

# The Beginning and Ending Stages of Cosmic Reionization

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Yue et al., 2013b, MNRAS 433, 1556

Yue et al., 2014, MNRAS 440, 1263

Xu et al., 2014, ApJ, 781, 97

# Outline

- Cosmic reionization
- The missing source for the near-infrared background (NIRB) fluctuations on large scales
- The direct collapse black hole (DCBH) interpretation
- The DCBH formation during the early EoR

*After the brief DCBH era...*

- The island model for the late EoR
  - The excursion set theory
  - The bubbles-in-island effect
  - The ionizing background
  - The percolation criterion and the neutral islands statistics
- Summary

# What is the Reionization Era?

## A Schematic Outline of the Cosmic History

Time since the  
Big Bang (years)

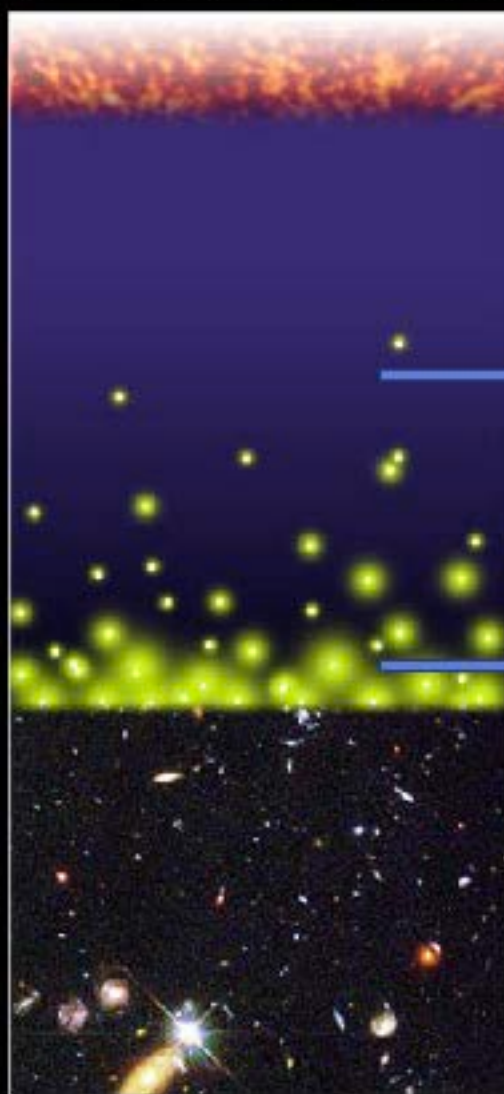
~ 300 thousand

~ 500 million

~ 1 billion

~ 9 billion

~ 13 billion



← The Big Bang

The Universe filled  
with ionized gas

← The Universe becomes  
neutral and opaque

The Dark Ages start

Galaxies and Quasars  
begin to form  
The Dark Ages ends

The Cosmic Renaissance  
The Dark Ages end

← Reionization complete,  
the Universe becomes  
transparent again

Galaxies evolve

The Solar System forms

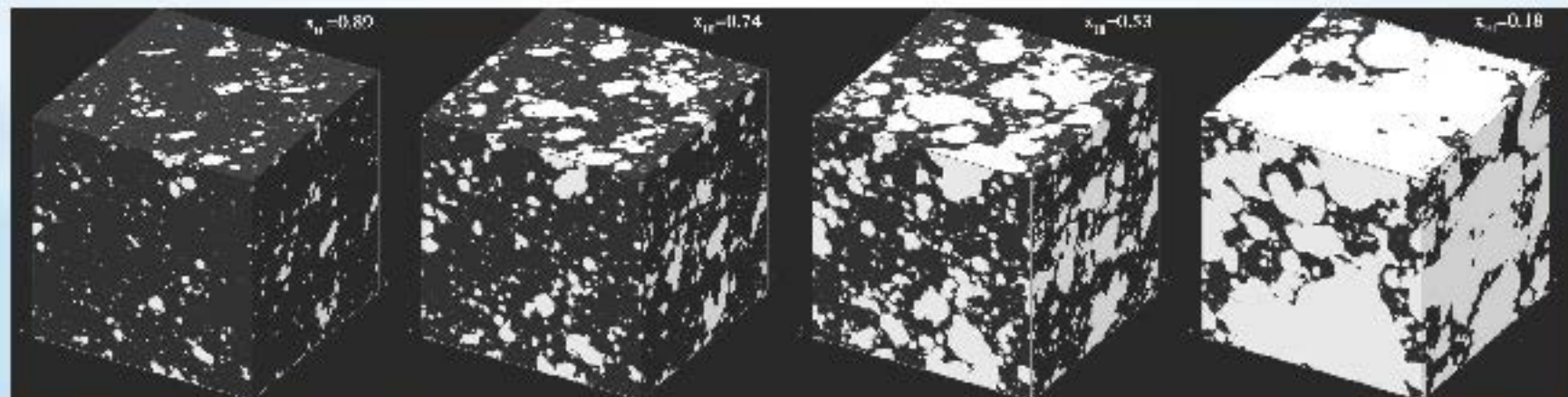
Today: Astronomers  
figure it all out!

DCBHs

Neutral  
Islands

# Theoretical works on reionization

- \* Simulations: e.g. Battaglia+2013; Ciardi+2012; Iliev+2012
- \* Analytical models: e.g. Furlanetto+2004 (“bubble model”)
- \* Semi-numerical simulations: e.g. Mesinger & Furlanetto 2007; Alvarez+2009

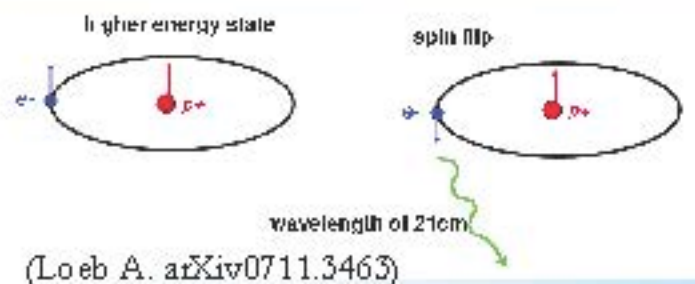


(From Mesinger & Furlanetto 2007 ApJ, 669, 663)

# Probes of cosmic reionization

- \* The polarization data of CMB from PLANCK  $\rightarrow z \sim 11.1$  (Planck Collaboration 2013);
- \* The Gunn-Peterson tests  $\rightarrow$  very nearly complete at  $z \sim 6$  (Fan et al. 2006);
- \* High redshift galaxy surveys (e.g. Lehnert+2010);
- \* The kSZ effect (e.g. Zahn et al. 2012);
- \* The 21cm probe of HI.

EDGES, PAPER, GMRT, LWA, MWA,  
LOFAR, 21CMA, HERA, SKA



# The Near-Infrared Background from the Epoch of Reionization (EoR)

## \* *Near-infrared:*

- \* wavelength range:  $\sim 0.7 - 8 \mu\text{m}$
- \* for galaxies with  $z > \sim 6$ , the bulk of the emission is redshifted into near-infrared band

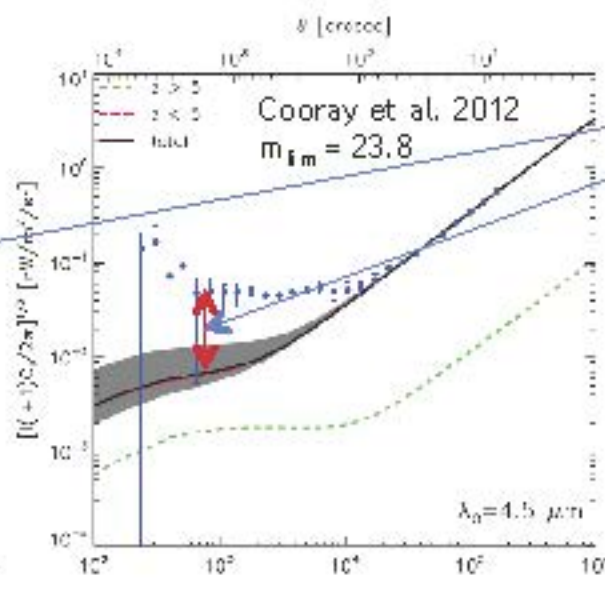
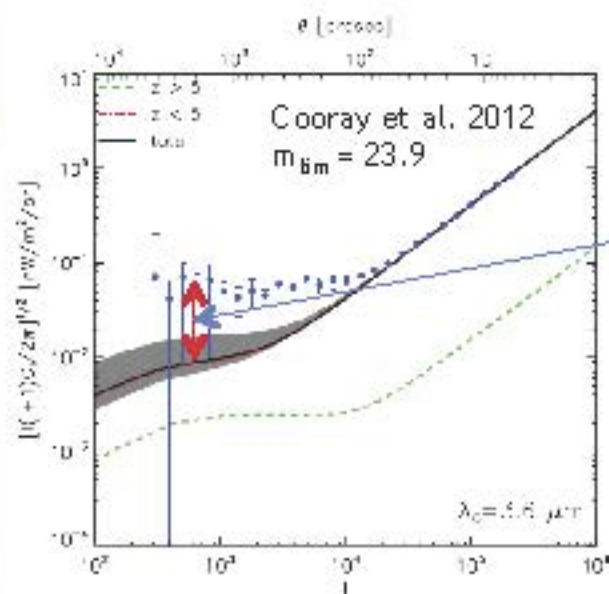
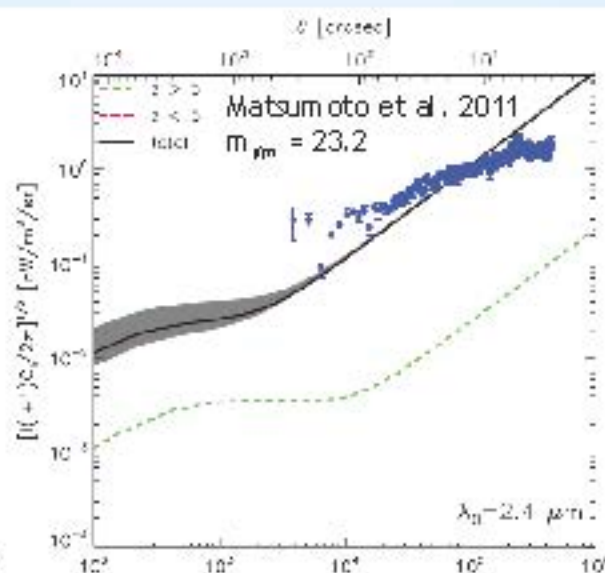
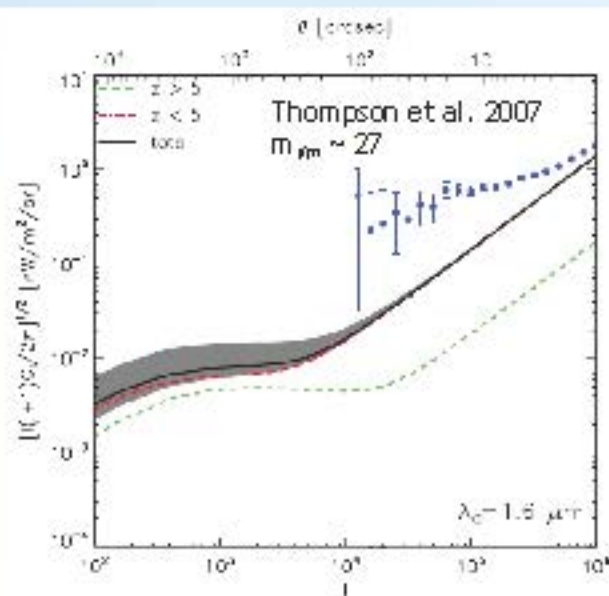
## \* *Background:*

- \* most of galaxies that contribute to reionization are below the limiting magnitude
  - \* their cumulative radiation forms a background
- \* The NIRB as a probe to the first galaxies and black holes.

# The Near-Infrared Sky

- \* Foreground (zodiacal light, strongest,  $\sim 10^4$  times the signal, but smooth at the observed sky region)
- \* + possible light from stars/ISM in our Milky Way
- \* + bright galaxies (point sources)
- \* + *faint galaxies, first stars... (of interests)*

# The angular fluctuations of source-subtracted NIRB



The contribution from low- $z$  galaxies  
- Helgason et al. (2012)

The contribution from high- $z$  galaxies  
- Yue et al. (2013a)

Unknown sources?

# The Clues

- \* Cluster on large scales
  - \* Faint (at least  $> \sim 25$  at  $3.6$  and  $4.5 \mu\text{m}$ ,  $> \sim 29 - 30$  at the H-band)
  - \* Negligible contribution to reionization, i.e.  $f_{\text{esc}} \sim 0$
  - \* Be consistent with the constraints on the CXB level
- *One possible explanation: diffuse intrahalo light of galaxies at  $z \sim 1 - 4$  (Cooray et al. 2012a, Nature, 490, 514)*

# Compton-thick, intermediate-mass black holes

- ✓ Sufficiently intense Lyman-Werner (LW) radiation  $\rightarrow$  H<sub>2</sub> cooling is quenched
- ✓ Pristine gas in halos with  $T_{\text{vir}} \geq 10^4$  K  $\rightarrow$  atomic H cooling sustain an almost isothermal collapse preventing gas fragmentation into smaller sub-units.
- $\rightarrow$  Rapid ( $\approx 1$  Myr) formation of a direct collapse black hole (DCBH) of mass  $10^{4-6} M_{\text{sun}}$  (Bromm & Loeb 2003; Begelman et al. 2006; Lodato & Natarajan 2006; Regan & Haehnelt 2009 .....).

*As the gas continues to fall onto the BH ...*

$\rightarrow$  *probably Compton-thick!*

# Spectrum of a Compton-thick accreting BH

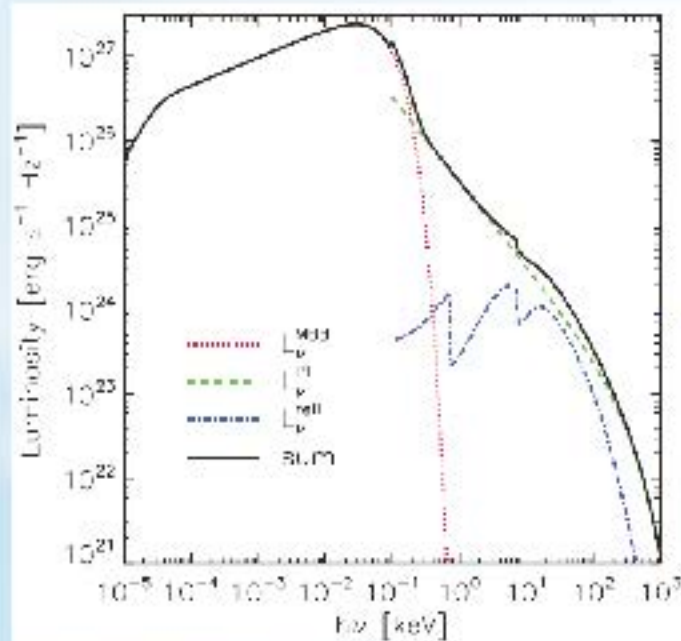
\* The primary spectrum

$$L_\nu = L_\nu^{\text{MBB}} + L_\nu^{\text{PL}} + L_\nu^{\text{refl}}$$

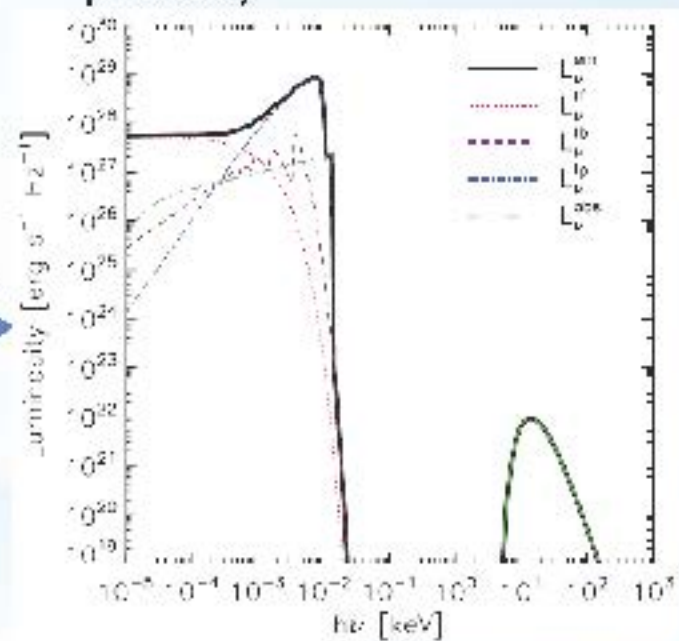
absorption  
scattering

\* The emergent spectrum

‘nebular emission’ (free-free, free-bound, two-photon)



a massive  
accreting  
envelope



$$M_{\text{BH}} = 10^6 M_{\text{sun}} \text{ and } N_{\text{H}} = 1.5 \times 10^{25} \text{ cm}^{-2}$$

# NIRB and CXB fluctuations from DCBHs

\* The emissivity:

$$\epsilon_\nu(z) = \frac{1}{4\pi} \int_{z_{\text{start}}}^z L_\nu^{\text{em}}(M') \frac{dn_{\text{BH}}}{dz'}(z') dz',$$

\* The BH power spectrum:

$$P_{\text{BH}}(k, z) \approx P(k, z) b_{\text{eff}}^2(z),$$

\* The angular power spectrum:

$$C_l^{\text{NIRB}} = \int_{z_{\text{end}}}^{z_{\text{start}}} dz \frac{[ \nu \epsilon_\nu(z) e^{-\tau(\nu_0, z)} ]^2}{H(z) r^2(z) (1+z)^4} P_{\text{BH}}(z),$$

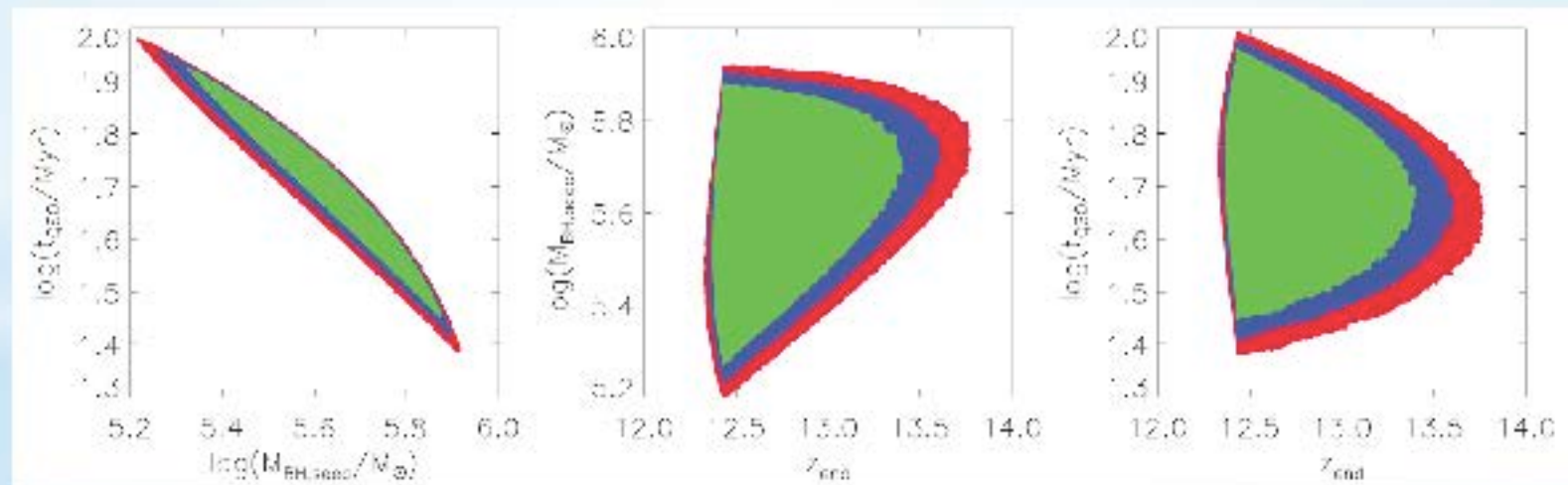
# Fitting the NIRB fluctuation data

- \* Model parameters:

- \*  $M_{\text{BH,seed}}$ ,  $t_{\text{QSO}}$ ,  $z_{\text{end}}$ ,  $N_{\text{H}}$  (the NIRB is very insensitive to  $N_{\text{H}}$  once the BH is Compton-thick, but the CXB level is sensitive to it)

- \* Limits:

- \* Apparent magnitude  $m > 30$  at the H-band of *HST*/WFC3
- \*  $m > 27$  at 3.6 and 4.5  $\mu\text{m}$  band of *Spitzer*/IRAC

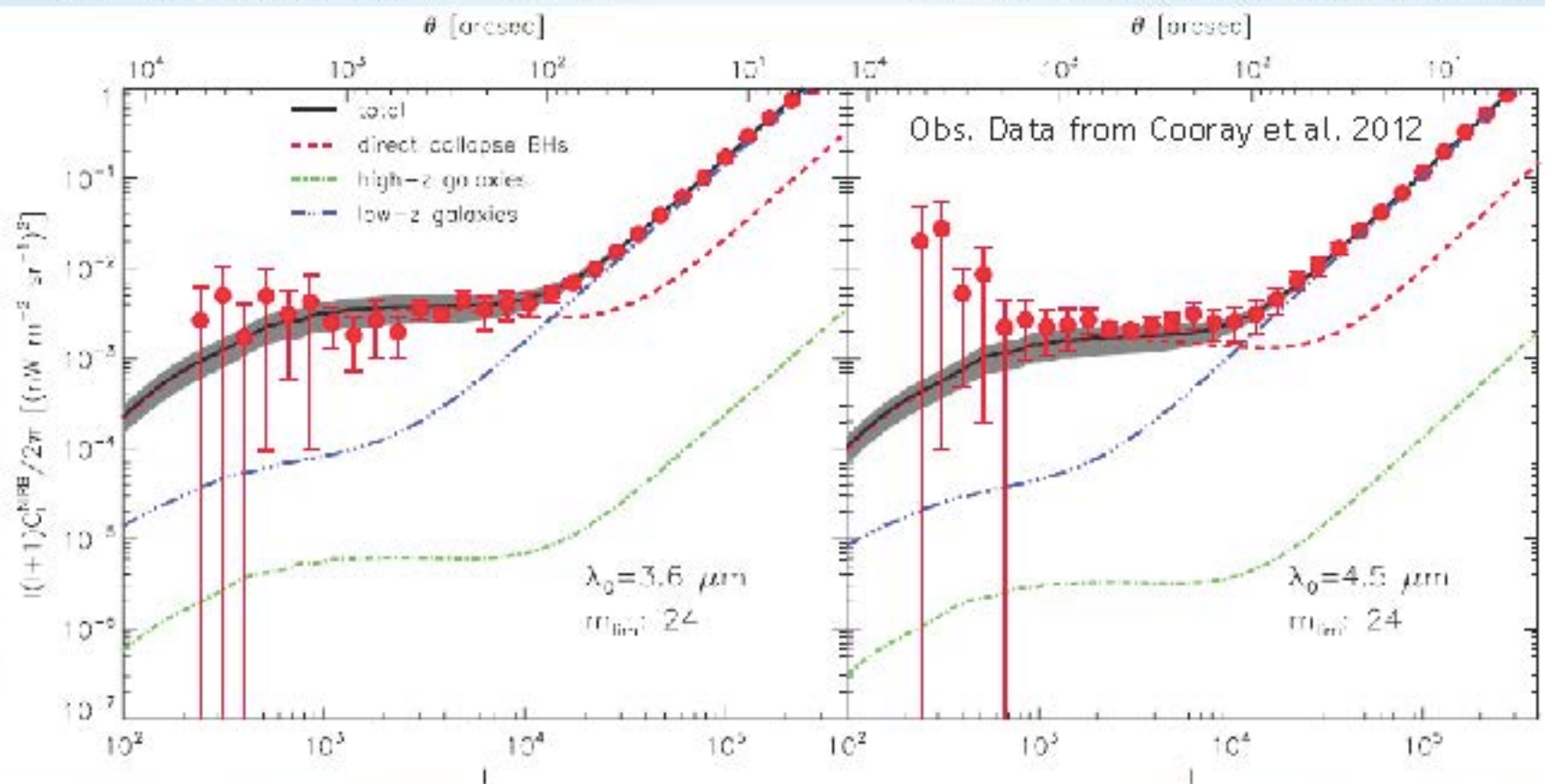


Best fitting parameters:  $M_{\text{BH,seed}}: 10^{5.85} \text{ Msun}$ ,  $t_{\text{QSO}}: 10^{1.48} \text{ Myr}$ ,  $z_{\text{end}}: 12.44$

# The predicted source-subtracted NIRB spectrum

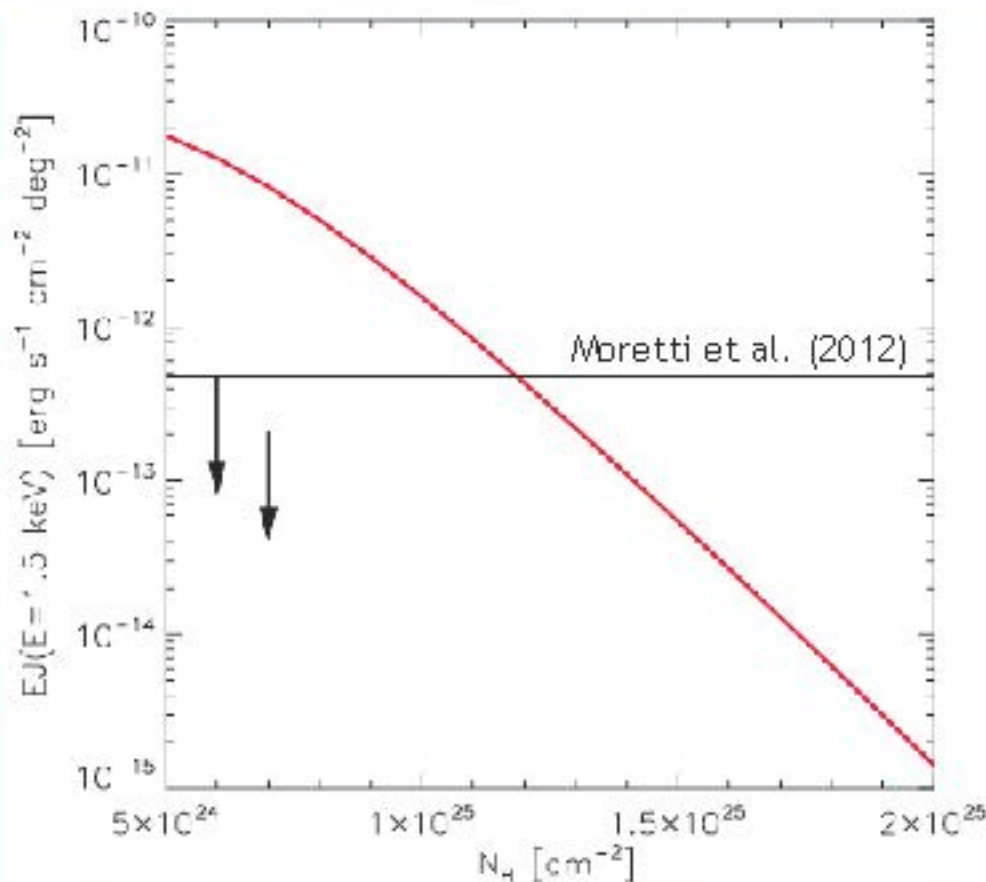
Faint galaxies at  $z > 5$ : Yue et al. 2013a

Faint galaxies at  $z < 5$ : Helgason et al. 2012



Best fitting parameters:  $M_{\text{BH,seed}}: 10^{5.85} \text{ Msun}$ ,  $t_{\text{QSO}}: 10^{1.48} \text{ Myr}$ ,  $z_{\text{end}}: 12.44$

# Constraints on the $N_H$ by CXB level

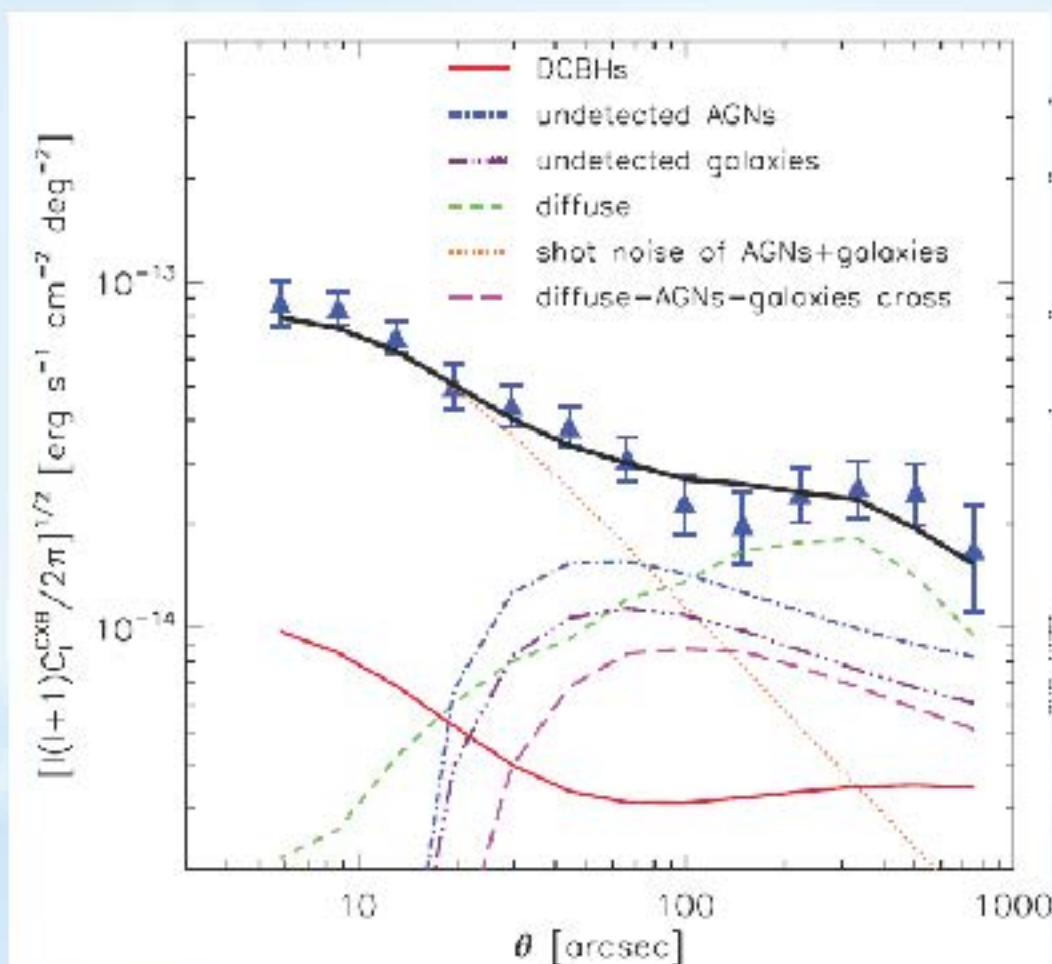


\* The DCBH contribution to the unresolved CXB for the best fitting parameters

→  $N_H > 1.2-1.3 \times 10^{25} \text{ cm}^{-2}$  is required

(Compton-thick)

# The angular power spectrum of the CXB

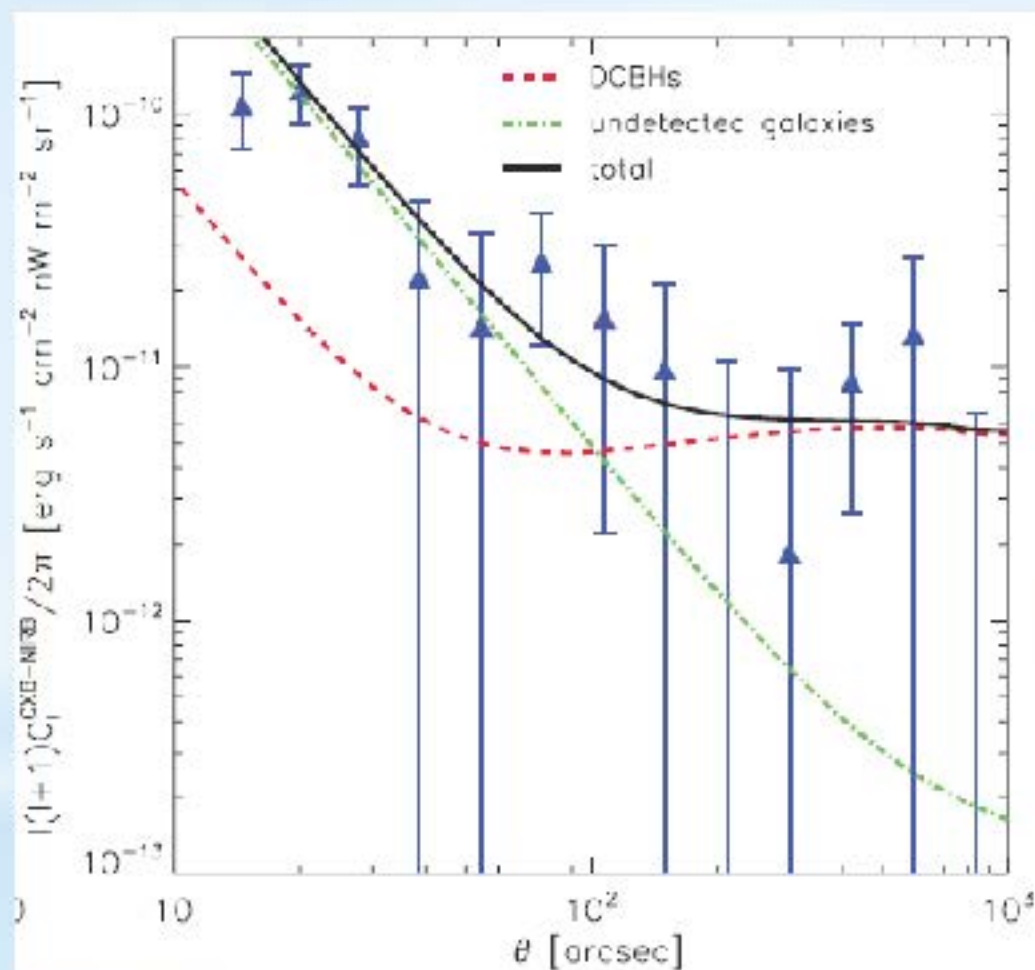


Data and other contributions  
from Cappelluti et al. 2012

- The contribution of DCBHs to CXB fluctuations is negligible with respect the other sources at all angular scales.

$$N_H = 1.5 \times 10^{25} \text{ cm}^{-2}$$

## CXB (0.5-2.0 keV)-NIRB(4.5 $\mu$ m) cross-correlation



$$N_H = 1.5 \times 10^{25} \text{ cm}^{-2}$$

Data from Cappelluti et al. 2013

- Undetected low- $z$  galaxies dominates at small scales ( $< 100$  arcsec) (shot noise)
- DCBHs dominate at large scales ( $> 100$  arcsec)

(unknown source reported by Helgason et al. 2014)

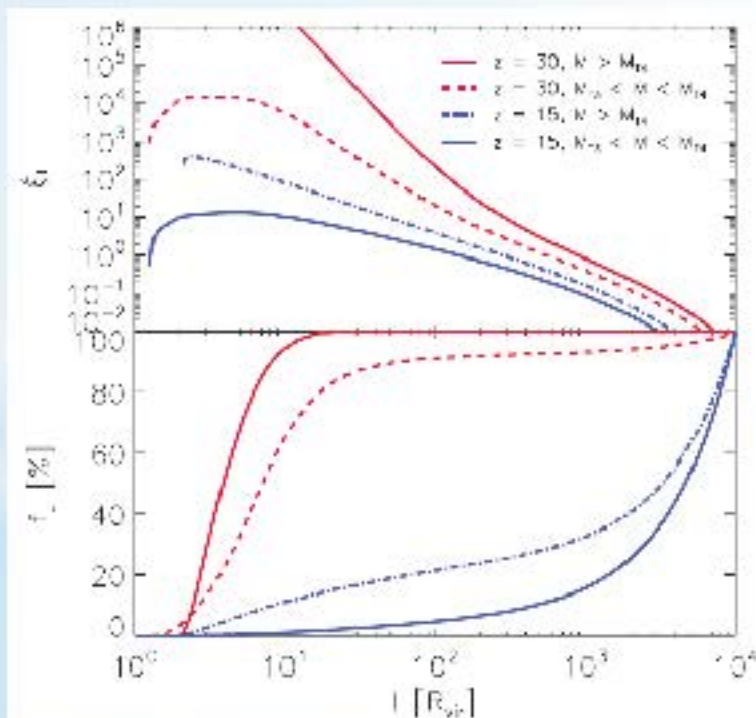
# The DCBH formation history

- \* At the early EoR, galaxies were rare but highly clustered, and they triggered the formation of the earliest DCBHs.
- \* Once the first DCBHs formed, they emit much more LW photons than galaxies
- \* They then trigger a runaway process of further DCBH formation, producing a sudden rise in their cosmic mass density.
- \* The formation is quenched by reionization after  $z \sim 13$

# Various feedbacks

## \* Positive feedback:

- \* Strong clustering
- \* High LW luminosity of DCBHs themselves



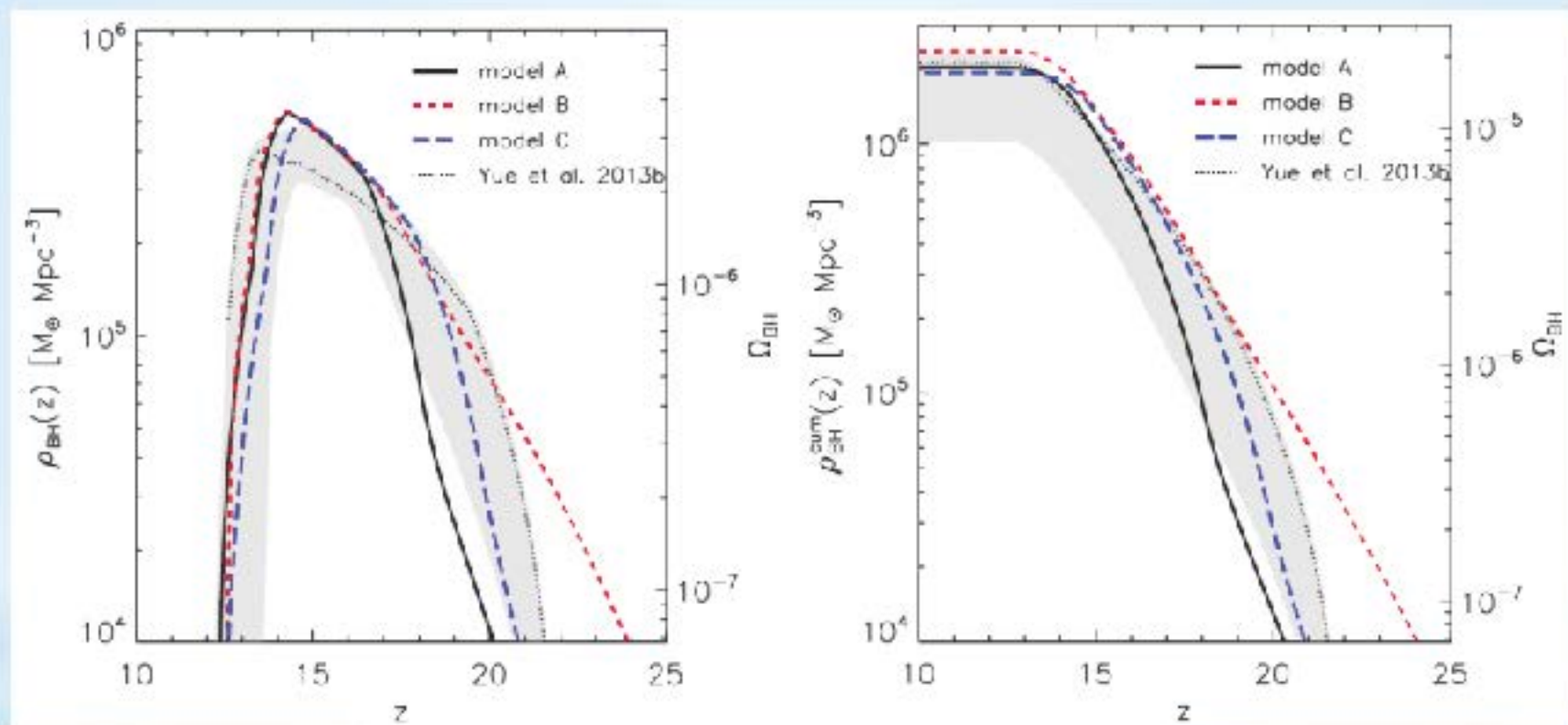
## \* Negative feedbacks:

- \* Genetic enrichment
- \* Wind enrichment
- \* Photo-evaporation by reionization

## \* The DCBH formation rate:

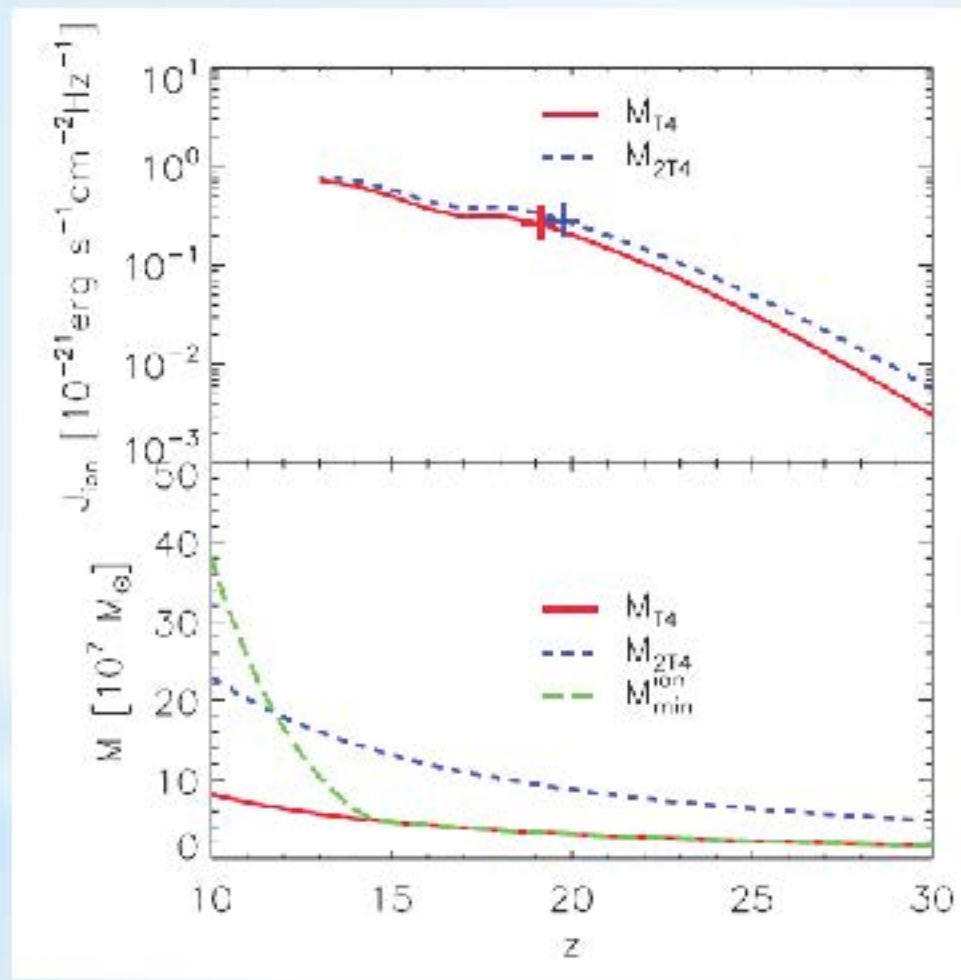
$$\frac{dn_{\text{BH}}}{dz}(z) = [1 - p_g(M'_{\text{T4}}, z')] p_{\text{J-w}}(M'_{\text{T4}}, z') \times \int_{M_{\text{T4}}}^{M_{\text{T4}}'} dM \frac{dn}{dM}(z) \frac{dP}{dz'}(z', M'_{\text{T4}} | M, z) \frac{dz'}{dz}.$$

# The DCBH abundance



| Model | $J_{\text{LW}}^{\text{crit}}$ | $l_{\text{min}}$ | $f_{\text{ion}}$ | Genetic enrichment |
|-------|-------------------------------|------------------|------------------|--------------------|
| A     | 50                            | 1                | 0.2              | Yes                |
| B     | 150                           | 1                | 0.3              | No                 |
| C     | 30                            | 2                | 0.3              | Yes                |

# The role of photo-evaporation



- Clustering of  $M_{\text{T4}}$  halos
- The DCBH era ends at  $z \sim 13$ .
- leaves behind a large number of ‘fossil’ DCBH seeds ( $\Omega_{\text{BH}} \sim 10^{-5}$ ).

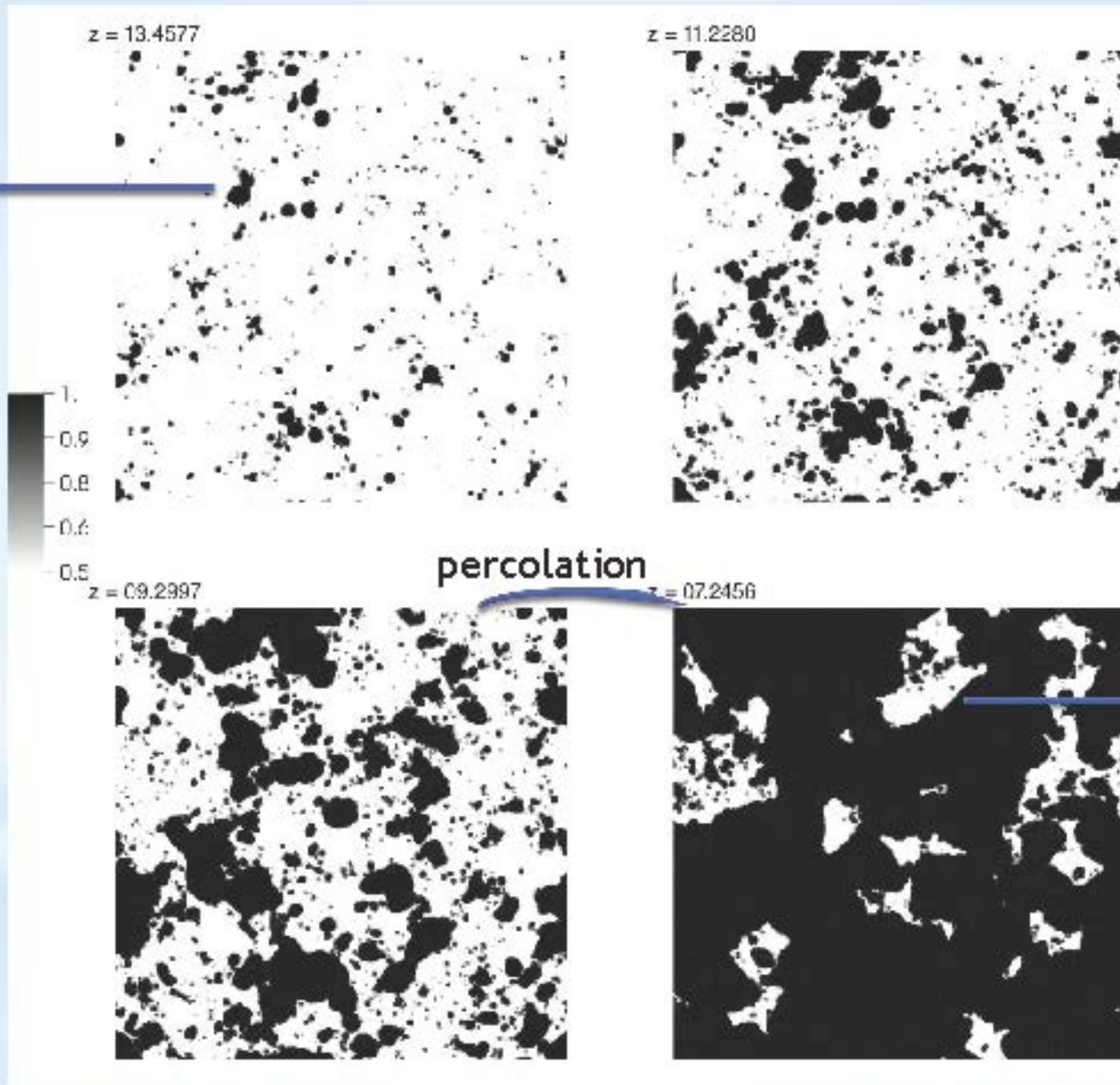
The DCBHs play no active role in driving reionization.

But reionization ends the DCBH era.



The epoch of reionization

bubbles



islands

# Analytical models of reionization

- \* Early stage – the “bubble model” (Furlanetto et al. 2004)

  - growing ionized bubbles

- \* Late stage – the “island model”

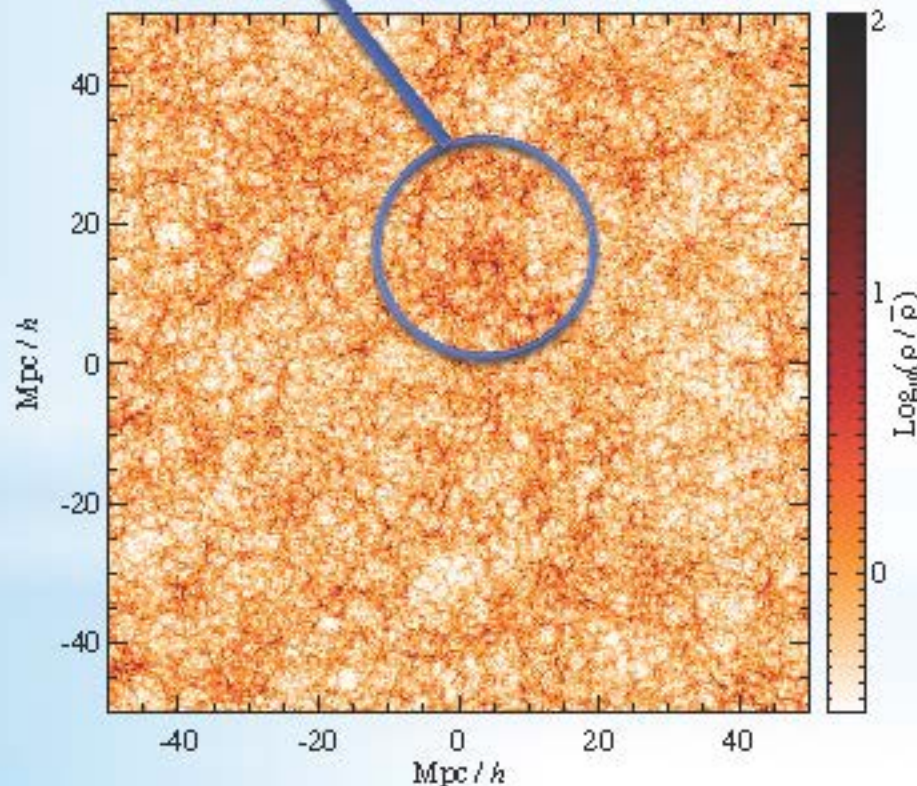
  - shrinking neutral islands

**→ Both based on the excursion set theory**

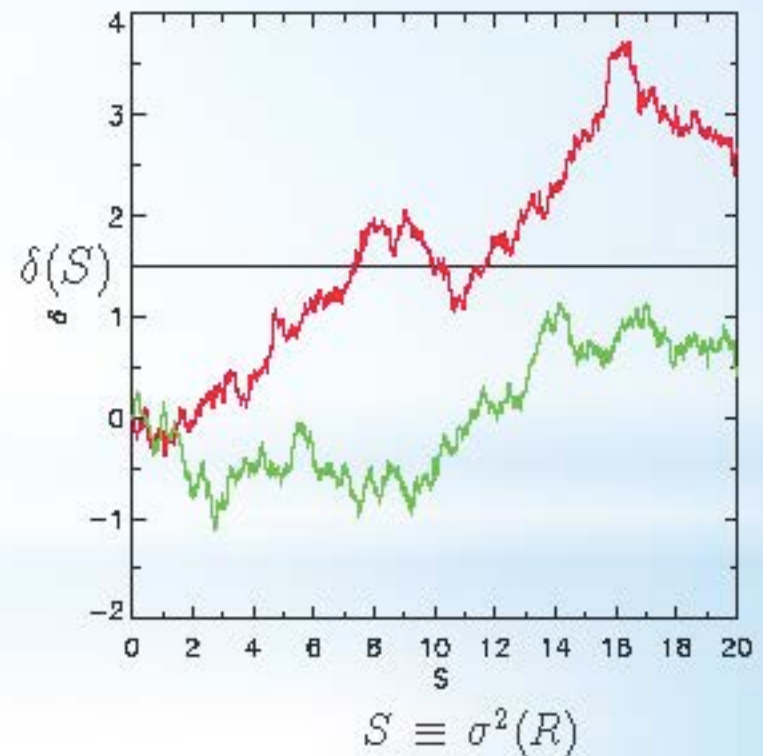
# The Excursion Set Theory of Halo Model

(Bond et al. 1991, Lacey & Cole 1993)

$$\delta(\vec{x}; R) \equiv [\rho(\vec{x}) - \rho_M] / \rho_M$$

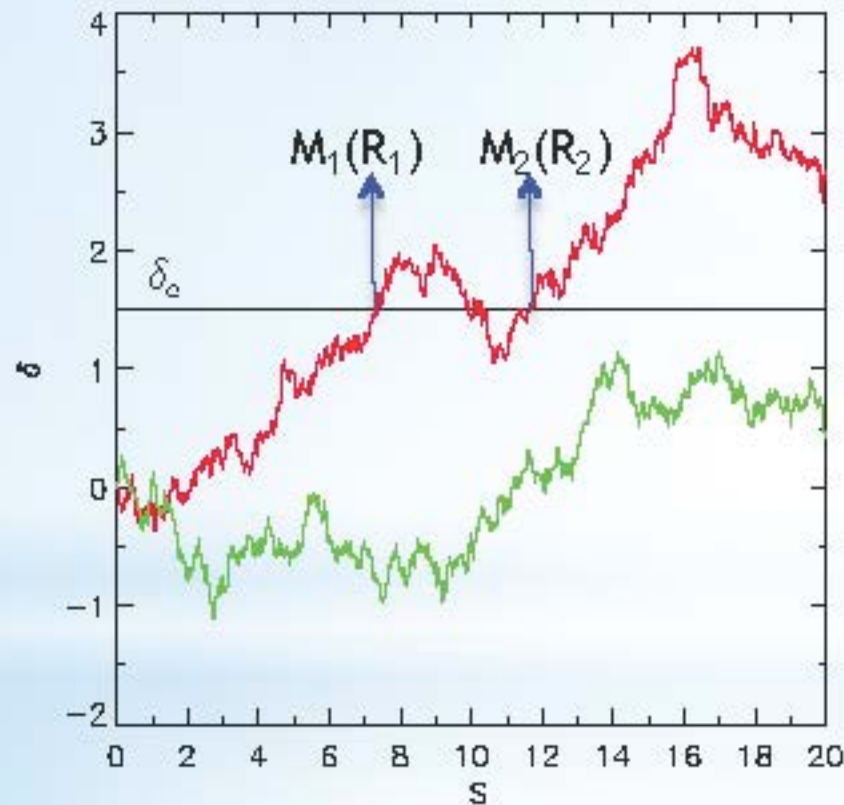


- k-space top-hat window function



# The Excursion Set Theory of Halo Model

(Bond et al. 1991, Lacey & Cole 1993)



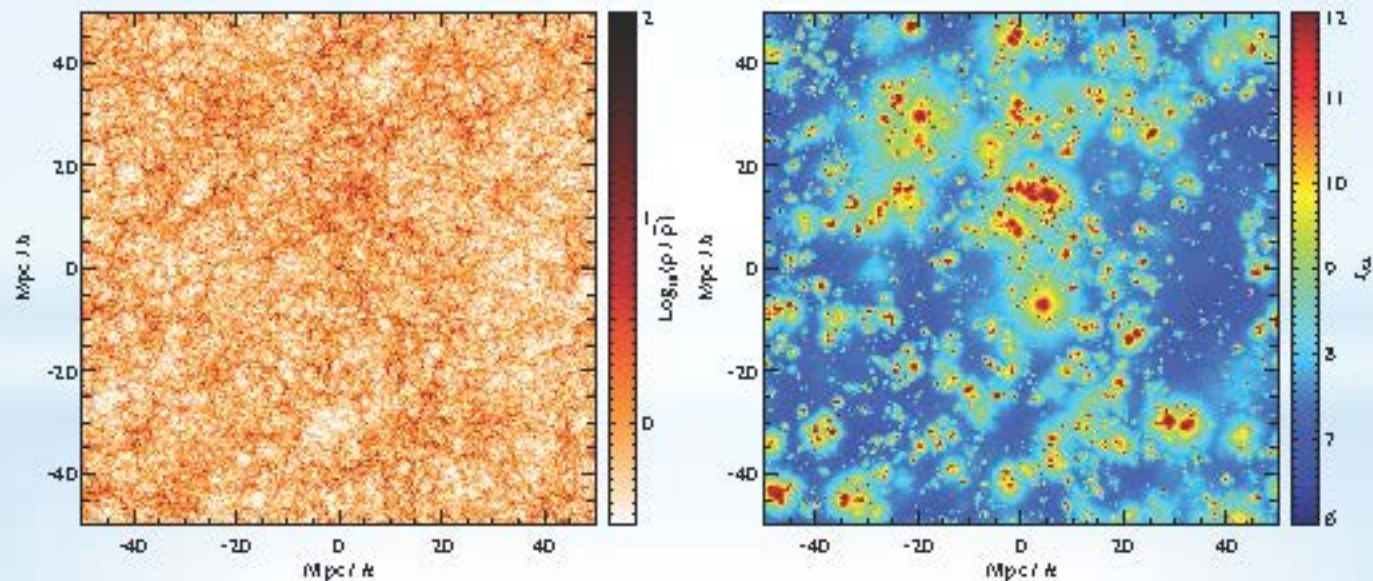
- *The cloud-in-cloud problem*



- Solving a diffusion equation  $\rightarrow$   
“*first-crossing distribution*”  $\rightarrow$   
halo mass function

# Why excursion set theory?

→ *The reionization field follows the density field on large scales (Battaglia et al. 2013)*



(From Battaglia et al. 2013 ApJ, 776, 81)

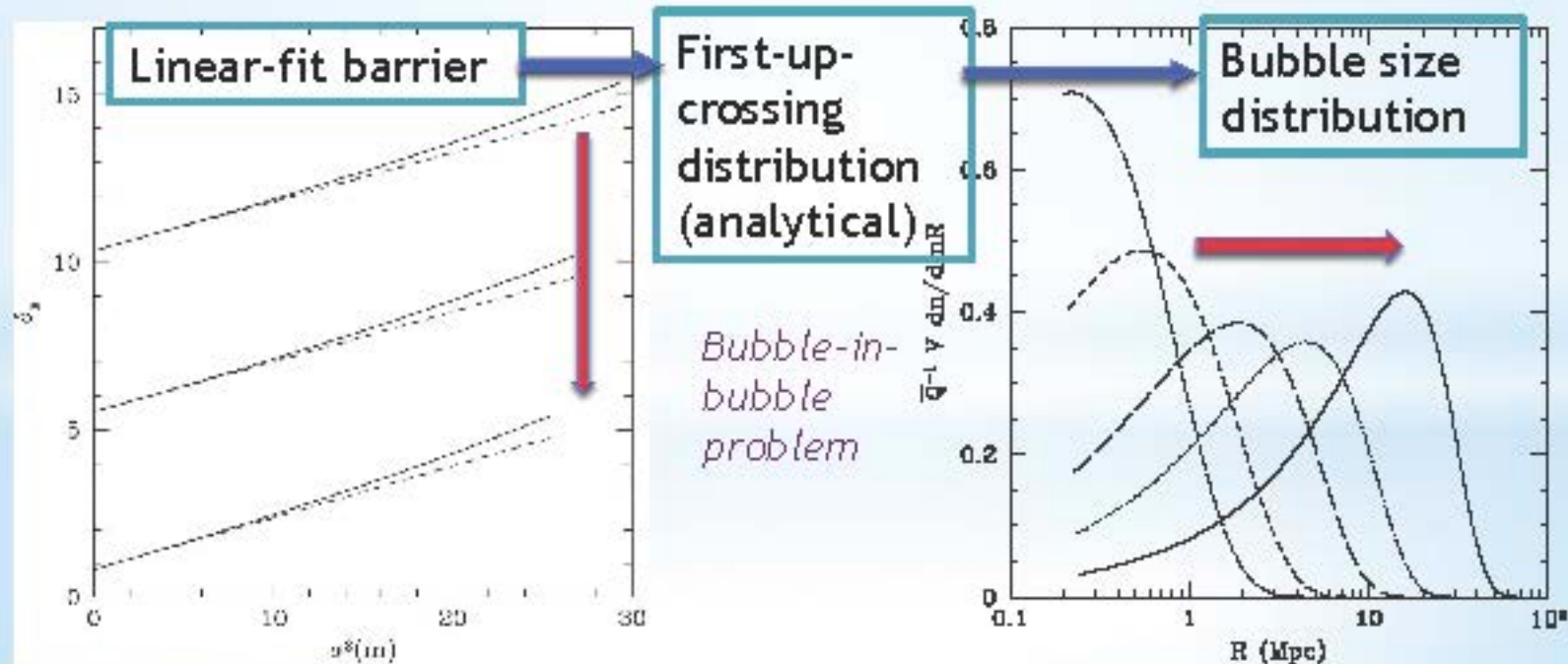
# The Excursion Set Approach for ionized bubbles

## - The bubble model of reionization

(Furlanetto et al. 2004)

- \* Relate the ionization field to the initial density field
- \* Ask whether an isolated region of mass  $M$  can be fully self-ionized.

$$f_{\text{coll}} > f_x - \zeta^{-1} \longrightarrow \delta_m > \delta_x(m, z) \quad \delta_c(z) \quad \sqrt{2K(\zeta)} [\sigma_{\text{min}}^2 \quad \sigma^2(m)]^{1/2}$$



## However, after percolation...

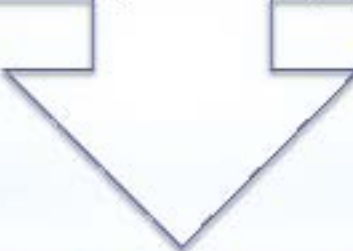
1. The isolated and spherical assumption for the ionized bubbles breaks down

→ the neutral islands are more isolated

2. The existence of an ionizing background

→ the shape of barriers could be changed

(the linear fit may not apply)



### The island model

It would be relatively easier for the upcoming instruments to probe the signal at the late reionization stages.

# The Island Model

- \* Negative island barrier (“inside-out” reionization)
- \* Island mass scales are identified by *first-down-crossings* through the island barrier (but not the “never-up-crossing” distribution).
- \* With the inclusion of an ionizing background, the condition of keeping from being ionized:

$$\xi f_{\text{cull}}(\delta_M; M, z) + \frac{\Omega_m}{\Omega_b} \frac{N_{\text{back}} m_{\text{II}}}{M X_{\text{II}} (1 + \bar{n}_{\text{rec}})} < 1,$$

→ The island barrier:

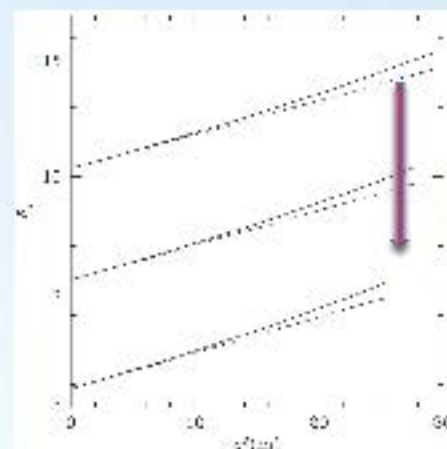
$$\delta_M < \delta_I(M, z) \equiv \delta_c(z) - \sqrt{2[S_{\text{max}} - S(M)]} \operatorname{erfc}^{-1}[K(M, z)],$$

$$K(M, z) = \xi^{-1} \left[ 1 - \boxed{N_{\text{back}}} (1 + \bar{n}_{\text{rec}})^{-1} \frac{m_{\text{H}}}{M (\Omega_b / \Omega_m) X_{\text{H}}} \right].$$

the integral number of background ionizing photons consumed by an island during the time interval between the setup of an ionizing background and the redshift under consideration.

# The Island Model

\* Define the “**background onset time**” as the time at which the barrier curve passes through the origin point on the  $\delta - S$  plane



$$\delta_I(S=0; z=z_{\text{back}}) = \delta_o(z_{\text{back}}) - \sqrt{2 S_{\text{max}}(z_{\text{back}})} \operatorname{erfc}^{-1}(\xi^{-1}) = 0.$$

\* We take  $\{f_{\text{esc}}, f_*, N_{\gamma/\text{H}}, \bar{n}_{\text{rec}}\} = \{0.2, 0.1, 4000, 1\}$  as the fiducial set of parameters, so that  $\xi = 40$  and  $z_{\text{back}} = 8.6$ .

- \* Solving for the first-down-crossing distribution (Zhang & Hui 2006):  
(the “island-in-island” problem is naturally solved)

$$f_I(S_I) = -g_1(S_I) - \int_0^{S_I} dS' f_I(S') [g_2(S_I, S')],$$

$$g_1(S_I) = \left[ \frac{\delta_I(S_I)}{S_I} - 2 \frac{d\delta_I}{dS_I} \right] P_0[\delta_I(S_I), S_I], \quad P_0(\delta, S) = \frac{1}{\sqrt{2\pi S}} \exp\left(-\frac{\delta^2}{2S}\right)$$

$$g_2(S_I, S') = \left[ 2 \frac{d\delta_I}{dS_I} - \frac{\delta_I(S_I) - \delta_I(S')}{S_I - S'} \right] P_0[\delta_I(S_I) - \delta_I(S'), S_I - S'],$$

- \* The mass function of islands:

$$\frac{dn}{d \ln M_I}(M_I, z) = \bar{\rho}_{m,0} f_I(S_I, z) \left| \frac{dS_I}{dM_I} \right|$$

- \* The volume fraction of neutral regions:

$$Q_V^I = \int dM_I \frac{dn}{dM_I} V(M_I).$$

# The bubbles-in-island effect

\* Solving for a two-barrier problem:

1 - The first down-crossing distribution of random walks w.r.t. **island barrier**:

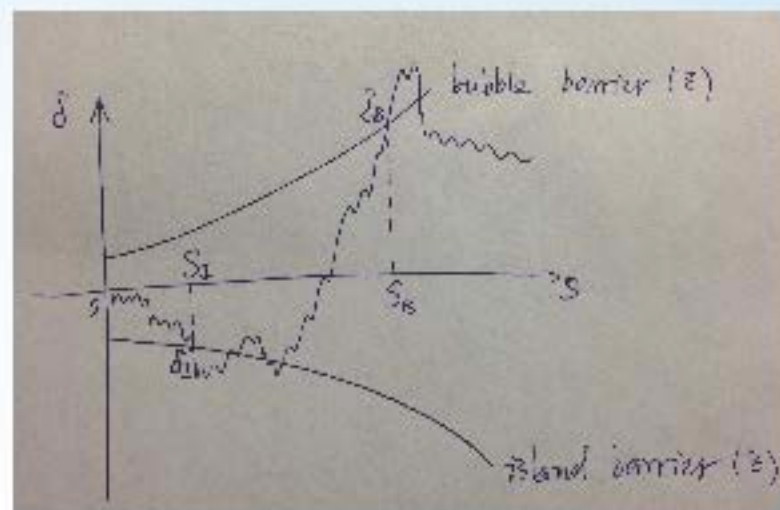
$$f_I(S_I, z)$$

2 - The conditional first up-crossing

distribution w.r.t. **bubble barrier**:  $f_B[S_B, \delta_B | S_I, \delta_I]$

\* The effective bubble barrier:

$$\delta'_B = \delta_B(S + S_I) - \delta_I(S_I) \quad \text{where } S = S_B - S_I.$$



# The bubbles-in-island effect

\* The bubbles-in-island fraction:

$$q_B(S_I, \delta_I; z) = \int_{S_I}^{S_{\max}(\xi, M_{\min})} [1 + \delta_I D(z)] f_B[S_B, \delta_B | S_I, \delta_I] dS_B.$$

\* The neutral island mass function:

$$\frac{dn}{dM}(M, z) = \frac{dn}{dM_1} \frac{dM_1}{dM} = \frac{\bar{\rho}_{m,0}}{M_1} f_1(S_1, z) \left| \frac{dS_1}{dM_1} \right| \frac{dM_1}{dM}.$$

$$M = M_1(S_1) [1 - q_B(S_1, \delta_I; z)]$$

# The ionizing background

- \* Considering the effect of *Lyman limit systems* on the mean free path of ionizing photons, the comoving number density of background ionizing photons is

$$n_{\gamma}(z) = \int_z \bar{n}_{\text{H}} \left| \frac{df_{\text{coll}}(z')}{dz'} \right| f_{*} N_{\gamma/\text{H}} f_{\text{esc}} \exp \left[ -\frac{l(z, z')}{\lambda_{\text{mfp}}(z)} \right] dz',$$

- \* With the MHR00 model for the volume-weighted density distribution of the IGM (Miralda-Escude et al. 2000),

$$P_V(\Delta) d\Delta = A_0 \exp \left[ -\frac{(\Delta^{-2/3} - C_0)^2}{2 (\delta_0/3)^2} \right] \Delta^{-\beta} d\Delta$$

the mean free path of ionizing photons can be written as

$$\lambda_{\text{mfp}} = \frac{\lambda_0}{[1 - F_V(\Delta_{\text{crit}})]^{2/3}},$$

# The ionizing background

- \* The critical relative density for a clump to self-shield

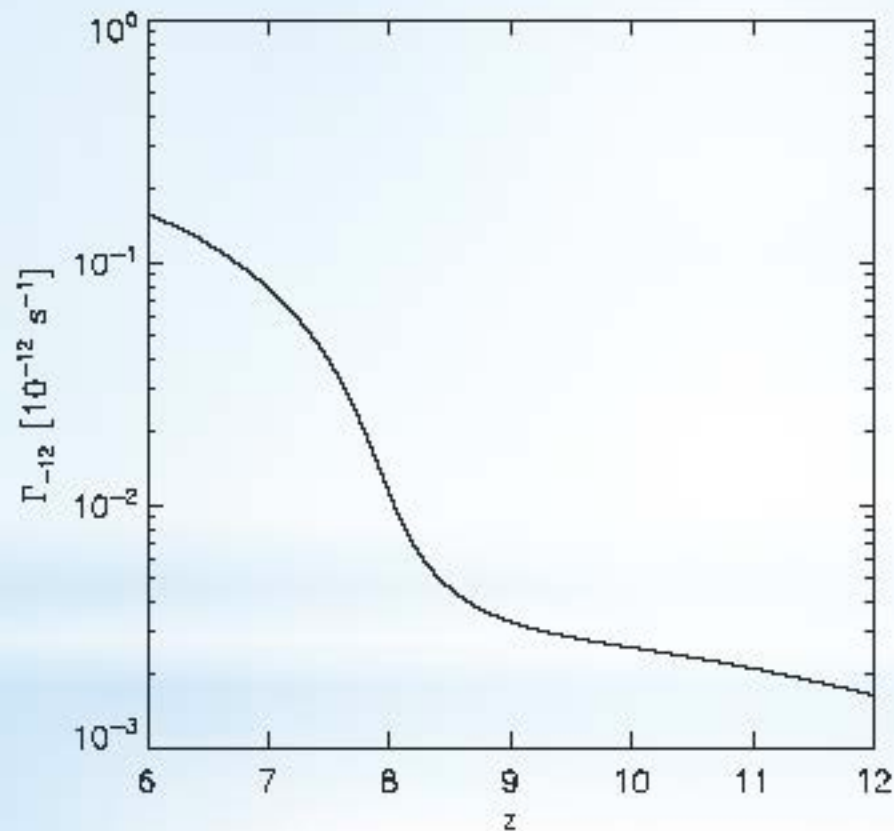
$$\Delta_{\text{crit}} = 36 \Gamma_{-12}^{2/3} T_4^{2/15} \left( \frac{\mu}{0.61} \right)^{1/3} \left( \frac{f_{\varepsilon}}{1.08} \right)^{-2/3} \left( \frac{1+z}{8} \right)^{-3}$$

- \* The HI photoionization rate  $\Gamma_{\text{HI}}$  is related to the total number density of ionizing photons  $n_{\gamma}$  by

$$\Gamma_{\text{HI}} = \int \frac{dn_{\gamma}}{d\nu} (1+z)^3 c \sigma_{\nu} d\nu,$$

- \* Scaling the hydrogen photoionization rate to be  $\Gamma_{\text{HI}} = 10^{-12.8} \text{ s}^{-1}$  at redshift 6, as suggested by recent measurements from the Ly- $\alpha$  forest (Wyithe & Bolton 2011; Calverley et al. 2011)

# The ionizing background



\* Consistent with our definition of the “background onset time”

# The island-vS model

- \* We assume that the photons consumed by an island at any instant is proportional to its surface area, then

$$N_{\text{back}} = \int F(z) \Sigma_I(t) dt, \quad F(z) = \bar{n}_\gamma(z) (1+z)^3 c/4.$$

- \* For a spherical island,

$$\bar{n}_H(R)(1 + \bar{n}_{\text{rec}}) 4\pi R^2 (-dR) = F(z) \frac{4\pi R^2}{(1+z)^2} dt,$$

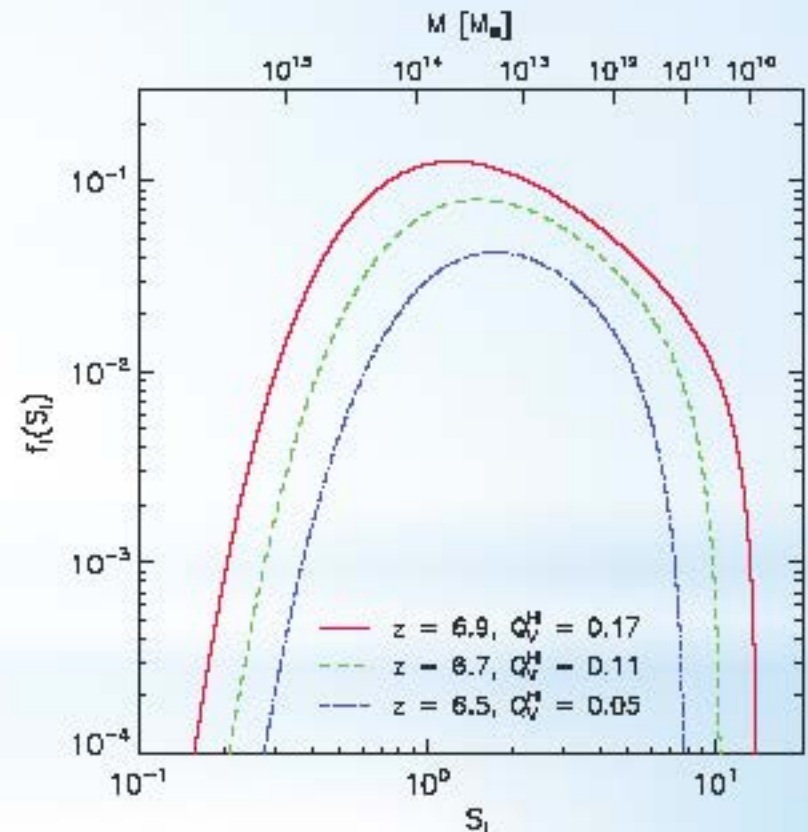
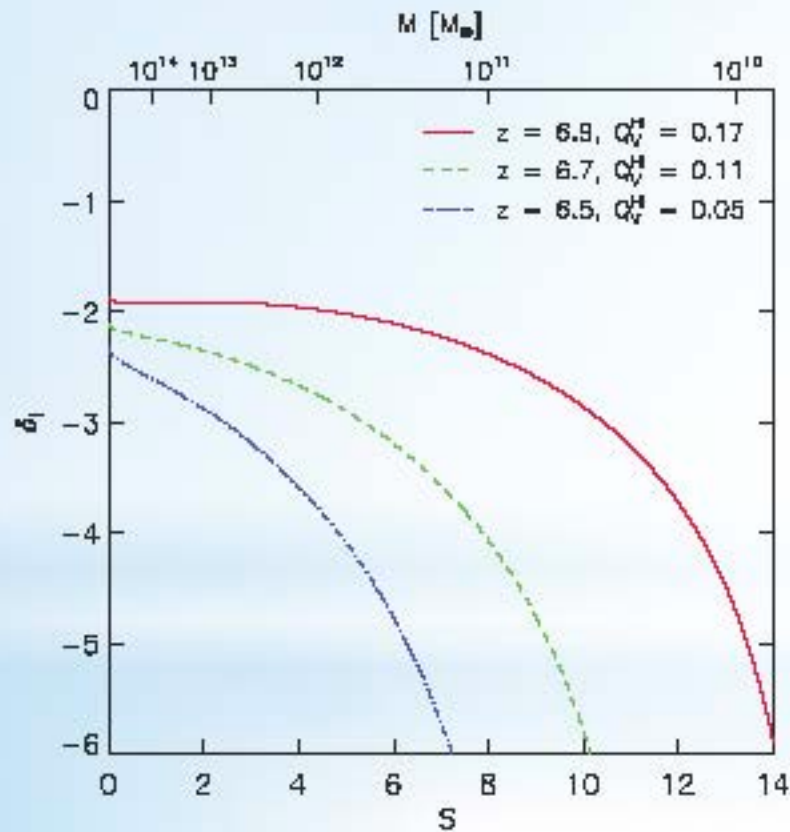
$$\Delta R \equiv R_i - R_f = \int_z^{z_{\text{back}}} \frac{F(z)}{\bar{n}_H(1 + \bar{n}_{\text{rec}})} \frac{dz}{H(z)(1+z)^3},$$

- \* The total number of background ionizing photons consumed is

$$N_{\text{back}} = \frac{4\pi}{3} (R_i^3 - R_f^3) \bar{n}_H(1 + \bar{n}_{\text{rec}}),$$

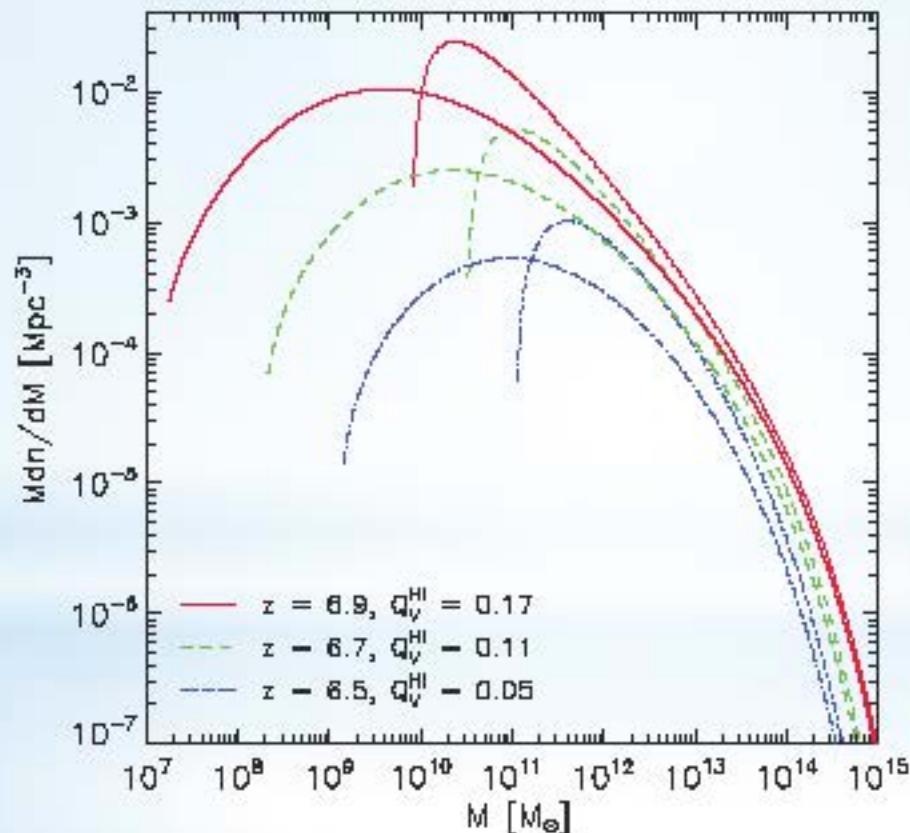
# The island- $\nu S$ model

- the island barrier and first down-crossing distribution



# The island- $\nu$ S model

## - the host island mass function

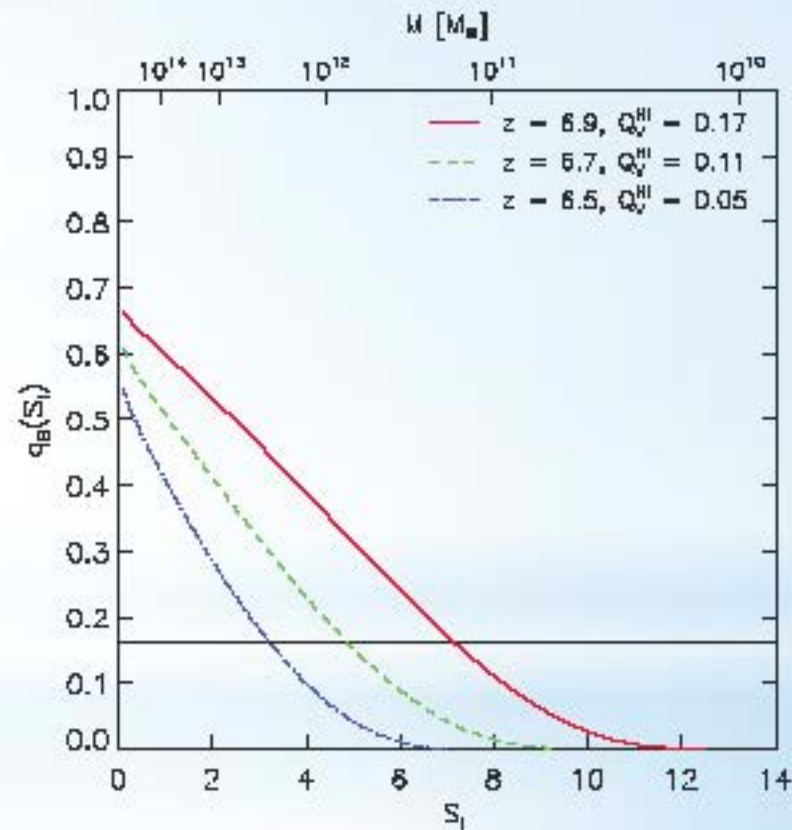
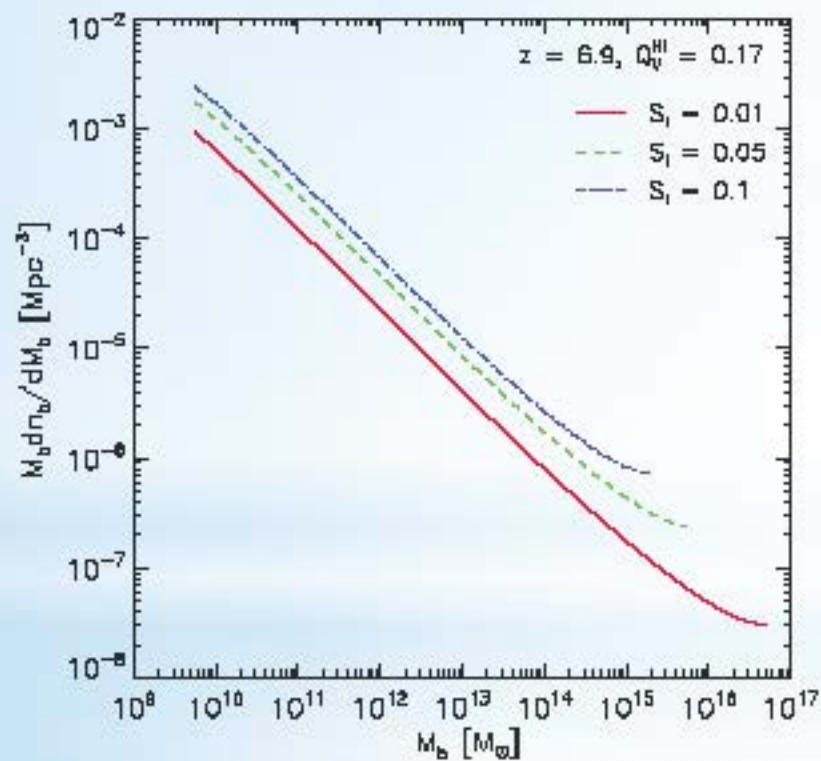


➤ The shrinking hosts

$$\frac{M_f}{M_i} = \left(1 - \frac{\Delta R_i}{R_i}\right)^3$$

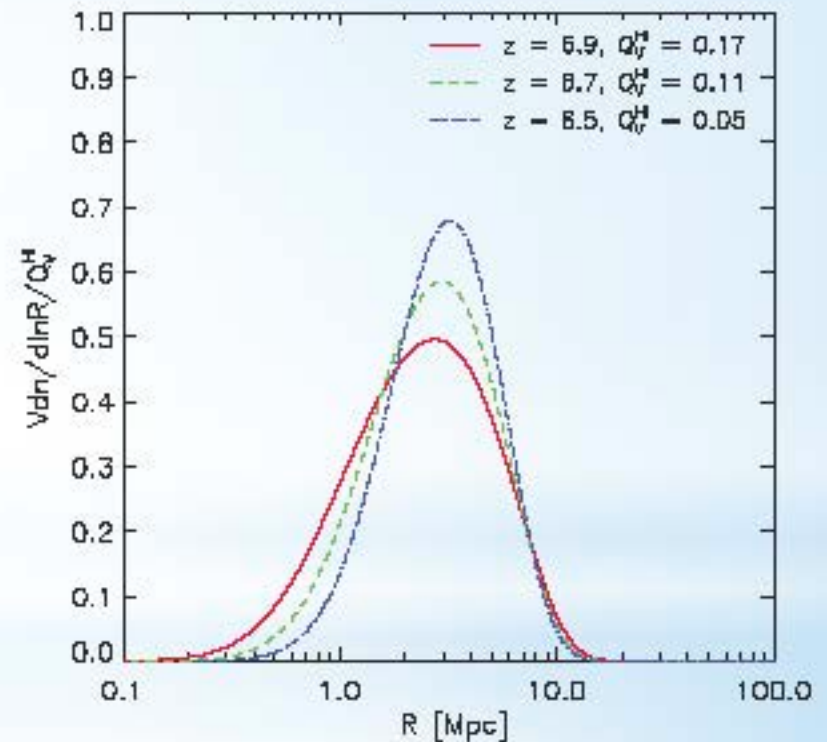
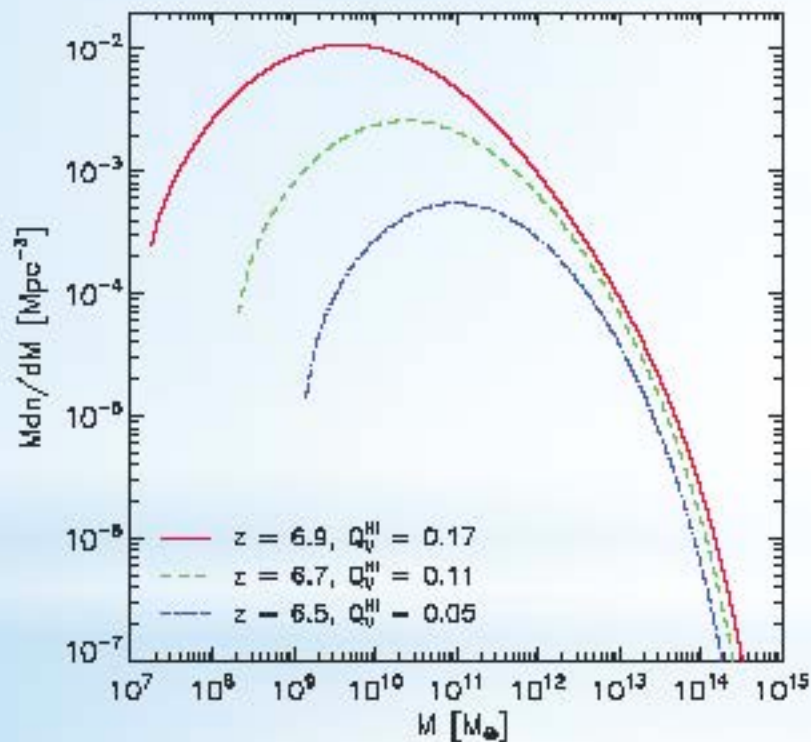
# The island- $\nu$ S model

- the bubbles-in-island

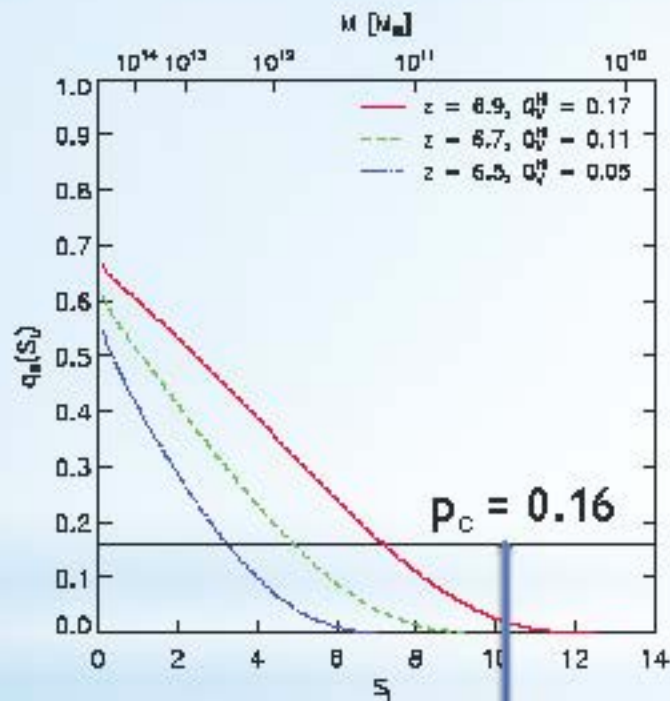


# The island- $\gamma$ S model

## - the mass function and size distribution



# The problem of large bubbles-in-island fraction



- \* Host islands → overestimate the neutral fraction
- \* Neutral islands → not the real image
- \* Difficult to visually identify the host islands
- \* Break down of bubble model inside islands

# The role of percolation threshold $p_c$

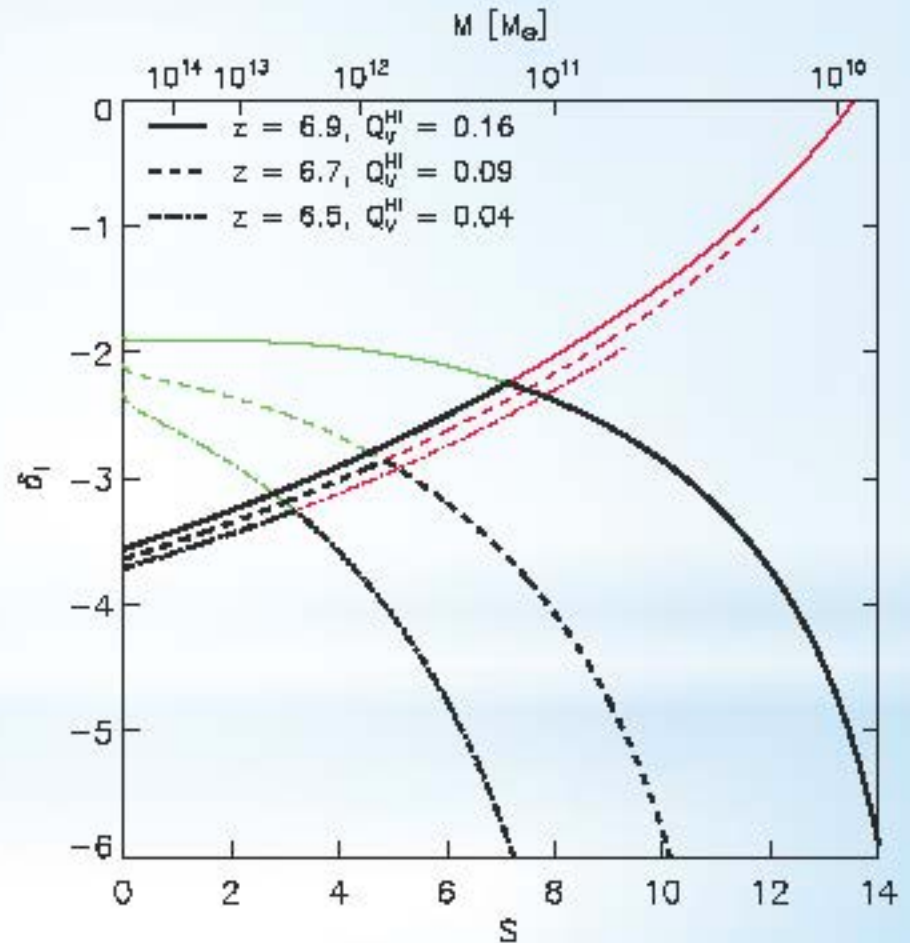
- \* The bubble model regime:  $z > z_{Bp}$  ( $x_{HI} < p_c$ )
- \* The island model regime:  $z < z_{Ip}$  ( $x_{HI} < p_c$ )
- \* The background onset redshift:  $z_{Bp} > z_{back} > z_{Ip}$
- \* The definition of bona fide neutral islands:  $q_B < p_c$

# The percolation criterion

\* The additional barrier is obtained by solving

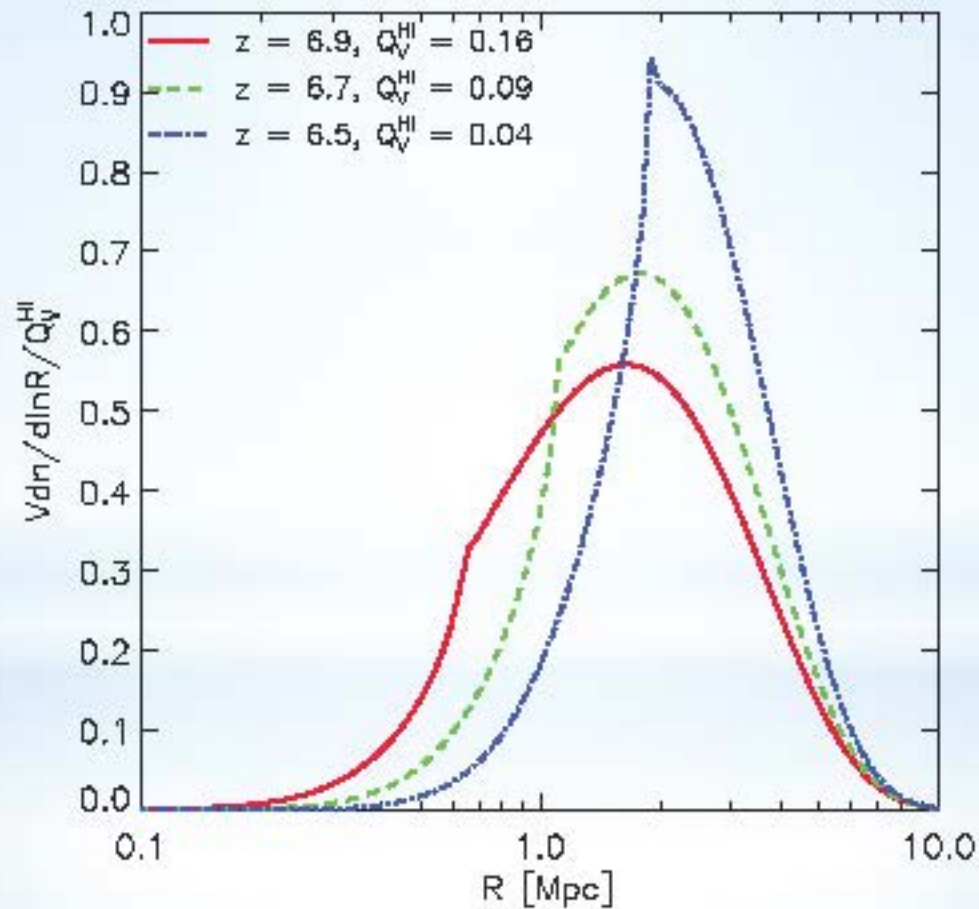
$$q_B(S_l, \delta_l; z) < p_c$$

\*  $p_c = 0.16$  for Gaussian random fields



# Results

- the size distribution with  $p_c$  cutoff



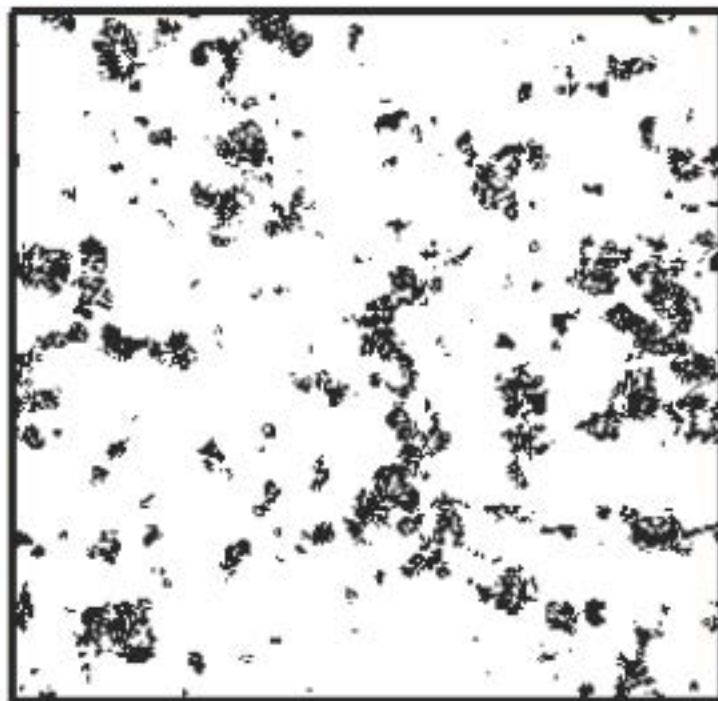
# Summary

- \* It is challenging to explain the measured level of NIRB fluctuations, the source-subtracted signal is  $>10$  times higher than expected from distant galaxies below the detection limit.
- \* We propose that the Compton-thick, intermediate-mass BHs formed through direct collapse are possible sources.
- \* This model predicts the NIRB angular power spectrum and the NIRB-CXB cross-correlation agrees well with observations.
- \* DCBHs themselves are bright at the LW band, they can trigger the formation of more other DCBHs, and a runaway formation process is ignited.
- \* This process is quenched by the reionization after  $z \sim 13$ .

# Summary

- \* **An analytical model of neutral islands during the late EoR based on the excursion set theory.**
  - \* An island barrier on the density contrast for the islands to remain neutral, with the inclusion of an ionizing background.
  - \* An island was identified when the random walk first-down-crosses the island barrier.
  - \* We took into account the effect of bubbles-in-island by computing the conditional first up-crossing distribution.
  - \* An semi-empirical way to determine the intensity of the ionizing background self-consistently.
  - \* A percolation criterion was applied to find bona fide neutral islands
- \* **The size distribution of neutral islands shows a peak indicating a characteristic scale of the islands, and it does not change much with redshift.**

THANK YOU!



512 Mpc,  $1024^3$

## low redshift faint galaxies ( $z < 5$ )

\* Helgason et al. 2012:

- \* get the number count of galaxies in multiband observations of LFs
- \* extrapolate to faint magnitude
- \* reconstruct the NIRB fluctuations
- \* galaxies clustering is from the halo model

# high redshift faint galaxies ( $z > 5$ )

\* Yue et al. 2013:

- \* reionization (the escape fraction, Mitra et al. 2012, 2013)

- \* observed LFs

- \* simulations (Salvaterra et al. 2011)

- $10 h^{-1} \text{Mpc}$ , including gas dynamics, radiative cooling, supernova explosion, photoionization and heating, and a detailed treatment of chemical enrichment.

- agree well with the observed LF!

# A toy model - island- $\gamma$

- The ionizing photons permeated through the neutral islands with a uniform density (X-rays)
  - Extremely large mean free path
  - Neglecting the absorption by dense clumps
- The averaged number density of the background ionizing photons

$$n_{\gamma} = \bar{n}_{\text{H}} f_{\text{coll}}(z) f_{*} N_{\gamma/\text{H}} f_{\text{esc}} - (1 - Q_{\text{V}}^{\text{I}}) \bar{n}_{\text{H}} (1 + \bar{n}_{\text{rec}}),$$

# A toy model - island-V

$$\delta_M < \delta_I(M, z) \equiv \delta_c(z) - \sqrt{2[S_{\max} - S(M)]} \operatorname{erfc}^{-1}[K(M, z)],$$

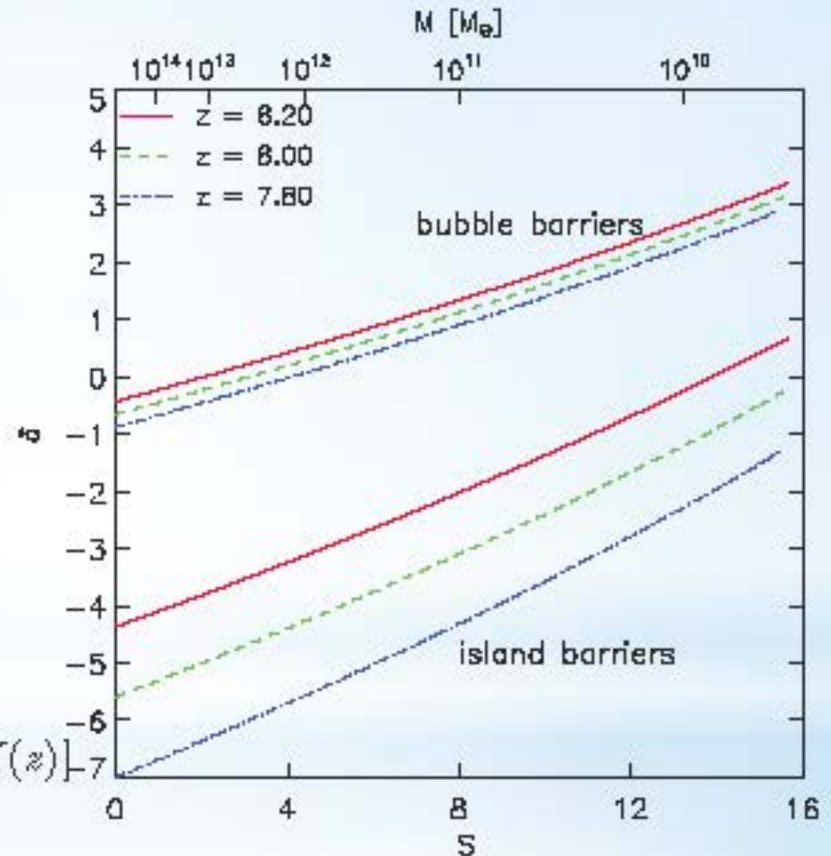
$$K(M, z) = \xi^{-1} \left[ 1 - N_{\text{back}}(1 + \bar{n}_{\text{rec}})^{-1} \frac{n_H}{M(\Omega_b/\Omega_m)X_H} \right].$$

➤  $N_{\text{back}}$  proportional to  $V$

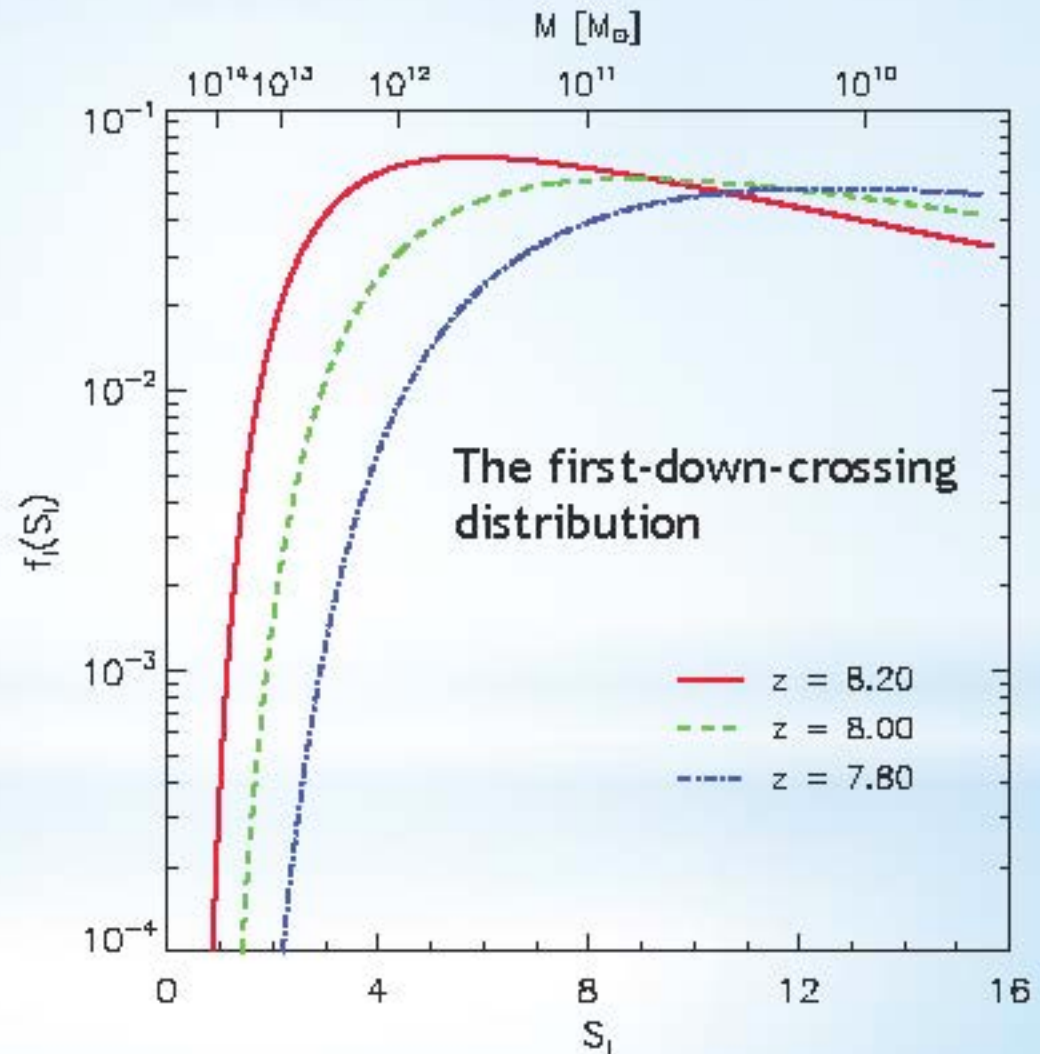
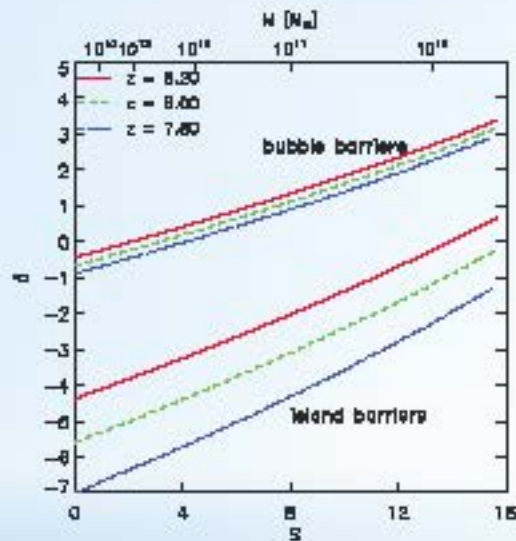
$$N_{\text{back}}/M = n_\gamma/\bar{\rho}_m$$



$$\delta_I(M, z) = \delta_c(z) - \sqrt{2[S_{\max} - S(M)]} \operatorname{erfc}^{-1}[K(z)]$$

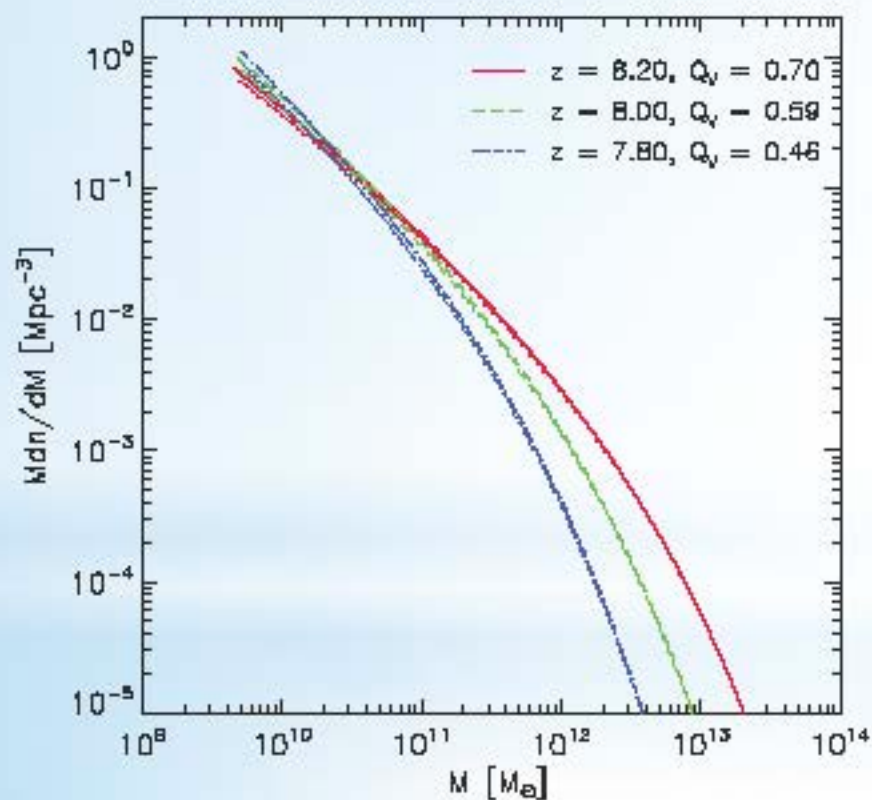


# A toy model - island-V



# A toy model - island-Y

Mass function of islands



Size distribution of islands

