Searching for 5-dimensional Nontrivial UV Fixed Point

Jin-Beom Bae

Ewha Woman's University
& Institute for Basic Sciences

Based on ArXiv:1412.6549
work with Soo-Jong Rey

October 28, 2015

Motivations

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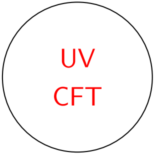

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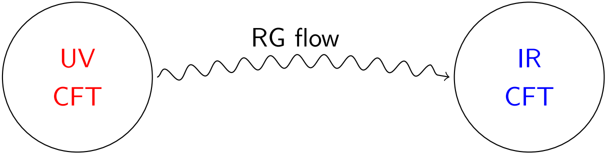

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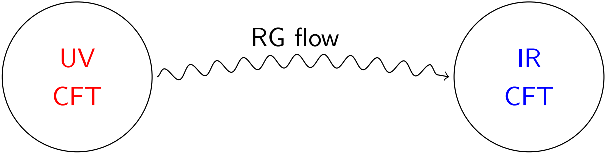

3D : Free Theory

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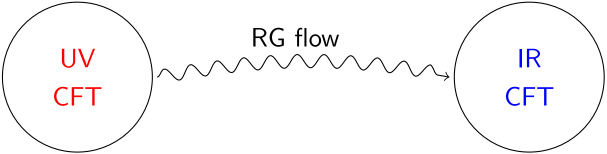

Relevant Deformation$(D > \Delta_{\mathcal{O}})$

3D : Free Theory $\rightsquigarrow$ Interacting Theory

φ^4-theory

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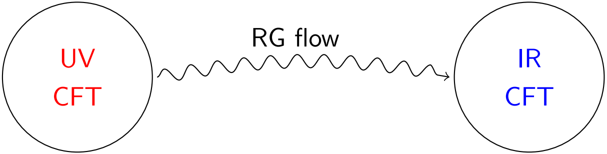

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3-dimensional Ising theory
Second order Phase Transition

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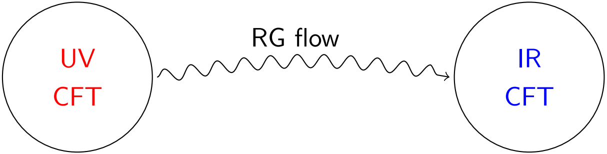

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Large-N limit

$\Downarrow$

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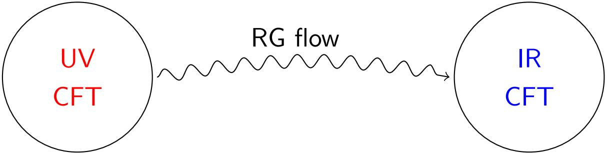

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Vasiliev Higher Spin theory on AdS_4

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⋆ The beta function of $O(N)$ ϕ^4-theory : $(\varepsilon = 4 - D)$

$$\beta(\lambda) = -\varepsilon\lambda + (N+8)\frac{\lambda^2}{8\pi^2} + \mathcal{O}(\lambda^3)$$

Fixed point at $\lambda^* = 0$(Gaussian) and $\lambda^* = \frac{8\pi^2}{N+8}\varepsilon$ (Wilson-Fischer).

Critical Exponents from ϵ-expansion

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These quantities are related to the *critical exponents* η and ν, by $\Delta_{\phi} = \frac{D}{2} - 1 + \frac{\eta}{2}$ and $\Delta_{\phi^2} = D - \frac{1}{\nu}$.

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$\star$ This ε-perturbation carried out up to 7-loop. [Guida, Zinn-Justin 98]

For $\epsilon = 1$, Borel resummation technique available.

The Experiment Result

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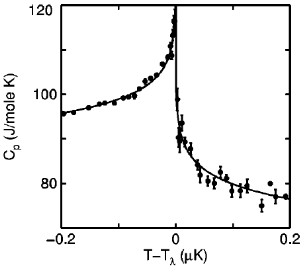
⋆ Helium-4 Specific heat measurement [Lipa, Swanson, Nissen, Chui, Israelsson 96]



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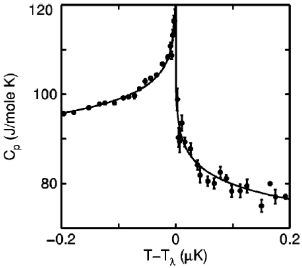

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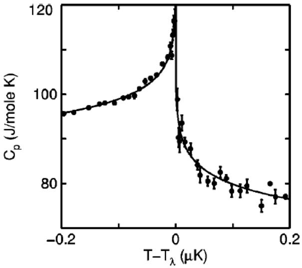

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⋆ By scaling relation $2-\alpha=\nu D$, measured exponent α is related to the $\nu(0.6709)$. This measured value agrees to the ε-expansion result of $O(2)$ model in 3-dimension(0.671 ± 0.005).

Solving 3D Ising theory(Old & New)

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Critical exponent			
η			
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High order
ε-expansion

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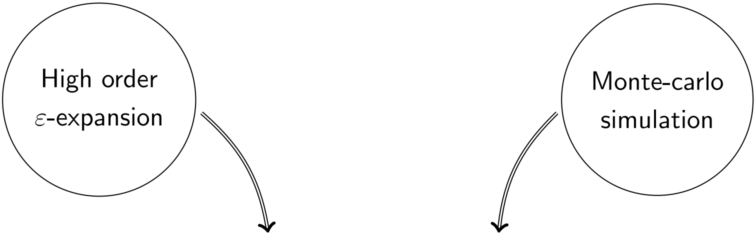

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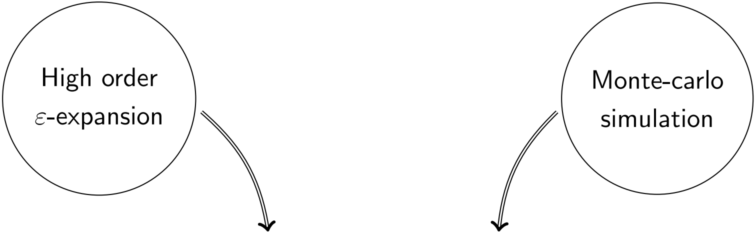

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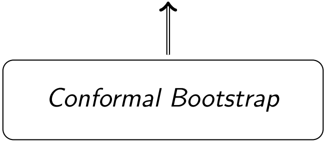

Solving 3D Ising theory(Old & New)

High order ε-expansion

Conformal bootstrap independently provides highly accurate numerical value of the critical exponents!

Monte-carlo simulation

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi 14]

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Conformal Bootstrap Program

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⋆ Consider *four point correlation function* of same scalar operators $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4) \rangle$ in Conformal Field Theory

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Is it consistent with Unitarity and Crossing symmetry ?

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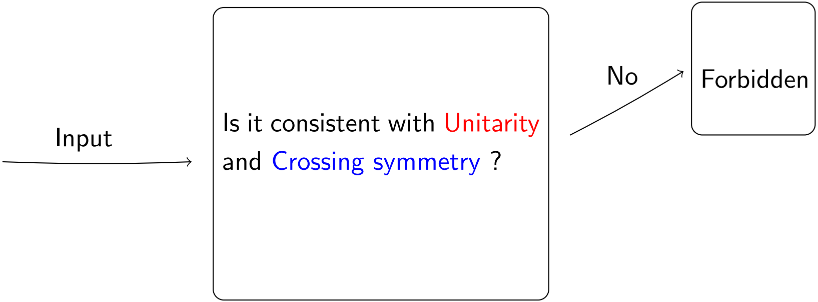

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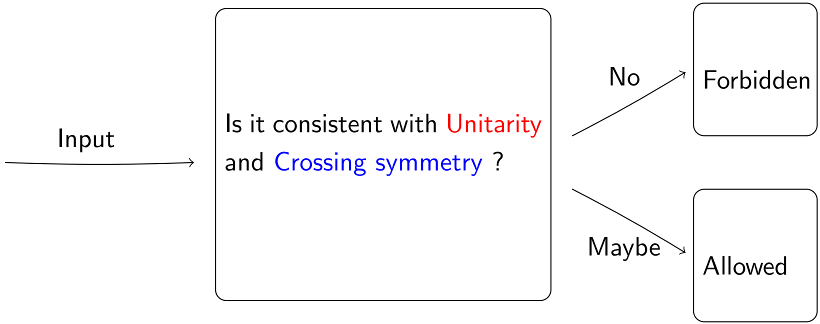

Applications of conformal bootstrap program

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Bootstrapping various dimensions

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- ⋆ (Today) UV fixed point of $D = 5$ $O(N)$ class.

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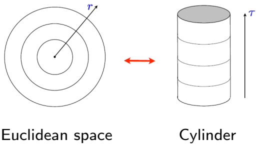
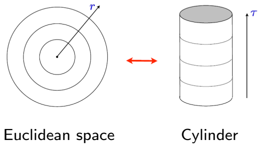


Radial Quantization in CFT

D-dimensional Flat space



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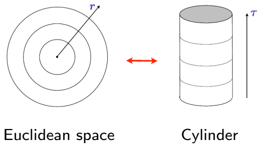

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$$d\tau^2 + d\Omega^2 = ds^2_{cyl}$$

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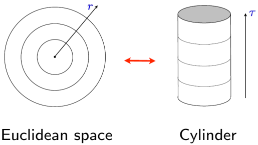

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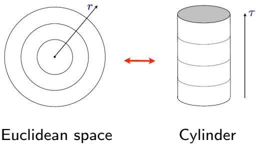

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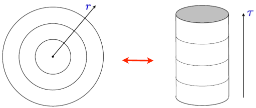

Euclidean space　　Cylinder

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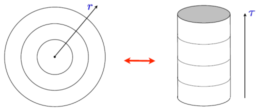

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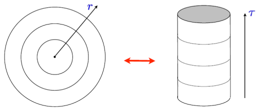
→ Every quantum states in conformal field theory are labeled by dilatation eigenvalue Δ and spin l.

⋆ P^μ acts as *raising operator* while K^μ acts as *lowering operator*.

$$D(P^\mu|\Omega\rangle) = (\Delta+1)|\Omega\rangle, \quad D(K^\mu(|\Omega\rangle) = (\Delta-1)|\Omega\rangle$$

Radial Quantization in CFT



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⋆ Time translation along τ direction replaced by dilatation along r direction in flat space.

→ Every quantum states in conformal field theory are labeled by dilatation eigenvalue Δ and spin l.

⋆ P^μ acts as *raising operator* while K^μ acts as *lowering operator*.

$$D(P^\mu|\Omega\rangle) = (\Delta+1)|\Omega\rangle, \quad D(K^\mu(|\Omega\rangle) = (\Delta-1)|\Omega\rangle$$

⋆ The *primary state* $|\Omega^P\rangle$ is defined by $K^\mu(0)|\Omega^P\rangle = 0$.

Correlation function in CFT

⋆ Conformal symmetry fixes structure of the correlation functions.

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$$\phi(x)\times\phi(y) = \sum_{\mathcal{O}} f_{\phi\phi\mathcal{O}}P(x-y,\partial_y)\mathcal{O}(y)$$

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⋆ Applying 12/34 channel OPE twice,

$$\langle \overbrace{\phi(x_1)\phi(x_2)}\overbrace{\phi(x_3)\phi(x_4)} \rangle = \sum_{\mathcal{O}} (f_{\phi\phi\mathcal{O}})^2 \frac{g_{\Delta,l}(u,v)}{x_{12}^{\Delta_\phi} x_{34}^{\Delta_\phi}} = \frac{G(u,v)}{x_{12}^{\Delta_\phi} x_{34}^{\Delta_\phi}}$$

Function $g_{\Delta,l}(u,v)$ is called by *conformal block*.

Expression of conformal block

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⋆ Closed form of the even dimensional conformal block :

[Dolan, Osborn 00]

$$g_{\Delta,l}(z,\bar{z}) = \begin{cases} f_{\Delta+l}(z)f_{\Delta-l}(\bar{z}) + (z \leftrightarrow \bar{z}) & D=2 \\ \frac{z\bar{z}}{z-\bar{z}}[k_{\Delta+l}(z)k_{\Delta-k-2}(\bar{z}) - (z \leftrightarrow \bar{z})] & D=4 \\ k_{\beta}(x) \equiv x^{\frac{\beta}{2}}_{2} F_1(\frac{\beta}{2},\frac{\beta}{2};\beta;x) & \end{cases}$$

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⋆ This closed form is solution of conformal Casimir equation.

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$$\frac{1}{2}M^{AB}M_{BA}\mathcal{O}_\Delta^{(l)} = \frac{1}{2}c_{\Delta,l}\mathcal{O}_\Delta^{(l)}, \quad c_{\Delta,l} = l(l+D-2) + \Delta(\Delta-D)$$

$$\longrightarrow \mathcal{D}_{z,\bar{z}}G_{\Delta,l} = \frac{1}{2}c_{\Delta,l}G_{\Delta,l}, \quad G_{\Delta,l} \sim u^{\frac{1}{2}(\Delta-l)}(1+\mathcal{O}(u,1-v))$$

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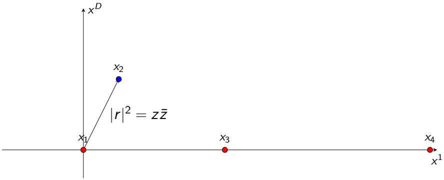
⋆ For odd dimension, no closed form expression is know. Alternatively, numerical value of conformal block is available via recursion relation.

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi 12], [Kos, Poland, Simmons-Duffin 13]

Dynamical Variables

Dynamical Variables

⋆ Conformal symmetry fix the insertion points x_1, x_3, x_4.



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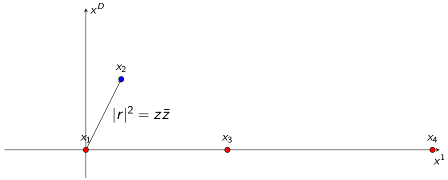

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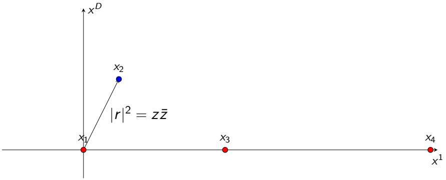

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⋆ Conformal block is two-variable function for *arbitrary spacetime dimension except* $D = 1$. Therefore, higher dimensional extension is very straightforward.

Bootstrap Constraint

Bootstrap Constraint

⋆ Unitarity : $\Delta \geq \frac{D}{2} - 1$ for spin 0 field,

$\Delta \geq D + l - 2$ for spin l field. [Minwalla 97]

$f^2_{\phi\phi\mathcal{O}} > 0.$

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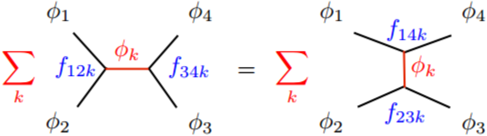
⋆ Crossing symmetry of the 4-point correlation function :



$$\sum_{\mathcal{O}} (f_{\phi\phi\mathcal{O}})^2 \frac{g_{\Delta,l}(u,v)}{x_{12}^{\Delta_\phi} x_{34}^{\Delta_\phi}} \qquad \sum_{\mathcal{O}} (f_{\phi\phi\mathcal{O}})^2 \frac{g_{\Delta,l}(v,u)}{x_{23}^{\Delta_\phi} x_{14}^{\Delta_\phi}}$$

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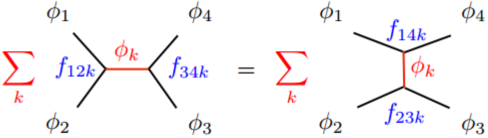

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⋆ The *Sum Rule* from Unitarity and Crossing symmetry constraint :

[Rattazzi, Rychkov, Tonni, Vichi 08]

$$u^\Delta - v^\Delta = \sum_{\substack{\Delta, l \\ \Delta \geq \Delta_{unit}}} f^2_{\phi\phi\mathcal{O}} \left(v^\Delta g_{\Delta,l}(u,v) - u^\Delta g_{\Delta,l}(v,u) \right)$$

Bootstrap Constraint II

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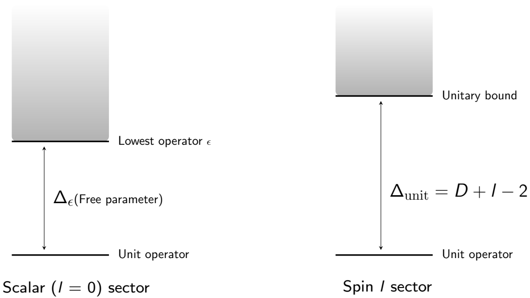
⋆ Intermediate states(in OPE) are consist of



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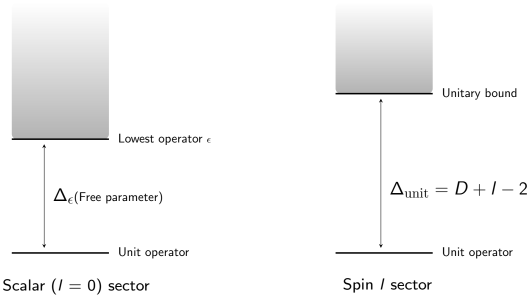

⋆ The final *Sum Rule* in bootstrap program :

$$u^\Delta - v^\Delta = \sum_{\substack{\Delta, l \\ \Delta \geq \Delta_\epsilon (l=0), \\ \Delta \geq \Delta_{unit} (l \neq 0)}} f^2_{\phi\phi\mathcal{O}} \left(v^\Delta g_{\Delta,l}(u,v) - u^\Delta g_{\Delta,l}(v,u) \right)$$

Solving Constraints

Solving Constraints

⋆ The Full Bootstrap Constraint : *Sum Rule*

$$\underbrace{u^{\Delta} - v^{\Delta}}_{\mathcal{F}_0(u,v)} = \sum_{\eth(\Delta,l)} \underbrace{f^2_{\phi\phi\mathcal{O}}}_{>0} \underbrace{\left(v^{\Delta} g_{\Delta,l}(u,v) - u^{\Delta} g_{\Delta,l}(v,u)\right)}_{\mathcal{F}(u,v)}$$

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⋆ The Full Bootstrap Constraint : *Sum Rule* $\bigoplus$ it's *Descendants*

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$$\underbrace{\partial_z^m \partial_{\bar{z}}^n \mathcal{F}_0(z,\bar{z})}_{\mathcal{F}_0^{n,m}(z,\bar{z})} = \sum_{\eth(\Delta,l)} \underbrace{f^2_{\phi\phi\mathcal{O}}}_{>0} \underbrace{\partial_z^m \partial_{\bar{z}}^n \mathcal{F}(z,\bar{z})}_{\mathcal{F}^{n,m}(z,\bar{z})}$$

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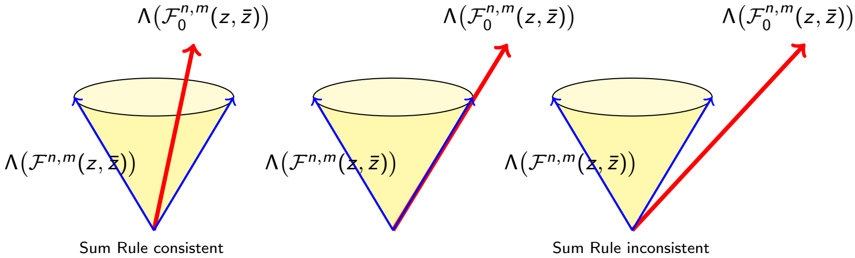
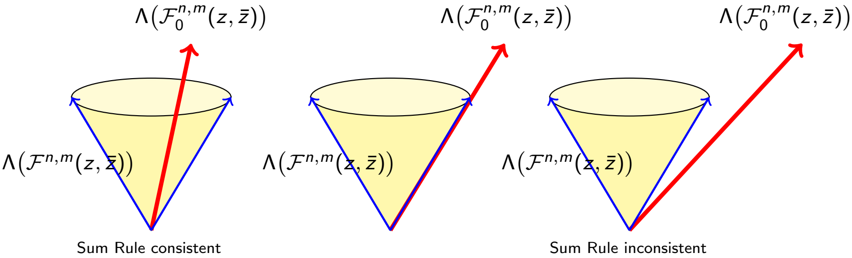


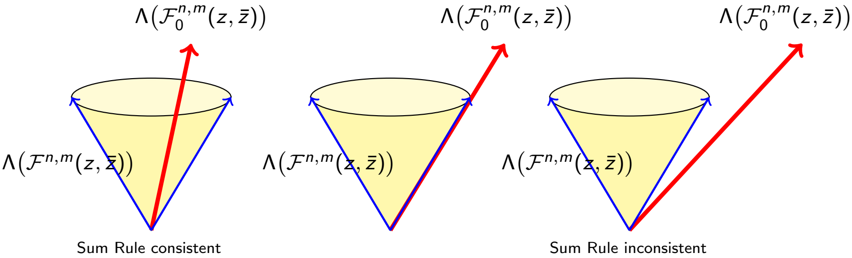
Solving Constraints



$\star$ RHS of sum rule : Linear combination (with positive coefficient $f^2_{\phi\phi\mathcal{O}}$) of $\Lambda\big(\mathcal{F}^{n,m}(z,\bar{z})\big)$ forms cone.

$\star$ LHS of sum rule : Red vector.

Solving Constraints



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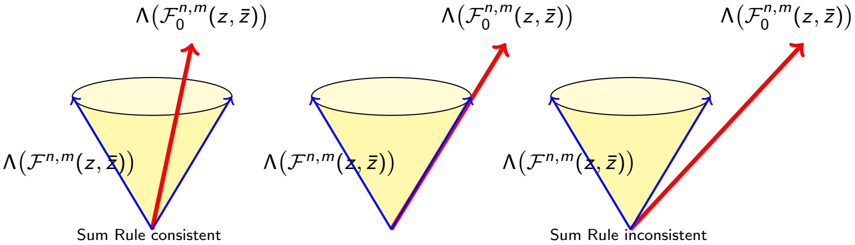
$\star$ LHS of sum rule : Red vector. [Rattazzi, Rychkov, Tonni, Vichi 08]

Linear Programming

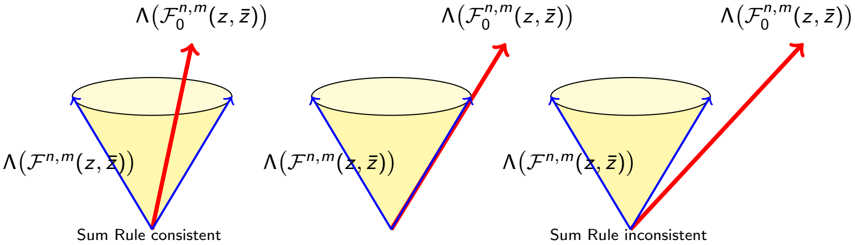
Find a vector $\vec{c} = (c_1, c_2, \cdots, c_n)$ which minimize $\vec{c}^T \cdot \vec{b} = c_1 b_1 + c_2 b_2 + \cdots + c_n b_n$ subject to constraints $\vec{b}^T \cdot A > \vec{a}$.

Solving Constraints

Solving Constraints

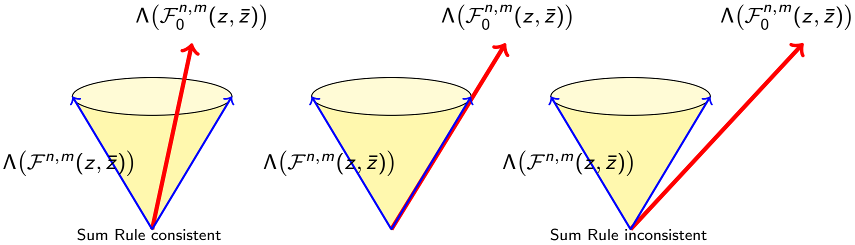


Solving Constraints



- If the red vector always inside the cone regardless of $c_{m,n}$, that conformal field theory is *consistent* with bootstrap constraints.

Solving Constraints



• If the red vector always inside the cone regardless of $c_{m,n}$, that conformal field theory is *consistent* with bootstrap constraints.

• If there is a set $c_{m,n}$ which makes red vector outside the cone, that conformal field theory is *inconsistent* with bootstrap constraints.

Conformal Bootstrap in a nutshell

Conformal Bootstrap in a nutshell

Basic Strategy

Conformal Bootstrap in a nutshell

Basic Strategy

Inputs : Δ_ϕ, Δ_ϵ

$\longrightarrow$

Conformal Bootstrap in a nutshell

Basic Strategy

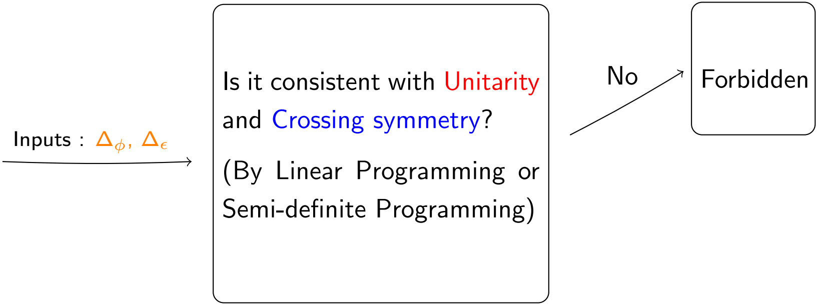
Inputs : $\Delta_\phi, \Delta_\epsilon$ →

Is it consistent with Unitarity and Crossing symmetry?

(By Linear Programming or Semi-definite Programming)

Conformal Bootstrap in a nutshell

Basic Strategy



Conformal Bootstrap in a nutshell

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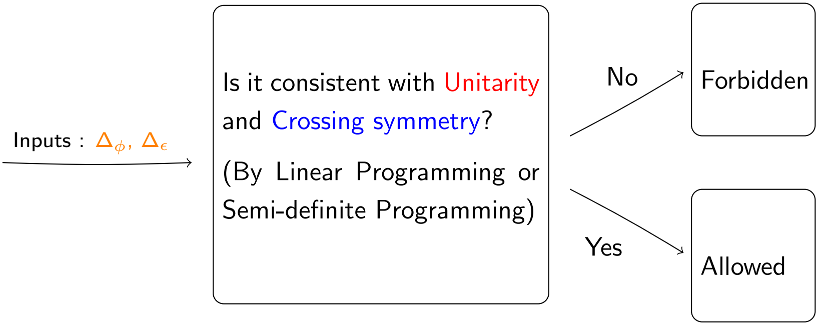

Bootstrapping 3D $\mathbb{Z}_2$ theory

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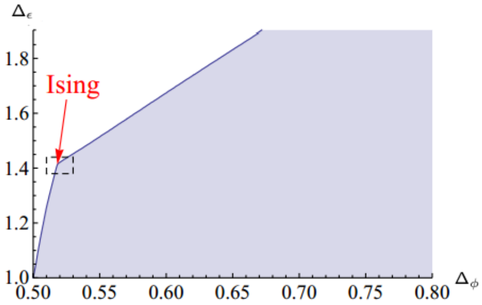
⋆ Bootstrapping 3D $\mathbb{Z}_2$ theory shows

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi 12]



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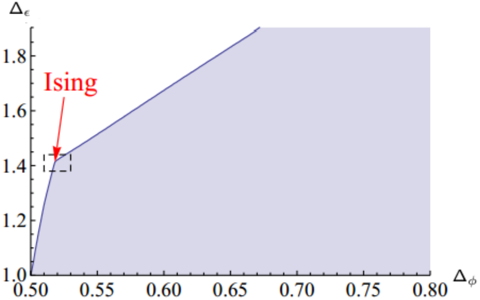

$\star$ The boundary has kink, whose location is $(\Delta_\phi, \Delta_\epsilon) = (0.518, 1.413)$. This numerical value indeed agrees to scaling dimension of relevant primary operators (ϕ, ϵ).

Sum Rule in $O(N)$ global symmetry

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⋆ Operator product of two primary scalar fields decomposed by

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⋆ Reflecting this structure, sum rule is promoted :

[Kos, Poland, Simmons-Duffin, 13]

$$\boxed{\sum_S c_{\Delta,l} V_{S,\Delta,l} + \sum_T c_{\Delta,l} V_{T,\Delta,l} + \sum_A c_{\Delta,l} V_{A,\Delta,l} = 0}$$

where

$$V_{S,\Delta,l} = \begin{pmatrix} 0 \\ \mathcal{F}^-_{\Delta,l}(u,v) \\ \mathcal{F}^+_{\Delta,l}(u,v) \end{pmatrix}, \quad V_{T,\Delta,l} \begin{pmatrix} \mathcal{F}^-_{\Delta,l}(u,v) \\ (1-\frac{2}{N})\mathcal{F}^-_{\Delta,l}(u,v) \\ -(1+\frac{2}{N})\mathcal{F}^+_{\Delta,l}(u,v) \end{pmatrix}, \quad V_{A,\Delta,l} = \begin{pmatrix} -\mathcal{F}^-_{\Delta,l}(u,v) \\ \mathcal{F}^-_{\Delta,l}(u,v) \\ -\mathcal{F}^+_{\Delta,l}(u,v) \end{pmatrix}$$

$$\mathcal{F}^{\pm} \equiv v^{\Delta} G_{\Delta,l}(u,v) \pm u^{\Delta} G_{\Delta,l}(v,u)$$

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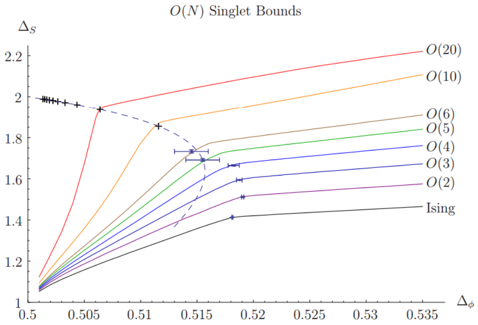

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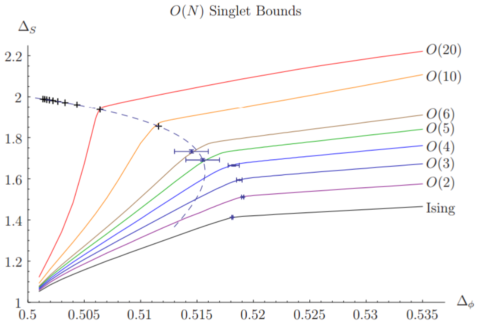

⋆ The boundary shows kink again, whose location agrees to the critical exponents η, ν.

Bootstrapping 3D $O(N)$ theory, Mixed correlator

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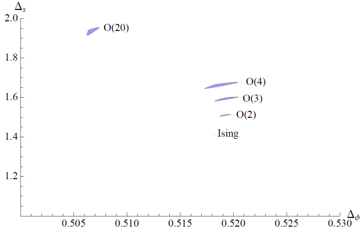
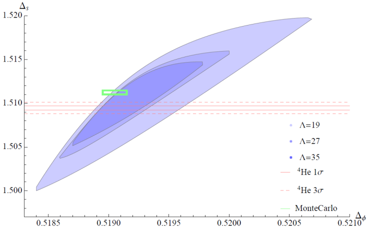
⋆ Bootstrapping 3-dimensional $O(N)$ theory with the mixed correator $\langle \phi_i \phi_j \phi_k \phi_l \rangle, \langle \phi_i \phi_j ss \rangle, \langle ssss \rangle$ shows [Kos, Poland, Simmons-Duffin, Vichi, 15]



$O(N)$ bootstrap with mixed correlator



$O(N = 2)$ bootstrap with mixed correlator(Zoomed in)

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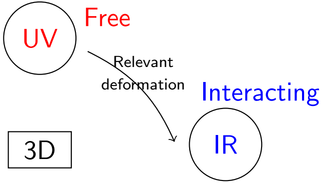

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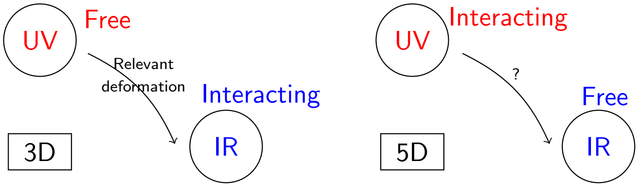

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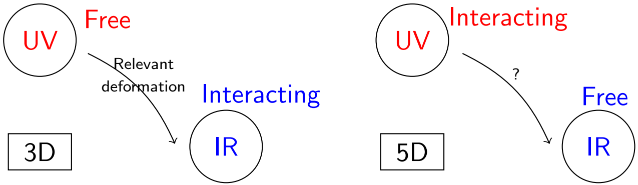

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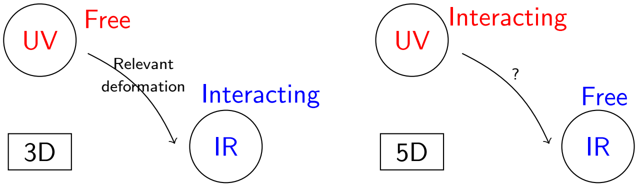

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⋆ Mean Field Theory expect only free theory at above upper critical dimension($D_c = 4$).

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⋆ The bulk theory of $O(N)$ symmetric theory conjectured to be higher-spin theory in AdS_{D+1}. [Klebanov, Polyakov 02]

$$\Big\langle exp\Big(\int d^D x \; h_0^{(\mu_1\mu_2\cdots\mu_s)} \underbrace{\overset{\mathrm{Tr}[\varphi\partial_{\mu_1}\partial_{\mu_2}\cdots\partial_{\mu_s}\varphi]}{J_{(\mu_1\mu_2\cdots\mu_s)}}}_{\text{O(N) singlet conserved current}} \Big)\Big\rangle = e^{W[h_0]}$$

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⋆ We have two solutions $\Delta_+ = D-2$ and $\Delta_- = 2$, which indicates UV/IR fixed point of D-dimensional $O(N)$ theory. This is consistent with unitary bound as far as $2 < D < 6$.

5-dimensional UV fixed point at large N

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⋆ Hubbard-Stratonovich transformation is applied for large-N limit.

$$S = \int d^D x \Big(\frac{1}{2} (\partial \phi^i)^2 + \frac{1}{2} \sigma \phi^i \phi^i - \frac{\sigma^2}{4\lambda} \Big)$$

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This computation is valid for arbitrary D. Among them, not only $2 < D < 4$, but also $4 < D < 6$, anomalous dimension is positive. (Consistence with unitary bound.)

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$\star$ Likewise 3-dimensional IR fixed point, can we see appearance of UV fixed point via conformal bootstrap?

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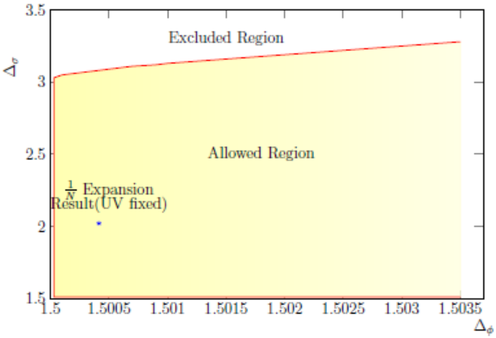
⋆ The straightforward application of numerical conformal bootstrap into 5-dimensional $O(N = 500)$ theory gives



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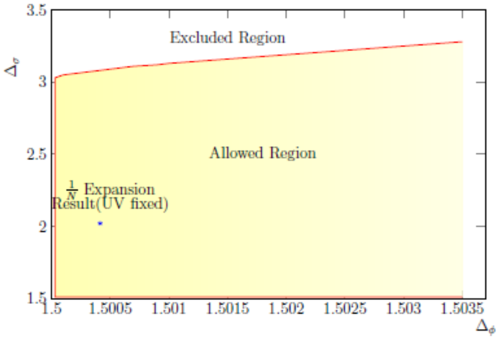

⋆ This does not specify UV fixed point on its boundary as 3-dimensional bootstrap does.

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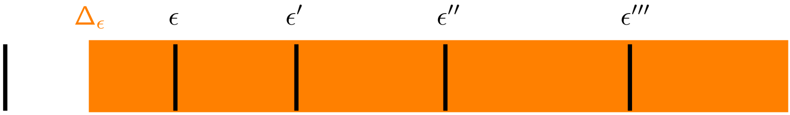

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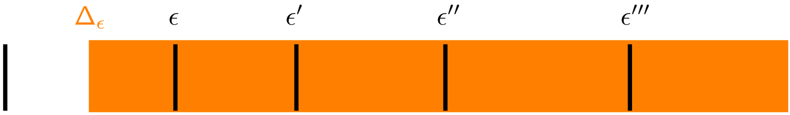

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Basic Strategy

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Three inputs

Δ_ϕ, Δ_{min}, Δ_{gap}

$$\longrightarrow$$

Fixed Δ_{gap}

Conformal Bootstrap in nutshell

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→

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Is it consistent with Unitarity and Crossing symmetry ?

(By Linear Programming or Semi-definite Programming)

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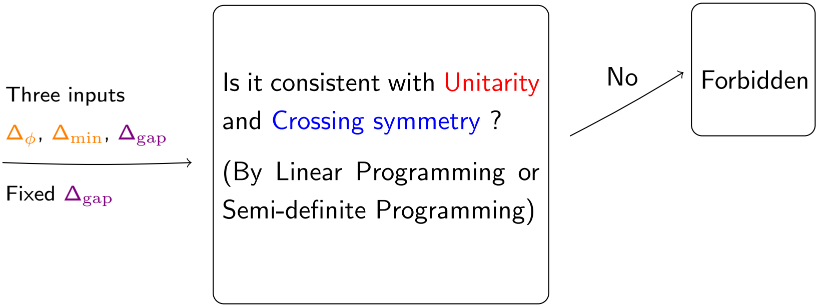

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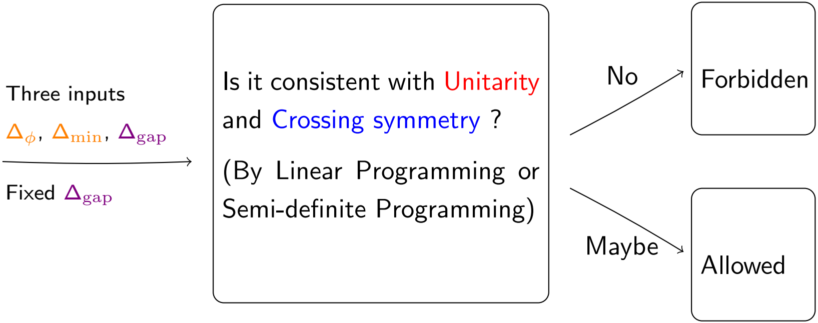

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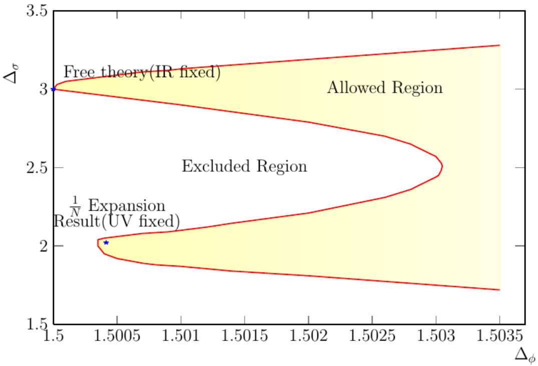

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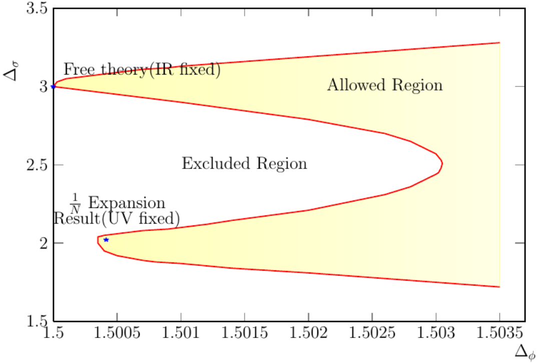

⋆ The location of low tip agrees to the result of $\frac{1}{N}$ expansion via Hubbard-Stratonovich transformation.

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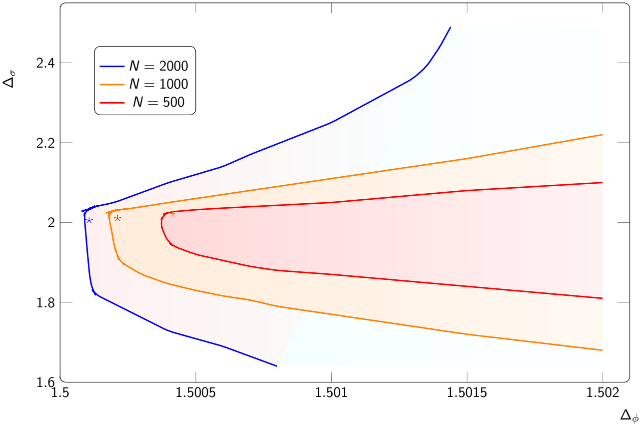

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⋆ Mixed Bootstrap Program on 5-dimensional $O(N)$?