Core-collapse supernovae hybrid stars quark matter connection

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Ordinary Phase Transition

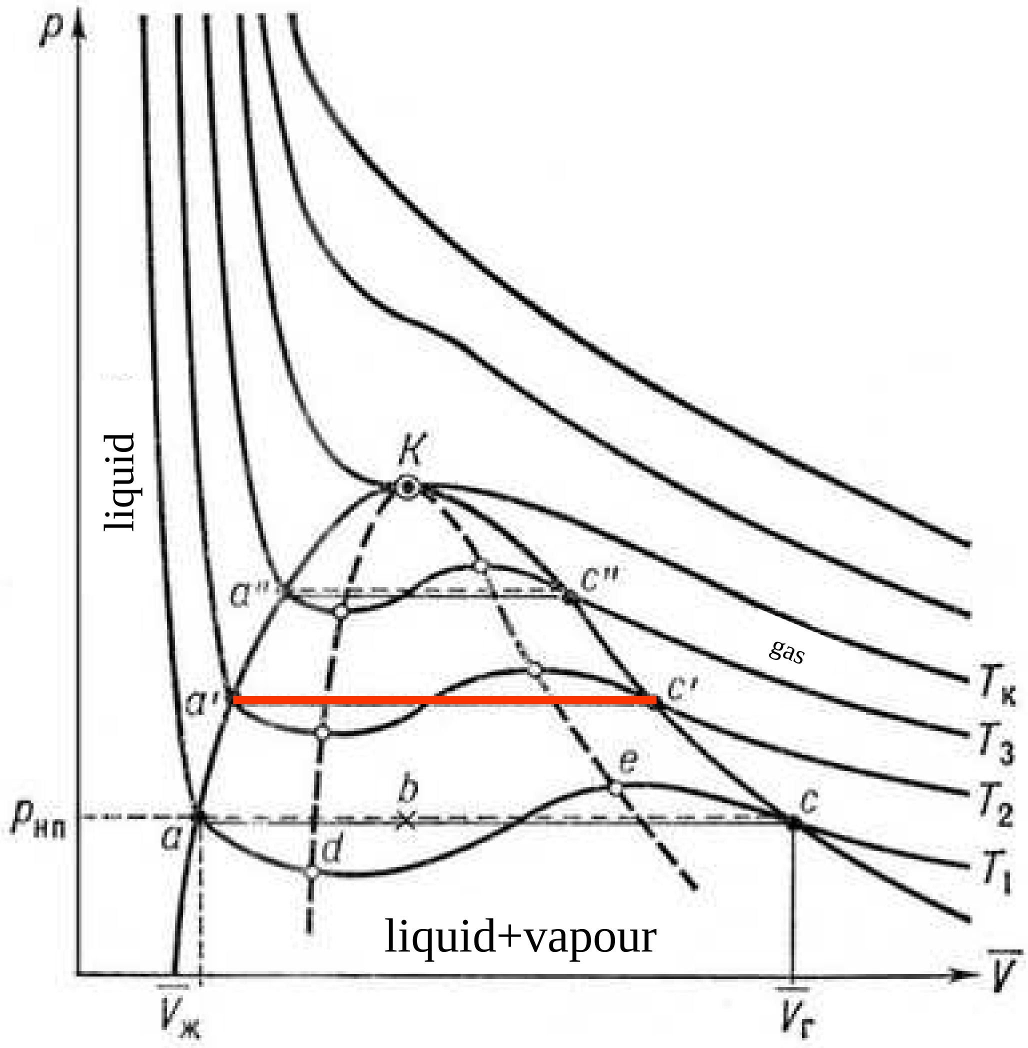
One-component matter
Maxwell-type PT

Phase coexistence conditions:

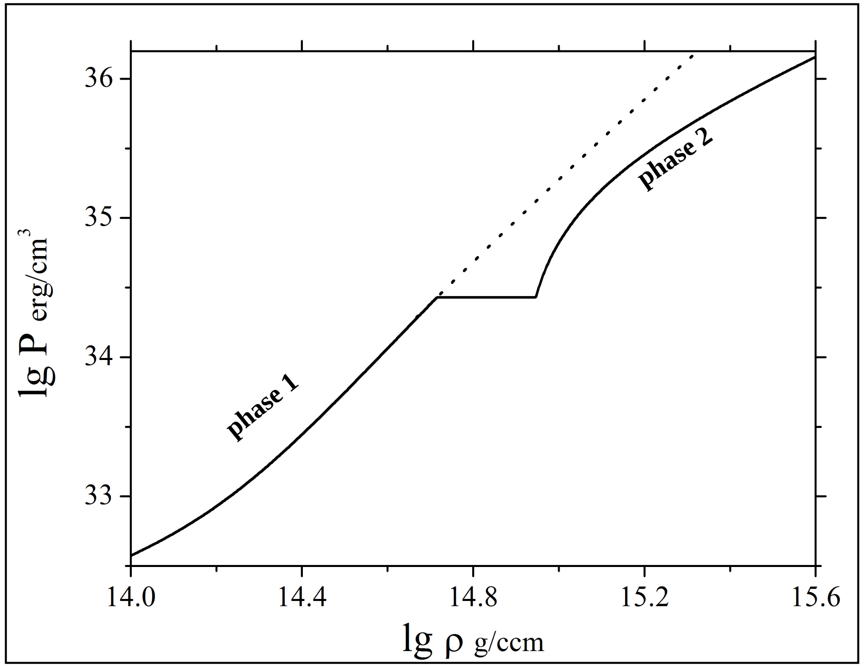
$$\begin{cases} P_I(\rho_I, T) = P_{II}(\rho_{II}, T) \\ \mu_I(\rho_I, T) = \mu_{II}(\rho_{II}, T) \end{cases}$$

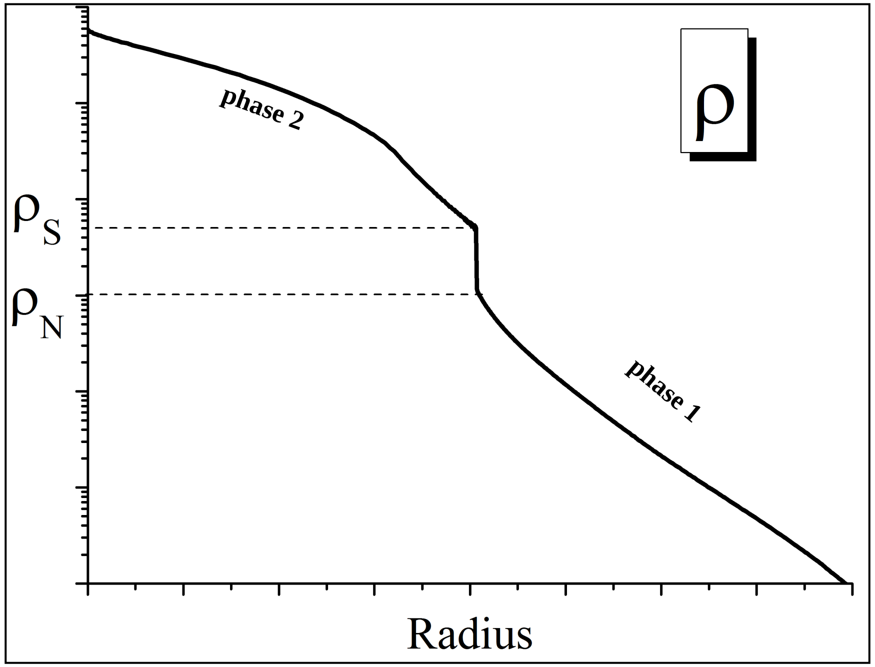
$$\rho = \chi\rho_I + (1-\chi)\rho_{II}$$

$$\chi = \frac{V_I}{V}, \quad V = V_I + V_{II}$$



Maxwellian-type phase transition causes a density jump inside the star

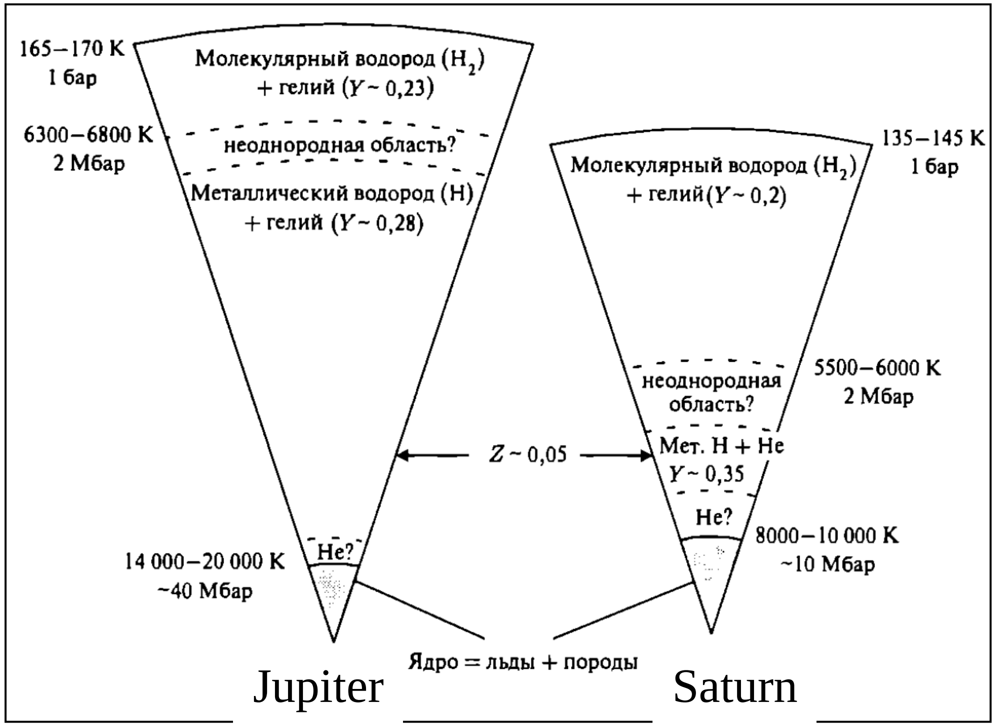




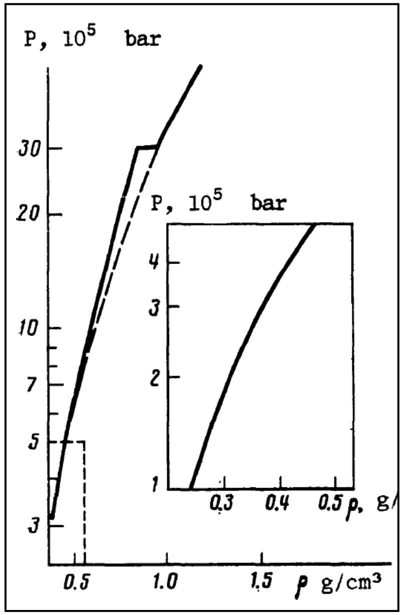
$$\lambda_c = \frac{\rho_S}{\rho_N} = \frac{3}{2}$$

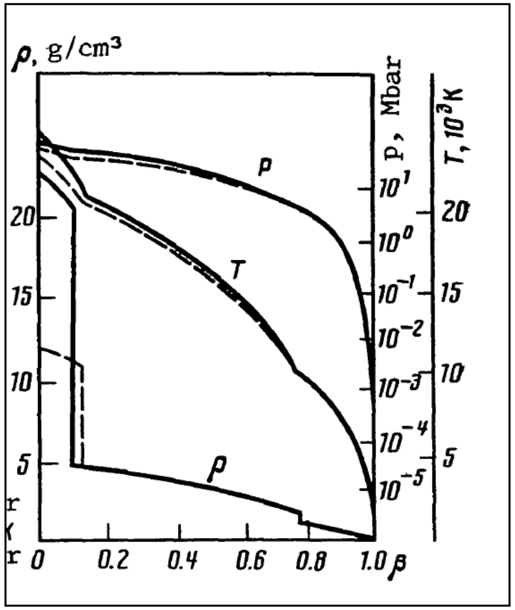
W.H. Ramsey, MNRAS 110 (1950) 325
M.J. Lighthill, MNRAS 110 (1950) 339

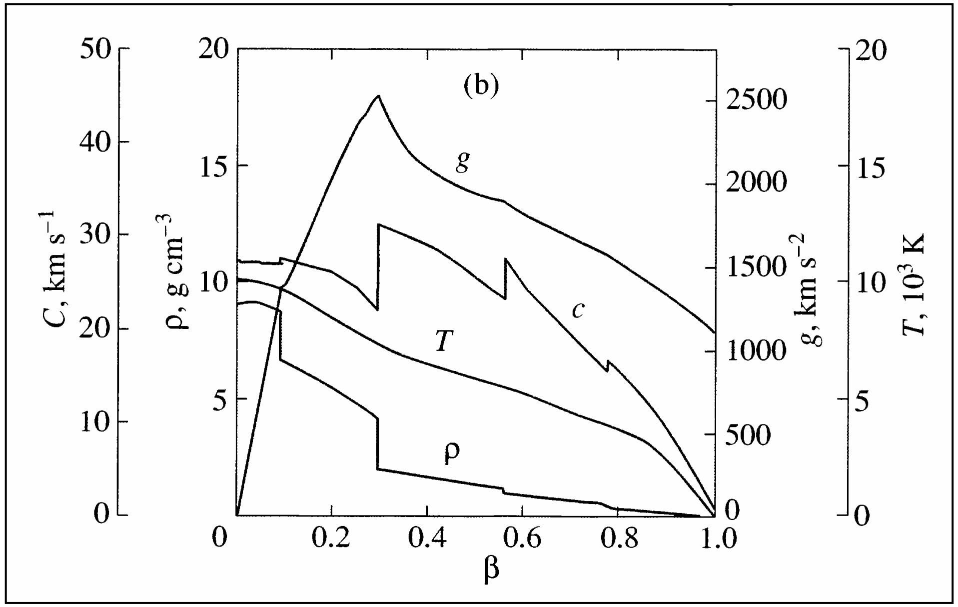
Giant Planets and (may be?) Brown Dwarfs



From “Systems of Jupiter and Saturn” by Kuskov, Dorofeeva, Kronrod and Makalkin URSS 2009 (on russian)





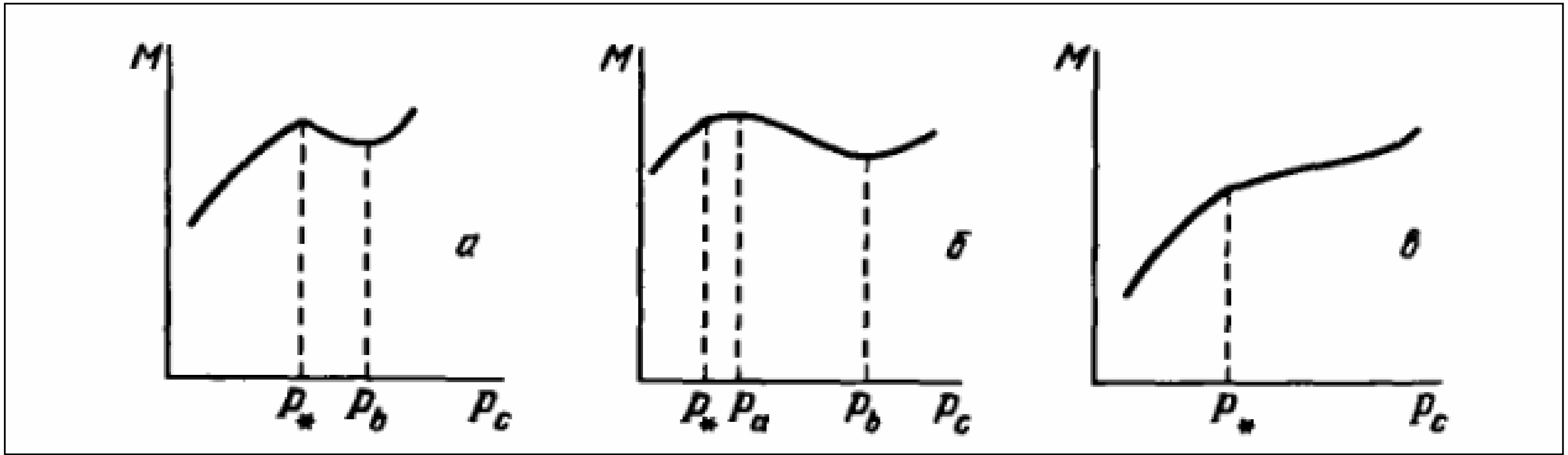


T. V. Gudkova, V. N. Zharkov, and V. V. Leont'ev

Soviet Astronomy Letters, Vol.14, NO. 2/MAR, P. 157, 1988

T. V. Gudkova* and V. N. Zharkov

Astronomy Letters, Vol. 29, No. 10, 2003, pp. 674–694.



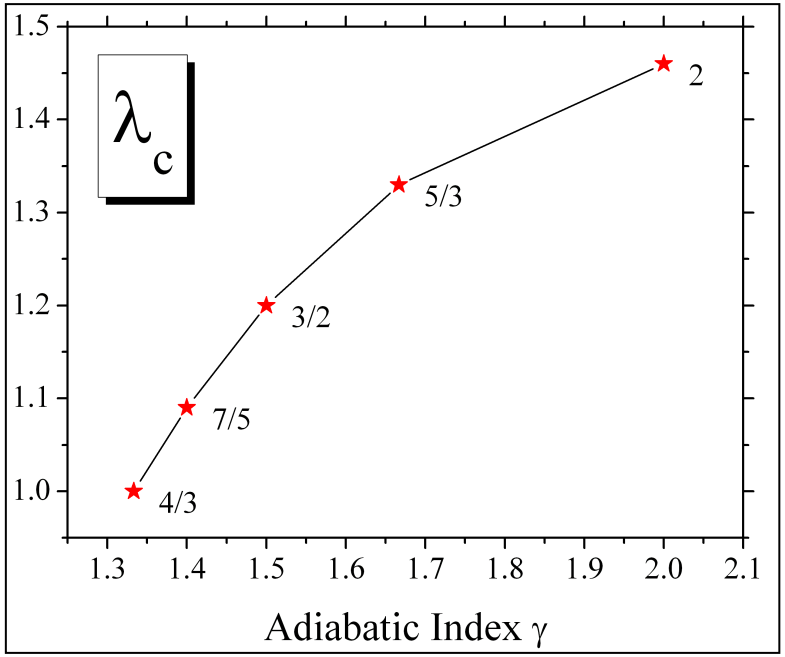
$\lambda > \dfrac{3}{2}$	$\lambda_c < \lambda < \dfrac{3}{2}$	$\lambda < \lambda_c$

S. Blinnikov (1975)

$$\frac{\partial M}{\partial P_c} > 0$$

$$\gamma = \left(\frac{\partial \ln P}{\partial \ln \rho} \right)_S$$

The stiffness of EOS

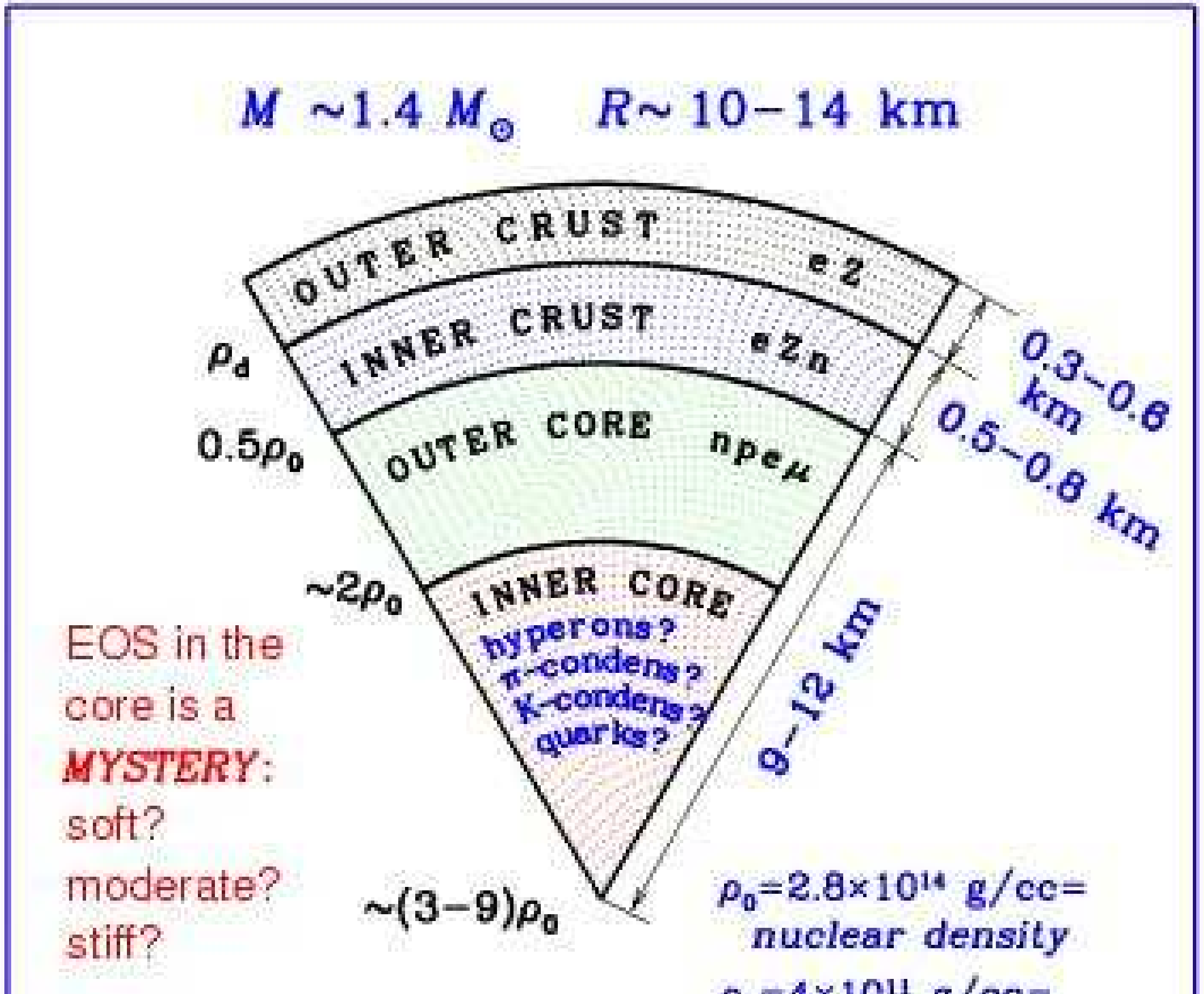


Possible phase transitions in the matter at high density

$$\lambda^{rel} = \frac{\varepsilon_2}{\varepsilon_1}$$

$$\lambda_c^{rel} = \frac{3}{2}\left(1 + \frac{P_*}{\varepsilon_1}\right)$$

Z.F. Seidov (1971)

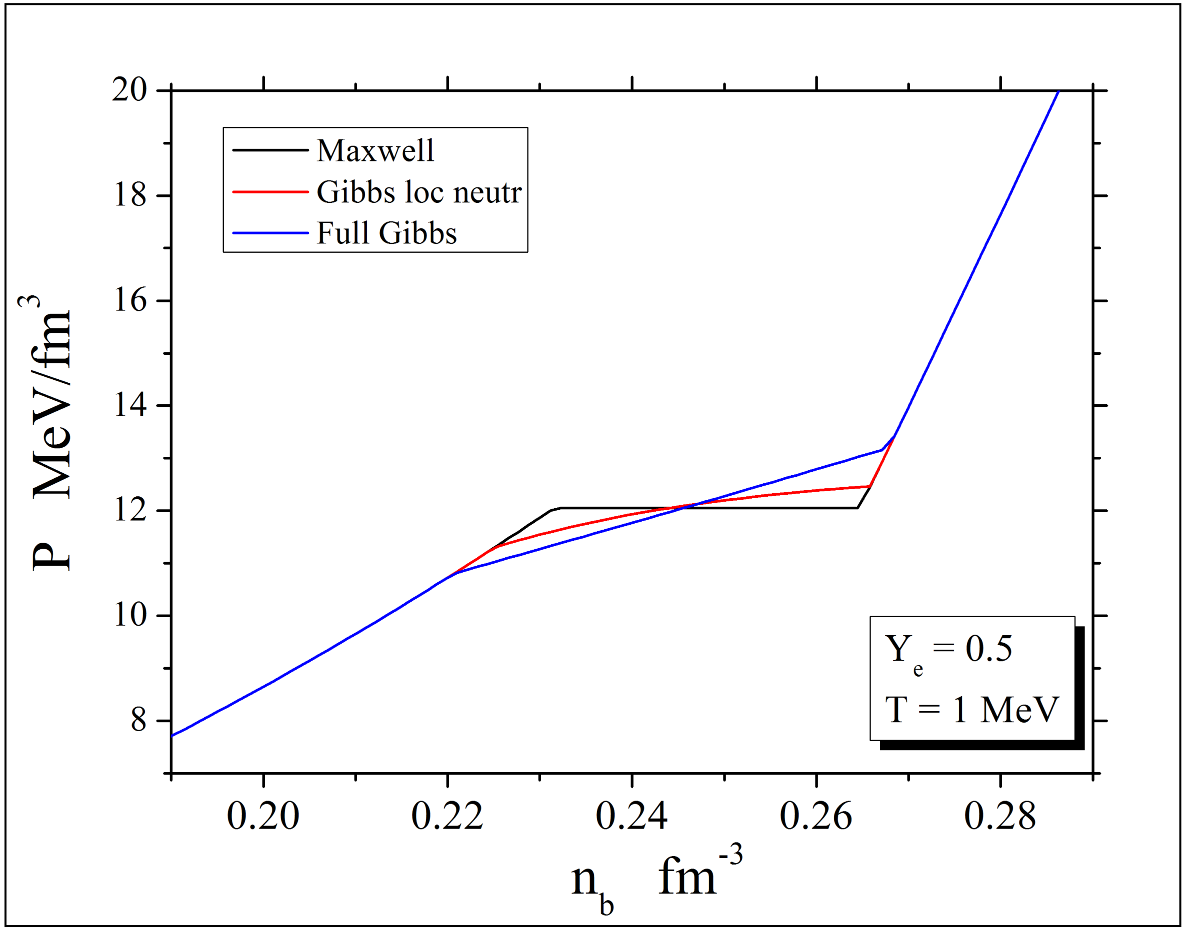


Adapted from D.G. Yakovlev

More than one conserving charge – Gibbs PT

Global or local charge neutrality
N.K. Glendenning (1992)

$$P_I(n_n, n_p, n_e^I, T) = P_{II}(n_{up}, n_{dw}, n_s, n_e^{II}, T)$$
$$\mu_n = \mu_{up} + 2\mu_{dw}$$
$$\mu_p = 2\mu_{up} + \mu_{dw}$$
$$\mu_e^I = \mu_e^{II}$$
$$\mu_s = \mu_{dw}$$



Interacting phases

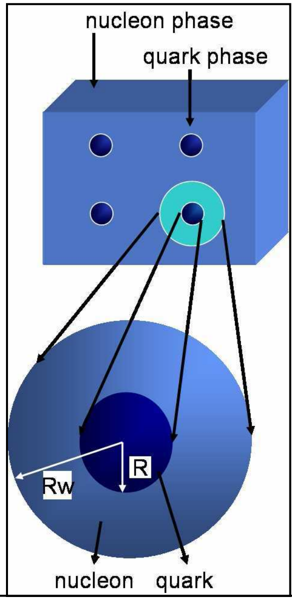
$$F = V_I f_I(\{n_I\},T) + V_{II} f_{II}(\{n_{II}\},T) + F_{mix}(\{n_I\},\{n_{II}\},T,V_I,V_{II})$$

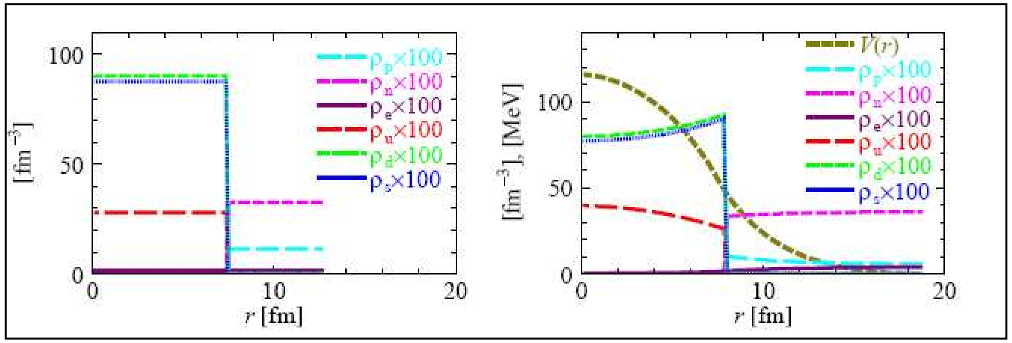
$$F_{mix} = F_S + F_C$$

$$P_I = P_{II}$$

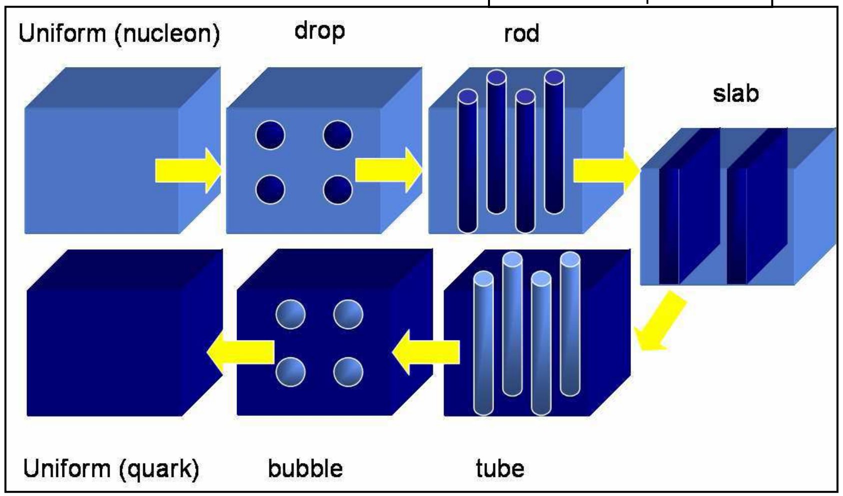
$$\mu_I^1 = \mu_{II}^1$$

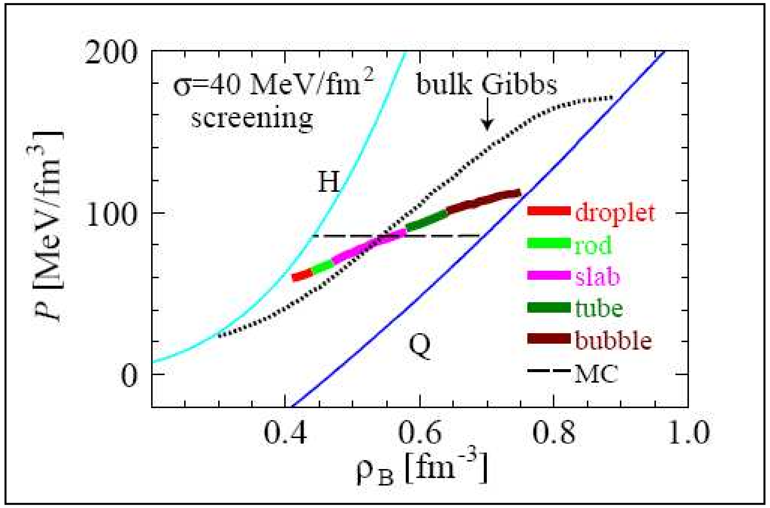
$$\mu_I^2 = \mu_{II}^2$$





T. Endo at. al. (2006)





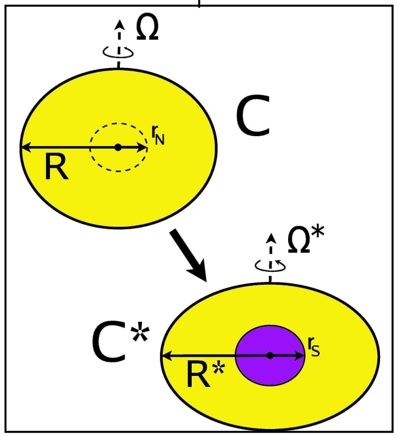
Phase Transition

Change of angular momentum (starquakes)

Maximum mass of neutron star

Gravitational energy release

Supernova

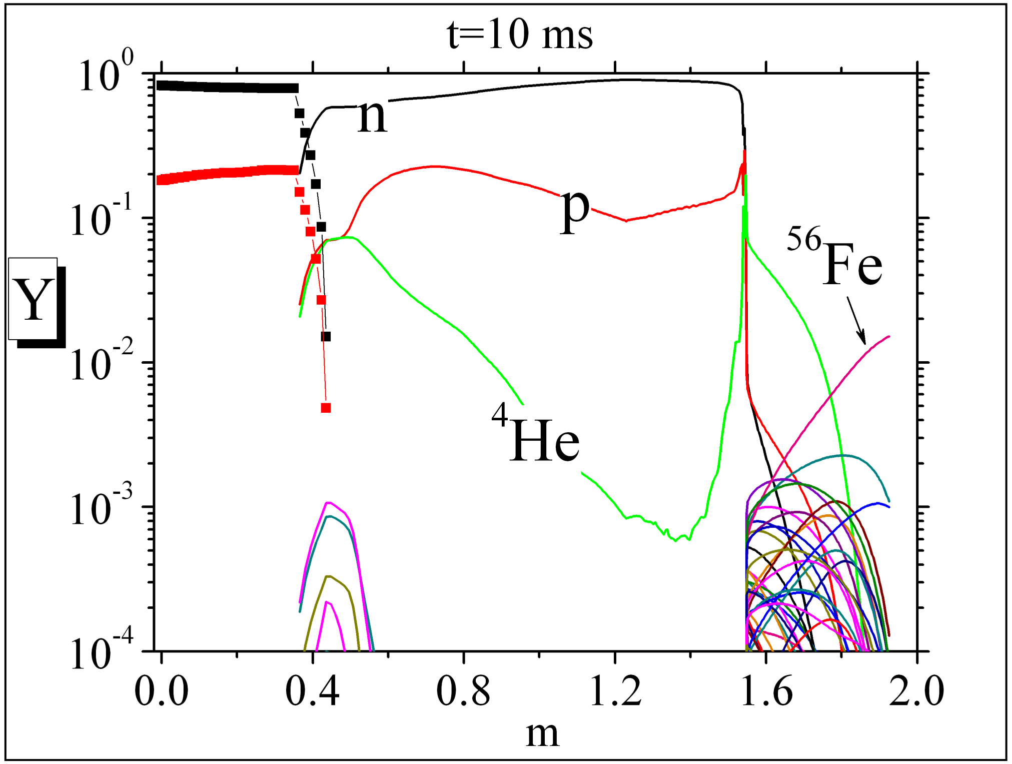


$$\Delta I/I_0 \simeq (-9\times 10^{-6})(2.94/\gamma_{\rm m}-1)(r_{\rm m}/1\ {\rm km})^5.$$

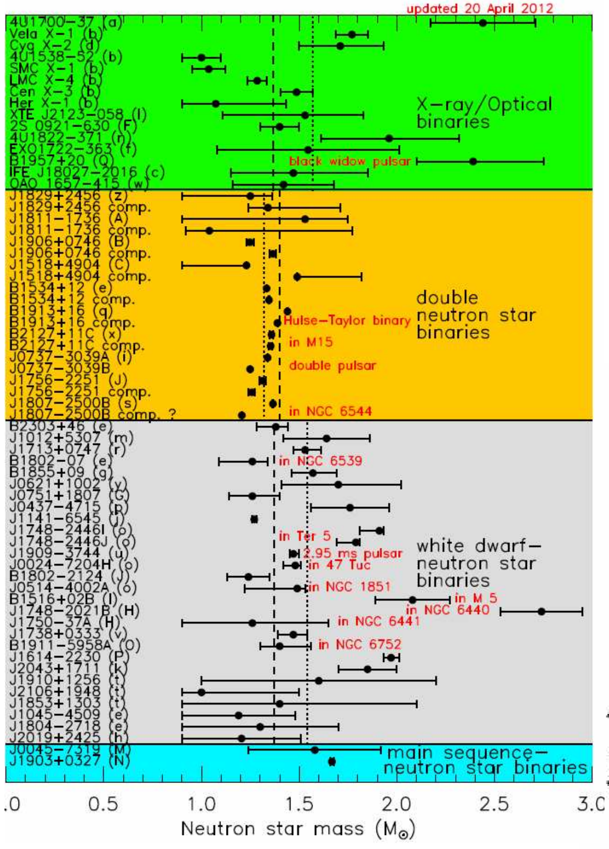
$$\Delta\Omega/\Omega \simeq -\Delta I/I \simeq 10^{-5}$$

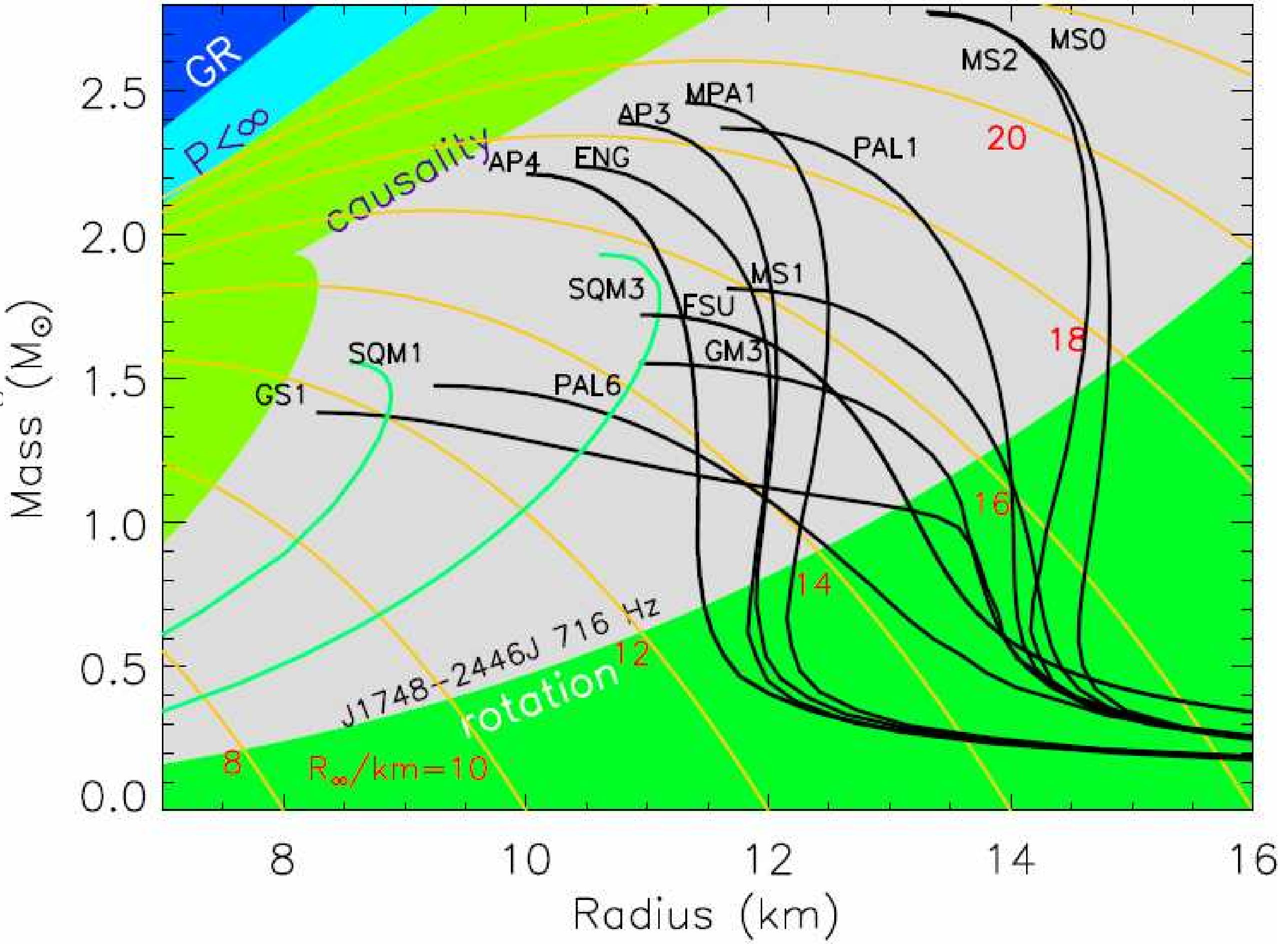
J. L. Zdunik, M. Bejger P. Haensel (2005)

$$\Delta E \simeq (4.1\times 10^{45})(2.94/\gamma_{\rm m}-1)(r_{\rm m}/1\ {\rm km})^7\ {\rm erg},$$



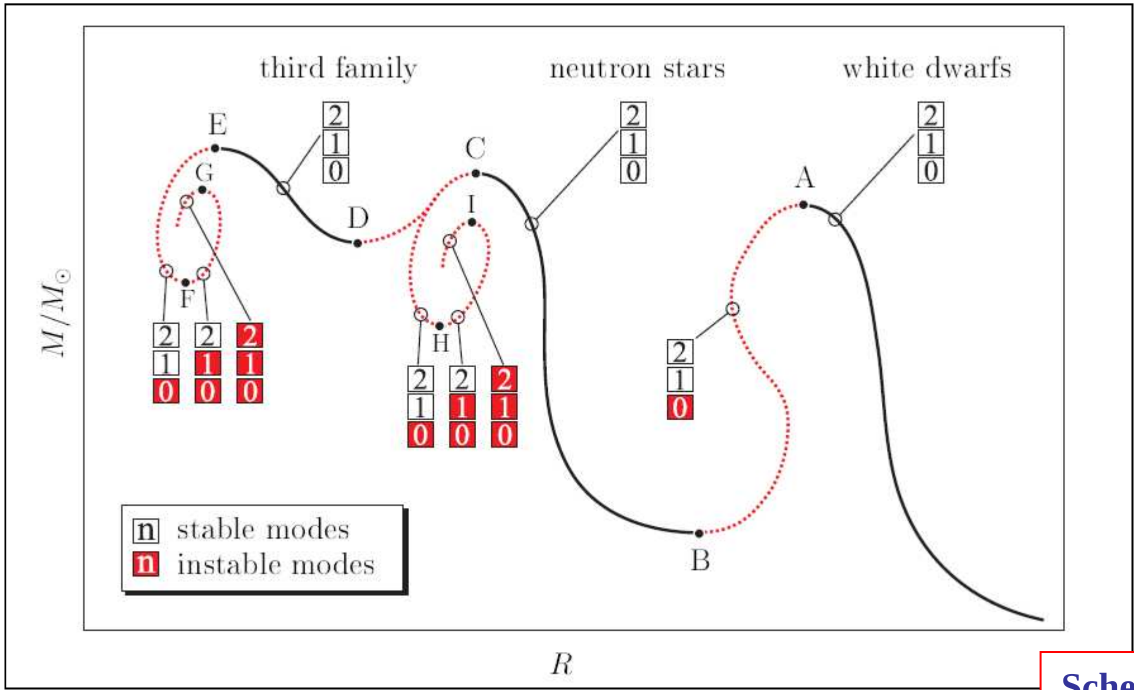
Maximum neutron star mass





J.M. Lattimer

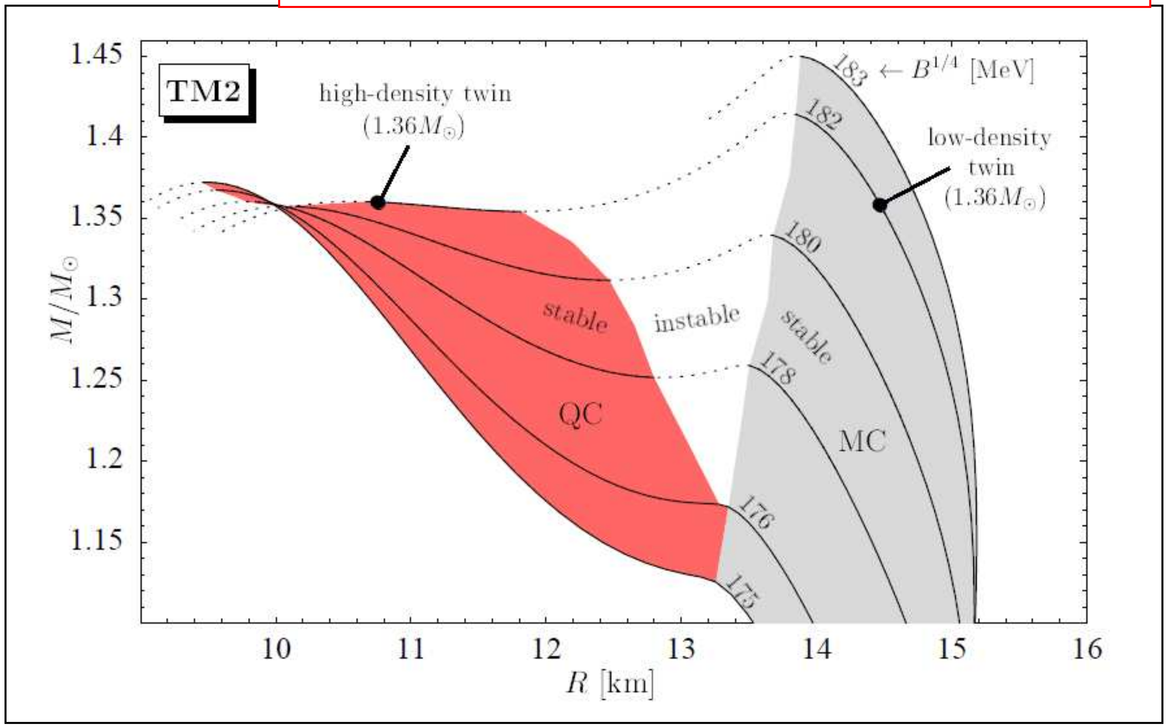
Annual Review of Nuclear and Particle Science, vol. 62, issue 1, pp. 485-515 (2012)



$$\Delta E \simeq (4.1 \times 10^{45})(2.94/\gamma_{\rm m} - 1)(r_{\rm m}/1\ {\rm km})^7\ {\rm erg},$$

Schertler, Greiner, Schaffner-Bielich, Thoma (2000)

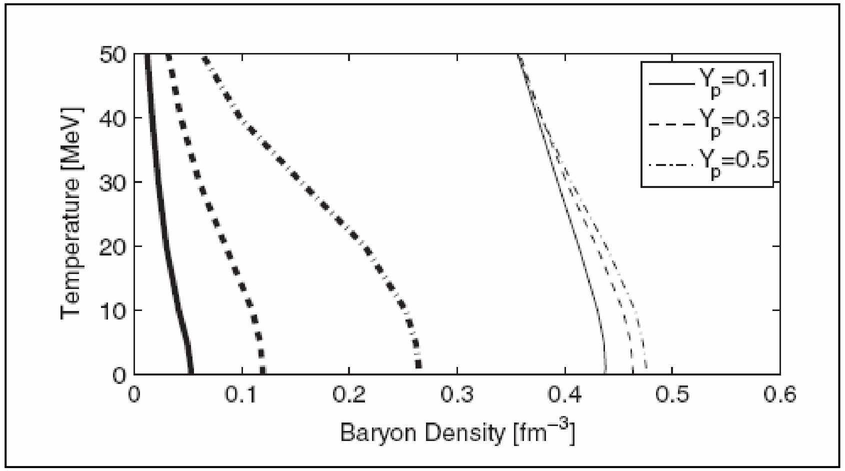
Accretion-induced
Thermal evolution
Slowing down rotation

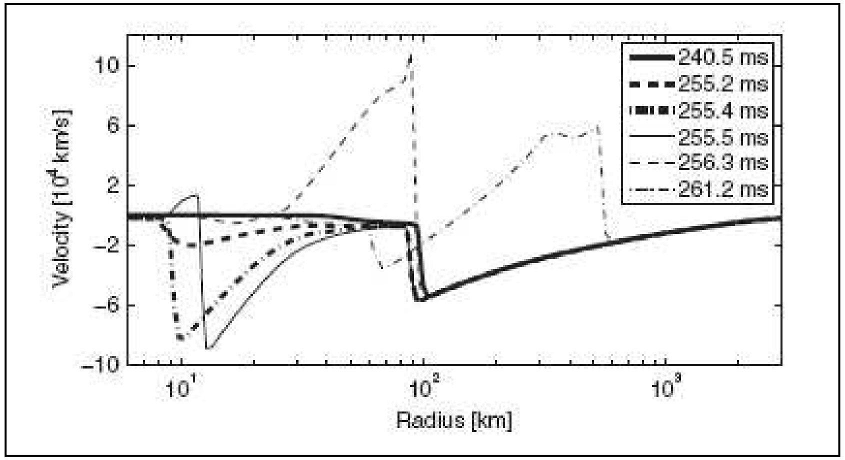


PRL 102, 081101 (2009) PHYSICAL REVIEW LETTERS week ending 27 FEBRUARY 2009

Signals of the QCD Phase Transition in Core-Collapse Supernovae

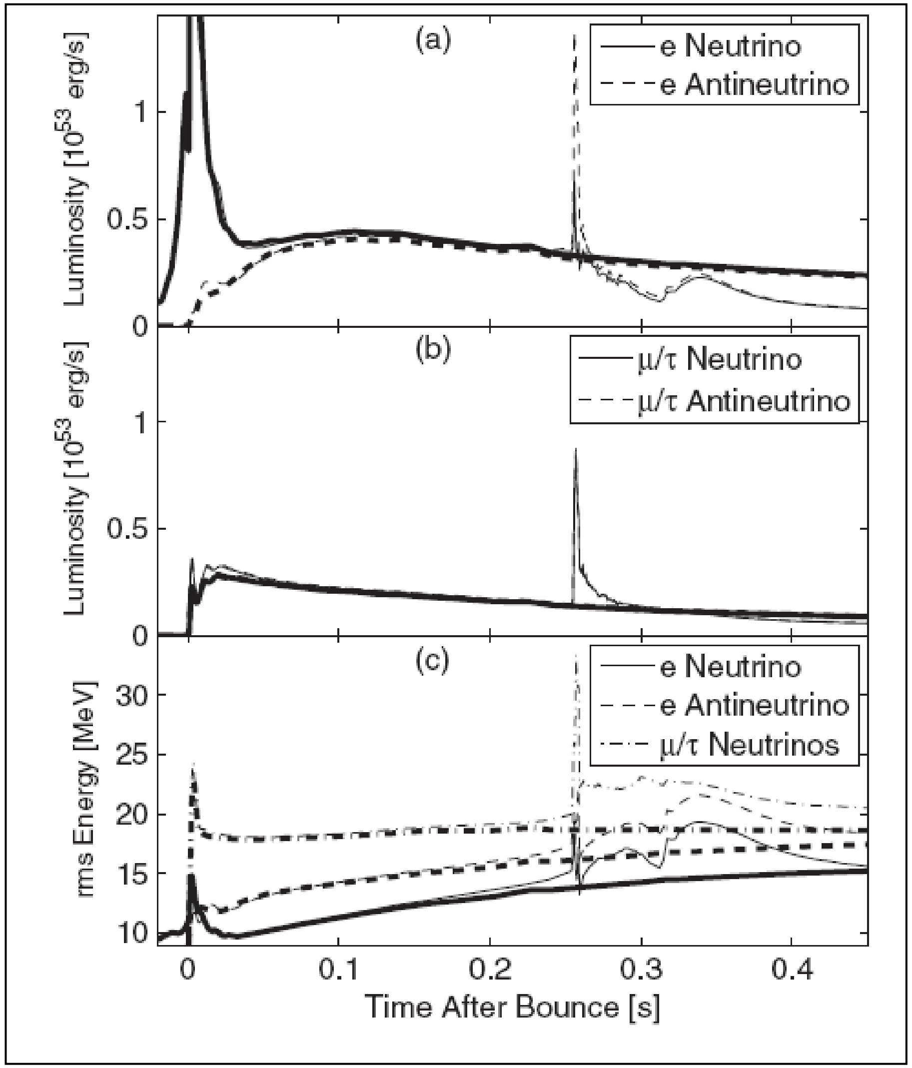
I. Sagert,[1] T. Fischer,[3] M. Hempel,[1] G. Pagliara,[2] J. Schaffner-Bielich,[2] A. Mezzacappa,[4]
F.-K. Thielemann,[3] and M. Liebendörfer[3]





Prog.	EOS	t_{pb}	M_Q	M_{mix}	M_{pns}	E_{expl}	BE	M_G
[$M_\odot$]		[ms]	[$M_\odot$]	[$M_\odot$]	[$M_\odot$]	[10^{51}erg]	[10^{53}erg]	[$M_\odot$]
10	$eos1$	255	0.850	0.508	1.440	0.44	3.40	1.25
10	$eos2$	448	1.198	0.161	1.478	1.64	3.19	1.30
15	$eos1$	209	1.146	0.320	1.608	0.42	4.08	1.38
15	$eos2$	330[a]	1.496	0.116	1.700	...[b]	4.28	1.46

[a]moment of black hole formation
[b]black hole formation before explosion



Mass-radius diagram for hybrid stars

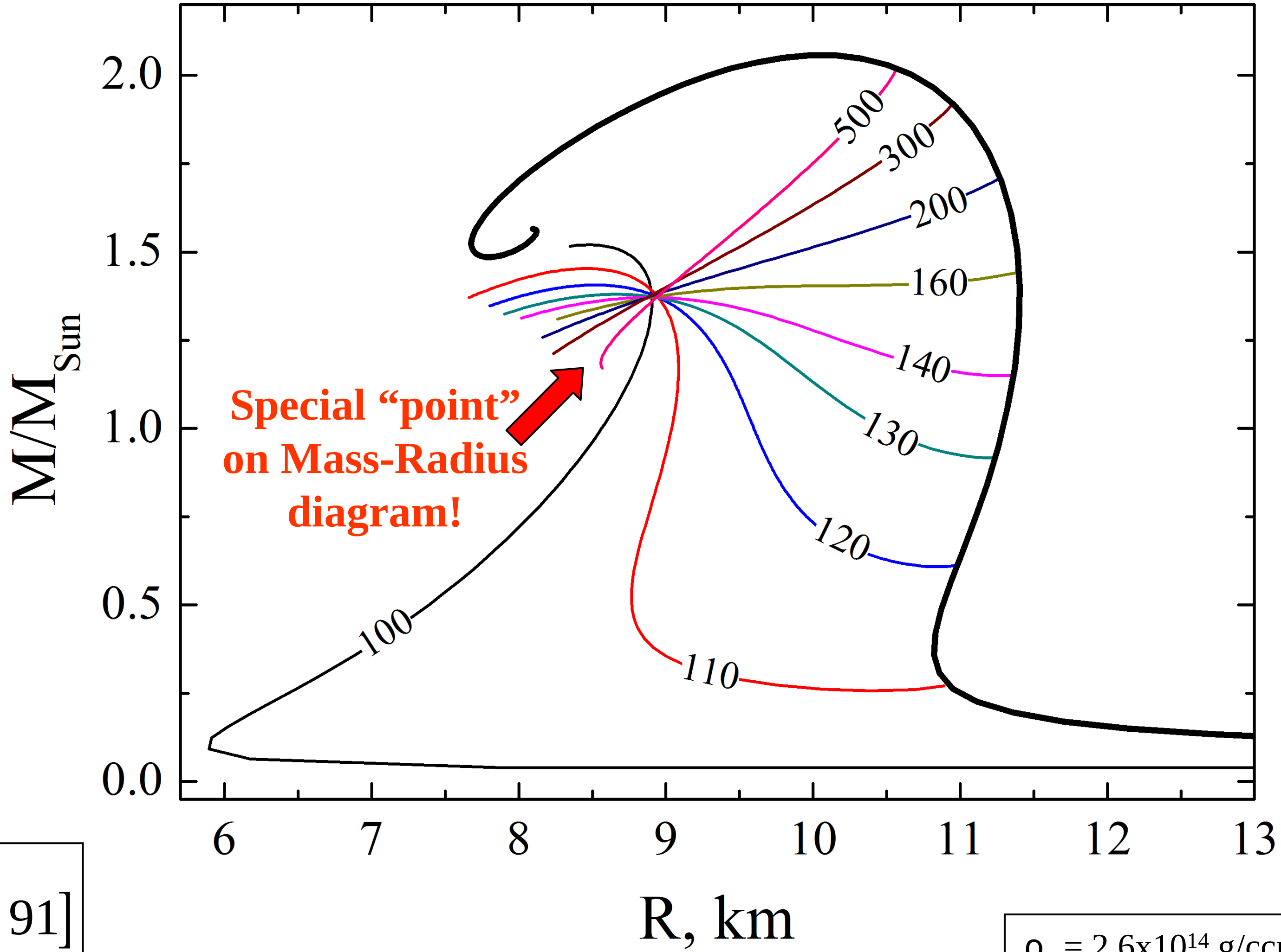
Phase transition:

Nuclear Matter

Quark Matter

$$P_q = \frac{1}{3}(E - 4B)$$

$$\frac{\rho_{pt}}{\rho_n} \approx -3 + \ln[B - 91]$$

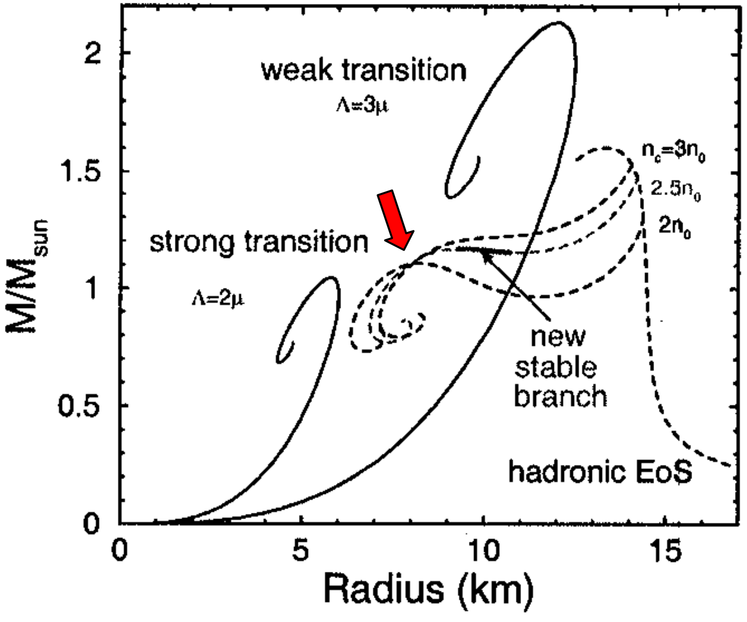


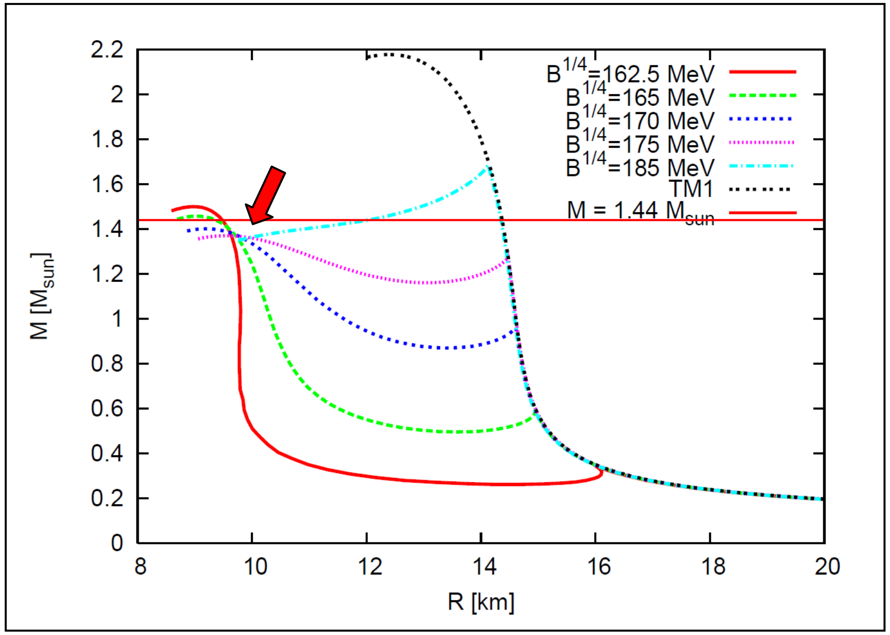
$\rho_n = 2.6x10^{14}$ g/ccm

Values of B are in MeV/fm^3

EOS for pure nuclear matter is from Douchin & Haensel, Astron. Astrophys., 380 151-167 (2001)

Fraga et al. Nucl. Phys. A, 702 (2002)

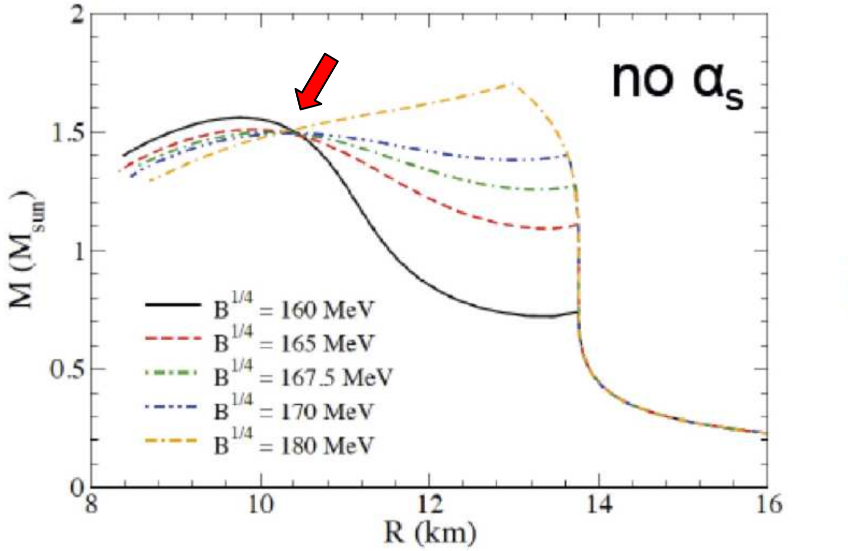


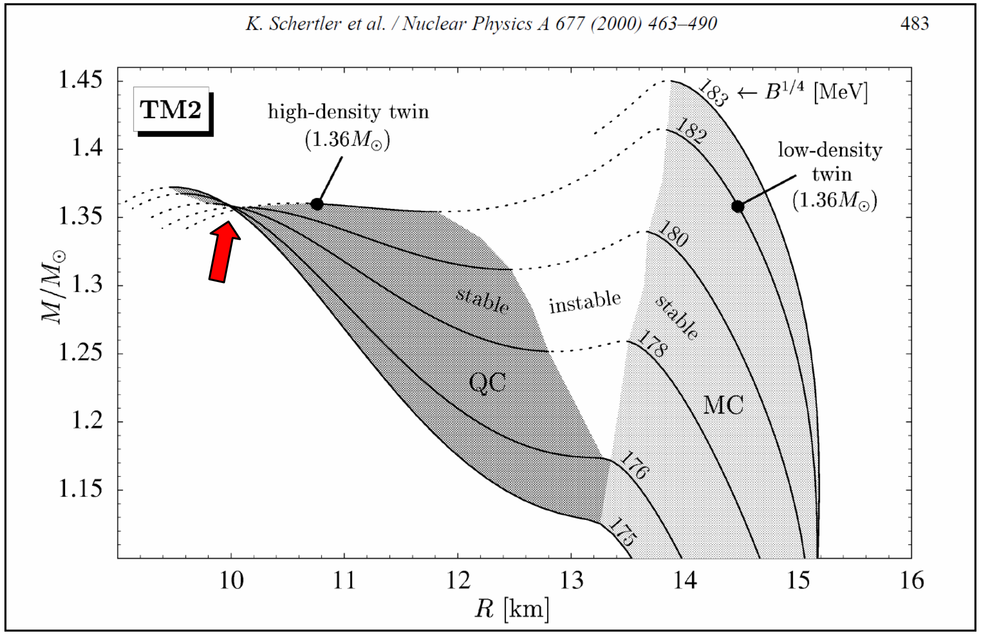


Sagert et al. J Phys G. 36, 6 (2009)

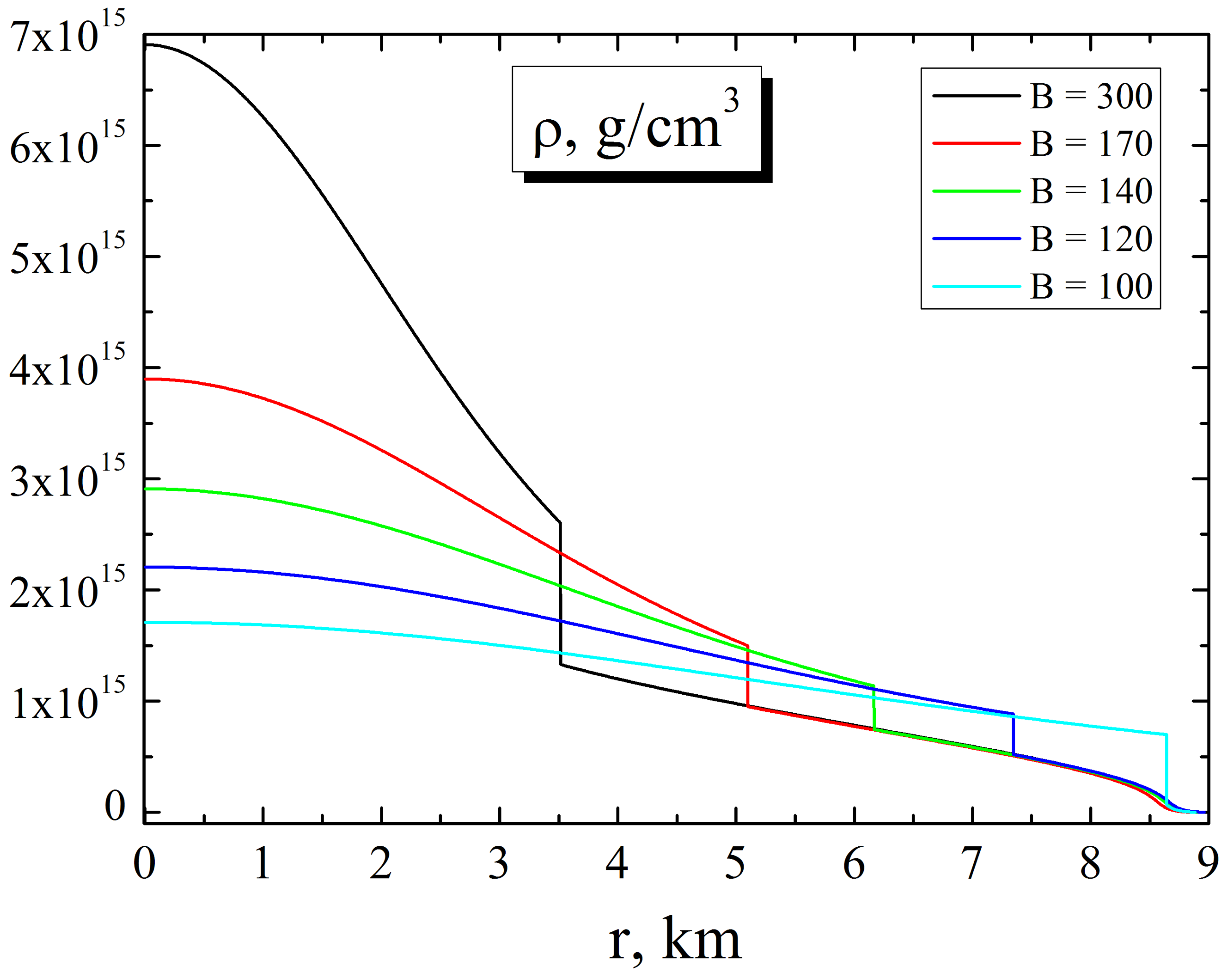
The are other examples of special point on M-R diagram!

Schramm et al arXiv:1310.5804





What is the structure of the stars at special point?



The stars have different quark cores but very similar envelope

The conditions for the envelope to remain unchanged at varying B:

OVT-equations of equilibrium

$$\begin{cases} \dfrac{dP}{dr} = -\dfrac{G(P+E)\left(m + \dfrac{4\pi r^3}{c^2}P\right)}{c^2 r\left(r - \dfrac{2Gm}{c^2}\right)}, \\ \dfrac{dm}{dr} = 4\pi r^2 \dfrac{E}{c^2}. \end{cases}$$

$$\Delta P = \left(\frac{\partial P}{\partial r}\right)_1 \delta r = \frac{1}{\lambda}\left(\frac{\partial P}{\partial r}\right)_2 \delta r.$$

$$\Delta m = 4\pi r_*^2 \frac{E_1}{c^2}\delta r = 4\pi r_*^2 \frac{E_2}{c^2}\left[1 - (\lambda - 1)\frac{P_2}{E_2}\right]\frac{\delta r}{\lambda}.$$

Phase equilibrium

$$\begin{cases} P_{pt} = P_1(n_1) = P_2(n_2) \\ \mu_{pt} = \dfrac{P_1 + E_1}{n_1} = \dfrac{P_2 + E_2}{n_2} \end{cases}$$

$$E_2 = E_2(n_2, B)$$

$$\Delta P = \frac{\delta B}{\lambda - 1}\left(\frac{\partial E_2}{\partial B}\right), \quad \lambda \equiv \frac{n_2}{n_1}.$$

$$\begin{cases} \Delta P = \left(\dfrac{dP}{dr}\right)_2 \delta r + \left(\dfrac{dP}{dP_c}\right)_{r,B} \delta P_c + \left(\dfrac{dP}{dB}\right)_{r,P_c} \delta B, \\ \Delta m = \left(\dfrac{dm}{dr}\right)_2 \delta r + \left(\dfrac{dm}{dP_c}\right)_{r,B} \delta P_c + \left(\dfrac{dm}{dB}\right)_{r,P_c} \delta B. \end{cases}$$

The whole set of the equations:

$$\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial E_2}{\partial B}\right)\partial B,$$

$$-\frac{dP}{dr}\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial P}{\partial P_c}\right)_{r,B}\delta P_c+\left(\frac{\partial P}{\partial B}\right)_{r,P_c}\delta B,$$

$$-\frac{dm}{dr}\left[\frac{P_2+E_2}{E_2}\right]\left[\frac{\lambda-1}{\lambda}\right]\delta r=\left(\frac{\partial m}{\partial P_c}\right)_{r,B}\delta P_c+\left(\frac{\partial m}{\partial B}\right)_{r,P_c}\delta B.$$

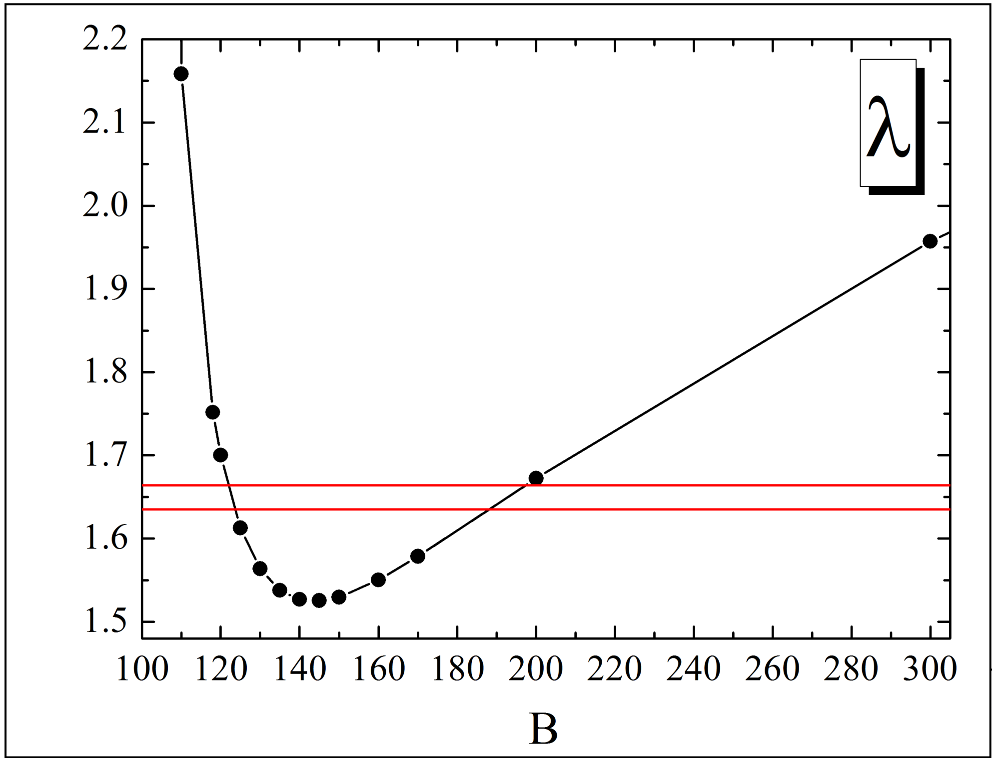
Non-trivial solution
Determinant = 0!

All the quantities in main equation refer to quark phase only!

The main equation

$$\left(\frac{\partial P}{\partial P_c}\right)_{r,B}\left[\frac{dm}{dr}\frac{(P_2+E_2)}{E_2\dfrac{dP}{dr}}\left(\frac{\partial E_2}{\partial B}\right)+\left(\frac{\partial m}{\partial B}\right)_{r,P_c}\right]=\left(\frac{\partial m}{\partial P_c}\right)_{r,B}\left[\left(\frac{\partial E_2}{\partial B}\right)+\left(\frac{\partial P}{\partial B}\right)_{r,P_c}\right].$$

This is the main equation. If it fulfilled at some point, the whole mass M and radius R of the star will remain constant with small changes in B. Pay attention that λ dropped out from the main equation! Thus this property does not depend on the parameters of star’s envelope.



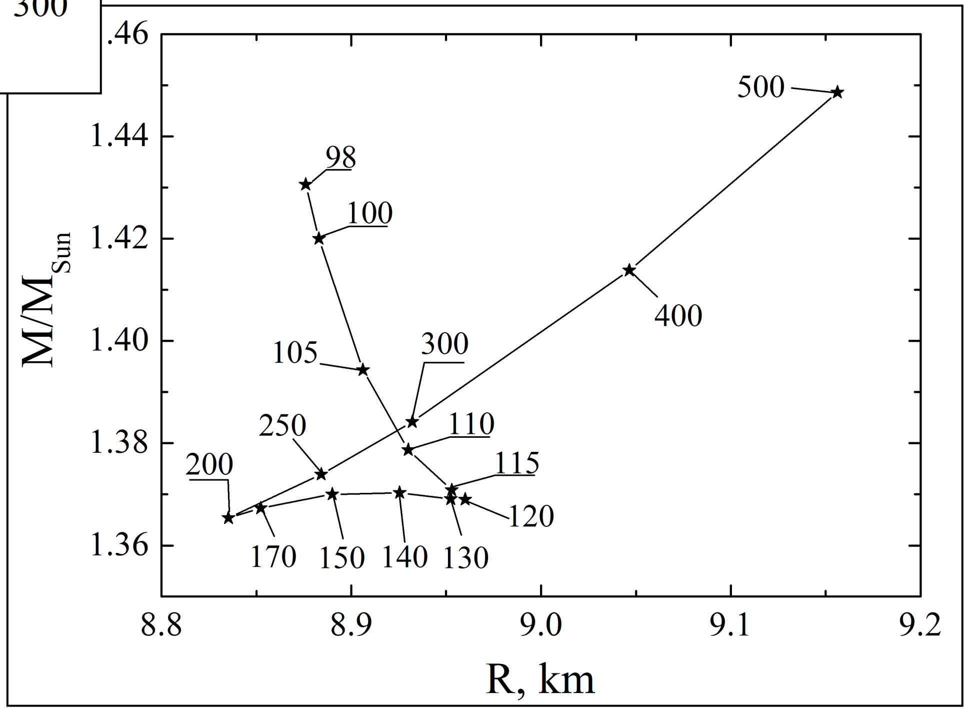
The reasons for the existence of a special point on the M-R diagram of hybrid stars:

1. **The linearity of quark EOS**

$$P = \alpha(E - E_0)$$

2. **The “phase diagram” of quark matter**

Global structure of fixed points



The condition that a small shift from a fixed point bring us to another fixed point

$$\lambda = \frac{u(3-v)\left[(1+v)^2(3-v)+8(1-v^2)\alpha-(3-v)^2\alpha^2\right]}{(1+v)(7v^2-6v+3)+8(1-v^2)(3-v)\alpha-(3-v)^3\alpha^2}$$

$$\lambda \equiv \frac{n_2}{n_1}$$

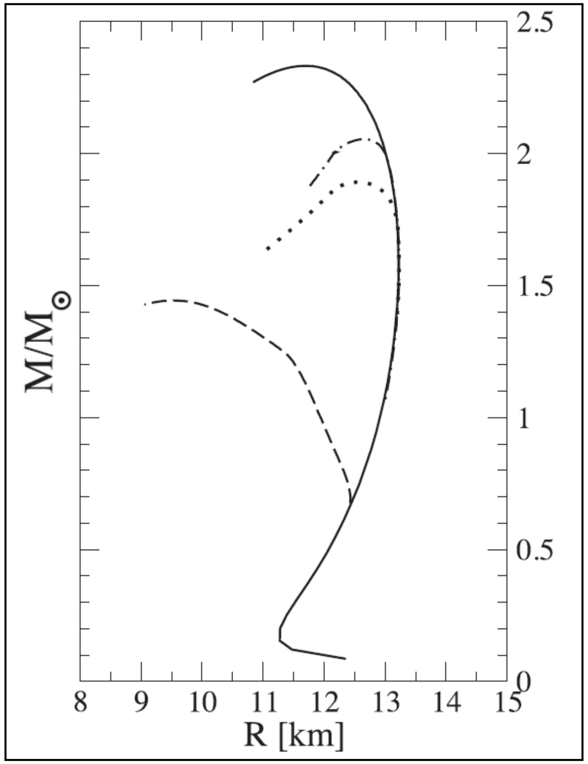
Large scale

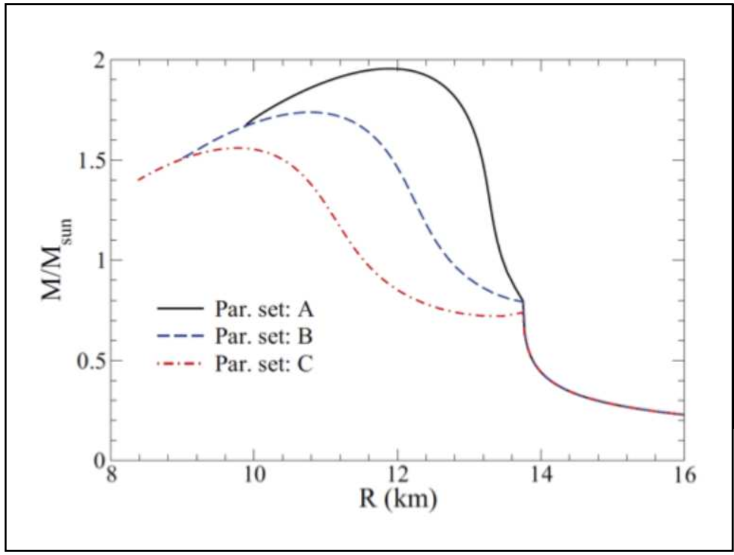
For a given EOS the point on M-R diagram where the main equation holds we call fixed point. This is a local property.

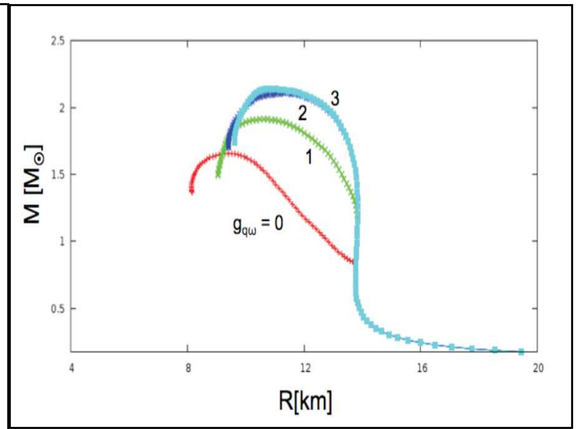
Open questions:

Other (non-linear) EOS as the solution of the main equation?
Other topology of fixed-points because different envelope?

Calculations with non-linear quark EOS:







Schramm et al. arXiv:1306.0989v2

Bombachi and Lagoteta, MNRASL 433, L79-L82 (2013)

Linear EOS (P being a linear function of ρ) is characteristic of a simplest bag model of quark matter that assumes massless quarks, but it also holds with very high accuracy for more realistic bag model with massive s-quark (Zdunik, 2000). **Zdunik and Haensel A&A 551, A61 (2013)**

The existence of special point means that the hybrid stars with the same Mass and Radius can have nevertheless absolutely different inner structure depending on EOS