Holographic vortex pair annihilation in superfluid turbulence

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String Seminars@Kavli IPMU

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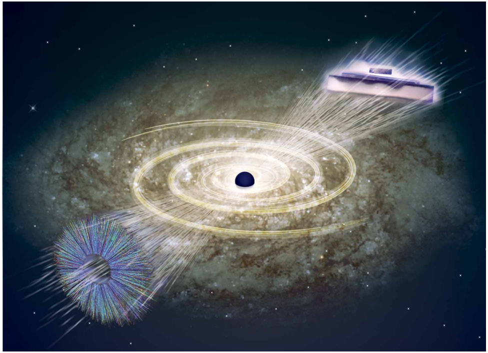
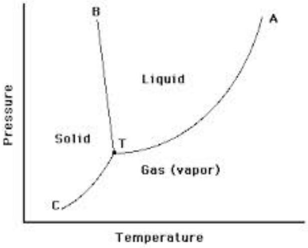
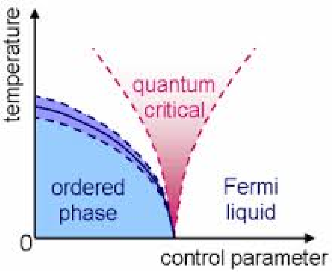
LIGO connected to LHC by holography!

Black hole can also answer condensed matter questions!



- The physical world is partially unified by remarkable RG flow in QFT
  - High Energy Physics: IR→UV(Reductionism)
  - Condensed Matter Physics: UV→IR(Emergence)
    - Thermal Phase Transition
    - Quantum Phase Transition





- Another seemingly distinct part is gravitation, which is understood as geometry by general relativity

Remarkably, with AdS/CFT correspondence, general relativity can also geometrize renormalization flow in particular when the quantum field theory is strongly coupled, namely

$$GR = RG.$$

In this sense, the world is further unified by AdS/CFT duality. This talk will focus on its particular application to condensed matter physics by general relativity.

1. Motivation and introduction

2. Holographic model of superfluids

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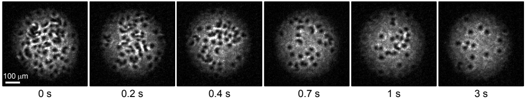
5. Conclusion and outlook

[Shin *et.al.* arXiv:1403.4658]



Gross-Pitaevskii equation

$$(i-\eta)\hbar\partial_t\varphi = \left(-\frac{\nabla^2}{2m} + V(x,y,t) + g|\varphi|^2 - \mu\right)\varphi$$

Here comes AdS/CFT I

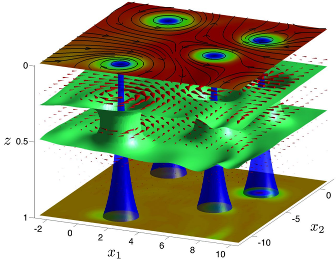
It is a machine, mapping a hard quantum many-body problem to an easy classical few-body one.

- Strongly coupled systems
- Non-equilibrium behaviors

Here comes AdS/CFT I

[Adams, Chesler, Liu, arXiv:1212.0281]

- Kolmogorov scaling law: $\epsilon_{kin}(k) \sim \varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$,
- A direct energy cascade from IR to UV.



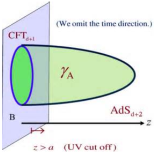
But the temporal variation of vortex number density $n(t)$ is more easily accessible by cold atom experiments. So here we are!

What AdS/CFT is I: Dictionary

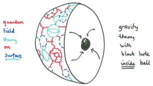
$$Z_{CFT}[J] = S_{AdS}[\phi](J = \phi)$$

What AdS/CFT is II: Implications

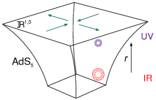
- Entanglement entropy for boundary QFT is equal to the extremal surface area in the bulk gravity



- Finite temperature field theory with finite chemical potential is dual to charged black hole

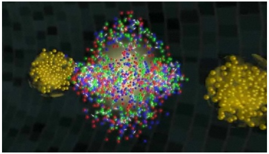


- AdS boundary corresponds to QFT at UV fixed point and the bulk horizon corresponds to IR fixed point

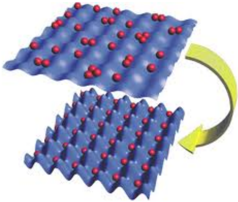


Towards applied AdS/CFT

- AdS/QCD



- AdS/CMT
  Non-Fermi liquids, superfluids and superconductors, charge density waves, thermalization and many-body localization...



- AdS/???

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- Action of model

[Hartnoll, Herzog, and Horowitz,arXiv:0803.3295,0810.6513]

$$S = \frac{1}{16\pi G}\int_{\mathcal{M}} d^4x\sqrt{-g}[R+\frac{6}{L^2}+\frac{1}{q^2}(-\frac{1}{4}F_{ab}F^{ab}-|D\Psi|^2-m^2|\Psi|^2)]. \quad (1)$$

- Background metric

$$ds^2 = \frac{L^2}{z^2}[-f(z)dt^2 - 2dtdz + dx^2 + dy^2], f(z) = 1 - (\frac{z}{z_h})^3. \quad (2)$$

- Heat bath temperature

$$T = \frac{3}{4\pi z_h}. \quad (3)$$

- Equations of motion

$$D_aD_a\Psi - m^2\Psi = 0, \nabla_a F^{ab} = i(\bar{\Psi}D^b\Psi - \Psi\overline{D^b\Psi}). \quad (4)$$

- Asymptotical behavior at AdS boundary

$$A_\nu = a_\nu + b_\nu z + o(z), \tag{5}$$

$$\Psi = \frac{1}{L}[\phi z + z^2\psi + o(z^2)]. \tag{6}$$

- AdS/CFT dictionary

$$\langle J^\nu \rangle = \frac{\delta S_{ren}}{\delta a_\nu} = \lim_{z\to 0} \frac{\sqrt{-g}}{q^2} F^{z\nu}, \tag{7}$$

$$\langle O \rangle = \frac{\delta S_{ren}}{\delta \phi} = \lim_{z\to 0}[\frac{z\sqrt{-g}}{Lq^2}\overline{D^z\Psi} - \frac{z\sqrt{-\gamma}}{L^2q^2}\bar{\Psi}]$$

$$= \frac{1}{q^2}(\bar{\psi} - \dot{\bar{\phi}} - ia_t\bar{\phi}), \tag{8}$$

where

$$S_{ren} = S - \frac{1}{Lq^2}\int_{\mathcal{B}} \sqrt{-\gamma}|\Psi|^2 \tag{9}$$

is the renormalized action by holography.

Phase transition to a superfluid

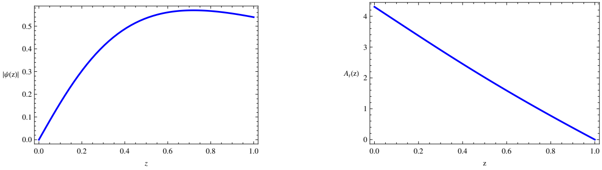

Figure: The profile of amplitude of scalar field and electromagnetic potential for the superconducting phase at the charge density $\rho = 4.7$.

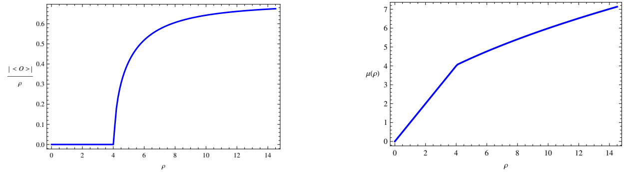

Figure: The condensate and chemical potential as a function of charge density with the critical charge density $\rho_c = 4.06(\mu_c = 4.07)$.

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Quantized vortex in superfluids

With the superfluid velocity defined as

$$\mathbf{u} = \frac{\mathbf{j}}{|\psi|^2}, \mathbf{j} = \frac{i}{2}(\bar{\psi}\partial\psi - \psi\partial\bar{\psi}), \tag{10}$$

the winding number w of a vortex is determined by

$$w = \frac{1}{2\pi}\oint_\gamma d\mathbf{x} \cdot \mathbf{u}, \tag{11}$$

In particular, close to the core of a single vortex with winding number w, the condensate

$$\bar{\psi} \propto (\mathbf{z} - \mathbf{z_0})^w, w > 0 \tag{12}$$

$$\psi \propto (\mathbf{z} - \mathbf{z_0})^{-w}, w < 0 \tag{13}$$

with $\mathbf{z}$ the complex coordinate and $\mathbf{z_0}$ the location of the core.

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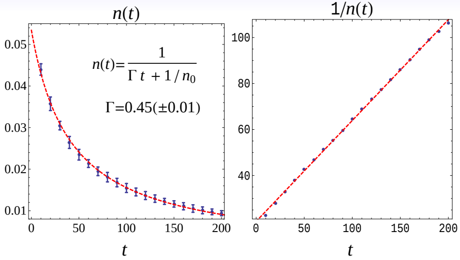

Figure: The temporal evolution of averaged vortex number density in the turbulent superfluid over 12 groups of data with randomly prepared initial conditions at the chemical potential $\mu = 6.25$

$$\frac{dn(t)}{dt} = -\Gamma n(t)^2, \tag{14}$$

where $\Gamma = \frac{vd}{2}$ with v the velocity of vortices and d cross section if the vortices can be regarded as a gas of particles.

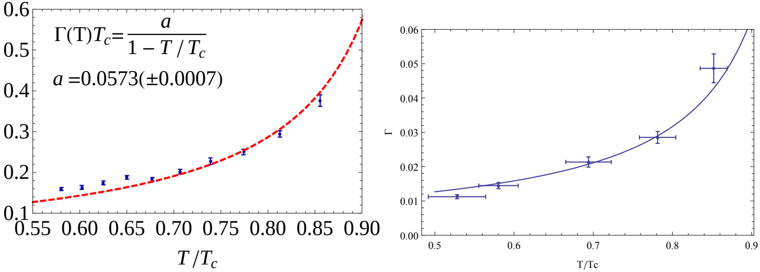

Figure: The variation of decay rate with respect to the temperature. The data near the critical point is fit by the effective field formula $\Gamma \propto |O|^{-2}$ [Chesler, Lucas, arXiv:1411.2610].

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Conclusion

- The decrease of vortex number can be well described by two-body decay due to vortex pair annihilation from a very early time on.
- The decay rate increases with the temperature.
- The decay rate near the critical temperature is in good agreement with the effective field theory calculation and the preliminary experimental data.
- Holography offers a first principles method for one to understand vortex dynamics by its gravity dual and may have an important impact on the upcoming experiments.

Outlook

- Low temperature behavior, where T^2 behavior can be reproduced?
- Other phenomena related to vortex dynamics such as snake instability, where the challenge arises mainly in the non-trivial boundary conditions.
- Back reaction effect, where one is require to go to fully numerical relativity regime.

Thanks for your attention!