

# Probing cosmic string properties through Pulsar Timing Array

Use of **anisotropy** of the gravitational wave background

Sachiko Kuroyanagi  
(Nagoya U.)

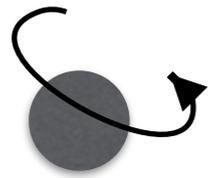
in collaboration with  
K. Takahashi, H. Kumamoto, N. Yonemaru

2015/10/28@IPMU

# Pulsar timing array

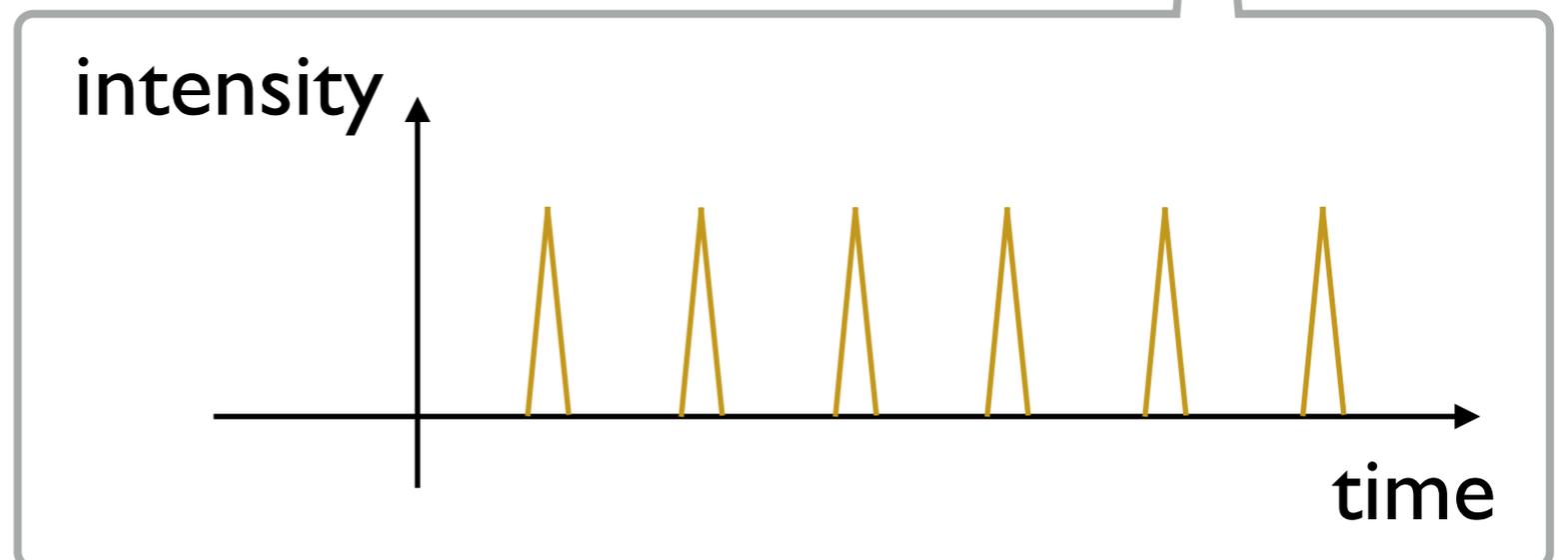
- Robust and unique test of gravitational waves

millisecond pulsar



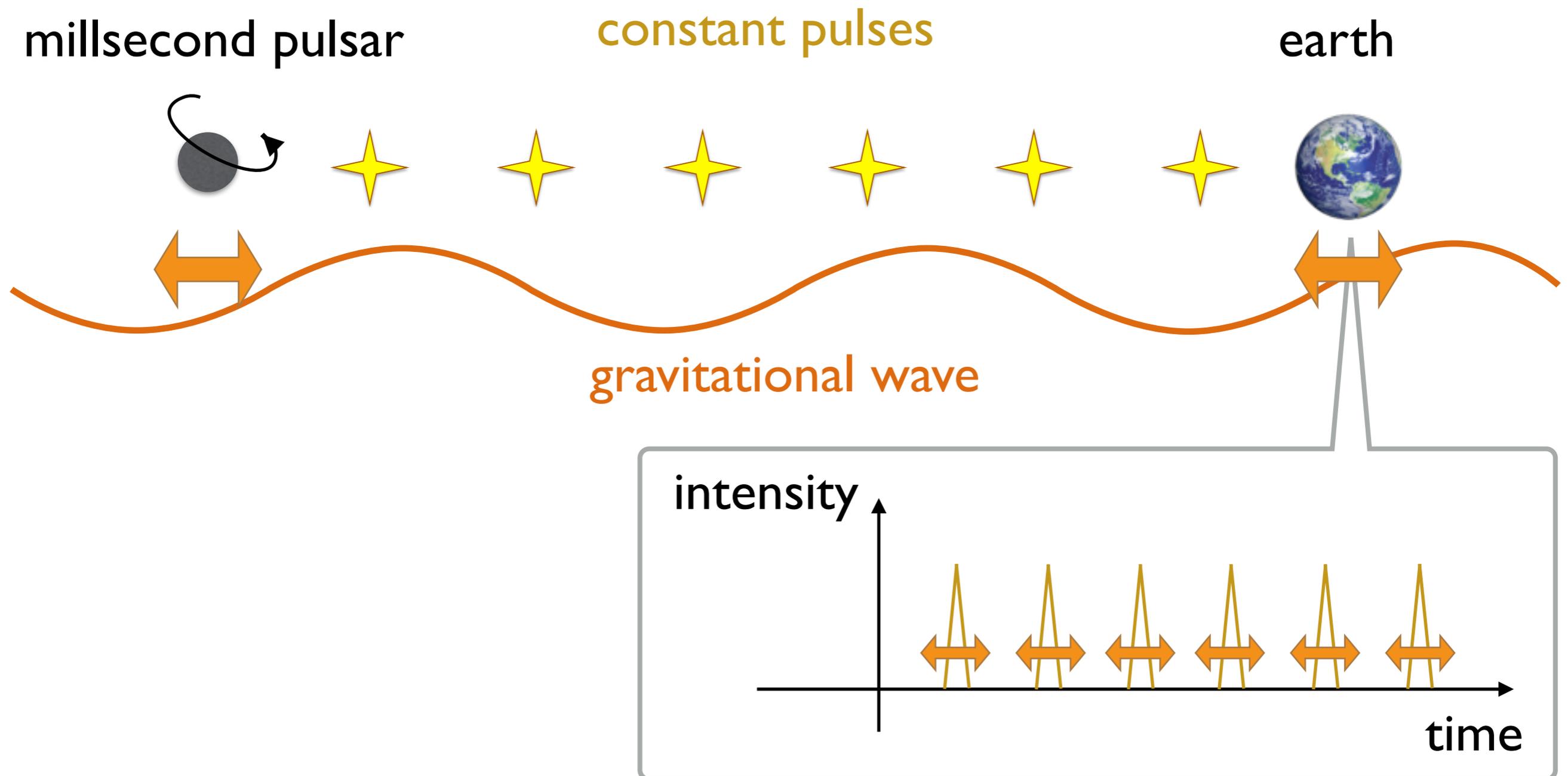
constant pulses

earth

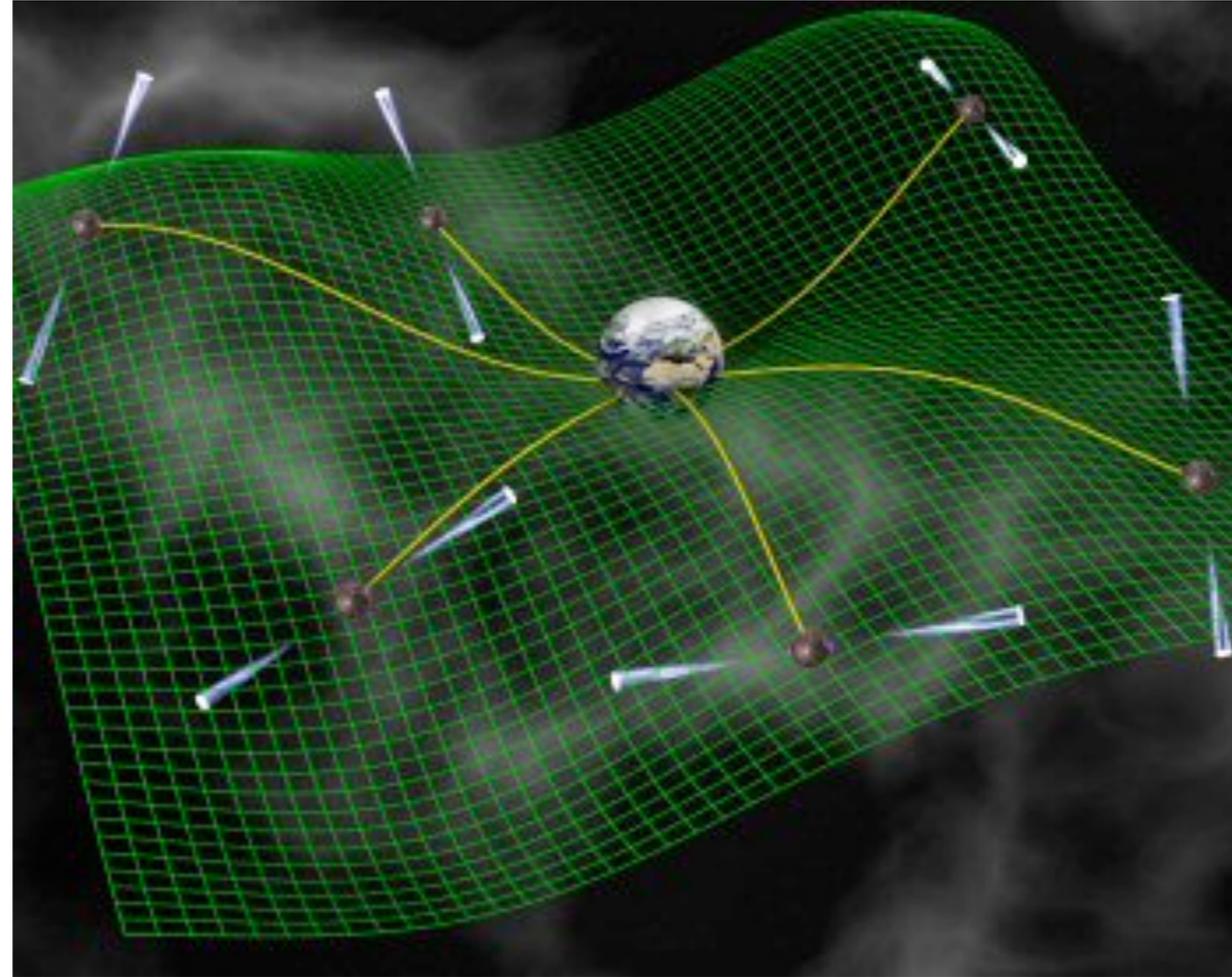


# Pulsar timing array

- Robust and unique test of gravitational waves

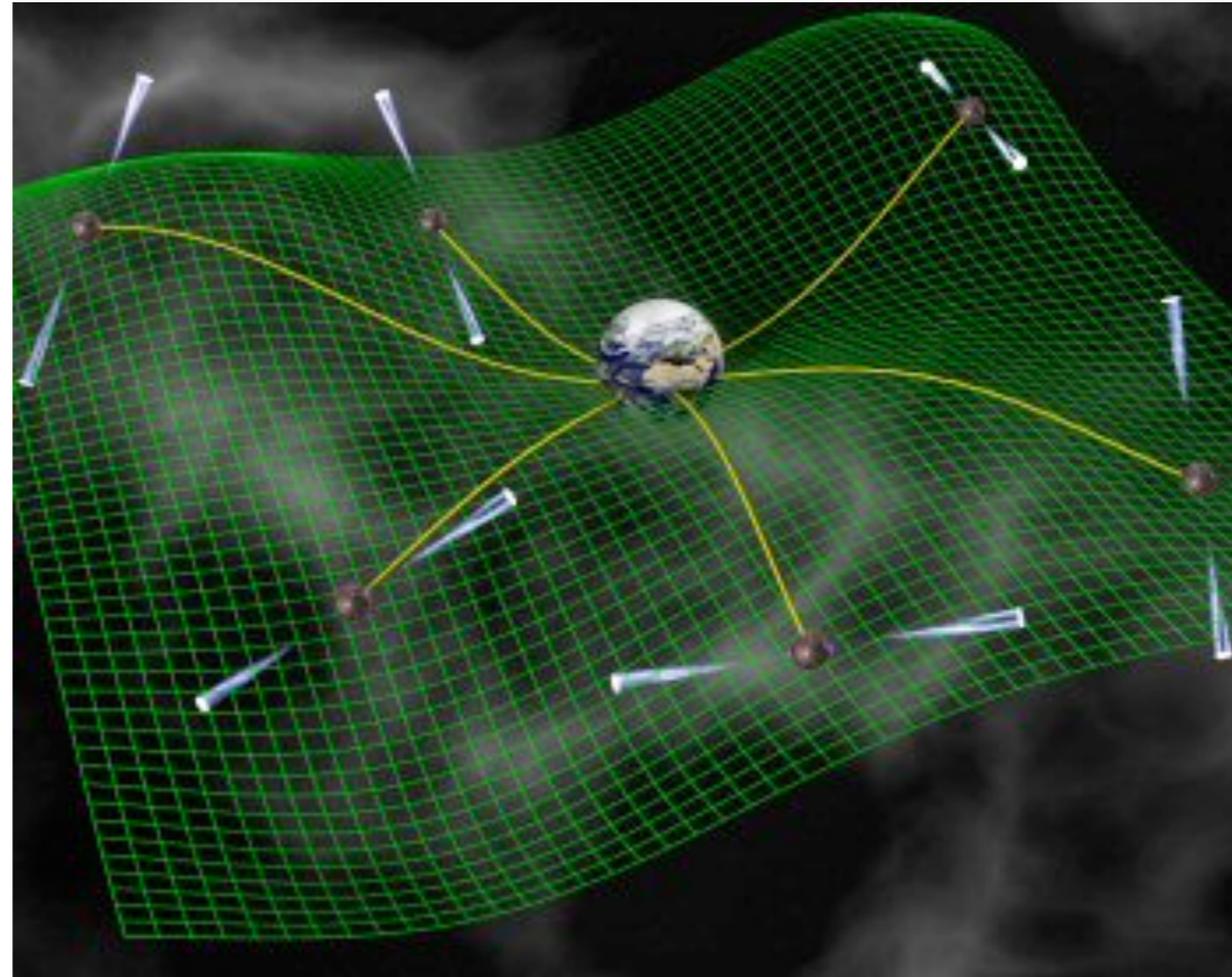


# Pulsar timing array



- Correlation analysis with multiple pulsars to remove noises associating with individual pulsars
- Frequency  $\propto$  (Observation time)<sup>-1</sup>

# Pulsar timing array



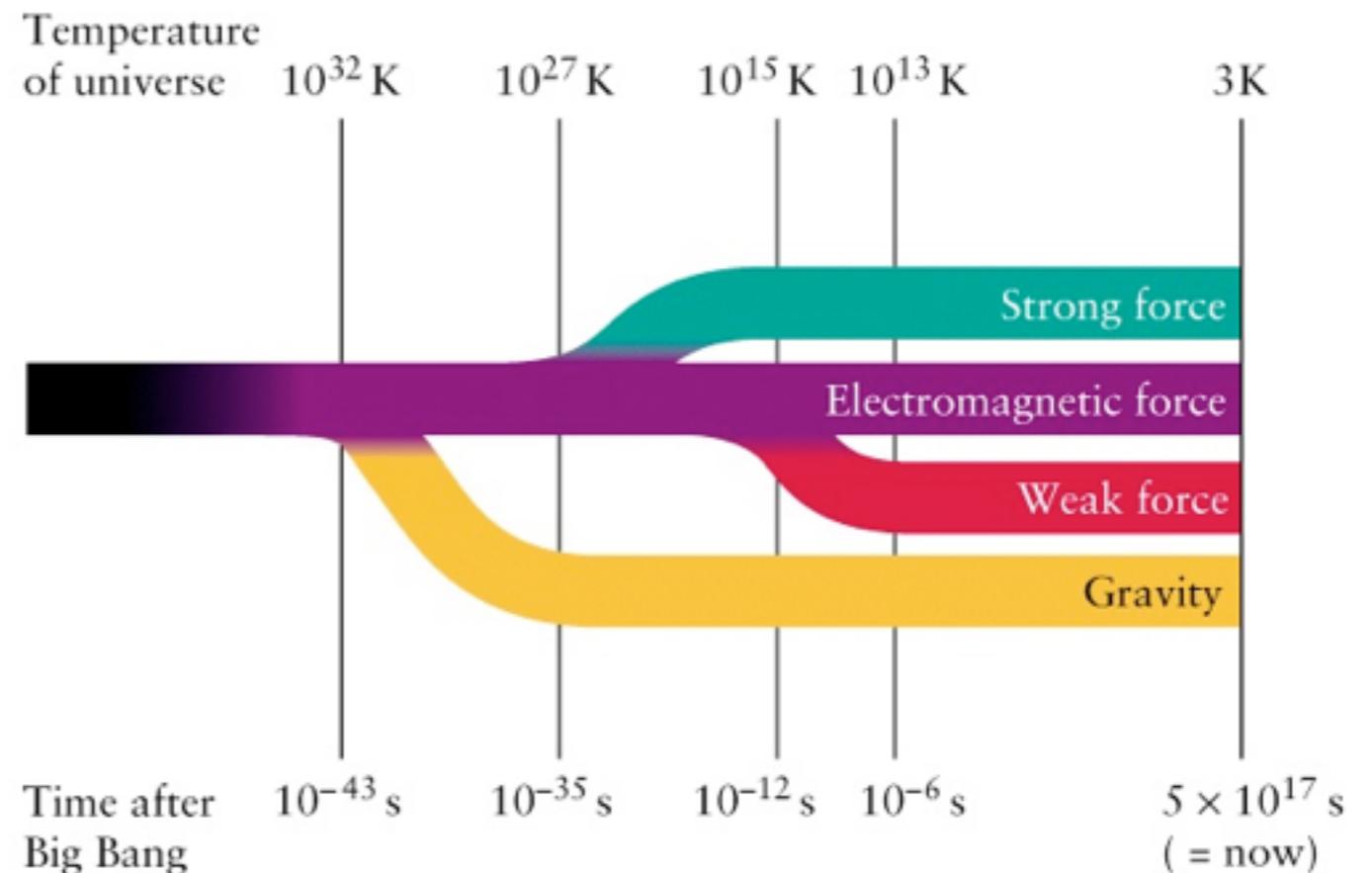
- expected to detect gravitational waves from super massive black hole binaries or maybe from exotic sources (e.g. **cosmic strings**)

# Cosmic string ?

One dimensional topological defect generated in the early universe

## Generation mechanism

1: Phase transition



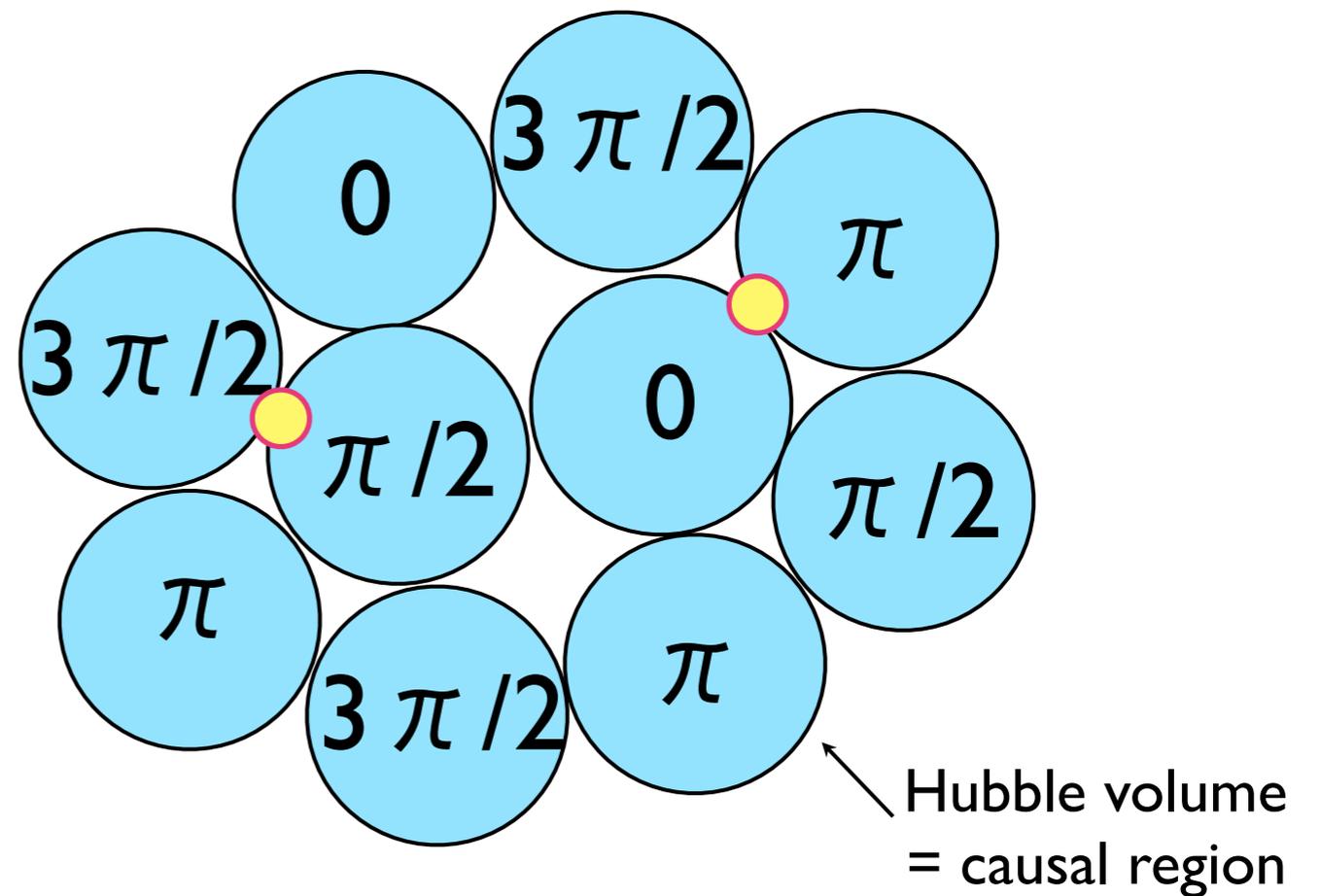
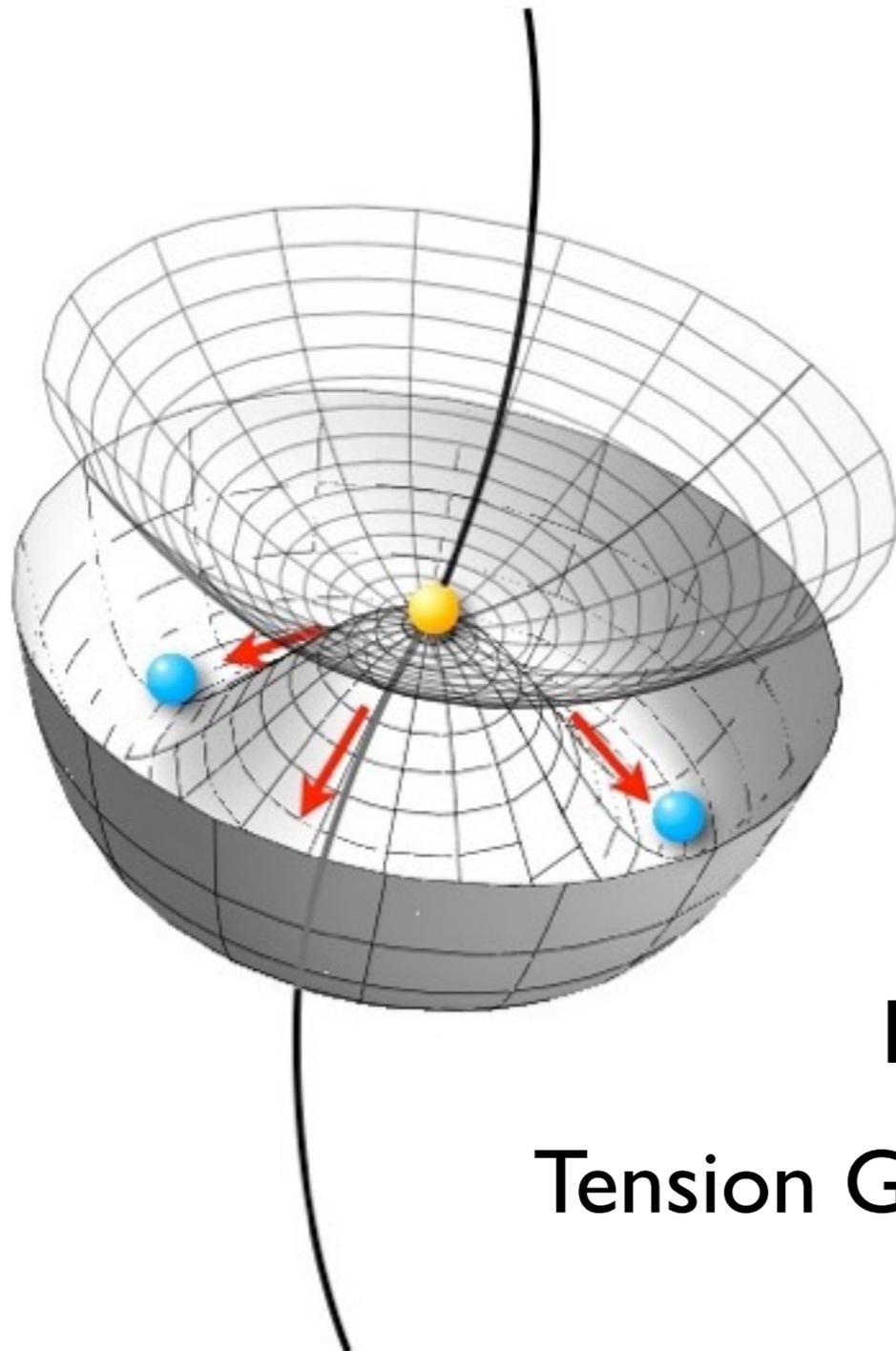
2: Cosmic superstrings

Cosmological size D-strings or F-strings remains after inflation

→ could provide some insight into fundamental physics

# Generation mechanism I: phase transition

The Universe has experienced symmetry breakings.

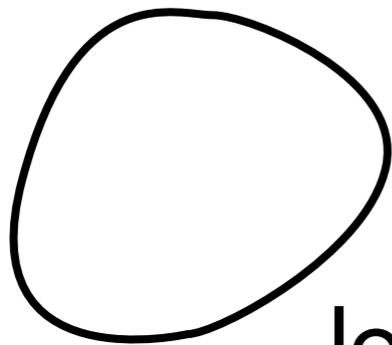
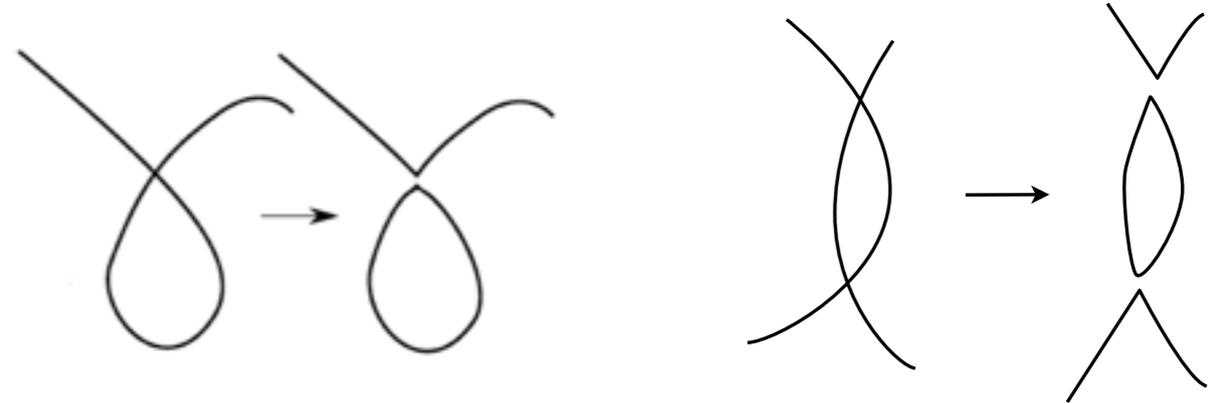


High energy vacuum remains at the center

Tension  $G\mu \sim$  potential energy of the true vacuum  
 $\sim$  the energy scale of the phase transition

# Cosmic string ?

infinite string becomes a loop by **reconnection**

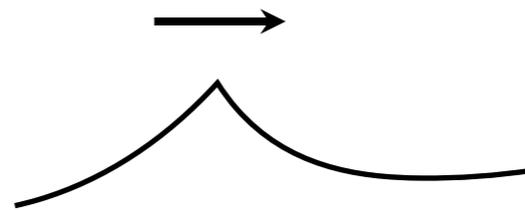


loop

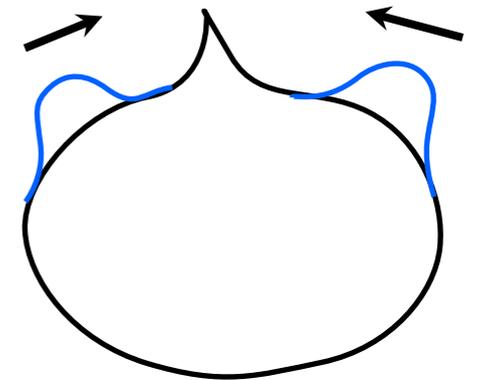
strings emit **gravitational waves**  
especially from singular structures

infinite  
string

kink



cusp



loops lose energy and shrink by emitting  
gravitational waves and eventually **evaporate**

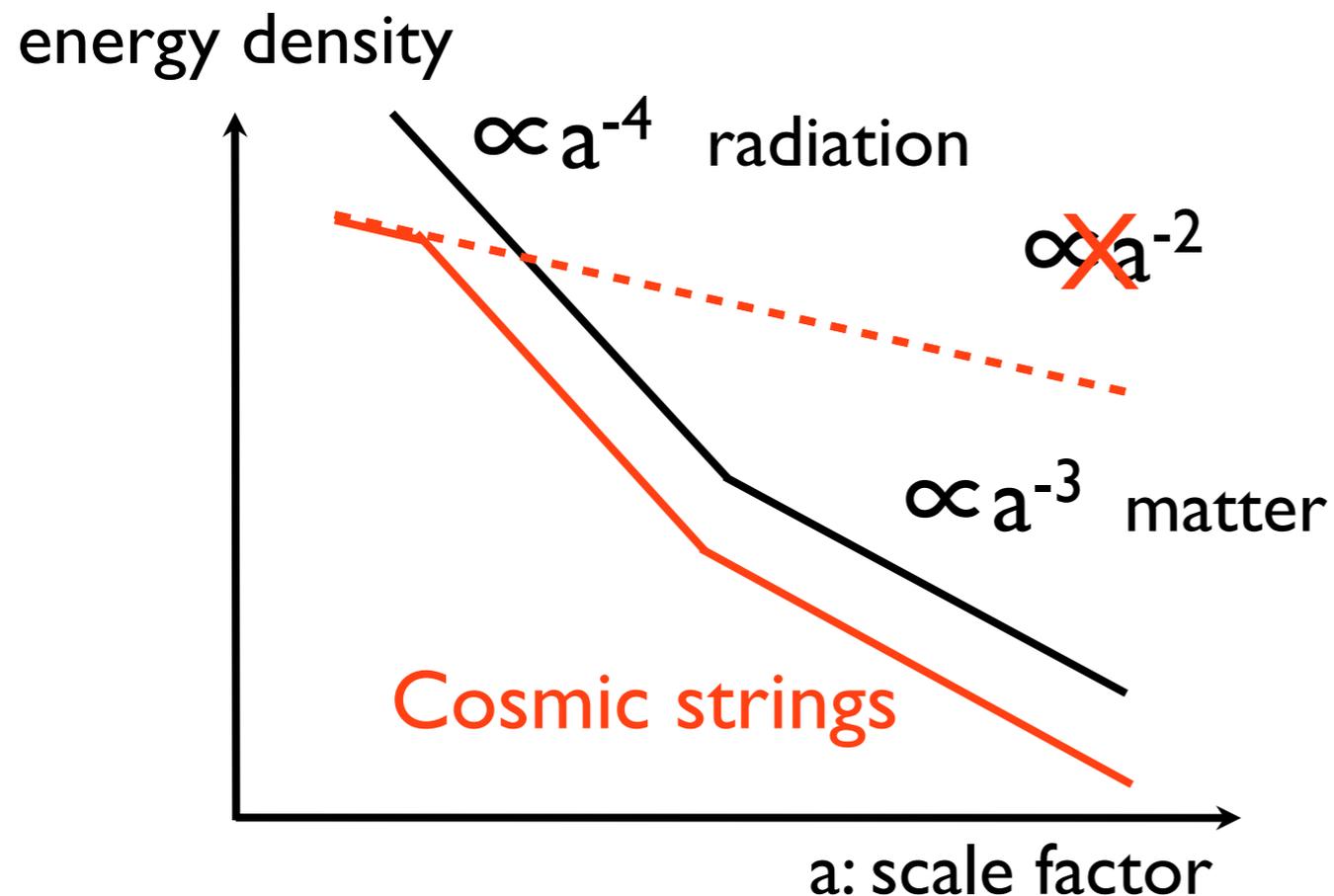
# Evolution of cosmic strings

The energy density of cosmic strings

$$\sim (\text{line density} \times \text{length})/\text{volume} \propto a^{-2} \quad \times$$

## Scaling law

The Universe always has  $O(1-10)$  strings per horizon



Increase of string length  
by the horizon growth

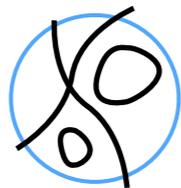
Higher reconnection rate  
more efficient generation of loops  
more energy release by GWs

Loss of infinite string length  
by generation of loops

# Cosmic string loops generate scale invariant GWs

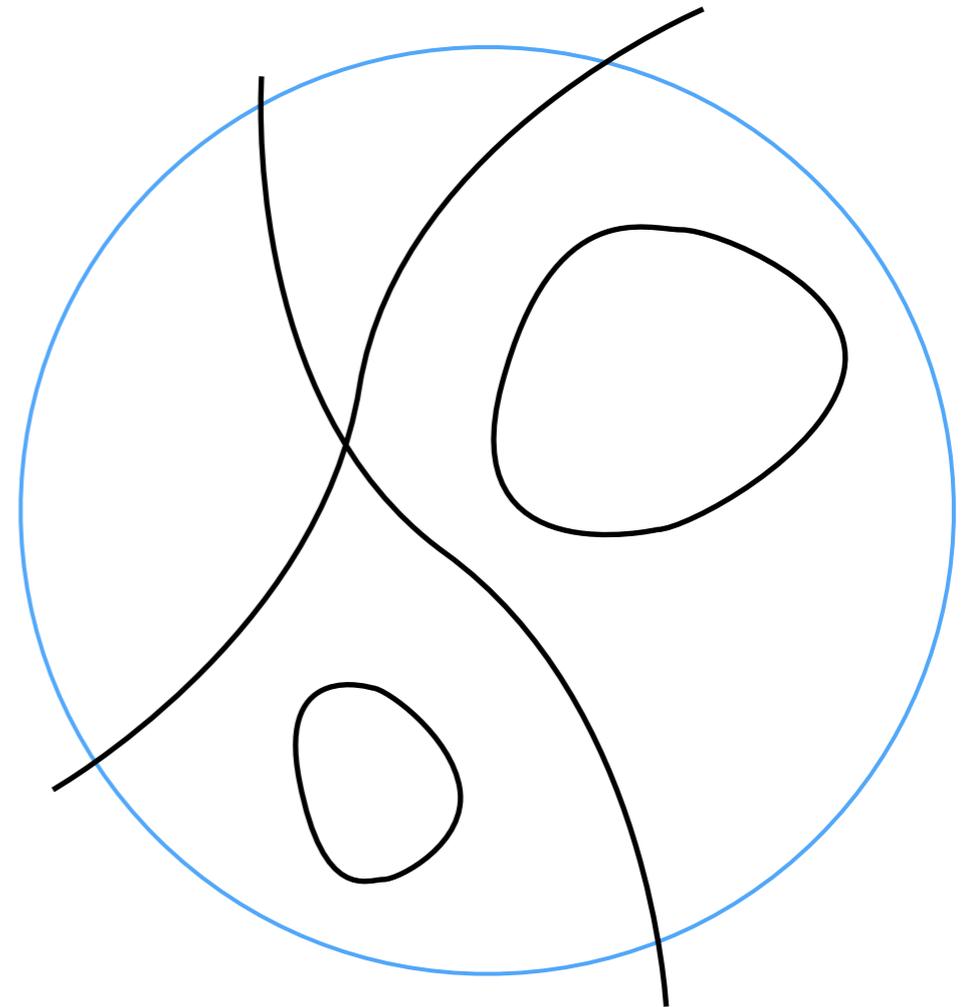
Early Universe

horizon



→ high frequency GWs

Late Universe



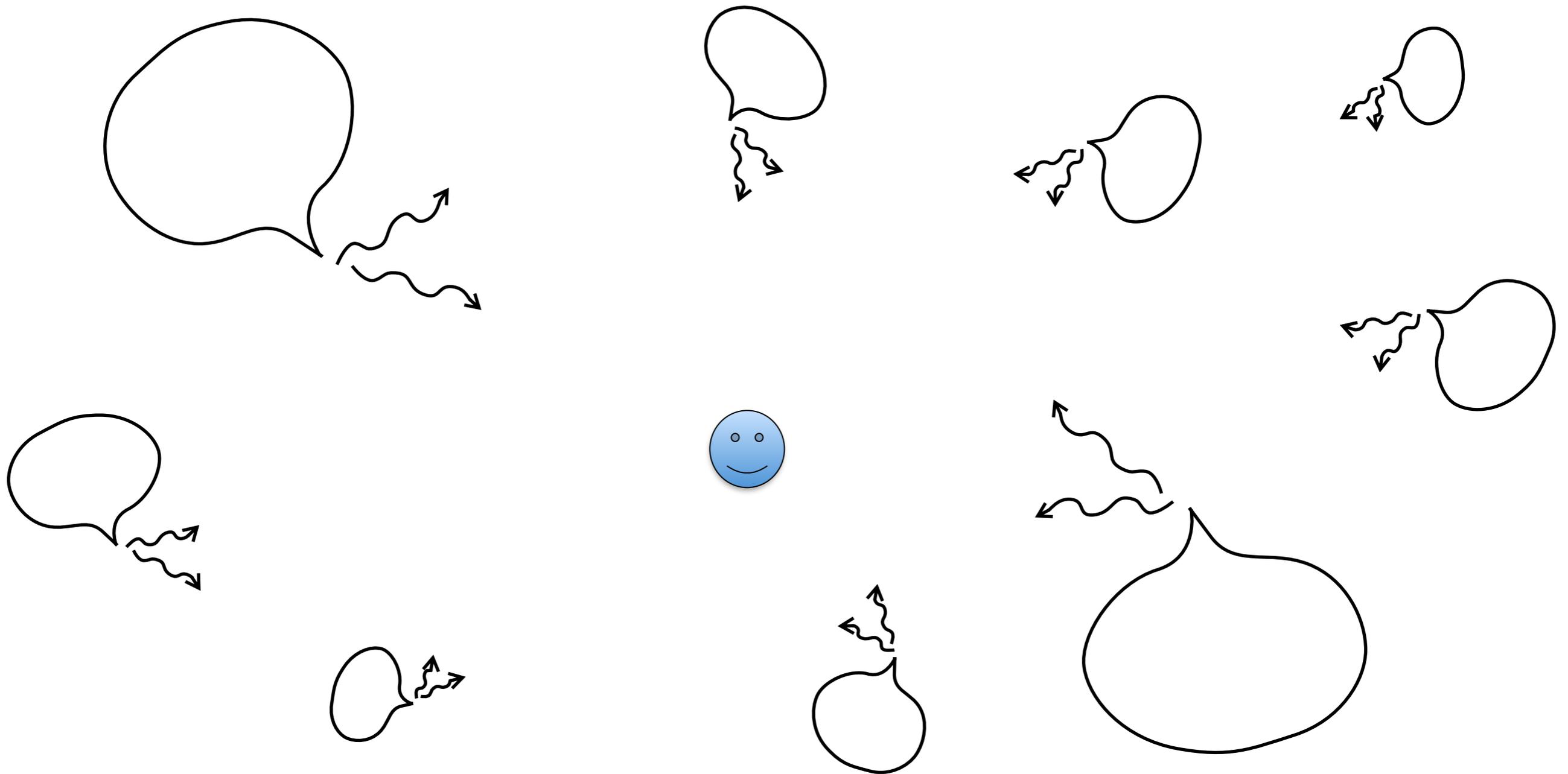
→ low frequency GWs

GW frequency  $\sim 1/L$

GW amplitude  $\Omega_{\text{GW}} \equiv \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln k} \sim \text{constant}$

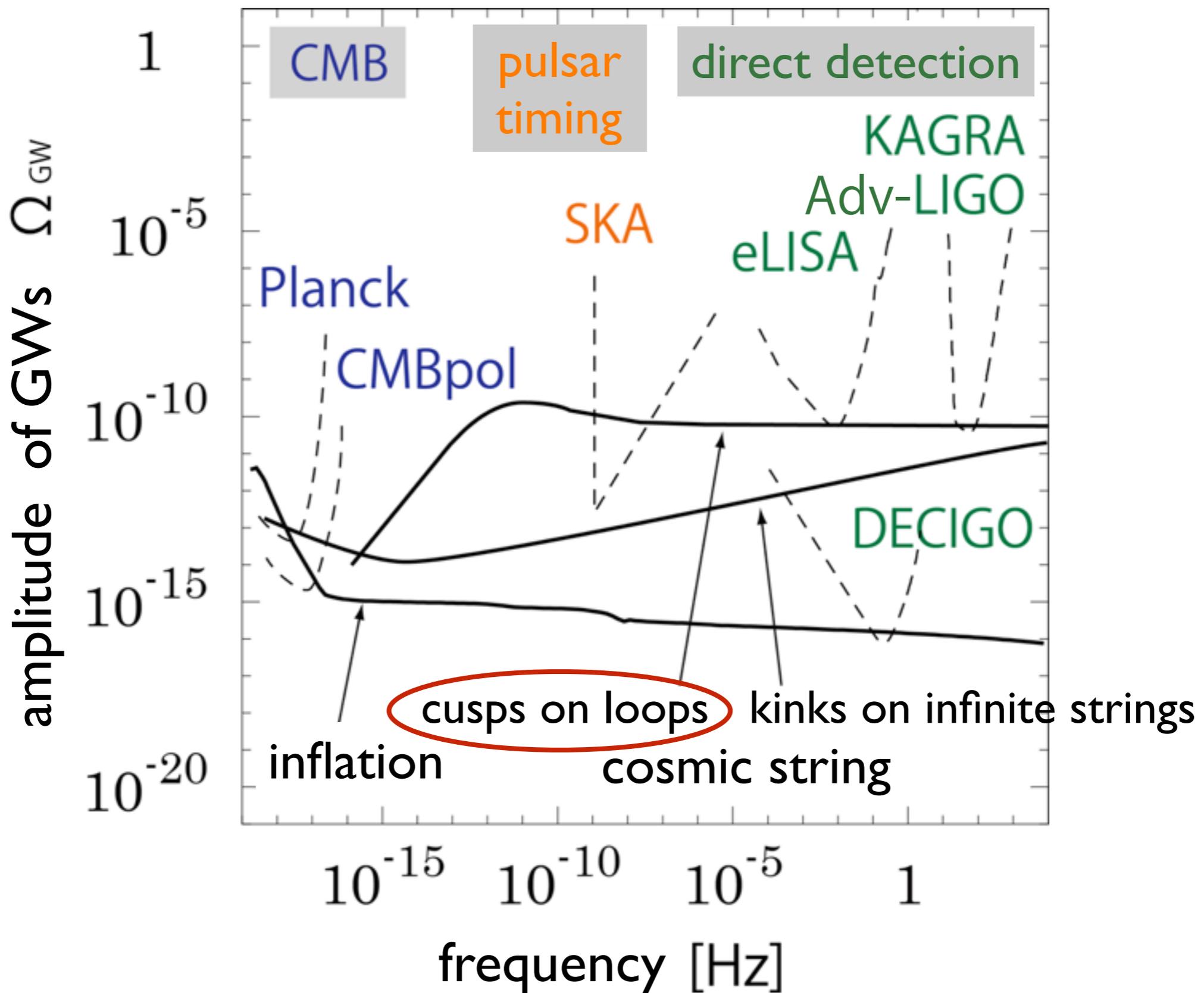
$\rho_c \propto a^{-4}$  for RD  
 $\rho_{\text{GW}} \propto a^{-4}$

# Gravitational waves from cosmic string loops



Gravitational waves coming from different directions overlap each other and form **gravitational wave background**

# Spectrum and Sensitivities

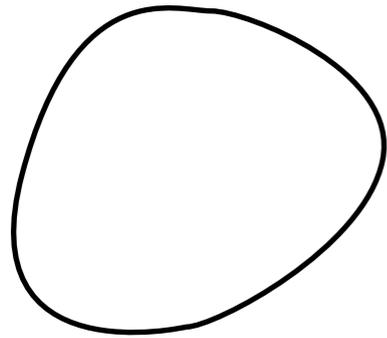


# What determines the GW amplitude?

3 main parameters to characterize cosmic string

- $G\mu$  : tension = line density  
Generation mechanism
- $\alpha$  : initial loop size  $L \sim \alpha H^{-1}$   
Network evolution
- $p$  : reconnection probability  
Phase transition origin:  $p=1$   
Cosmic superstring:  $p \ll 1$

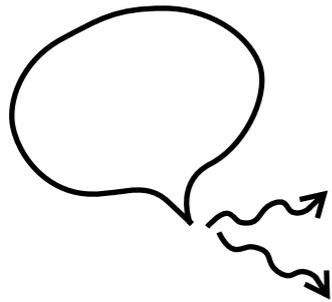
# Evolution of a loop



Initial loop length =  $\alpha t_i$

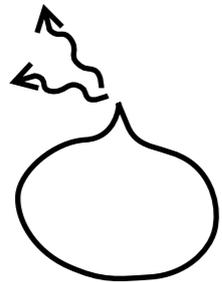
$t_i$  : time when the loop formed

**GW power**  $P = \Gamma G \mu^2$   $\Gamma$ : numerical constant  $\sim 50-100$



From the energy conservation law  
(energy of loop at time  $t = \mu l$ )

$$= (\text{initial energy of the loop} = \mu \alpha t_i) - (\text{energy released to GWs} = P \Delta t)$$



**Loop length at time  $t$**

$$l(t, t_i) = \alpha t_i - \Gamma G \mu (t - t_i)$$



**Lifetime of the loop** =  $\frac{(\text{initial loop energy})}{(\text{energy release rate per time})}$

$$= \frac{\mu \alpha t_i}{\Gamma G \mu^2} = \frac{\alpha t_i}{\Gamma G \mu}$$

0



# Amplitude of GW background

$\propto$  amplitude of single burst  $h \propto G\mu$

$\propto$  number density of loops  $n_l$

depends on **Lifetime of the loop**  $= \frac{\alpha t_i}{\Gamma G\mu}$

$$\alpha \ll \Gamma G\mu$$

Loops evaporate soon  
after the generation

$$n_l \propto \text{const}$$

$$\alpha \gg \Gamma G\mu$$

Loops remain for a while  
 $\rightarrow$  the string network  
consists of old and new loops

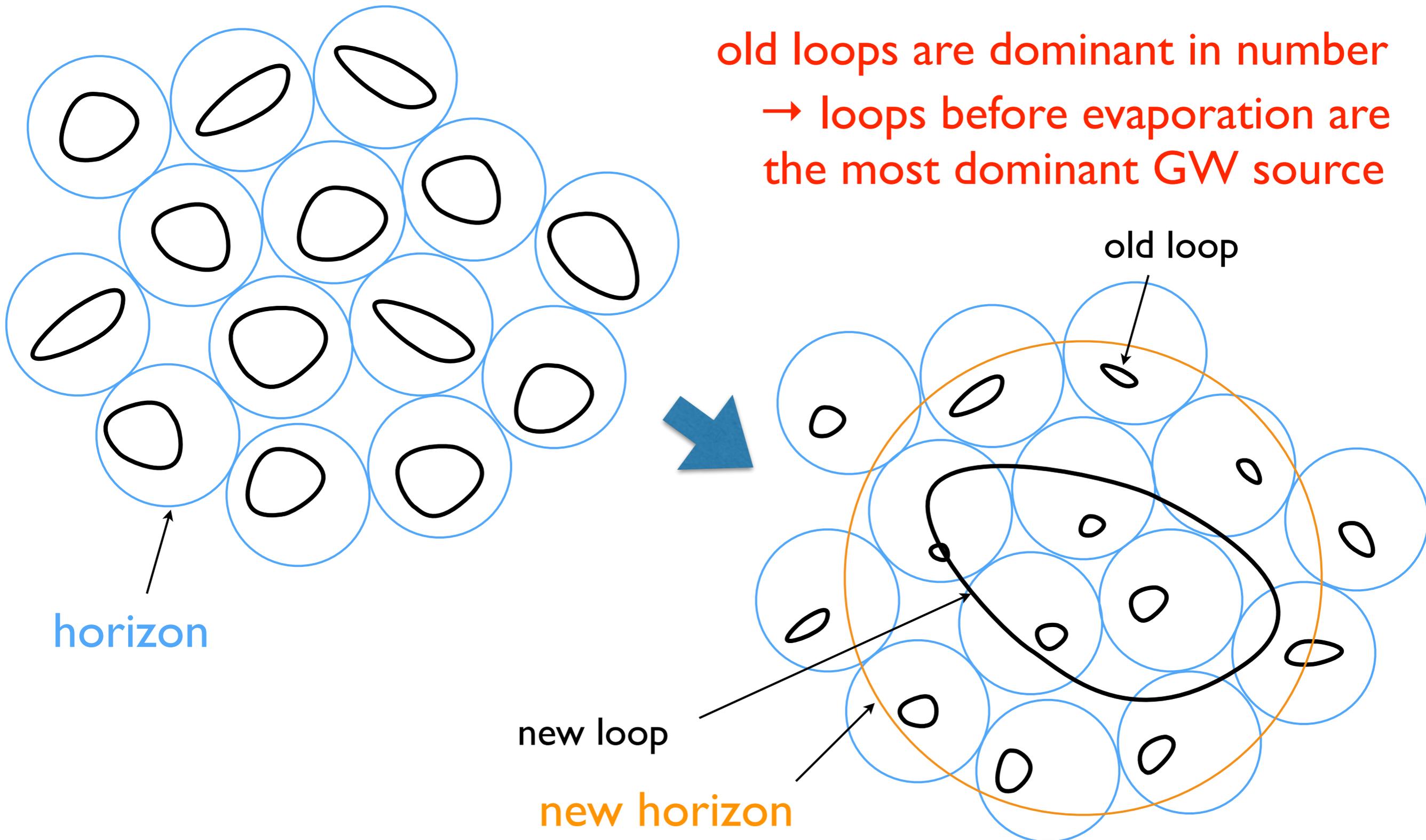
$$n_l \propto \alpha^{1/2}$$

# Number density of loops

$$\alpha \gg \Gamma G \mu$$

→ the network consists of old and new loops

old loops are dominant in number  
→ loops before evaporation are the most dominant GW source



# GW spectrum

$$\alpha \gg \Gamma G\mu$$

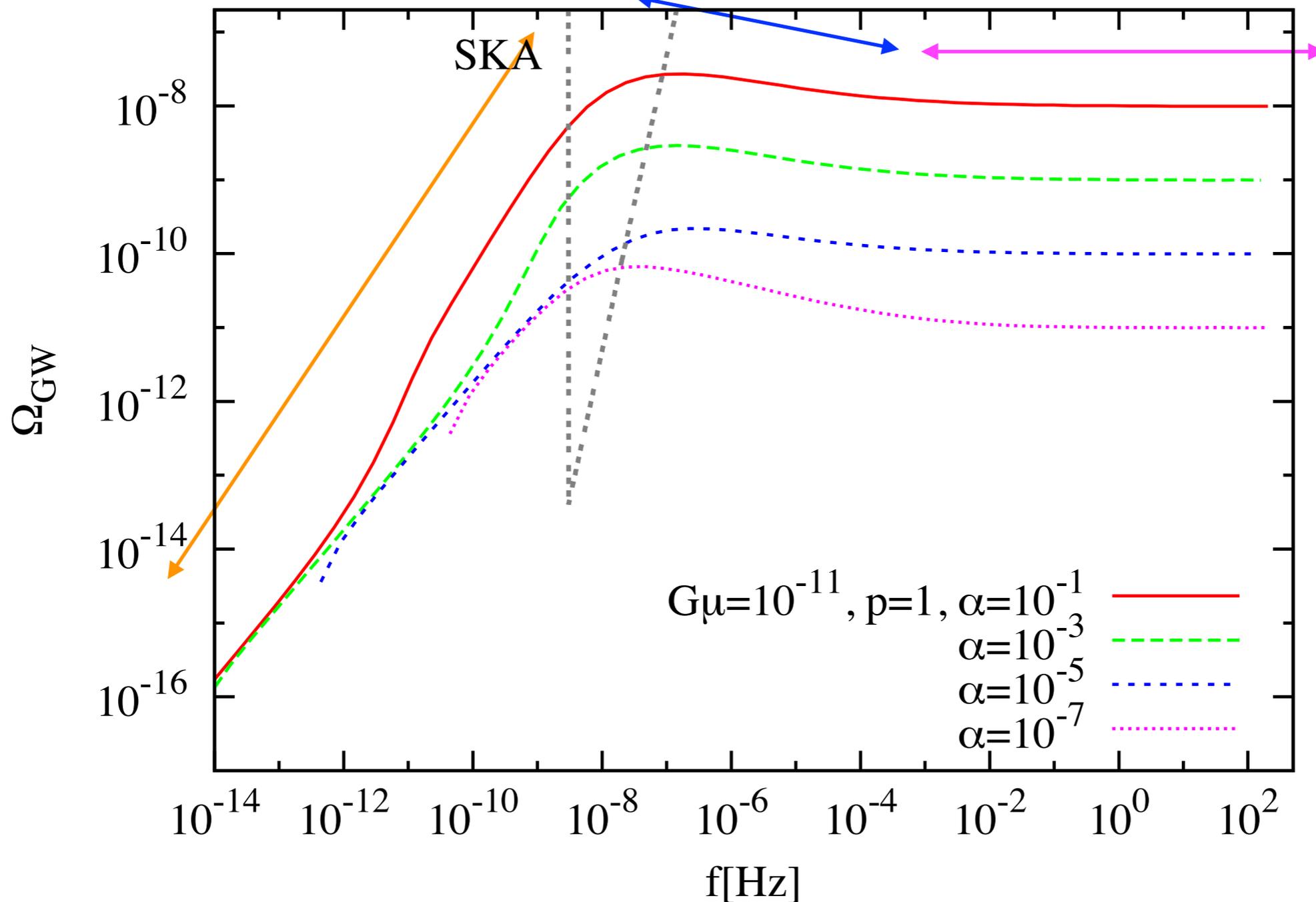
Loops remain for a while

$$n_l \propto \alpha^{1/2}$$

loops which evaporate in MD

GWs from loops which still exists

loops which evaporate in RD

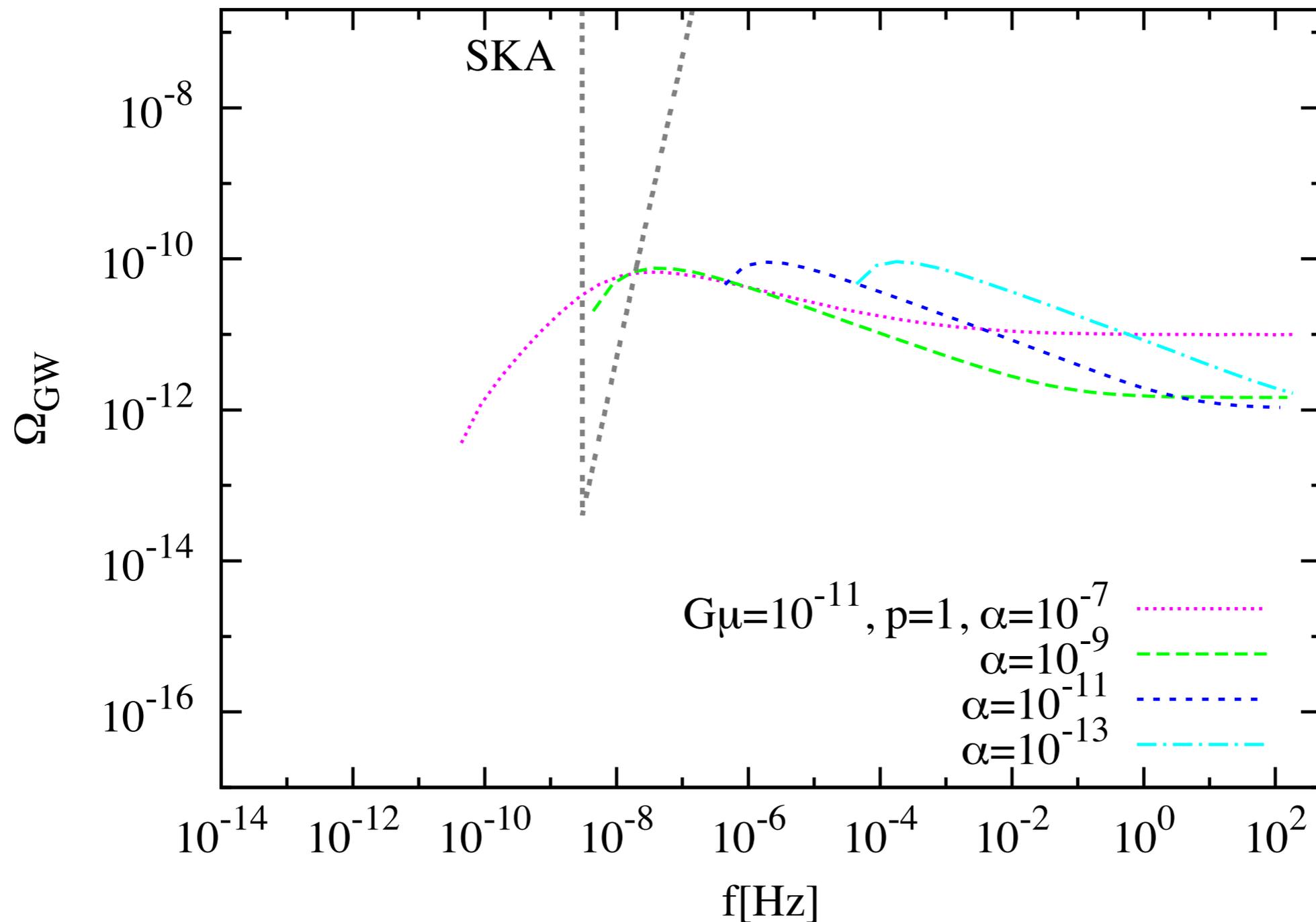


# GW spectrum

$$\alpha \ll \Gamma G\mu$$

Loops evaporate soon after  
the generation

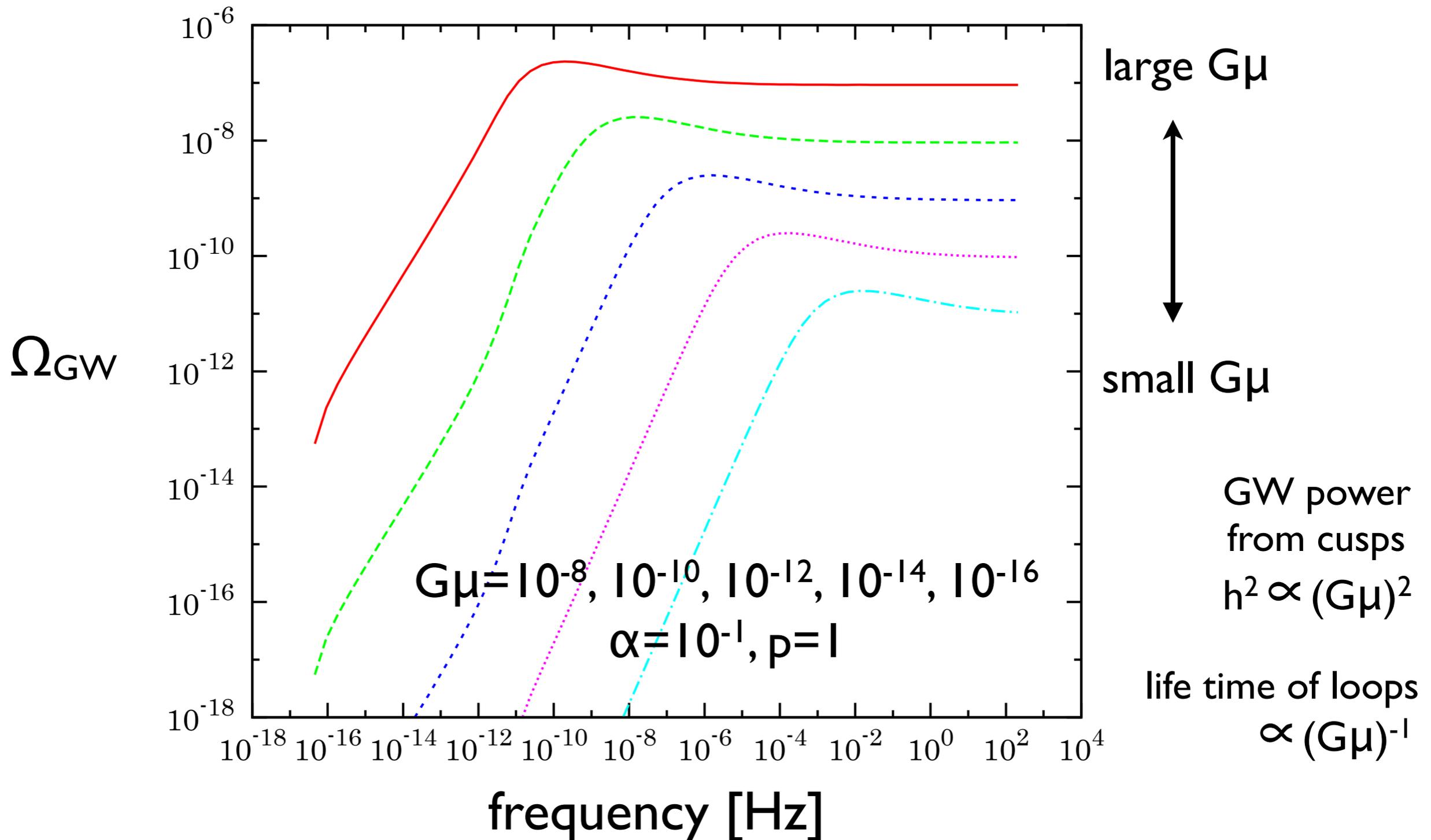
$$n_l \propto \text{const}$$



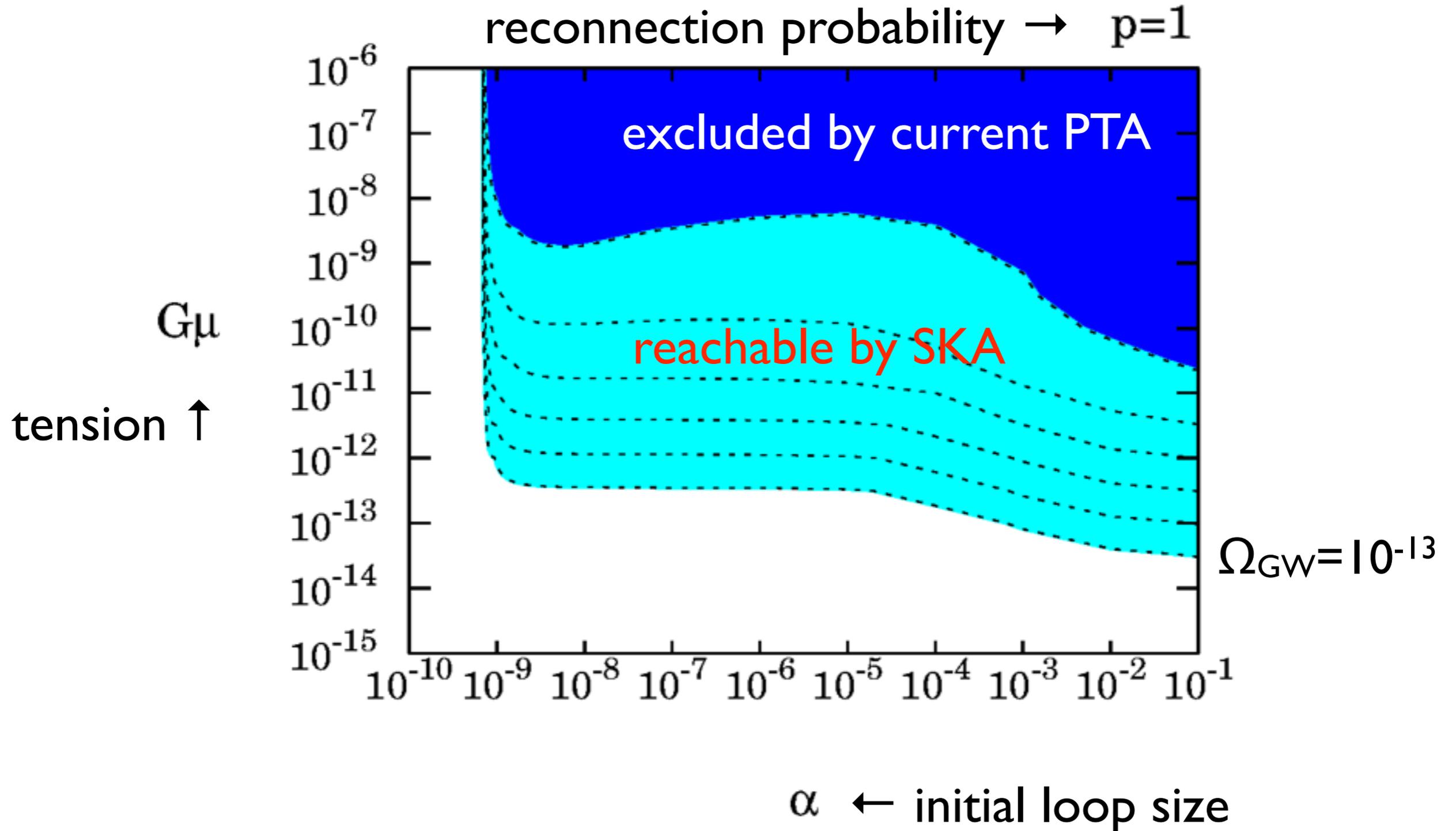
# GW spectrum

$$\alpha \gg \Gamma G\mu$$

dependence on  $G\mu$

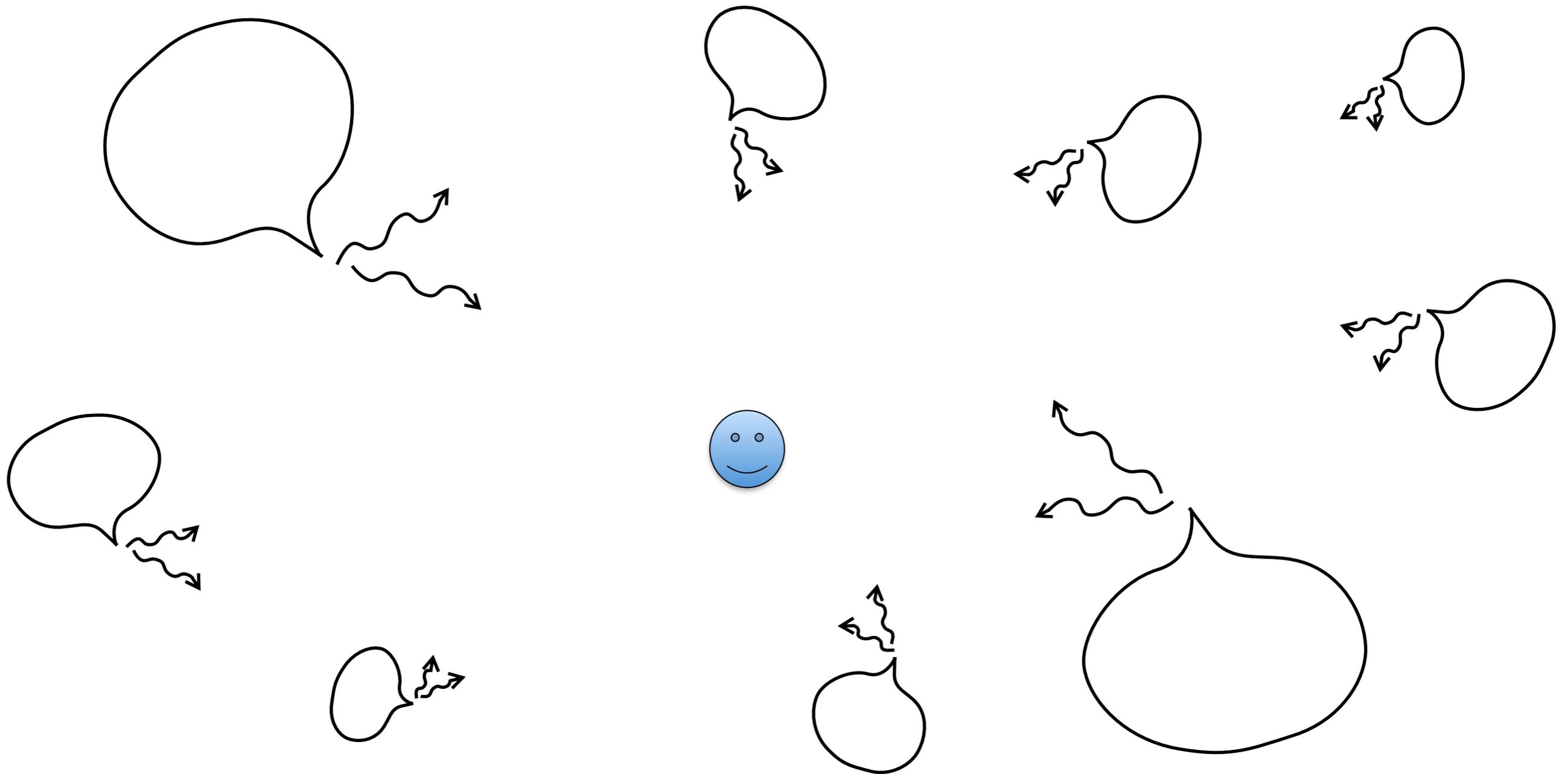


# Detectability in $G\mu$ - $\alpha$ plane



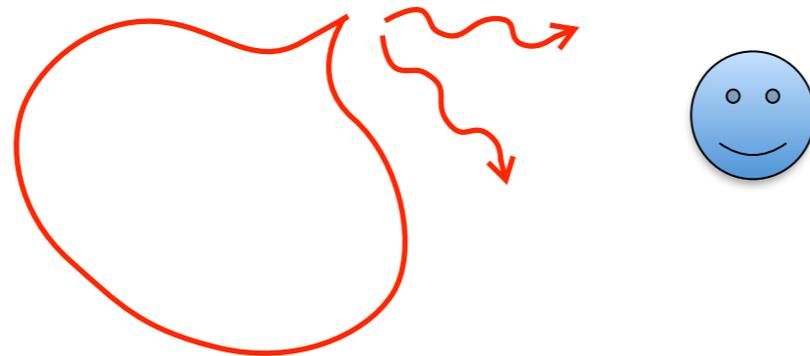
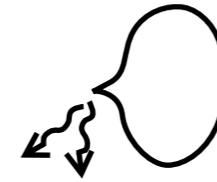
SKA will cover a large parameter space of cosmic string parameters

# Gravitational waves from cosmic string loops



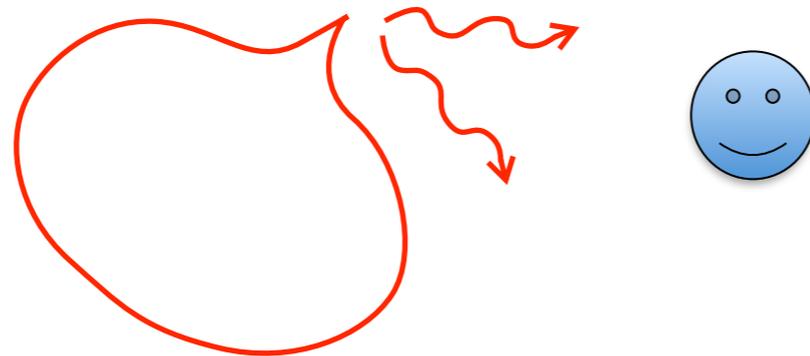
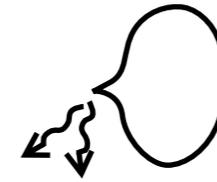
Gravitational waves coming from different directions overlap each other and form **gravitational wave background**

# Anisotropy of gravitational wave background



If there is very few loops,  
gravitational wave background becomes **anisotropic**

# Anisotropy of gravitational wave background



**Can we extract information of cosmic strings from anisotropy of the gravitational wave background?**

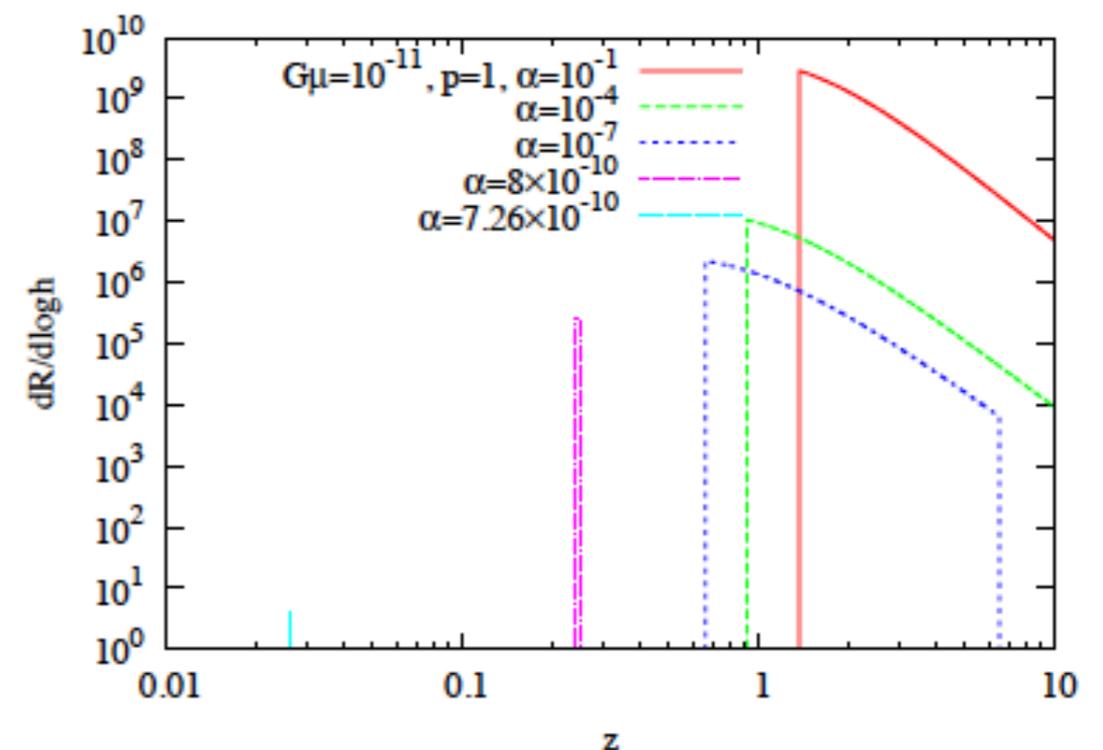
**Anisotropy level** depends on distribution of loops

→ depends on properties of cosmic string

3 main parameters to characterize cosmic string

- $G\mu$  : tension = line density → Generation mechanism
- $\alpha$  : initial loop size  $L \sim \alpha H^{-1}$  → **Network evolution**
- $p$  : reconnection probability → Phase transition origin:  $p=1$   
Cosmic superstring:  $p \ll 1$

Assuming the values of parameters and using theoretical evolution model, we can predict number density and redshift distribution of gravitational wave bursts from cosmic string loops



# Anisotropy test

- Formulations are constructed by Mingarelli et. al., PRD 88, 062005 (2013)

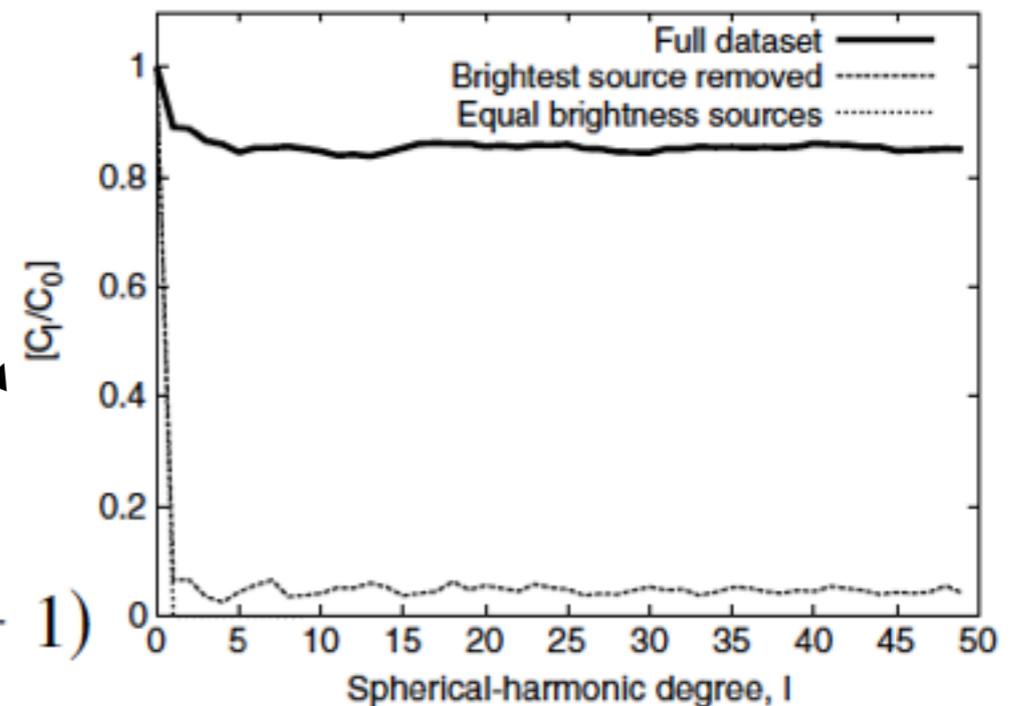
GW amplitude:  $\Omega_{\text{gw}}(f) = \frac{8\pi^2}{3H_0^2} f^3 H(f) \int d\hat{\Omega} P(\hat{\Omega})$

$P(\hat{\Omega}) \equiv \sum_{lm} c_l^m Y_l^m(\hat{\Omega})$  ← Spherical harmonic expansion

- Simulation study in a context of GWs from SMBH binaries Taylor & Gair, PRD 88, 084011 (2013)



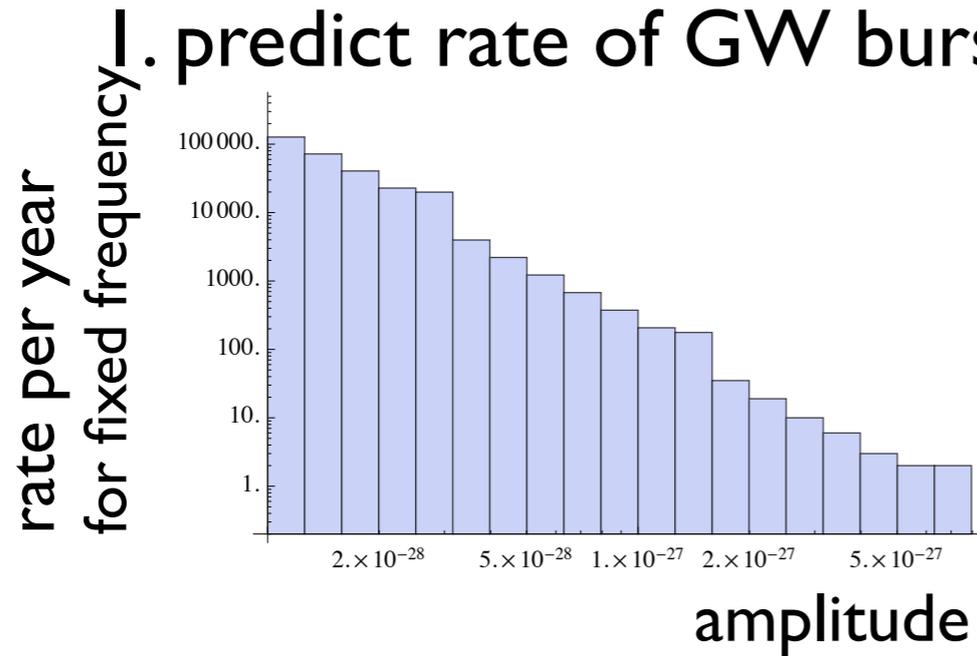
Anisotropy test is possible for detection with SN > 10



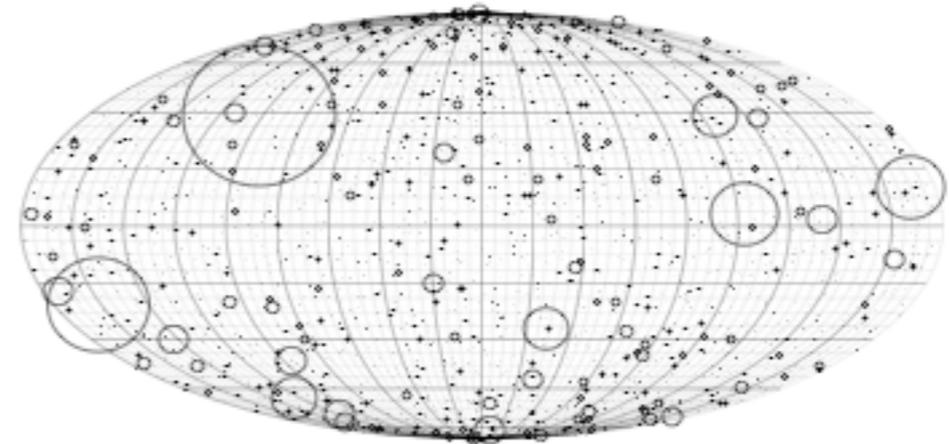
angular power spectrum:  $C_l = \sum_m |c_{lm}|^2 / (2l + 1)$

# Predictions for cosmic strings

1. predict rate of GW bursts



2. randomly distribute the source

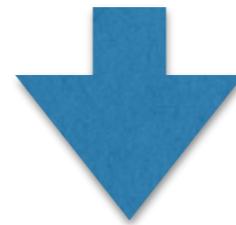


3. make 100 realizations

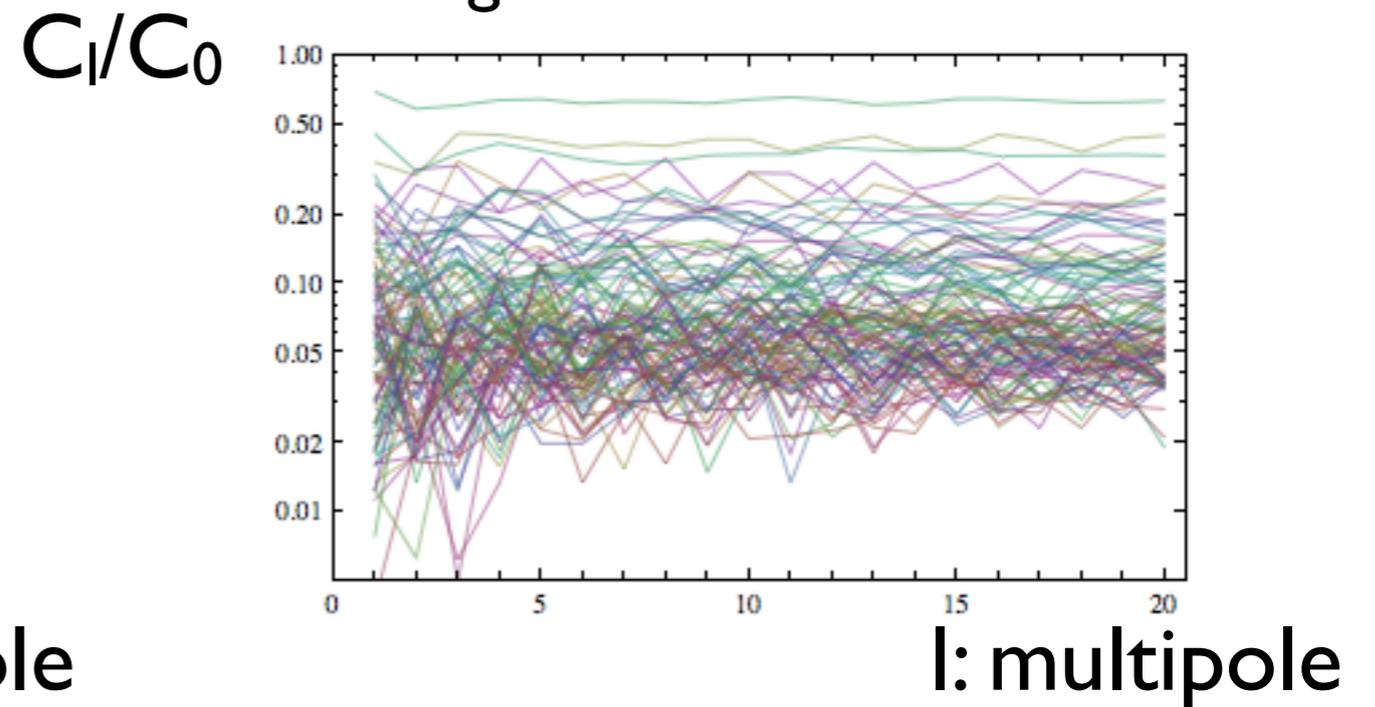
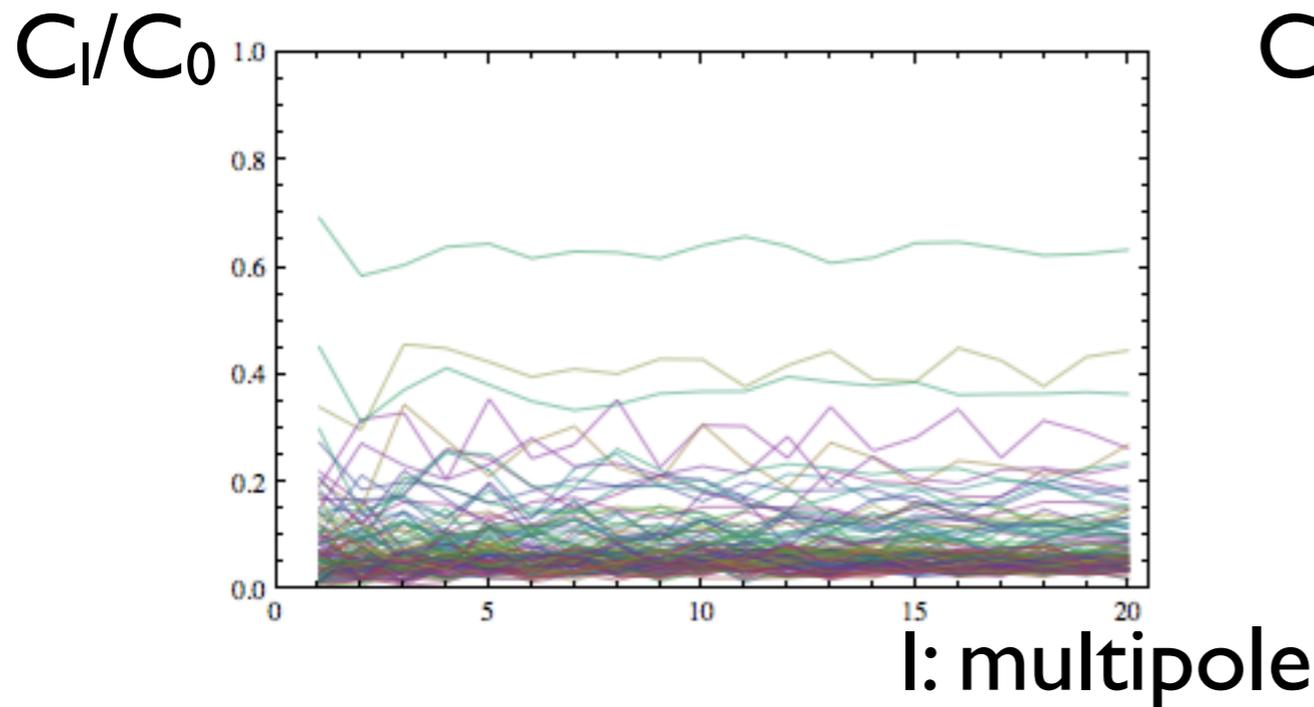
$$C_l = \sum_m |c_{lm}|^2 / (2l + 1)$$

4. calculate anisotropy levels

$$c_{lm} = \sum_{i=1}^N \rho_i Y_{lm}(\hat{\Omega}_i)$$



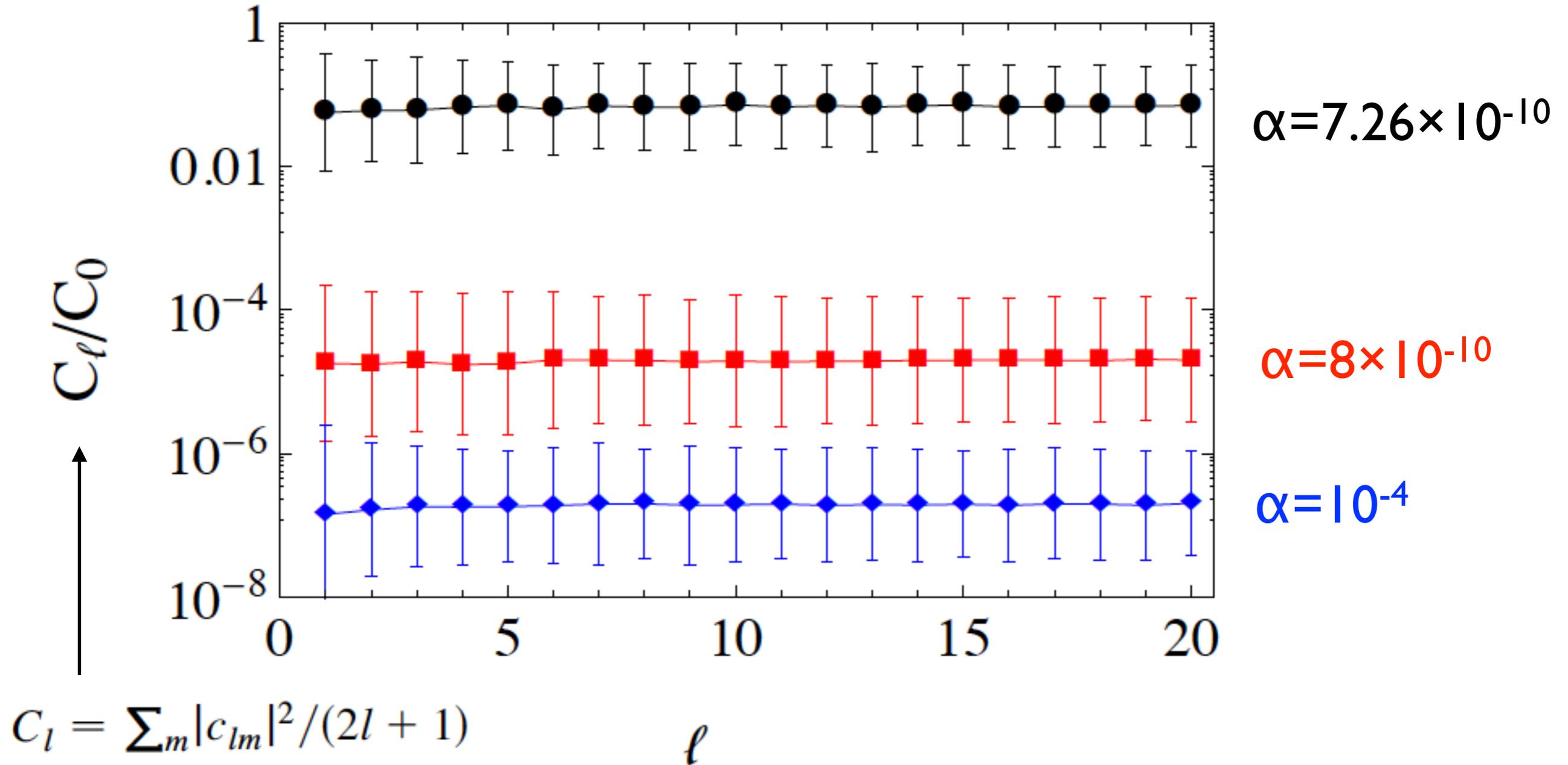
log normal distribution?



assumed  $f=(10\text{yr})^{-1}$   
 $G\mu=10^{-11}$ ,  $p=1$

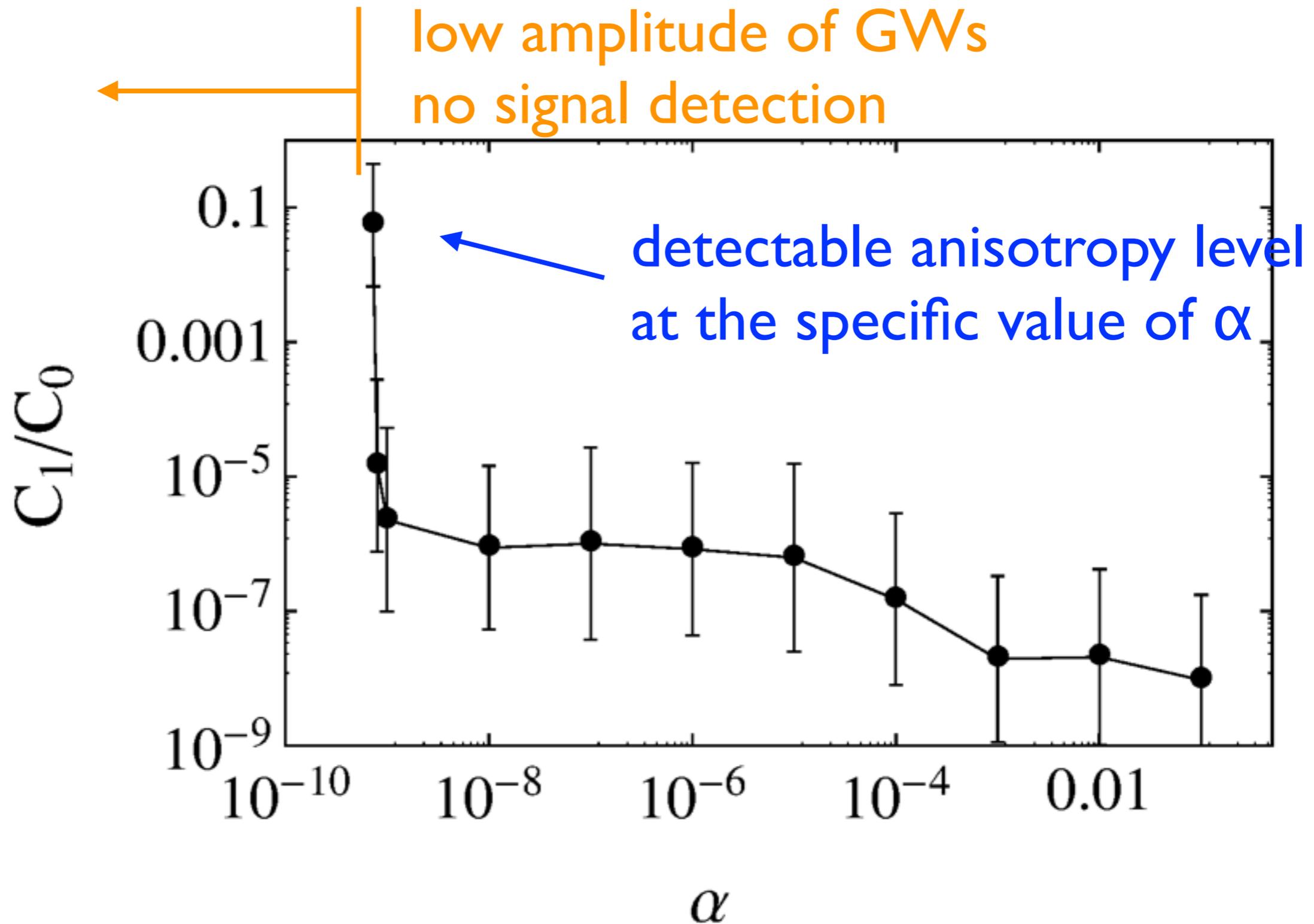
# Anisotropy level

with  $2\sigma$  error bar



We get large anisotropy for smaller value of  $\alpha$   
(smaller initial loop size)

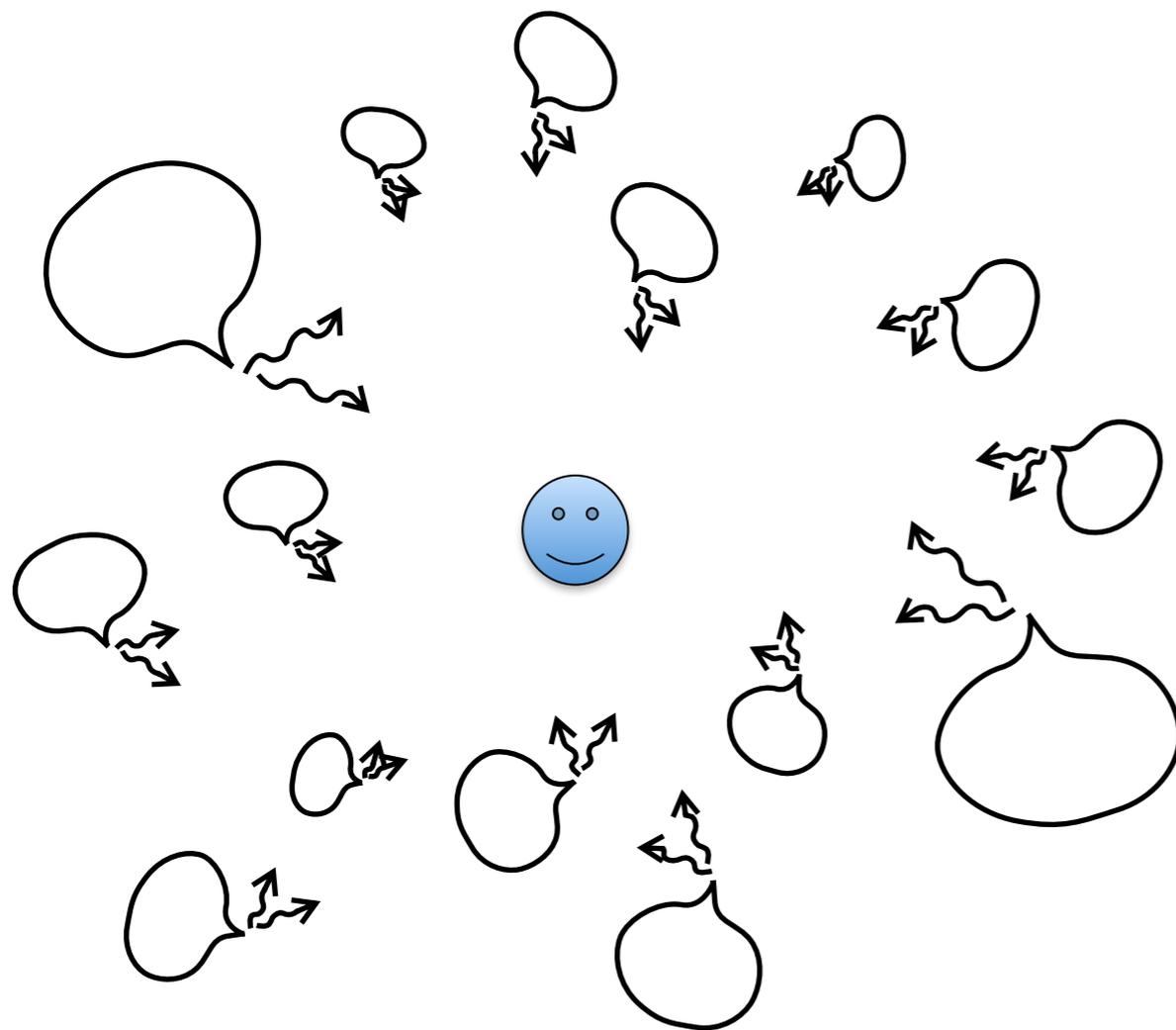
# Dipole component $\alpha$ dependence



# Interpretation

**large  $\alpha$**  = long lifetime

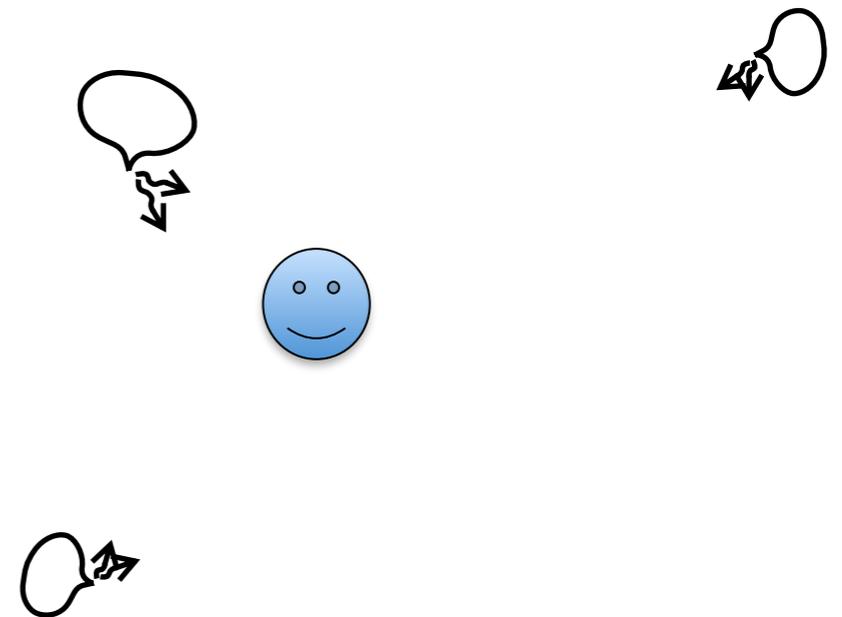
network consists of  
mixture of old and new loops



→ isotropic GWs

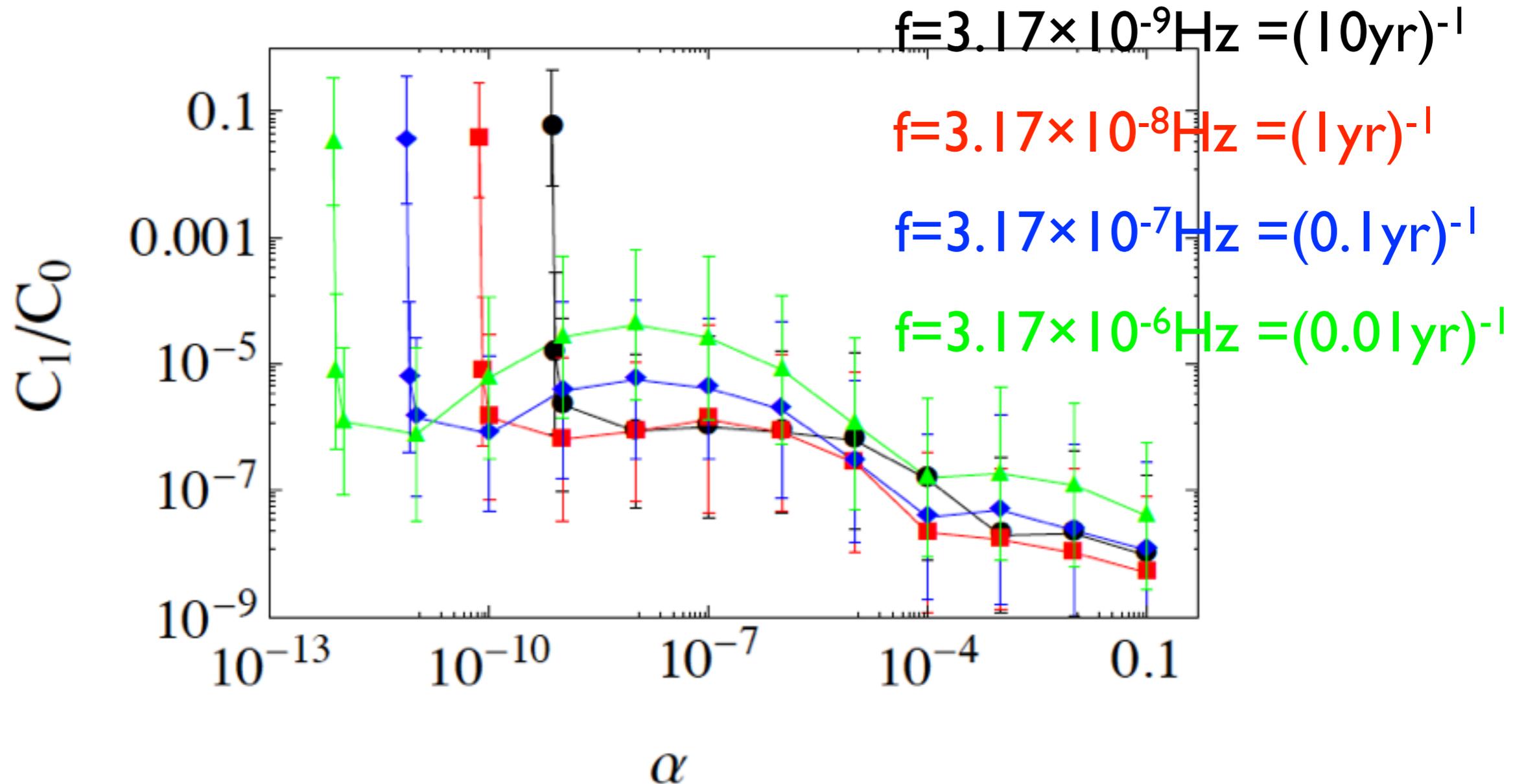
**small  $\alpha$**  = short lifetime

loops evaporate soon after  
their formation



→ anisotropic GWs

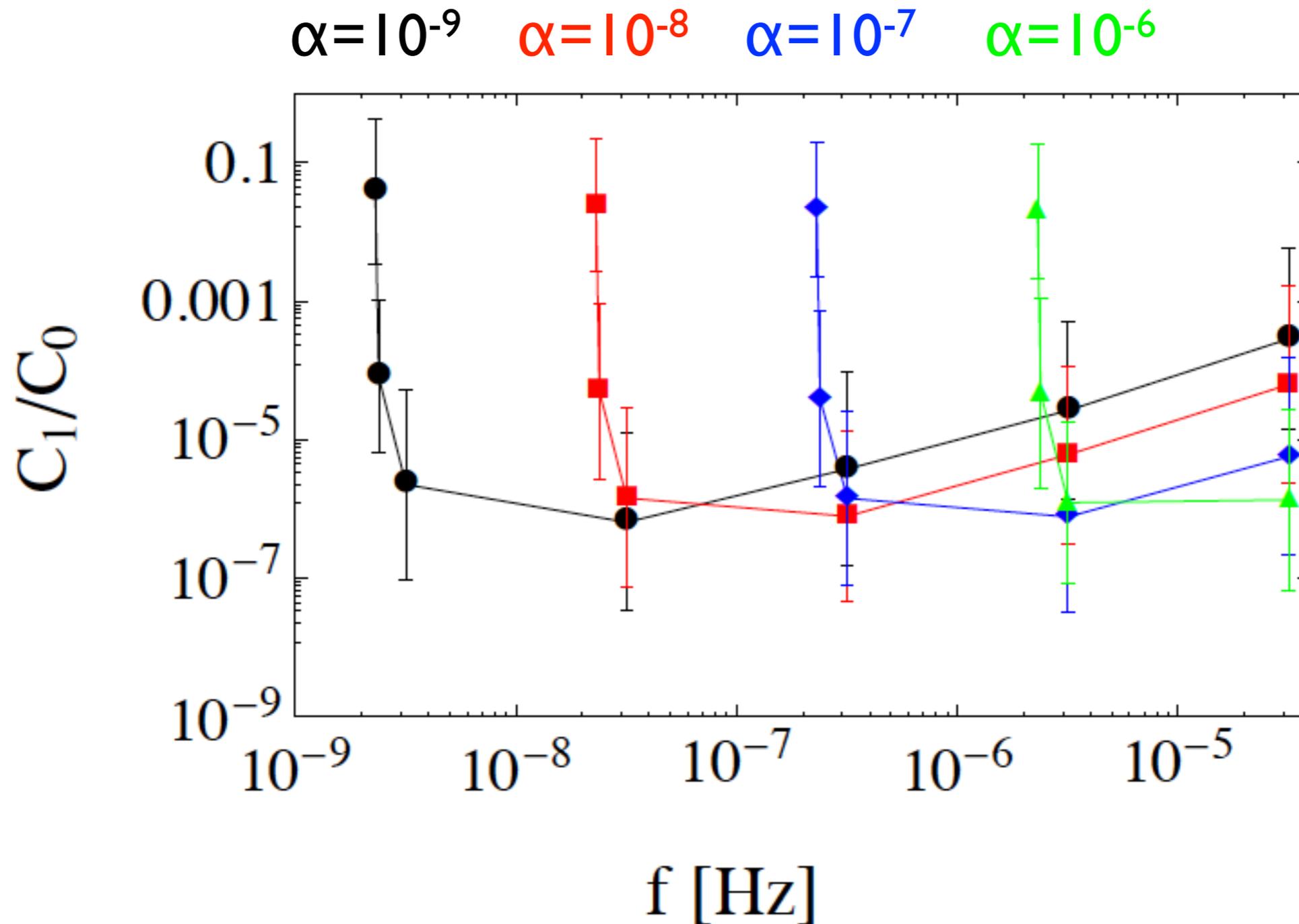
# Observation frequency dependence



The peak position changes for different observation frequency  
When loops evaporate soon after the formation by emitting GWs,

$$\text{frequency of GWs} \sim (\text{initial loop size})^{-1} \propto \alpha^{-1}$$

# Observation frequency dependence



By checking anisotropy for different frequency bands, it may be possible to obtain implication on the value of  $\alpha$

# Summary

- Testing the existence of cosmic string by PTA is important for obtaining implication on fundamental physics.
- SKA will cover a large parameter space of cosmic string parameters.
- Anisotropy of the gravitational wave background can be used to extract information on the initial loop size, which is important for understanding cosmic evolution of string network.