Stability in the Landscape

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Models for Landscapes

The landscape gains its traction from the problem of the cosmological constant. At a theoretical level, don't have examples of theories with exponentially large numbers of metastable vacua found through any systematic analysis.

Models:

- String theories with fluxes. Many possible types of fluxes, taking discrete values – many possible "vacuum states" (Bousso-Polchinski)
- Theories with many fields (e.g. Arkani-Hamed, Dimopoulos, Kachru).

Flux vacua.

Only framework we have at present in which to meaningfully address is string theory. Big challenges:

- Stabilization of moduli
- Need for a controlled expansion

KKLT

Proposal in which *all* moduli stabilized, possibility to study in a systematic approximation with large R, approximate supersymmetry.

There are many questions one can ask about the KKLT framework, but not our focus today. Does make the existence of a landscape at least plausible. Suggests a strategy to determine statistics of states (esp. Douglas, Denef).

Well studied examples: Type IIB compactified on Calabi-Yau spaces (and related F theory constructions). Numbers of three form fluxes given by various topological numbers ($h_{2,1}$), $F_{i,\bar{j}\bar{k}}, H_{i,\bar{j},\bar{k}}$. $h_{2,1}$ can be quite large (100's, 1000's). Total fluxes constrained, but can also be large. Combination (N types of fluxes, M values) give of order N^M possibilities.

In these cases, also many light fields if fluxes turned off (fluxes paired with "moduli"). Most are massive (not moduli) in presence of fluxes, but still many light fields *if* radii are large,

$$V = N\frac{M^2}{R^6} \tag{1}$$

in string units. Require $R \gg (\sqrt{N}M)^{1/3}$.

Multiple Fields

Through much of this talk, study theories with many fields as models for a landscape, We see some overlap of the two ideas in the IIB flux vacua. More comments later.

Consider a field theory with N fields. Suppose, first,

$$V(\phi_i) = \sum v_i(\phi_i) \tag{2}$$

If, say, v_i typically has m stationary points, then m^N stationary points of V. Expect of order 2^{-N} stable.

E.g. if m=3, 2 minima, 1 maximum, $3^N \rightarrow 2^N$.

Classical Stability

Now couple the fields (Easther et al). If couplings between all fields substantial, scalar mass matrix a random matrix with all elements chosen from identical, independent (say Gaussian) distributions. Then probability that all eigenvalues positive behaves as

$$P(\mu_i^2 > 0) = e^{-cN^2}. \tag{3}$$

This is a dramatic suppression. If bounded potentials, of order one minimum. If unbounded, some sort of runaway likely.

If, instead, there is some sort of locality in the *index space* (μ_{ij}^2 in some sense sparse), suppression is only e^{-aN}.

Quantum Stability

Even if the system is classically stable, there is an issue of quantum stability. A particular vacuum, say with small c.c., is not the lowest energy state. It may be surrounded by a large – exponentially large – number of negative energy states. Decays to every state must be suppressed.

Without supersymmetry, we will shortly see this leads to a huge suppression of the chances of stability. Some general principle needed if the very idea of a landscape is to make sense.

Plan for the rest of the talk

1. Non-supersymmetric model for quantum stability: Greene, Kagan, Masoumi, Mehta, Weinberg et Xiao
2. Supersymmetry and Stability
3. Returning to classical stability: Marsh McAlister, Wrase, “Wasteland of Random Supergravities" (also Bachlechner)
4. Puzzles in the Supersymmetric case
5. Failures of unitarity and their resolution
6. A sensible, if not friendly, landscape

Tunneling in Theories with Many Fields

Greene et al landscape model: N scalar fields, ϕ_i, potential $V(\phi_i)$. Expand about presumed minimum:

$$V = \left(\sum_i \mu_i^2 \phi_i^2 + \sum_{ijk} \gamma_{ijk} \phi_i \phi_j \phi_k + \sum_{ijkl} \lambda_{ijkl} \phi_i \phi_j \phi_k \phi_\ell \right) \qquad (4)$$

μ_i^2, γ_{ijk},λ_{ijkl} random variables,

$$0 < \mu_i^2 < M^2; \;\; -M < \gamma_{ijk} < M; \;\; -1 < \lambda_{ijk} < 1. \qquad (5)$$

M is some fixed mass which will scale out of our problem.

Ask what fraction of states have bounce action for *all* possible bounce solutions greater than some fixed, large number B_0. This assumption is self consistent. If all tunneling amplitudes are small, all bounce actions are large, the semiclassical analysis is justified. Ignore gravity for the moment.

Greene et al: computer simulations. Searched for stationary points of the action. Looked for nearby critical points with low barriers. Applied a crude model for the bounce action. Up to $N = 10$.

1. The distance to the nearest stationary point behaves as

$$\phi_{top} \approx 0.5 N^{-1.15} : \tag{6}$$

2. The height of the lowest stationary point behaves as:

$$V_{top} \approx 0.2 N^{-3.16} \tag{7}$$

3. The lowest bounce action scales as:

$$B \approx N^{-2.7}. \tag{8}$$

These scalings are similar for both cubic and quartic potentials.

Try to understand these results by simple statistical reasoning.

For large N, typically one has a few $\mu_i^2 \sim \frac{1}{N}$. First assume that the smallest bounce action (and lowest barrier and shortest distance to tunnel) are obtained in a straight line (in field space) in one of these direction. Call $i = 1$ the direction with smallest μ^2. Let's assume, first, that the lowest bounce action is obtained by a straight line trajectory in the 1 direction. The important cubic and quartic couplings are then $\gamma_{111} \equiv \gamma$, $\lambda_{1111} \equiv \lambda$ and these will typically be of order 1. In this case, the cubic term dominates, and

$$\phi_{top} = \frac{2}{3}\frac{\mu^2}{\gamma} \quad V_{top} = -\frac{4}{27}\left(\frac{\mu^6}{\gamma^2}\right), \tag{9}$$

with corrections of order $1/N$.

Need to correct for the possibility that the cubic and quartic couplings fluctuate downward, in which case one of the larger masses may dominate. A more careful analysis gives, for the median values of ϕ_{top}, for large N:

$$\phi_{top} = .924\ N^{-1} \tag{10}$$

and

$$V_{top} = 0.284 N^{-3}. \tag{11}$$

This is compatible with the results of Greene et al up to corrections of order $1/N$.

Scaling of the Bounce Action

Greene et al make a crude approximation, by analogy with the thin wall approximation:

$$B = \frac{\pi^2}{2} \sigma R^3 \tag{12}$$

σ is a one dimensional bounce action and R is taken as N-independent. Given the N scaling of the barrier height and width,

$$\sigma \propto \int d\phi \sqrt{V} \sim N^{-5/2} \tag{13}$$

roughly as they find.

Knowing that the tunneling trajectories are dominated by small μ^2, we can do a more systematic calculation of the large N tunneling behavior. Interested in

$$V = \mu^2\phi^2 - \gamma\phi^3 \tag{14}$$

Scaling arguments give

$$\phi(r) = \frac{\mu^2}{\gamma}\phi_0(r\mu), \tag{15}$$

where ϕ_0 is the bounce for the potential $V = \phi^2 - \phi^3$, and the bounce action scales as μ^2/γ^2. Sarid has studied this problem numerically, obtaining

$$B = 2.376 \times 2\pi^2\frac{\mu^2}{\gamma^2}. \tag{16}$$

The median bounce action at large N:

$$B_{med} = \frac{97.5}{N} \quad (17)$$

So if $N = 100$, for example, the exponential of the typical bounce action is $\mathcal{O}(1)$.

Seek the probability that the lowest action satisfies

$$B > B_0 \tag{18}$$

for some constant B_0.

$$P(B > B_0) = P(w > w_0) = \begin{cases} w_0 < 1 & \left(1 - \frac{w_0}{3}\right)^N \\ \\ w_0 > 1 & \left(\frac{2}{3\sqrt{w_0}}\right)^N \end{cases} \tag{19}$$

where $B \equiv 2.376 \times 2\pi^2 w$. Requiring that the B_0 give a lifetime for the shortest tunneling amplitude longer than the age of the universe (not in our past last cone) gives $w_0 > 5.7$. To get some feeling for numbers, taking $N = 100$, this is a suppression of order 10^{-56}.

Implications of Quantum Instability

So we see that the requirement to avoid decay is that *all* masses be larger than some minimal value, μ_0^2. This is stronger than the requirement of classical stability. It can lead to further e^{-bN^2} or e^{-bN} suppression depending on the assumptions about the distribution of masses (to which we will return).

Stability with Supersymmetry

With exact supersymmetry in flat space, the vacuum is stable. This can be understood as a consequence of the existence of global supercharges, obeying the familiar algebra:

$$\{Q_\alpha, \bar{Q}_{\dot\beta}\} = 2P^\mu(\sigma_\mu)_{\alpha\dot\beta} \tag{20}$$

As a result, there are *no tachyons* and *no tunneling* (Weinberg, 1981).

Stability with slightly broken supersymmetry

Classical stability: With (slightly) broken supersymmetry, expect only a few states with masses of order $m_{3/2}$ potentially tachyonic.

Quantum stability: expect tunneling still vanishes or highly suppressed. Two classes of trajectories: directions with fields much more massive than $m_{3/2}$, and directions with masses of order $m_{3/2}$. For the former, for a broad class of models (Festuccia, Morisse, M.D.), one has a general formula:

$$\Gamma \propto e^{-2\pi^2 \left(\frac{M_p^2}{m_{3/2}^2} \right)} \quad (21)$$

For the latter, anything is possible, but as for classical stability, only a few trajectories must be suppressed; don't expect e^{-aN} type suppression of stability.

Approximate Supersymmetry

How might approximate supersymmetry arise in a landscape?

Douglas and Denef considered the likelihood in classes of flux vacua, one had approximate supersymmetry, simply as a result of random choices of flux (we'll call this “tuned supersymmetry"). Found low scale unlikely, roughly

$$P(F) \sim |F|^6 \tag{22}$$

Result understood in terms of a low energy theory with a light field (goldstino), with a uniform distribution of superpotential parameters (as complex numbers) (Z Sun, M.D.)

$$W = W_0 + \gamma Z + \mu Z^2 + \dots \tag{23}$$

So supersymmetry rare as a random phenomenon. In this context, can't provide a *natural* explanation of the hierarchy.

A different possibility: exponential separation of susy scale (dynamical supersymmetry breaking). Supersymmetry a good symmetry at some high energy scale; low energy theory breaks supersymmetry

$$F = Me^{-\frac{8\pi^2}{g^2(M)}} \tag{24}$$

In this case, if g^2 roughly uniformly distributed, roughly equal probability of susy breaking per decade. (Not necessarily a prediction of low energy supersymmetry breaking).

Our stability discussion suggests that theses states might be special.

A Puzzle With Stability (work in progress)

McAlister et al, studied states with approximate supersymmetry, and N massive fields in the context of tuned supersymmetry. Here “massive" means heavy compared to any scale of supersymmetry breaking. Found an exponential suppression of classical stability, i.e. only an order e^{-cN} fraction of the states identified by Douglas and Denef are classically stable.

Rather surprising. Expect there is one (or at most a few) light fields. Z: the “Goldstino supermultiplet". (With assumption of uniform distribution of parameters, multiple light fields very unlikely). Low energy theory: superpotential and Kahler potential, with a small number of parameters (random numbers, one would expect more or less uniformly distributed). Why so sensitive to the number of heavy fields?

Not difficult to isolate the origin of the problem. Suppose we have a superpotential, including the heavy fields:

$$W = XF + \sum_{i=1}^{N} \left(\frac{1}{2} m_i^2 \Phi_i \Phi_i + \gamma_i X^2 \Phi_i \right). \quad (25)$$

Integrating out the massive fields generates a correction to the Kahler potential:

$$\delta K = \sum_{i=1}^{N} \frac{|\gamma_i^2||X^\dagger X|^2}{m_i^2}. \quad (26)$$

This leads to a correction to the mass of X. One finds:

$$m_X^2 = -\sum_{i=1}^{N} \frac{|\gamma_i|^2}{4 m_i^2}.$$

This contribution, is a sum of a large number of terms, each of which is negative so potentially problematic. If each term is chosen independently from an identical, N-independent, distribution, for large N this grows linearly with N. To avoid a tachyon, it is necessary that nearly every term fluctuate to a low value; this leads to an e^{-aN} suppression.
Unitarity and perturbativity, however, restrict the possibilities.

Unitarity

Consider the process $X + X \to X + X$ (where here X denotes the scalar in the multiplet) at energies high compared to the masses of the N ϕ_i's. Then the potential includes:

$$V(X) = \sum_{i=1}^{N} |\gamma_i|^2 |X|^4. \tag{27}$$

The cross section, for energies large compared to the masses of the N fields, behaves as

$$\sigma(X + X \to X + X) = \frac{(\sum_{i=1}^{N} |\gamma_i|^2)^2}{s}. \tag{28}$$

With these assumptions, this violates partial wave unitarity for large N by a factor N^2. So require some modification.
Before considering flux landscapes and landscape models more generally, it is interesting to consider what happens in critical string theories with large numbers of fields.

A string computation

While we are not able to compute large numbers of couplings directly in a state with many fluxes turned on, we can do something much simpler to explore correlations of couplings. We can consider critical strings, compactified in such a way that there are large numbers of *fields*, and place bounds on sums of squared-couplings of the types encountered above. We will see no evidence for growth of such sums.

Assuming this is general, we will draw some conclusions about classical stability in the landscape.

Study compactification of the heterotic string on a Calabi-Yau space with large Euler number. While there is not a systematic large N computation (as is also true for most landscape constructions) we can still reasonably ask whether large numbers appear in perturbative computations. This would imply, in a manner analogous to our landscape discussion, that a valid perturbation theory would require a large radius or very small g (by powers of N).

Take famous case of quintic in CP^4 101 27's (corresponding to 101 complex structure moduli) Compute the cubic terms in the superpotential. Of order 10^5 independent 27^3 couplings.

Can readily bound certain combinations of couplings. Consider a four point function of vertex operators:

$$\langle V_a(z_1) V_b(z_2) V_a(z_3) V_b(z_4) \rangle \tag{29}$$

If the leading term in the operator product expansion of V_a and V_b is:

$$G(z_1, z_2, z_3, z_4)_{ab} = V_a(z_1) V_b(z_2) = c_{abc} \frac{V_c}{|z_1 - z_2|^2}, \tag{30}$$

If we take $z_1 \rightarrow z_2$, $z_3 \rightarrow z_4$, then

$$G(z_1, z_2, z_3, z_4)_{ab} = \frac{\sum_{c=1}^{N} c_{abc}^2}{|z_1 - z_3|^2 |z_1 - z_2|^2 |z_3 - z_4|^4} \tag{31}$$

So if we can estimate or bound the four point function, we can bound the couplings of the fields a, b to other fields.

Study Green's function involving two fermions and two bosons. For definiteness and because of its simplicity, we work in the fermionic formulation for the gauge degrees of freedom, and in the R-NS formulation for the right moving fermions. Then the spatial coordinates can be grouped as $y^i, y^{\bar{i}}, x^\mu$, where the i, $\bar{i}$ are complex indices for the six dimensional Kahler manifold, and μ are four dimensional Minkowski indices. The left moving fermions are $\lambda^i, \lambda^{\bar{i}}, \lambda^a$, where the a's are $O(10)$ indices. Space-time spinor operators can be taken as $S^0_\alpha, S^i_\alpha, S^{\bar{0}}_\alpha, S^{\bar{i}}_\alpha$, where α are four dimensional spinor indices. The 0 and $\bar{0}$ indices correspond to the covariantly constant spinor. At large radius, the theory is nearly free, and these operators reduce to their free field forms.

Bosonizing the right moving fermions,

$$\psi^i = e^{i\phi_i} \tag{32}$$

for the fermions with indices in the compact space, whereas for the ψ's with Minkowski indices

$$(\psi_1 + i\psi_2) = e^{i\xi_1} \ \ (\psi_3 + i\psi_4) = e^{i\xi_2}. \tag{33}$$

$$S^0 = ce^{\frac{i}{2}(\phi_1+\phi_2+\phi_3)} e^{\frac{i}{2}(\pm\chi_1\pm\chi_2)} \quad S^{\bar{0}} = ce^{-\frac{i}{2}(\phi_1+\phi_2+\phi_3)} e^{\frac{i}{2}(\pm\chi_1\pm\chi_2)} \tag{34}$$

where in the first case there are an even number of plus signs, the second an odd number. The S_i's are given by

$$S^i_\alpha = ce^{\frac{i}{2}(\phi_1+\phi_2-\phi_3)} e^{\frac{i}{2}(\pm\chi_1\pm\chi_2)} \quad S^{\bar{i}}_\alpha = ce^{-\frac{i}{2}(\phi_1+\phi_2-\phi_3)} e^{\frac{i}{2}(\pm\chi_1\pm\chi_2)} \tag{35}$$

(this is $S^3,\ S^{\bar{3}}$; other values of the index are obtained by changing the placement of the minus sign in the first exponent).

Decomposing the 27 into representations of $O(10) \times U(1)$,

$$27 = 16_{-1/2} + 10_1 + 1_{-2} \tag{36}$$

the boson vertex operators for particles in the 1 can be taken to be :

$$V_B = \lambda^{\bar{i}} \lambda^{\bar{j}} \psi^k \chi^{(\alpha)}_{k\bar{i}\bar{j}} \tag{37}$$

and its complex conjugate.

$\chi^{(\alpha)}_{k\bar{i}\bar{j}}$ is a harmonic (2,1) form. It is related to the corresponding fluctuation in the metric, δg^{α}_{ij} through

$$\delta g^{(\alpha)}_{ij} = \chi^{\bar{m}(\alpha)}_{\bar{k}\bar{l}} \Omega^{\bar{k}\bar{l}\bar{n}} g_{j\bar{m}} g_{i\bar{n}}. \tag{38}$$

where Ω_{ijk} is the covariantly constant three form. We will write the fermion vertex operator, for particles in the 10 representation, as:

$$V_F = \epsilon^{i}_{\alpha} \lambda^{a} \lambda^{i} S^{j}_{\alpha} \delta g^{(\alpha)}_{ij}. \tag{39}$$

We consider the scattering of one fermion corresponding to the (α)'th $2,1$ form with a scalar corresponding to the β'th $2,1$ form, to produce the same fermion and boson (elastic scattering). This arises from the cubic terms in the superpotential,

$$\gamma_{\alpha\beta\gamma}\Phi^{\alpha}\Phi^{\beta}\Phi^{\gamma}. \tag{40}$$

The coefficient of

$$\frac{1}{|z_1 - z_3|^4 |z_1 - z_2|^2 |z_3 - z_4|^2} \tag{41}$$

is

$$\mathcal{A} = \int d^6 y \delta g_{ij}^{(\alpha)} \delta g_{kl}^{(\beta)} \left(\delta g^{*(\alpha)ij} \delta g^{*(\beta)kl} - \delta g^{*(\alpha)ik} \delta g^{*(\beta)jl} \right) \tag{42}$$

The δg's are normalized to unity. As a result, the integral does not show growth with the number of complex structure moduli ($h_{2,1}$ of the manifold). On the other hand

$$\mathcal{A} = \sum_k |\gamma_{ijk}|^2, \quad (43)$$

which is now of order 1 rather than of order N.

Lessons

While not a direct calculation in actual flux vacua, strong evidence that with large numbers of fields, couplings are correlated; it is simply not true that all possible couplings are independent of the number of fields. Supports the notion of some sort of locality in *index space*.

Unitarity/Perturbativity More Generally

Requiring that in the effective field theory with N fields (non-supersymmetric) corrections to amplitudes not grow with N, yields for couplings of type $\lambda^{(n)}\phi^n$, assuming uniform scalings.

$$\lambda^{(n)} \sim N^{-n/2}. \tag{44}$$

For terms $\gamma^{(n)}\Phi^n$ in a superpotential in the supersymmetric case,

$$\gamma^{(n)} \sim N^{-\frac{n-1}{2}}. \tag{45}$$

Returning to tuned Supersymmetry

For tuned supersymmetry these scalings imply:

1. The N massive fields have masses of order $1/\sqrt{N}$
2. Before integrating out these fields to obtain the effective theory for Z, the leading term in the Kahler potential dominates, and one has a "Polonyi" model.
3. Integrating out the massive fields yields an order 1 correction to the mass of Z (as opposed to N, $\sqrt{N}$, etc. (We have checked that the statistics of the mass matrix for the ϕ_i's does not enhance this term).

So we do not expect a strong suppression of stability in this case (though recall that such states are unlikely in any case).

Flux Landscapes: Scaling with R

We have seen that in order to have large numbers of fields, one requires:

$$R \gg (\sqrt{N}M)^{1/3} \tag{46}$$

This also leads to suppression of couplings. In the interaction of complex structure moduli, each additional field (in the IIB case) adds an additional $1/R$ factor.

$$\lambda^{(n)} \sim R^{-4-n} \tag{47}$$

It is possible that R has to be somewhat larger in order to obtain a sensible unitary structure at low energies.

It appears a tall order to obtain vast numbers of states at large R. So whether N fields are, indeed, a good model for a landscape would seem an open question.

One Guide: Behavior with R

For small R (R of order the fundamental scale) typically not N special states in flux compactifications. Results of large R analysis should be compatible. Not clear what would play the role of the parameter N. Possible applications:

1. No supersymmetry: very high degree of suppression, since don't expect any enhancement with N at small R (not clear what would be the parameter which plays the role of N).
2. Approximate (tuned) supersymmetry: Already very rare, without accounting for stability. Small R: again not clear what would play the role of N in accounting for suppression of stability. Results above suggest that there might be no such suppression.
3. Exponential hierarchies (dynamical supersymmetry breaking): no light scalar fields partnered with Goldstino, so problem of stability does not arise.

So we tentatively conclude, for non-supersymmetric states:

1. Classical stability of non-supersymmetric states is subject to at least exponential suppression with the number of fields, and possibly much more.
2. In the framework of flux landscapes, even this analysis requires the existence of vast numbers of stationary points at large R. All of this suggests that there is of order one potentially metastable state per choice of flux – or less, if potential unbounded below or subject to runaway.
3. Quantum stability of non-supersymmetric states differing in flux is likely to give rise to further exponential suppression.

For supersymmetric states:

1. Approximate supersymmetry leads to order one chance of classical and quantum stability. This seems to be the case even if supersymmetry is tuned.
2. From this we might conclude (now much more tentatively) that *some* degree of supersymmetry is likely among states in any would-be landscape.