Solitons on intersecting D3-branes

based on

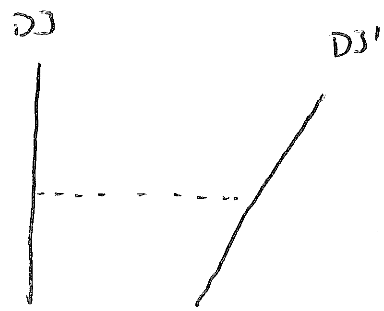
Cottrell, AH, Pillai 1406.5872

Cottrell, AH, Pettengill, Pillai in prep

Basic issue.

Consider a D3 and a D3'

D3: 01 45
D3': 01 67



$\rightarrow\!|\ \Delta X_8 = \alpha' v\ |\!\leftarrow$

- 8 supercharges
- parameters g_s, α', v
- Low energy spectrum
  - 3+1 $N=4$ $U(1)$ on D3 & D3'
  - 33' strings B, C (mass $= v$) $N=2$ $D=4$ hypers reduced to 1+1

question: is there a sensible $\alpha' \to 0$

keeping v, g_s fixed?

- Seem sensible enough. Keep strings with mass $m^2 \sim \mathcal{O}(v^2)$ or less. Discard states with mass order $\mathcal{O}(\frac{1}{\alpha'})$

- Supersymmetric, gauge invariant Lagrangian with the appropriate field content and global symmetry easy to write down.

(1402.6327)

- Minton, Polchinski, and Sun asked.

  Does this Lagrangian support BPS soliton corresponding to a D1 stretched between D3 & D3'

  - mass $\sim \frac{1}{g} v < \frac{1}{\alpha'}$
  - Appears to be required by duality
  - Answer appears to be a "No"

Outline of this talk.

(1) Brief review of Minton, Polchinski, & Sun

(2) Perspective from the non-abelian flux construction 1406.5872

(3) Perspective from gauge-gravity duality.

- our analysis suggests that the effective field theory of interest <u>exists</u> and is a consistent dynamical system, <u>contrary to MPS</u>

Minton - Polchinski - Sen

$$S = \frac{1}{g_{ym}^2} \int dx^0 dx^1 dx^4 dx^5 \quad -\frac{1}{4} F_{ab} F^{ab} + (\ldots)$$

$$+ \frac{1}{g_{ym}^2} \int dx^0 dx^1 dx^6 dx^7 \quad -\frac{1}{4} F'_{ab} F'^{ab} + (\ldots)$$

$$+ \frac{1}{g_{ym}^2} \int dx^0 dx^1 \quad - \left| \left(\partial_m + \frac{i(A_m - A_m')}{2} \right) B \right|^2$$

$$- \frac{|\phi_s - \phi_s'|^2}{2} |B|^2$$

B, C electrically coupled to A, A'

$$- \left| \left(\partial_m - \frac{i(A_m - A_m')}{2} \right) C \right|^2$$

$$- \frac{|\phi_s - \phi_s'|^2}{2} |C|^2$$

Turns out the kink in B, C

magnetically coupled.

$$\partial_a \tilde{F}^{ab} = \frac{1}{2} \epsilon^{bc} \partial_c \left(|C|^2 - |B|^2 \right) \delta^2(x_4, x_5)$$

$$\partial_m \tilde{F}'^{mn} = \frac{1}{2} \epsilon^{np} \partial_p \left(|C|^2 - |B|^2 \right) \delta^2(x_6, x_7)$$

$$Q_m = \frac{1}{2} \left[\left(|C|^2 - |B|^2 \right)_{\infty} - \left(|C|^2 - |B|^2 \right)_{-\infty} \right]$$

Looked like they had found the magnetic soliton, but there was a problem.

if $|C|^2 - |B|^2 \neq 0$ semi infinitely

$|C|^2 + |B|^2 \neq 0$

$m^2 \sim v^2$

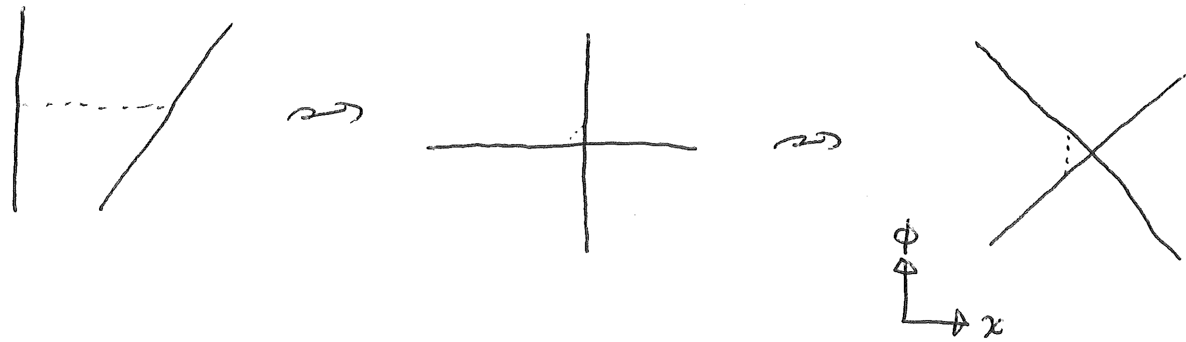
$\hookrightarrow$ kink has $\int_{\text{sem infinite}} v^2 \left(|C|^2 + |B|^2\right) = \infty$

Not the correct BPS mass.

MPS tried to "correct" this by generalizing the metric of the B, C target space
(kahler)

Despite their best efforts, the sigma model target space metric was singular at large field values $\Longrightarrow$ failure of Decoupling

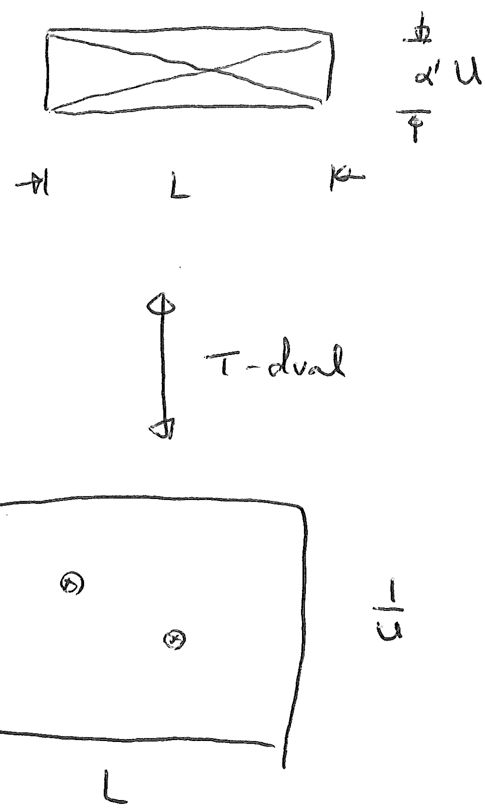
The "Tilted brane" construction



$\phi = c\,(x+iy)$

c: $(\text{MASS})^2$

Hashimoto & Taylor,
9703217



<u>Spectrum</u> (van Baal, 1984)

$U(1) \times U(1) \subset U(2)$

Tower of states

$m^2 \sim cN + v^2$

$c \to 0$ Tower $\Rightarrow$ plane wave.

$c \to \infty$ Decouple all but $N=0$

Q: Can we add a magnetic monopole to this background?

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D3: $\quad 01 \quad \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}_c$

D3' $\quad 01 \quad \begin{pmatrix} 2 \\ 4 \end{pmatrix}_c \begin{pmatrix} 3 \\ 5 \end{pmatrix}_{-c}$

D1 $\quad 0 \qquad 6$

$A_1 \; A_2 \; A_3 \qquad \Phi_1 \; \Phi_2 \; \Phi_3$

Problem naturally falls into the problem of finding (1/4) BPS solutions to SYM in 6d (Corrigan, Goddard, Fairlie, Nuyts; Bak, Lee, Park; AH, Ooguri, Yamazaki)

$\not{F} \chi = 0$ $\qquad$ BPS bound

$\Gamma^{1236} \chi = \Gamma^{1245} \chi = 0$

$$\frac{1}{4} \mathrm{Tr}\, F_{ab}^2 > \frac{1}{16} \epsilon_{abcdef} T_{ab} \mathrm{Tr}(F_{cd} F_{ef})$$

$F_{34} = -F_{65}$

$+$ 6 similar

$F_{36} = F_{45} - F_{12}$

$T_{12} = -1$

$T_{36} = -1$

$T_{45} = 1$

if $\Phi_4 = \Phi_5 = 0$: Prasad Sommerfield solution

$\Phi_4 + i\Phi_5 = c\,(x_1 + i x_2)\,\sigma_3$: Tilted brane.

$A_\mu = 0$

Can we find a solution which incorporates both the monopole charge and the tilt?

- Brute force?
- Generalize Nahm construction?

Couldn't do it (so far)

Argument that a solution exists

1) Start with PS solution $\phi_4 = \phi_5 = 0$
2) Simply set $\phi_4 + i\phi_5 = c\,(x_1 + i x_2)\,\sigma_3$

This is NOT a solution, but

1) Carries magnetic charge
2) Satisfies but does not saturate the BPS bound.

The solution we are after should be achievable by "cooling" this solution.

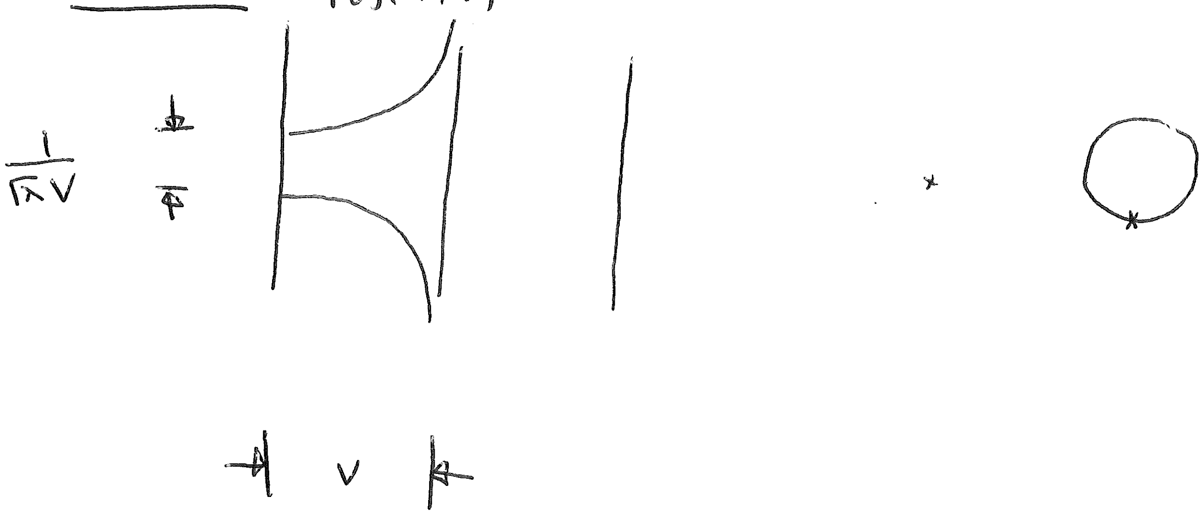
Perspective from Holography

N D3 $\leftarrow$ $AdS_5 \times S_5$

1 D3$'$ $\leftarrow$ probe

parameters: V, g_s, λ, $\alpha' \to 0$

Schwarz 1705.7444



DBI simplifies

$$\phi = V - \frac{1}{\sqrt{\lambda}} \frac{1}{r}$$

$$x = R^2 \phi = \sqrt{\lambda}\, \alpha' \phi$$

MASS: $\frac{1}{g} \sqrt{\lambda}\, V$

Seem Reasonably straightforward to accomodate the tilt.

$$X^6 = r\cos\theta$$
$$X^4 = r\sin\theta\cos\phi$$
$$X^5 = r\sin\theta\sin\phi\cos\alpha_1$$
$$X^7 = r\sin\theta\sin\phi\sin\alpha_1\cos\alpha_2$$
$$X^8 = r\sin\theta\sin\phi\sin\alpha_1\sin\alpha_2\cos\alpha_3$$
$$X^9 = r\sin\theta\sin\phi\sin\alpha_1\sin\alpha_2\sin\alpha_3$$

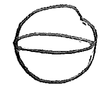
$\alpha_1, \alpha_2, \alpha_3 = 0$

ϕ - symmetric

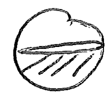
$r = \sqrt{\lambda}\,\alpha' u$

$u(\rho, z)$ $\theta(\rho, z)$

90°



other tilt:



$$u \sin\theta = c\rho$$

$$u \cos\theta = v$$

$$\tan\theta = \frac{c\rho}{v}$$

DBI action for $u(\rho, z)$ and $\theta(\rho, z)$ is <u>very</u> complicated.

<u>SUSY helps dramatically</u>

$$2\pi\alpha' F_{\phi z} = R^2 \left(u \sin^2\theta - \rho \partial_\rho u \right) \sec\theta$$

$$2\pi\alpha' F_{\rho\phi} = -\frac{R^2}{u\rho} \left(u^2\rho^2 + \sin^2\theta \right) \partial_z u \sec\theta$$

$$\left.\begin{aligned} \partial_\rho \theta &= \frac{u - \rho\partial_\rho u}{u\rho} \tan\theta \\ \partial_z \theta &= -\frac{1}{u} \tan\theta \, \partial_z u \end{aligned}\right\} \quad u \sin\theta = c\rho$$

Bianchi identity $\quad \phi_6 = u\cos\theta$

$$\left(\frac{1}{\rho} \partial_\rho \rho \partial_\rho + \partial_z^2 \right) \phi + \partial_z \frac{c^2}{(c^2\rho^2 + \phi_6^2)^2} \partial_z \phi_6 = 0$$

$\Rightarrow$ No square roots

$\Rightarrow$ Non-linear

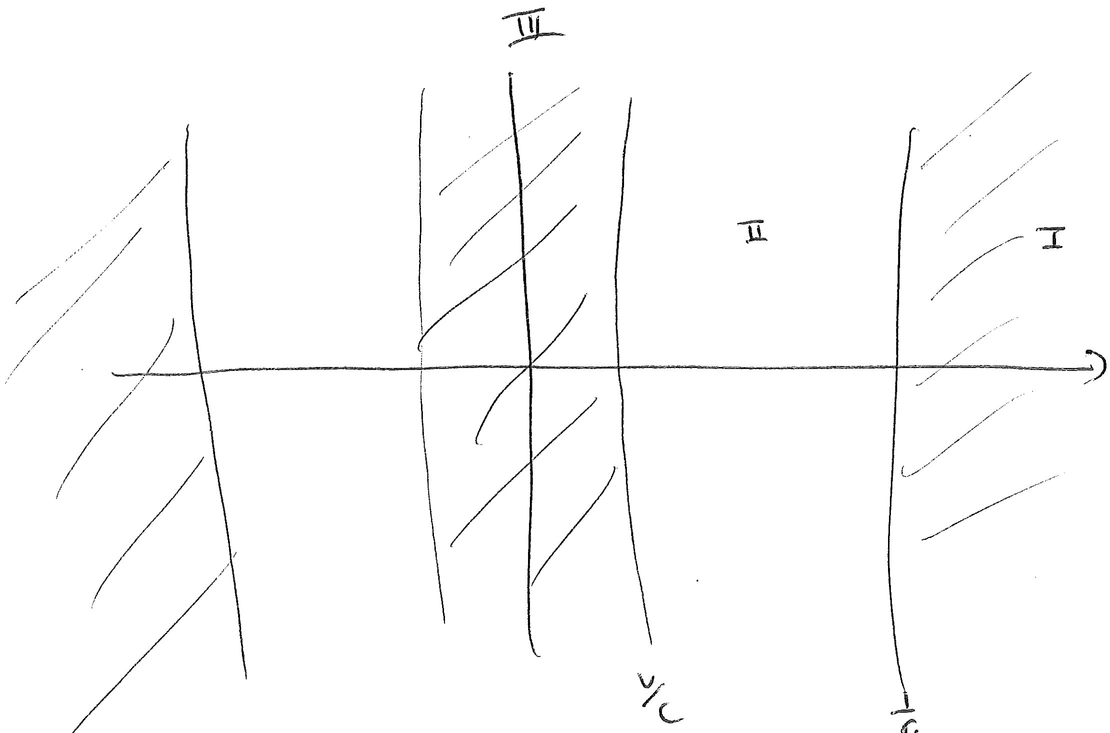
Can try to get some feel by studying linearized equation for "small" $\delta\phi$

$$\phi_6 = v + \delta\phi_6$$

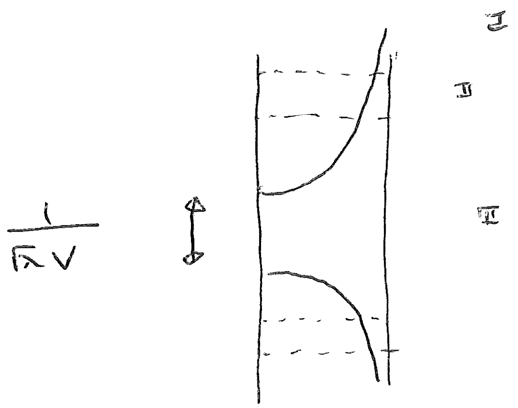
$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \partial_3^2\right)\phi_6 + \frac{c^2}{(c^2\rho^2+v^2)^2}\,\partial_\theta^2\phi_6$$

Laplace Equation on

$$-dt^2 + (dx^3)^2 + \left(1 + \frac{c^2}{(v^2+(c\rho)^2)^2}\right)(d\rho^2 + \rho^2 d\varphi^2)$$



$\underbrace{\qquad\qquad}_{\text{exists for}}$ $c > v^2$



Energy: integrate fixed ϕ from $\phi = 0$ to $\phi = v$

Gives $m = \frac{1}{3} \sqrt{\lambda}\, v$

When do linear analysis break down?

when $\rho < \frac{v}{c}$

or

when $\rho < \frac{1}{\sqrt{\lambda} v}$

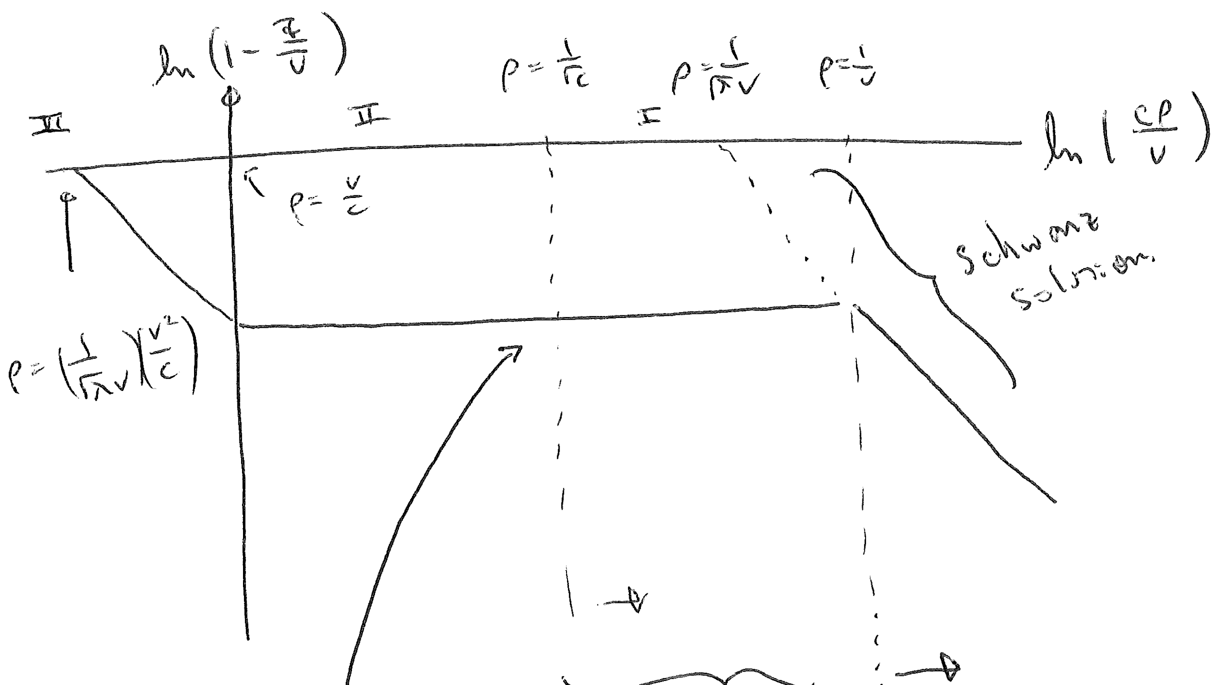
Now, imagine pushing $c \to \infty$ keeping

v, λ fixed

Consider now taking $\frac{c}{v^2} \gg \lambda$

metric in ρ direction rescaled by $\frac{c}{v^2}$ in Region III

useful way to plot Φ



in $c \to \infty$ keeping v, $\frac{c\rho}{v}$ fixed, this part gets pushed to infinity

AdS Boundary. No flux

flux extend in $\hat{z}$ direction

Conclusion : by scaling

$\tan\theta = \alpha' c$

c - fixed

$\alpha' \to 0$

create MPS with ∞ tower of

B, C states with mass

$$m^2 = cN + v^2$$

which appears to avoid soliton issues

is this tower regulating the UV

on MPS anticipated as necessary

$c \to \infty$ limit would decouple these tower

states

Perfectly sensible in gravity dual.

Perhaps we should look to fix MPS.