The $AdS_5 \times S^5$ string field theory vertex and integrability

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Outline

Study $\mathcal{N}=4$ Super-Yang-Mills theory — a 4D gauge theory which is a conformal theory...

Key questions:

- Find the spectrum of conformal weights
  $\equiv$ eigenvalues of the dilatation operator
  $\equiv$ (anomalous) dimensions of operators

$$\langle O(0)O(x)\rangle = \frac{1}{|x|^{2\Delta}}$$

  The dimensions are complicated functions of the coupling:

$$\Delta = \underbrace{\Delta_0(\lambda)}_{planar} + \underbrace{\frac{1}{N_c^2}\Delta_1(\lambda) + \dots}_{nonplanar} \qquad \text{where } \lambda \equiv g_{YM}^2 N_c$$

- Find the OPE coefficients C_{ijk} defined through

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$\boxed{\mathcal{N} = 4 \text{ Super Yang-Mills theory}} \quad \equiv \quad \boxed{\text{Superstrings on } AdS_5 \times S^5}$

The AdS/CFT dictionary

Operators in $\mathcal{N} = 4$ SYM	$\longleftrightarrow$	(quantized) string states in $AdS_5 \times S^5$
Single trace operators	$\longleftrightarrow$	single string states
Multitrace operators	$\longleftrightarrow$	multistring states
Large N_c limit	$\longleftrightarrow$	suffices to consider single string states
Operator dimension	$\longleftrightarrow$	Energy of a string state in $AdS_5 \times S^5$
Nonplanar corrections	$\sim$	string interactions
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- Consider a closed string in $AdS_5 \times S^5$:
- The embedding coordinates of the point (τ, σ) are *quantum fields* $X^\mu(\tau, \sigma)$ on the worldsheet which has the geometry of a cylinder
- String theory in $AdS_5 \times S^5 \equiv$ a specific two dimensional quantum field theory defined on a cylinder (worldsheet QFT) with the Lagrangian induced by the geometry of $AdS_5 \times S^5$
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1. **Anomalous dimensions in the planar limit:**

   $\equiv$ energy levels of a single string in $AdS_5 \times S^5$
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2. **Nonplanar corrections to the dilatation operator or OPE coefficeints:**

   $\equiv$ string interactions
   $\equiv$ the specific 2D QFT on a string 'pants' topology:

   This is the **string field theory vertex** ← focus of this talk

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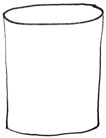

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   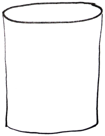

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   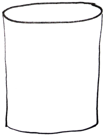

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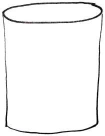

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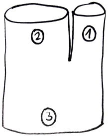

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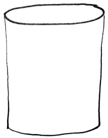

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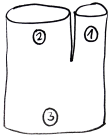

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Integrability?

- Typically interacting QFT's can only be studied via perturbation theory...

  difficult and practically hopeless to get exact answers...

- There exists a subclass of interacting (massive) two-dimensional QFT's which are integrable — exhibit additional conserved quantities
- Many quantities for these theories can be determined **exactly** without recourse to perturbation theory
- This typically involves solving functional equations of some sort...
- The worldsheet QFT of the string in $AdS_5 \times S^5$ is an integrable QFT! (but of a highly nonstandard type..)

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Part II

Motivation

- We have a very good understanding of **the spectrum** of a string on $AdS_5 \times S^5$
- This is due to **the integrability** of the worldsheet theory

Key question:

- How to describe string **interactions** for a generic integrable worldsheet theory
- Previously we knew how to proceed only for a **free** worldsheet theory
  - massless free bosons and fermions in the case of flat spacetime
  - massive free bosons and fermions in the case of pp-wave background geometry

Disclaimer: Here we only consider light-cone String Field Theory

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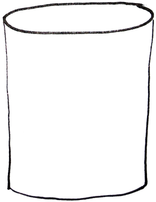

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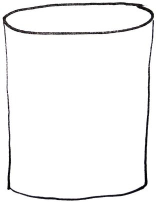

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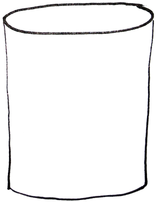

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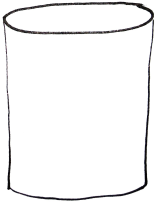

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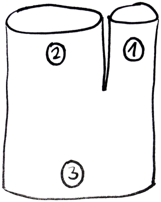

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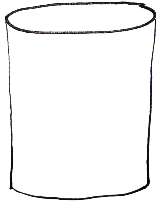

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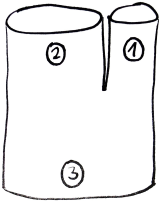

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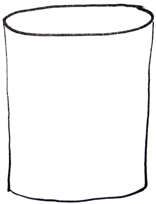

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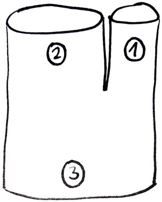

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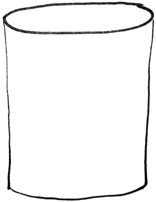

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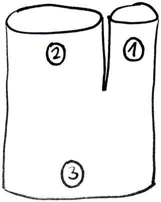

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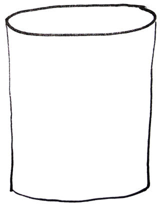

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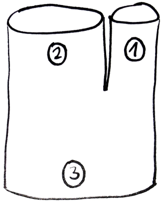

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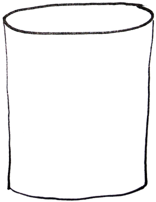

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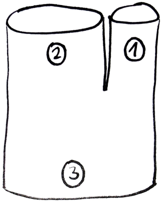

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Look for guidance from the case of two known problems:

1. Integrable bootstrap for the **spectal problem**...
2. Integrable bootstrap for **form factors**...

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I) solve the theory on an infinite plane

1. Particle momenta are completely unconstrained!
2. We may perform analytic continuation into the complex plane (of appropriate rapidities)
3. We get **crossing** equation
4. This together with unitarity...
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Works equally for relativistic and $AdS_5 \times S^5$ case...

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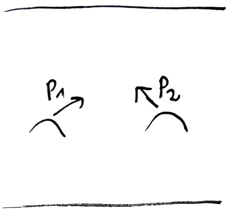

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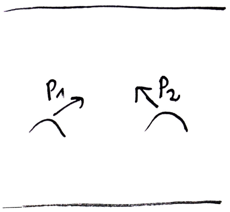

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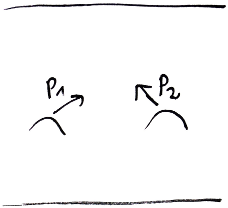

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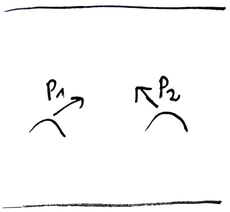

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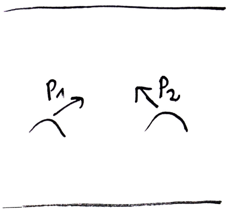

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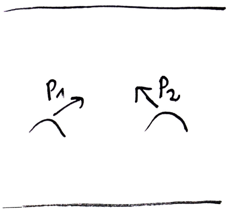

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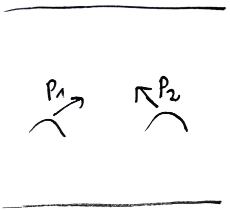

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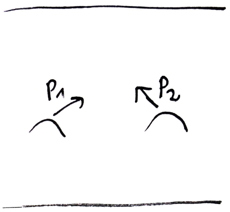

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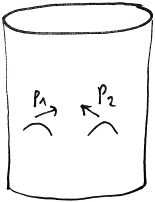

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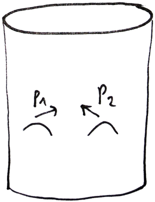

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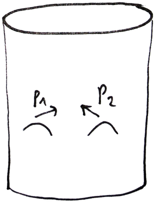

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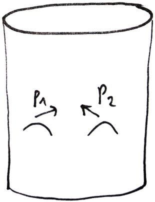

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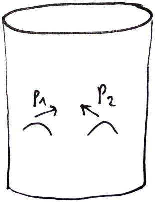

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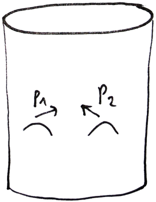

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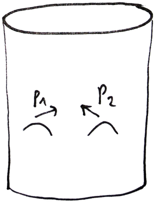

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III) Include leading wrapping corrections...

— generalized Lüscher formulas

IV) Resum all wrapping corrections

— Thermodynamic Bethe Ansatz
$\longrightarrow$ Quantum Spectral Curve

Comments

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  (despite numerous subtleties and novel features)
- **Key role** of the infinite plane $\longrightarrow$ only there do we have crossing+analyticity which allows for solving for the S-matrix
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How to solve the spectral problem?

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— generalized Lüscher formulas

IV) Resum all wrapping corrections

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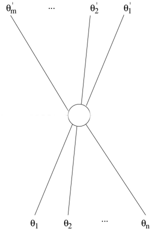
Proceed to form factors...

Form factors



- Form factors are expectation values of a local operator sandwiched between specific multiparticle *in* and *out* states $\quad p_k = m \sinh \theta$
- Form factors in infinite volume (on an infinite plane) satisfy a definite set of functional equations $\quad \langle \varnothing | \mathcal{O}(0) | \theta_1, \ldots, \theta_n \rangle \equiv f(\theta_1, \ldots, \theta_n)$

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- Solutions explicitly known for numerous relativistic integrable QFT's

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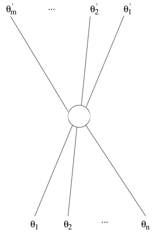

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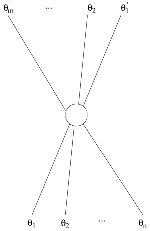

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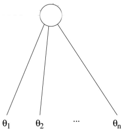

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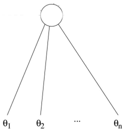

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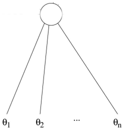

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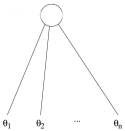

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- The form factor axioms do not depend at all on the specific local operator inserted...
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Finite volume $\equiv$ form factors on a cylinder

- Up to wrapping corrections ($\sim e^{-mL}$), very simple way to pass to finite volume (cylinder of circumference L): Pozsgay, Takacs

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Bajnok, RJ, Wereszczyński

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coincides exactly with a classical computation of a 'diagonal' form factor

figure from Zarembo 1008.1059

- Also seen at weak coupling!

Hollo, Jiang, Petrovsky

- Similarities with the String Field Theory vertex...

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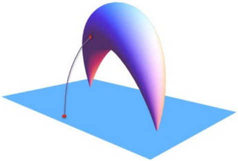

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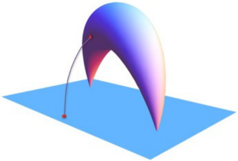

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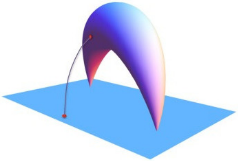

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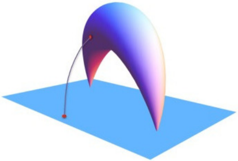

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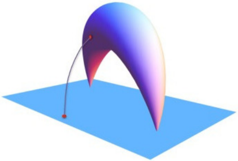

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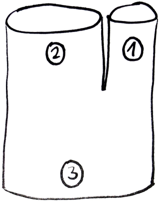
Proceed to the light cone String Field Theory vertex...

Light-cone String Field Theory Vertex



String Field Theory vertex describes the splitting/joining of 3 strings with generic sizes $J_1 + J_2 = J_3$

Comments:

1. The lengths here are directly the R-charges w.r.t. $U(1) \subset SO(6)$ (these are not spin-chain lengths)
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   – this setup encompasses any possible configuration of excitations...

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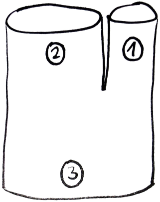

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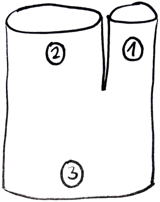

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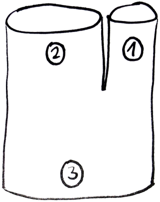

String Field Theory vertex describes the splitting/joining of 3 strings with generic sizes $J_1 + J_2 = J_3$

Comments:

1. The lengths here are directly the R-charges w.r.t. $U(1) \subset SO(6)$ (these are not spin-chain lengths)
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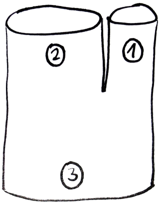

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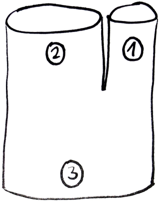

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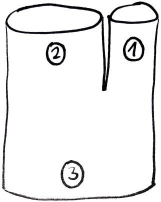
Our goal: Concentrate on defining the string field theory vertex for a generic integrable worldsheet theory

Light-cone String Field Theory Vertex – the pp-wave



- pp-wave SFT vertex $\equiv$ free massive boson ϕ (or fermion) on this geometry
- impose continuity conditions for ϕ and $\Pi \equiv \partial_t \phi$
- ϕ expressed in terms of $\cos \frac{2\pi n}{L_r}$ and $\sin \frac{2\pi n}{L_r}$ modes...

  looks like an inherently finite-volume computation...
- solution is surprisingly complicated...

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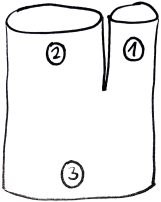

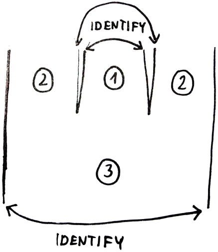

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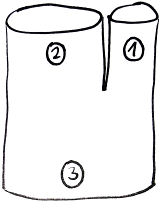

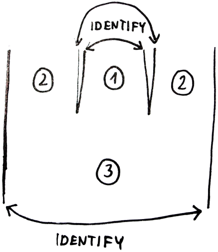

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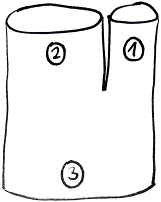

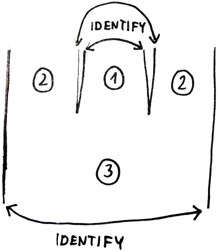

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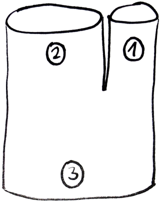

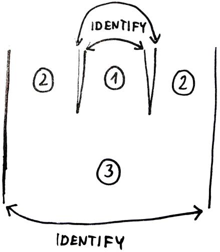

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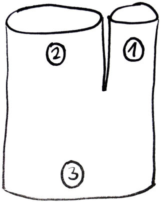

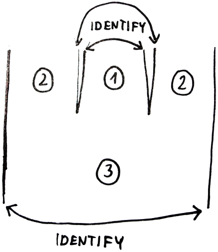

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- Implement these relations as operator equations acting on a state $|V\rangle \in \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \mathcal{H}_3$ — the SFT vertex
- The state has the form

$$|V\rangle = (\mathit{Prefactor}) \cdot \exp\left\{\frac{1}{2}\sum_{r,s=1}^{3}\sum_{n,m} N^{rs}_{nm}\, a_n^{+(r)} a_m^{+(s)}\right\}|0\rangle$$

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He, Schwarz, Spradlin, Volovich

$\rightarrow$ Lucietti, Schafer-Nameki, Sinha

- Exact expressions involve novel special functions $\Gamma_\mu(z)$

Lucietti, Schafer-Nameki, Sinha

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$$N^{rs}_{mn} = \left[\frac{\sqrt{(\omega^r_m + \mu\alpha_r)(\omega^s_n + \mu\alpha_s)}}{\omega^r_m + \omega^s_n} - \frac{\sqrt{(\omega^r_m - \mu\alpha_r)(\omega^s_n - \mu\alpha_s)}}{\omega^r_m + \omega^s_n} \right] \cdot (simple)$$

- Instead of integer mode numbers use rapidities... $p_k = M \sinh\theta_k$

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- The integer mode numbers (characteristic of finite volume) are completely inessential – they only obscure a simple underlying structure
- Pole at $\theta_1 = \theta_2 + i\pi$ (position of kinematical singularity as for form factors!) $\longrightarrow$ there should be some underlying axioms...
- Still some surprising features — the $\sin \frac{p_k L_1}{2}$ factors

Light-cone String Field Theory Vertex – the pp-wave

- In the pp-wave times, people used simplified expressions for N^{rs}_{nm} neglecting exponential $e^{-\mu\alpha_r}$ terms $\alpha_r = L_r/L_3$
  (these are exactly wrapping terms e^{-ML_r}!!)
- Going to an exponential basis (BMN basis) one got in this limit

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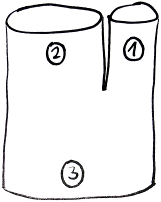
Step I) The decompactified string vertex



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Formulate functional equations...

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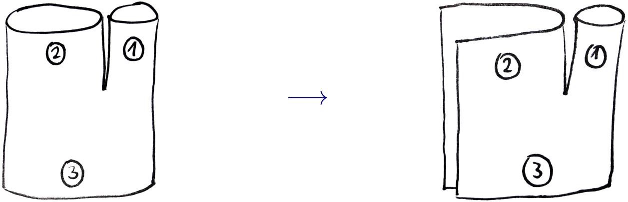

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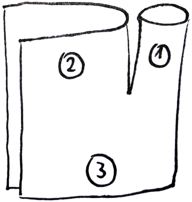

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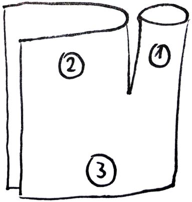

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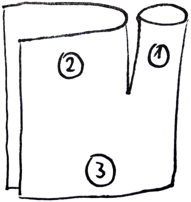

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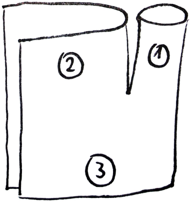

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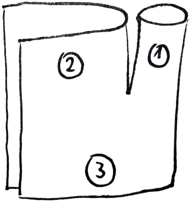

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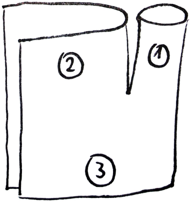

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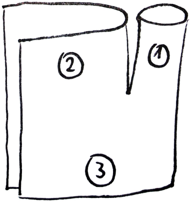

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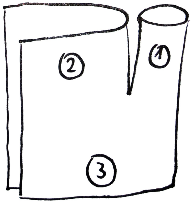

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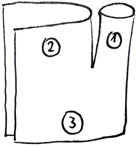
Functional equations for the (decompactified) string vertex

written here for two incoming particles and, for the moment, free theory



- The exact pp-wave solution, involving the $\Gamma_\mu(\theta)$ special function solves these equations and can be reconstructed from them!

$$n(\theta)n(\theta+i\pi) = -\frac{1}{2\pi^2} ML \sinh\theta \sin\frac{p(\theta)L}{2}$$

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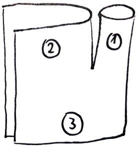

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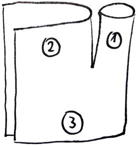

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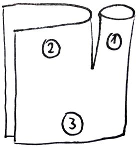

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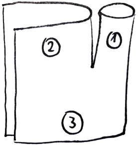

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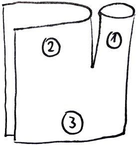

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Analyticity conditions

- We have to supplant the functional equations with some analyticity conditions
- We consider the simplest case of the Neumann coefficient $N^{33}(\theta_1, \theta_2)$ (more precisely the SFT amplitude with two incoming particles and vacuum on the remaining strings)
- By examining the pp-wave case we deduced the following condition:

  • The Neumann coefficient $N^{33}(\theta_1, \theta_2)$ should have zeroes on the real axis and not on the 'crossing line' $Im\,\theta = \pi$
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Step II) The string vertex — back to finite volume

We considered so far the 'decompactified string vertex'...

but ultimately we are interested in the finite volume one...

Use the same prescription (Pozsgay-Takacs) as for form factors...

This means that we neglect exponential corrections for strings #2 and #3 but **keep** all size dependence of string #1... (i.e. infinite set of wrapping corrections)

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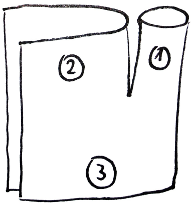

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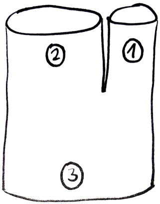

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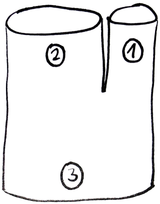

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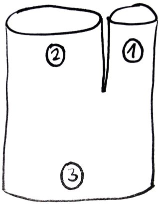

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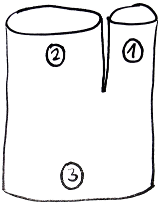

but ultimately we are interested in the finite volume one...

Use the same prescription (Pozsgay-Takacs) as for form factors...

This means that we neglect exponential corrections for strings #2 and #3 but **keep** all size dependence of string #1...
(i.e. infinite set of wrapping corrections)

What happens in $AdS_5 \times S^5$?

Novel kinematics

- Complex rapidities are defined on a covering of an elliptic curve
- The momentum p is *not* a well defined function
- Only e^{ip} is a well defined elliptic function
- The phase factors $e^{ip\mathbf{L}}$ make sense directly only for integer $\mathbf{L}$ which is nice from the point of view of $\mathcal{N}=4$ SYM...

Complicated dynamics

- The S-matrix does not depend on the difference of rapidities
- The S-matrix is nondiagonal which drastically complicates solving form factor axioms (which are a special case of our SFT axioms)

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The $AdS_5 \times S^5$ elliptic curve

- Excitations of the string in $AdS_5 \times S^5$ have a nonstandard dispersion relation

$$E = \sqrt{1 + 16 g^2 \sin^2 \frac{p}{2}} \qquad \text{where} \quad g^2 = \frac{\lambda}{16\pi^2} \equiv \frac{g_{YM}^2 N_c}{16\pi^2}$$

- This can be uniformized through elliptic functions
- The natural space for rapidities is the covering space of an elliptic curve – the complex plane RJ
- $z \to z+1$ is the novel periodicity, while $z \to z + \tau/2$ is the AdS analog of the crossing transformation in relativistic theories
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$$e^{-ip} = q^{\frac{1}{2}} \cdot \frac{\theta_0\left(z - \frac{1}{2} + \frac{\tau}{4}\right)\theta_0\left(z - \frac{1}{2} - \frac{3\tau}{4}\right)}{\theta_0^2\left(z - \frac{1}{2} - \frac{\tau}{4}\right)}$$

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The $AdS_5 \times S^5$ Neumann coefficient

- Consider arguably the simplest SFT amplitude with two particles on the incoming string #3 and vacua on the outgoing strings..

we will call it still as Neumann coefficient

- This corresponds to the N^{33}_{nm} pp-wave Neumann coefficient ($N^{33}(z_1, z_2)$ in our notation)
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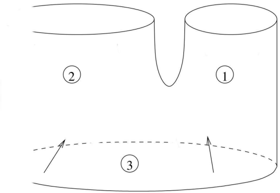

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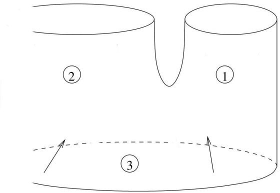

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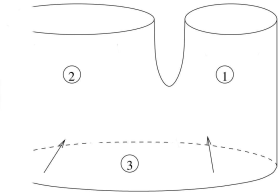

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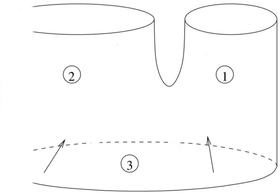

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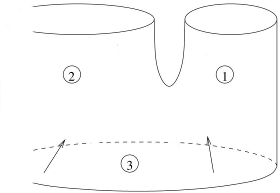

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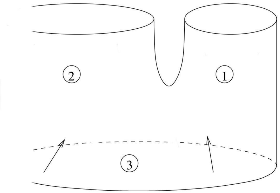

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$$\begin{aligned} \mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1, i_2} &= S^{kl}_{i_1 i_2}(z_1, z_2)\mathbf{N}^{33}_{\mathbf{L}}(z_2, z_1)_{l,k} \\ \mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1, i_2} &= e^{-ip(z_1)\mathbf{L}}\mathbf{N}^{33}_{\mathbf{L}}(z_2, z_1 - \tau)_{i_2, i_1} \\ \mathrm{res}_{z'=z}\, \mathbf{N}^{33}_{\mathbf{L}}(z' + \tau/2, z)_{\bar{i}, i} &= \left(1 - e^{ip(z)\mathbf{L}}\right) \end{aligned}$$

- Suppose that we know a solution of 2-particle form factor equations in AdS $\mathbf{F}(z_1, z_2)_{i_1, i_2}$ s.t.

$$\mathbf{F}(z + \tau/2, z)_{i_1, i_2} = \delta_{\bar{i}_1, i_2}$$

- Then we can solve the SFT vertex equations by

$$\mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1 i_2} = \mathbf{F}(z_1, z_2)_{i_1, i_2} \cdot N^{33}_{\mathbf{L}}(z_1, z_2)$$

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- $N^{33}_{\mathbf{L}}(z_2, z_1)$ incorporates all $\mathbf{L}$ dependence (all wrapping corrections w.r.t. string $\#1$) at any coupling
- Conversely, if we have *any* solution of the SFT axioms, then the ratio

$$\frac{\mathbf{N}^{33}_{\mathbf{L}}(z_1, z_2)_{i_1 i_2}}{N^{33}_{\mathbf{L}}(z_1, z_2)}$$

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- We will solve the equations following the general structure of the pp-wave answer:

$$N^{33}(\theta_1, \theta_2) = \frac{2\pi^2}{L} \cdot \underbrace{\frac{1 + \tanh\frac{\theta_1}{2}\tanh\frac{\theta_2}{2}}{M\cosh\theta_1 + M\cosh\theta_2}}_{P(\theta_1,\theta_2)} n(\theta_1)n(\theta_2)$$

- The denominator generalizes directly to the AdS case – however it in addition to the kinematical singularity pole at $\theta_1 = \theta_2 + i\pi$, it has another pole at $\theta_1 = -\theta_2 + i\pi$
- The $\tanh\frac{\theta_i}{2}$ factors in the numerator exactly cancel the unwanted pole
- Use the following ansatz in the general $AdS_5 \times S^5$ case...

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The kinematical $AdS_5 \times S^5$ Neumann coefficient — the $n(z)$ part

- $n(z)$ satisfies in particular

$$n(z)n(z+\tau/2) \propto \sin\frac{pL}{2}$$

- This already implies a set of zeroes – we should distribute them on the real line we consider $L = 2n$

$$n(z) \propto \sqrt{\frac{L}{2}} \prod_{k=1}^{n-1} \frac{\sqrt{1+16g^2\sin^2\frac{\pi k}{L}} - E(z)}{4g\sin\frac{\pi k}{L}}$$

- $n(z)$ also satisfies a monodromy property

$$n(z+\tau) = e^{-ip(z)L} n(z)$$

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We performed a number of checks:

1. We verified that our formula has the correct pp-wave limit
2. The L dependence in the weak coupling limit agrees with spin chain calculations
3. We observe 'vanishing of monodromy' in the asymptotic large L limit i.e. for any L we have

$$\lim_{\varepsilon \to 0^+} \frac{n(z+\tau-i\varepsilon)}{n(z+i\varepsilon)} = e^{-ip(z)L}$$

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Conclusions & outlook

- We propose a framework for formulating functional equations for string interactions (light cone string field theory vertex) when the worldsheet theory is **integrable**
- This approach should work in particular for strings in the full $AdS_5 \times S^5$ geometry
- A key step is the existence of an infinite volume setup, which allows for formulating functional equations incorporating e.g. crossing
- We reproduced pp-wave string field theory formulas for the Neumann coefficients
- We solved for the 'kinematical' part of the $AdS_5 \times S^5$ Neumann coefficient describing exact volume dependence (for even L) at any coupling – may describe all order wrapping w.r.t. one string

- Solve the form factor equations — to obtain the matrix part...
- Understand links with the subsequent 'hexagon' approach of Basso, Komatsu, Vieira

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