Grand Unification & Supersymmetry at High Scales

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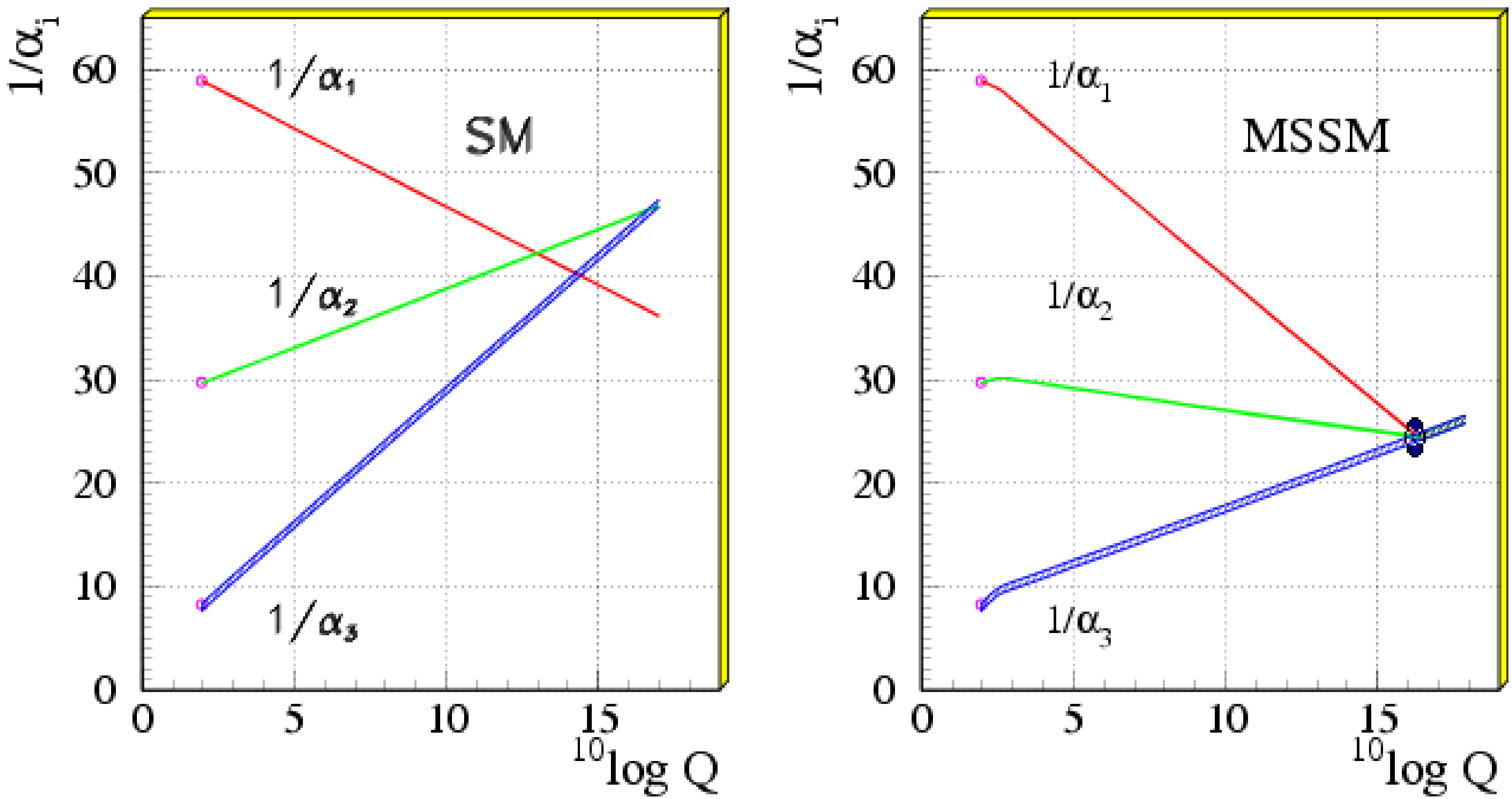
UV Completion of the Standard Model

- Structure of Standard Model of Particle Physics points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...)
  GUT groups: SU(5), SO(10), ...

- Strong theoretical arguments for supersymmetry at "high" energy scales (extra dimensions, strings; SM and gravity)

- Energy scale of grand unification: $\Lambda_{\mathrm{GUT}} \simeq 10^{15} \ldots 10^{16}\ \mathrm{GeV}$
  energy scale of supersymmetry breaking: $\Lambda_{\mathrm{SB}} \simeq$??
  Electroweak hierarchy problem? This talk: $\Lambda_{\mathrm{SB}} \sim \Lambda_{\mathrm{GUT}}$

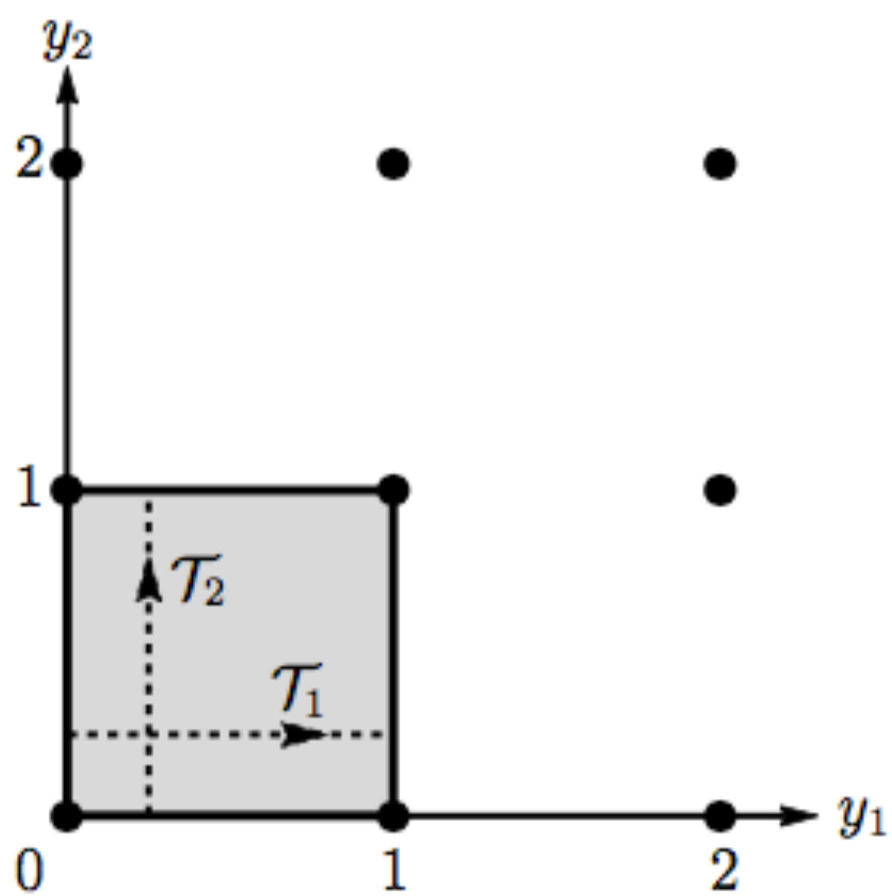


Effective interactions strengths (couplings α_i) depend on energy; unification at GUT scale in SM or supersymmetric extension; hint for supersymmetry? Contributions of additional particles? What is the energy scale of unification?

Supersymmetric Unification in 6d

WB, Dierigl, Ruehle, Schweizer arXiv: 1506.05771, 1507.06819, 1603.00654, ...

Supersymmetric GUTs strongly suggest extra space dimensions ("Orbifold GUTs",... [Kawamura '00; Hall, Nomura '01;... Hebecker, Trapletti '04;...]). Start from toy model: 6d supergravity with $U(1)$ gauge field



compactification on **torus**, metric:

$$(g_6)_{MN} = \begin{pmatrix} r^{-2}(g_4)_{\mu\nu} & 0 \\ 0 & r^2(g_2)_{mn} \end{pmatrix}$$

quantized flux:

$$\langle F \rangle = d\langle A \rangle = f v_2 = \text{const}$$

$$\frac{q}{2\pi} \int_{T^2/\mathbb{Z}_2} \langle F \rangle = \frac{qf}{4\pi} \equiv -N \in \mathbb{Z}$$

on **orbifold** Wilson lines are discrete, i.e. fractional localized flux:

$$W_i = \exp\left(iq \oint_{\mathcal{C}_i} A^{\mathrm{orb}} \right) = e^{iq\pi c_i}\,, \quad c_i = \frac{k_i}{q}\,, \quad k_i \in \mathbb{Z}$$

additional **bulk flux** is **quantized** (v_2 volume form on orbifold):

$$\langle F \rangle = d\langle A \rangle = f v_2 = \mathrm{const}$$

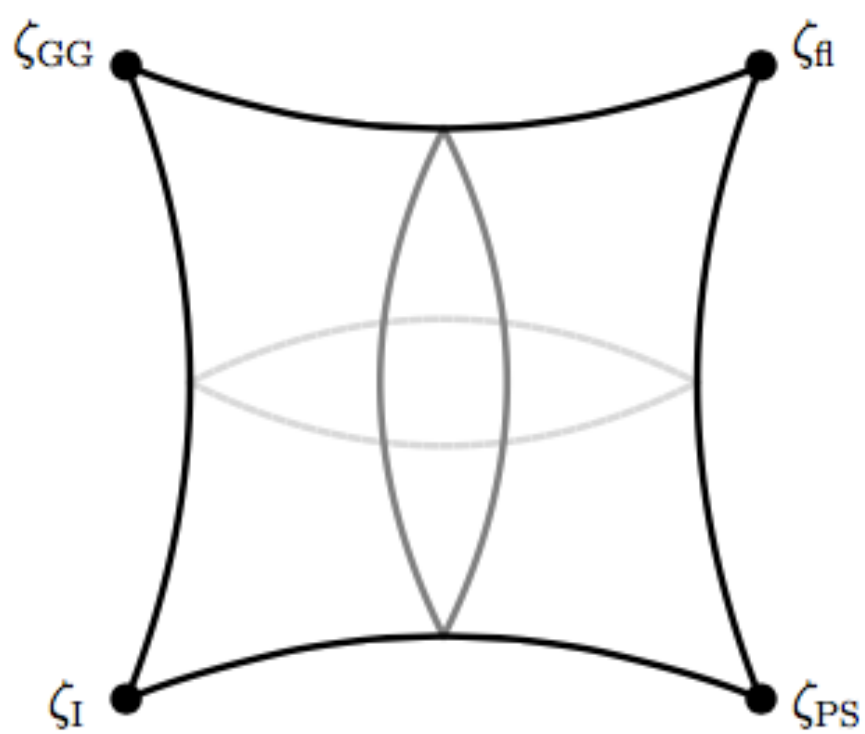
$$\frac{q}{2\pi} \int_{T^2/\mathbb{Z}_2} \langle F \rangle = \frac{qf}{4\pi} \equiv -N \in \mathbb{Z}$$

total background field: $\langle A \rangle + A^{\mathrm{orb}}$; bulk flux generates **chiral massless fermions** due to index theorem:

$$n_L = N\,, \quad N > 0$$

Split Symmetries

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with Standard Model group as intersection; bulk fields 45, 16, 16*, 10's [Asaka, WB, Covi '02,'03]; full 6d gauge symmetry:



$$SO(10) \times U(1)_A$$

N 16's, i.e. quark-lepton generations, from flux (N quanta; Witten '84, ...):

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}}\ [G_{\mathrm{SM}}]$$

“Matter field” $\psi \sim \mathbf{16}$ has charge q; all other (Higgs) fields have charge zero; flux induced zero modes $\psi_i, \ i = 1 \ldots N$ form complete GUT generations; split multiplets for other fields:

$$H_1 \supset H_u \,, \quad H_2 \supset H_d \,, \quad \Psi \supset D^c, N^c \,, \quad \Psi^c \supset D, N$$

Flux **breaks supersymmetry** [Bachas ’95], i.e. soft SUSY breaking only for quark-lepton families; Higgs fields massless at tree-level:

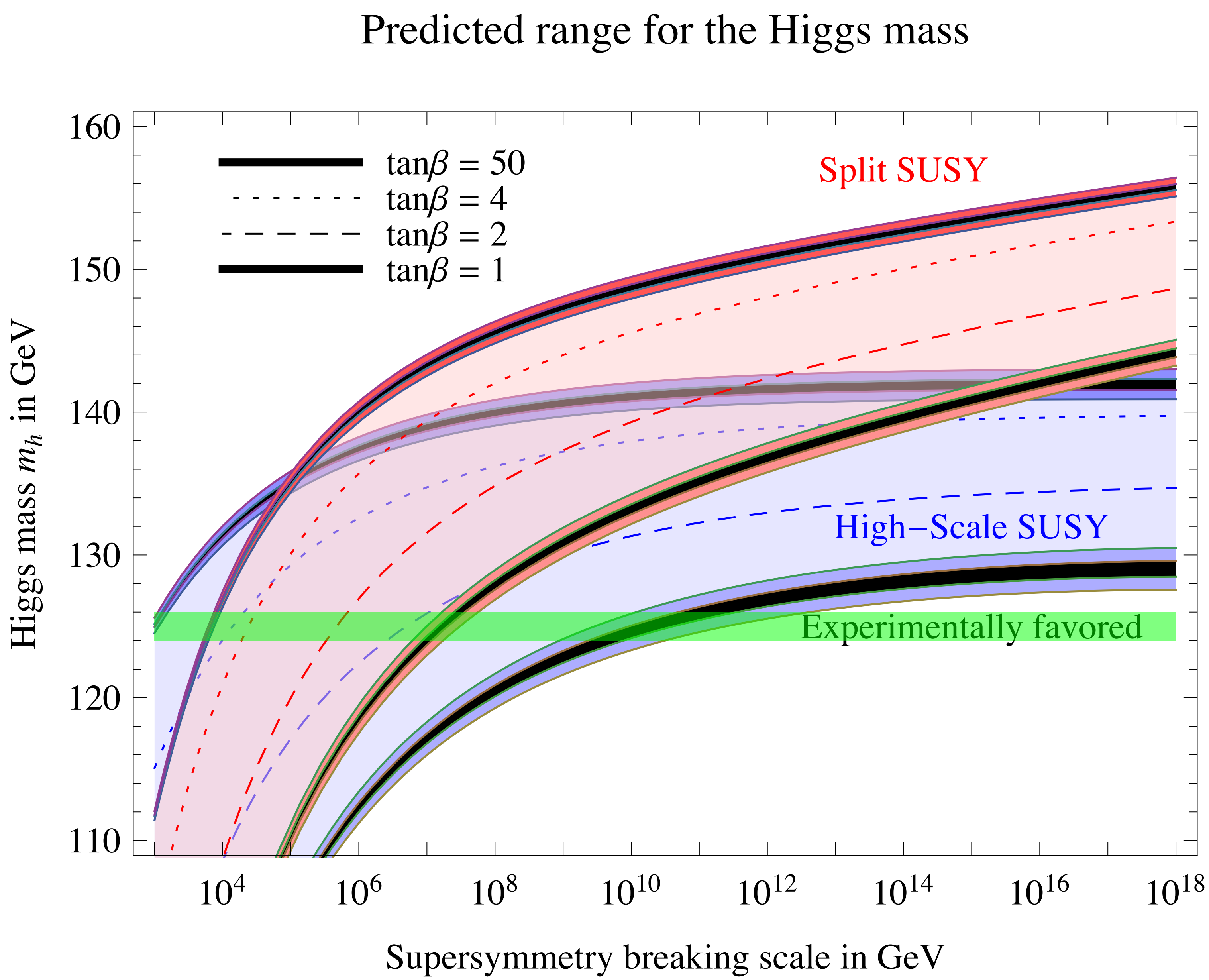
$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15}\mathrm{GeV})^2 \ ,$$
$$m_{3/2} \sim 10^{14} \ \mathrm{GeV} \ , \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 \gg m_{3/2} \gg m_{1/2}, m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (cf. “split/spread SUSY” [Arkani-Hamed, Dimopoulos ’04; Hall, Nomura ’11]):

- supersymmetry breaking is large for scalar quarks and leptons *because* they form *complete* GUT multiplets
- supersymmetry breaking is small for gauge and Higgs fields *because* they form *incomplete* GUT multiplets

[important: anomaly cancellation ....; embedding into F-theory ....]

Vacuum stability & SUSY at high scales



[Degrassi et al '12]

Matching of SM Higgs coupling to MSSM at SUSY breaking scale for `Split SUSY' (one Higgs doublet, higgsinos and gauginos light) and `High-scale SUSY' (one Higgs doublet light). Is the SUSY breaking scale necessarily much below the GUT scale?

Extrapolating the THDM to the GUT scale

Bagnaschi, Brummer, WB, Voigt, Weiglein arXiv: 1512.07761

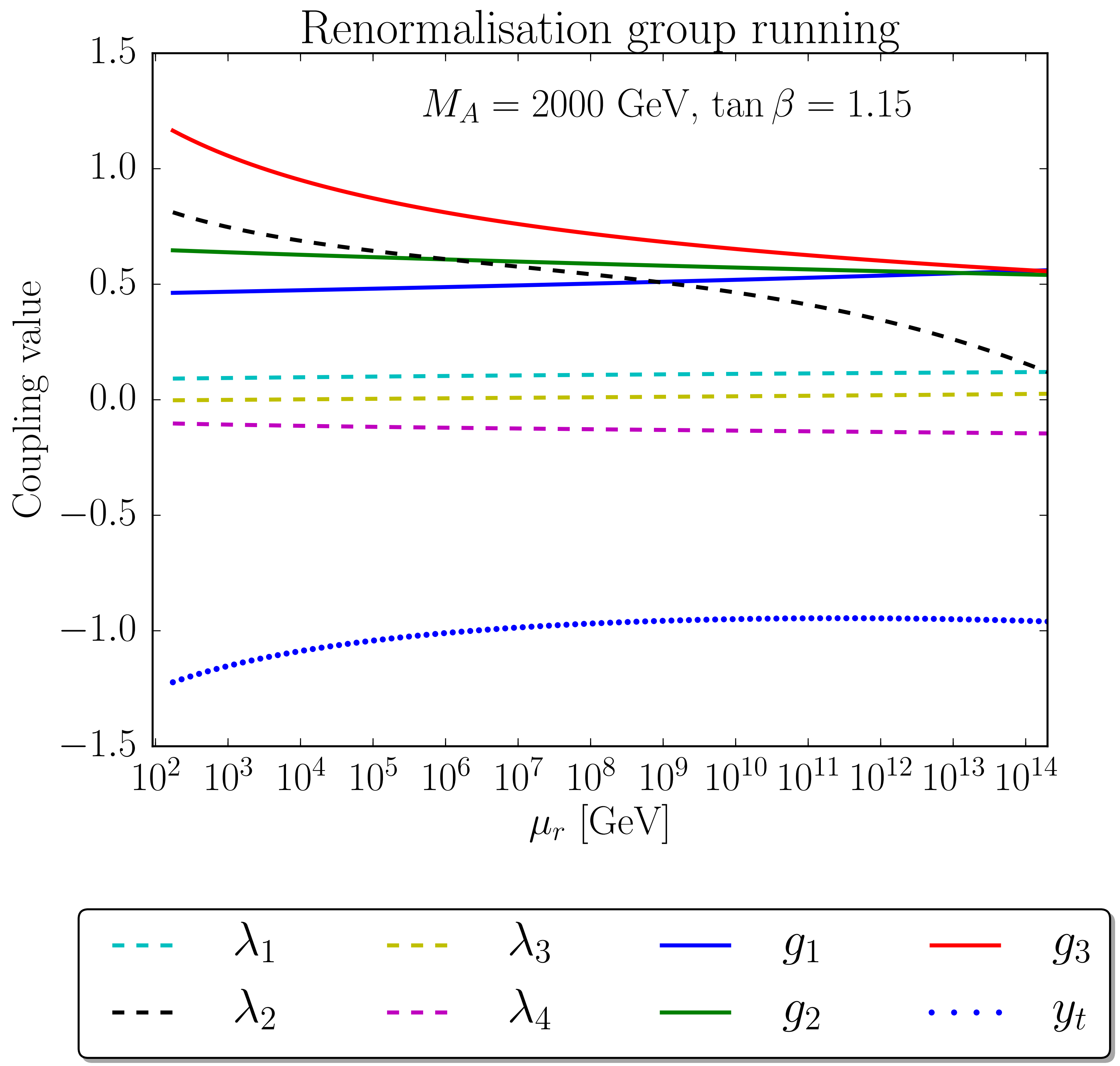
Is SUSY breaking at the GUT scale consistent with RG running of couplings and vacuum stability? Excluded for one light Higgs doublet! 6d GUT model suggests to consider 2 light Higgs doublets [Gunion, Haber '03... Lee, Wagner '15])

$$V = m_1^2\, H_1^\dagger H_1 + m_2^2\, H_2^\dagger H_2 - \left(m_{12}^2\, H_1^\dagger H_2 + \text{h.c.}\right) + V_4\,,$$

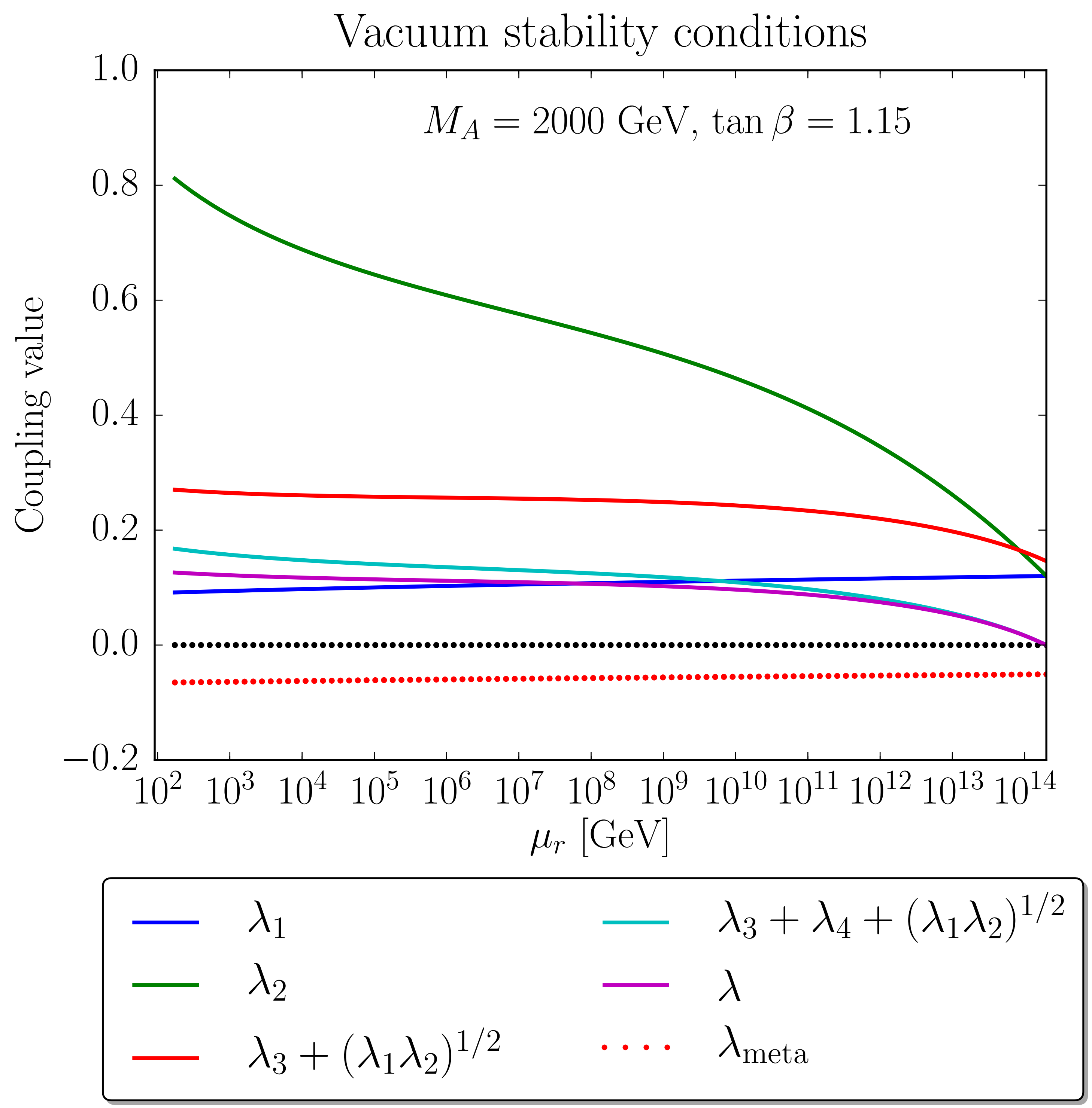
$$V_4 = \frac{\lambda_1}{2}(H_1^\dagger H_1)^2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3(H_1^\dagger H_1)(H_2^\dagger H_2) + \lambda_4|H_1^\dagger H_2|^2$$
$$+ \left(\frac{\lambda_5}{2}(H_1^\dagger H_2)^2 + \lambda_6(H_1^\dagger H_2)(H_1^\dagger H_1) + \lambda_7(H_1^\dagger H_2)(H_2^\dagger H_2) + \text{h.c.}\right)$$

Matching conditions at SUSY breaking scale determine quartic couplings:

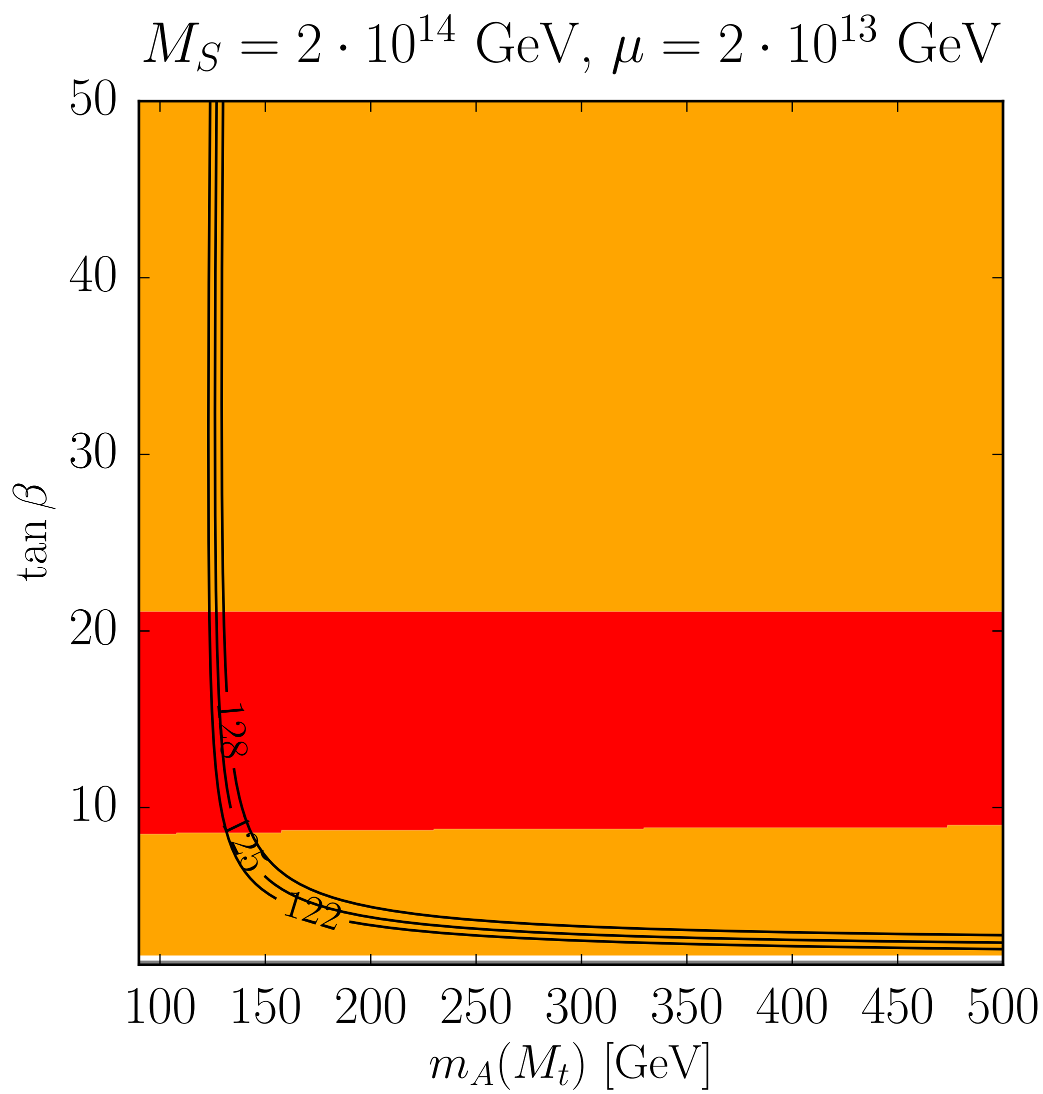
$$\lambda_1 = \frac{1}{4}\left(g^2 + {g'}^2\right)\,, \quad \lambda_2 = \frac{1}{4}\left(g^2 + {g'}^2\right)$$
$$\lambda_3 = \frac{1}{4}\left(g^2 - {g'}^2\right)\,, \quad \lambda_4 = -\frac{1}{2}g^2\,, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0$$

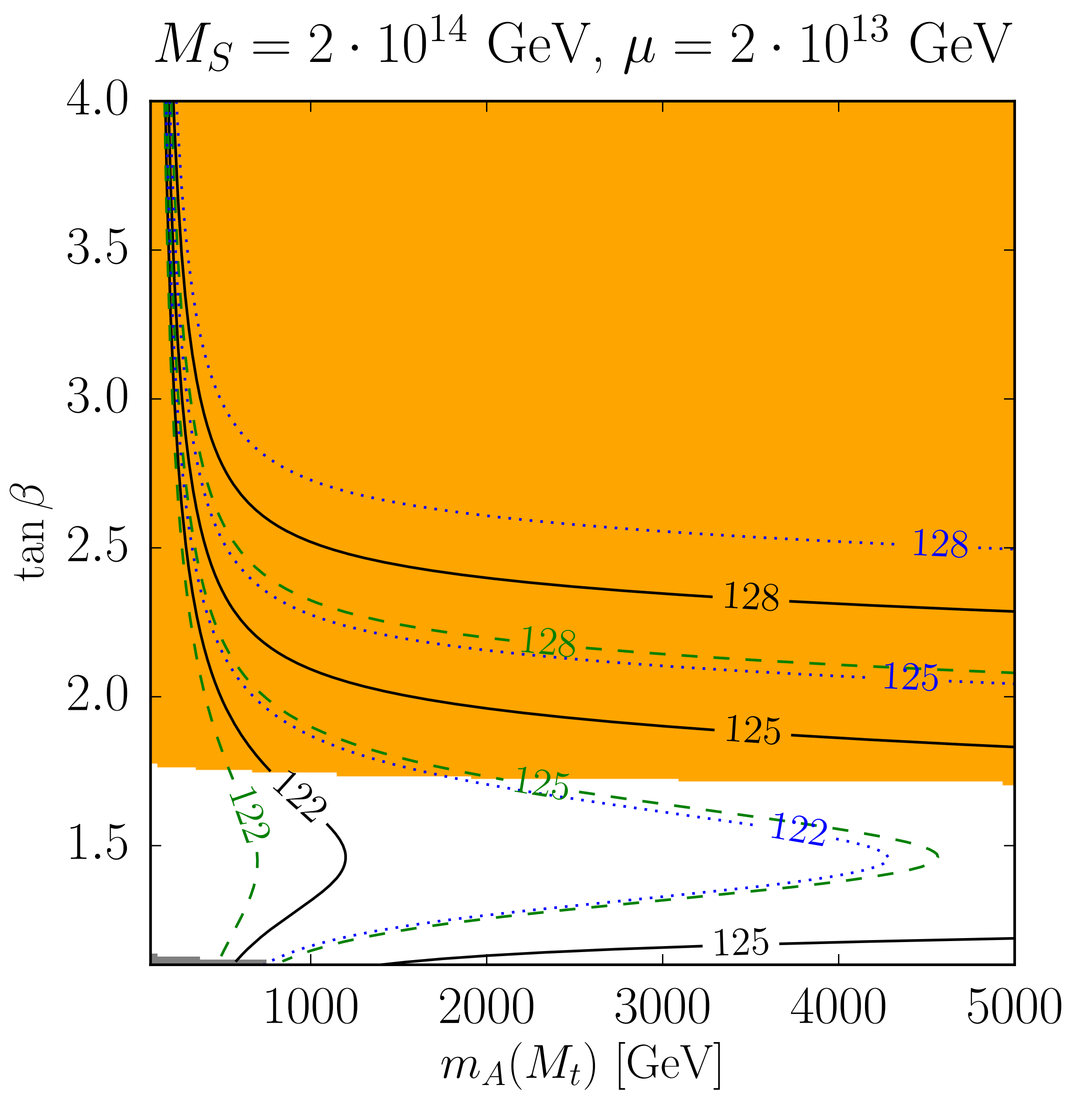


example of RG running of gauge, Yukawa and quartic couplings; reasonable gauge coupling unification



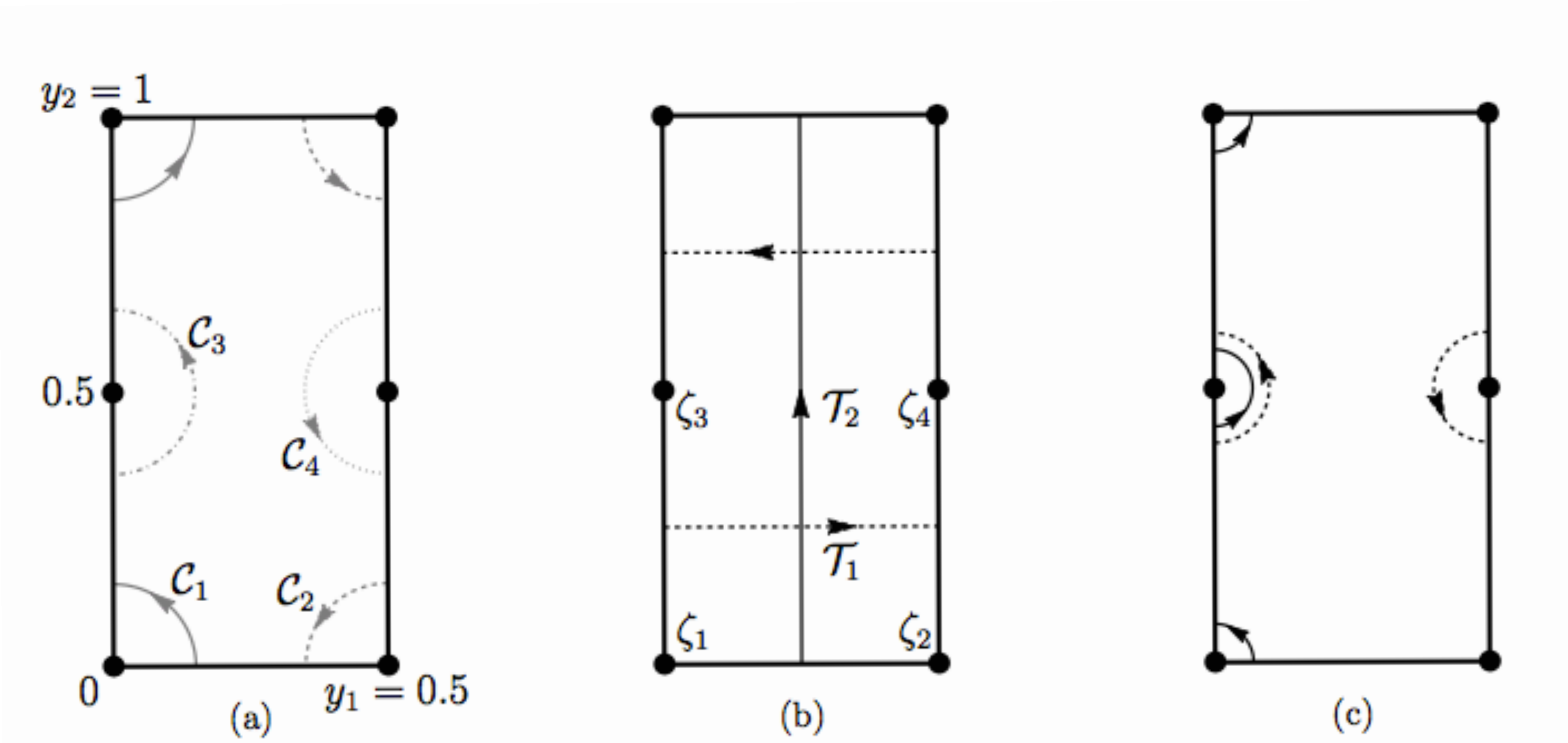
for this example vacuum stability conditions are fulfilled; additional Higgs bosons are heavy!





result of parameter scan; red: excluded by vacuum stability; orange: meta-stable vacuum; large tanβ excluded, small tanβ allowed with $M_A > 1$ TeV; light higgsino possible, split SUSY inconsistent!

Wave functions & Yukawa couplings



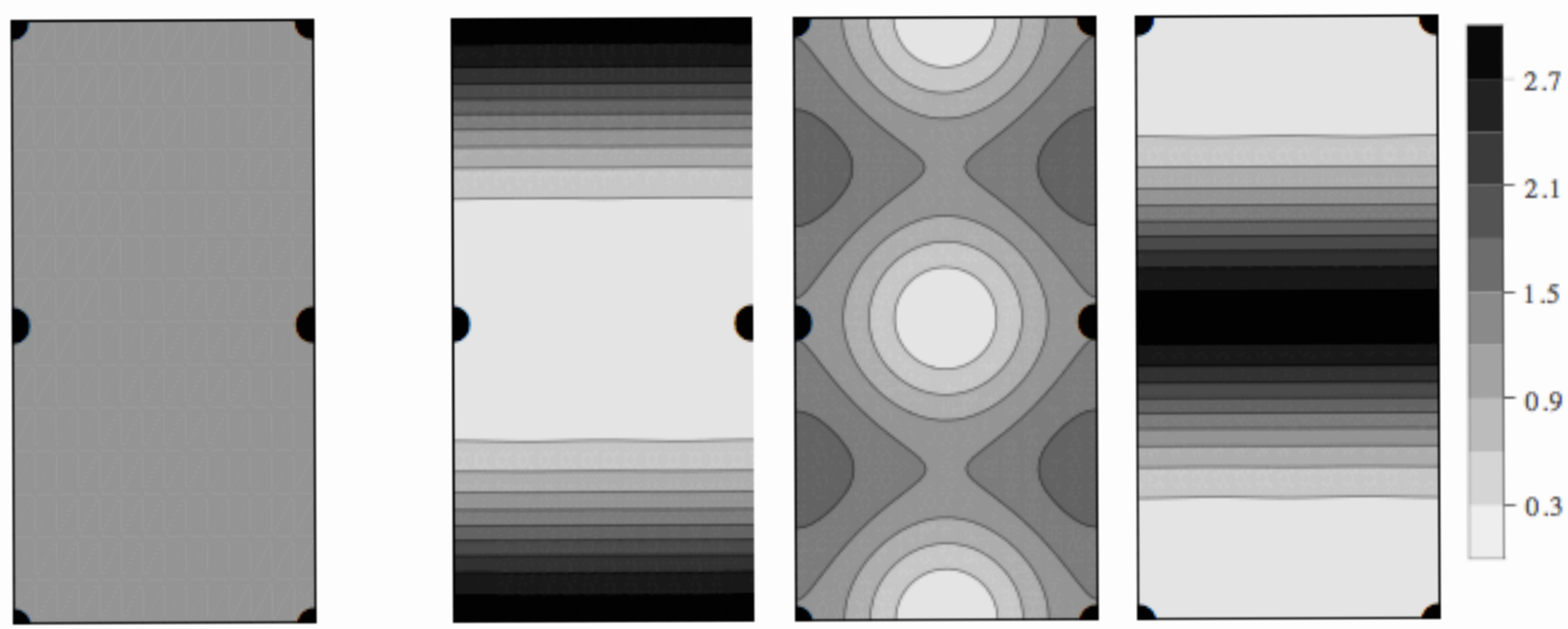
on **orbifold** projection of \`torus Wilson lines' to \`canonical Wilson lines' around orbifold fixed points:

$$\mathcal{T}_1 = \mathcal{C}_3 + \mathcal{C}_4 = -(\mathcal{C}_1 + \mathcal{C}_2)\,, \quad \mathcal{T}_2 = \mathcal{C}_1 + \mathcal{C}_3$$

Flux implies fermion zero modes; important information in wave functions (see [Bachas '95; Cremades et al '04; Braun et al 07; Abe et al. '13]), characteristic zeros at fixed points; spectrum for scalars and fermions:

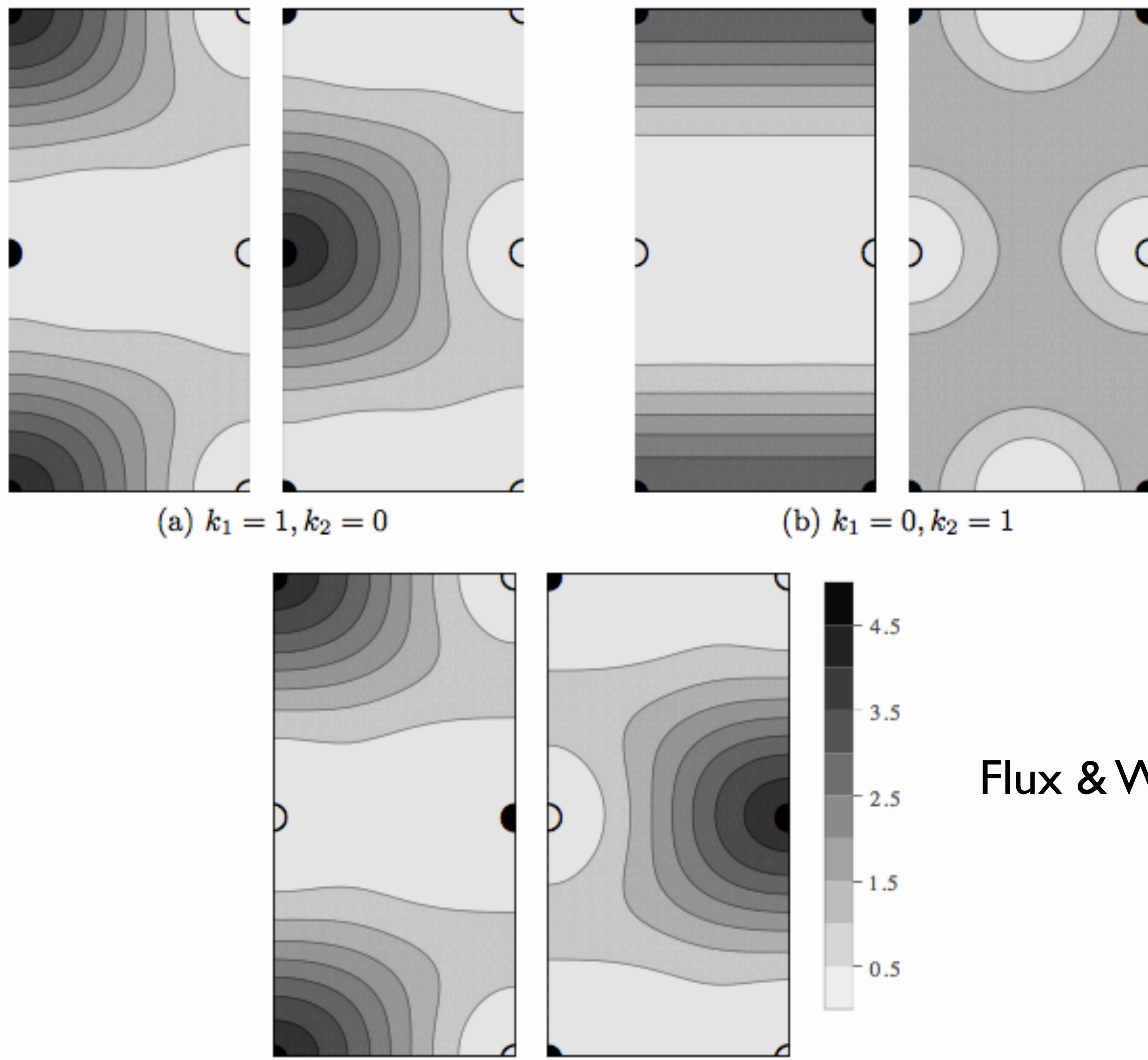
$$\text{bosons}: \quad m_n^2 = \frac{8\pi|N|}{V_2}\left(n+\frac{1}{2}\right), \quad N = 0, \pm 1, \pm 2, \ldots, \quad n = 0, 1, 2, \ldots$$

$$\text{fermions}: \quad m_n^2 = \frac{8\pi|N|}{V_2}\left(n+\frac{1}{2}\mp\frac{1}{2}\right)$$



(a) No flux (b) $N=2, j=0$ (c) $N=2, j=1$ (d) $N=2, j=2$

Flux, no Wilson lines



(a) $k_1 = 1, k_2 = 0$

(b) $k_1 = 0, k_2 = 1$

(c) $k_1 = 1, k_2 = 1$

Flux & Wilson lines

Realistic case of SO(10) breaking: Yukawa couplings from superpotential at fixed points (restricted by geometry, 14 parameters),

$$\begin{aligned} W_{\rm FP} = \delta_{\rm I}\ & (h_u^{\rm I}\mathbf{16}\ \mathbf{16}\ H_1 + h_d^{\rm I}\mathbf{16}\ \mathbf{16}\ H_2 + h_n^{\rm I}\mathbf{16}\ \mathbf{16}\ \Psi^*\Psi^*) \\ & +\delta_{\rm GG}(h_u^{\rm GG}\mathbf{10}\ \mathbf{10}\ H_{\mathbf{5}} + h_d^{\rm GG}\mathbf{5}^*\mathbf{10}\ H_{\mathbf{5}^*} + h_\nu^{\rm GG}\mathbf{5}^* n^c H_{\mathbf{5}} + h_n^{\rm GG} n^c n^c N N) \\ & +\delta_{\rm PS}(h_u^{\rm PS}\mathbf{4}\ \mathbf{4}^*\Delta_1 + h_d^{\rm PS}\mathbf{4}\ \mathbf{4}^*\Delta_2 + h_n^{\rm PS}\mathbf{4}^*\mathbf{4}^* F\ F\ ) \\ & +\delta_{\rm fl}\ (h_d^{\rm fl}\tilde{\mathbf{10}}\ \tilde{\mathbf{10}}\ H_{\mathbf{5}} + h_u^{\rm fl}\tilde{\mathbf{5}}^*\tilde{\mathbf{10}}\ H_{\mathbf{5}^*} + h_e^{\rm fl}\tilde{\mathbf{5}}^* e^c H_{\mathbf{5}} + h_n^{\rm fl}\tilde{\mathbf{10}}\ \tilde{\mathbf{10}}\ \tilde{T}^*\tilde{T}^*) \end{aligned}$$

Mass matrices after vev's of Higgs fields; flavour structure given by matrices c_{ij}^α, products of wave functions at the fixed points, determined by flux and symmetry breaking (texture zero's):

$$\begin{aligned} W = & (h_u^{\rm I} c_{ij}^{\rm I} + h_u^{\rm GG} c_{ij}^{\rm GG} + h_u^{\rm PS} c_{ij}^{\rm PS} + h_u^{\rm fl} c_{ij}^{\rm fl})\ H_u q_i u_j^c \\ & + (h_d^{\rm I} c_{ij}^{\rm I} + h_d^{\rm GG} c_{ij}^{\rm GG} + h_d^{\rm PS} c_{ij}^{\rm PS} + h_d^{\rm fl} c_{ij}^{\rm fl})\ H_d q_i d_j^c \\ & + (h_d^{\rm I} c_{ij}^{\rm I} + h_d^{\rm GG} c_{ij}^{\rm GG} + h_d^{\rm PS} c_{ij}^{\rm PS} + h_e^{\rm fl} c_{ij}^{\rm fl})\ H_d e_i^c l_j \\ & + (h_u^{\rm I} c_{ij}^{\rm I} + h_\nu^{\rm GG} c_{ij}^{\rm GG} + h_u^{\rm PS} c_{ij}^{\rm PS} + h_d^{\rm fl} c_{ij}^{\rm fl})\ H_u l_i n_j^c \\ & + (h_n^{\rm I} c_{ij}^{\rm I} + h_n^{\rm GG} c_{ij}^{\rm GG} + h_n^{\rm PS} c_{ij}^{\rm PS} + h_n^{\rm fl} c_{ij}^{\rm fl})\ n_i^c n_j^c N N \end{aligned}$$

Stabilizing the compact dimensions

Consider bosonic part of 6d supergravity action [Nishino, Sezgin '86, ... , Aghababai, Burgess, Parameswaran, Quevedo '03,... , Lee, Nilles, Zucker '04, ..., Braun, Hebecker, Trapletti '07,...], with dilaton, vector and tensor fields, and bulk matter field:

$$S = \int \left(\frac{1}{2} R - \frac{1}{2} d\phi \wedge * d\phi - \frac{1}{2} e^{2\phi} H \wedge * H - \frac{1}{2} e^{\phi} F \wedge * F \right)$$

$$F = dA \,, \quad H = dB - \omega_{3L} + \omega_{3G} \,,$$

$$\omega_{3L} = \mathrm{tr} \left(\omega \wedge d\omega + \frac{2}{3} \omega \wedge \omega \wedge \omega \right) \,, \quad \omega_{3G} = A \wedge F \,,$$

crucial: cancellation of **all** anomalies (bulk, fixed-point and flux zero-modes of matter field) by Green-Schwarz mechanism:

$$S_{\mathrm{GS}}[A, B] = - \int \left(\frac{\beta}{2} A \wedge F + \alpha \delta_O A \wedge v_2 \right) \wedge dB$$

Moduli: 2 axions, dilaton and radion,

$$t = r^2 e^{-\phi} \,, \quad s = r^2 e^{\phi}$$

Supersymmetric low-energy effective Lagrangian, given in terms of Kahler potential, gauge kinetic function and superpotential (defined at orbifold fixed points):

$$K = -\ln(S + \bar{S} + iX^S V) - \ln(T + \bar{T} + iX^T V) - \ln(U + \bar{U})\,,$$
$$S = \frac{1}{2}(s + ic)\,, \quad T = \frac{1}{2}(t + ib)\,,$$
$$X^T = -i\frac{f}{\ell^2}\,, \quad X^S = -i\frac{N+1}{(2\pi)^2}$$

U is shape modulus; Killing vectors due to quantized flux and Green-Schwarz term, note opposite signs! Gauge kinetic function [cf. Ibanez, Nilles '87]:

$$H = h_S S + h_T T\,, \quad h_S = 2\,, \quad h_T = -\frac{2\ell^2}{(2\pi)^3}$$

Note **opposite sign** of the two contributions! Result: no scale model with gauged shift symmetry, involving S and T!

Gauge invariant KKLT-type superpotential at fixed points:

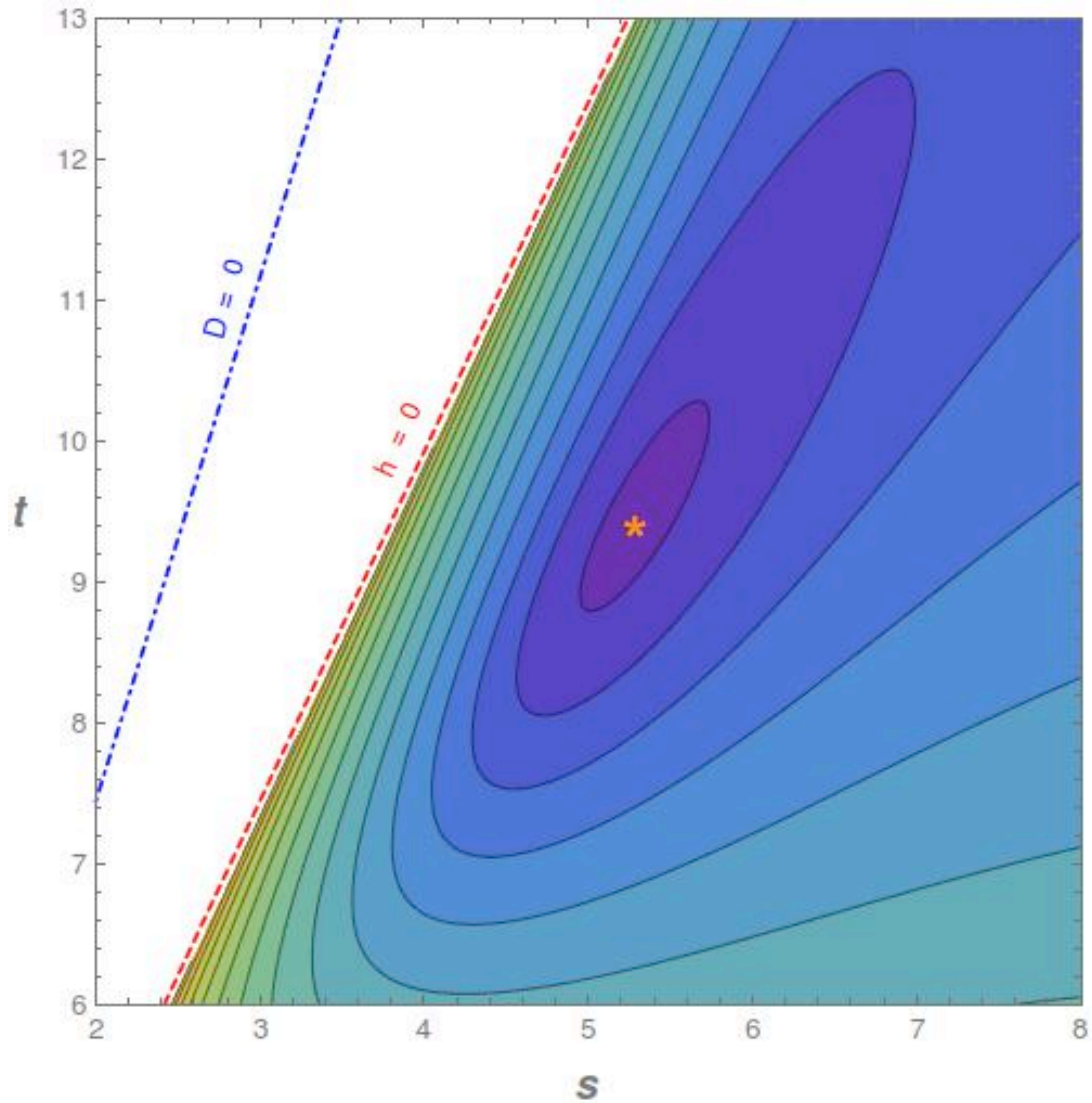
$$W(Z) = W_0 + W_1 e^{-aZ} \ , \quad Z = iX^T S - iX^S T$$

Scalar potential involving F- and D-terms:

$$V = V_F + V_D = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) + \frac{1}{2h} D^2 \, ,$$

$$D = iK_i X^i = -\frac{i}{s} X^S - \frac{i}{t} X^T \, .$$

Due to flux AND quantum corrections to gauge kinetic function and Killing vectors, **de Sitter vacua exist** without further F-term uplift (e.g. Polonyi)! Size of extra dimensions determined by parameters of superpotential; example: $W_0 \sim W_1 \sim 10^{-3} \, , a \sim 1 \to r\ell \sim 10^2$, i.e. GUT scale extra dimensions. Hence most basic ingredients of 6d compactifications sufficient to obtain de Sitter vacua and moduli stabilization!



de Sitter (Minkowski) metastable minimum of scalar potential, lifetime essentially infinite; vector boson and **all** moduli have GUT scale masses (no light axion)!

Conclusions

- Supersymmetric extensions of Standard Model strongly motivated, but what is the scale of SUSY breaking??
- Higher-dimensional GUT model with flux suggests GUT scale for SUSY breaking; emerging low energy spectrum reminiscent of `spread' SUSY
- Extrapolations of THDMs to GUT scale consistent with RG running and vacuum stability
- Anomalous U(1) allows de Sitter vacua
- Possible discoveries at LHC: Higgs bosons, light higgsinos

Backup Slides

Low energy signatures

- Possible discoveries at the LHC: additional Higgs bosons, light higgsino (dark matter?)

- Characteristic (but unobservable?) signature for origin of quark-lepton generations: universal contact interaction,

$$\mathcal{L}_{\rm uni} = \frac{q^2 g^2}{2m_A^2} \bar{\Psi}\gamma^\mu \Psi \bar{\Psi}\gamma_\mu \Psi \,, \quad \Psi = \text{all quarks and leptons}$$

- Very light (pseudo)scalar particles (axions etc)? Maybe from further embedding into string theory

Decomposition of fields at fixed points:

$$
\begin{aligned}
G_{\rm PS}: \quad & \mathbf{45} \to (\mathbf{15},\mathbf{1},\mathbf{1}) \oplus (\mathbf{1},\mathbf{3},\mathbf{1}) \oplus (\mathbf{1},\mathbf{1},\mathbf{3}) \oplus (\mathbf{6},\mathbf{2},\mathbf{2}) \\
& \mathbf{10} \to (\mathbf{1},\mathbf{2},\mathbf{2}) \oplus (\mathbf{6},\mathbf{1},\mathbf{1}) \\
& \mathbf{16} \to (\mathbf{4},\mathbf{2},\mathbf{1}) \oplus (\mathbf{4}^*,\mathbf{1},\mathbf{2})\,, \quad \mathbf{16}^* \to (\mathbf{4}^*,\mathbf{2},\mathbf{1}) \oplus (\mathbf{4},\mathbf{1},\mathbf{2}) \\
G_{\rm GG}, G_{\rm fl}: \quad & \mathbf{45} \to \mathbf{24_0} \oplus \mathbf{1_0} \oplus \mathbf{10_4} \oplus \mathbf{10^*_{-4}} \\
& \mathbf{10} \to \mathbf{5_2} \oplus \mathbf{5^*_{-2}} \\
& \mathbf{16} \to \mathbf{5^*_3} \oplus \mathbf{10_{-1}} \oplus \mathbf{1_{-5}}\,, \quad \mathbf{16}^* \to \mathbf{5_{-3}} \oplus \mathbf{10^*_1} \oplus \mathbf{1_5}
\end{aligned}
$$

Assign **charge** q **to "matter field"** $\psi \sim \mathbf{16}$, charge zero to the other fields, introduce background flux for anomalous $U(1)_A$: leads to **4d zero modes** $\psi_i \sim \mathbf{16}$, i.e. **complete GUT multiplets**, and **split multiplets** for other fields:

$$
H_1 \supset H_u\,, \quad H_2 \supset H_d\,, \quad \Psi \supset D^c, N^c\,, \quad \Psi^c \supset D, N\,,
$$

i.e. **doublet-triplet splitting** for Higgs doublets, and fields to break $B-L$. Anomaly cancellation similar to U(1) case: **Green-Schwarz term cancels 4d anomalies of zero modes** $\psi_i \sim \mathbf{16}$, fixed point anomalies more complicated; axion couples also to standard model gauge fields

Different projections of 10-plets at the fixed points restrict allowed Higgs couplings and yield two Higgs doublets as zero modes:

$$10 \to \begin{cases} H_1 \supset H_{\mathbf{5}} \supset H_u,\ H_2 \supset H_{\mathbf{5}^*} \supset H_d & \text{at } \zeta_{\mathrm{GG}}\,, \\ H_1 \supset H_{\tilde{\mathbf{5}}^*} \supset H_u,\ H_2 \supset H_{\tilde{\mathbf{5}}} \supset H_d & \text{at } \zeta_{\mathrm{fl}}\,, \\ H_{1,2} \supset \Delta_{1,2} = (H_u, H_d) \sim (\mathbf{1},\mathbf{2},\mathbf{2}) & \text{at } \zeta_{\mathrm{PS}} \end{cases}$$