Why screening mechanism does not solve CC problem

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CC problem

$$R_{\mu\nu} - R\frac{g_{\mu\nu}}{2} + \Lambda g_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}$$

- First guess, Λ must be at Planck scale
- Second gues, maybe some particle physics (SUSY?) scale (comparable to energy-momentum tensor)
- Completely wrong guess;)

- Why it is not large?
- Why it is not zero?
- Why it is comparable to the energy density of the matter NOW?

Some numbers

- Comparison with experimental bound on photon mass

$$m_\gamma^2 < O(10^{-50}) \quad \mathrm{GeV}^2$$
$$\Lambda \sim O(10^{-84}) \quad \mathrm{GeV}^2$$

- Is this comparison cheating? One cannot experiment the mass of photon beyond the scale of universe (c.f. coincidence problem)

- Compared with Planck scale^2, it is 120 digits off;)

- Vacuum energy-density near weak scale,

$$\delta\Lambda = m_{\mathrm{weak}}^4 / M_{pl}^2 \sim 10^{55} \Lambda_{\mathrm{obs}}$$

Screening mechanism

- The CC has a contribution from matter and this is the origin of CC problem

$$\Lambda_{\rm eff} = \Lambda_0 - M_{pl}^{-2}\langle T^{\mu}{}_{\mu}\rangle$$

- Non-zero CC means, universe would be de-Sitter space (if without matter densities)

$$ds^2 = \frac{-d\eta^2 + dx_i^2}{H^2\eta^2} \qquad H^2 = \Lambda_{\rm eff}$$

- (Some say) field theories in dS space is very weird
- In particular, lighter (scalar) fields may be unstable → de-Sitter symmetry might be broken
- Does back-reaction may resolve the issue?

One scenario

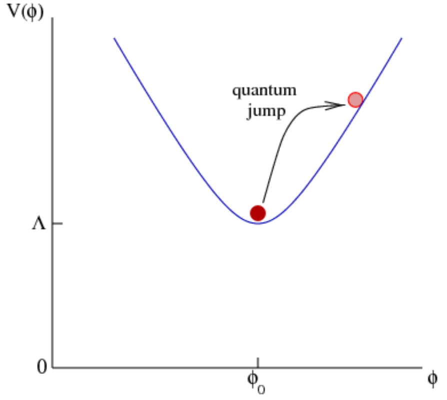
- Consider (minimally coupled) massless scalar in de-Sitter space
- Claimed that there is no de-Sitter invariant state (due to IR divergence)
- Pick a de-Sitter non-invariant state

$$\langle \phi^2(\eta, t) \rangle \sim H^2 \log(-\eta H)$$

- If interaction (e.g. $\lambda \phi^4$) exists, it will back-react to the CC in a time-dependent way

$$\langle T^\mu_{\ \mu} \rangle \sim \lambda H^4 \log^2(-\eta H) \sim \lambda H^6 t^2$$

$$\Lambda_{\rm eff} = \Lambda_0 - M_{pl}^{-2} \lambda H^4 \log^2(-\eta H)$$



- Cf: Eternal inflation
- Cute isn't it?

But…

- Such a self-tuning scenario will never solve the CC problem

- Among others…

- Weinberg's no-go theorem
- Polchinski's too late argument

- Both do apply! (Let alone, actually the sign was wrong…)

Weinberg’s no-go theorem

Self-tuning model will always show run-away behavior

Assumptions

- General covariance
- Massless graviton
- Finite number of fields below cutoff
- No negative norm states
- Constant fields at late times

Weinberg's no-go theorem

Self-tuning model will always show run-away behavior

- Suppose self-tuning is caused by a scalar operator O
- Consider 1PI effective action $\Gamma[O, g_{\mu\nu}]$
- Assuming fields/metric are constant at late time

$$\frac{\delta\Gamma(O, g_{\mu\nu})}{\delta O}|_{const} = 0 \qquad \frac{\delta\Gamma[O, g_{\mu\nu}]}{\delta g_{\mu\nu}}|_{const} = 0$$

- Natural self-tuning requires

$$g_{\mu\nu}\frac{\delta\Gamma[O, g_{\mu\nu}]}{\delta g_{\mu\nu}}|_{const} = f\frac{\delta\Gamma(O, g_{\mu\nu})}{\delta O}|_{const}$$

- But this means effective potential shows run-away

$$\Gamma[O, g_{\mu\nu}] = \int d^4x\sqrt{g}e^{-fO} + \text{derivatives}$$

Polchinski's too late argument

The effects of self-tuning is always too small to cancel CC

- The effects of back-reaction from matter are always too late
- Suppose the matter feels CC and try to back-react through the EM tensor

$$\delta T_{\mu\nu} = O(\Lambda_0^2)$$

- The back-reaction gives the correction to the CC as

$$\delta\Lambda \sim M_{pl}^{-2}\langle T^{\mu}_{\ \mu}\rangle \sim M_{pl}^{-2}\Lambda_0^2$$

- This is too small!!

Polchinski's too late argument

The effects of self-tuning is always too small to cancel CC

- In other words, to cancel the cosmological constant today, the effects must occur when the size of universe is around a meter

$$\delta\Lambda \sim M_{pl}^{-2}\langle T^{\mu}_{\mu}\rangle \sim M_{pl}^{-2}\Lambda_0^2$$

- But at that time, most of energy of our universe comes from matter/radiation. There is no (reasonable) mechanism (1) that detects the CC at that time and (2) cancels so that it is almost zero today
- Cannot work without fine-tuning;)

CC problem (recap)

$$R_{\mu\nu} - R\frac{g_{\mu\nu}}{2} + \Lambda g_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}$$

- Why it is not large?
- Why it is not zero?
- Why it is comparable to the energy density of the matter NOW?

- The self-tuning mechanism does not solve the CC problem (of our universe)
- A-principle?
- Nevertheless I study screening effects in 2D gravity (because it is fun as a field theory problem, and may enhance our understanding of quantum gravity anyway…)

Is cosmological constant screened in Liouville gravity with matter?

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2D classical gravity and CC problem

$$S = \int d^2x \sqrt{-g} \frac{1}{2\kappa}(R - 2\Lambda)$$

- Einstein term is topological
- Without matter, non-zero CC does not allow ANY classical solution

$$\Lambda g_{\mu\nu} = 0$$

- Must be fine-tuned? A-principle?
- Closed string tachyon in critical string theory

2D quantum gravity and CC problem

- Situation is not that dangerous in quantum gravity
- At least matter central charge $-\infty < c_{\mathrm{matter}} \leq 1$

- Formal quantum gravity path integral

$$\int \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{-S[g_{\mu\nu}]-S_{\mathrm{matter}}} = \int \mathcal{D}\Phi e^{-S[\Phi]-S_{\mathrm{matter}}}$$

- Take conformal gauge $g_{\mu\nu} = e^{2\Phi}\hat{g}_{\mu\nu}$
- Jacobian gives kinetic terms for the Liouville field

$$S[\Phi] = \int d^2x\sqrt{\hat{g}}\left(\frac{1}{4\pi b^2}\partial^\mu\Phi\partial_\mu\Phi + \frac{Q}{4\pi b}\Phi\hat{R} + \frac{\Lambda}{\kappa}e^{2\Phi}\right)$$

$$Q = b + b^{-1} \qquad 6Q^2 = 25 - c_{\mathrm{matter}}$$

2D Liouville gravity and CC problem

- So, 2D quantum gravity is ordinary field theory in a fixed background metric $\hat{g}_{\mu\nu}$

$$S[\Phi] = \int d^2x \sqrt{\hat{g}} \left(\frac{1}{4\pi b^2} \partial^\mu \Phi \partial_\mu \Phi + \frac{Q}{4\pi b} \Phi \hat{R} + \frac{\Lambda}{\kappa} e^{2\Phi} \right)$$

- (Renormalized) cosmological constant appears in Liouville potential
- Non-zero CC is not a problem

- Through Liouville equation, it is related to the "actual" curvature of the physical metric $g_{\mu\nu} = e^{2\Phi} \hat{g}_{\mu\nu}$

$$-\frac{1}{4\pi b^2} \Box \Phi + \frac{Q}{4\pi b} \hat{R} + \frac{2\Lambda}{\kappa} e^{2\Phi} = 0$$

Classical Liouville theory and CC

- Suppose matter central charge is (negatively) large
- Classical approximation is valid in Liouville sector

$$b \to 0 \qquad -\frac{1}{4\pi b^2}\Box\Phi + \frac{Q}{4\pi b}\hat{R} + \frac{2\Lambda}{\kappa}e^{2\Phi} = 0$$

- Similar to semi-classical Einstein gravity + quantum matter

$$\Lambda_{eff} = \Lambda_0 - 2\kappa\langle T^{\mu}_{\ \mu}\rangle$$

- A crucial difference (not well recognized):
  - Positive cc (positive energy) → AdS
  - Negative cc (negative energy) → dS

CdL-like instanton in Liouville theory

- Decay of metastable vacua in Liouville theory (Zamolodchikov^2, Nakayama)

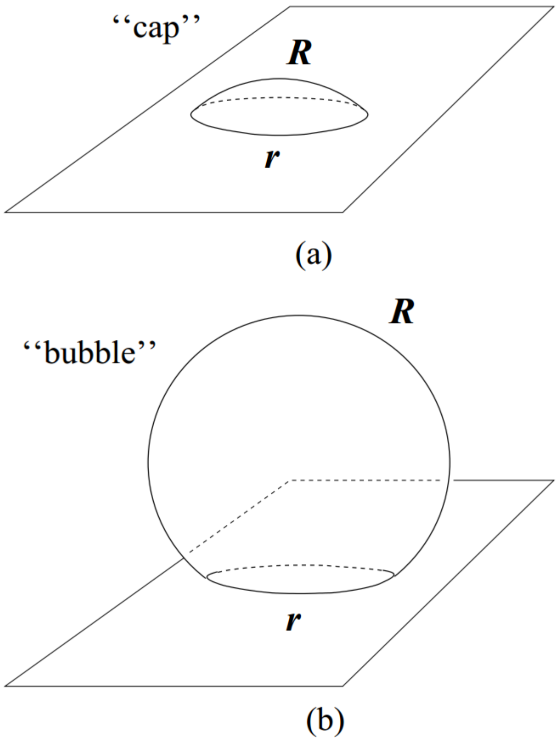
- It is through the CdL-like instanton.

(from AdS-> dS, flat -> dS)

- Euclidean path integral formalism

$$P = \left(1 + \frac{\sigma^2}{b^{-2}\Lambda}\right)^{-b^{-2}}$$

- Semi-classical computation seems in agreement with matrix model



Quantum matter in 2D de-Sitter space

- Semi-classical approach $$ds^2 = e^{2\phi}\hat{g}_{\mu\nu}dx^\mu dx^\nu = \frac{-d\eta^2 + dx^2}{H^2\eta^2}$$
- Fix background Liouville field (de-Sitter space)
- Study matter quantum effects
- Is CC screened (un)like in higher dimensions?

- Example: massless scalar (w/wo interaction)

- Massless scalar in d=2 is "conformal", and dS space is conformally flat, so is it trivial?
- There is an IR divergence (due to zero modes)
- Analytic continuation from sphere misses some physics

Massless scalar in 2D de-Sitter space

$$S = \int d^2x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

- How to treat IR divergence $\quad (\partial_\mu \partial^\mu)^{-1} = ?$
- Treat zero mode separately (works in Euclidean theory, e.g. on sphere)

$$\phi = \phi_0 + \bar{\phi}(x)$$

- Polyakov regularization (used in string theory, somehow unpopular in cosmology)

$$\langle \phi(x)\phi(y) \rangle = \log \left(\frac{(x-y)^2}{e^{\Phi(x)+\Phi(y)}} \right)$$

- Cut-off (consistent with EOM, but breaks dS symmetry)

$$\langle \phi(x)\phi(y) \rangle = \log \left(\frac{(x-y)^2}{H^{-2}} \right)$$

- Bose-Fermi correspondence (later)

Cut-off prescription in $\lambda\phi^4$ theory

- Use cut-off propagator $\langle\phi(x)\phi(y)\rangle = \log\left(\frac{(x-y)^2}{H^{-2}}\right)$
- Compute the energy-momentum tensor

$$\langle T^{\mu}{}_{\mu}\rangle = -\langle\lambda\phi^4\rangle$$

- Lowest order @ 2-loop

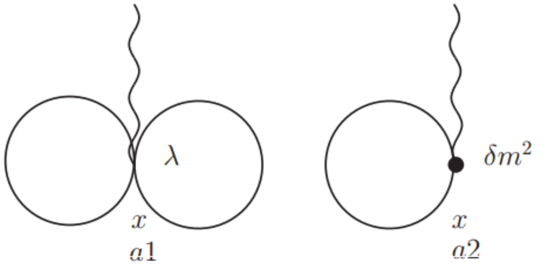


Figure 1: Order λ corrections to the EM tensor.

- This may not be universal (may be changed by operator renormalization), but anyway it gives

$$\Lambda_{\rm eff} = \Lambda_0 - \frac{\kappa}{2}\langle T^{\mu}{}_{\mu}\rangle = \Lambda + \frac{\kappa\lambda}{32\pi^2}\log^2(-H\eta)$$

Cut-off prescription in $\lambda\phi^4$ theory

- Similarly @ 3-loop

$$\delta\Lambda \sim \frac{1}{8\pi}\frac{\lambda^2}{(4\pi)^2 H^2}\log^4(-\eta H)$$

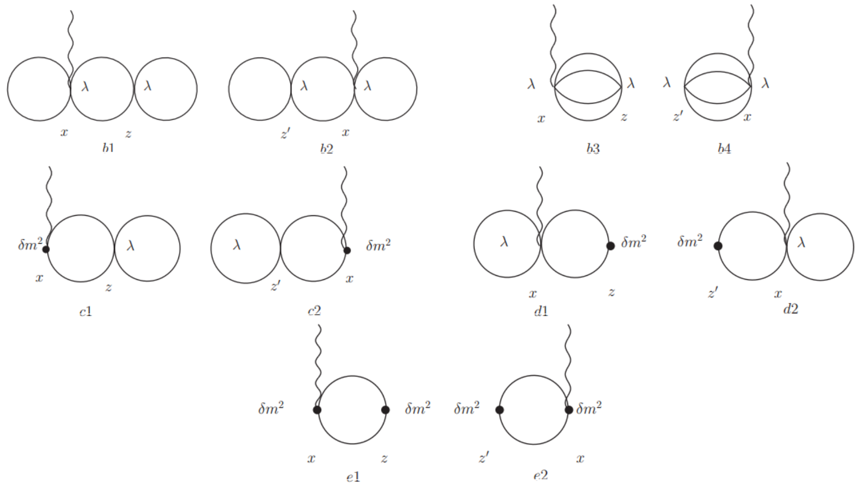


Figure 3: Order λ^2 corrections to the EM tensor $T_{\mu\nu\text{pot}}$.

- The structure is very similar to that in d=4
- BUT, since cc is negative in de-Sitter space, the effect is screening! (rather than anti-screening in d=4)
- Note this mechanism does not solve our "CC problem"
- In d=2, no Planck scale, so it may work?

What will happen eventually?

- This perturbative result is puzzling
- In IR, the massless $\lambda\phi^4$ theory will be identified with critical Ising model, and it is equivalent to massless Majorana fermion
- Massless fermion does not show any IR pathology in dS space
- Perturbation must break down, or the screening effect may be just artefact of renormalization ambiguity, choice of state etc…

Cut-off propagator and Bose/Fermi correspondence

- Bose/Fermi correspondence in dS space
- We again use cut-off regularization

$$\langle \phi(x)\phi(y)\rangle = \log\left(\frac{(x-y)^2}{H^{-2}}\right)$$

$$:\bar{\psi}\psi: = \frac{\eta H}{\pi\gamma}\cos 2\sqrt{\pi}\phi\ , \quad :\bar{\psi}\gamma_5\psi := i\frac{\eta H}{\pi\gamma}\sin 2\sqrt{\pi}\phi$$

$$:\bar{\psi}\gamma_\mu\psi: = \frac{\eta\epsilon_{\mu\nu}}{\sqrt{\pi}}\partial^\nu\phi\ , \qquad :\bar{\psi}\gamma_\mu\gamma_5\psi := \frac{\eta}{\sqrt{\pi}}\partial_\mu\phi$$

- Extra factor of η rectifies de-Sitter breaking of scalar propagator to make fermion correlators dS invariant

(Non-)equivalence of Sine-Gordon and massive Thirring in dS space

- Sine-Gordon action

$$S_{gSG} = \frac{1}{2}\int d\tau dx \left(\partial_\tau\phi\partial_\tau\phi - \partial_x\phi\partial_x\phi - \frac{\lambda}{\tau^2}\cos(\beta\phi)\right)$$

- Treat Sine potential term in perturbation
- Cut-off propagator (hiddenly) breaks dS invariance
- Fermionization makes de-Sitter breaking manifest

$$S_{dual} = \int d\tau dx \frac{i}{\tau}\left(\bar{\psi}\gamma_0\partial_\tau\psi - \bar{\psi}\gamma_1\partial_x\psi + \frac{1}{2\tau}\bar{\psi}\gamma_0\psi\right) - \frac{g}{2}(j_0 j_0 - j_1 j_1) - \lambda\frac{\bar{\psi}\psi}{\tau^{2+\beta/2\sqrt{\pi}}}$$

- The fermion mass must be $\frac{\bar{\psi}\psi}{\tau^2}$ for the classical de-Sitter symmetry
- Duality does not hold in de-Sitter space (Bander)??

How to cure the duality and dS invariance

- But there is a quick fix
- Start with manifestly dS invariant massive Thirring model

$$S_{dual} = \int d\tau dx \frac{i}{\tau}\left(\bar{\psi}\gamma_0\partial_\tau\psi - \bar{\psi}\gamma_1\partial_x\psi + \frac{1}{2\tau}\bar{\psi}\gamma_0\psi\right) - \frac{g}{2}(j_0 j_0 - j_1 j_1) - \lambda\frac{\bar{\psi}\psi}{\tau^2}$$

- Perform bosonization

$$S_{dualgSG} = \frac{1}{2}\int d\tau dx \left(\partial_\tau\phi\partial_\tau\phi - \partial_x\phi\partial_x\phi - \frac{m\tau^{\beta/2\sqrt{\pi}}}{\tau^2}\cos(\beta\phi)\right)$$

- Essentially time-dependent renormalization of coupling constant (IR counter-terms)
- Time dependence in coupling constant cancels against the de-Sitter breaking in IR regularization
- With this renormalization, there is no screening of CC
- No eternal inflation (of course, you could pick up a state that breaks dS invariance…)

Conclusion

- In Liouville gravity, negative energy → de-Sitter
- In perturbation theory , $\lambda\phi^4$ interaction screens CC in Liouville gravity

- In Sine-Gordon theory, we can find a renormalization scheme, where de-Sitter invariance is intact and CC is not screened.
- In view of Landau-Ginzburg correspondence, I suspect de-Sitter invariance may be restored in $\lambda\phi^4$ theory at least in the IR limit…
- Is SG soliton stable in dS space?