Scaling laws for thermoelectric transport at quantum criticality

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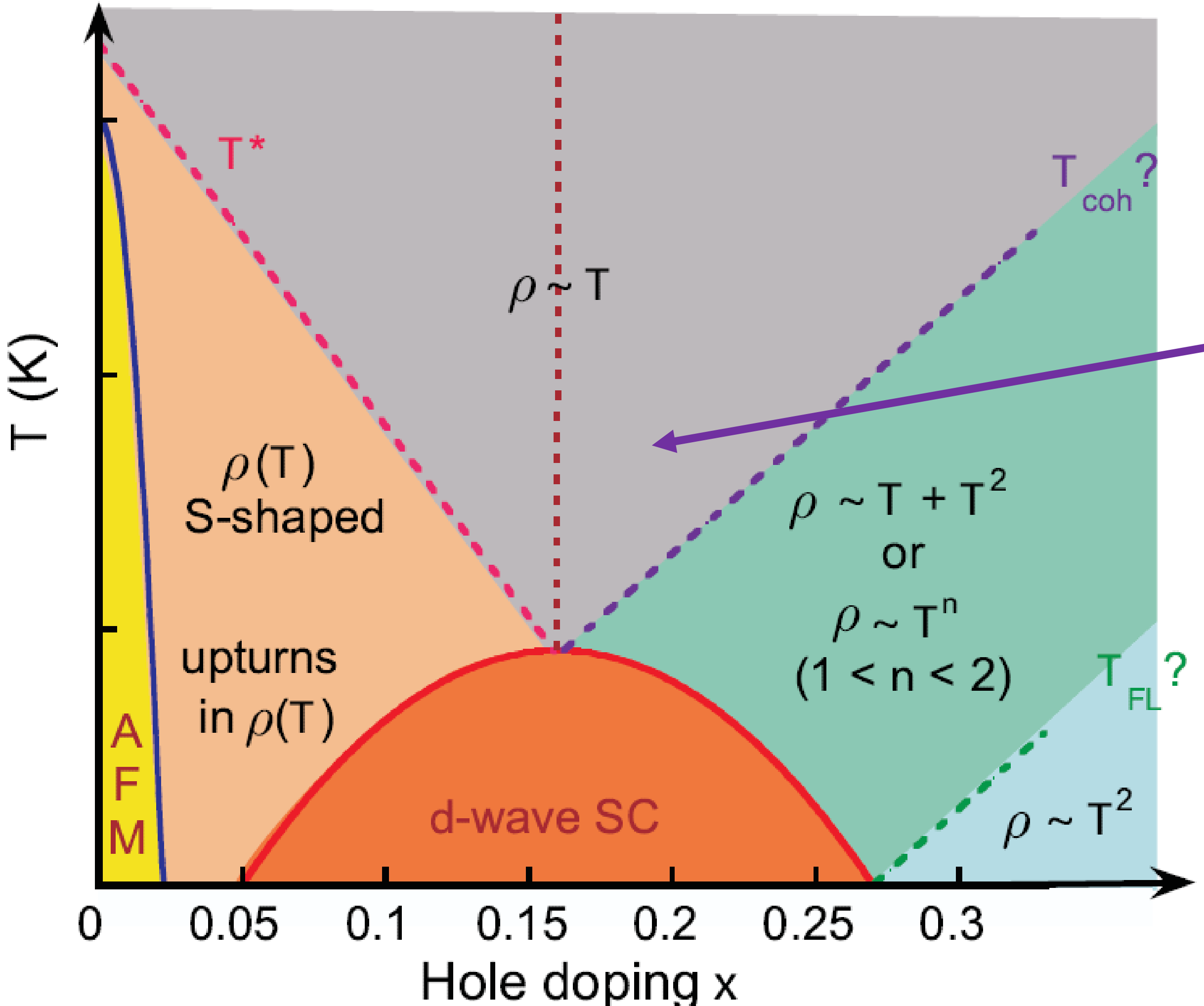
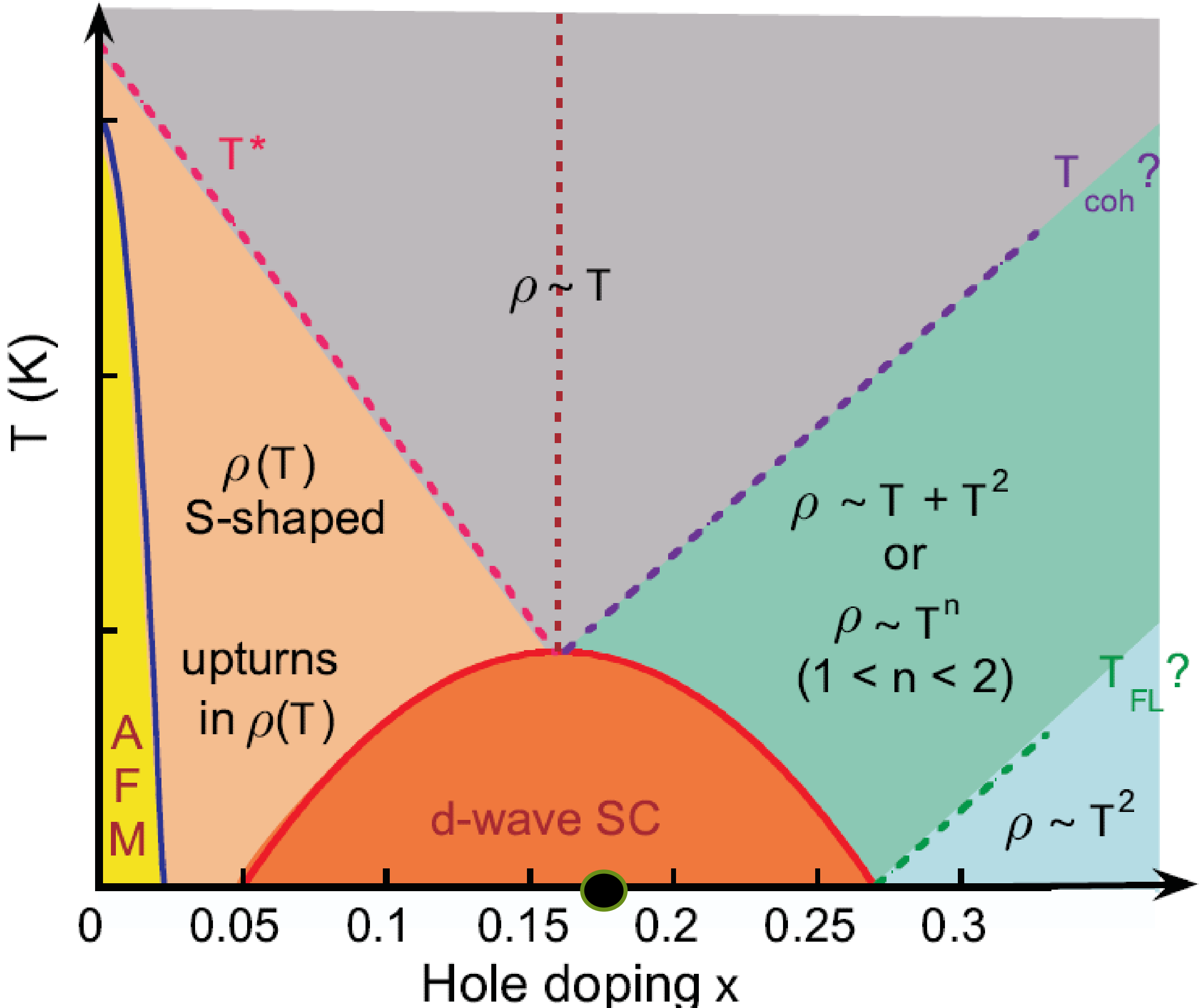
Schematic Phase Diagram of cuprates



Strange Metal Phase
These are the degrees of freedom that condense to form superconductor.

Strange Metal / QCP



One popular scenario:

Linear resistivity
driven by Quantum Critical
Fluctuations?

Why QCP?

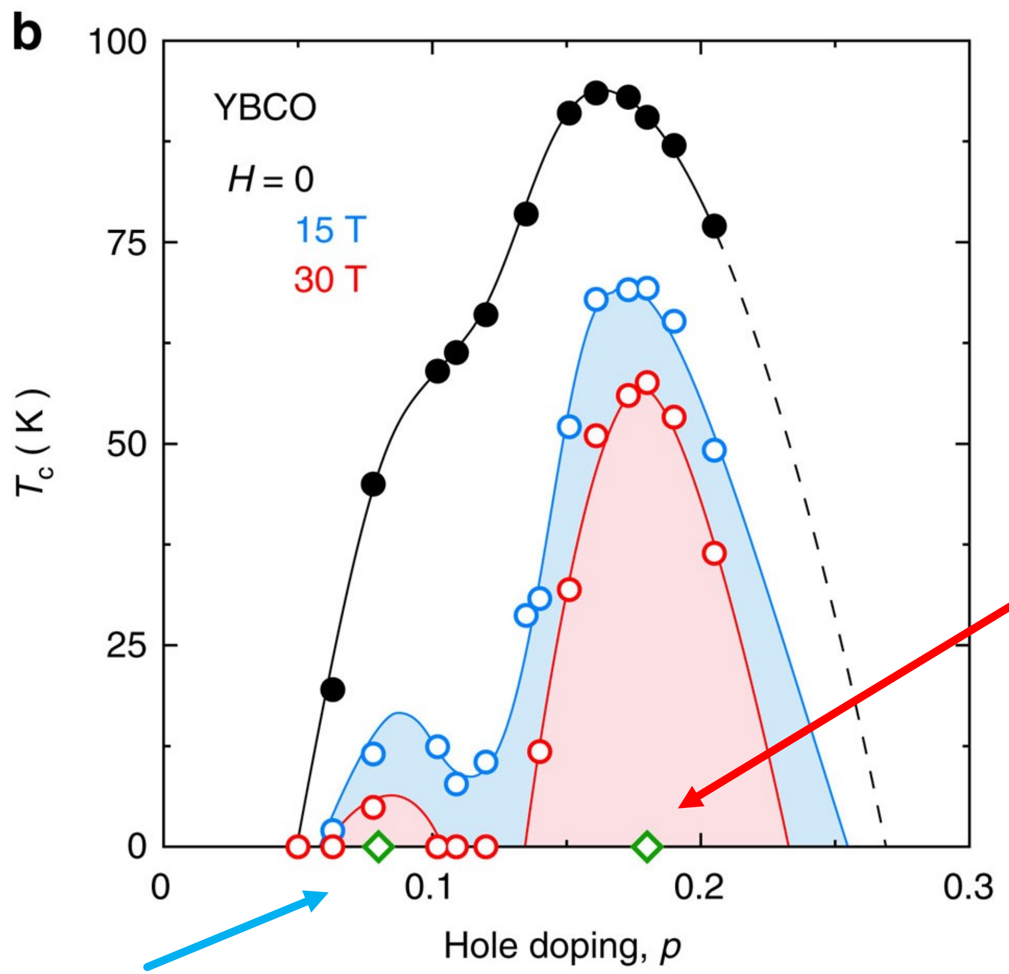
Non Fermi-liquid like non-elephant biology.
Defined by what it is not.

Landau: low energy degrees of freedom weakly coupled fermions
implies Fermi liquid behavior!

QCP allows extra light degree of freedom (order parameter)
and so in principle allows departure from Fermi liquid.

Many different possibilities for underlying microscopic mechanism!

Multiple critical points seem to exist!



Cuprates have seemingly two QCPs!

Rearrangment of electronic structure? Presumably the one responsible for strange metal

Magnetic Rearrangements?

(Grissonnanche et al, 2014)

Two basic scenarios

1) Charged sector with approximately conserved momentum current interacts with QCP whose Green's functions are governed by scale invariance of the QCP. E.g. marginal Fermi liquid

2) Charged sector itself is quantum critical. All transport phenomena obey scaling laws. Temperature dependence essentially governed by dimensional analysis. Incoherent Metal.

We study this second scenario.

Transport governed by scaling

Simplest regime:

Temperature >> critical temperature

Transport coeff = Tempature^{power} + ….

Power determined by dimensional analysis

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

Dynamical Critical Exponent.

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

Examples:

$$i\frac{d}{dt}\psi = -\frac{\psi''}{2m} \qquad x \to \lambda x, \; t \to \lambda^2 t \qquad z = 2$$

$$\frac{d^2}{dt^2}\psi = -\psi'' \qquad x \to \lambda x, \; t \to \lambda t \qquad z = 1$$

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta$$

Hyperscaling Violating Exponent.

Dimensional Analysis at QCP

Entropy S. Counts microstates.
Dimensionless number.
s should scale as volume.

$$[s] = d - \theta$$

$$s \sim \xi^{-d+\theta}$$

Volume scaling violated
if every time the volume appears
(e.g. the metric appears)
it is accompanied with extra
powers of the correlation length.

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta \qquad [\varepsilon] = d + z - \theta$$

$\boldsymbol{\theta}$ = anomalous dimension of energy density/current.

Ex: Stat Mech critical systems above critical dimension
where mean field applies.

Dimensional Analysis at QCP

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta$$

$$[n] = d - \theta + \Phi$$

Anomalous Scaling of Charge Density

$$n \sim s\,\xi^{-\Phi}$$

(Gouteraux et al, AK)

(Φ required in holographic models)

Dimensional Analysis at QCP

$$L \sim A\, j$$

$$[B] = 2 - \Phi$$

$$[E] = 1 + z - \Phi$$

Anomalous Coupling
to E&M Fields.

Dimensional Analysis at QCP

E, B always appears as g E, g B where
g is a dimensionful coupling.

$$[B] = 2 - \Phi$$

$$[E] = 1 + z - \Phi$$

Examples:

Φ generically non-zero in holographic models

(AK; Gouteraux et. al.)

Holography: Large class of strongly correlated quantum systems whose dynamics can be solved analytically by mapping to dual gravitational description. Toy models.

Non-zero Φ recently been demonstrated in large classes of standard large N field theories.

(AK; 1504.02478)

Upshot: Scaling fixed by 3 parameters

$$[x] = -1 \qquad [t] = -z$$

$$[s] = d - \theta \qquad [\varepsilon] = d + z - \theta$$

$$[n] = d - \theta + \Phi$$

$$[B] = 2 - \Phi$$

$$[E] = 1 + z - \Phi$$

Scaling and the Cuprates.

If we try to explain scaling in the cuprates,
is non-zero Φ needed?

Is there a simple physical observable whose dimension is zero unless Φ is non-zero?

$$[\kappa] = d - \theta + z - 2$$ thermal conductivity

$$[\sigma] = d + 2\Phi - \theta - 2$$ electric conductivity

$$[L] = \left[\frac{\kappa}{\sigma T}\right] = -2\Phi$$ Lorenz ratio

Thermoelectric transport

Electric current

$$\begin{pmatrix} j \\ j^Q \end{pmatrix} = \begin{pmatrix} \sigma & T\alpha \\ T\alpha & T\overline{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

Energy (heat) current

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Lorenz ratio and scaling

Non-zero Φ implies: $L \sim T^{-2\,\Phi/z}$

Sharp contrast to Wiedemann-Franz law:

$$L = \frac{\pi^2}{3}\,\frac{1}{e^2}$$

True in metals. Both heat and charge get transported by electrons. Electron charge fixes ratio.

Non-zero Φ can be interpreted as scale-dependent charge!

Experimental Data on Lorenz Ratio?

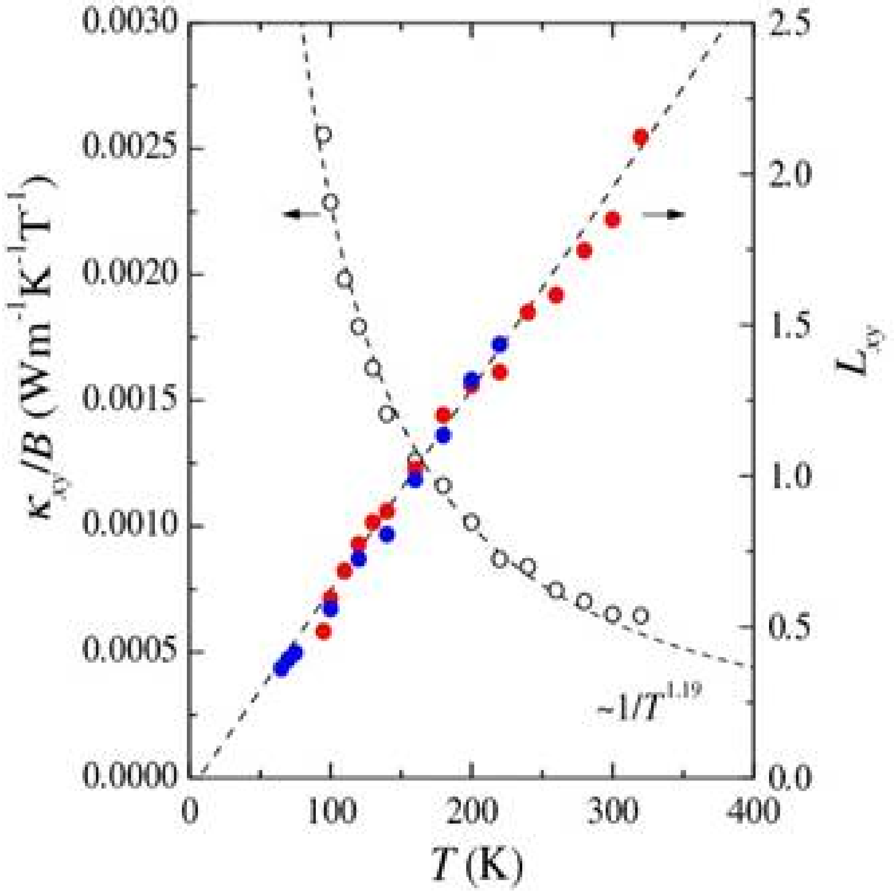
Concern: Thermal conductivity receives contributions from all degrees of freedom including phonons.

Expect system to be: QCP + neutral heat bath

(can carry spin, but no charge)

Isolate: **Hall Lorenz ratio**.

Wiedemann-Franz Law Violation



(Zhang, Ong, Xu, Krishana, Gagnon, Taillefer, PRL 84, 2000)

Caveat: Recently Matusiak et al have reported results differing by an order of magnitude)

Are we in business?

QCP at high temperatures governed by
simple power laws governed by dimensional analysis.

$$\sigma_{xx} \sim T^{(d+2\phi-\theta-2)/z} \qquad \kappa_{xx} \sim T^{(d-\theta+z-2)/z}$$

$$\alpha_{xx} \sim T^{(d+\phi-\theta-2)/z}$$

To fix remaining powers one needs to spell out a few dynamical assumptions.

Dynamical Assumption (I)

(I) QCP is time reversal invariant

$\sigma_{xy}, \alpha_{xy}, \kappa_{xy}$ are time reversal odd

These Hall-type conductivities must be proportional to background magnetic field.

$$\sigma_{xy} \sim BT^{(d+3\phi-\theta-4)/z}$$

$$\alpha_{xy} \sim BT^{(d+2\phi-\theta-3)/z}$$

$$\kappa_{xy} \sim BT^{(d+\phi-\theta+z-4)/z}$$

Dynamical Assumption (II)

(II) QCP has broken particle-hole symmetry

Otherwise: $\sigma_{xy}, \alpha_{xy}, \kappa_{xy}$ are particle-hole symmetric, but magnetic field is not.

Would be proportional to B and charge density.

$$\sigma_{xy} \sim BnT^{(2\phi-4)/z}$$

$$\alpha_{xy} \sim BnT^{(\phi-3)/z}$$

$$\kappa_{xy} \sim BnT^{(z-4)/z}$$

Alternative scaling.
Requires non-zero n
QCP not along physical axis.

Dynamical Assumption (III)

(III) Currents not dominated by (almost) conserved momentum

Conductivities dominated by scaling in quantum critical regime.

No Drude peak with width much smaller than the temperature.

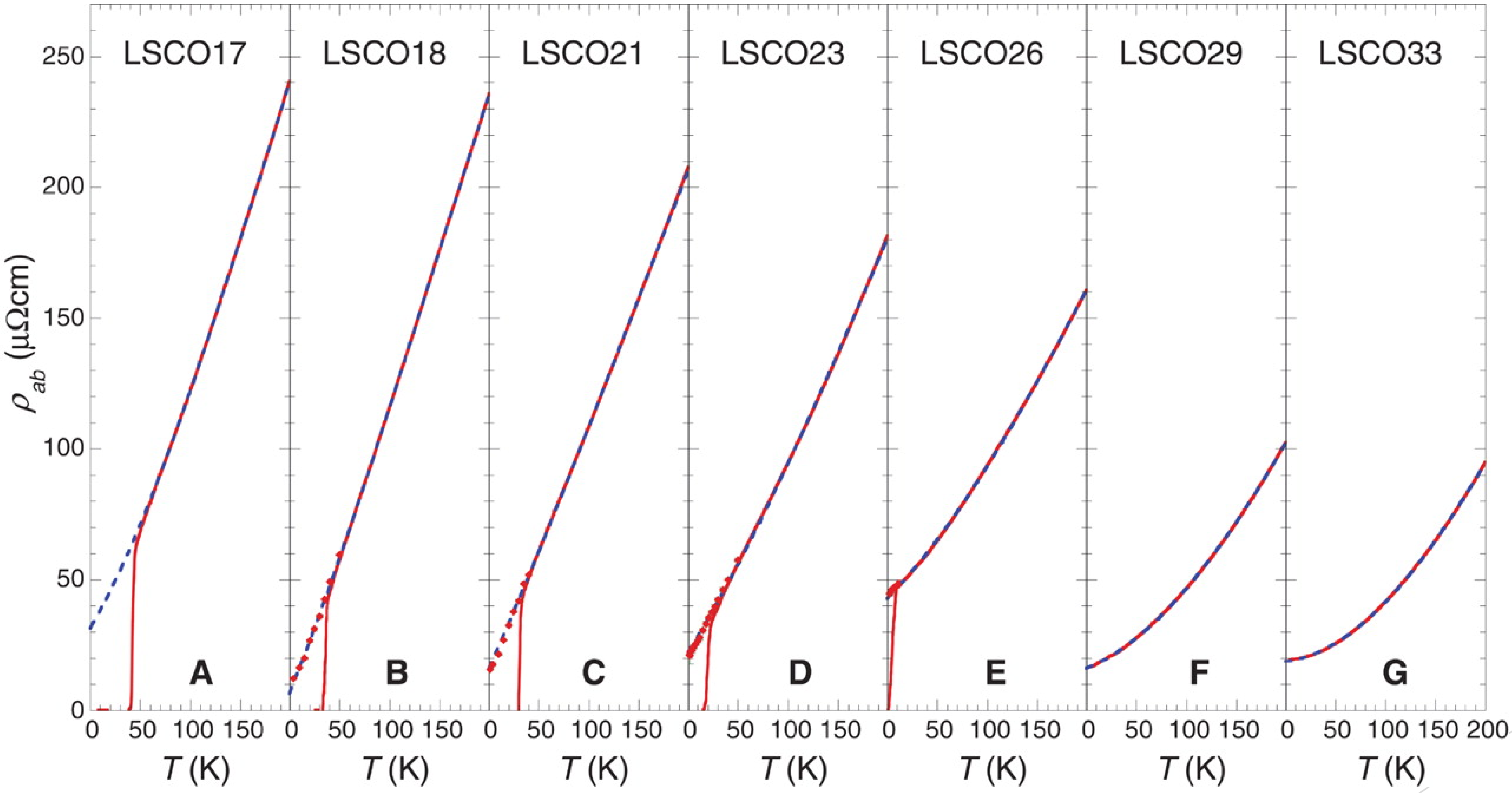
Test scaling experimentally: Inputs

Need 3 experimentally well established scalings to pin down the three exponents.

1) Lorenz Ratio linear in T

$$z = -2\phi$$

2) Linear Resistivity

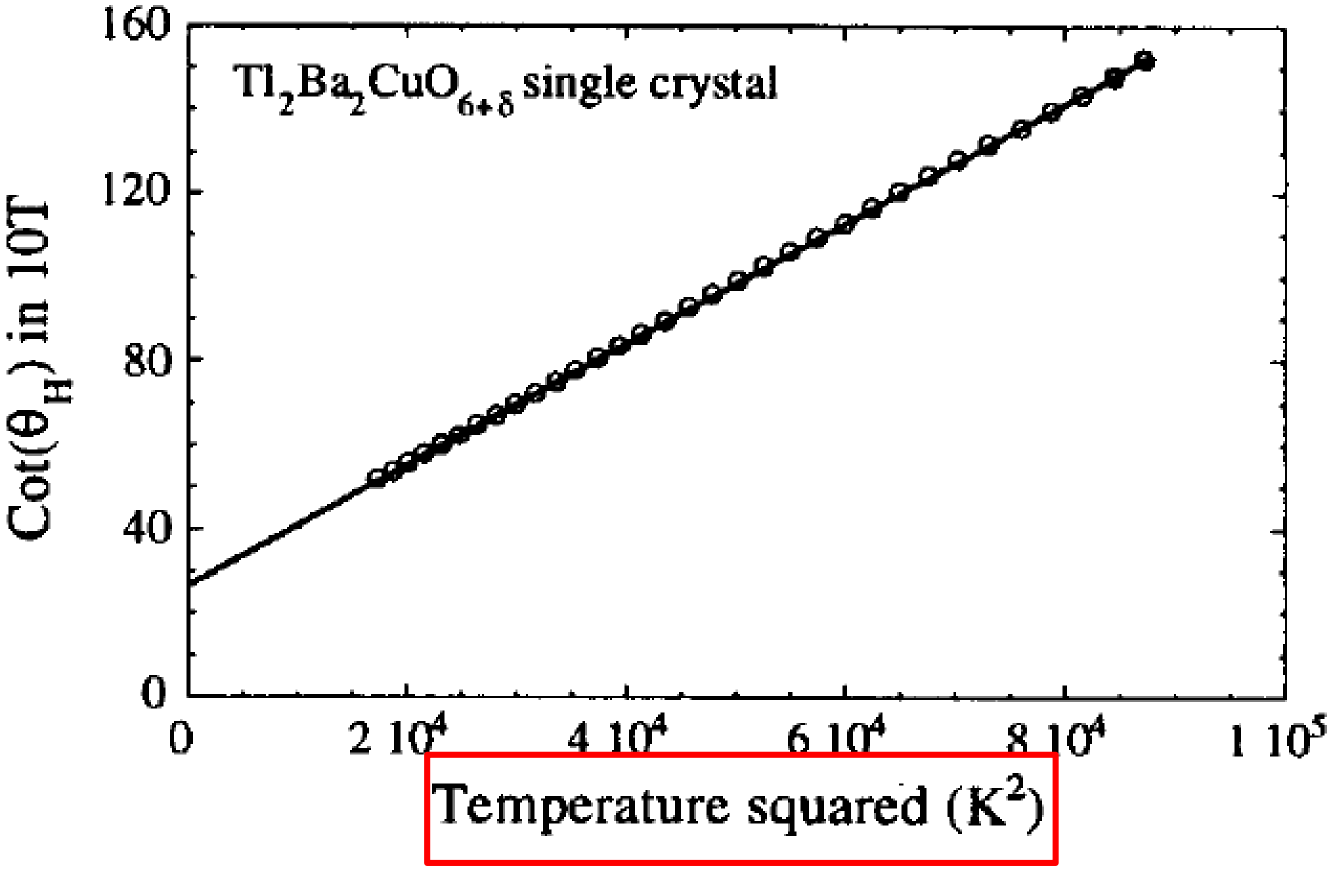


$\theta = 0$

Cooper et al, Science (2009)

$\sigma_{xx} \sim T^{(d+2\phi-\theta-2)/z}$

3) Hall Angle



$$\cot(\theta_H) = \frac{\sigma_{xx}}{\sigma_{xy}}$$

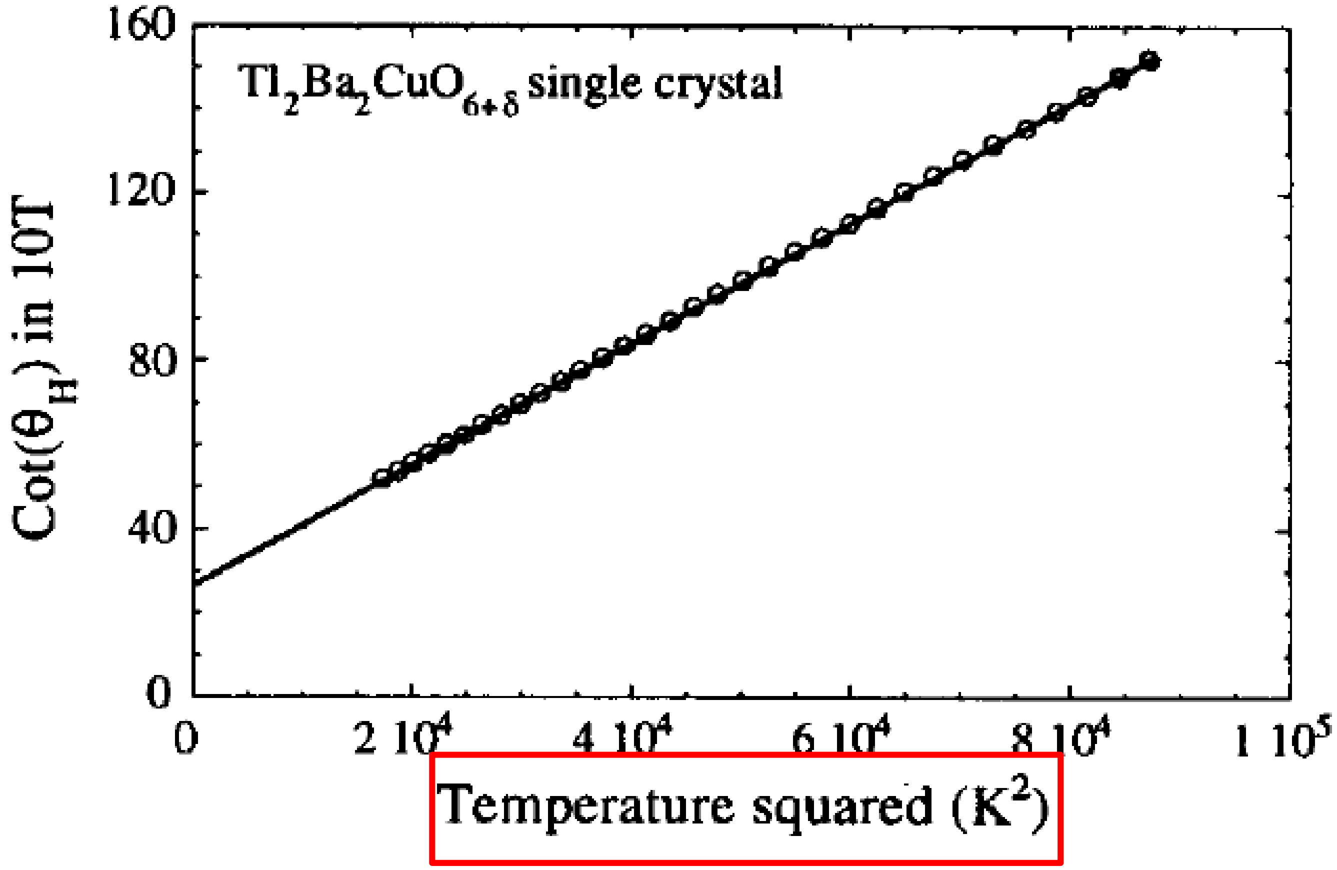
$$\sigma_{xy} \sim BT^{(d+3\phi-\theta-4)/z}$$

$$z = \frac{4}{3}$$

$$\phi = -\frac{2}{3}$$

(Tyler and Mackenzie, 1997)

Alternate Scenario



(Tyler and Mackenzie, 1997)

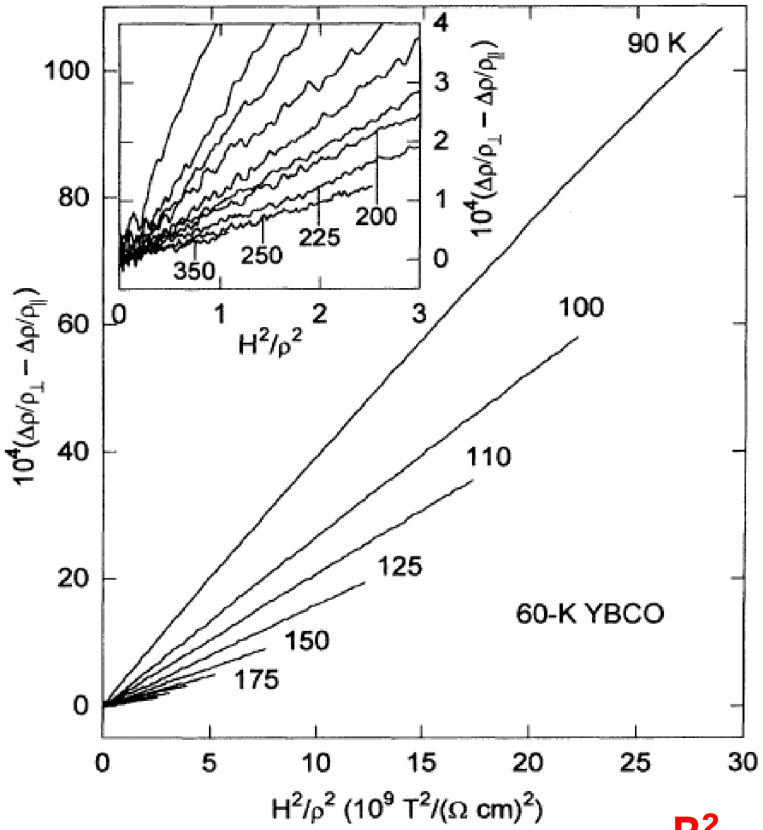
$$\cot(\theta_H) = \frac{\sigma_{xx}}{\sigma_{xy}}$$

$$\sigma_{xy} \sim BnT^{(2\phi-4)/z}$$

$$z = 2$$
$$\phi = -1$$

Prediction 1: Magnetoresistance

Δρ



B^2

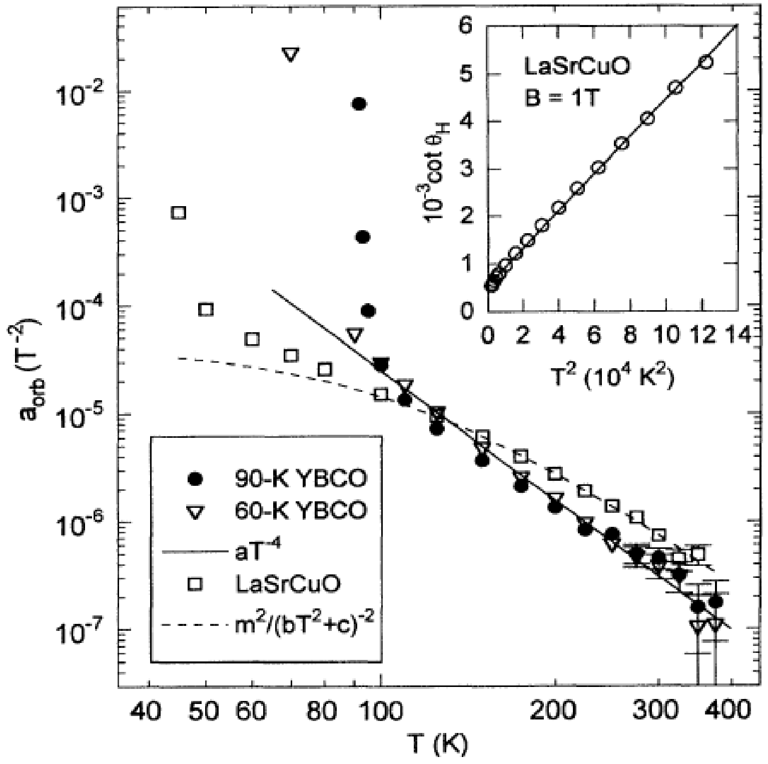
(Harris et al, 1996))

Δρ

$$\rho = \rho_{B=0} + B^2\rho_{(2)} + \cdots$$

Time reversal

Magnetoresistance



Scaling implies:

$$\frac{\Delta\rho}{\rho_{B=0}} \sim \frac{B^2}{T^4}$$

Perfectly agrees with experimental data!

(Harris et al, 1996))

Magnetoresistance

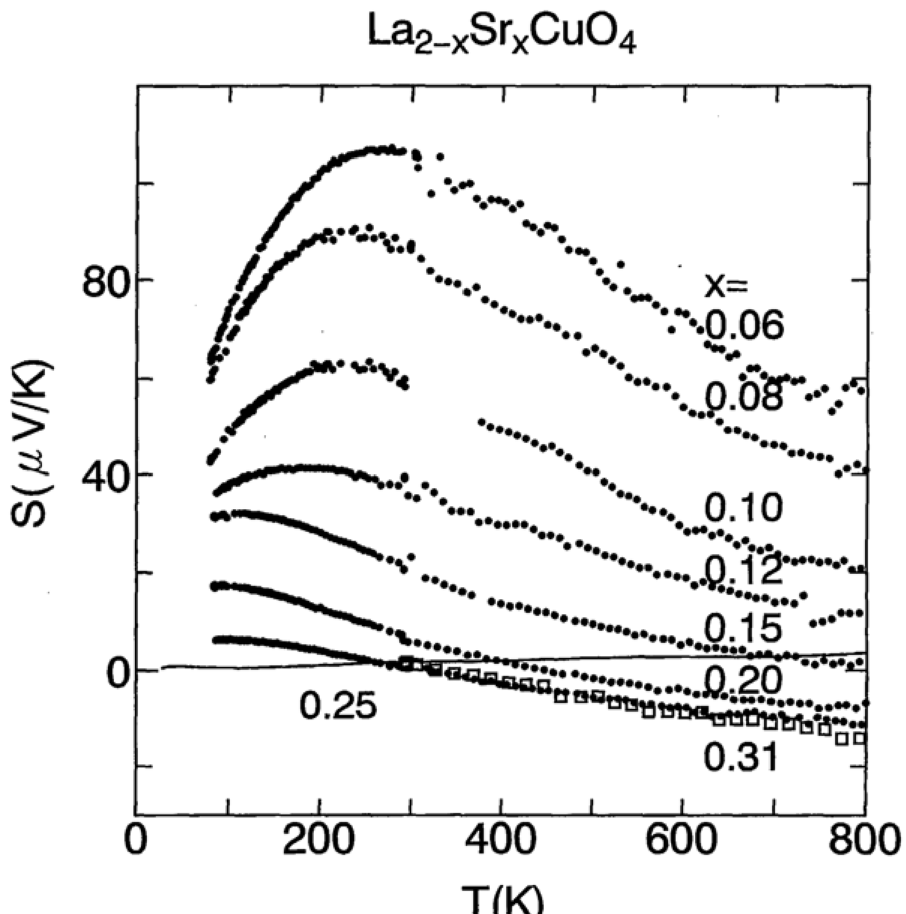
Alternative Scenario (z=2, Φ=-1) predicts:

$$\frac{\Delta\rho}{\rho_{B=0}} \sim \frac{B^2}{T^3}$$

Genuinely new observable. Non-trivial.

Prediction 2: Thermoelectric

Typically measured as Seebeck:



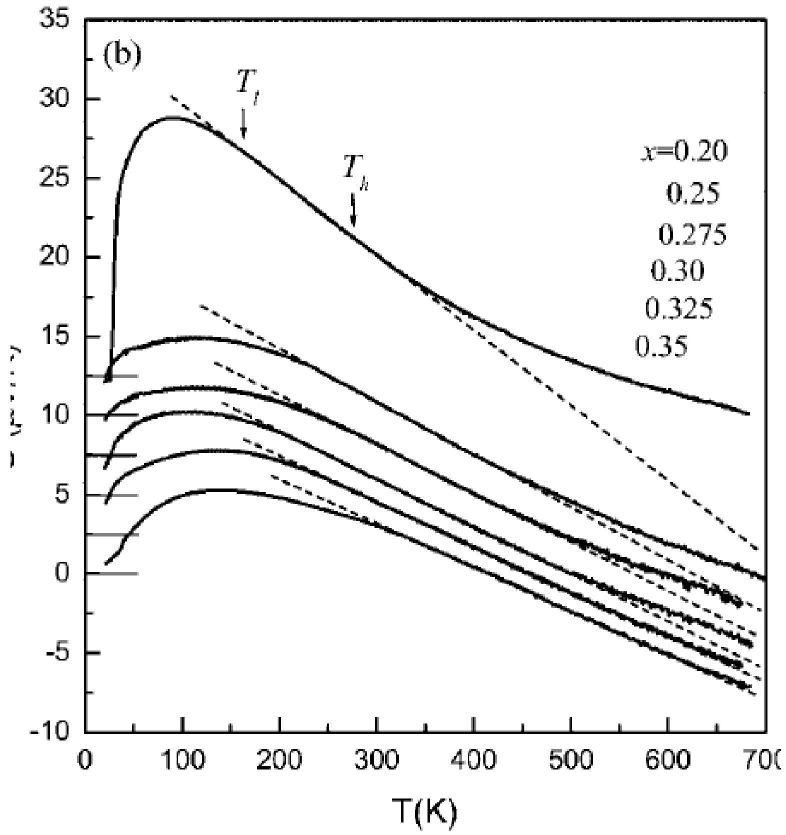
$$S \equiv \frac{\alpha_{xx}}{\sigma_{xx}} \sim -T^{1/2}$$

(find E so that no current flows in response to T-gradient)

No fit to shape of data attempted in early experimental work.

(Nishikawa et al, 1994)

Prediction 2: Thermoelectric coefficient



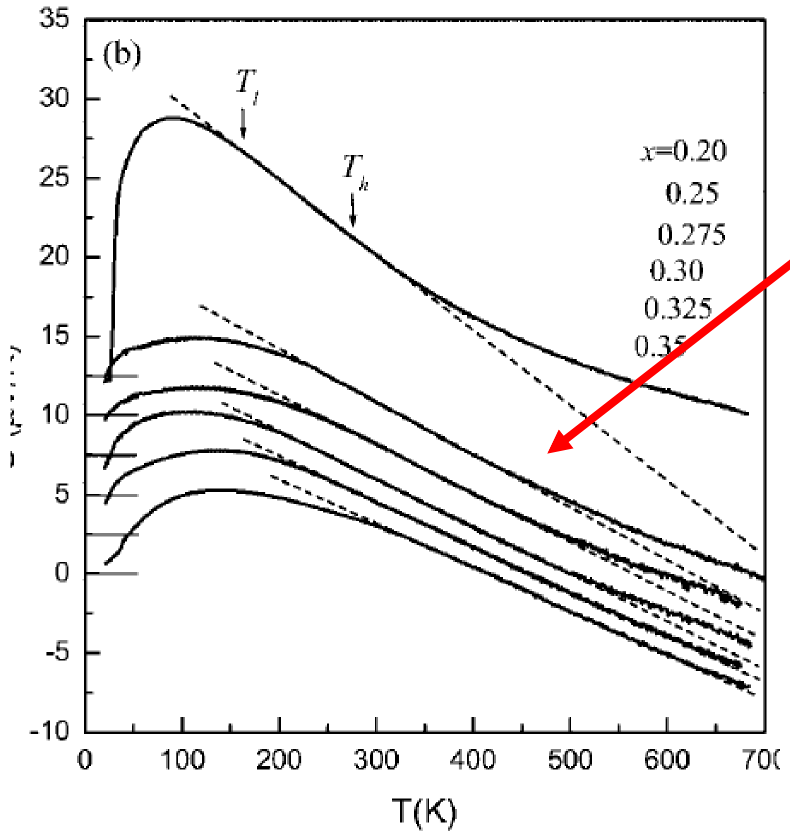
(Kim et al, 2004)

Ten years later data looks much cleaner !

The published linear fit clearly doesn't capture high T.

Does this look like const.-$\sqrt{T}$?

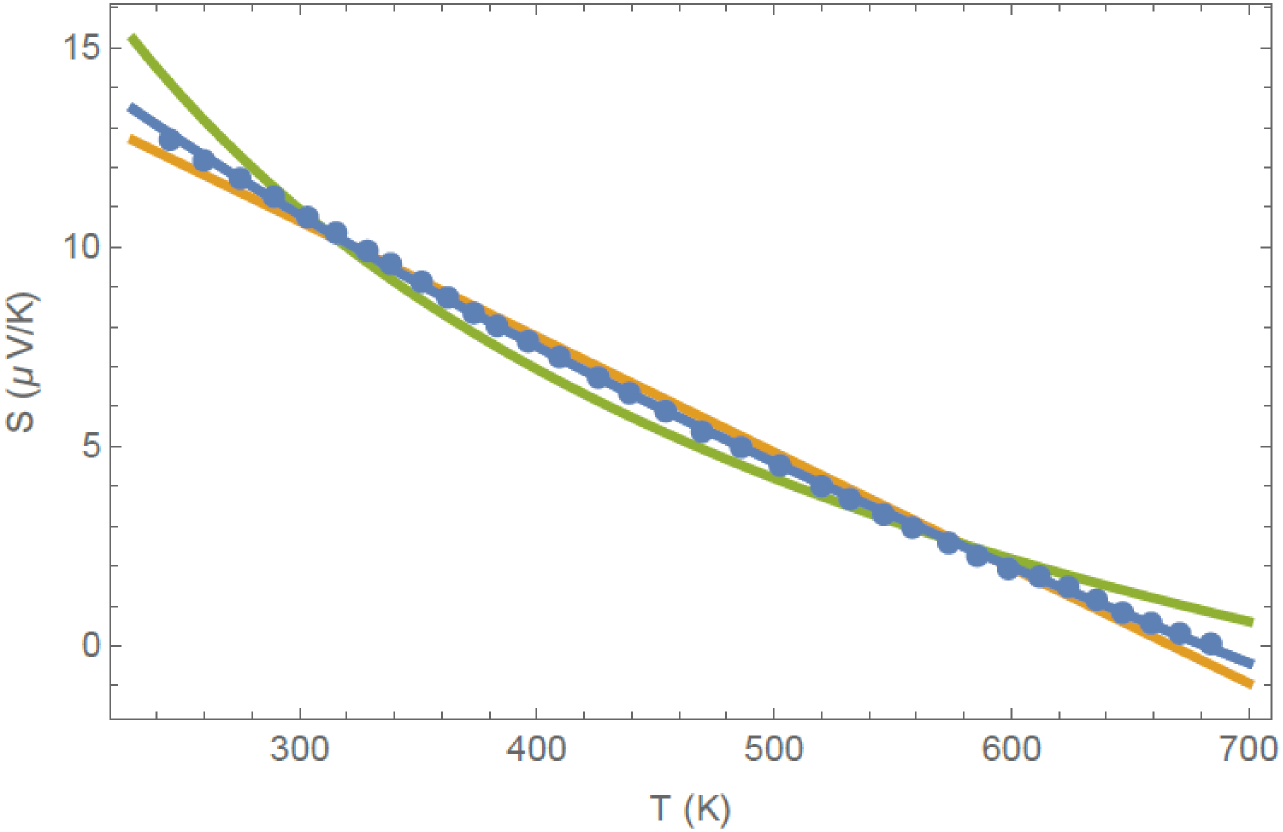
Prediction 2: Thermoelectric coefficient



(Kim et al, 2004)

Use Mathematica to
pick out points along
the x=0.25 curve and attempt
our own fit!

Seebeck Coefficient



$a - b\,T^{1/2}$

fits data head on!

$a - b\,T^{1}$ **and**

$a - b\,T^{-1/2}$

don't.

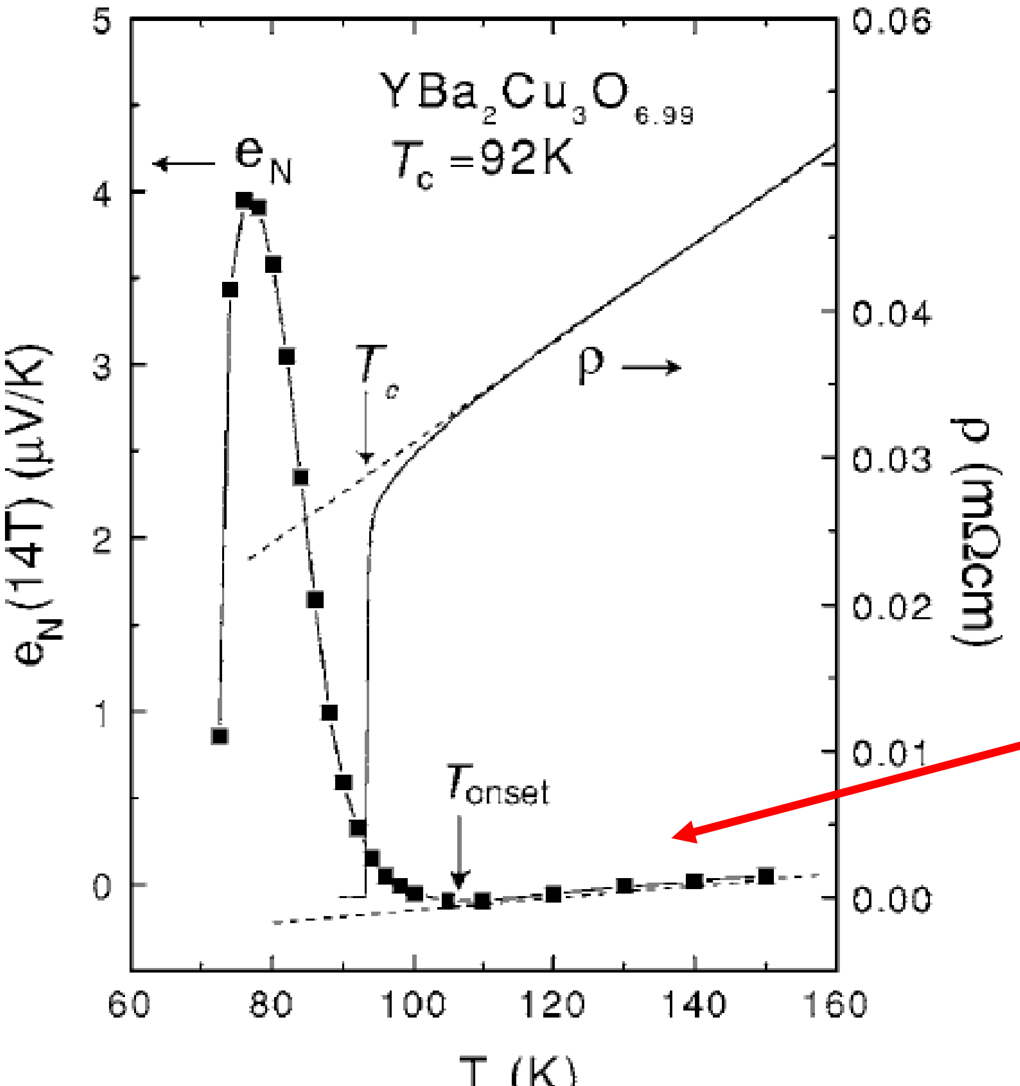
Best fit: $a - b\,T^{m}$ with: a=32, b=1.2, m=.49

Hall-Thermoelectric or Nernst

$$\nu \equiv \frac{1}{B}\left(\frac{\alpha_{xy}}{\sigma_{xx}} - S\tan\theta_H\right) \sim T^{-3/2}$$

No sufficiently high quality high T data exists to extract power law with confidence. Existing data consistent with our scaling.

Nernst data

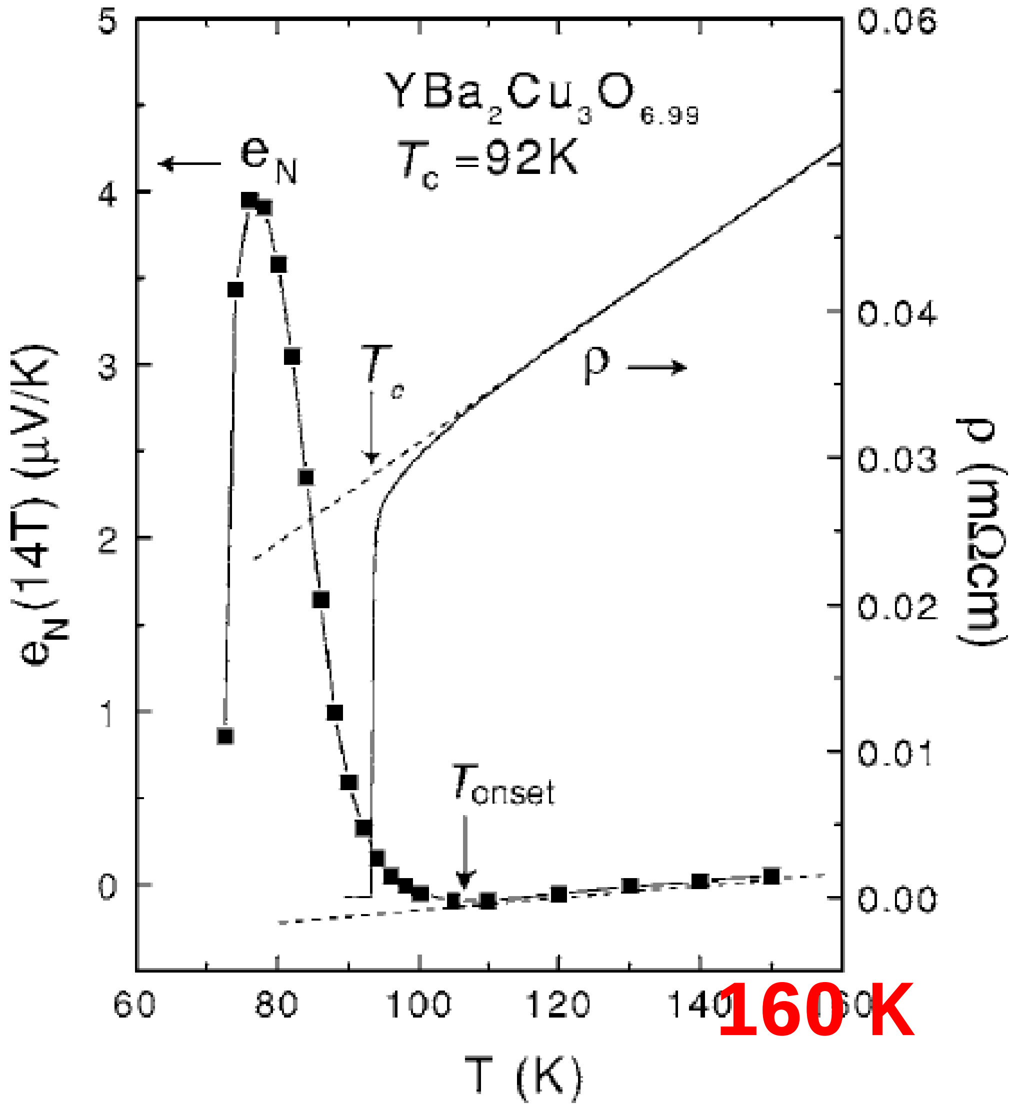


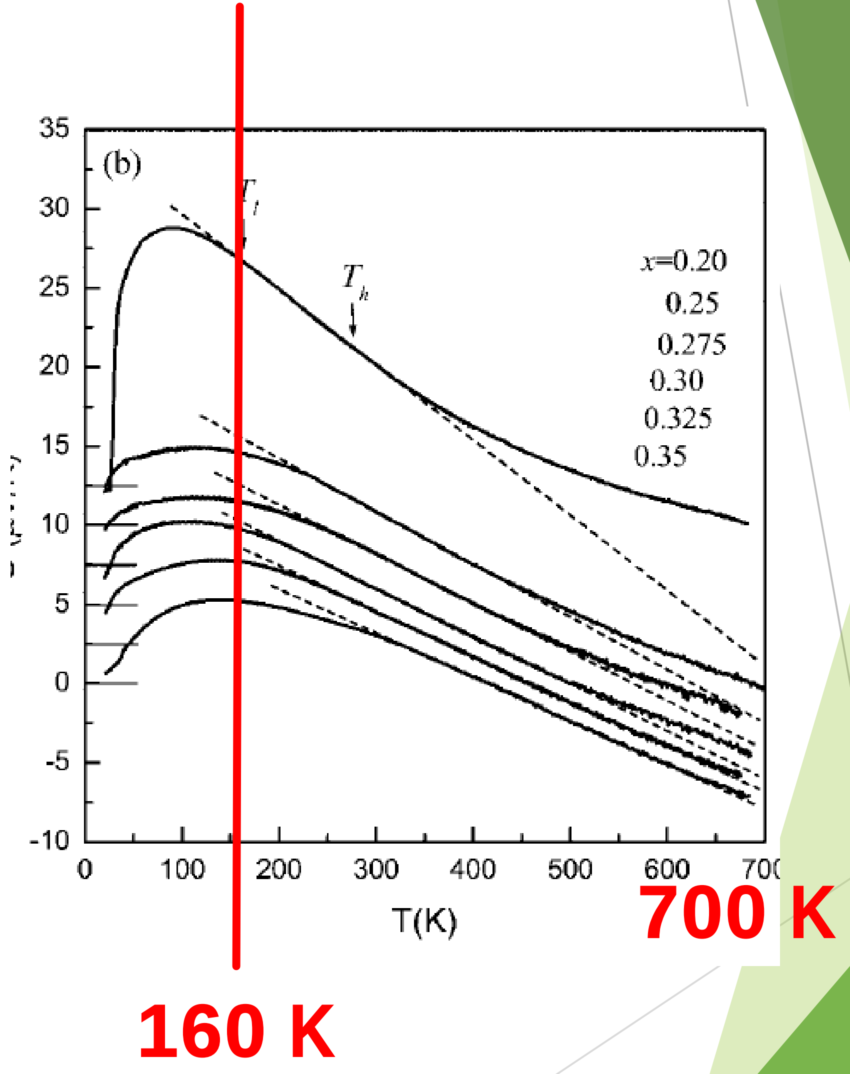
Need higher T!

Hard to fit a power law to this!

(Wang et al, 2006)

Nernst vs thermoelectric





Doping dependence

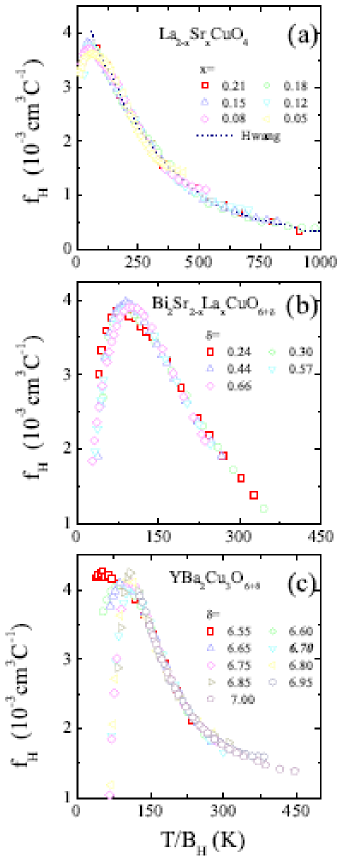
Two dimensionfull parameters, T and Δ

Pseudogap. Scales as energy. Parametrizes distance to optimal doping.

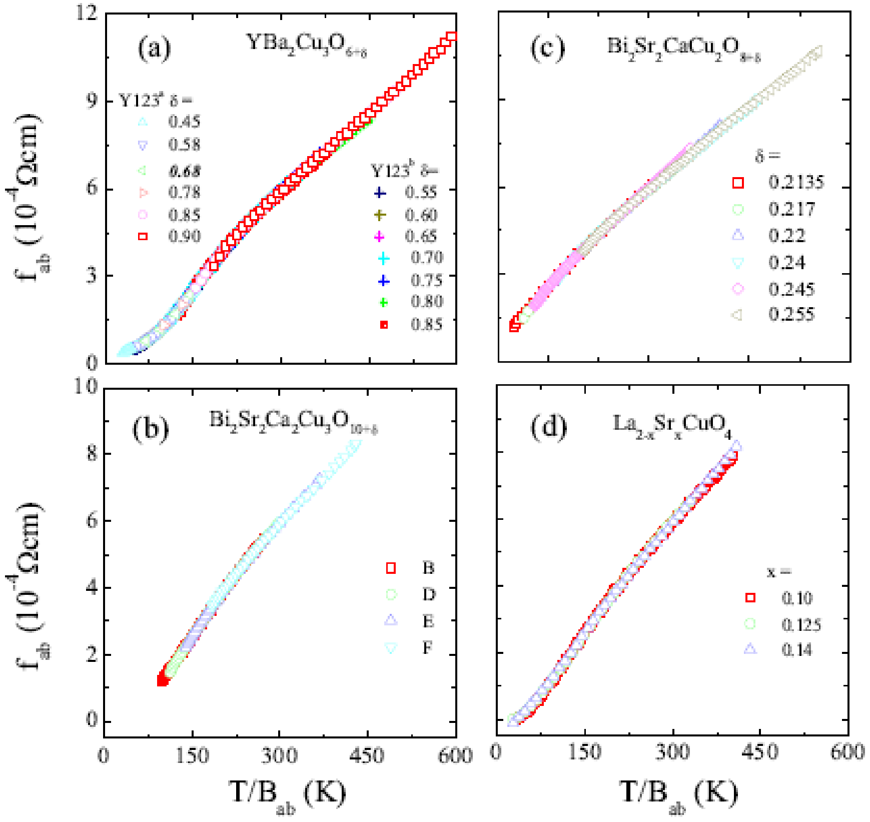
Expect: all transport scaling function of T/Δ

Scaling hypothesis tested by Luo et al, PRB 09

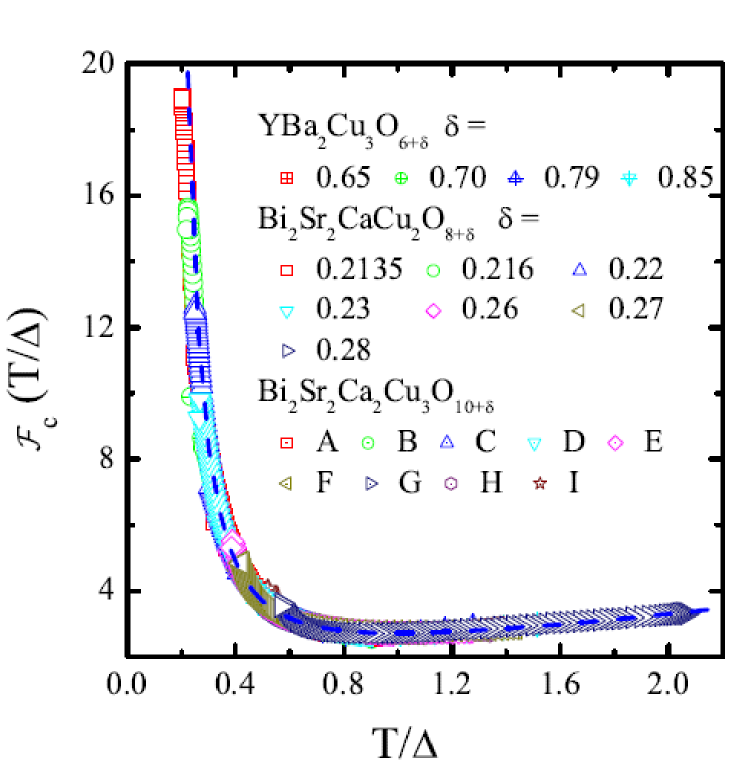
Scaling hypothesis



Hall



in plane resistivity

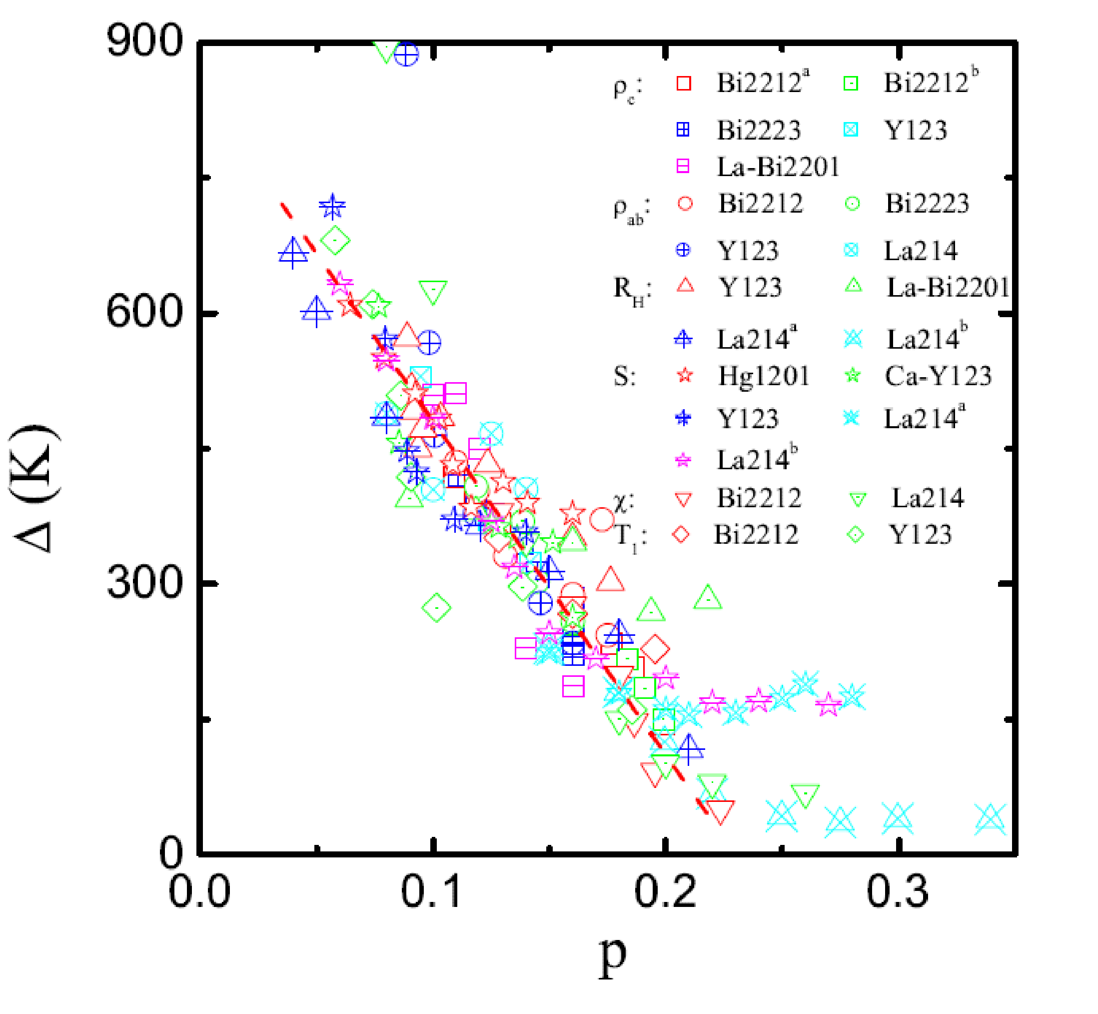


out of plane resistivity

Curves collapse for doping dependent gap!

Our exponents?

According to our scaling n and T have the same units.



Predicts pseudogap should close as T* ~ (p-pc)

Executive Summary so far

Scaling hypothesis works for transport!

Thermodynamics

Scaling gives consistent picture of transport!

Our Scaling Analysis also makes predictions for Thermodynamic properties.

Heat Capacity ~ $T^{3/2}$
Magnetic Susceptibility ~ $T^{-3/2}$

Not born out by data. Thermo mostly "conventional"

Boring Thermo, anomalous transport?

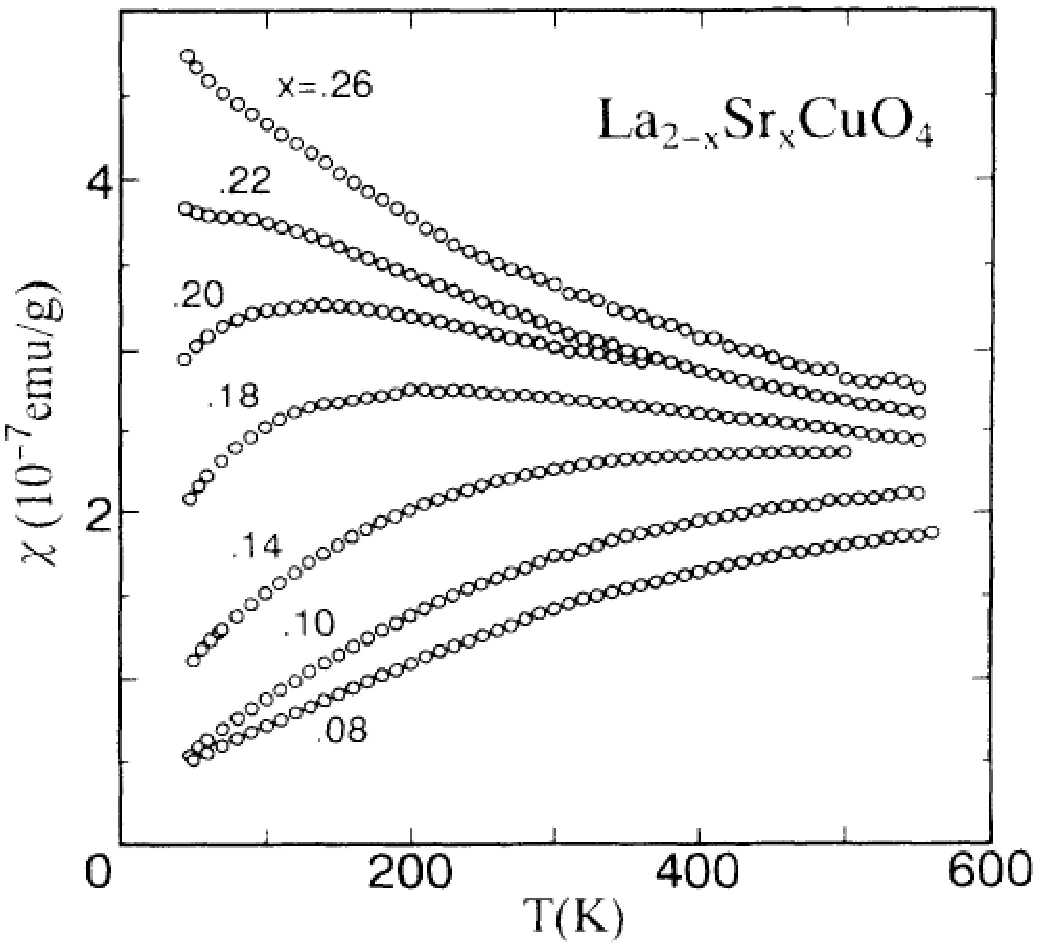
Suggests interplay between neutral sector (thermo) and charged sector (quantum criticial, dominates transport)

Makes clear predictions which quantities should be governed by scaling analysis.

Data leaves room for a significant contribution:

Heat Capacity ~ $T^{3/2}$
Magnetic Susceptibility ~ $T^{-3/2}$

Example: Magnetic Susceptibility



Magnetic Susceptibility
$\sim T^{-3/2}$ + other ?

(Nakano et al, 1994)

Neutral versus charged sector

Lots of options:

- Localized degrees of freedom (spins?)

Dimension of magnetic moment not linked to dimension of current!

- Neutral degrees of freedom (spinons?)

- Charge carriers localized in momentum space (density wave?)

Outlook

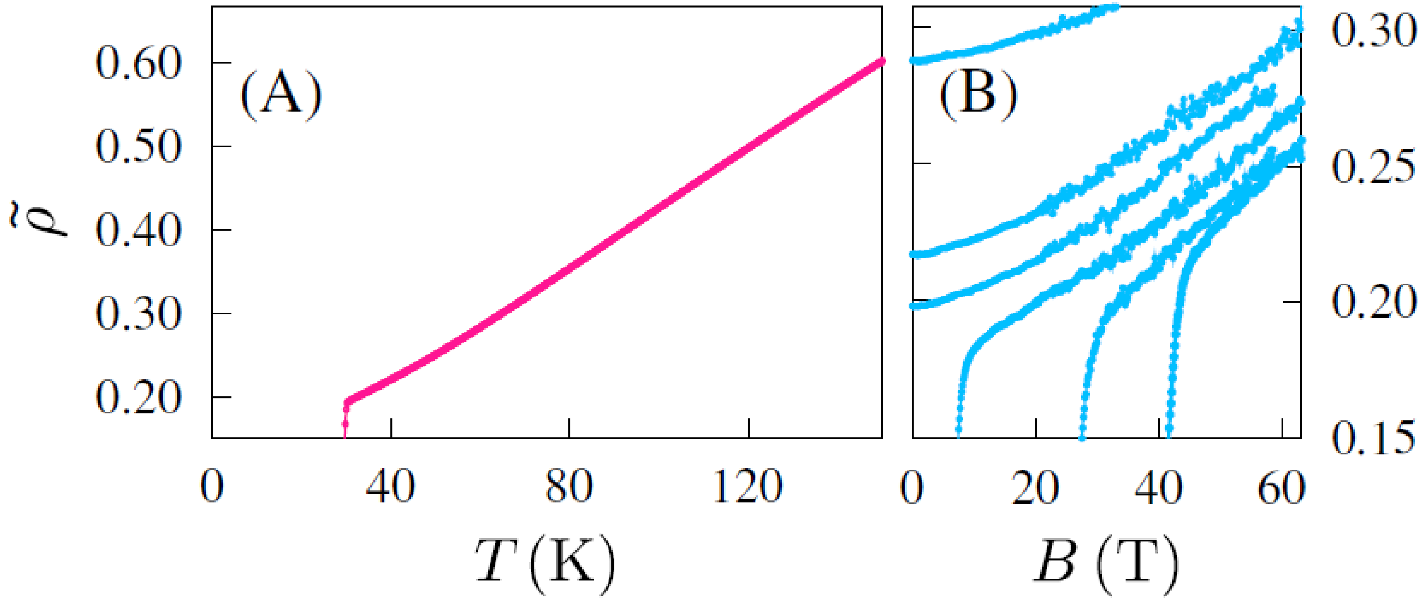
Predictions for the future

Scaling hypothesis in the cuprates should
be easy to test with future precision experiments
of transport at T >> Tc

Most pressing: clarity on Lorenz ratio.
Nernst effect.

Other Materials: Iron Pnictides

Not all the same. Example: $BaFe_2(As_{1-x}P_x)_2$



$$\rho_{xx} \sim \sqrt{T^2 + B^2}$$

(Hayes et al, 2014)

Iron Pnictide

- $\rho_{xx} \sim \sqrt{T^2 + B^2}$ implies [B]=[T]
- Apparently we still have linear resistivity
- Assume θ=0

$$z = 4, \qquad \phi = -2$$

Makes lots of predictions!
More measurements are needed.
Lorenz??

Summary

Holography suggests that NR scale invariant theories generically allow for anomalous a dimension of the current operator.

Allowing for this anomalous dimension one obtains a surprisingly successful fit for transport in the cuprates.

Scaling hypothesis should be easy to test in new transport experiments (higher T, other materials).