

Reheating and Primordial Gravitational Waves in Generalized Galilean Genesis



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[S. Nishi, T. Kobayashi, [arxiv: 1501.02553 [hep-th]]

[S. Nishi, T. Kobayashi, [arxiv: 1601.06561 [hep-th]]

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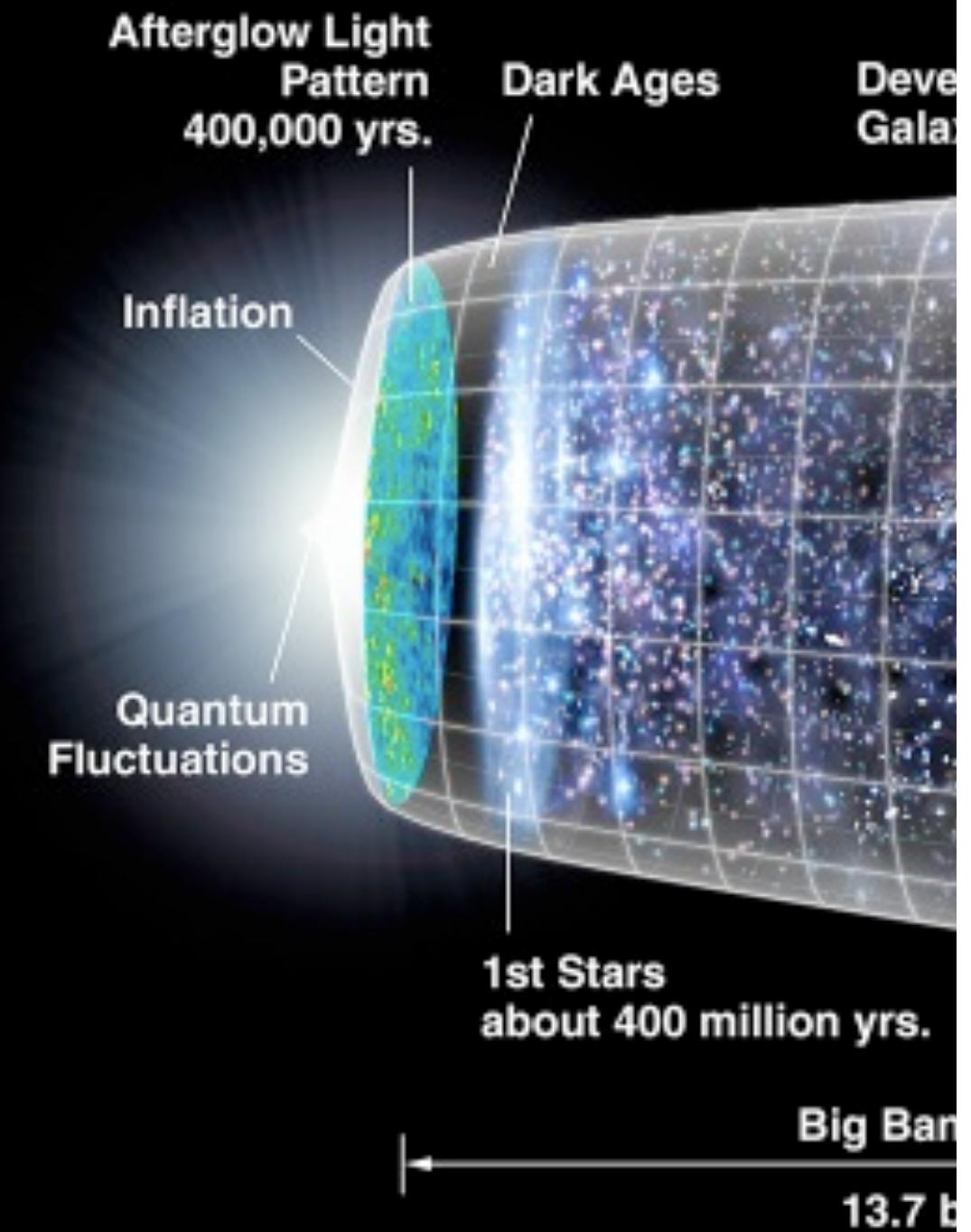
Outline



- **Introduction**
- Original model of Galilean Genesis
- Generalized Galilean Genesis
- Reheating phase
- Gravitational waves
- Conclusions

Introduction

- There are many kinds of models which explain the early universe.
- Inflation explains the observational result well.
- Galilean Genesis is an alternative to Inflation.



Galilean Genesis - Horndeski theory

- The most general scalar-tensor theory
- Field eqs. have no 3rd and higher derivative terms
- Generalized Galilean Genesis is subclass of this theory.

$$S_{\text{Hor}} = \int d^4x \sqrt{-g} \left\{ G_2(\phi, X) - G_3(\phi, X) \square \phi + G_4(\phi, X) R \right. \\ \left. + G_{4X} [(\square \phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2] + G_5(\phi, X) G^{\mu\nu} \nabla_\mu \nabla_\nu \phi \right. \\ \left. - \frac{1}{6} G_{5X} [(\square \phi)^3 - 3 \square \phi (\nabla_\mu \nabla_\nu \phi)^2 + 2 (\nabla_\mu \nabla_\nu \phi)^3] \right\} \\ X := -g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi / 2$$

[G. W. Horndeski (1974)]
[T. Kobayashi, M. Yamaguchi and J. Yokoyama (2014)]

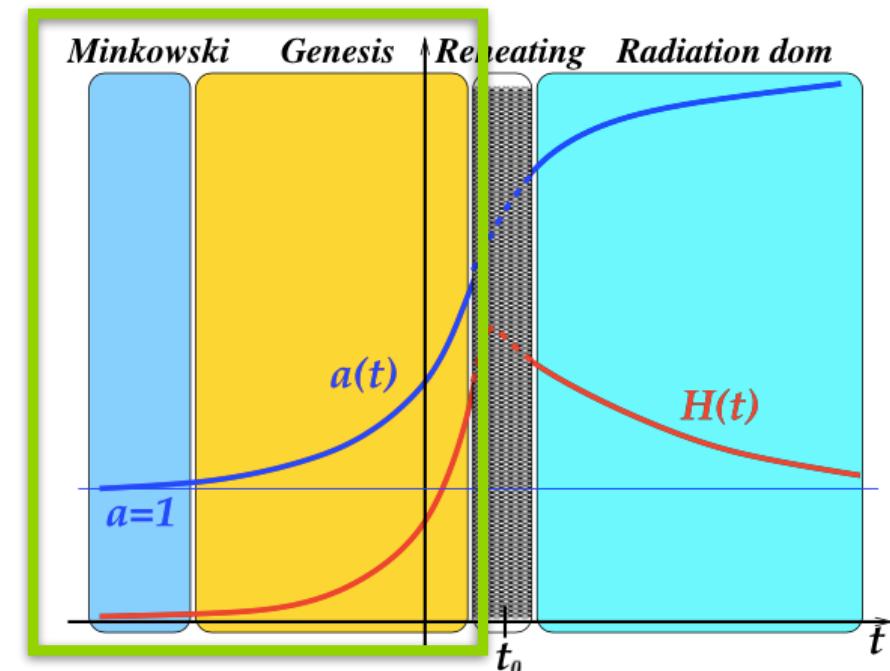
Introduction - motivation

- Only inflation can explain the early universe?

compare genesis to the other inflation models and discuss observational implications.

In the previous study...

- Background evolution
- Perturbations
 - Scalar, Tensor



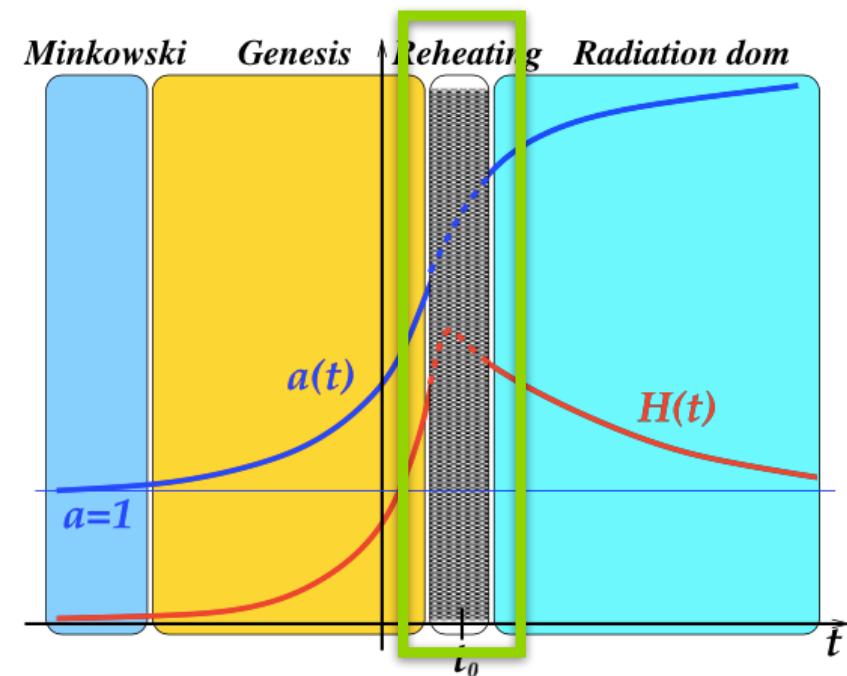
Introduction - motivation

- Only inflation can explain the early universe?

compare genesis to the other inflation models and discuss observational implications.

In this talk...

- Matter creation
- Gravitational Waves



Outline

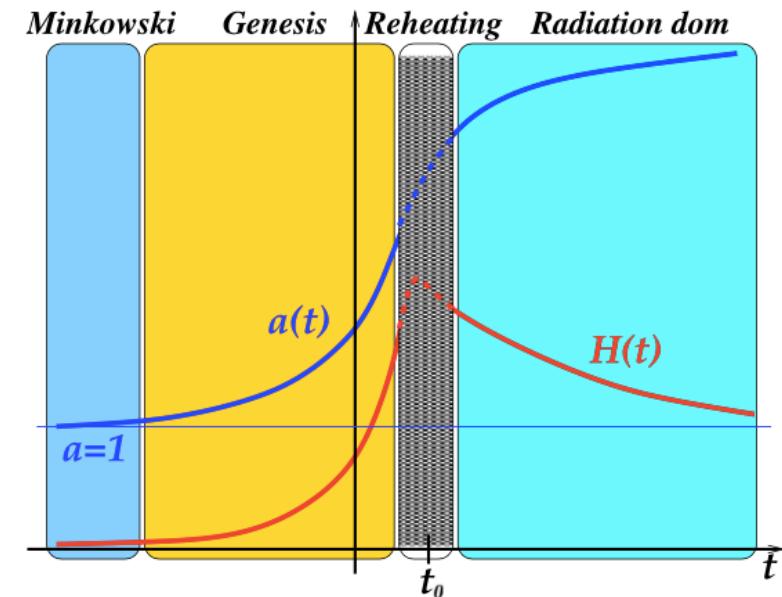


- Introduction
- **Original model of Galilean Genesis**
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Galilean Genesis - Previous work

- Alternative to inflation model
- Null Energy Condition is violated stably

- Previous work
 - action



$$\mathcal{S} = \int dx^4 \sqrt{-g} \left[f^2 e^{2\phi} (\partial\phi)^2 + \frac{f^3}{\Lambda^3} (\partial\phi)^2 \square\phi + \frac{f^3}{2\Lambda^3} (\partial\phi)^4 \right]$$

Galilean Genesis - Previous work

- solutions

$t \rightarrow -\infty$

$$e^{\lambda\phi} \propto (-t)^{-1}$$

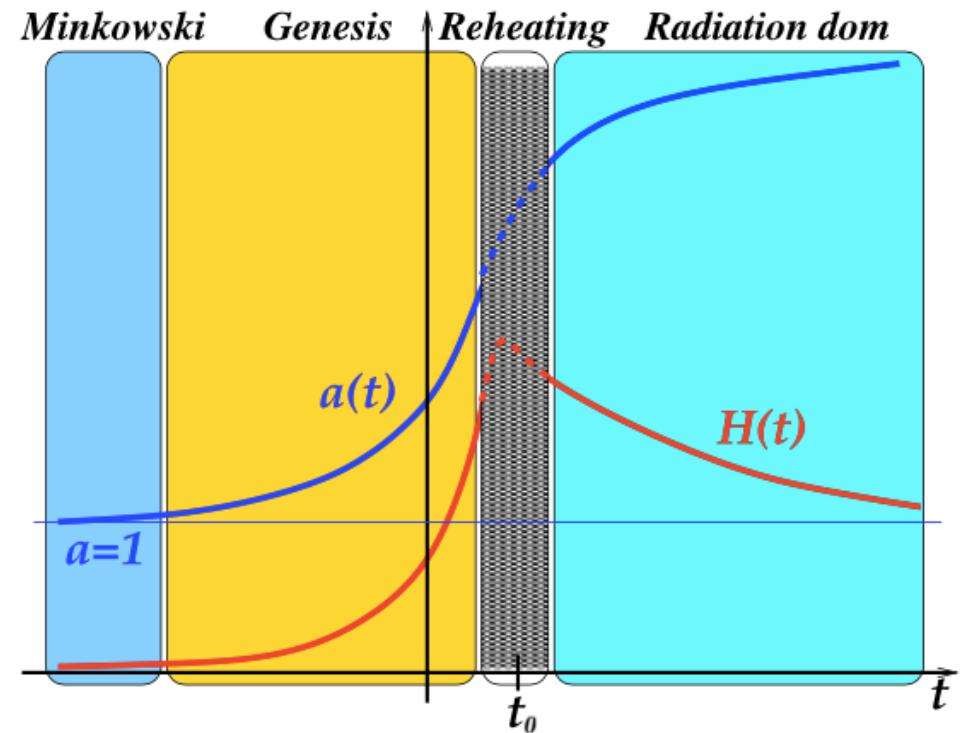
$$H(t) \simeq -\frac{f^2}{3M_{Pl}^2} \frac{1}{H_0^2 t^3}$$

$$a(t) \simeq 1$$

-> Minkowski space-time

$t \rightarrow t_0$ (numerical analysis)

$$a(t) = \exp \left[\frac{8f^2}{3H_0^2 M_{Pl}^2} \frac{1}{(t_0 - t)^2} \right]$$



$$H(t) \simeq \frac{16f^2}{3M_{Pl}^2} \frac{1}{H_0^2(t_0 - t)^3}$$

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Galilean Genesis - Generalized model



- include the various models of Genesis
- parameter α
arbitrary function $g_i(Y)$

$$G_2 = e^{2(\alpha+1)\lambda\phi} g_2(Y), \quad G_3 = e^{2\alpha\lambda\phi} g_3(Y),$$
$$G_4 = \frac{M_{\text{Pl}}^2}{2} + e^{2\alpha\lambda\phi} g_4(Y), \quad G_5 = e^{-2\lambda\phi} g_5(Y). \quad Y := e^{-2\lambda\phi} X$$

- Example - Original model

$$g_2 = 2f^2Y + 2\frac{f^3}{\Lambda^3}Y^2, \quad g_3 = 2\frac{f^3}{\Lambda^3}Y, \quad g_4 = g_5 = 0, \quad \alpha = \lambda = 1$$

Galilean Genesis



solutions $(-\infty < t < 0)$

- Friedmann eq. $\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi} \hat{\rho}(Y_0) \simeq 0$

$$\hat{\rho}(Y) := 2Yg'_2 - g_2 - 4\lambda Y (\alpha g_3 - Yg'_3)$$

- $\hat{\rho} = 0$ $Y_0 = e^{-2\lambda\phi} X = const.$

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \propto \frac{1}{(-t)^{2\alpha+1}}, \quad a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \right]$$

- Evolution eq. NEC violated
$$\mathcal{P} \simeq 2\mathcal{G}(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi} \hat{p}(Y_0) \simeq 0 \quad \hat{p} < 0$$

Galilean Genesis



solutions $(-\infty < t < 0)$

- Friedmann eq. $\mathcal{E} \simeq e^{2(\alpha+1)\lambda\phi} \hat{\rho}(Y_0) \simeq 0$

$$\hat{\rho}(Y) := 2Yg'_2 - g_2 - 4\lambda Y (\alpha g_3 - Yg'_3)$$

- $\hat{\rho} = 0$ $Y_0 = e^{-2\lambda\phi} X = const.$

higher order of t^{-1}

$$e^{\lambda\phi} \propto (-t)^{-1}, \quad H(t) \propto \frac{1}{(-t)^{2\alpha+1}}, \quad a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \right] + \dots$$

- Evolution eq.

NEC violated

$$\mathcal{P} \simeq 2\mathcal{G}(Y_0)\dot{H} + e^{2(\alpha+1)\lambda\phi} \hat{p}(Y_0) \simeq 0$$

$$\hat{p} < 0$$

Background



- Numerical analysis

corresponding to

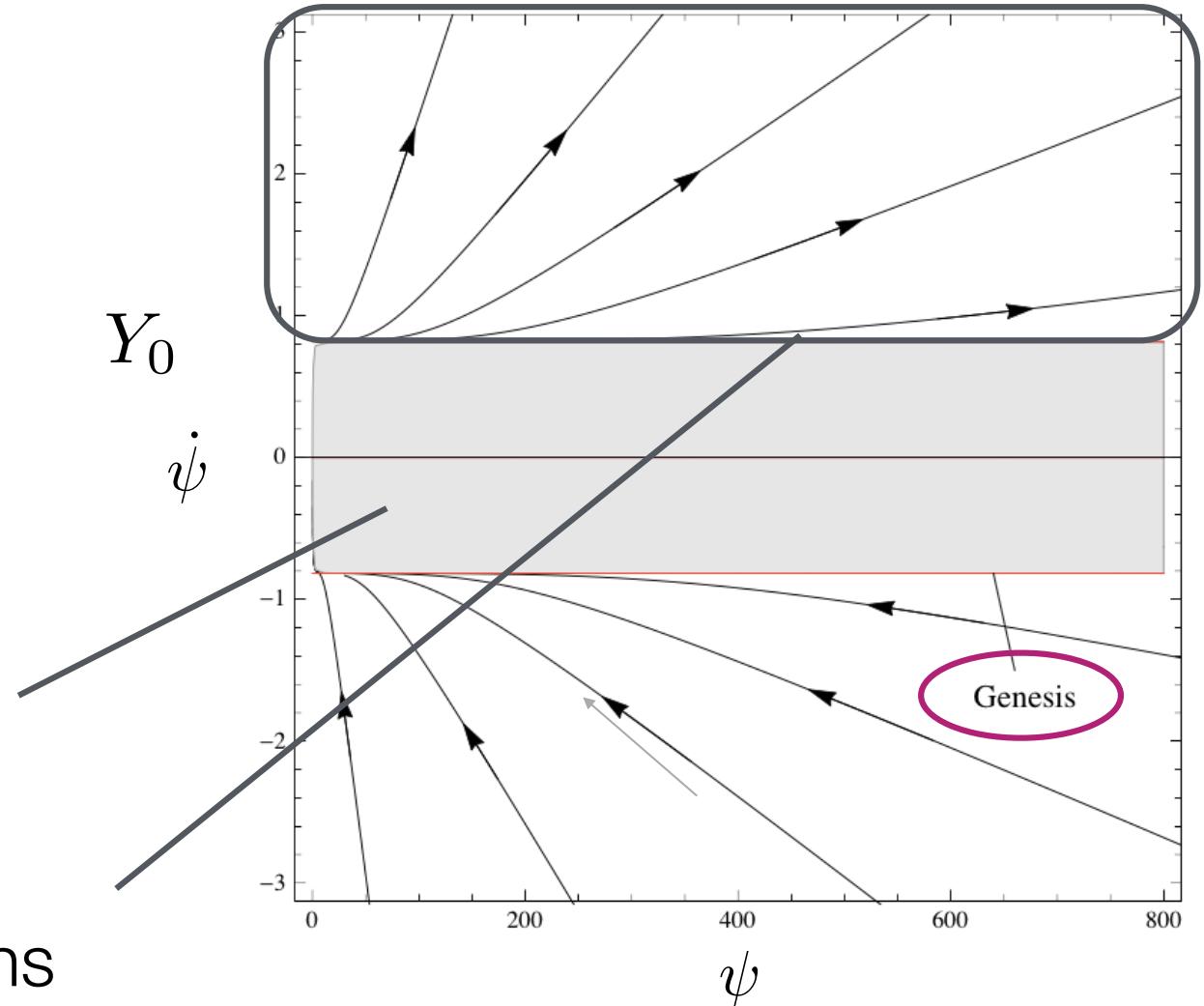
$$\rightarrow \psi = e^{-\lambda\phi}$$

$$Y_0 \propto \dot{\psi}$$

no solution of $H(t)$

time reversal solutions

$$\begin{aligned}\alpha &= 1 \\ g_2 &= -Y + Y^2 \\ g_3 &= Y \\ g_4 &= 0 \\ g_5 &= 0\end{aligned}$$



Perturbation (tensor)



Action

$$\mathcal{S}_T^{(2)} = \frac{1}{8} \int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2} (\nabla^2 h_{ij})^2 \right]$$

$\simeq const.$

- > in Minkowski spacetime fluctuation do not grow
- > too small to detect...

Stability

sound speed $\rightarrow c_t^2 = \frac{\mathcal{F}_T}{\mathcal{G}_T} = \frac{M_{\text{Pl}}^2 + 4\lambda Y_0 g_5(Y_0)}{\mathcal{G}(Y_0)}$

$$\mathcal{G}(Y_0) > 0, \quad M_{\text{Pl}}^2 + 4\lambda Y_0 g_5(Y_0) > 0$$

Perturbation (scalar)



- Lagrangian

$$\mathcal{L}_\zeta = \mathcal{A}(Y_0)(-t)^{2\alpha} \left[\dot{\zeta}_k^2 - k^2 c_s^2 \zeta_k^2 \right]$$

- Wave eq.

$$\ddot{\zeta}_k - \frac{2\alpha}{(-t)} \dot{\zeta}_k + k^2 c_s^2 \zeta_k = 0$$

- solution

$$\zeta_k = \frac{1}{2} \sqrt{\frac{\pi}{\mathcal{A}(Y_0)}} (-t)^\nu H_\nu^{(1)}(\omega_k(-t)), \quad \nu = \frac{1}{2} - \alpha$$

- $0 < \alpha < \frac{1}{2}$: decaying mode + const.
- $\alpha > \frac{1}{2}$: growing mode + const.

Perturbation (scalar)



- $0 < \alpha < \frac{1}{2}$ $\mathcal{P}_\zeta(k) = \frac{c_s^{-2\nu} 2^{2\nu-2} \Gamma(\nu)^2}{\pi^3 \mathcal{A}(Y_0)} k^{3-2\nu}$

$$n_s = 2\alpha + 3$$

- $\alpha > \frac{1}{2}$ $\mathcal{P}_\zeta(k) = \frac{c_s^{2\nu} 2^{-2\nu-3} \Gamma(\nu)^2 (-t_{end})^4}{\pi^3 \mathcal{A}(Y_0)} k^{3+2\nu}$

$$n_s = 5 - 2\alpha$$

genesis phase
ends at t_{end}

- $\alpha = 2$: flat spectrum
- $\alpha \neq 2$: introducing the curvaton field

Perturbation (scalar)

- Stability
 - Lagrangian

$$\mathcal{L}_\zeta = \mathcal{A}(Y_0)(-t)^{2\alpha} \left[\dot{\zeta}_k^2 - k^2 c_s^2 \zeta_k^2 \right]$$

- sound speed

$$c_s^2 = \frac{\mathcal{F}_s}{\mathcal{G}_s} = \frac{\xi'(Y_0)\hat{p}(Y_0)}{\xi(Y_0)\hat{\rho}'(Y_0)} = \text{const.}, \quad \xi(Y) := -\frac{Y\mathcal{G}(Y)}{\hat{p}(Y)}$$

->

$$\hat{\rho}'(Y_0) > 0, \quad \xi'(Y_0) < 0$$

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Matter Creation - scenario

- Massless scalar field matter χ is generated.

$$\mathcal{L}_\chi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi$$

- Genesis at the end of genesis

$$a \simeq a_G \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \right] = \delta_* \ll 1$$

Kination

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2}R + X \quad (\text{X : Kinetic term})$$



Matter Creation

- Massless scalar field

How \mathcal{L} changes?

introduce \downarrow in $g_2(Y)$

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{Pl}^2 R + f^2 \frac{e^{2\pi}}{1 + \beta e^{2\pi}} (\partial\pi)^2 + \frac{f^3}{\Lambda^3} (\partial\pi)^2 \square\pi + \frac{f^3}{2\Lambda^3} (\partial\pi)^4 \right]$$

- Genesis



Kination

[D. Pirtskhalava, L. Santoni, E. Trincherini, P. Uttayarat (2014)]

$$\mathcal{L} \simeq \frac{M_{Pl}^2}{2} R + X \quad (X : \text{Kinetic term})$$

$$L \sim \alpha(-\tau) = \delta_* \ll 1$$

time



Genesis



Kination

Radiation

Matter Creation - scenario



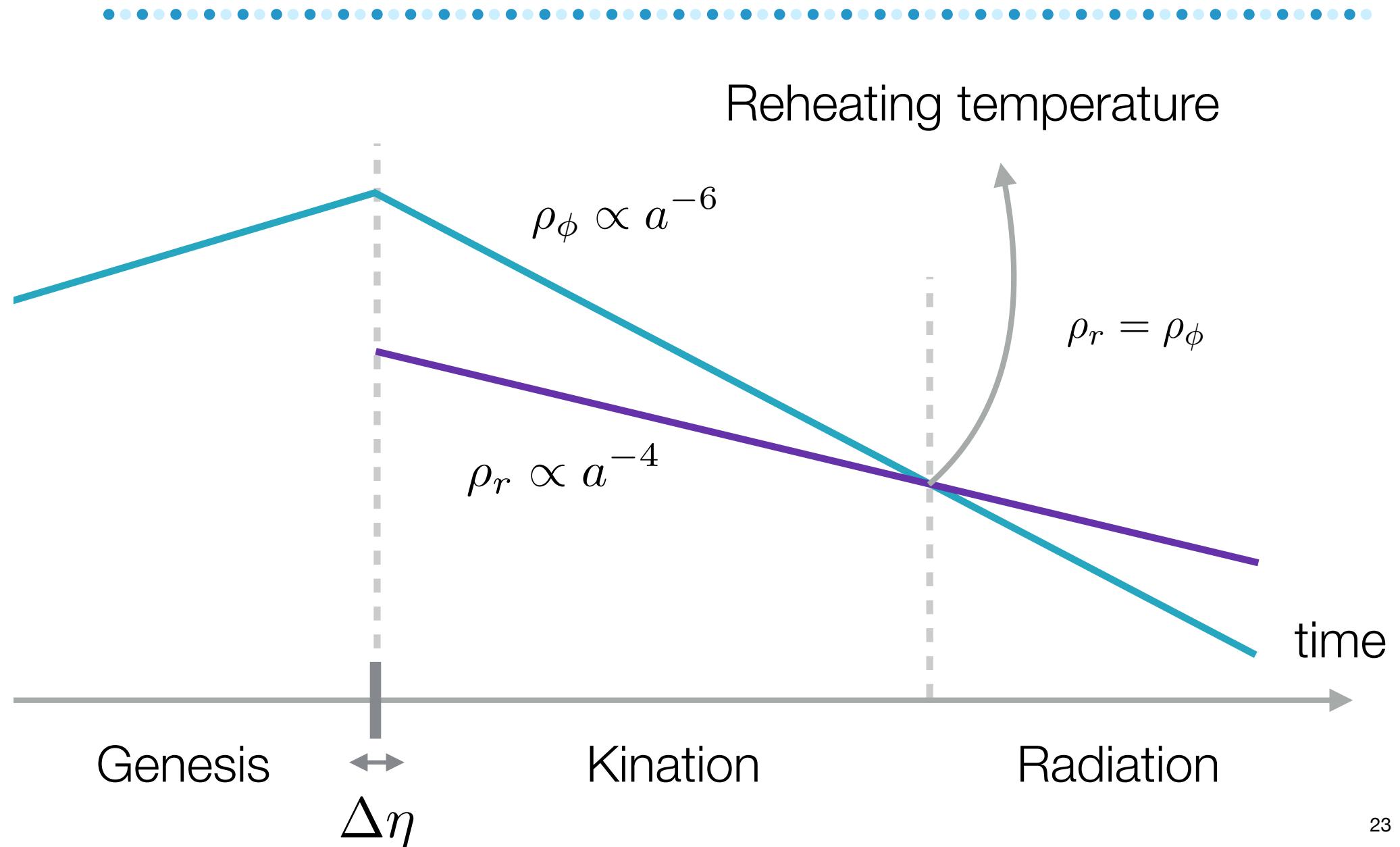
- Scalar field ϕ (generate the Genesis phase)
in kination phase

$$\rho_\phi \propto a^{-6} \quad \longleftrightarrow \quad \mathcal{L} \simeq \frac{M_{Pl}^2}{2} R + X$$

- Scalar field χ (created matter)
energy density of radiation

$$\rho_r \propto a^{-4} \quad \longleftrightarrow \quad \rho_\chi = \frac{1}{2\pi^2 a^4} \int_0^\infty k^3 |\beta_k(\infty)|^2 dk$$

Matter Creation - scenario



Matter Creation



- Solution of χ

$$a(\eta)\chi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2k}}e^{ik\eta} + \frac{\beta_k(\eta)}{\sqrt{2k}}e^{-ik\eta}$$

- Definition of β_k and energy density

$$\beta_k(\eta) = -\frac{i}{2k} \int_{-\infty}^{\eta} e^{-2iks} \frac{a''}{a} ds \quad \rho_{\chi} = \frac{1}{2\pi^2 a^4} \int_0^{\infty} k^3 |\beta_k(\infty)|^2 dk$$



$$\rho_{\chi} = -\frac{1}{128\pi^2 a^4} \int_{-\infty}^{\infty} d\eta_1 \int_{-\infty}^{\infty} d\eta_2 \ln(m|\eta_1 - \eta_2|) V'(\eta_1) V'(\eta_2)$$

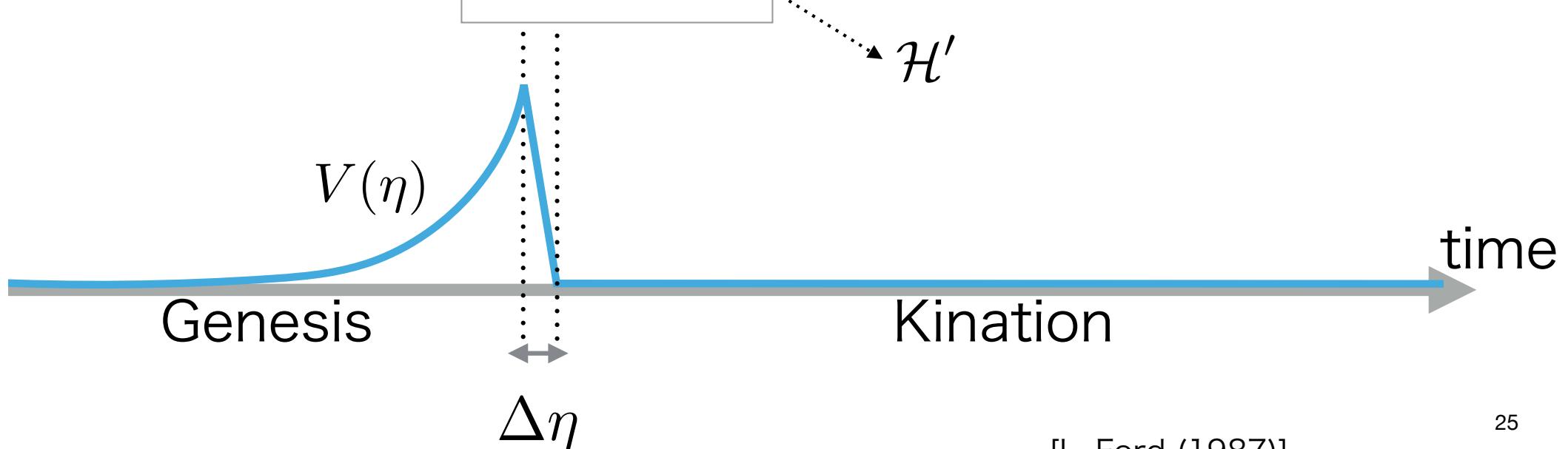
Matter Creation



$$\rho_\chi = -\frac{1}{128\pi^2 a^4} \int_{-\infty}^{\infty} d\eta_1 \int_{-\infty}^{\infty} d\eta_2 \ln(m|\eta_1 - \eta_2|) V'(\eta_1) V'(\eta_2)$$

- genesis ends at $\eta = \eta_*$
kination starts at $\eta = \eta_* + \Delta\eta$

- seek $V(\eta) = \boxed{\frac{f''f - (f')^2/2}{f^2}}$ $f(\eta) := a^2(\eta)$



Matter Creation



$$\rho_r = -\frac{1}{128\pi^2 a^4} \int_{-\infty}^{\infty} d\eta_1 \int_{-\infty}^{\infty} d\eta_2 \ln(m|\eta_1 - \eta_2|) V'(\eta_1) V'(\eta_2)$$

- assume

$$V(\eta) = \frac{f'' f - (f')^2 / 2}{f^2}$$

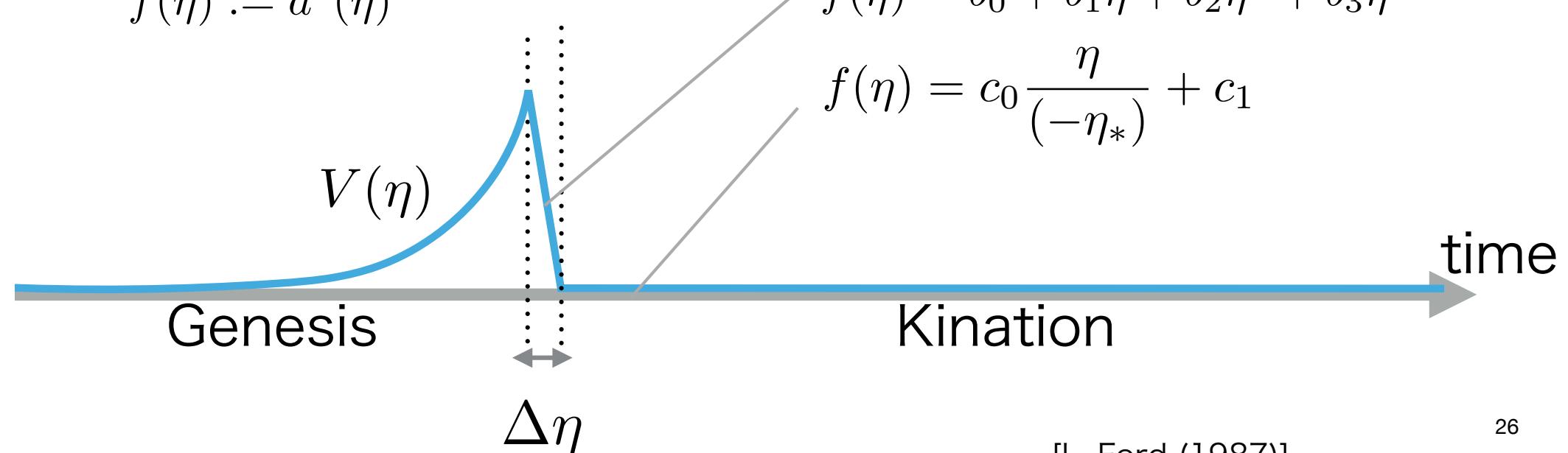
$V(\eta)$ approximately a straight line



$$f(\eta) := a^2(\eta)$$

$$f(\eta) = b_0 + b_1 \eta + b_2 \eta^2 + b_3 \eta^3$$

$$f(\eta) = c_0 \frac{\eta}{(-\eta_*)} + c_1$$



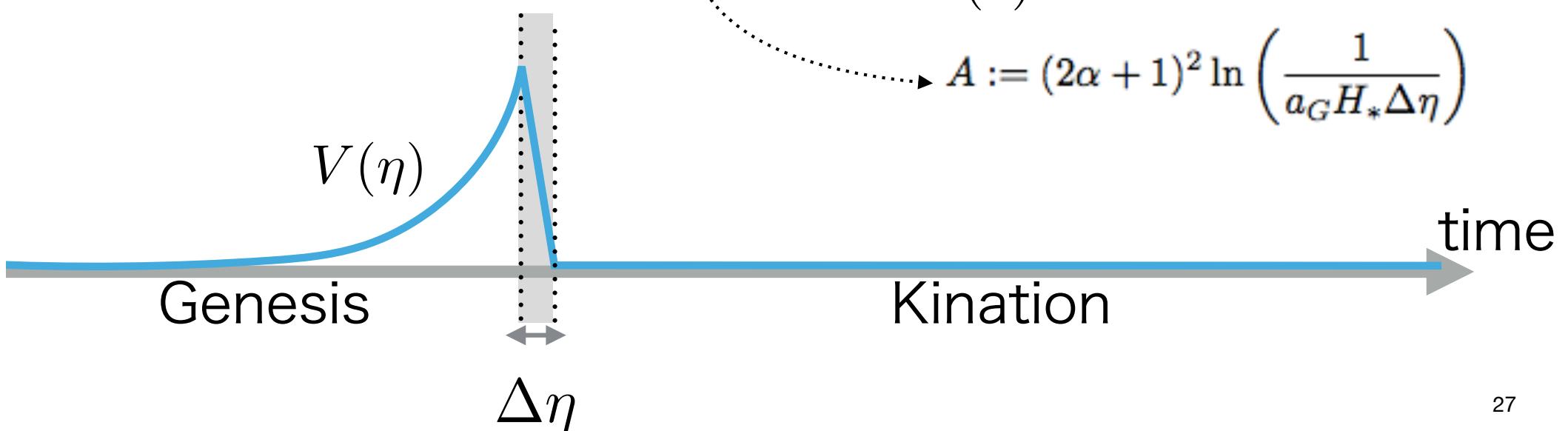
Matter Creation

- Therefore...

Matter χ is generated in $\Delta\eta$

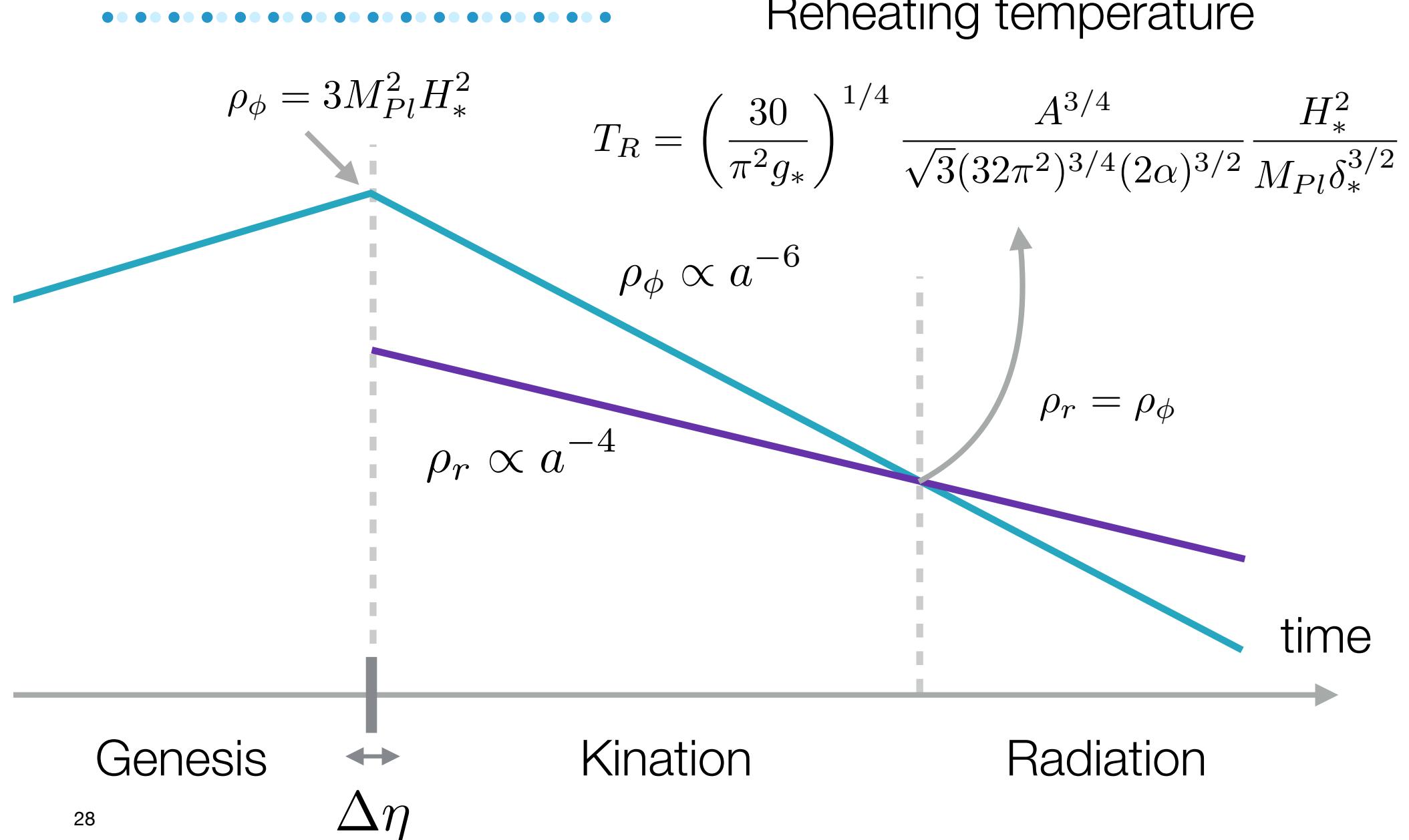
$$\rho_\chi = \frac{(2\alpha + 1)^2}{32\pi^2} \ln\left(\frac{1}{a_G H_* \Delta\eta}\right) \frac{h_0^2}{(-t_*)^{4(\alpha+1)}} \left(\frac{a_G}{a}\right)^4$$

assume $\mathcal{O}(1)$



in inflation $T_R \sim \frac{H_{inf}^2}{M_{Pl}}$

Matter Creation



Matter Creation - summary



- massless scalar generated in $\Delta\eta$
- How we set the end of genesis ($\eta = \eta_*$) determine ρ_χ and T_R .
- H_* of genesis can be smaller than that of inflation.

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Gravitational Waves - previous study

- from the quadratic action

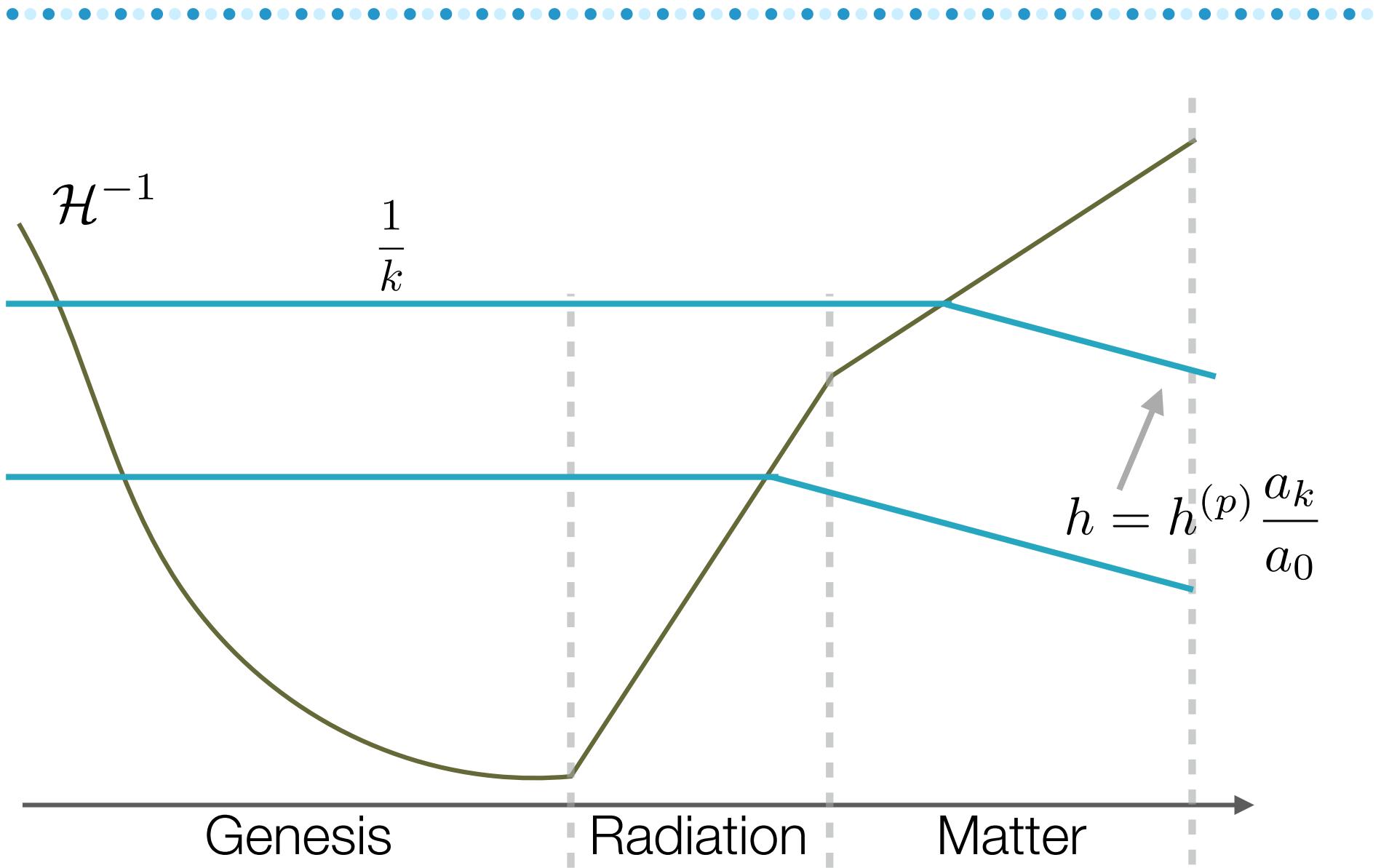
$$\mathcal{S}_T^{(2)} = \frac{1}{8} \int dt d^3x a^3 \left[\mathcal{G}_T \dot{h}_{ij}^2 - \frac{\mathcal{F}_T}{a^2} (\nabla^2 h_{ij})^2 \right]$$

$\simeq const.$

in Minkowski space-time GWs do not grow
→ too small to detect

- In this study...
 - Power spectrum grows during high frequency area In this scenario.

Gravitational Waves



Gravitational Waves - power spectrum

- Power spectrum

Horizon cross

$$\Omega_{\text{gw}} = \Omega_{\text{gw}}^{(p)}(k) \times \begin{cases} \frac{k_R}{k} \frac{k_{\text{eq}}^2}{k_R^2} \frac{k_0^4}{k_{\text{eq}}^4} & (k_R < k < k_*) \\ \frac{k_{\text{eq}}^2}{k^2} \frac{k_0^4}{k_{\text{eq}}^4} & (k_{\text{eq}} < k < k_R) \\ \frac{k_0^4}{k^4} & (k_0 < k < k_{\text{eq}}) \end{cases}$$

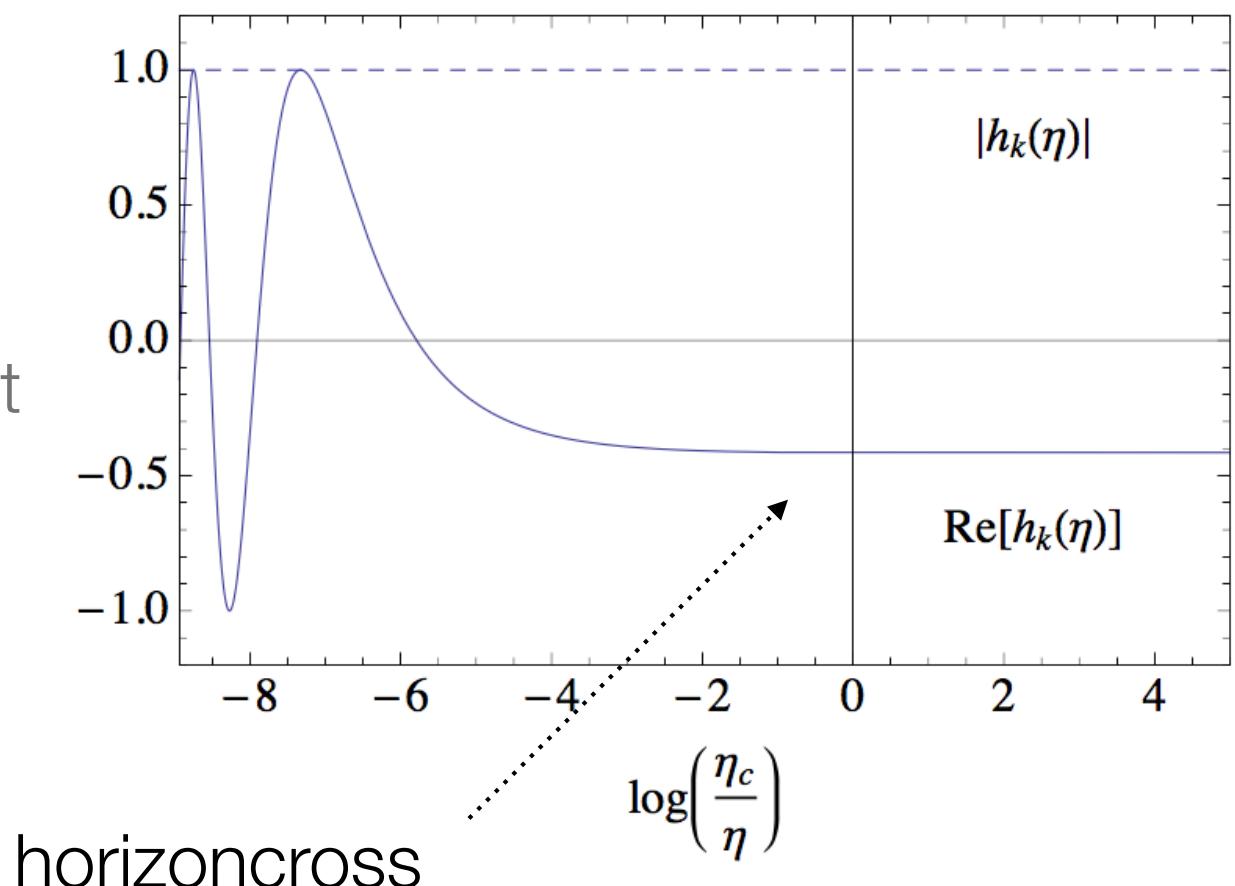
Kination
Radiation
Matter

Gravitational Waves

- h_k do not grow in genesis.

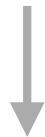
$$h_k = \frac{1}{a} \sqrt{\frac{2}{Gc_t k}} e^{-ic_t k\eta}$$

- $|h_k|$ do not change at the horizoncross.



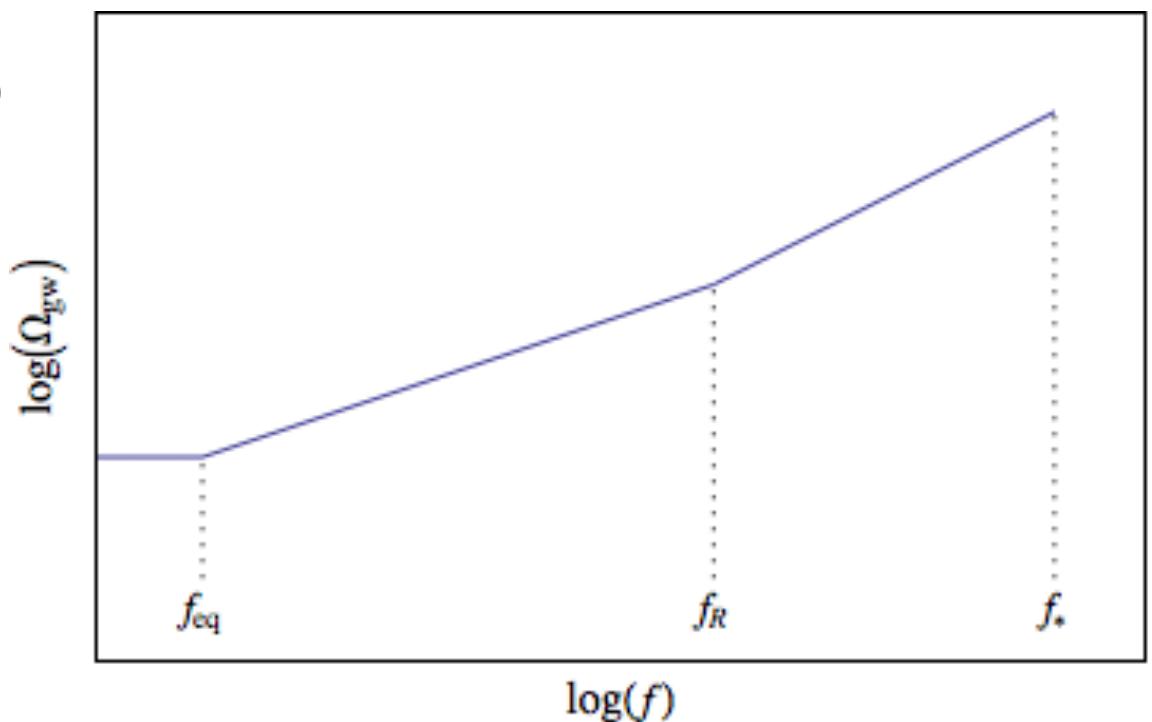
Gravitational Waves

$$\Omega_{\text{gw}} = \begin{cases} \propto k^3 & (k_R < k < k_*) \\ \propto k^2 & (k_{\text{eq}} < k < k_R) \\ \text{const.} & (k_0 < k < k_{\text{eq}}) \end{cases}$$



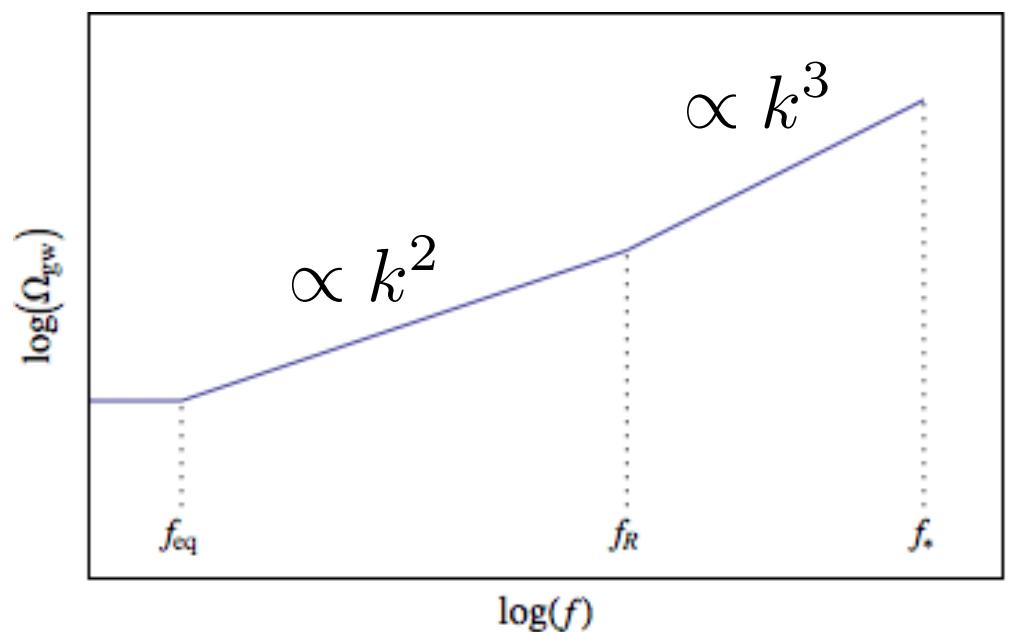
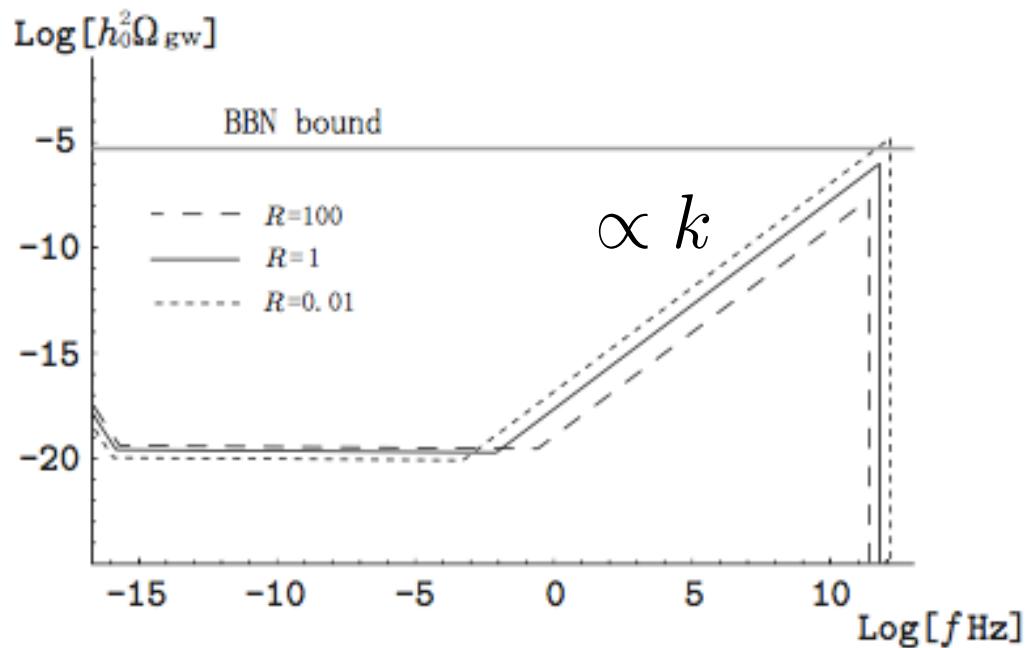
$$\Omega_{gw}(k_R) \simeq \frac{\delta_*^2 T_R^4}{M_{Pl}^2 H_*^2} \times 10^{-2}$$

$$\Omega_{gw}(k_*) \simeq \frac{H_*^5}{M_{Pl}^3 \delta_*^2 T_R^2} \times 10^{-7}$$



Gravitational Waves

- Inflation
- Genesis



[H. Tashiro, T. Chiba, M. Sasaki, (2012)]

Gravitational Waves



- genesis

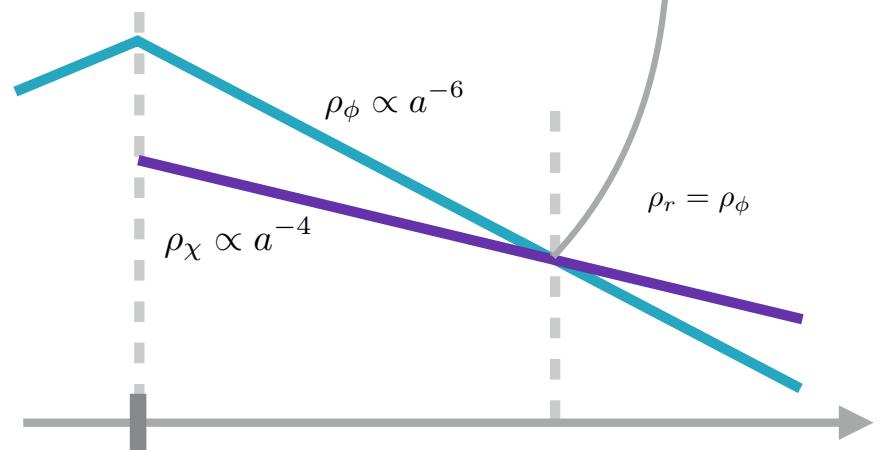
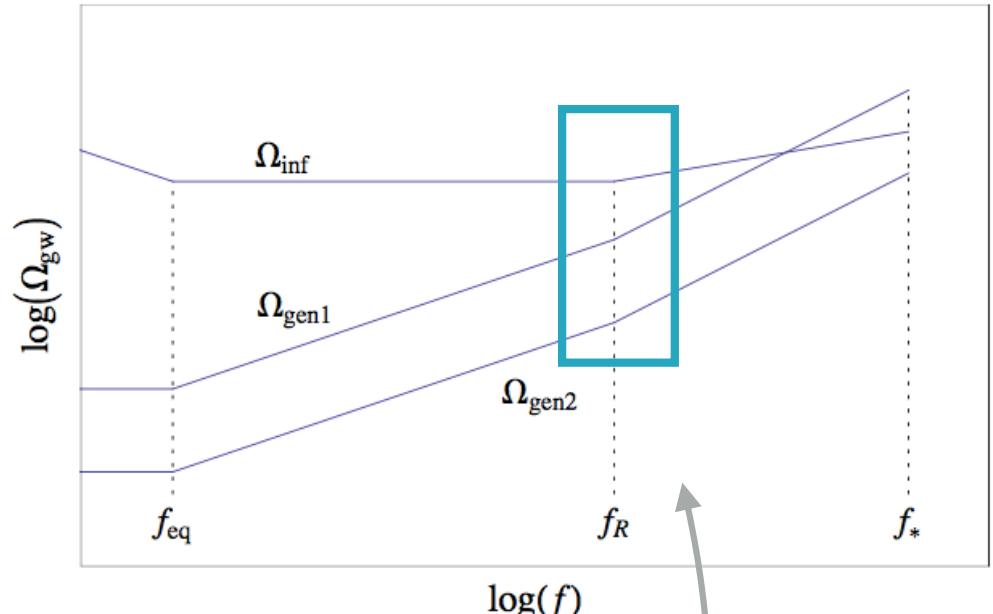
$$\Omega_{gw}(k_R) \simeq \left(\frac{H_*}{M_{Pl}} \right)^2 \left(\frac{a_R}{a_G} \right)^{-4} \times 10^{-5}$$

$$\Omega_{gw}(k_*) \simeq \left(\frac{H_*}{M_{Pl}} \right)^2 \left(\frac{a_R}{a_G} \right)^2 \times 10^{-5}$$

- inflation

$$\Omega_{gw}^{inf} \simeq \left(\frac{H_{inf}}{M_{Pl}} \right)^2 \times 10^{-5}$$

Ω_{gw}^{gen} can not be larger than Ω_{gw}^{inf}



Gravitational Waves

- General cases

$$\Omega_{\text{gw}}(f) = 10^{-31} \cdot 3^{-\frac{1}{2+\alpha}} \left(\frac{32\pi^2}{A}\right)^{\frac{1+2\alpha}{2(2+\alpha)}} \left(\frac{\pi^2 g_*}{30}\right)^{\frac{1+\alpha}{2(2+\alpha)}} \tilde{h}^{\frac{1}{2+\alpha}} \left(\frac{T_R}{M_{pl}}\right)^{\frac{\alpha}{2+\alpha}} \left(\frac{f}{100 \text{ Hz}}\right)^3$$

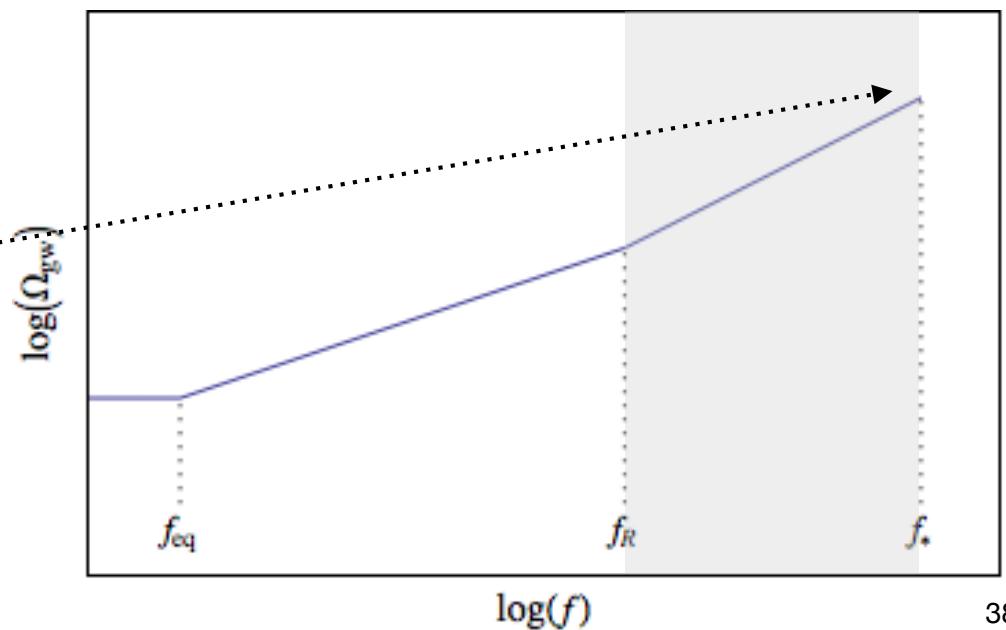
-> fix α and energy scales

in any model

$$\Omega_{gw} \sim 10^{-12}$$

$$f = 100 \text{ MHz}$$

$$f_R \simeq 0.026 \left(\frac{g_*}{106.75}\right)^{1/6} \left(\frac{T_R}{10^6 \text{ GeV}}\right) \text{ Hz}$$



Matter Creation - conditions

- for the end of genesis

assume

$$h_0 = BM_{Pl}^{-2}\mu^{-2\alpha+2}$$

$$\begin{aligned} a \simeq a_G & \left[1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \right] \\ &= \delta_* \ll 1 \end{aligned}$$



$$\frac{H_*}{M_{Pl}} \ll B^{-1/2\alpha} \left(\frac{\mu}{M_{Pl}} \right)^{(\alpha-1)/\alpha}$$

- scale factor grows

$$a_R > a_G$$



$$\frac{H_*}{M_{Pl}} < \left(\frac{96\pi^2}{A} \right)^{(2\alpha+1)/2} B \left(\frac{\mu}{M_{Pl}} \right)^{2(1-\alpha)}$$

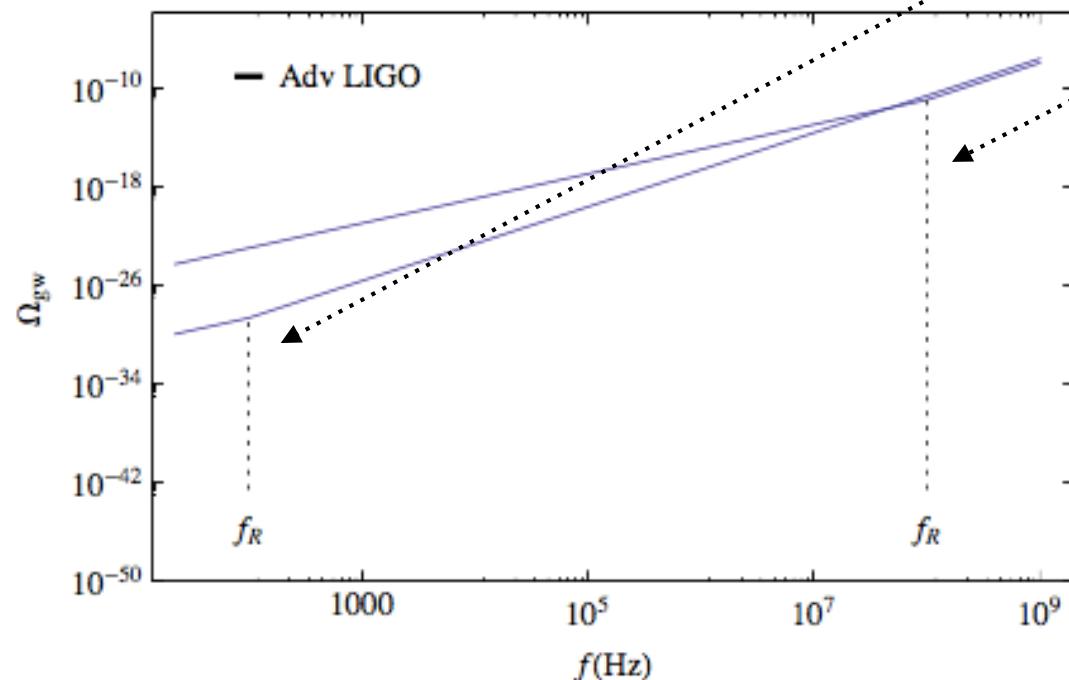
Gravitational Waves - example 1

- Original model

$$g_2 = -2f^2Y + \frac{2f^3}{\Lambda^3}Y^2, \quad g_3 = \frac{2f^3}{\Lambda^3}Y,$$

$$g_4 = g_5 = 0, \quad \lambda = 1, \quad \alpha = 1,$$

$$\frac{\mu}{\Lambda} = 10^2 \text{ and } T_R \sim 10^{10} \text{ GeV}$$



$$\frac{\mu}{\Lambda} = 1 \text{ and } T_R \sim 10^{16} \text{ GeV}$$

$$\frac{H_*}{M_{Pl}} \ll B^{-1/2\alpha} \left(\frac{\mu}{M_{Pl}} \right)^{(\alpha-1)/\alpha}$$

$$\frac{H_*}{M_{Pl}} < \left(\frac{96\pi^2}{A} \right)^{(2\alpha+1)/2} B \left(\frac{\mu}{M_{Pl}} \right)^{2(1-\alpha)}$$

Gravitational Waves - example 2

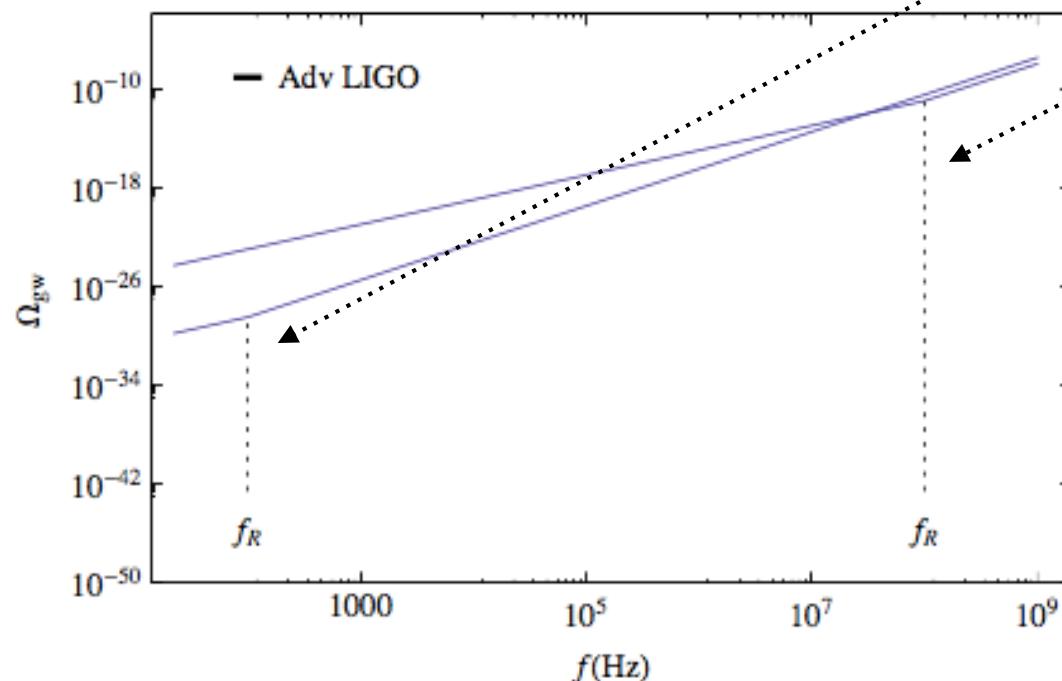


- $\alpha = 2$ (the scale invariant curvature perturbation)

$$g_2 = 2f^2Y + \frac{2f^3}{\Lambda^3}Y^2, \quad g_3 = \frac{2f^3}{\Lambda^3}Y,$$

$$g_4 = g_5 = 0, \quad \lambda = 1,$$

$$\frac{M_{Pl}\mu^2}{\Lambda^3} = 10^6 \quad \text{and} \quad T_R \sim 10^{10} \text{GeV}$$



$$\frac{M_{Pl}\mu^2}{\Lambda^3} = 1 \quad \text{and} \quad T_R \sim 10^{16} \text{GeV}$$

$$\frac{H_*}{M_{Pl}} \ll B^{-1/2\alpha} \left(\frac{\mu}{M_{Pl}} \right)^{(\alpha-1)/\alpha}$$

$$\frac{H_*}{M_{Pl}} < \left(\frac{96\pi^2}{A} \right)^{(2\alpha+1)/2} B \left(\frac{\mu}{M_{Pl}} \right)^{2(1-\alpha)}$$

Gravitational Waves

- density perturbation

$$\mathcal{P}_\zeta \simeq 10^{-11} \left(\frac{g_*}{106.75} \right)^{3/8} \left(\frac{M_{\text{Pl}} \mu^2}{\Lambda^3} \right)^{1/2} \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^{3/2}$$

↗ $\frac{M_{\text{Pl}} \mu^2}{\Lambda^3} \sim 10^4 \left(\frac{T_R}{10^{10} \text{ GeV}} \right)^{-3}$

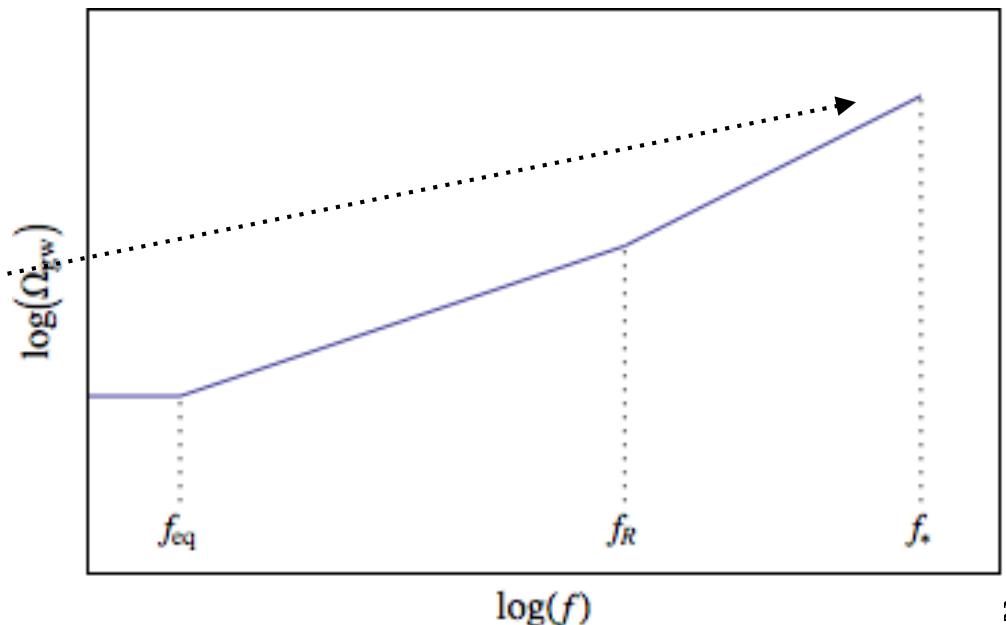
$$\mathcal{P}_\zeta \sim 10^{-9}$$

- gravitational waves

assume $T_R \sim 10^{10} \text{ GeV}$

$$\Omega_{gw} \sim 10^{-13}$$

$$f = 100 \text{ MHz}$$



Inflation and Genesis

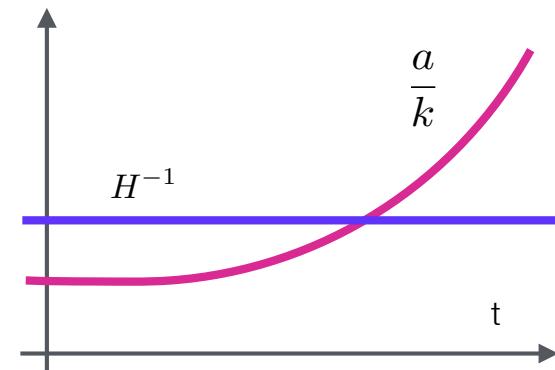


Background

Inflation

- Exponentially expansion.

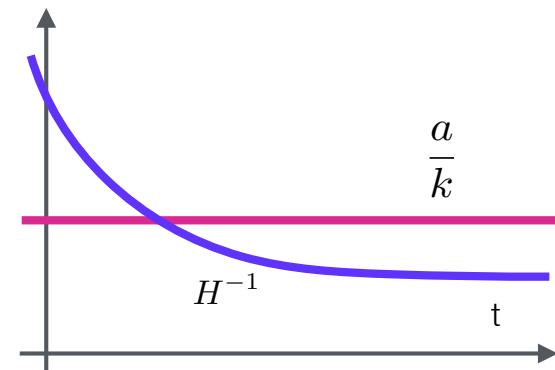
$$a(t) = a(t_i)e^{H_{inf}(t-t_i)}$$



Genesis

- Our universe started from Minkowski space-time.

$$a(t) \simeq 1 + \frac{1}{2\alpha} \frac{h_0}{(-t)^{2\alpha}} \quad (-\infty < t < 0)$$



($t \rightarrow -\infty$)

Inflation and Genesis



Scalar perturbation

Inflation

- Flat spectrum

In many models of
Galilean Genesis, $a=1$.
 $a=2$ in a few models.

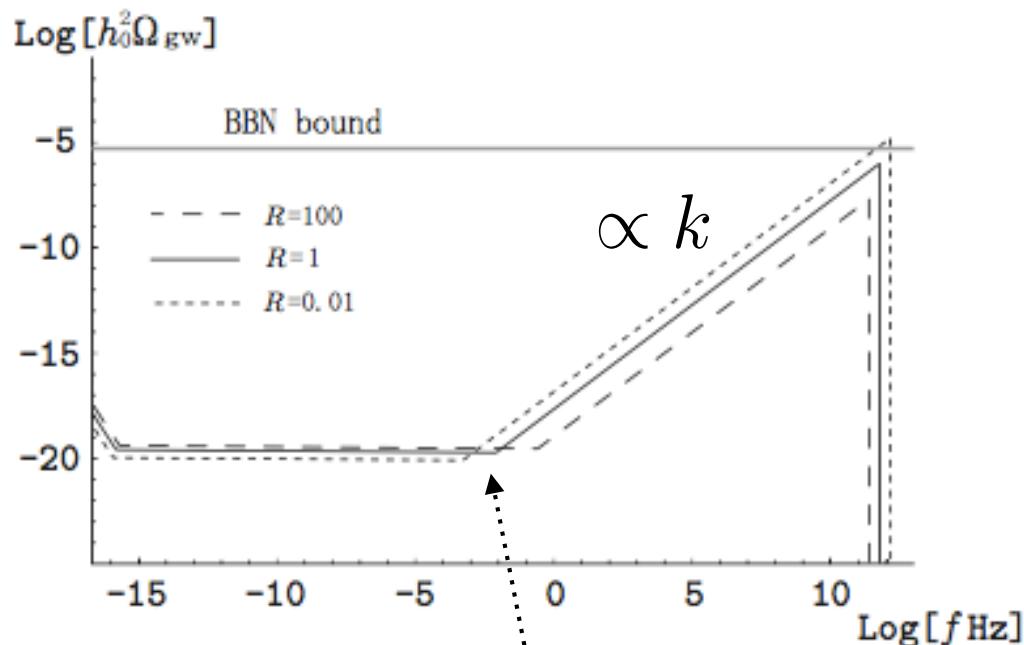
Genesis

- $a=2 \rightarrow$ We obtain the flat spectrum without curvaton.

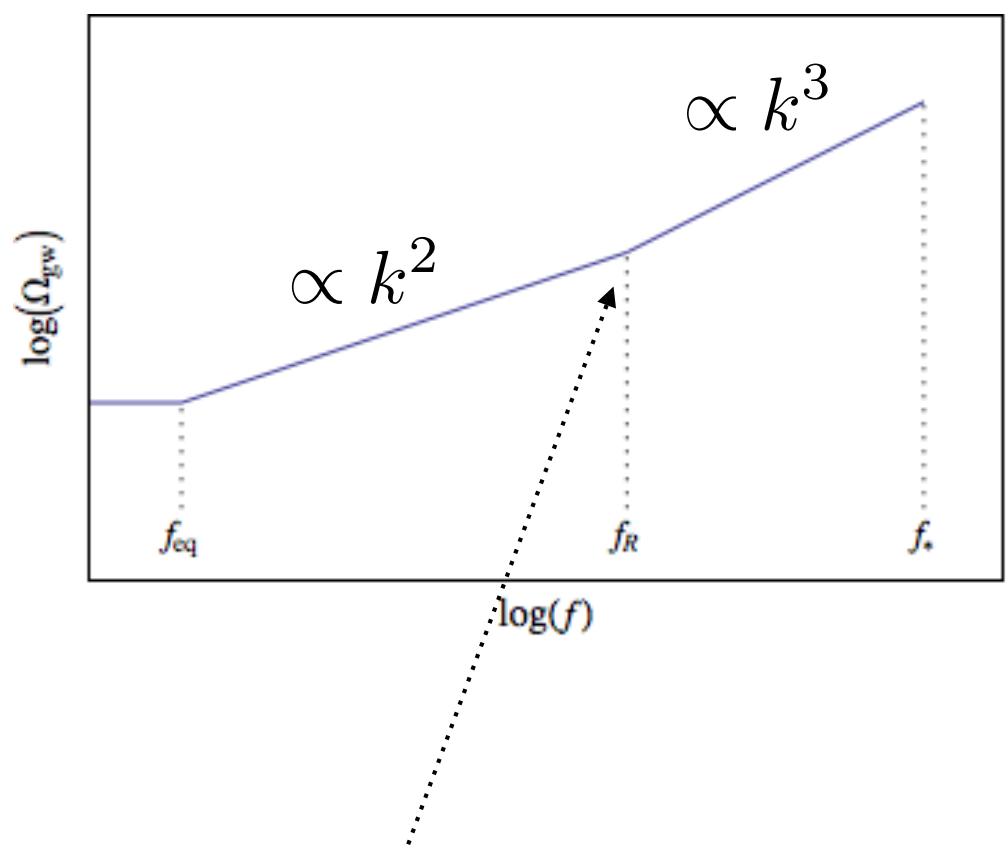
Inflation and Genesis



- Inflation
- Genesis



[H. Tashiro, T. Chiba, M. Sasaki, (2012)]



Ω_{gw}^{inf} can be larger than Ω_{gw}^{gen}

Conclusion

- H_* of genesis can be smaller than that of inflation.
- The shape of power spectrum is different between inflation and Genesis.
- We can find $\Omega_{gw} \sim 10^{-12}$ at $f = 100\text{MHz}$ in any model of Genesis.