...in the Alphabet of Bsm Curiosities...

$\boldsymbol{A}$ is for $\boldsymbol{axion}$

- particle from Beyond-the-Standard-Model, but *light*. (So forget usual EFT)
- very light ($m \sim 10^{-4}$ eV), very weakly coupled ($\lesssim 10^{-12}$), *theoretically beloved* (pseudo) scalar
- one parameter model: couplings $\propto$ mass (for QCD axion
  Axion-Like-Particles = ALPS = same Lagrangian, couplings free)
- $m_a \sim m_\nu$, but *COLD* Dark Matter $\Rightarrow$ *distinguish from WIMPs using LSS data?*

Sacha Davidson *IPN de Lyon/CNRS*

arXiv:1405.1139 , 1307.8024 with M Elmer, in progress with T Schwetz

Outline: to distinguish axions from WIMPs with Large Scale Structure data?

1. remember the QCD axion...
   - astrophysical constraints

2. the story of the Universe (according to axions)
   - inflation and the birth of the axion: let suppose inflation first...
   - the QCD phase transition: the axion gets a mass
   - redshift as Cold Dark Matter: field, and particles from strings

3. structure formation with axion Dark Matter : distinguishing from WIMPs?
   - Sikivie's scenario and the Bose Einstein Condensate
   - D'après moi, principle is simple $\begin{cases} \text{axion field} \approx \text{fluid} \\ \text{the stress} - \text{energy tensor is different} \end{cases}$
   - linear fluctuation growth same for axions and WIMPs
   - non-linear structure formation: does the field fragment into drops?

Strong CP problem,the chiral anomaly and axion models

Peccei Quinn
Kim , ShifmanVainshteinZakharov
DineFischlerSrednicki,Zhitnitsky
Srednicki NPB85

Problem:can put a renormalisable, CPV interaction for gluons in QCD:

$$-\underbrace{\frac{1}{4}G^A_{\mu\nu}G^{\mu\nu A}}_{\vec{E}^2+\vec{B}^2} - \underbrace{\boldsymbol{\theta}\frac{g_s^2}{32\pi^2}G^A_{\mu\nu}\widetilde{G}^{\mu\nu A}}_{\vec{E}\cdot\vec{B}} + \sum_i \overline{q}_i(\not{D} - m_i)q_i \qquad A: 1..8, \quad \widetilde{G}^{\mu\nu} = \varepsilon^{\alpha\beta\mu\nu}G_{\alpha\beta}$$

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But $\theta \lesssim 10^{-10}$ from non-observation of electric dipole moment of neutron

Pich deRafael
Pospelov, Ritz

How to get rid of θ?

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1. Can rotate away with the chiral anomaly $(\delta\mathcal{L} \propto \theta\partial_\mu J_5^\mu = \theta\frac{g_s^2 N}{8\pi^2}G\widetilde{G})$ for massless quarks!

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2. but SM quarks are not massless :(
$$m\overline{q_L}q_R \to e^{i\theta/2}m\overline{q_L}q_R$$

3. add ... quarks with a mass invariant under chiral rotns!
$\Rightarrow$ introduce new quarks, and new complex scalar $\Phi = |\Phi|e^{ia/f}$, such that $\Phi \to e^{-i\theta/2}\Phi$, whose vev ($\sim 10^{11}$ GeV) gives mass to new quarks
$$\mathcal{L} = \mathcal{L}_{SM} + \partial_\mu\Phi^\dagger\partial^\mu\Phi + i\overline{\Psi}\not{D}\Psi + \{\lambda\Phi\overline{\Psi}\Psi + h.c.\} + V(\Phi)$$

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4. θ is gone, $|\Phi|$ and new quarks are heavy...remains at low energy a, the axion.

Remains the axion at low energy

1. Traded CPV parameter θ for a dynamical field a (with potential min at 0)

2. a was phase of $\Phi \sim f e^{ia/f}$, $f \sim 10^{11}$ GeV, but...
   only new particle at low-energy is the (pseudo-) goldstone a

$$\text{mixes to pion} \quad : \quad m_a \sim \frac{m_\pi f_\pi}{f} \simeq \; 6 \times 10^{-5} \; \frac{10^{11} \text{ GeV}}{f} \, \text{eV}$$

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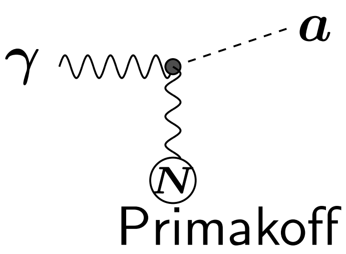
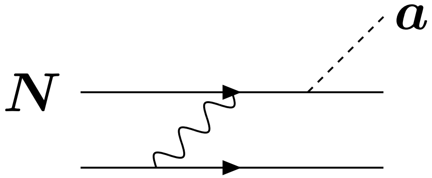
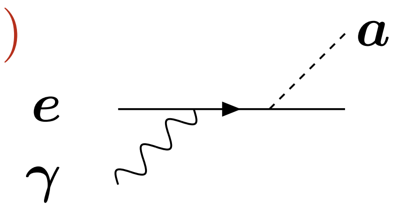
3. light, feebly coupled $\Rightarrow$ produce in sun, He-burning stars(g_{ae}), supernovae(g_{aN})...
   upper bound on coupling to avoid rapid stellar energy loss:

Raffelt...



$m_a \lesssim 10^{-2}$ eV $\qquad (f_{PQ} \gtrsim 10^9 \text{ GeV})$





The story of the (QCD) axion Universe

Non-thermal axion production: *Cold* Dark Matter!

1. *Lets suppose...* in the beginning, there was inflation

   avoids CMB bounds on isocurvature fluctuations :

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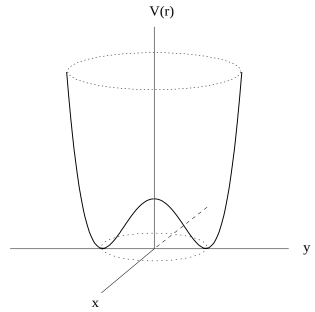

2. Then the axion is born

$$\Phi \to f e^{ia/f} \quad (f \sim 10^{11}\ \text{GeV})$$

   * a massless, random $-\pi f \leq a_0 \leq \pi f$ in each horizon

$$\langle a_0^2 \rangle_{U\ today} \sim \pi^2 f^2/3$$

   * ...one string/horizon

PreskillWiseWilcek
AbbottSikivie
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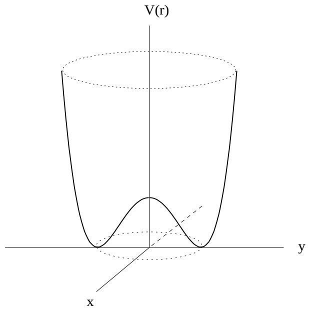

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3. Laaaater: QCD Phase Transition ($T \sim 200$ MeV): ...m_π (tilt mexican hat)

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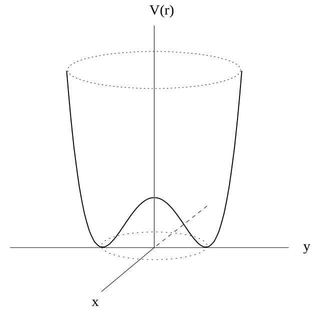

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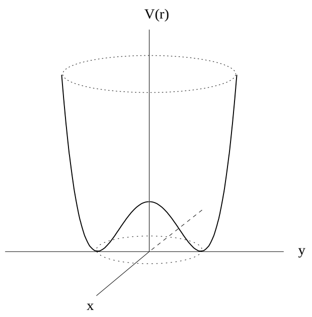

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Hiramatsu etal,etal+Saikawa

axion after inflation $\Rightarrow$ **oscillating axion field + cold particles** redshift like CDM

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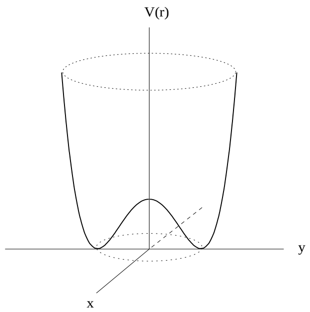

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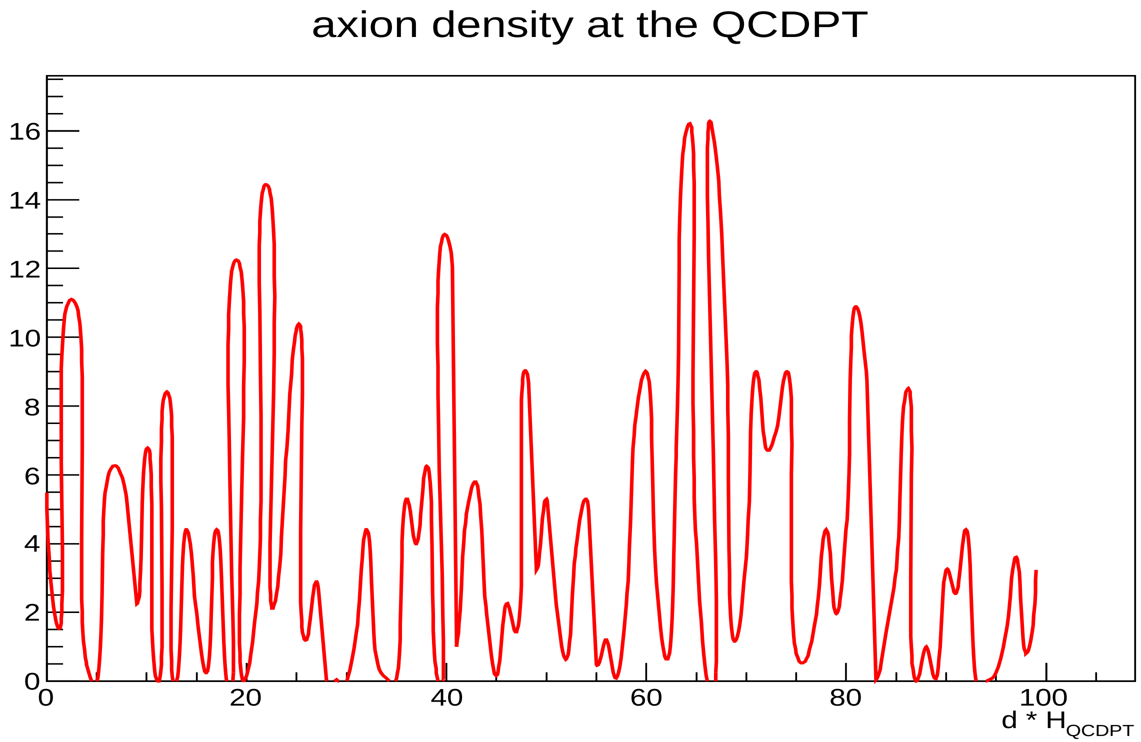
Inhomogeneities are $\mathcal{O}(1)$ on the QCD horizon scale: axion "miniclusters"

Hogan+Rees
Tkachev+Kolb

$a(\vec{x}, t)$ random from one horizon(~ 5km) to next; $\rho_a(\vec{x}, t) \simeq m_a^2 a^2(\vec{x}, t)$



anticipate what they will do: frozen til $\rho_{mat} = \rho_{rad}$, then collapse.

Summary so far...

- QCD axion solves the strong CP problem
- for $m_a < 10^{-2}$ eV, stars live long enough (not cooled to fast)
- if born after inflation
  avoid isocurvature bound from PLANCK
  get correct Ω_{CDM} for $m_a \sim 10^{-4}$ eV
- a CDM candidate should:

⋆ redshift as $1/R^3$ — true for axion field and cold particles (from strings)
⋆ grow $\delta\rho/\rho$, on LSS scales, like WIMPs — (true, see later)
$\Rightarrow$ axion is CDM

From the QCD Phase Transition to today

What does gravity do with axions?

(? distinguish from WIMPs in Large Scale Structure Data?)

Structure formation with axions: Sikivie's Scenario

Sikivie,Yang
Erken,Sikivie,Tam,Yang
Bannik,Sikivie

1. Consider DM axions... $HUGE$ occupation number of low-$\vec{p}$ modes.
   a) This enhances interaction rates.
   b) In (thermal) equilibrium, would form a Bose Einstein Condensate.

Sikivie's Scenario

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1. Consider DM axions... $HUGE$ occupation number of low-$\vec{p}$ modes.

2. at $T_\gamma \sim$ keV, gravitational interaction rate $> H$, so "gravitational thermalisation" causes axions to form a "Bose-Einstein Condensate"

$$\Gamma_{grav} \sim \frac{mG_N \rho_a R^3}{R} \sim \frac{G_N m_a^2 n_a}{H^2}$$

(QFT confirmation: Saikawa etal)

3. axion **BEC** can support vortices, which allow caustics in the galactic DM distribution. $\Leftrightarrow$ **axion DM signature?**

Rindler-Daller+Shapiro
Saikawa etal
SD+Elmer,SD
Berges+Jaeckel
...
Guth etal

I am confused...

1. what is a Bose Einstein Condensate?

   coherent scalar field carrying conserved particle number...

   but is it constant everywhere = coherent state of zero-mode particles?

   Or not neccessarily?

2. Are we talking about fields or particles? Does it matter?

3. What is thermalisation? How to quantify?

4. ...vortices in BECs...happen when? Why?

5. what observables are we trying to compute anyway?

$\Rightarrow$ ask the path integral! The path integral knows everything...

(usually tells you nothing because can't compute...

but axion most weakly coupled model I ever met, if perturbation theory works for QED, surely it works for axion?

Ask the Path Integral:

What are relevant variables and equations to describe axion evolution?
Suppose two CDM axion populations are classical field and distribution of cold particles (from strings).

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Suppose two CDM axion populations are classical field and distribution of cold particles (from strings).

- variables = expectation values of n-pt functions ($a \equiv$ axion)

$$\langle a \rangle \leftrightarrow \text{ classical field} = \text{misalignment axions } a_{cl}$$

$$\langle a(x_1)a(x_2) \rangle \leftrightarrow \text{ (propagator)} + \text{distribution of particles } f(x,p)$$

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- get Eqns of motion for expectation values in Closed Time Path formulation

Einsteins Eqns with $T^{\mu\nu}(a_{cl}, f)$ + quantum corrections(λ, G_N)

⇒**leading order is simple:** Einsteins Eqns with $T^{\mu\nu}(a_{cl}, f)$.
In practise: compute $T^{\mu\nu}$ in usual 2nd quantised QFT, as expectation of the operator in a coherent state + bath of particles

Rediscovering...stress-energy tensors

non-rel axion particles are dust, like WIMPs:

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j \\ & \end{bmatrix}$$

compare to perfect fluid: $T_{\mu\nu} = (\rho + P)U_\mu U_\nu - P g_{\mu\nu}$. $P_{int} \propto \lambda^2 \to 0$, nonrel $\Rightarrow P \ll \rho, U = (1, \vec{v}), |\vec{v}| \ll 1$

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Classical field in non-relativistic limit

$$T_{\mu\nu} = \begin{bmatrix} \rho & \rho\vec{v} \\ \rho\vec{v} & \rho v_i v_j + \Delta T_{ij} \end{bmatrix} \qquad \Delta T^i_j \sim \partial_i a \partial_j a \ , \ \lambda a^4$$

Sikivie

⋆ "extra" pressure with classical field... *not need Bose Einstein condensation!*

BE condensate described (at leading order) as a non-relativistic classical field. Misalignment axions already a non-rel. classical field. No need to form a BE condensate ?

⋆ classical field is single-valued, like fluid..not phase space! (more later...)

⇒ is structure formation different?

density fluctuations of small amplitude (linear eqns in fourier space)

large scale init cdns: "inflationary" , adiabatic density fluctuations (inherited at QCDPT)

Eqns of motion: Einsteins Eqns and $T^{\mu}{}_{\nu;\mu} = 0$. For linear adiabatic perturbations:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_N \overline{\rho}\delta + c_s^2 \frac{k^2}{R^2(t)}\delta = 0 \qquad \left(\delta \equiv \frac{\delta\rho(\vec{k},t)}{\overline{\rho}(t)} \right)$$

(H = Hubble rate, extra pressures in $c_s \simeq \partial P/\partial\rho$)

on LSS scales, $k^2 \to 0$, *same equation/dynamics as WIMPs*

Ratra, Hwang+Noh

density fluctuations of small amplitude (linear eqns)

large scale init. cdns: "inflationary" , adiabatic density fluctuations (inherited at QCDPT)

Eqns of motion: Einsteins Eqns and $T^{\mu}_{\nu;\mu} = 0$. For linear adiabatic perturbations:

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?short distance differences:

pressure and a Jeans length ($R_{Jeans} \sim \sqrt{\frac{H}{m}} \times$ horizon)

$R_{Jeans} \ll$ comoving QCD-horizon $\Rightarrow$ miniclusters are "frozen" (not damp or grow)

Distinguishing axion-field vs WIMPs in *non*-linear structure formation?

• extra pressures in fluid eqns for non-relativistic axion field (black=eqns for dust) :

$$T^{\mu}_{\ \nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 & V_N(r) = -\frac{GM(r)}{r} \\ \rho \partial_t \vec{v} + \rho (\vec{v} \cdot \nabla) \vec{v} = -\rho \nabla V_N \pm \ \text{extra pressures from field} & \end{cases}$$

• axion field is 1-pt function, single-valued ≈ fluid
usual CDM is particles, described by phase-space (fluid approx breaks down at shell-crossing)

⇒ **hack a structure formation code to run fluid DM** (or field:Broadhurst etal)
compare to N-body **(phase-space)** **code**

Trying to learn something analytically...(confusion in progress)

- fluid eqns for non-relativistic axion field (black=eqns for dust) :

$$T^{\mu}{}_{\nu;\mu} = 0 \quad \Leftrightarrow \quad \begin{cases} \partial_t \rho + \nabla \cdot (\rho \vec{v}) = 0 \\ \\ \rho \partial_t \vec{v} + \rho(\vec{v} \cdot \nabla)\vec{v} = -\rho \nabla V_N + \rho \nabla \left(\frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |g| \frac{\rho}{m^2} \right) \end{cases}$$

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- equivalent to non-relativistic eqns for axion field

$$a = \frac{1}{\sqrt{2m}} \left(\phi e^{-imt} + \phi^* e^{+imt} \right) \ , \ \phi(\vec{r}, t) = \sqrt{\frac{\rho}{m}} e^{-iS(\vec{r},t)} \ , \ \vec{v} = -\frac{1}{m} \nabla S \ , \ V_N = -\frac{GM(r)}{r} \ , \ g = -\frac{1}{(3! f^2)}$$

self-interaction pressure *inwards*: $\frac{\partial}{\partial r} r^{-n} < 0$

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$*$ fluid parameters single-valued ($\Rightarrow$ shocks, etc.) ... different from $f(x,p)$

"Bose Stars" in GR (eg Liebling, Palenzuela): solns for classical field coupled to GR

Rindler-Daller+Shapiro, Chavanis, ...: stationary,rotating solns, with $g, m \sim 10^{-22}$ eV to give galactic mass/radius

Broadhurt etal: numerics for the $m \sim 10^{-22}$ eV case

I fix m, g for QCD axion($m \sim 10^{-4}$ eV, $f \sim 10^{11}$ GeV); what sized solution?

To model Andromeda (today, not formation) with an axion field?

two issues: how does overdensity collapse? What are "stable" solutions?

Euler Eqn for the non-relativistic axion field:

$$\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v} \quad = \quad \nabla\Big(- V_N + \frac{\nabla^2 \sqrt{\rho}}{2m^2 \sqrt{\rho}} + |g| \frac{\rho}{m^2} \Big)$$

$$V_N = -\frac{GM(r)}{r}$$
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Neglect LHS (v constant?):

To model Andromeda with an axion field?

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Neglect LHS (v constant?):

1. balance gravity with gradient pressure in object of mass M:

   $\frac{1}{R^2} \simeq \frac{m^2 M}{m_{pl}^2 R} \quad \Rightarrow \quad \frac{m_{pl}^2}{m^2}\frac{1}{M} \sim R_{Jeans}$

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Chavanis, Barranco,...

$$R \sim 10^7 \text{ cm} \sim 10^{-3} R_\odot \quad , \quad \rho \sim 0.2 \frac{\text{g}}{\text{cm}^3} \Leftrightarrow M \sim 10^{20}\text{g} \sim 10^{-14} M_\odot$$

Andromeda : $M \sim 10^{12} M_\odot$, flat rotn curves to 100s kpc

(allowing rotation does not seem to make heavier solutions?)

CITE

Speculations : the dynamics of axion-field-CDM in galaxy formation

Back to "miniclusters" = the $\mathcal{O}(1)$ fluctuations on QCD horizon scale, from from QCD PT 'til $\rho_a \sim \rho_{rad}$.

At matter-radiation-equality, these "miniclusters" ($M \sim 10^{-8} M_\odot$, $R \sim 10^9$ km), decouple from Hubble flow and collapse. (recall: stable clumps were $10^{-13} M_\odot$)

Then...what?(recall: stable clumps were $10^{-13} M_\odot$)

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Then...what?(recall: stable clumps were $10^{-13} M_\odot$)

1. form a black hole?
   $\Rightarrow$ axion-field CDM participates in galaxy formation as BHs?

2. ...? gravitational binding energy has to go somewhere = gradients... axion field configuration "fragments" into $\sim 10^{-14} M_\odot$ drops ?
   $\Rightarrow$ axion-field dark matter today is a phase space distribution of drops with $R \sim 100$ km, $m \sim 10^{-14} M_\odot$ ("MACRO"s: seems allowed by microlensing, CMB,other?)

3. or could the field "evaporate" into axion particles? (?phase transition?)

$\Rightarrow$? in all cases, misalignment axions look like WIMPs in LSS data?

(but different for ADMX)

Summary

The QCD axion solves the strong CP problem, is consistent with astrophysics and laboratory constraints for $m_a \lesssim 10^{-2}$ eV.

Non-thermal production mechanisms in cosmology can generate $\Omega_{CDM} \sim 0.25$. If the axion is born after inflation, two populations arise at the QCD Phase Transition: the classical "misalignment" field, and cold particles from the decay of strings. They could give Ω_{CDM} for $m_a \sim 10^{-4}$ eV.

The particles and field redshift like CDM, and grow small inhomogeneities (linear eqns) like CDM.
But the field differs from WIMPs during non-linear structure formation:
1) behaves like a fluid,
2) has extra pressures and viscosities

$\Rightarrow$ *numerical galaxy formation?*
(analytics suggests the field fragments into drops?)

To distinguish axion from WIMP CDM:
direct detection (of axions from strings), axion effects on γ propagation? ...
??? Large Scale Structure data? Not if field in form of drops?

Questions...

If PQPT before inflation...

1. no miniclusters, $\mathcal{O}(1)$ inhomogeneities at mat-rad equality are muuuch bigger — do they fragment into drops?

2. no axions from strings — what can ADMX see?

Backup

Using $T^{\mu\nu}{}_{;\nu} = 0$ vs Eqns of motion of the field ϕ

Why not study eqns of motion of axion field cpled to gravity? (Sikivie etal, Saikawa etal, Guth etal...)

$$(\Box - m^2)a(y) \sim m^2 G_N \int \frac{d^3x a^2(x)}{x-y} a(y) \quad \Rightarrow \quad i\frac{\partial f(x,p)}{\partial t} \sim m G_N \int \frac{d^3k}{k^2} a^4$$

Both obtained from $T^{\mu\nu}{}_{;\nu} = 0$ and Poisson Eqn ($\rightarrow$ dynamics is equivalent?)

$$\begin{aligned} T^{\mu\nu}{}_{;\nu} &= \nabla_\nu[\nabla^\mu\phi\nabla^\nu\phi] - \nabla_\nu[g^{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\phi\nabla_\alpha\phi - V(\phi)\right)] \\ &= (\nabla_\nu\nabla^\mu\phi)\nabla^\nu\phi + \nabla^\mu\phi(\nabla_\nu\nabla^\nu\phi) - g^{\mu\nu}\nabla_\nu\nabla^\alpha\phi\nabla_\alpha\phi + g^{\mu\nu}V'(\phi)\nabla_\nu\phi \\ 0 &= \nabla^\mu\phi[(\nabla_\nu\nabla^\nu\phi) + V'(\phi)] \end{aligned}$$

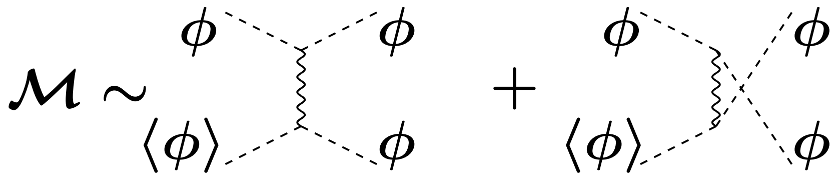
1. eqns for $T_{\mu\nu} \sim \phi^2$ solvable during linear structure formation. $\delta \equiv \delta\rho(\vec{k}, t)/\overline{\rho}(t)$ is labelled by momentum k of graviton = momentum diff between fields.

2. axions making up classical field do not have a phase space distribution (classical field is like a fluid; one velocity at given pt). Eqn for $f(x,p)$ deceptive: looks like Boltzmann, but...
   - $T_{\mu\nu} \neq \int f p_\mu p_\nu$
   - "collision term" linear in cpling coherent, deterministic = no entropy production?

3. "better" handle on IR divs: ensures that long-wave-length gravitons see large objects (like MeV photons see the proton, and not quarks inside)

Moving axions between field and bath with gravity? (in galaxy today)

at $\mathcal{O}(G_N^2)$, quantized GR ($v \sim 10^{-3}$ in cm frame)



$$\sigma = \frac{G_N^2 m^2 \pi}{8 v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2$$

Dewitt

IR cutoff of graviton momenta $\sim H$?

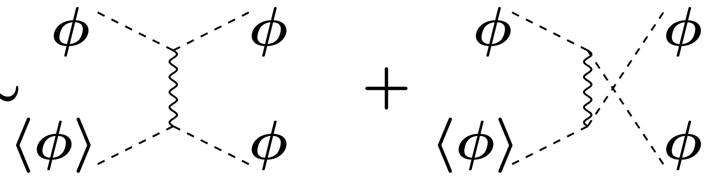
$$\sigma \sim \frac{G_N}{v^2}$$

...but this is for empty U containing two axions...

Moving axions between field and bath with gravity? (in galaxy today)

at $\mathcal{O}(G_N^2)$, quantized GR ($v \sim 10^{-3}$ in cm frame)

$\mathcal{M} \sim$



$$\sigma = \frac{G_N^2 m^2 \pi}{8v^4} \int \sin\theta d\theta \left(\frac{1}{\sin^2(\theta/2)} + \frac{1}{\cos^2(\theta/2)} \right)^2 \to 10^4 \frac{m^2}{m_{pl}^4} \qquad (m \sim 10^{-5} eV)$$

graviton couples to $T^{\mu\nu}$! Only sees single axion when can look inside box $\delta^3 \sim 1/(mv)^3 \Rightarrow$ IR cutoff of graviton momenta $\sim mv$.

$$\text{probability} = \left| \sum \text{ indistinguisable amplitudes} \right|^2$$

graviton of 10 metre wavelength interacts coherently with all axions in 10 metre cube $\leftrightarrow T_{\mu\nu}$. (like MeV γ scatters off proton and not individual quarks inside).

To estimate rate, account for high axion occupation # (in galaxy today)

to estimate evaporation/condensation rate, must take into account high occupation number of axions:

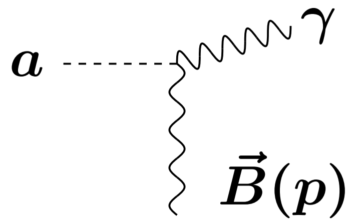
$$\frac{\partial}{\partial t} n = \int \Pi_i \widetilde{d^3 p_i} \tilde{\delta}^4 |\mathcal{M}|^2 \Big[f_1 f_2 (1 + f_3)(1 + f_4) - f_3 f_4 (1 + f_1)(1 + f_2) \Big]$$

$[...] \sim f^3$, so rate for individual axion to evaporate/condense

$$\Gamma \sim n_\phi \sigma_G f \sim 10^{13} \left(\frac{\rho_{DM}}{\rho_c} \right)^2 \left(\frac{m}{m_{pl}} \right)^3 H_0 \ll H_0$$

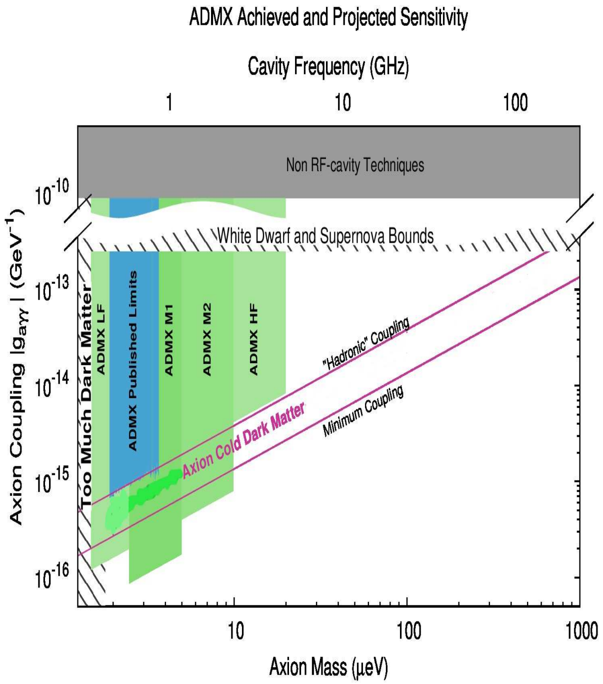
is negligeable...

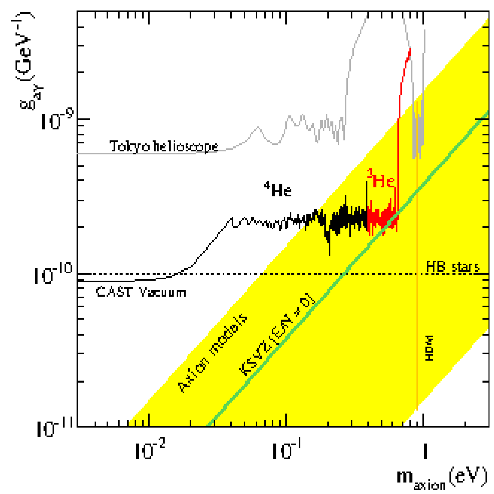
Direct detection (of axions)



1. $a \to \gamma$ conversion in $\vec{B}$ field. (with gradient, to transfer correct $\vec{p}$...a diff $\vec{B}$ for each m_a)
   (a) CernAxionSolarTel: LHC magnet, points at sun, convert solar a to γs (also Sumico)
   (b) ADMX: dark matter axions ($E_\gamma \sim m_a \sim$ microwave)
2. WIMP direct detection expts look for axions too!

Edelweiss,...





What is a Bose Einstein condensate? (I don't know. Please tell me if you do!)

Important characteristics of a BE condensate seem to be

1. a classical field,
2. carrying a conserved charge,
3. ? whose fourier modes are concentrated at a particular value — most of the "particles" who condense, should coherently do the same thing (but not necc the zero-momentum mode)

consistent with

- BE condensation in equilibrium stat mech, finite T FT, alkali gases.
- LO theory of BE condensates (Boguliubov $\to$ Pitaevskii) as a classical field

Particles vs fields

Develop field operator

$$\hat{a}(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ \hat{b}_{\vec{k}} \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + \hat{b}^\dagger_{\vec{k}} \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

then write the coherent state:

$$|a(\vec{x},t)\rangle \quad \propto \quad \exp\left\{ \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}} \right\} |0\rangle$$

which satisfies $\hat{b}_{\vec{q}}|a(\vec{x},t)\rangle = a(\vec{q},t)|a(\vec{x},t)\rangle$ (can check $\hat{b}_{\vec{q}}\{1 + \int \frac{d^3p}{(2\pi)^3} a(\vec{p},t) b^\dagger_{\vec{p}}\}|0\rangle = a(\vec{q},t)|0\rangle$)
where the classical field is

$$a(t,\vec{x}) = \frac{1}{[R(t)L]^{3/2}} \int \frac{d^3k}{(2\pi)^3} \left\{ a(\vec{k},t) \frac{\chi(t)}{\sqrt{2\omega}} e^{i\vec{k}\cdot\vec{x}} + a^*(\vec{q},t) \frac{\chi^*(t)}{\sqrt{2\omega}} e^{-i\vec{k}\cdot\vec{x}} \right\}$$

Brodsky+Heurer,Donoghue etal
Olive+Montonen...I+Z,C-T...

What is quantum?

Classical $=$ saddle-point configurations of the path integral
$\Rightarrow$ attribute dimensions to fields/parameters $\ni$ [action]$=$ E*t, and no $\hbar$ in selected classical limit (this is *not* unique)

Summary: particles or fields can be obtained in a "classical" ($=$ no $\hbar$) limit. However, $\hbar$ is differently distributed in the Lagrangian in the two limits, so to get from one to another requires $\hbar$...

in particular, to define a number of quanta, in the field picture, requires $\hbar$.

ex 1: massive scalar electrodynamics

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi - \tilde{m}^2 \phi^\dagger \phi - \frac{1}{4} FF \qquad , \quad D_\mu = \partial_\mu - i\tilde{e} A_\mu$$

Classical field limit: $[\phi, A] = \sqrt{E/L}$, $[m] = 1/L$, $[\tilde{e}] = 1/\sqrt{EL}$.

No $\hbar$ in classical EoM. OK that $[m^2] = 1/L^2$ because gravity couples is the stress-energy tensor, function of the fields.

If in Maxwells Eqns, want $j^0 = i\tilde{e}(\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi})$ to be eN/V, then need number of charge-carrying quanta $\Rightarrow e = \tilde{e}\hbar$.

De même, if classically m a particle mass, need $m = \tilde{m}\hbar$.

ex 2: the SHO Hamiltonian is (no $\hbar$)

$$H = \frac{1}{2m} P^2 + \frac{m\nu^2}{2} X^2$$

where ν is the oscillator frequency.

But to *quantise*, = introduce creation and annihilation ops, requires $\hbar$.
To write the total energy as $\omega(N + 1/2)$, requires $\hbar$ to convert frequency to energy $\omega = \hbar\nu$, and downstairs in the defn of N, because its the number of *quanta*.