ell = 2$



$$D \to \infty$$

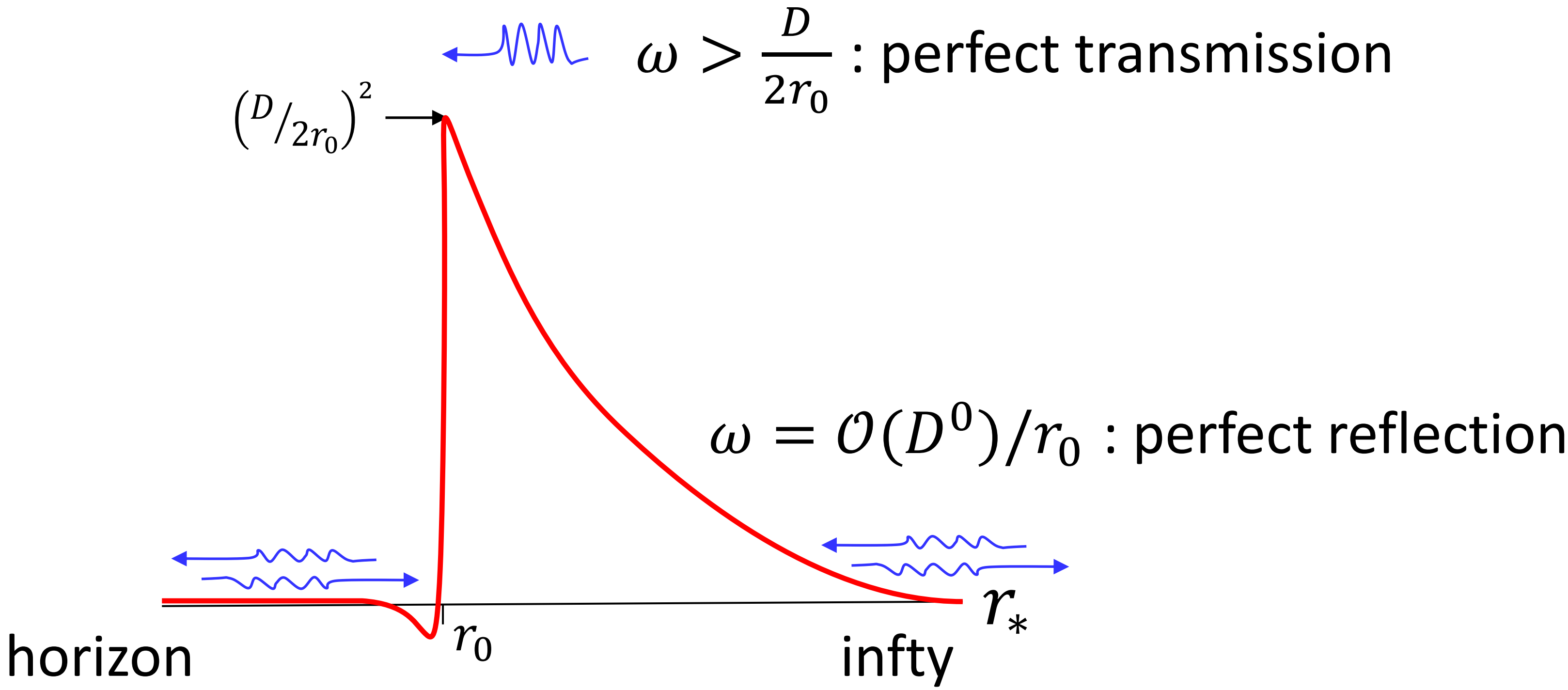
$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$



$\ell = \mathcal{O}(1)$

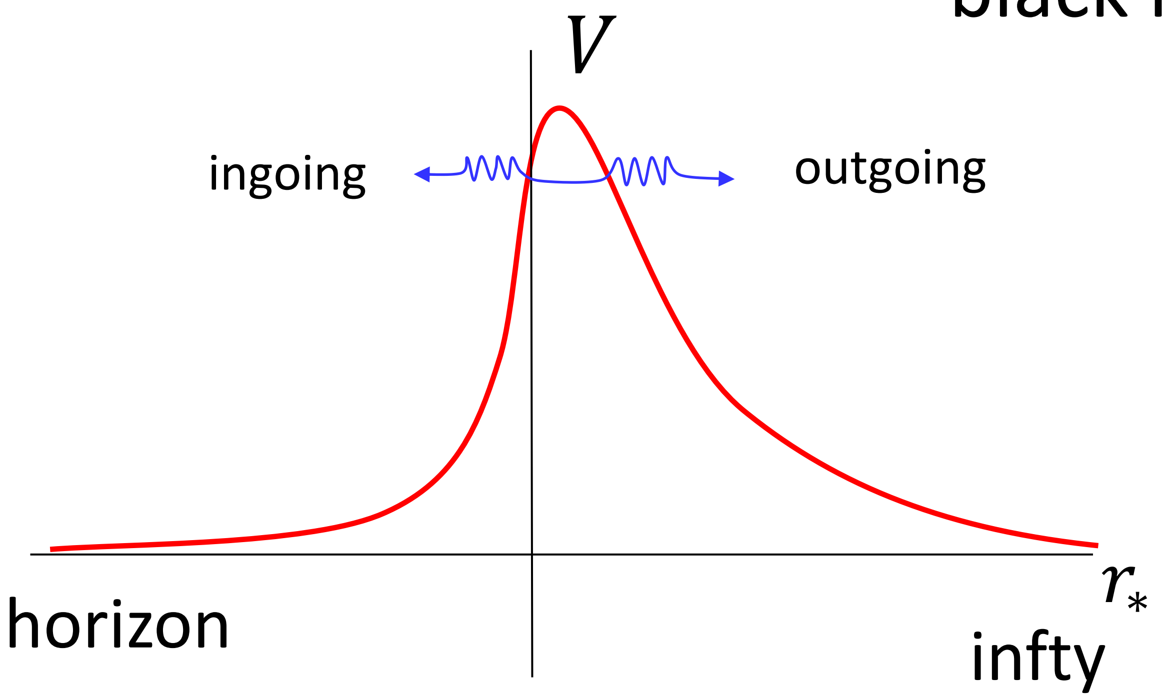
(for simplicity)

$$V(r_*) \to \frac{D^2}{4r_*^2}\Theta(r_* - r_0)$$



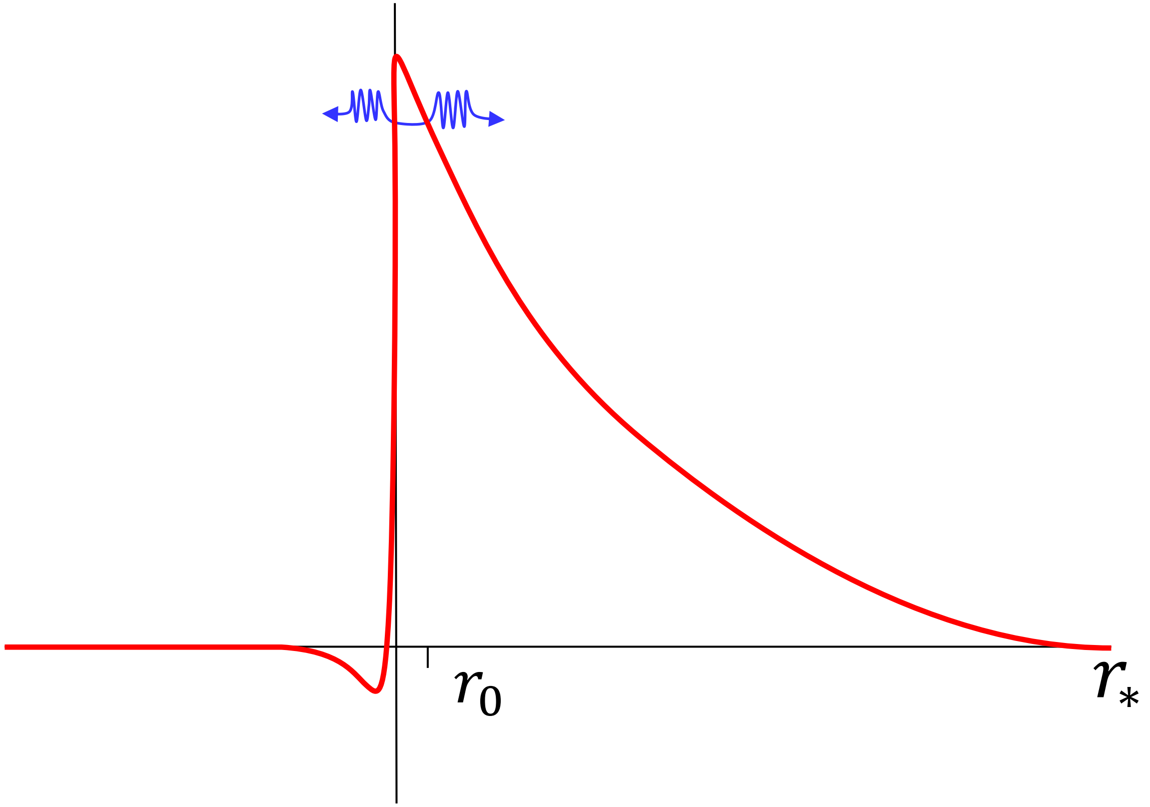
Quasinormal modes

Free, damped oscillations of black hole



Non-decoupled QNMs

$$\omega \sim D/r_0$$



Non-decoupled QNMs

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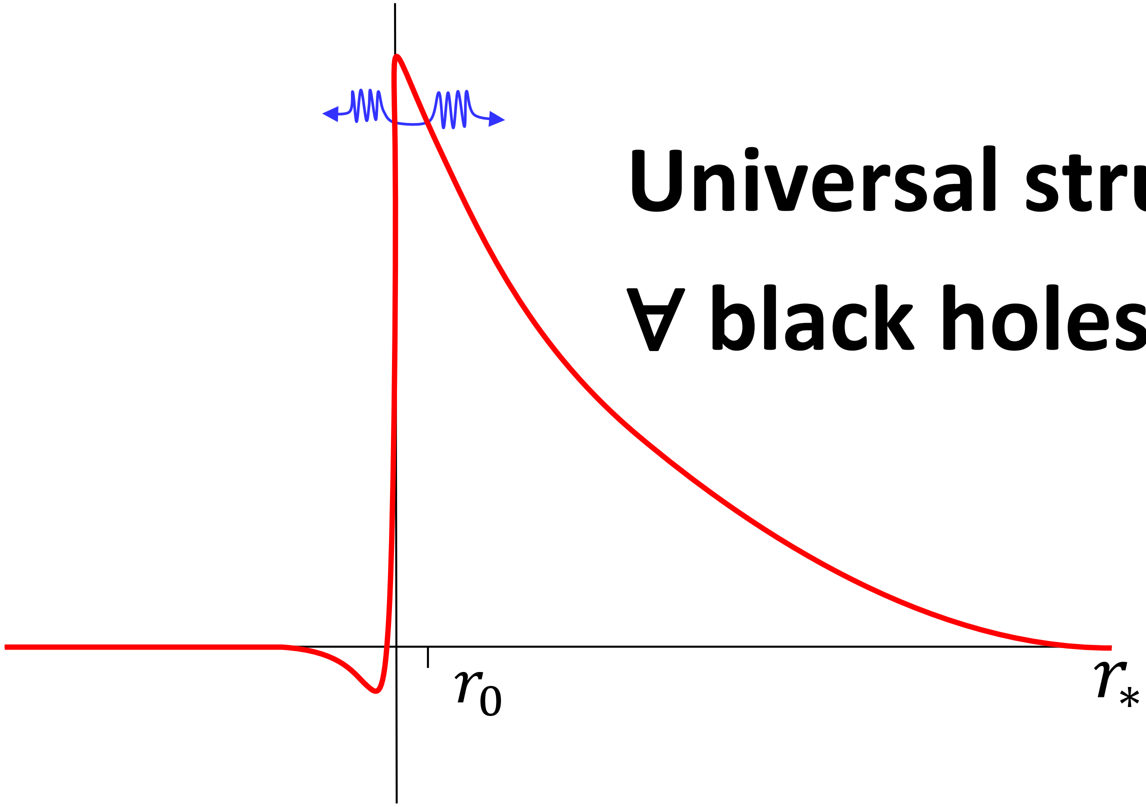
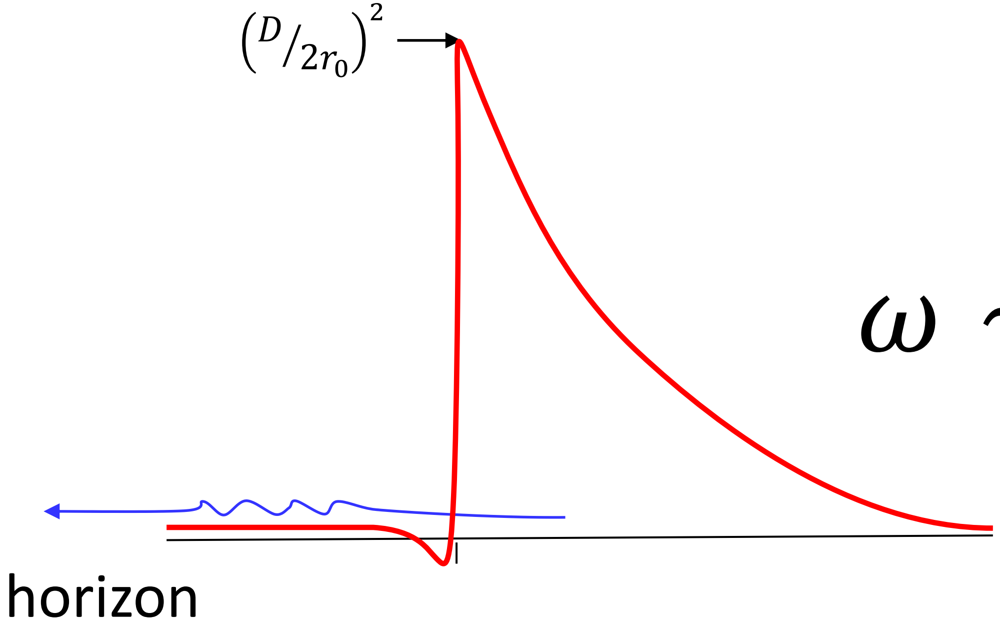


Universal structure

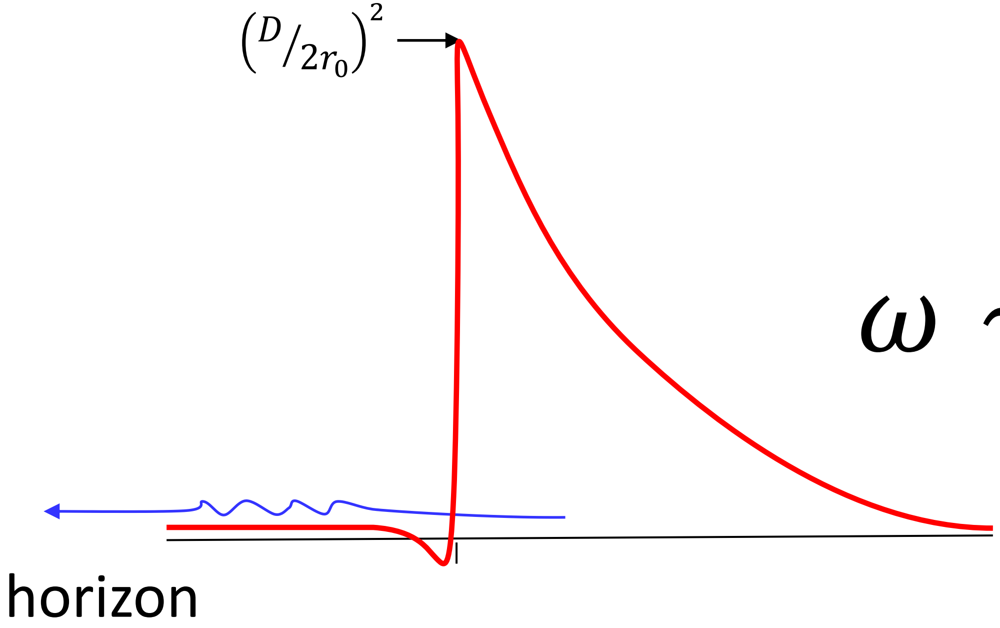
∀ black holes @ $D \to \infty$

Decoupled QNMs



$$\omega \sim \frac{1}{r_0} \ll \frac{D}{r_0}$$

Decoupled QNMs



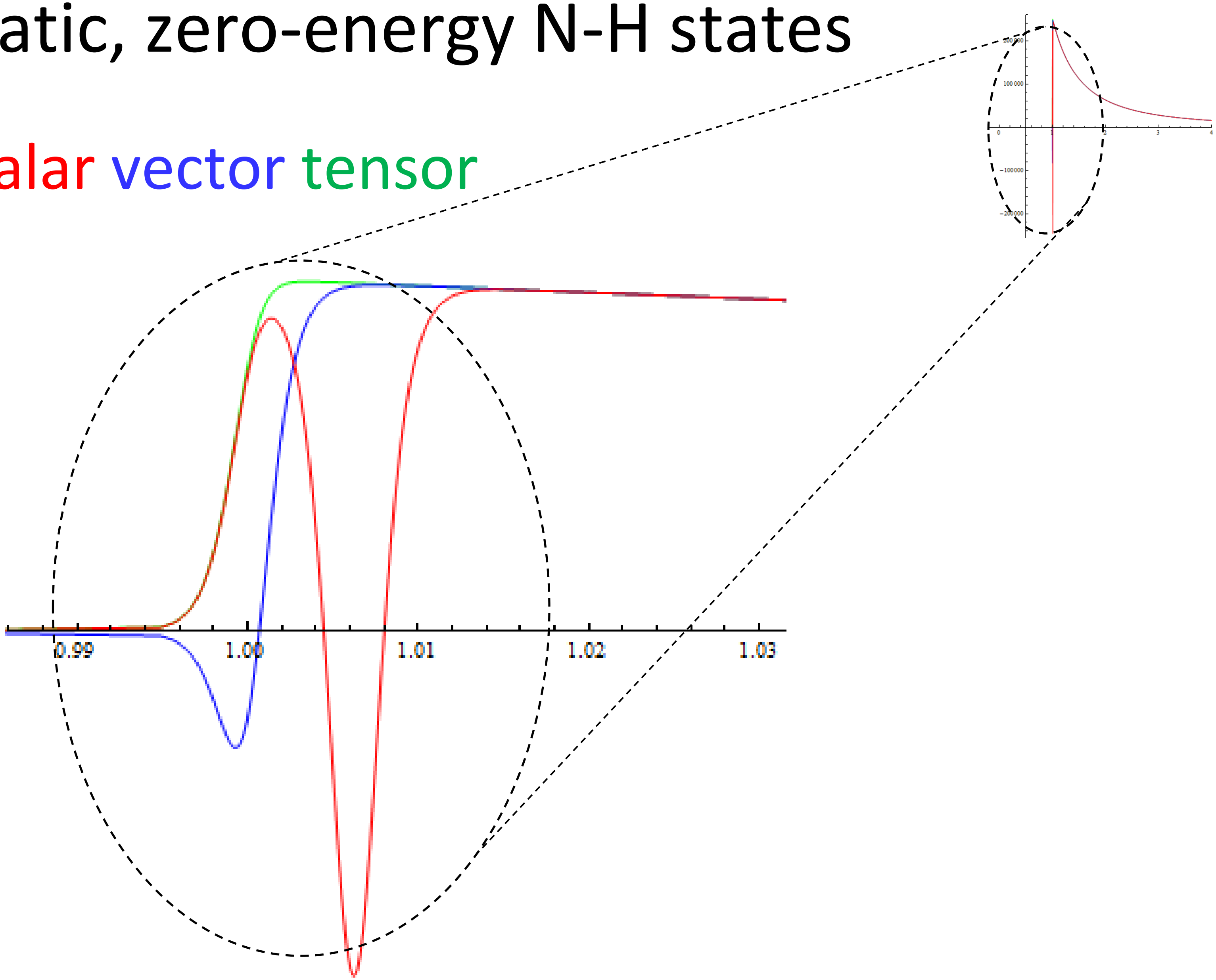
$$\omega \sim \frac{1}{r_0} \ll \frac{D}{r_0}$$

$\omega_{near} = \frac{\omega}{D} \to 0$: *static* N-H states

(leading 1/D order)

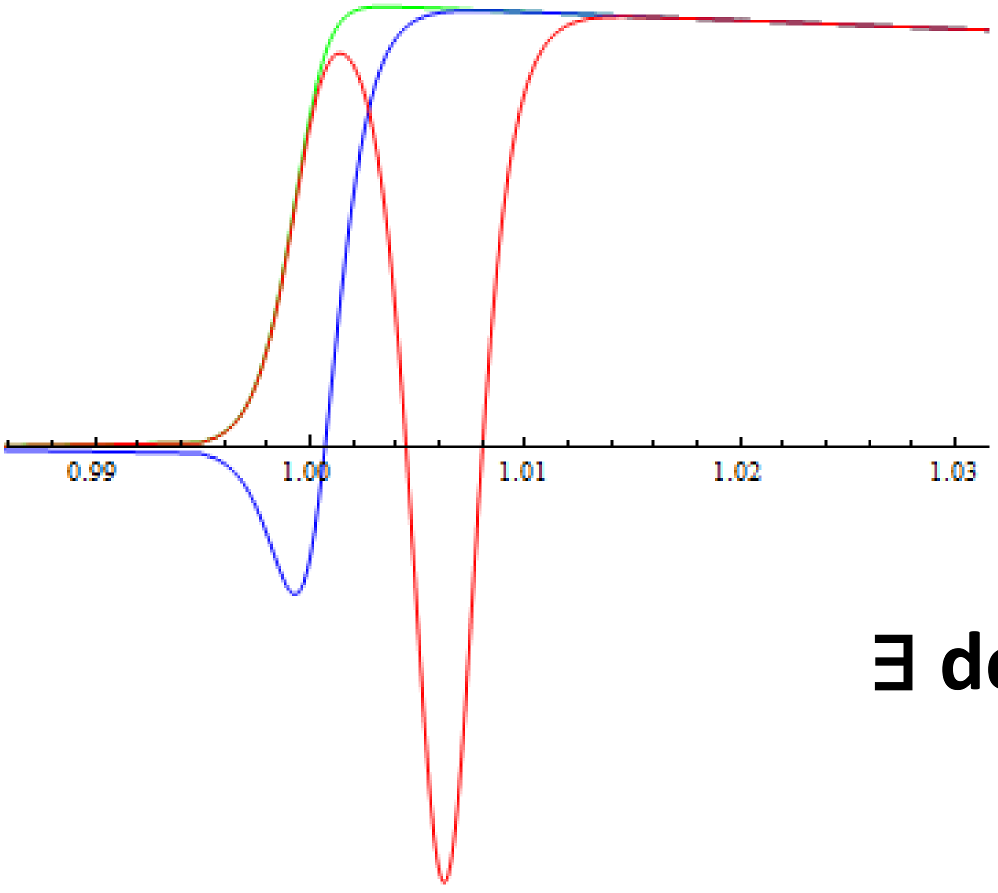
Static, zero-energy N-H states

scalar vector tensor



Static, zero-energy N-H states

scalar vector tensor



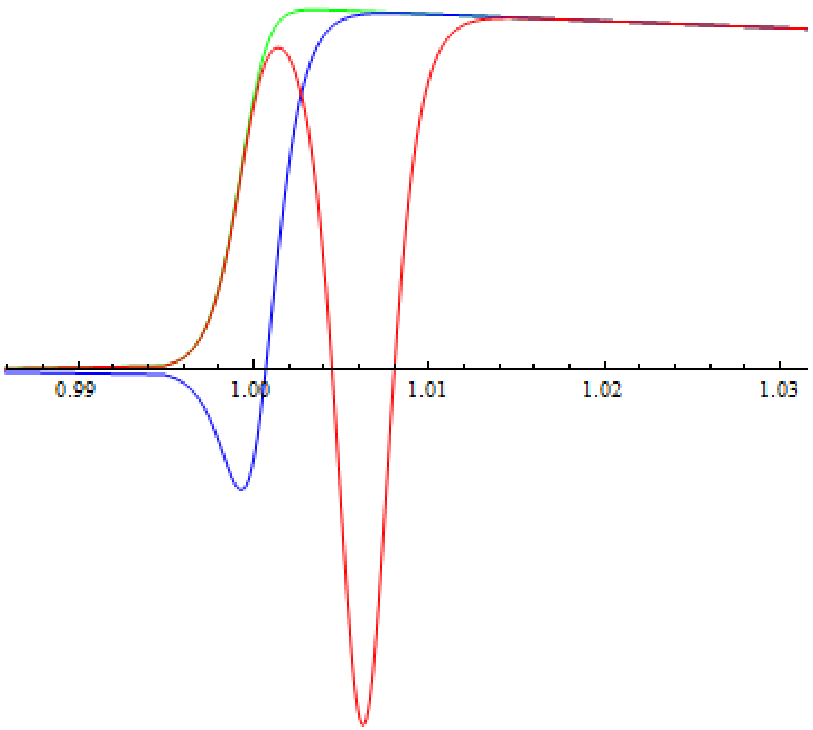
∃ zero-energy
scalar vector

not tensor

∃ decoupled states

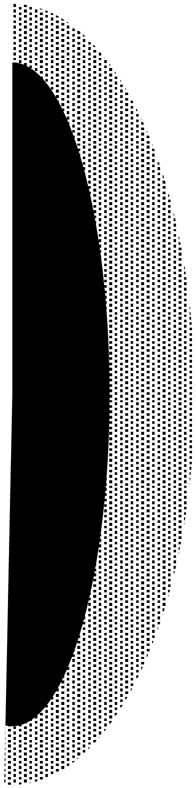
Decoupled QNMs

We've computed the QNM frequencies up to $1/D^3$



BH dynamics @ large D

BH excitations (quasinormal modes) in terms of near-horizon dynamics



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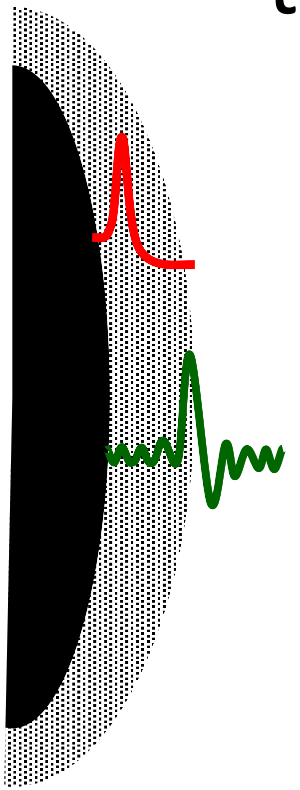


"Decoupled" states

strongly localized near the horizon

"Non-decoupled" states

communicate bh to asymptotic region

Quantitative accuracy

Decoupled modes $\boldsymbol{\omega r_0 = \mathcal{O}(1)}$

At $D = 100$: ($\ell = 2$ vector mode, purely imaginary)

$\mathrm{Im}\,\omega r_0 =$ -1.01044742 (analytical)

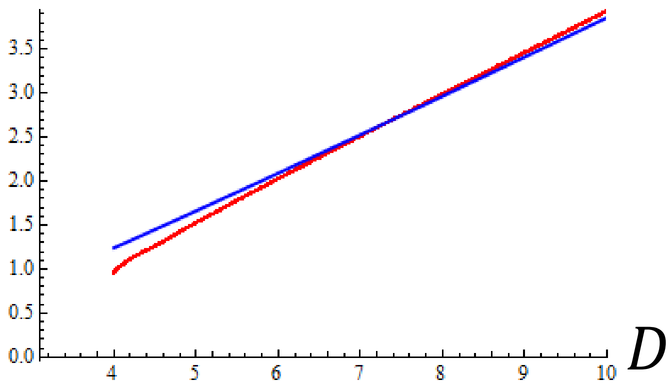
-1.01044741 (numerical *Dias et al*)

Quantitative accuracy

Non-decoupled modes $\boldsymbol{\omega r_0 = \mathcal{O}(D)}$

Re ωr_0: good at moderate D

Re ωr_0



$\ell = 2$

Im $\omega r_0 \sim D^{1/3}$: only good at *very* high D

Going fully non-linear

Non-linear theory of decoupled **zero-modes** (static deformations)

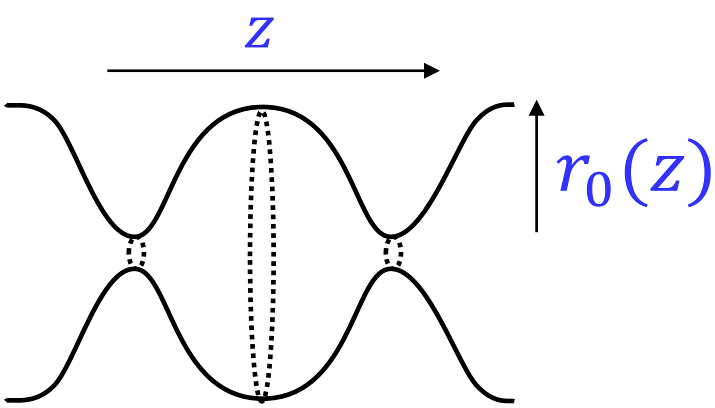
Radial direction solved analytically

reduce 2 dim PDE to ODE

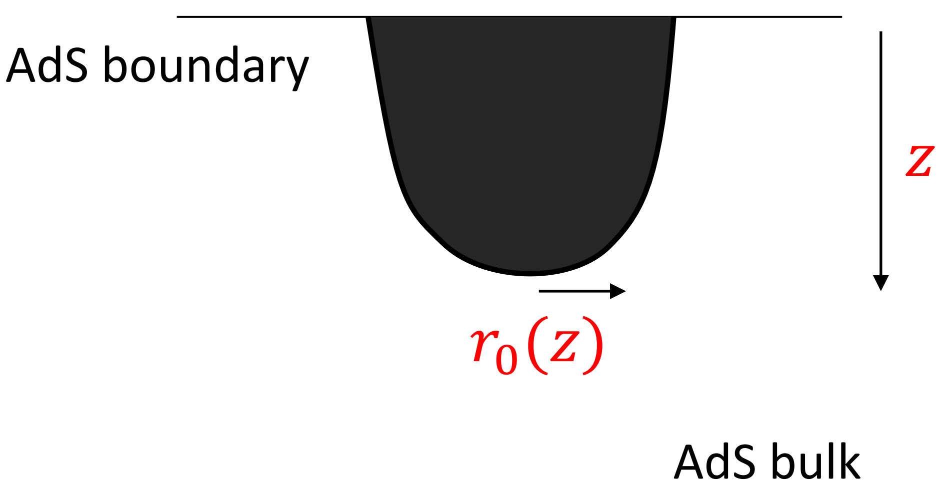
Obtain non-linear eq for zero mode (collective field)

Non-uniform black string

R Suzuki



Black droplet at AdS boundary



Outlook

Universal features @ large D

Far region

∀bhs: empty space

Near-horizon region

∀neutral bhs: 2D string bh

BH dynamics splits into:

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

scalar field **oscillations of a hole** in space

universal normal modes

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

localized in near-horizon region

$\omega r_0 = \mathcal{O}(D^0)$: decoupled dynamics

– **specific** of each bh
– less numerous
– ultraspinning instabilities in this sector
– hydro modes of black branes

$\omega r_0 = \mathcal{O}(D)$: non-decoupled dynamics

- **universal** normal modes of hole in space
- much more **numerous**
- describe **interaction** of bh **w/ environment**

Full non-linear dynamics

Stationary black holes

deformed rotating bhs

Time evolution

non-linear Gregory-Laflamme as 1+1 system

Towards a general theory of horizon dynamics @ large D