Pseudo Observables in Higgs Physics

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- Introduction
- General comments about PO
- PO in Higgs decays
- The $h \to 4f$ case
- Parameter counting, symmetry limits, dynamical constraints
- PO beyond decays
- Conclusions

based on
Gonzales-Alonso et al. 1412.6038; 1504.04018;
Bordone et al. 1507.02555
& several discussions within the HXSWG
Introduction

After the exciting discovery phase...

...we are entering into the era of precise measurements of the properties of the "Higgs particle" observed at 125 GeV.
Introduction

It's already quite clear that this particle is well compatible with the massive excitation of the (unique) Higgs field postulated within the SM:

\[
\mathcal{L}_{\text{Symm. Break.}}(\phi, A_a, \psi_i) = D\phi^+ D\phi - V(\phi) + \ldots
\]

\[
V(\phi) = -\mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2 + Y^{ij} \psi_L^i \psi_R^j \phi
\]

...but we are far from having established that there is nothing else beside the SM (or that the cut-off of SM viewed as an effective theory is very high)

On general grounds, it is natural to expect possible deviations from the SM in the Higgs sector

High-precision Higgs physics
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I. precise measurements of SM allowed processes (production & decay)

II. search for rare/exotic decay modes
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On general grounds, it is natural to expect possible deviations from the SM in the Higgs sector

Given the absence of clear NP directions, it's important to make these studies in general terms (with minimum theoretical bias)

I. precise measurements of SM allowed processes (production & decay)

II. search for rare/exotic h decay modes
Introduction

So far, possible non-standard properties of the Higgs boson (in process with a leading SM amplitude) have been analyzed from the experimental point of view using the so-called “kappa-formalism”:

\[
\sigma(ii \rightarrow h+X) \times \text{BR}(h \rightarrow ff) = \sigma_{ii} \frac{\Gamma_{ff}}{\Gamma_h} = \frac{\kappa_{ii}^2 \kappa_{ff}^2}{\kappa_h^2} \sigma_{\text{SM}} \times \text{BR}_{\text{SM}}
\]

Main virtues:

- Clean SM limit [best up-to-date TH predictions recovered for \(\kappa_i \rightarrow 1\)]
- Well-defined both on TH and EXP sides
- (almost) Model independent
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Main problem:

- **Loss of information** on possible NP effects modifying the kinematical distributions

N.B.: easy to conceive NP effects showing up mainly in kin. effects rather than in total rates (e.g. CPV)
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We need to identify a larger set of “pseudo-observables” able to characterize NP in the Higgs sector in general terms.
General comments about Pseudo Observables

Experimental data
- raw data,
- fiducial cross-sections,
- ...

Pseudo Observables
- masses, widths,
- slopes, ...

Lagrangian parameters
- Wilson coefficients,
- renormalization scale,
- running masses, ...

\[ L = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} \not{D} \psi + h.c. + \lambda_1 \psi_1 \psi_2 \phi + h.c. + |D_\mu \phi|^2 - V(\phi) \]
General comments about PO

- The goal of the PO is to provide a general encoding of the exp. results in terms of a limited number of “simplified” (idealized) observables of easy th. interpretation. [old idea - heavily used and developed at LEP times]

- The experimental determination of an appropriate set of PO will “help” and not “replace” any explicit NP approach to Higgs physics (including the EFT).

Experimental data  Pseudo Observables  Lagrangian parameters

The PO can be computed in terms of Lagrangian parameters in any specific th. framework (SM, SM-EFT, SUSY, ...)

\[ L = -\frac{1}{4} F_{mn} F^{mn} + i \sqrt{2} \bar{\psi} D \psi + \text{h.c.} \\
+ \lambda_1 y_1 \phi + \lambda_2 y_2 \phi^3 + \lambda_3 |\phi|^4 - V(\phi) \]
General comments about PO

- The goal of the PO is to provide a general encoding of the exp. results in terms of a limited number of “simplified” (idealized) observables of easy th. interpretation [old idea - heavily used and developed at LEP times]

- The experimental determination of an appropriate set of PO will “help” and not “replace” any explicit NP approach to Higgs physics (including the EFT)

- The PO should be defined from kinematical properties of on-shell processes (no problems of renormalization, scale dependence, ...)

- The theory corrections applied to extract them should be universally accepted as “NP-free” (soft QCD and QED radiation)
**General comments about PO**

**Example I:** The mass of a particle is a PO

Not always obvious how to extract it from data (**debate on Z line-shape**) and how to make it in a way that is useful for theoreticians (**top mass**).

The $M_Z$, $M_W$, $M_h$, determined by experiments are 3 well-defined PO and not fundamental couplings of the SM Lagrangian (or BSM models).

Either we predict them (**at a certain order**) in terms of other couplings or we use them to extract the couplings (**at a given order and at a given scale**...). This does not affect their experimental determination, while the way they are defined from data affect the way we compute them.
General comments about PO

Example II: The effective couplings of the Z boson

Parametrise the $Z f \bar{f}$ vertex as $\gamma \mu (g^f_V + g^f_A \gamma_5)$

$$\Gamma_f \equiv \Gamma (Z \to f \bar{f}) = 4 c_f \Gamma_0 (|g^f_V|^2 R^f_V + |g^f_A|^2 R^f_A) + \Delta_{\text{EW/QCD}}$$

Bardin, Grunewald, Passarino, '99

The pseudo-observables are defined as

$$g^f_V = \text{Re } g^f_V, \quad g^f_A = \text{Re } g^f_A$$

To be model-independent it is important to work with on-shell initial and final states.

Then a theorist can take their model, or their EFT, compute the contribution to these POs, and obtain the constraints on the model.
General comments about PO

There are two main categories:

A) “Ideal observables”

\[ M_W, \Gamma(Z \rightarrow ll), \ldots \]

\[ M_h, \Gamma(h \rightarrow \gamma\gamma), \Gamma(h \rightarrow 4\mu), \ldots \]

but also \[ d\sigma(pp \rightarrow hZ)/dm_{hZ} \ldots \]

B) “Effective on-shell couplings”

\[ g_Z^f, g_W^f, \ldots \]

This is the category we want to “extend” in order to describe non-standard effects in the Higgs sector.

- Both categories are useful
  (there is redundancy having both, but that's not an issue...).

- For B) one can write an effective Feynman rule, not to be used beyond tree-level
There is more to extract from data other than the $\kappa_i$.

Multi-body modes
e.g. $h \rightarrow 4\ell, \ell\ell\gamma, \ldots$

Two-body (on-shell) decays

[no polarization properties of the final state accessible]
e.g. $h \rightarrow \gamma\gamma, \mu\mu, \tau\tau, \text{bb}$

The $\kappa_i (\leftrightarrow \Gamma_j)$ is all what one can extract from data

[+ one more parameter if the polarization is accessible]
PO in Higgs decays

Multi-body modes
e.g. $h \rightarrow 4\ell$, $\ell\ell\gamma$, ...

$\downarrow$

Form factors $\rightarrow f_i(s)$ [E.g.: $s = m^2_{\ell\ell}$]

Two-body (on-shell) decays

[e.g. $h \rightarrow \gamma\gamma$, $\mu\mu$, $\tau\tau$, $bb$]

[no polarization properties of the final state accessible]

E.g.: $\mathcal{A}(h \rightarrow Z\,ee) \sim$

$$\epsilon^Z_\mu J^{e_L}_\mu \left[ f_1^{Ze_L}(q^2) g^{\mu\nu} + f_3^{Ze_L}(q^2) (p q g^{\mu\nu} - q^\mu p^\nu) + ... \right]$$

N.B.: There is noting “wrong” or “dangerous” in using $f.f.$, provided

- they are defined from on-shell amplitudes
  
  [hill-defined for $h \rightarrow WW^*$, $ZZ^*$ but perfectly ok for $h \rightarrow 4\ell$ ]

- no model-dependent assumptions are made on their functional form
**PO in Higgs decays**

**Multi-body modes**
e.g. \( h \rightarrow 4\ell, \ell\ell\gamma, \ldots \)

\[ \text{Form factors} \rightarrow f_i(s) \quad \text{[E.g.:} \quad s = m^2_{\ell\ell}] \]

**Two-body (on-shell) decays**

[no polarization properties of the final state accessible]
e.g. \( h \rightarrow \gamma\gamma, \mu\mu, \tau\tau, bb \)

\[ \kappa_i (\leftrightarrow \Gamma_i) \]

---

- No need to specify any detail about the EFT, but for the absence of light new particles → momentum expansion very well justified by the Higgs kinematic

- The \( \{\kappa_i, \varepsilon_i\} \) thus defined are well-defined PO → systematic inclusion of higher-order QED and QCD (soft) corrections possible (and necessary...)

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**Momentum expansion of the f.f. around leading poles**

E.g.: \( f_i^{\text{SM+NP}} = \frac{\kappa_i}{s - m_Z^2 + i m_z \Gamma_Z} + \frac{\varepsilon_i}{m_Z^2} + \text{O}(s/m_Z^4) \)

---

**Gonzales-Alonso et al.**

1412.6038
The $h \rightarrow 4f$ case
The $h \to 4f$ case

Two main hypotheses:

I. Fermion couples to the Higgs via helicity-conserving local currents
   [↔ neglect helicity-violating interactions, naturally linked to $m_f$ also BSM]

\[ G_{[JJh]} = \langle 0 \vert \mathcal{T} \left\{ J^\mu_f(x), J^\nu_{f'}(y), h(0) \right\} \vert 0 \rangle \]

The amplitude is fully determined by this Green function that contains long-distance modes (↔ non-local terms in $x$ and $y$ due to the exchange of EW gauge bosons)
& short-distance modes (↔ contact terms for $x$ or $y \to 0$)

Only 3 Lorentz structures allowed, e.g.:

\[ A = i \frac{2m_Z^2}{v_F} \sum_{e=e_L, e_R} \sum_{\mu=\mu_L, \mu_R} (\bar{e} \gamma_\alpha e)(\bar{\mu} \gamma_\beta \mu) \times \]
\[
\left[ F_1^{e\mu}(q_1^2, q_2^2) g^{\alpha\beta} + F_3^{e\mu}(q_1^2, q_2^2) \frac{q_1 \cdot q_2}{m_Z^2} g^{\alpha\beta} - q_2^{\alpha} q_1^{\beta} \right] + F_4^{e\mu}(q_1^2, q_2^2) \frac{\varepsilon^{\alpha\beta\rho\sigma} q_2 \rho q_1 \sigma}{m_Z^2} \]
The $h \to 4f$ case

Two main hypotheses:

I. Fermion couples to the Higgs via helicity-conserving local currents
   \[ \leftrightarrow \text{neglect helicity-violating interactions, naturally linked to } m_f \text{ also BSM} \]

\[ G_{[JJh]} = \langle 0 | \mathcal{T} \left\{ J^\mu_f(x), J^\nu_{f'}(y), h(0) \right\} |0\rangle \]

II. Expansion of $G_{[JJh]}$ neglecting short-distance modes corresponding to local operators with $d > 6$
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I. Fermion couples to the Higgs via helicity-conserving local currents
   $\leftrightarrow$ neglect helicity-violating interactions, naturally linked to $m_f$ also BSM

II. Expansion of $G_{[JJh]}$ neglecting short-distance modes corresponding to local operators with $d > 6$

non-local amplitude at the EW scale:

\[ G_{[JJh]} = \langle 0 | \mathcal{T} \left\{ J^\mu_f(x), J^\nu_{f'}(y), h(0) \right\} |0 \rangle \]
The $h \to 4f$ case

Following these hypotheses we can expand the form factors around the physical poles and retain only the leading terms (in the momentum expansion), e.g.:

$$A = -i \frac{2m_Z^2}{v_F} \sum_{e=e_{L,R}} \sum_{\mu=\mu_{L,R}} (\bar{\epsilon}_\gamma \bar{e}_e)(\bar{\mu}_\gamma \beta_\mu) \times$$

$$\left[ \left( \kappa_{Z_2} \frac{g_{Z}^e g_{Z}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \frac{\epsilon_{Z_2}}{m_Z^2} \frac{g_{Z}^e}{P_Z(q_2^2)} \right) + \left( \kappa_{Z_1} \frac{g_{Z}^e g_{Z}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \kappa_{Z_1}^\text{SM-1L} \epsilon_{Z_1} \right) + \kappa_{Z_2}^\text{SM-1L} \epsilon_{Z_2} \right] g^{\alpha\beta} + \kappa_{Z_2}^\text{SM-1L} \epsilon_{Z_2} \frac{q_1 \cdot q_2}{m_Z^2} \frac{g^{\alpha\beta} - q_2^\alpha q_1^\beta}{m_Z^2}$$

- The $\{\kappa_i, \epsilon_i\}$ are defined from the residues of the amplitude on the physical poles → well-defined PO that can be extracted from data and computed to desired accuracy in a given BSM framework

- By construction, the $g^f_Z$ are the PO from Z-pole measurements, while $\kappa_{\gamma\gamma}$ and $\kappa_{Z\gamma}$ are the standard “kappas” from on-shell $h \to \gamma\gamma$ and $h \to Z\gamma$

$$P_Z(q^2) = q^2 - m_Z^2 + im_Z \Gamma_Z$$
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$$A = i \frac{2m_Z^2}{v_F^2} \sum_{e=e_L, e_R} \sum_{\mu=\mu_L, \mu_R} (\bar{e} \gamma_\alpha e)(\bar{\mu} \gamma_\beta \mu) \times$$

$$\left[ \left( \kappa_{ZZ} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2) P_Z(q_2^2)} + \frac{\epsilon_{Ze}}{m_Z^2} \frac{g_Z^e}{P_Z(q_2^2)} + \frac{\epsilon_{Z\mu}}{m_Z^2} \frac{g_Z^\mu}{P_Z(q_1^2)} \right) g^{\alpha \beta} + \right.$$

$$\left. + \left( \epsilon_{ZZ} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2) P_Z(q_2^2)} + \kappa_{Z\gamma} \epsilon_{Z\gamma}^{SM-1L} \left( \frac{e_Q \mu g_Z^e}{q_2^2 P_Z(q_1^2)} + \frac{e_Q \epsilon g_Z^\mu}{q_2^2 P_Z(q_1^2)} \right) + \kappa_{\gamma \gamma} \epsilon_{\gamma \gamma}^{SM-1L} \frac{e^2 Q e Q}{q_1^2 q_2^2} \right) q_1 \cdot q_2 \cdot \frac{g^{\alpha \beta} - q_2^{\alpha} q_1^{\beta}}{m_Z^2} + \right]$$

$$\left. + \left( \epsilon_{Z\gamma}^{CP} \frac{g_Z^e g_Z^\mu}{P_Z(q_1^2) P_Z(q_2^2)} + \epsilon_{Z\gamma}^{CP} \left( \frac{e_Q \mu g_Z^e}{q_2^2 P_Z(q_1^2)} + \frac{e_Q \epsilon g_Z^\mu}{q_2^2 P_Z(q_1^2)} \right) + \epsilon_{\gamma \gamma}^{CP} \frac{e^2 Q e Q}{q_1^2 q_2^2} \right) \frac{\epsilon^{\alpha \beta \rho \sigma} q_2 \epsilon q_1 \sigma}{m_Z^2} \right]$$

$$P_Z(q^2) = q^2 - m_Z^2 + i m_Z \Gamma_Z$$

- The $\kappa_i$ are normalized such that the SM is recovered in the limit $\kappa_i \rightarrow 1$.
- The $\epsilon_i$ describe terms not present in the SM at the tree level (and always sub-leading): SM recovered for $\epsilon_i^{(SM)} = O(10^{-3}) \rightarrow 0$.
- To this amplitude we can apply a “radiation function” to take into account QED radiation → excellent description of SM (and NP) beyond the tree level.
The $h \rightarrow 4f$ case

“Dressing” with QED radiation → excellent description of SM beyond the tree level

Bordone, Greljo, G.I., Marzocca, Pattori, arXiv: 1507.02555
The $h \rightarrow 4f$ case

“Dressing” with QED radiation $\rightarrow$ excellent description of SM beyond the tree level & relevant impact for BSM @ NLO

Bordone, Greljo, G.I., Marzocca, Pattori, arXiv: 1507.02555
The "physical meaning" of the parameters appearing in this decomposition is not obvious at first sight, but it is actually quite simple:

\[
\mathcal{A} = \frac{2m_Z^2}{v_F} \sum_{e=e_L,e_R} \sum_{\mu=\mu_L,\mu_R} (\bar{e}_\gamma \alpha e)(\bar{\mu}_\gamma \beta \mu) \times \\
\left[ \frac{g_{ZZ}^c g_{Z}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \frac{\epsilon_{Ze}^c g_{Z}^\mu}{m_Z^2 P_Z(q_2^2)} + \frac{\epsilon_{Z\mu}^c g_{Z}^c}{m_Z^2 P_Z(q_1^2)} \right] g^{\alpha\beta} + \\
\left[ \kappa_{ZZ} \frac{\epsilon_{Ze} g_{Z}^c}{P_Z(q_1^2)P_Z(q_2^2)} + \kappa_{ZZ} \epsilon_{Z\gamma}^{SM-1L} \left( \frac{e Q_\mu g_{Z}^c}{q_2^2 P_Z(q_1^2)} + \frac{e Q_\gamma g_{Z}^\mu}{q_1^2 P_Z(q_2^2)} \right) + \kappa_{\gamma\gamma} \epsilon_{Z\gamma}^{SM-1L} e_{\gamma\gamma} \frac{e^2 Q_\gamma Q_{\mu}}{q_1^2 q_2^2} \right] \frac{q_1 \cdot q_2 g^{\alpha\beta} - q_2^\alpha q_1^\beta}{m_Z^2} + \\
\left[ \epsilon_{ZZ}^{CP} \frac{g_{ZZ}^c g_{Z}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \epsilon_{ZZ}^{CP} \left( \frac{e Q_\mu g_{Z}^c}{q_2^2 P_Z(q_1^2)} + \frac{e Q_\gamma g_{Z}^\mu}{q_1^2 P_Z(q_2^2)} \right) + \epsilon_{\gamma\gamma}^{CP} e_{\gamma\gamma} \frac{e^2 Q_\gamma Q_{\mu}}{q_1^2 q_2^2} \right] \frac{\epsilon_{\alpha\beta\rho\sigma} q_2 q_1}{m_Z^2}
\]

"double Z-pole"

\[\Gamma(h \rightarrow Z_LZ_L) = \frac{\Gamma(h \rightarrow 2e2\mu)[\kappa_{ZZ}]}{\mathcal{B}(Z \rightarrow 2e)\mathcal{B}(Z \rightarrow 2\mu)} = 0.209 |\kappa_{ZZ}|^2 \text{ MeV}\]

\[\Gamma(h \rightarrow Z_TZ_T) = \frac{\Gamma(h \rightarrow 2e2\mu)[\epsilon_{ZZ}]}{\mathcal{B}(Z \rightarrow 2e)\mathcal{B}(Z \rightarrow 2\mu)} = 0.0189 |\epsilon_{ZZ}|^2 \text{ MeV}\]

\[\Gamma^{CPV}(h \rightarrow Z_TZ_T) = \frac{\Gamma(h \rightarrow 2e2\mu)[\epsilon_{ZZ}^{CP}]}{\mathcal{B}(Z \rightarrow 2e)\mathcal{B}(Z \rightarrow 2\mu)} = 0.00799 |\epsilon_{ZZ}^{CP}|^2 \text{ MeV}\]
The “physical meaning” of the parameters appearing in this decomposition is not obvious at first sight, but it is actually quite simple:

\[
\mathcal{A} = i \frac{2m_Z^2}{v_F} \sum_{e=e_L,e_R} \sum_{\mu=\mu_L,\mu_R} (\bar{e} \gamma_{\alpha} e)(\bar{\mu} \gamma_{\beta} \mu) \times
\]

\[
\left[ \left( \kappa_{ZZ} \frac{g_{Z}^{e} g_{Z}^{\mu}}{P_{Z}(q_{1}^{2})P_{Z}(q_{2}^{2})} + \frac{\epsilon_{Ze} g_{Z}^{\mu}}{m_{Z}^{2} P_{Z}(q_{2}^{2})} + \frac{\epsilon_{Z\mu} g_{Z}^{e}}{m_{Z}^{2} P_{Z}(q_{1}^{2})} \right) \gamma_{\alpha} \right] \gamma_{\beta}
\]

\[
+ \left( \epsilon_{ZZ} \frac{g_{Z}^{e} g_{Z}^{\mu}}{P_{Z}(q_{1}^{2})P_{Z}(q_{2}^{2})} + \kappa_{Z\gamma} \epsilon_{Z\gamma}^{SM-1} \left( \frac{\epsilon_{Q\mu} g_{Z}^{e}}{q_{2}^{2} P_{Z}(q_{1}^{2})} + \frac{\epsilon_{Qe} g_{Z}^{\mu}}{q_{1}^{2} P_{Z}(q_{2}^{2})} \right) + \kappa_{\gamma\gamma} \epsilon_{\gamma\gamma}^{SM-1} \frac{e^{2} Q_{e} Q_{\mu}}{q_{1}^{2} q_{2}^{2}} \frac{q_{1} \cdot q_{2} g_{\alpha\beta} - q_{2}^{\alpha} q_{1}^{\beta}}{m_{Z}^{2}} \right)
\]

\[
+ \left( \epsilon_{ZZ}^{CP} \frac{g_{Z}^{e} g_{Z}^{\mu}}{P_{Z}(q_{1}^{2})P_{Z}(q_{2}^{2})} + \epsilon_{Z\gamma}^{CP} \left( \frac{\epsilon_{Q\mu} g_{Z}^{e}}{q_{2}^{2} P_{Z}(q_{1}^{2})} + \frac{\epsilon_{Qe} g_{Z}^{\mu}}{q_{1}^{2} P_{Z}(q_{2}^{2})} \right) + \epsilon_{\gamma\gamma}^{CP} \frac{e^{2} Q_{e} Q_{\mu}}{q_{1}^{2} q_{2}^{2}} \right) \frac{\epsilon_{\alpha\beta\rho\sigma} q_{2\rho} q_{1\sigma}}{m_{Z}^{2}}
\]

“single Z-pole”

\[
\Gamma(h \rightarrow Z \ell^{+} \ell^{-}) = 0.0366 |\epsilon_{Z\ell}|^2 \text{ MeV}
\]
The \( h \rightarrow 4f \) case

The PO are calculable in the (various) Higgs-EFT approaches (both linear and non-linear EFT)

\[
A = i \frac{2m_Z^2}{v_F} \sum_{e=\epsilon_L, \epsilon_R} \sum_{\mu=\mu_L, \mu_R} (\bar{e}\gamma_\alpha e)(\bar{\mu}\gamma_\beta \mu) \times \\
\left[ \left( \kappa_{ZZ} \frac{g_{\mu}^e g_{\nu}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \frac{\epsilon_Z e Z}{m_Z^2 P_Z(q_1^2)} + \frac{\epsilon_Z \mu}{m_Z^2 P_Z(q_1^2)} \right) g_{\alpha \beta} + \\
+ \left( \epsilon_{\mu \nu} \frac{g_{\mu}^e g_{\nu}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \kappa_{\gamma \gamma} e_{Z \gamma}^{SM-1L} \left( \frac{e Q_{\mu} Q_{\mu}}{q_1^2 P_Z(q_1^2)} + \frac{e Q_{\epsilon} Q_{\mu}}{q_1^2 P_Z(q_1^2)} \right) + \kappa_{\gamma \gamma} e_{\gamma \gamma}^{SM-1L} \frac{e^2 Q_{\epsilon} Q_{\mu}}{q_1^2 q_2^2} \right) \frac{q_1 \cdot q_2}{m_Z^2} g_{\alpha \beta} - q_2^\alpha q_1^\beta + \\
+ \left( \epsilon_{Z \gamma} \frac{g_{\mu}^e g_{\nu}^\mu}{P_Z(q_1^2)P_Z(q_2^2)} + \epsilon_{Z \gamma}^{CP} \left( \frac{e Q_{\mu} Q_{\mu}}{q_1^2 P_Z(q_1^2)} + \frac{e Q_{\epsilon} Q_{\mu}}{q_1^2 P_Z(q_1^2)} \right) + \epsilon_{\gamma \gamma}^{CP} \frac{e^2 Q_{\epsilon} Q_{\mu}}{q_1^2 q_2^2} \right) \frac{\epsilon_{\alpha \beta \rho \sigma} q_2 \rho q_1 \sigma}{m_Z^2} \right]
\]

In the limit where we consider Higgs-processes only, and we work at the tree-level in the EFT \( \rightarrow \) simple linear relation between PO and EFT couplings: one-to-one correspondence between PO and combinations of couplings of the most general Higgs EFT (non-linear EW symm. breaking, no custodial symm., no flavor symm., no CP symmetry).

But this does not hold beyond the tree-level.
Parameter counting, symmetry limits, dynamical constraints
**Parameter counting, symmetry limits, dynamical constraints**

Number of independent PO for $h \rightarrow 4\ell$ ($\ell=\text{e,}\mu,\nu$) + $\ell\ell\gamma + \gamma\gamma$: 

<table>
<thead>
<tr>
<th>Decay modes</th>
<th>\textbf{flavor +CP symm.}</th>
<th>\textbf{flavor non univ.}</th>
<th>\textbf{CP violation}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h \rightarrow \gamma\gamma, 2e\gamma, 2\mu\gamma$</td>
<td>$\kappa_{ZZ}, \kappa_{Z\gamma}, \kappa_{\gamma\gamma}$ (6)</td>
<td>$\epsilon_{Z\mu_L}, \epsilon_{Z\mu_R}$ (2)</td>
<td>$\epsilon_{Z\gamma}^{CP}, \epsilon_{Z\gamma}^{CP}, \epsilon_{\gamma\gamma}^{CP}$ (3)</td>
</tr>
<tr>
<td>$4e, 4\mu, 2e2\mu$</td>
<td>$\epsilon_{ZZ}, \epsilon_{Ze_L}, \epsilon_{Ze_R}$</td>
<td></td>
<td></td>
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Parameter counting, symmetry limits, dynamical constraints

Number of independent PO for $h \rightarrow 4\ell$ ($\ell=\text{e,}\mu,\nu$) + $\ell\ell\gamma + \gamma\gamma$:

<table>
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<tr>
<th>Decay modes</th>
<th>flavor +CP symm.</th>
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<tr>
<td>$h \rightarrow \gamma\gamma, 2e\gamma, 2\mu\gamma$</td>
<td>$\kappa_{ZZ}, \kappa_{Z\gamma}, \kappa_{\gamma\gamma}$</td>
<td>$\epsilon_{Z\mu L}, \epsilon_{Z\mu R}$</td>
<td>$\epsilon_{CP}^{Z\gamma}, \epsilon_{CP}^{Z\gamma}, \epsilon_{CP}$</td>
</tr>
<tr>
<td>$4e, 4\mu, 2e2\mu$</td>
<td>$\epsilon_{ZZ}, \epsilon_{ZL}, \epsilon_{ZR}$</td>
<td>(6)</td>
<td>(2)</td>
</tr>
<tr>
<td>$h \rightarrow 2e2\nu, 2\mu2\nu, e\nu\mu\nu$</td>
<td>$\kappa_{WW}$</td>
<td>$\epsilon_{Z\nu}, \Re(\epsilon_{W\mu L})$</td>
<td>$\epsilon_{WW}, \Im(\epsilon_{W\mu L})$</td>
</tr>
<tr>
<td></td>
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<td>$\epsilon_{\mu L}, \epsilon_{\mu R}$ (2)</td>
<td>$\epsilon_{CP}^{ZZ}, \epsilon_{CP}^{Z\gamma}, \epsilon_{CP}^{\gamma\gamma}$ (3)</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_{ZZ}, \epsilon_{Ze L}, \epsilon_{Ze R}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h \rightarrow 2e2\nu, 2\mu2\nu, e\nu\mu\nu$</td>
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<td>$\epsilon_{CP}^{WW}, \text{Im}(\epsilon_{W\mu L})$ (5)</td>
</tr>
<tr>
<td></td>
<td>$\epsilon_{WW}, \epsilon_{Z\nu e}, \text{Re}(\epsilon_{W\nu L})$</td>
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<td></td>
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<tr>
<td>all modes with custodial symmetry</td>
<td>$\kappa_{ZZ}, \kappa_{Z\gamma}, \kappa_{\gamma\gamma}$</td>
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<td>$\epsilon_{CP}^{ZZ}, \epsilon_{CP}^{Z\gamma}, \epsilon_{CP}^{\gamma\gamma}$</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>$\text{Re}(\epsilon_{W\nu L})$ (7)</td>
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20 (no symmetries) $\rightarrow$ 7 (CP + Lepton Univ + Custodial)
Parameter counting, symmetry limits, dynamical constraints

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The symmetry assumptions can be directly tested from data, focusing on specific kinematical distributions sensitive to the relevant PO's [e.g. CPV-violating observables & LFU tests $\rightarrow$ key role played by the “contact terms” ($\epsilon_{Z1}$)]
Parameter counting, symmetry limits, dynamical constraints

Computing the PO in specific EFT (e.g.: the linear EFT) we get additional dynamical constraints dictated by the specific extra dynamical assumption of the EFT employed (e.g.: \( h \) belongs to the \( SU(2)_L \) doublet breaking the EW symmetry)

E.g.:

\[
\begin{align*}
\kappa_{WW} - \kappa_{ZZ} &= -\frac{2}{g} \left( \sqrt{2} \epsilon_{W_L}^i + 2c_w \epsilon_{Z_L}^i \right)
\end{align*}
\]

- **N.B:** Custodial Symmetry does not imply \( \kappa_{WW} = \kappa_{ZZ} \)
- Using these relations we can (try...) to test if \( h \) belongs to a doublet simply using Higgs data

Residual accidental custodial symmetry at \( d=6 \)

G. Isidori – PO in Higgs decays  
IPMU July 2015

Contino et al., 1303.3876  
Pomarol, 1412.4410  
Gonzales-Alonso et al., 1412.6038
Parameter counting, symmetry limits, dynamical constraints

Computing the PO in specific EFT (e.g.: the linear EFT) we get additional dynamical constraints dictated by the specific extra dynamical assumption of the EFT employed (e.g.: h belongs to the SU(2)$_L$ doublet breaking the EW symmetry)

The most powerful of such constraints is the link between the contact terms and EW precision measurements performed at LEP:

EPWO + Linear EFT $\rightarrow$ small (tiny) & flavor-universal $\varepsilon_{Zl}$

Excellent opportunity to test from data (via $h \rightarrow 4l$) if $h$ belongs to a pure SU(2)$_L$ doublet

Contino et al., 1303.3876
Pomarol & Riva, 1308.2803

G.I., Manohar, Trott, 1305.0663
G.I., Trott, 1307.4051
Parameter counting, symmetry limits, dynamical constraints

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![Graph showing contact terms and EW precision measurements](image-url)
Parameter counting, symmetry limits, dynamical constraints

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Maim message: full complementary between PO approach and EFT.

- PO → inputs for EFT coupling fits
- EFT → predictions of relations between different PO sets (that can be tested)
PO beyond decays

\[ h \rightarrow Z f f \quad \text{vs.} \quad q q h \]
**PO beyond decays**

The same Green Function controlling \( h \rightarrow 4f \) decays is accessible also in \( pp \rightarrow hV \) and \( pp \rightarrow h \) via VBF, i.e. the two leading EW-type Higgs production processes (N.B.: this follows from “plain QFT” no need to invoke any EFT...)

\[
G_{[JJh]} = \langle 0 | \mathcal{T} \left\{ J_\mu^f(x), J_\mu^{f'}(y), h(0) \right\} |0\rangle
\]

But for two important differences:

- different flavor composition (q ↔ ℓ) \( \rightarrow \) 4 more param. for \( hZ \) + 4 for \( hW \) and VBF (no symm.) \( \rightarrow \) only 2 eff. combinations easily accessible

- different kinematical regime: momentum exp. not always justified
  (large momentum transfer)
The new parameters to be introduced are related to the momentum transfer associated to the quark-current ↔ variable related to the possible breakdown of the momentum expansion.

\[
\frac{1}{s - m_Z^2} \left[ g^Z_q \kappa_{ZZ} + \varepsilon_{Zq} (s - m_Z^2)/m_Z^2 + \ldots \right] s = (m_{hZ})^2
\]

Two (complementary) approaches:

- design kinematical cuts to remain in the region where the expansion works & introduce diagnostic tools to validate the result
- “ideal solution”: extract the shape of the distribution from data (only for the variables that can go into the large-momentum transfer region)

\[
\frac{[d\sigma(pp \rightarrow hZ)/dm_{hZ}]_{\text{exp}}}{[d\sigma(pp \rightarrow hZ)/dm_{hZ}]_{\text{SM}}}
\]
Conclusions

The 125 GeV scalar is certainly compatible with the properties of the SM Higgs boson, but we are still far from having explored its properties in great detail.

The PO represent a general tool for the exploration of such properties (in view of high-statistics data), with minimum loss of information and minimum theoretical bias.