Non-perturbative properties of large-scale structure formation and their implications to cosmology

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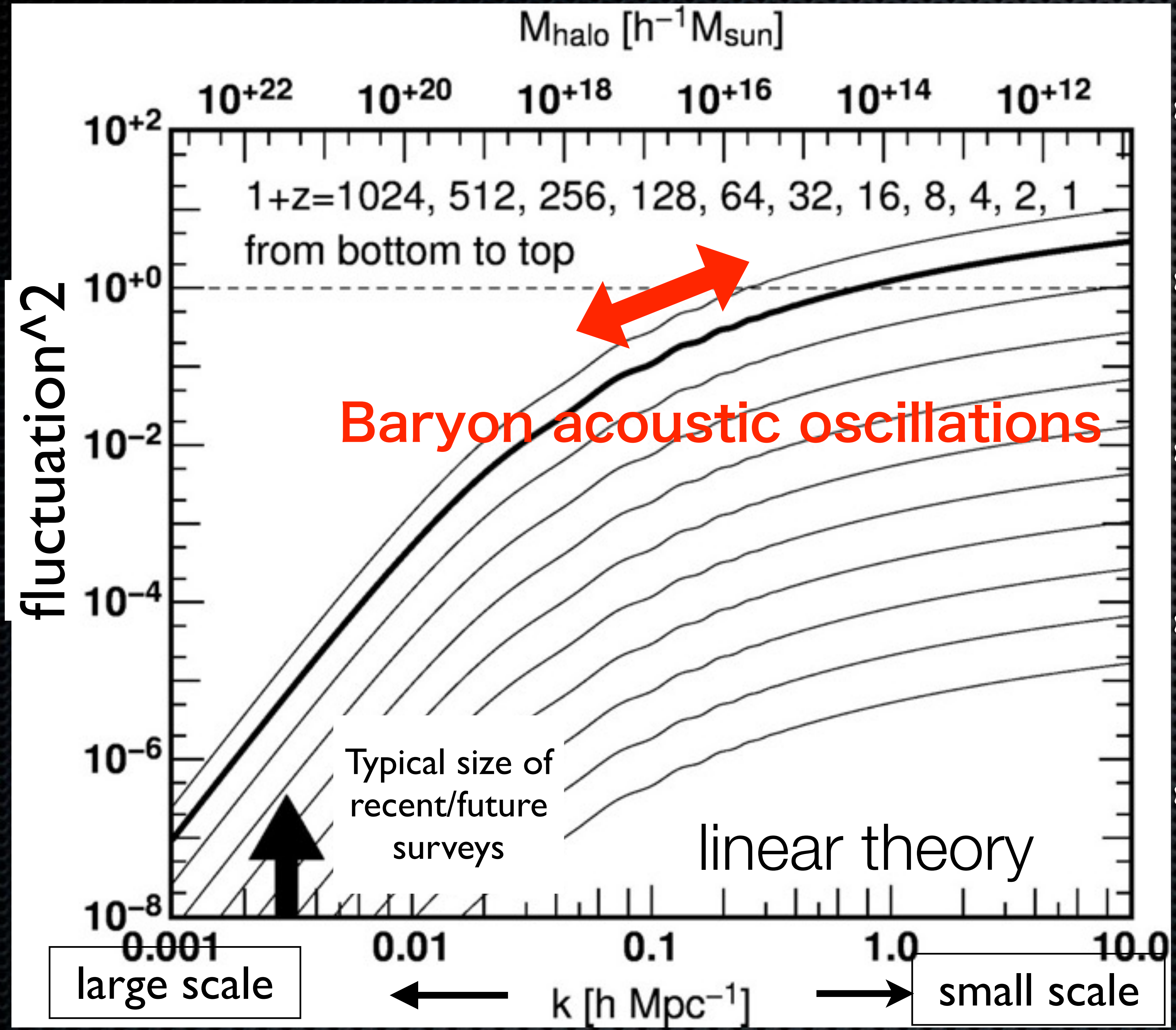
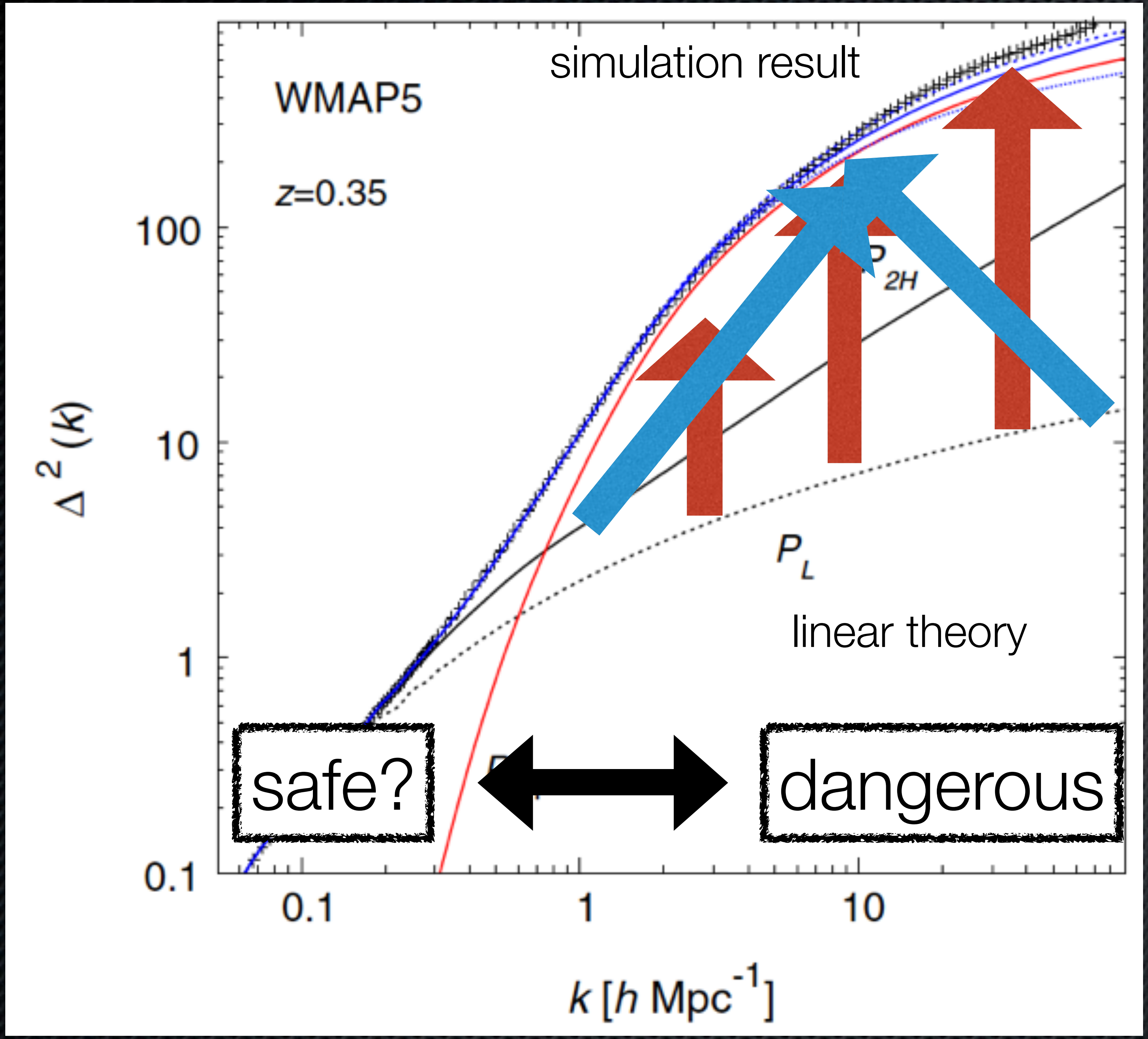
Scale of interest and growth of structure



- Entering into nonlinear stage from small scales
- Giga parsec-class big observations are ongoing/upcoming
- Accurate measurement of Baryon Acoustic Oscillations is a key goal
- The relevant scale is in the transition regime from linear to nonlinear
- Redshift-space distortions are cosmologically important/tractable up to these scales

Nonlinear growth: mode coupling btwn different scales



Valageas,TN, Taruya '13

- larger fluctuation → larger nonlinearity
- **modes at different k are no longer independent**
- fluctuations on small scales are subject to non gravitational physics (ex. cooling, feedback,...)
- Q: Is observation of large scale modes a faithful tracer of the cosmological model (Initial condition+Energy budget +gravitational law)?

Mode coupling: perturbative approach

Continuity+Euler+Poisson equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}] = 0,$$
$$\frac{\partial \boldsymbol{v}}{\partial t} + H\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{a}\nabla\phi,$$
$$\nabla^2\phi = 4\pi\mathcal{G}\bar{\rho}a^2\delta.$$

density fluctuations $\delta(\mathbf{x}) = \rho(\mathbf{x})/\bar{\rho} - 1$

peculiar velocity $\mathbf{v(x)}$ (single flow)

$\theta(\mathbf{x}) = \nabla\cdot\mathbf{v}(\mathbf{x})$ (velocity divergence)

(for Einstein-de Sitter universe; $\Omega_m=1, \Omega_\Lambda=0$)

$$a\frac{\partial\delta(\mathbf{k},a)}{\partial a} + \theta(\mathbf{k},a) = -\frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{k}_1 d^3\mathbf{k}_2\,\delta_D(\mathbf{k}-\mathbf{k}_{12}) \times \alpha(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a)$$
$$a\frac{\partial\theta(\mathbf{k},a)}{\partial a} + \frac{1}{2}\theta(\mathbf{k},a) + \frac{3}{2}\delta(\mathbf{k},a) = -\frac{1}{(2\pi)^{3/2}}\int d^3\mathbf{k}_1 d^3\mathbf{k}_2\,\delta_D(\mathbf{k}-\mathbf{k}_{12}) \times \beta(\mathbf{k}_1,\mathbf{k}_2)\theta(\mathbf{k}_1,a)\theta(\mathbf{k}_2,a),$$

Perturbative solution is known

$$\tilde{\delta}(\mathbf{k},\tau) = \sum_{n=1}^{\infty} a^n(\tau)\delta_n(\mathbf{k}), \qquad \tilde{\theta}(\mathbf{k},\tau) = -\mathcal{H}(\tau)\sum_{n=1}^{\infty} a^n(\tau)\theta_n(\mathbf{k})$$

$$\delta_n(\mathbf{k}) = \int d^3\mathbf{q}_1\ldots\int d^3\mathbf{q}_n\,\delta_D(\mathbf{k}-\mathbf{q}_{1\ldots n})F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n),$$
$$\theta_n(\mathbf{k}) = \int d^3\mathbf{q}_1\ldots\int d^3\mathbf{q}_n\,\delta_D(\mathbf{k}-\mathbf{q}_{1\ldots n})G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n)\delta_1(\mathbf{q}_1)\ldots\delta_1(\mathbf{q}_n)$$

$$F_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[(2n+1)\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + 2\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big],$$

$$G_n(\mathbf{q}_1,\ldots,\mathbf{q}_n) = \sum_{m=1}^{n-1}\frac{G_m(\mathbf{q}_1,\ldots,\mathbf{q}_m)}{(2n+3)(n-1)}\Big[3\alpha(\mathbf{k}_1,\mathbf{k}_2)F_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n) + 2n\beta(\mathbf{k}_1,\mathbf{k}_2)G_{n-m}(\mathbf{q}_{m+1},\ldots,\mathbf{q}_n)\Big],$$

Goroff+'86

Approximate symmetry of the gravitational dynamics

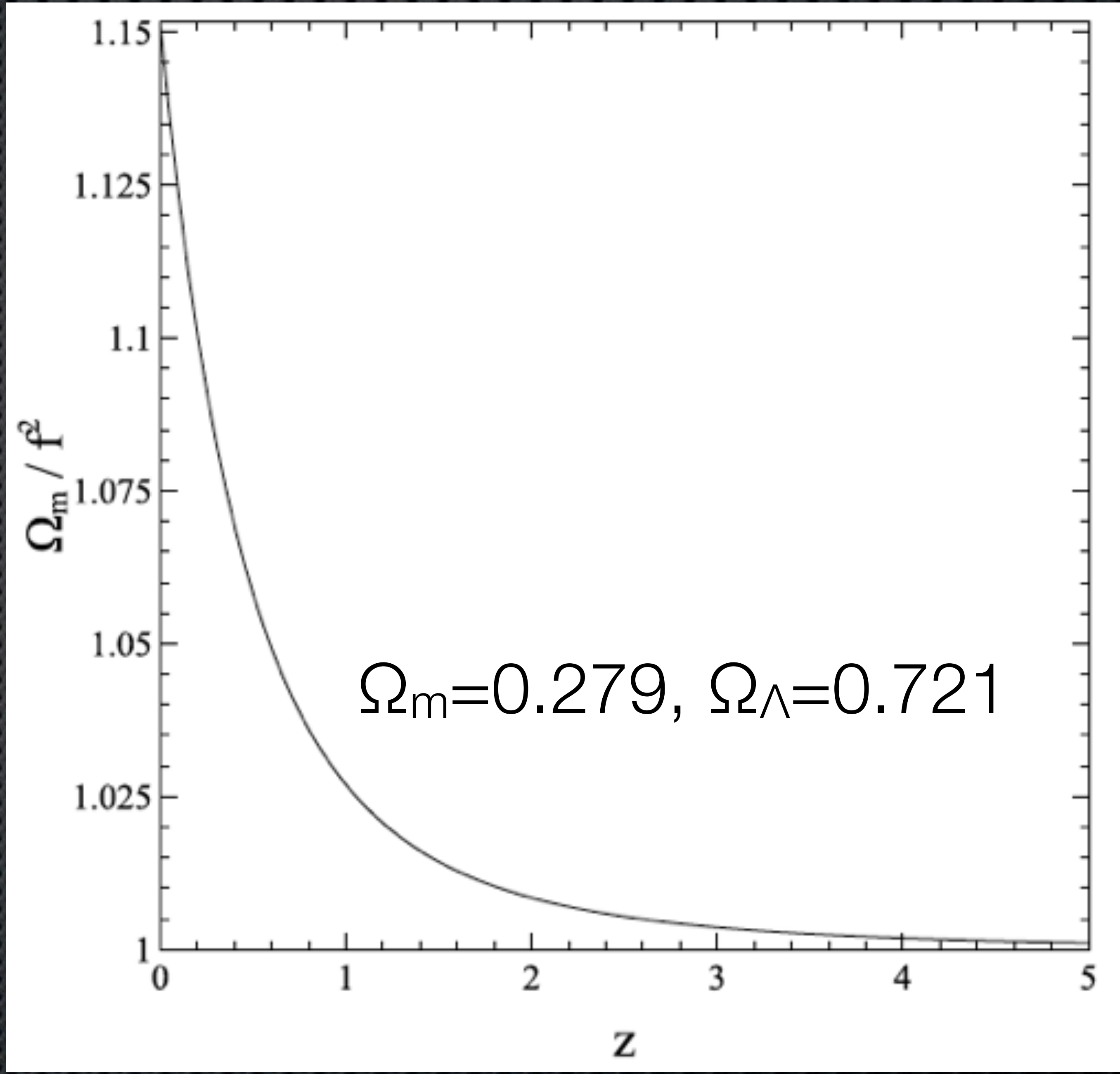
Continuity+Euler+Poisson equations

$$\frac{\partial \delta}{\partial t} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{v}] = 0,$$
$$\frac{\partial \boldsymbol{v}}{\partial t} + H\boldsymbol{v} + \frac{1}{a}(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\frac{1}{a}\nabla\phi,$$
$$\nabla^2\phi = 4\pi\mathcal{G}\bar{\rho}a^2\delta.$$

Change of variables

$$\eta = \ln D_+, \quad \boldsymbol{v} = \dot{a}f\mathbf{u}, \quad \phi = (\dot{a}f)^2\varphi$$

$$\frac{\partial \delta}{\partial \eta} + \nabla\cdot[(1+\delta)\mathbf{u}] = 0,$$
$$\frac{\partial \mathbf{u}}{\partial \eta} + \left(\frac{3\Omega_\mathrm{m}}{2f^2} - 1\right)\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\nabla\varphi,$$
$$\nabla^2\varphi = \frac{3\Omega_\mathrm{m}}{2f^2}\delta.$$



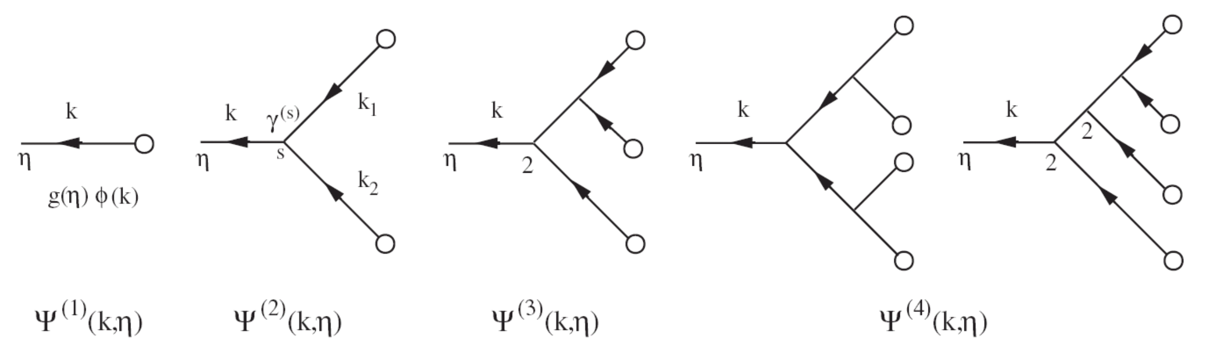
Explicit cosmology dependence lost when $\Omega_\mathrm{m}/f^2 \simeq 1$

EdS solutions for F_n, G_n are good approximations

Existing PT calculations are all within this approximation

New development in PTs

based on Crocce&Scoccimarro06



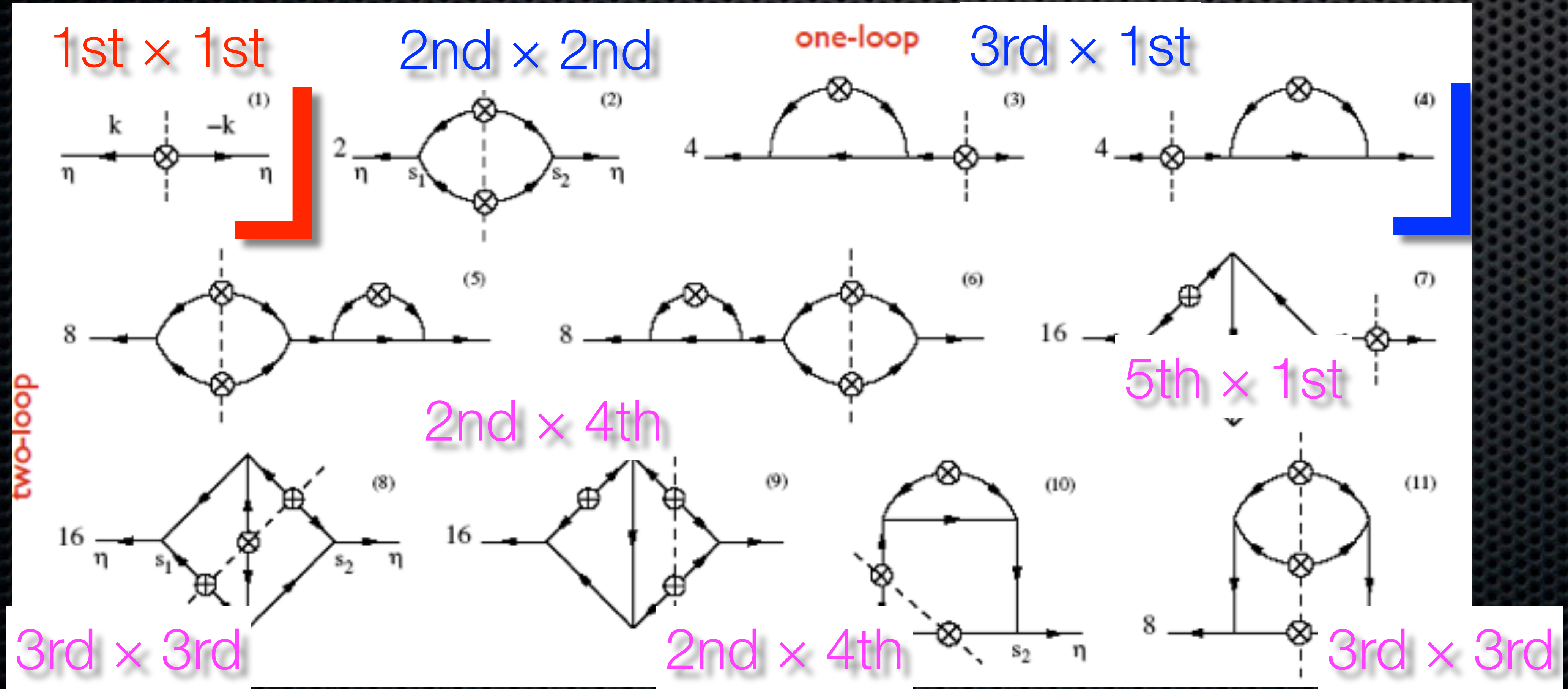
Crocce,Scoccimarro'06, Taruya,Hiramatsu'07, Matsubara'08,...

- "renormalization" techniques: group infinite diagrams and sum them up

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \delta^{(4)} + \ldots$$

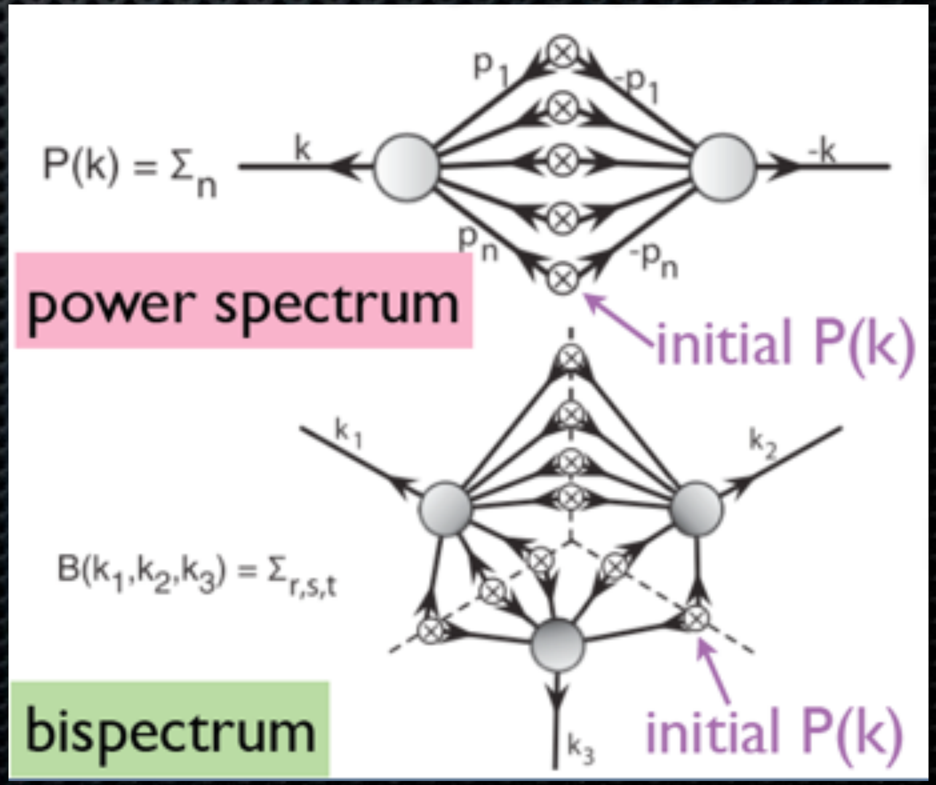
$$\theta = \theta^{(1)} + \theta^{(2)} + \theta^{(3)} + \theta^{(4)} + \ldots$$

power spectrum $P = P^{(11)} + P^{(22)} + P^{(31)} + P^{(13)} + \ldots$



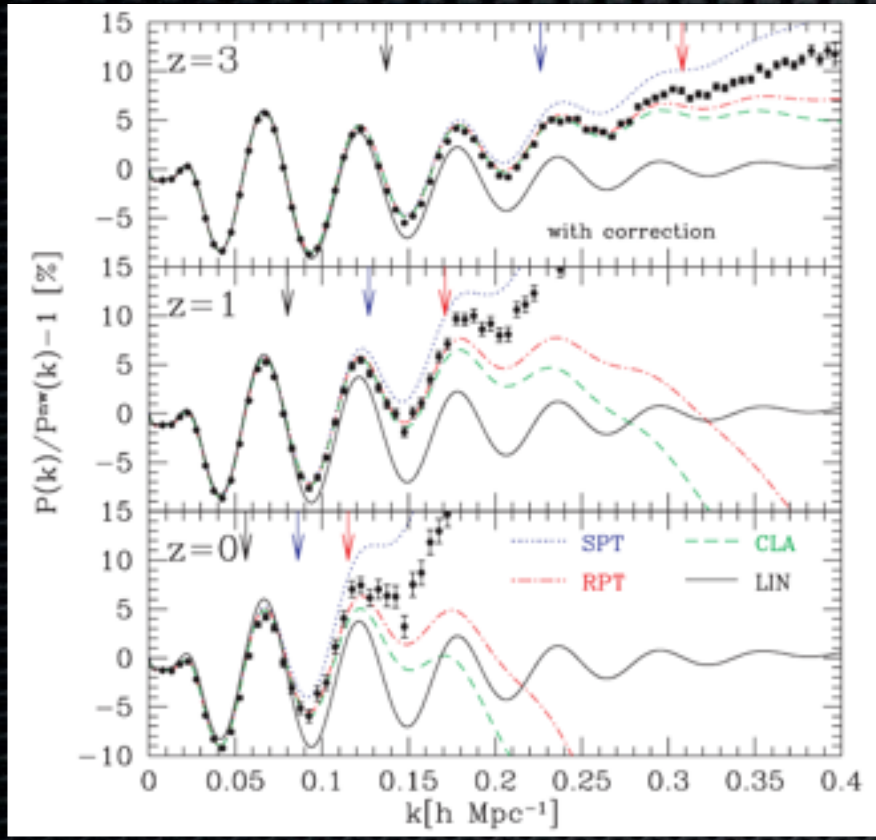
ex. Gamma expansion

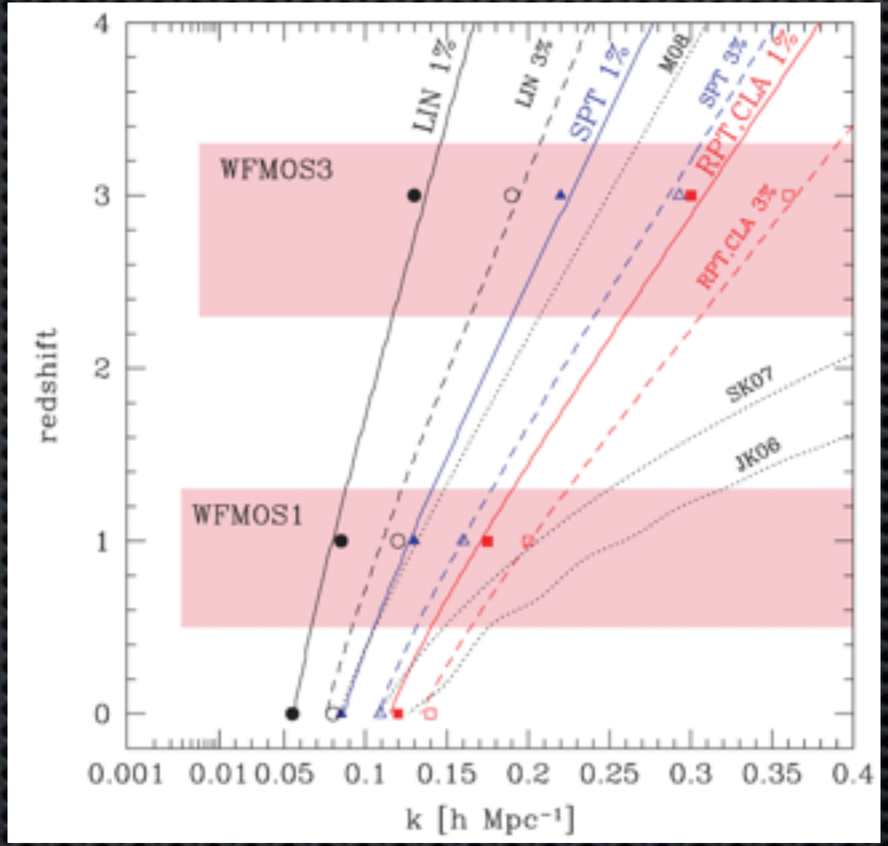
Bernardeau + '09



Accuracy/reliability

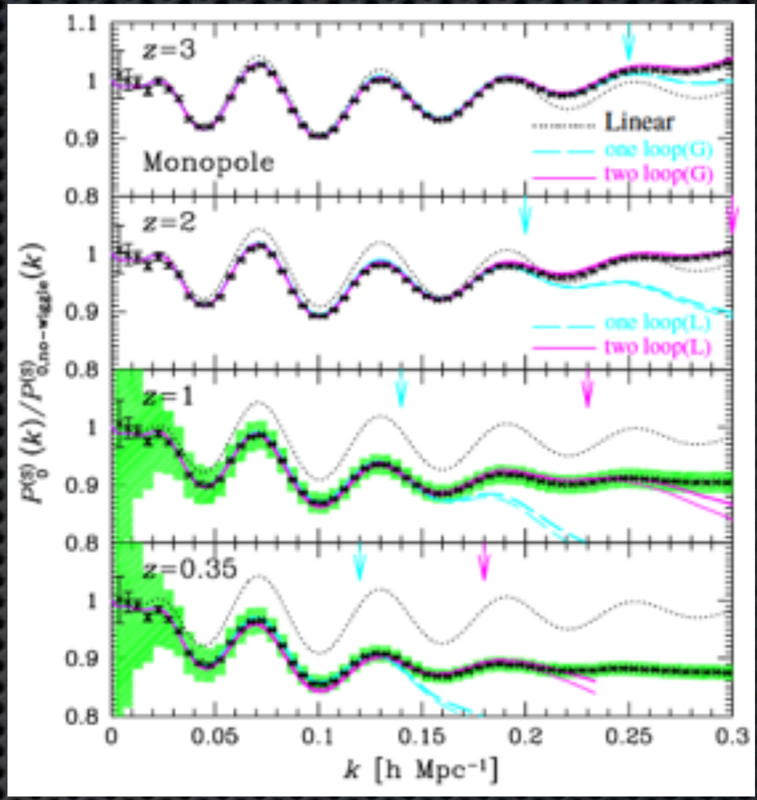
- Comparison with N-body simulations
  - for a given cosmological model
  - as a function of wavenumber k and redshift z





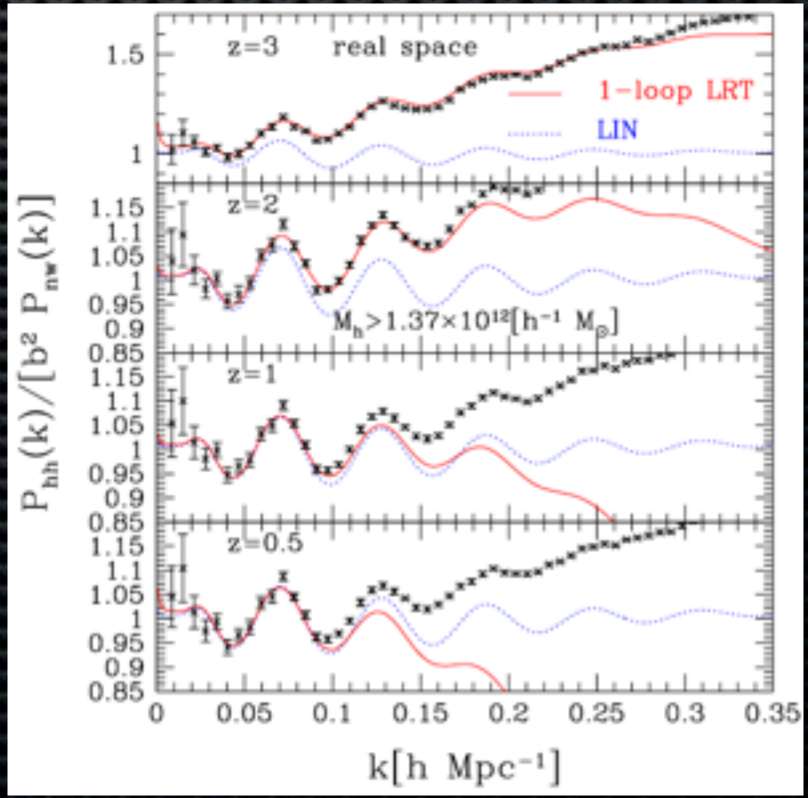
TN+'09

- extension to redshift space



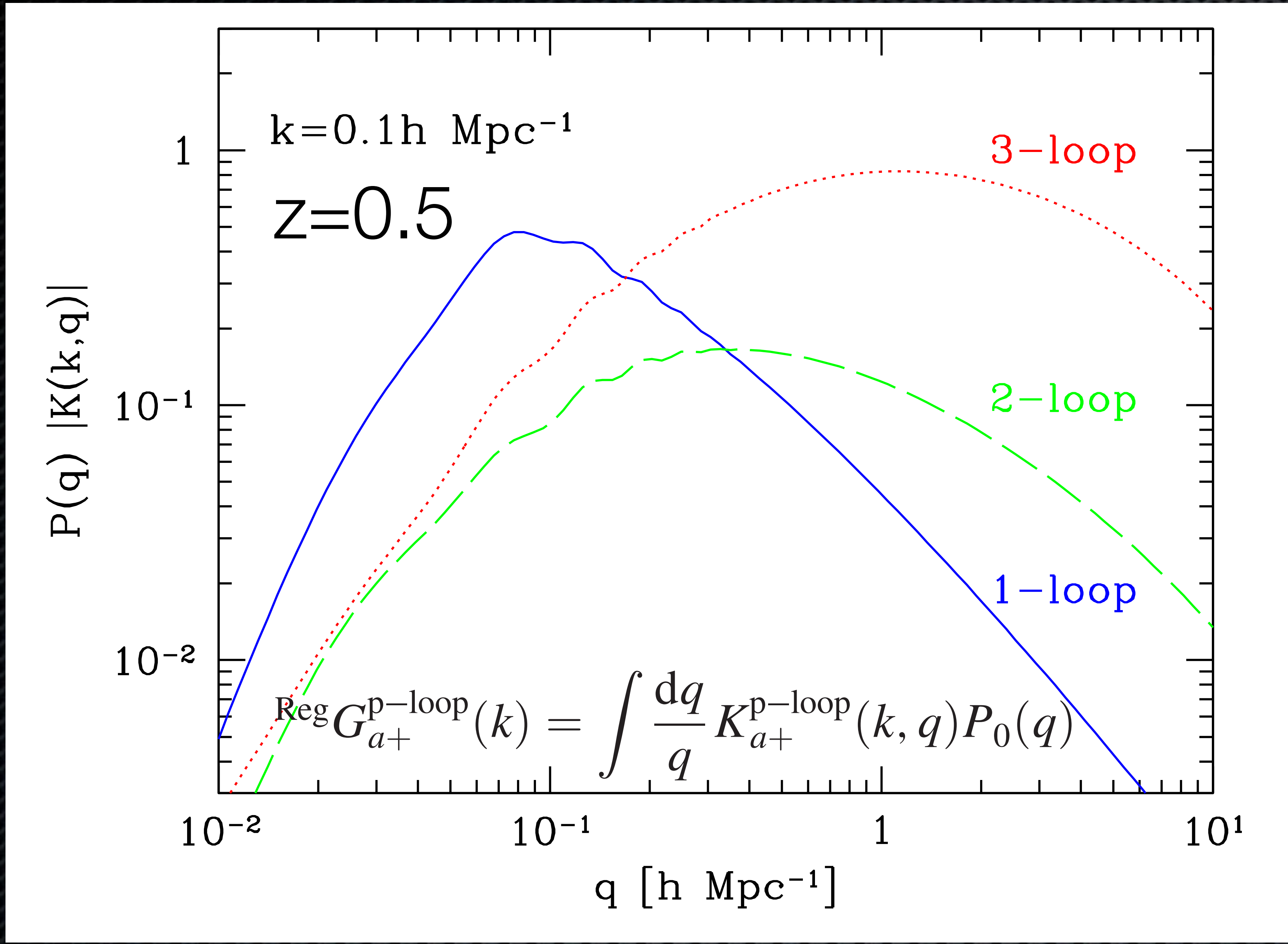
Taruya, TN, Bernardeau'13

- take account of halo bias



Sato, Matsubara'11

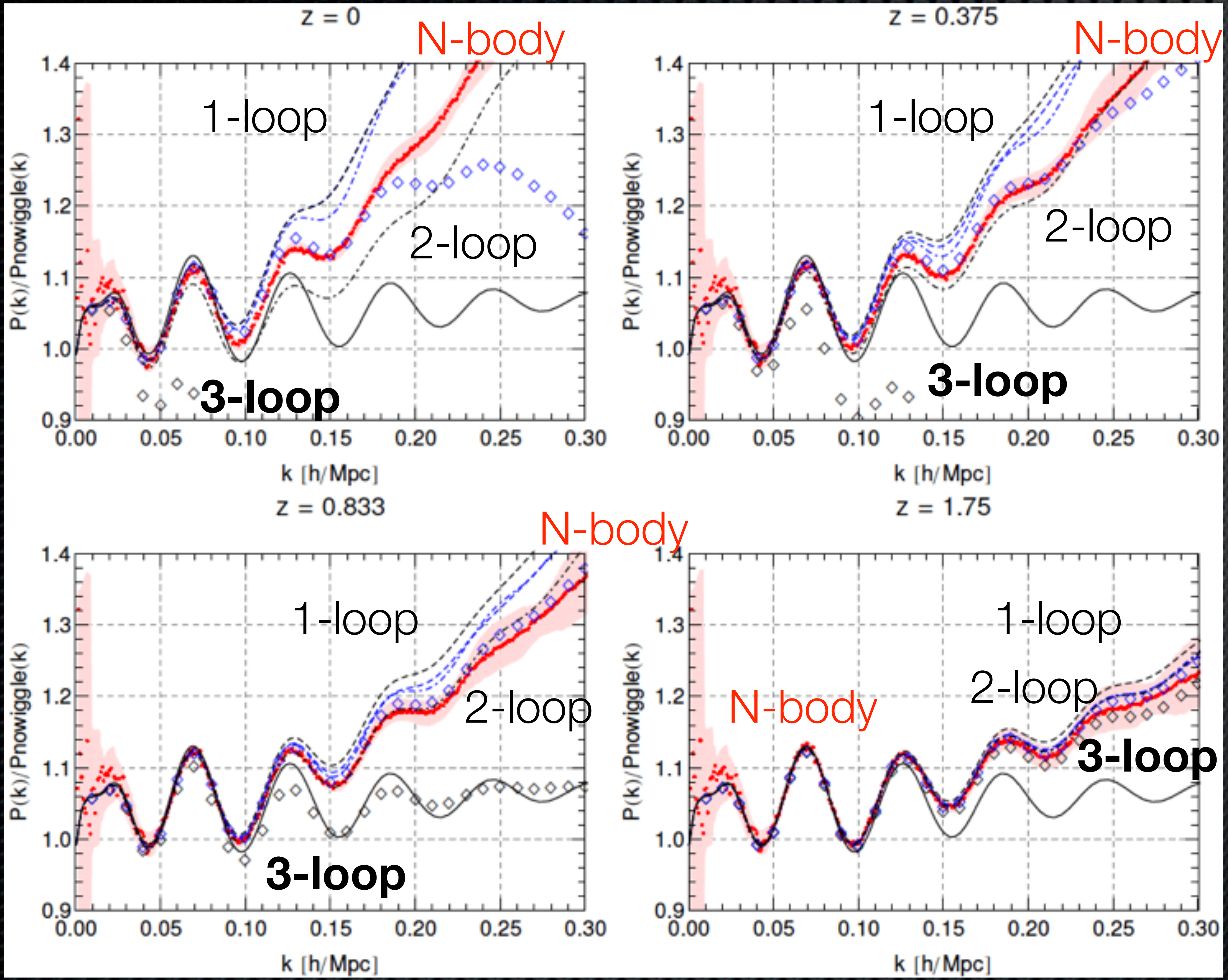
Mode coupling structure from PT



- effective 2-scale kernel function at the level of 2-point propagator
- How sensitive is a wave mode k in the final state to a wave mode q?
  - higher loops dominant @ high q
  - **3-loop > 2-loop over all scales!?**
- PT expansion converges?

Bernardeau, Taruya, TN '14

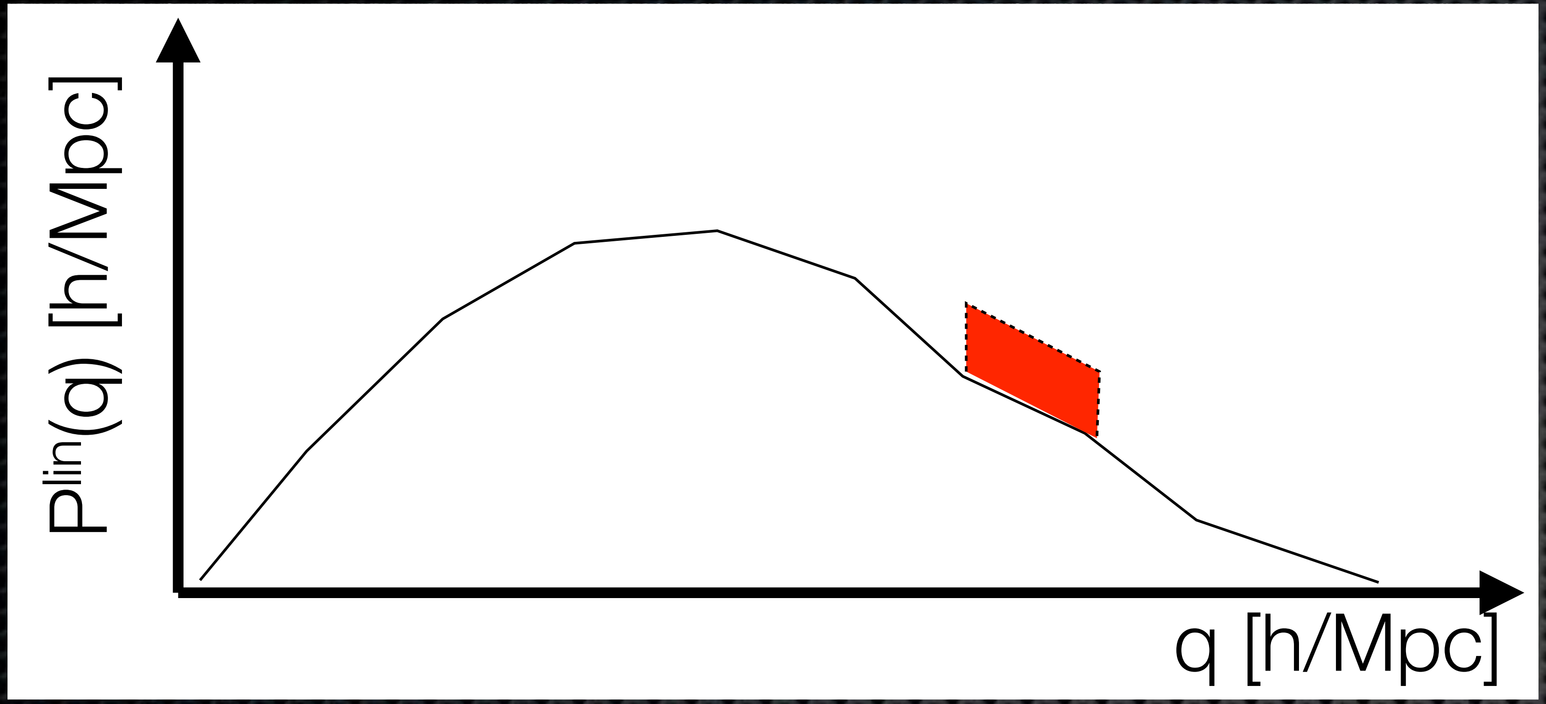
Difficulties beyond 2-loops?

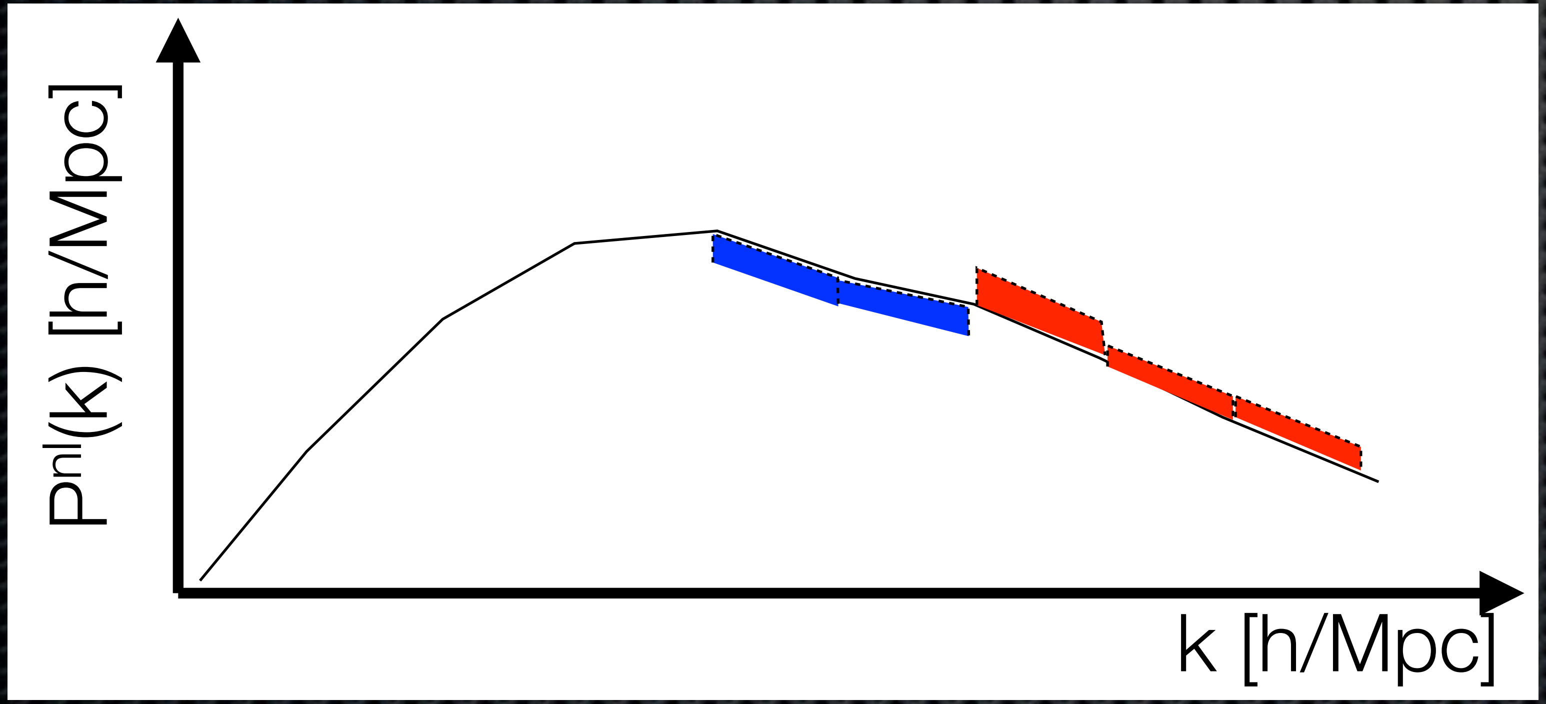


- renormalized PTs so far based on 1- or 2-loop calculations
- Standard PT up to the 3-loop was done recently, but…
  - Need some regularization for higher loop diagrams?
  - Give up PT calculations at low z?
- **PT crisis!?**

Blas, Garny & Konstandin '14

Direct measurement of the kernel function from N-body





- Want to see at the full order, not order by order

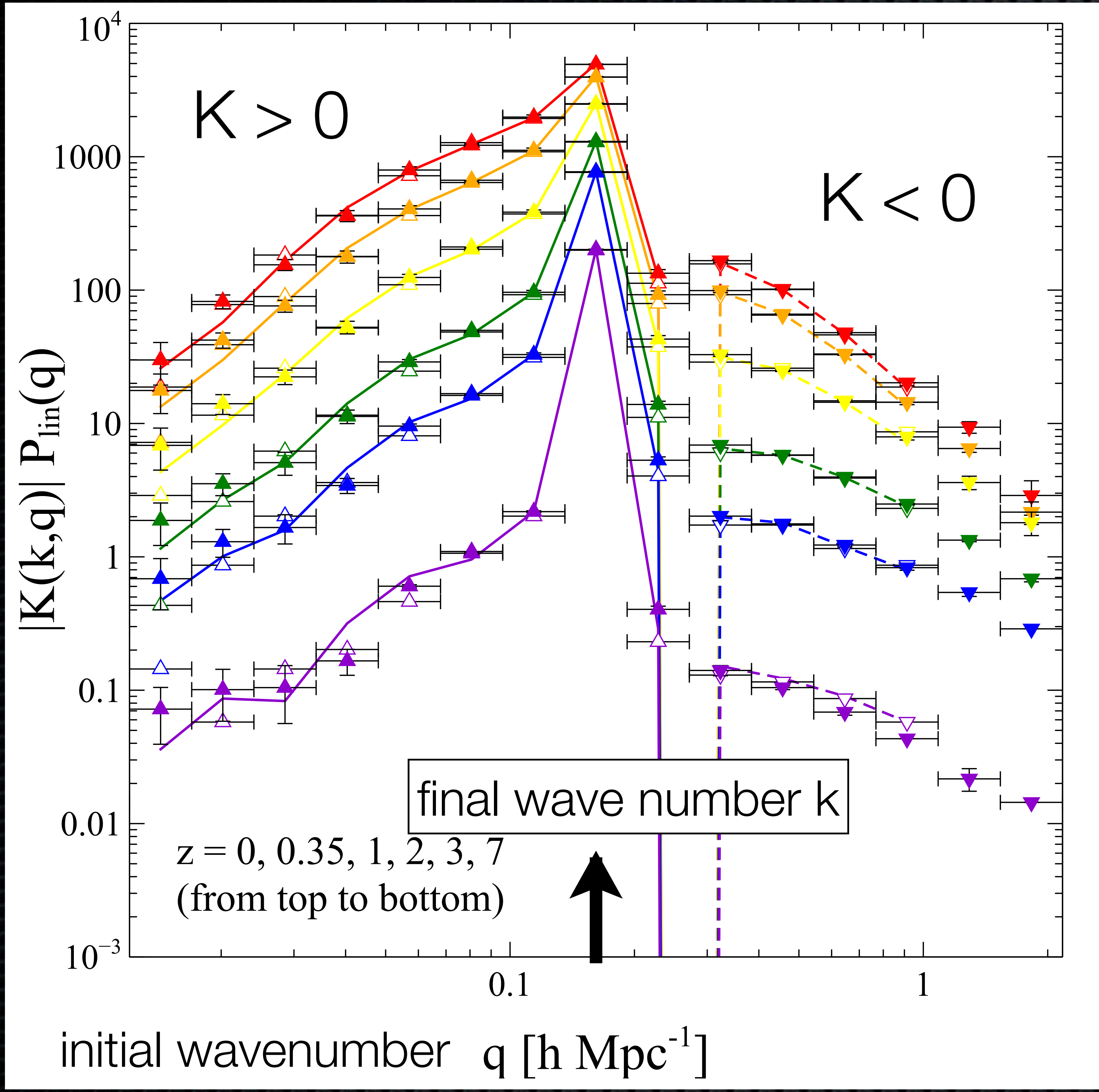
$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

$$\hat{K}_{i,j}P^{\rm lin}_j \equiv \frac{P^{\rm nl}_i[P^{\rm lin}_{+,j}] - P^{\rm nl}_i[P^{\rm lin}_{-,j}]}{\Delta \ln P^{\rm lin}\Delta \ln q}$$

name	box	particles	start-z	bins	runs	total
L9-N9	512	512^3	31	15	4	120
L9-N8	512	256^3	15	13	4	104
L10-N9	1024	512^3	31	15	1	30

TN, Bernardeau, Taruya '14

Measurement of the kernel function



$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

- Converged result against simulation volume and number of particles
  - broader on lower z
  - contribution from larger scales (q<k) dominant

name	box	particles	start-z	bins	runs	total
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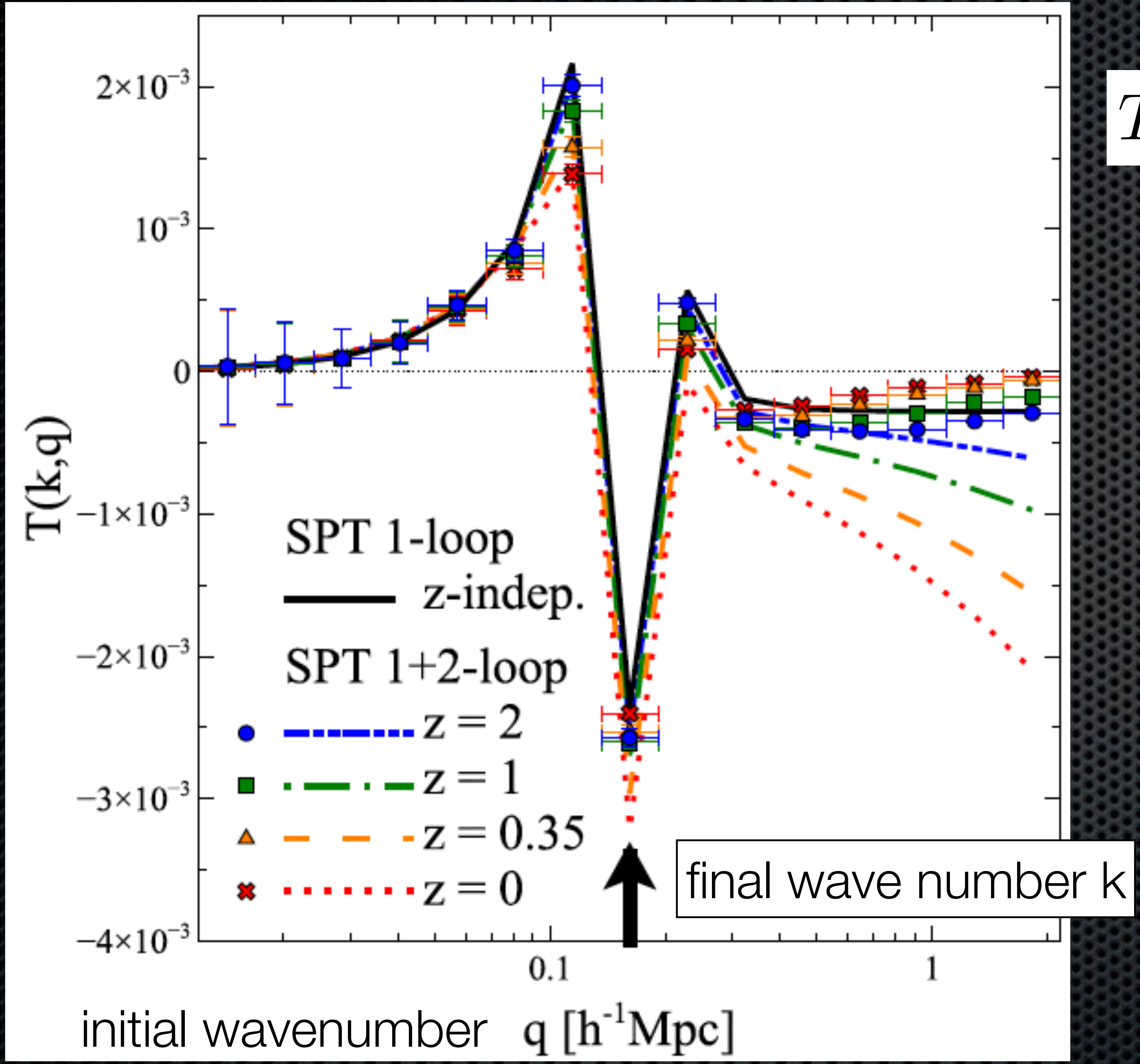
TN, Bernardeau, Taruya '14

Comparison with PT

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



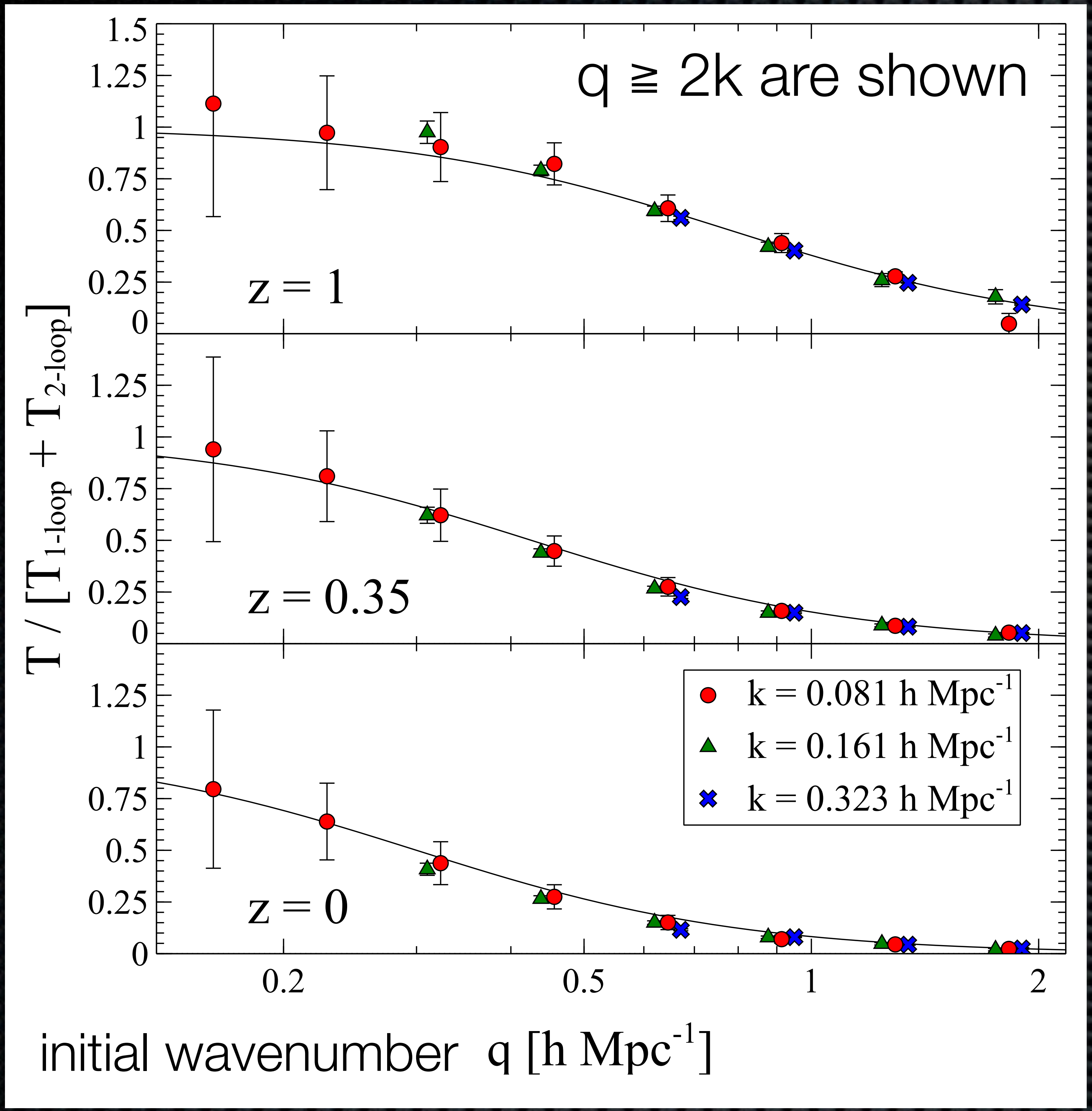
$$T(k,q) = [K(k,q) - K^{\rm lin}(k,q)]/[qP^{\rm lin}(k)]$$

- @ q < k
  - z-dependence hardly seen
  - 1-loop PT works excellently
- @ q ~ k
  - 2-loop surely improves the agreement
- @ q > k
  - approaches to 0 as decreasing z
  - **1-loop not good、2-loop worse!**

TN, Bernardeau, Taruya '14

Damping of coupling to short modes

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



$$T(k,q) = [K(k,q) - K^{\rm lin}(k,q)]/[qP^{\rm lin}(k)]$$

- Simple Lorentzian function fits the data very well

$$T^{\rm eff.}(k,q) = \left[T^{1-\rm loop}(k,q) + T^{2-\rm loop}(k,q)\right]\frac{1}{1+(q/q_0)^2}$$

$$q_0(z) = 0.3D_+^{-2}(z)h/\,\mathrm{Mpc}$$

  - k-independent damping (consistent with PT)
- Some mechanism suppresses the impact of small-scale physics to large scale fluctuations?
  - shell crossing? → Effective Field Theory approach?
  - can be explained within the single stream dynamics?

TN, Bernardeau, Taruya '14

Symmetry of the dynamics and consistency relations

- Gaussian initial condition is supported both theoretically and observationally
  - $P_{ini}=P_L$, $B_{ini}=0$, $T_{ini}=0$, …
- Higher-order polyspectra are generated **exactly by the same mechanism** as that causes nonlinear correction to the power spectrum

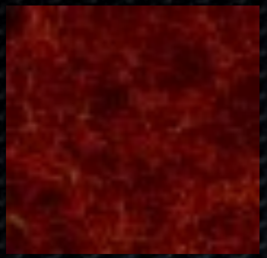
$$P_{1-\mathrm{loop}}(k) \ni 2\int \frac{\mathrm{d}^3q}{(2\pi)^3}[F_2(\mathbf{k}-\mathbf{q},\mathbf{q})]^2 P_L(|\mathbf{k}-\mathbf{q}|)P_L(q)$$

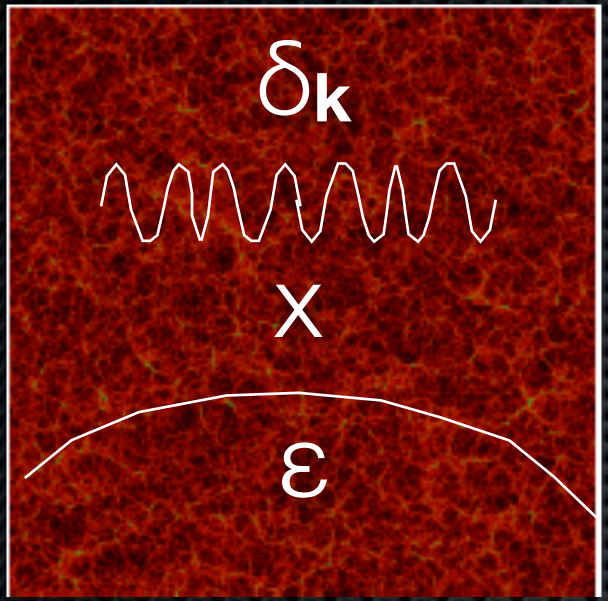
$$B(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = 2F_2^s(\boldsymbol{k}_1,\boldsymbol{k}_2)P_L(k_1)P_L(k_2) + (\mathrm{cyc.}),$$

  - There might exist some relations between different spectra at the full order
  - Their violation immediately means either
    - non-Gaussian initial condition
    - departure from GR
  - Higher-order spectra can be analytically computed using low-order spectra

Angular-averaged equal-time consistency relations

Valageas '14





2 views for the same system

$\{\bar\rho(t), a(t)\}$ $\{\bar\rho'(t), a'(t)\}$

add a uniform density

$\bar\rho a^3 = \bar\rho' a'^3 = \bar\rho_0, \quad a' = a[1-\epsilon(t)], \quad \bar\rho' = \bar\rho[1+3\epsilon(t)]$ $\epsilon(t) = D_+(t)\,\epsilon_0$

$\boldsymbol{x}' = (1+\epsilon)\boldsymbol{x}, \quad \delta' = \delta - 3\epsilon(1+\delta), \quad \boldsymbol{v}' = \boldsymbol{v} + \dot\epsilon a \boldsymbol{x}$ $\phi' = \phi - a^2(\ddot\epsilon + 2H\dot\epsilon)x^2/2$

$D'_+(t) = D_+(t) + y(t)$ $y(t) = \frac{13}{7}\epsilon_0 D_+(t)^2$

compensate by a negative fluctuation

Response of a hard mode k to a soft mode ε

$$\left.\frac{\partial\tilde\delta(\mathbf{k},t)}{\partial\epsilon_0}\right|_{\epsilon_0=0} = D_+(t)\left[\frac{13}{7}\frac{\partial\tilde\delta}{\partial\ln D_+} - \mathbf{k}\cdot\frac{\partial\tilde\delta}{\partial\mathbf{k}}\right]$$

linear order calculation for ε

(l+n)-pt function n-pt function

$$\langle\tilde\delta_{\boldsymbol{k}'_1}\ldots\tilde\delta_{\boldsymbol{k}'_\ell}\tilde\delta_{\boldsymbol{k}_1}\ldots\tilde\delta_{\boldsymbol{k}_n}\rangle'_{k'_j\to0} = \mathcal{L}'_1\cdots\mathcal{L}'_\ell\cdot\langle\tilde\delta_{\boldsymbol{k}_1}\ldots\tilde\delta_{\boldsymbol{k}_n}\rangle'.$$

soft modes hard modes

$$\mathcal{L}'_j = P_L(k'_j)\left[1 + \frac{13}{21}\frac{\partial}{\partial\ln D_+} - \frac{1}{3}\sum_{m=j+1}^{\ell}\frac{\partial}{\partial\ln k'_m} - \frac{1}{3}\sum_{i=1}^{n}\frac{\partial}{\partial\ln k_i}\right]$$

Testing angular-averaged equal-time consistency relations

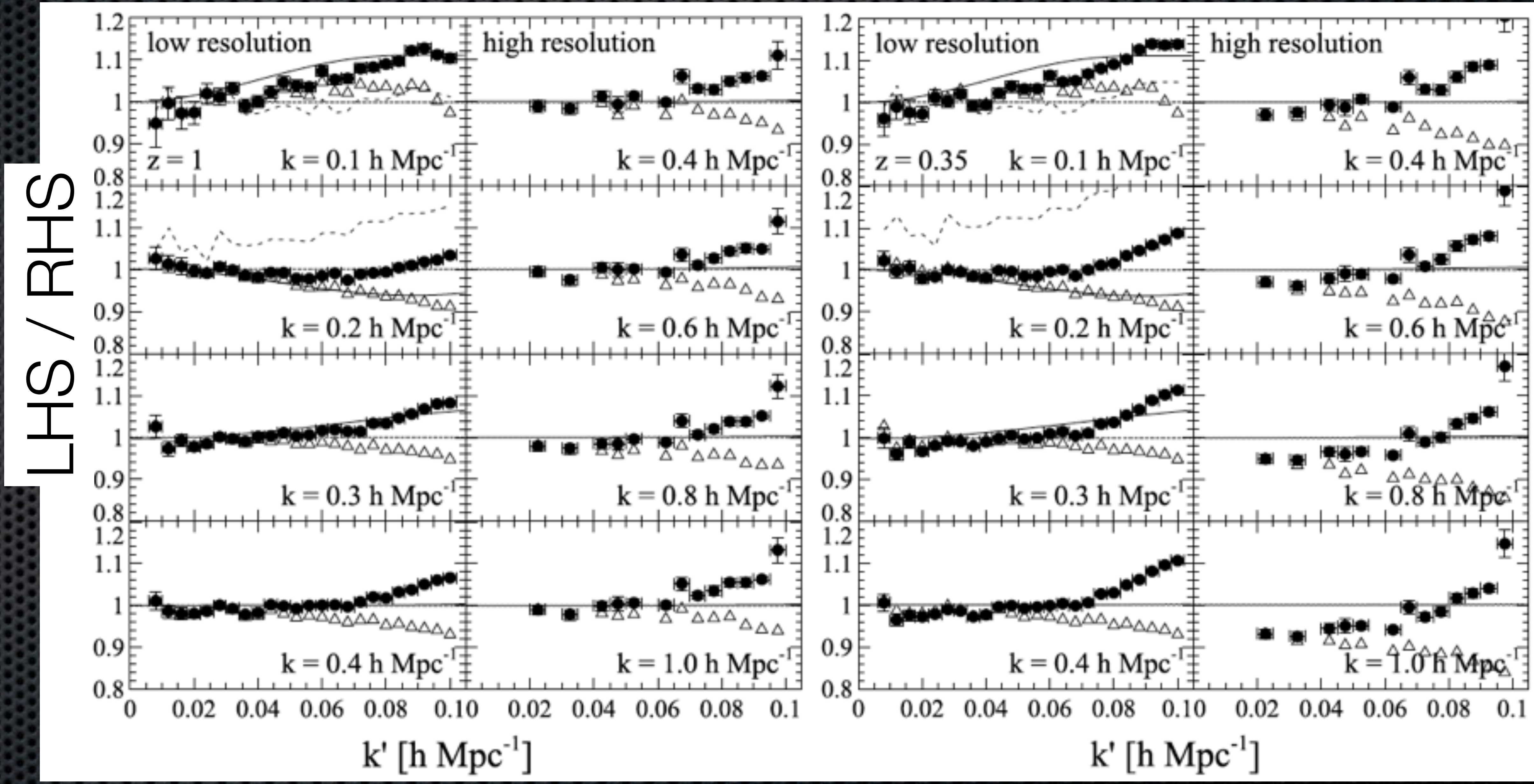
TN, Valageas '14

The lowest order version of the relation:

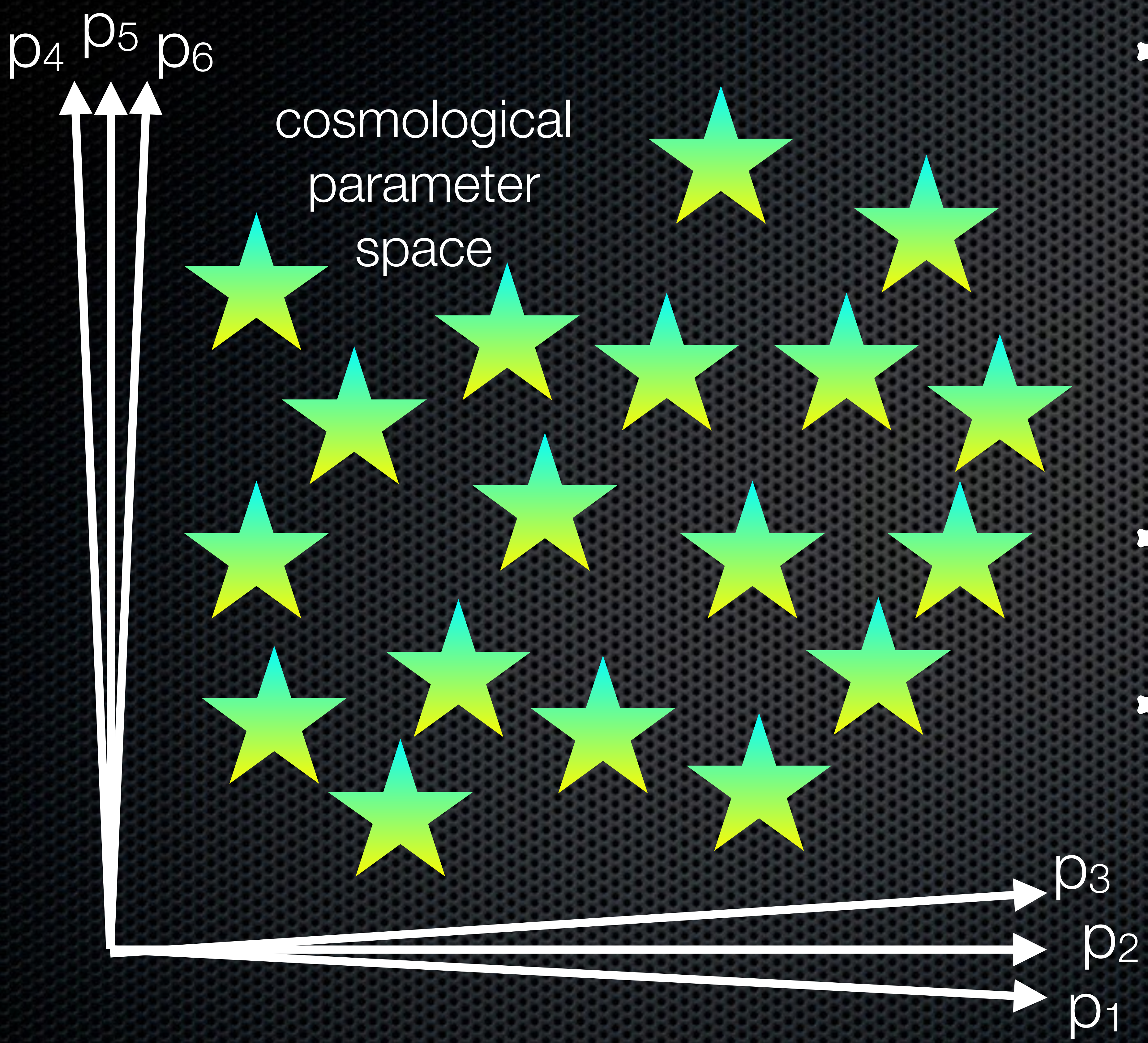
$$\bar{B}(k';k) \equiv \langle \overline{\tilde{\delta}_{\mathbf{k}'}} \tilde{\delta}_{\mathbf{k}-\mathbf{k}'/2} \tilde{\delta}_{-\mathbf{k}-\mathbf{k}'/2} \rangle'$$

$$\bar{B}(k';k)_{k'\to 0} = P_L(k') \left[1 + \frac{13}{21} \frac{\partial}{\partial \ln D_+} - \frac{1}{3} \frac{\partial}{\partial \ln k} \right] P(k)$$

- **Confirmed the relation numerically**
- 60 realization of (2Gpc/h)^3 simulations with 1024^3 particles
- relation holds within ~% accuracy at z=1
- at z=0.35, a sign of the breakdown of the relation is found at high k at ~7% level



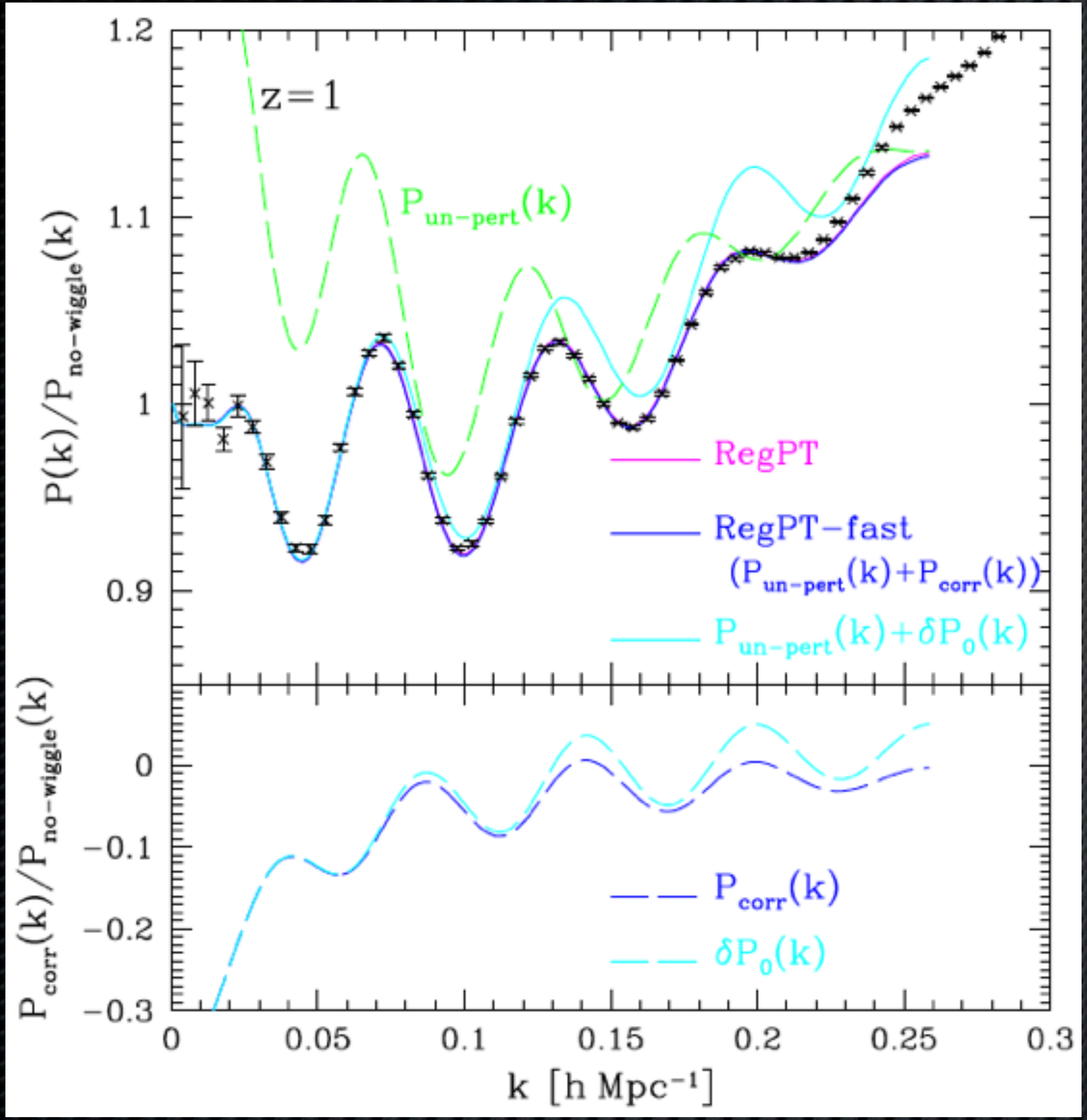
Cosmo Library (tentative title): mock simulation suits for SuMIRe

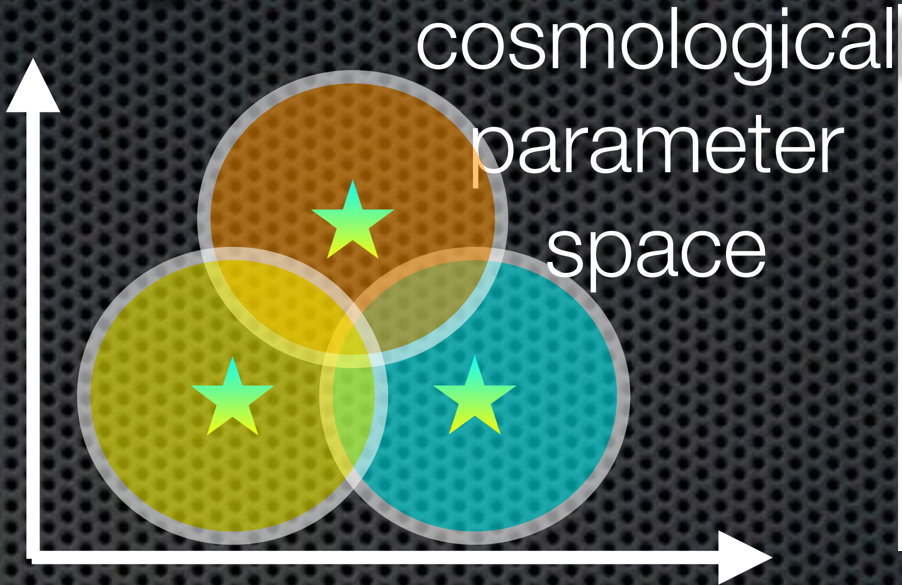


- **Fill up cosmological parameter space with N-body simulations**
  - $\mathcal{O}(1000)$ simulations for the base ΛCDM parameters
  - $\mathcal{O}(100)$ non-standard models (wCDM, non-Gaussianity, modified gravity, etc…)
- Study data convergence and cosmological dependencies
- Release basic data to the community
  - Light cone out put of halos/subhalos
  - Their merger history
  - weak-lensing convergence maps

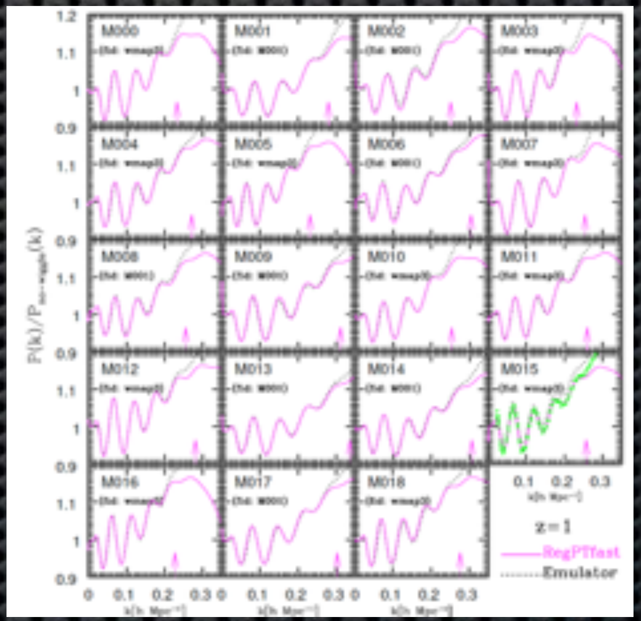
Application of the kernel：RegPTfast

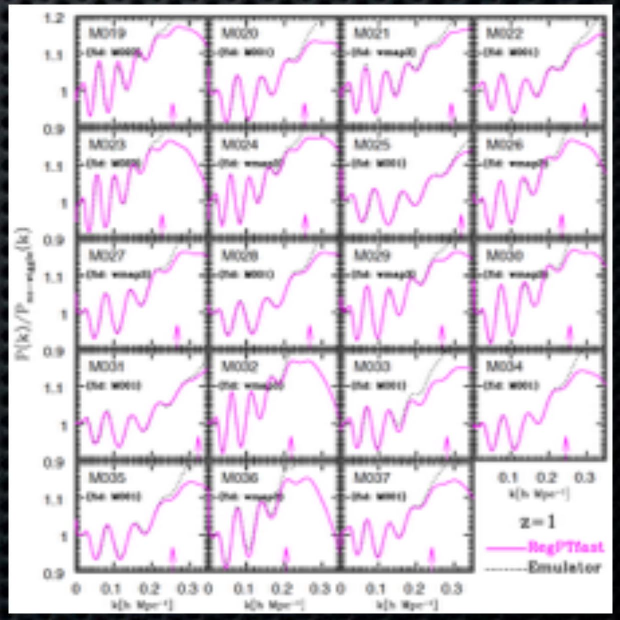
Taruya, Bernardeau, TN, Codis'12





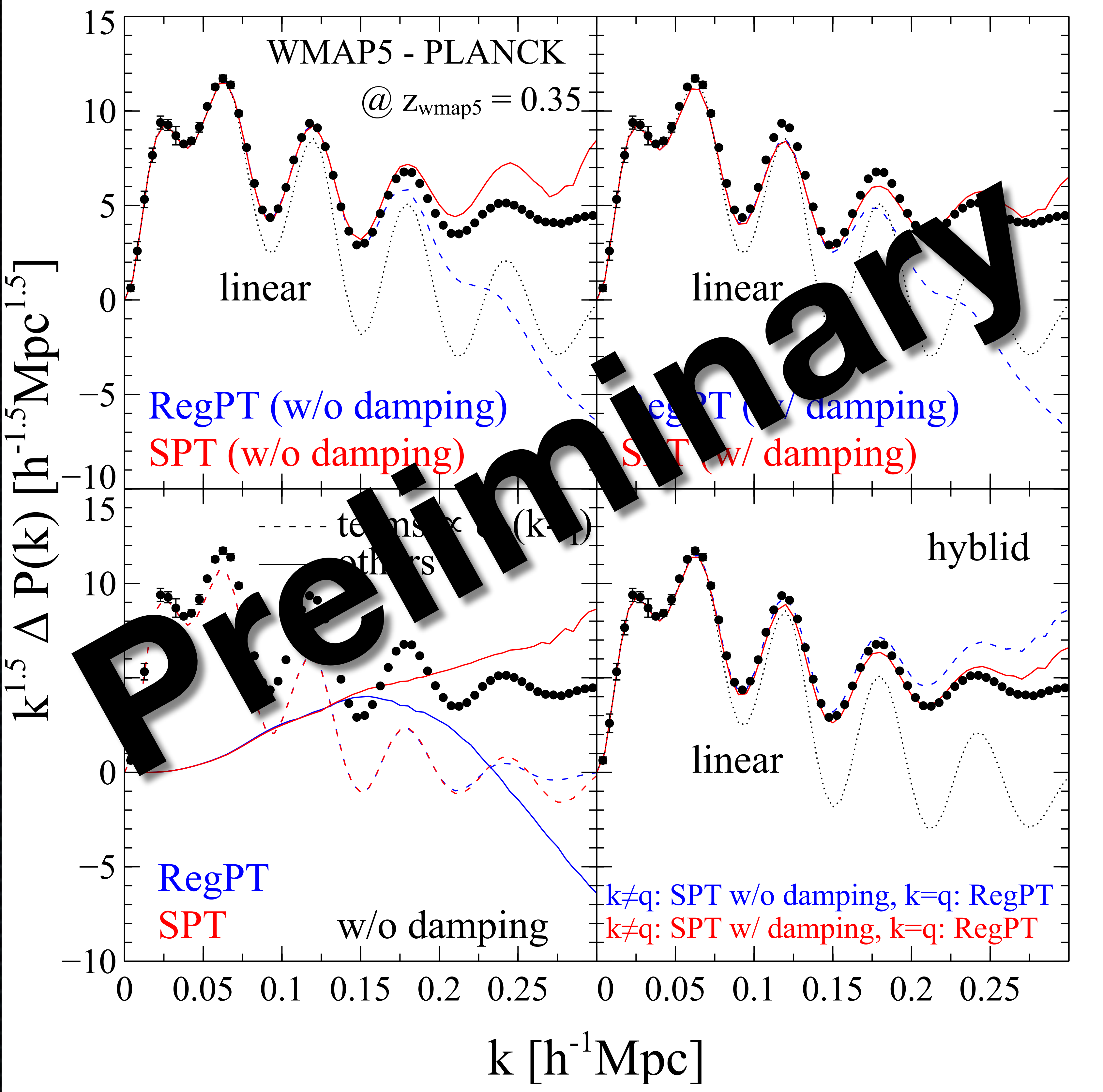
$$0.120 < \omega_{\rm m} < 0.155$$
$$0.0215 < \omega_{\rm b} < 0.0235$$
$$0.85 < n_{\rm s} < 1.05$$
$$-1.30 < w < -0.70$$
$$0.616 < \sigma_8 < 0.9$$





- Pre-computed kernels for 3 cosmological models
- Reconstruction of the nonlinear power spectrum using by interpolation using the kernel requires only 1D integrals. **~a few seconds**

Use of the kernel calibrated by N-body simulations

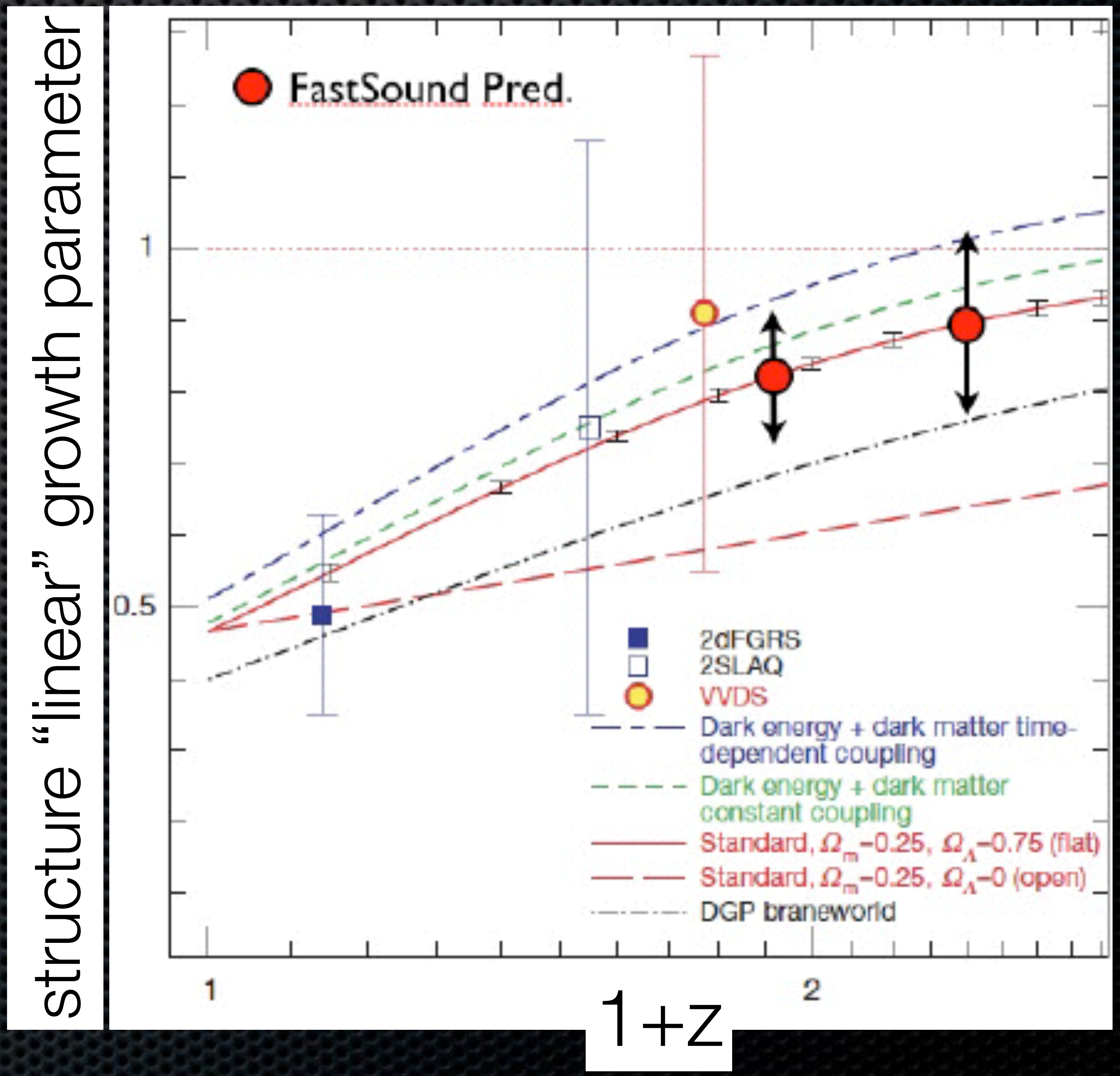


- Calibration of the kernel function with the Cosmo Library data
- RegPTfast-like reconstruction of the nonlinear power spectrum for other cosmological models
- The idea itself can be used not only for the power spectrum, but also for any statistical quantities
  - higher-order statistics
  - halo mass function
  - halo/subhalo bias

Testing modified gravity: toward model independent analysis

Taken from FastSound proposal

- “linear” growth parameter: f(z)=dlnD(z)/dlna
  - useful for constraining modified gravity scenarios
  - One of the most important goal together with measurement of DE EOS
- variety of gravitational law characterized by 1 parameter?
- Do we really measure “linear” thing?
- **Needs for new index**



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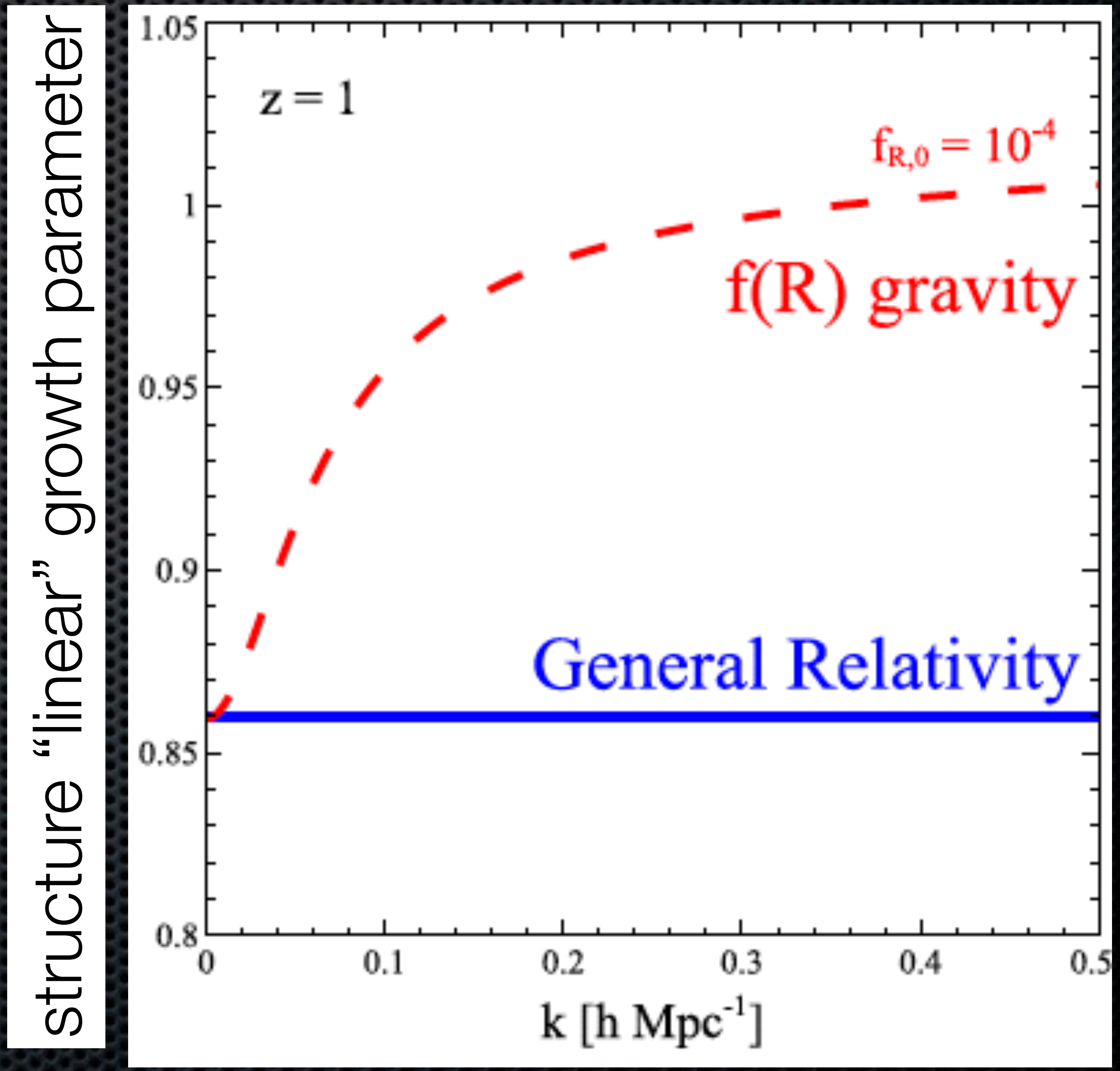

Testing modified gravity: toward model independent analysis

Blake +'11, WiggleZ, ~15k galaxies

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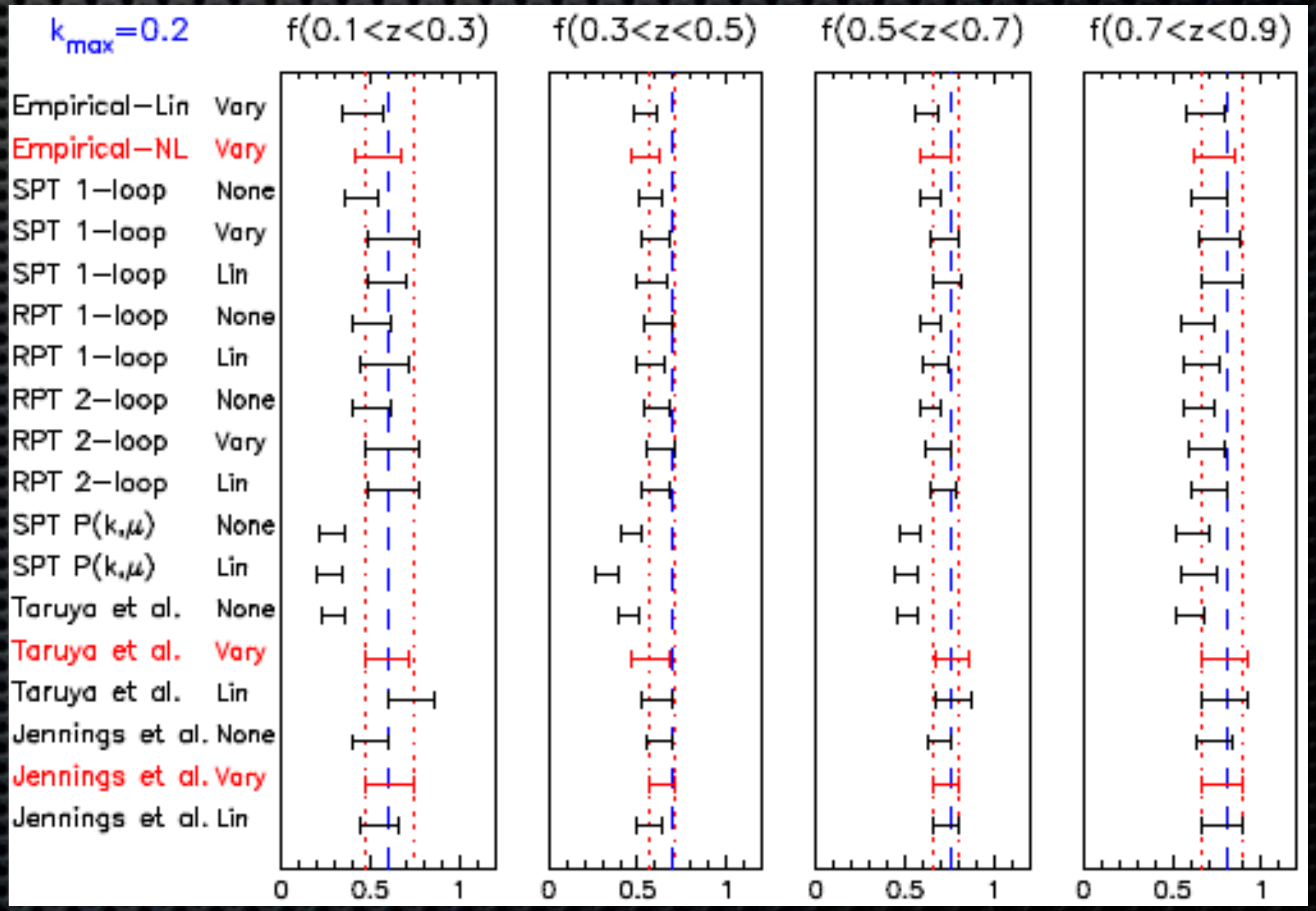
structure "linear" growth parameter



Trying various models for nonlinearity including ours, but…

Testing modified gravity: toward model independent analysis

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    - useful for constraining modified gravity scenarios
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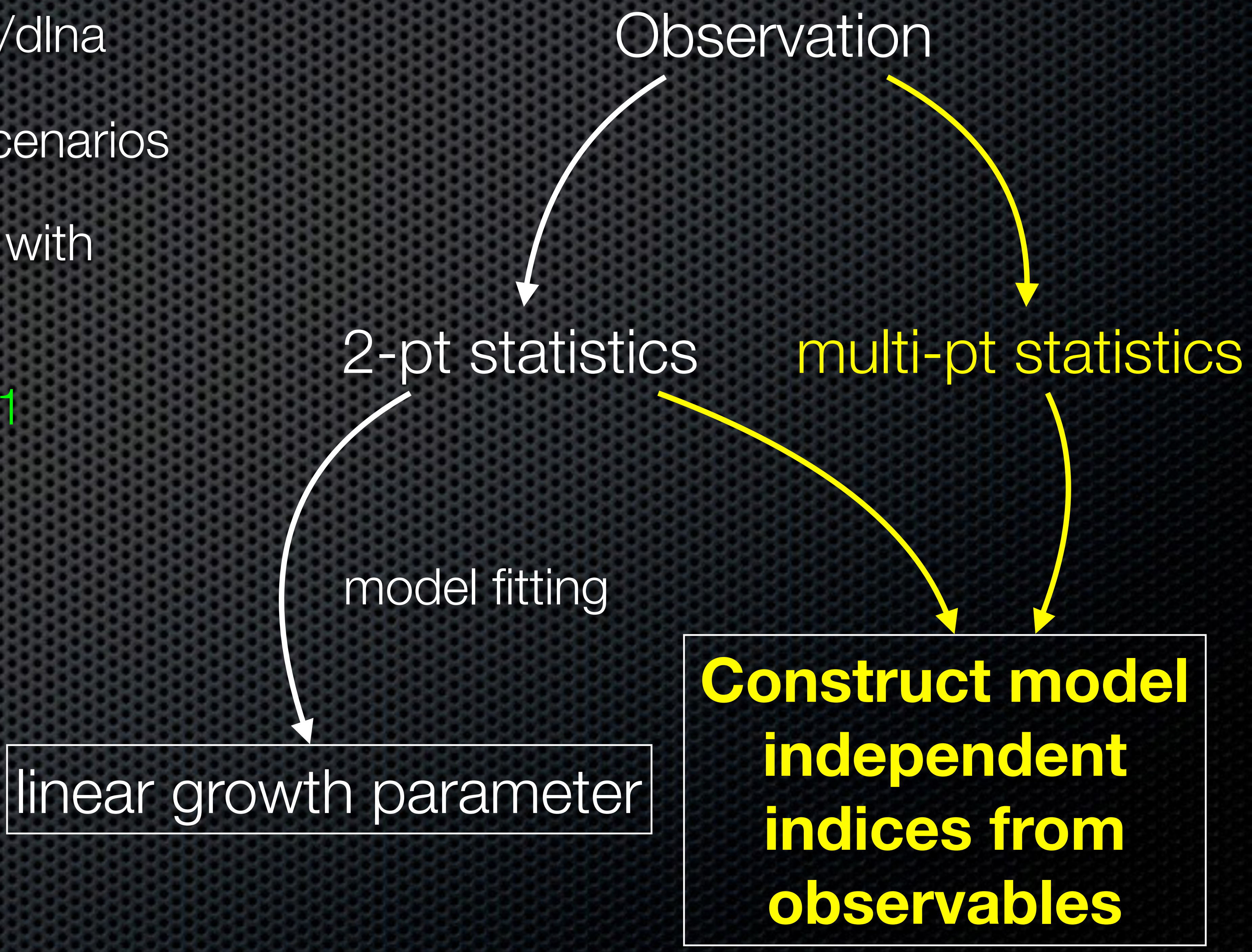
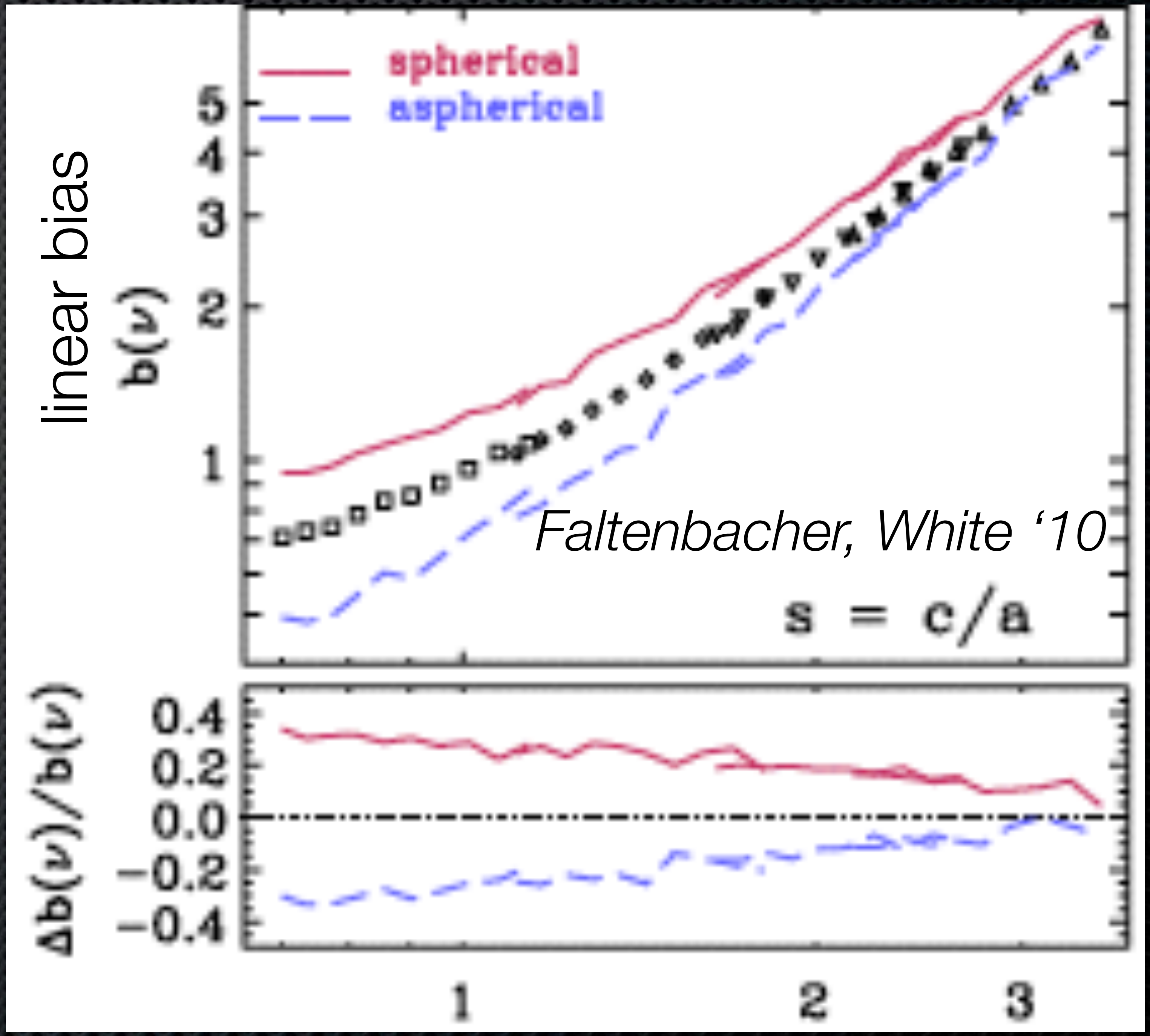


How to characterize galaxy bias?

- peaks or (sub)halos in sims or analytical models
  - the most trivial parameter: peak height or halo mass
  - Halo Occupation Distribution
- Other parameters within the halo picture
  - halo assembly bias (in a wider sense)
  - nonlinear generalization?
  - velocity structure inside a halo
- Faithful cosmological test possible under such complexities of galaxy bias?
- Test these issues with Cosmo Library

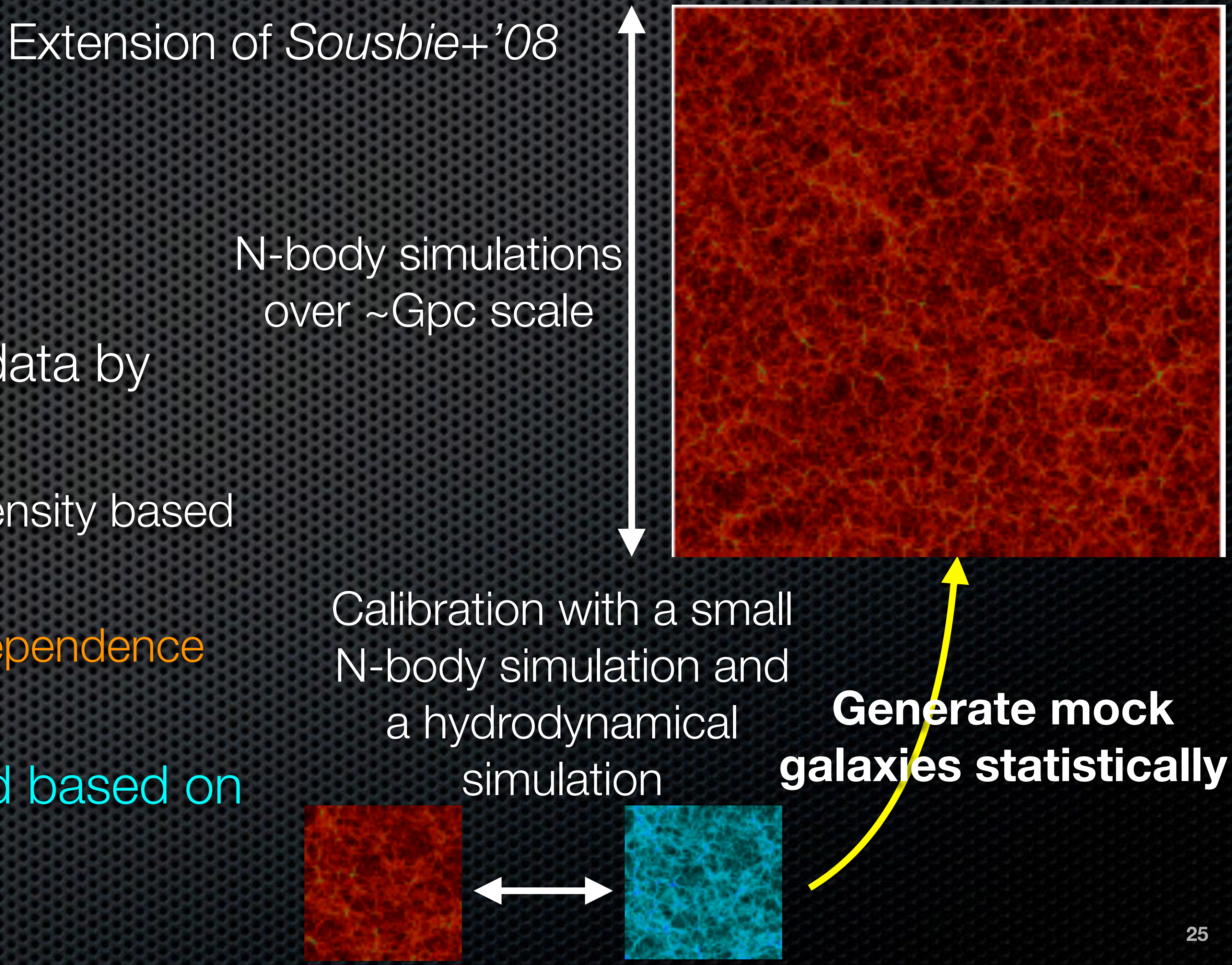


peak height

Toward more realistic mock catalogs

- N-body simulations
  - at most (sub)**halos**
- Galaxy formation
  - semi analytic model
  - hydrodynamical simulations
- Construction of large-scale mock data by combining different simulations
  - Existing method: local mapping of density based on a probabilistic approach
  - Nonlocal extension, environmental dependence taken into account
- Are statistical methods constructed based on halos still useful for "galaxies"?

Mock Local Universe Survey Constructor 2



Summary

- Direct measurement of the mode-coupling structure using $\mathcal{O}(100)$ N-body simulations
  - Is large-scale fluctuations around the BAO wiggles protected from small scale uncertainties?
  - generalize the discussion in 1D (k) to 2D (k, q)
  - q<k: PT is fine
  - q>k: Simulation data decays rapidly
  - Needs for an appropriate regularization scheme!
- Numerical confirmation of the consistency relation between the power and bispectra with 60 simulations
  - relation is confirmed within % accuracy at z=1
  - small but statistically significant violation is found at z=0.35
- Planing even bigger simulations for SuMIRe HSC/PFS with $\mathcal{O}(1000)$ N-body simulations