ON A CANONICAL QUANTIZATION OF PURE 3D ADS GRAVITY

With Jihun Kim

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OUTLINE

- CLASSICAL ADS GRAVITY AS CHERN-SIMONS SL(2,R)XSL(2,R)
- CLASSICAL PHASE SPACE OF SL(2,R) CONTAINS TEICHMULLER SPACE
- HOLOMORPHIC QUANTIZATION: WHICH NORM?
- A CONNECTION WITH CFTs

- GLOBAL DIFFEOMORPHISMS AND THE MODULAR GROUP ACTION ON WAVE FUNCTIONS

- NORMALIZABILITY OF THE WAVE FUNCTION

- AN IMPROVED CONNECTION WITH CFTs: HOLOGRAPHY AS SUPERSELECTION PROJECTION

- THE CASE OF $c<1$

- TENTATIVE CONCLUSIONS

CLASSICAL GRAVITY AS CHERN-SIMONS

$$A = e/l - \omega, \qquad \tilde{A} = e/l + \omega$$

$$S_E = S(A) - S(\tilde{A})$$

$$S = \frac{k}{4\pi} \int \mathrm{tr}\,(AdA + \frac{2}{3}AAA), \qquad k = \frac{l}{4G}$$

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IN CANONICAL QUANTIZATION 3D SPACE IS

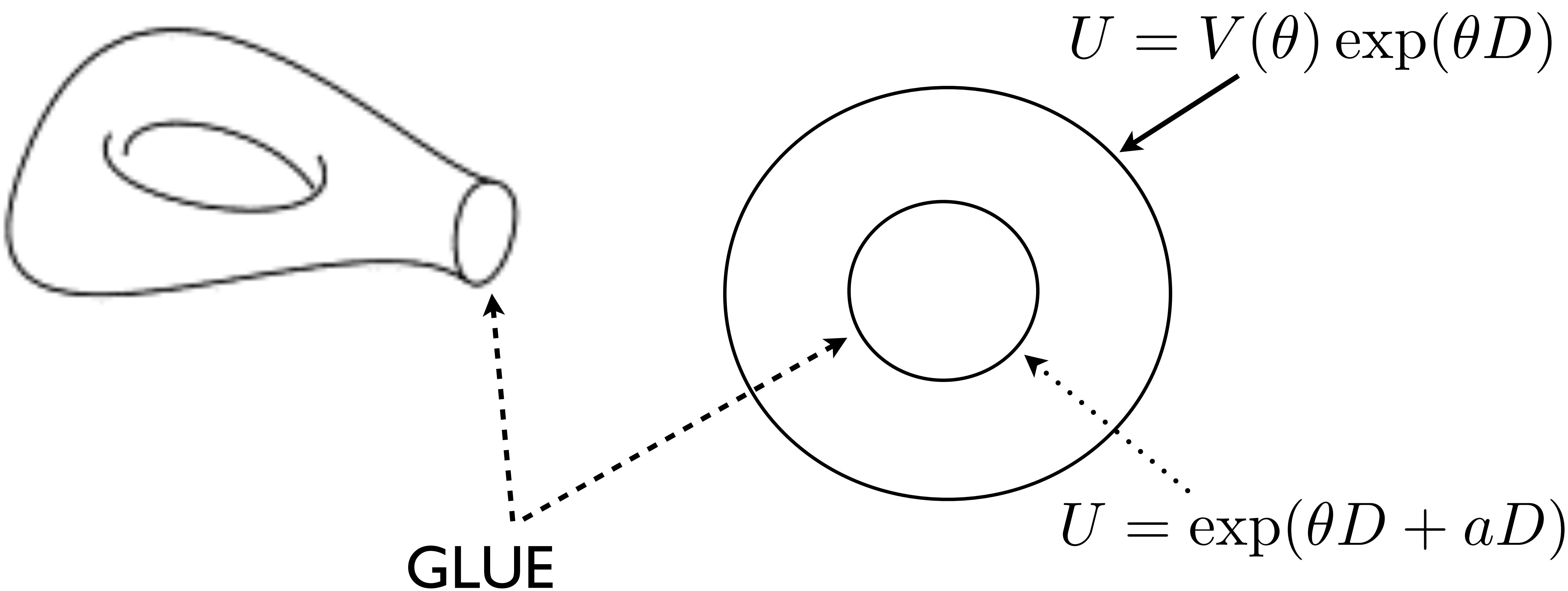
$$M = \Sigma \times R$$

CONSTRAINT EQUATION (GAUSS LAW)

$$F|_{\Sigma} = 0 \rightarrow A = dUU^{-1} \qquad \text{(locally)}$$

THE SPACE OF FLAT CONNECTIONS MODULO GAUGE TRANSFORMATIONS IS A DIRECT PRODUCT OF TWO SPACES: (EQUIVALENCE CLASSES OF) BOUNDARY GAUGE TRANSFORMATIONS TIMES A FINITE DIMENSIONAL SPACE

GEOMETRICALLY:



(IN)FINITE DIMENSIONAL SPACE WITH SEVERAL CONNECTED COMPONENTS.

WHEN ALL HOLONOMIES ARE HYPERBOLIC AND MANIFOLD HAS ONE BOUNDARY COMPONENT THIS SPACE IS

$$T_\Sigma \times \widehat{SL}(2,R)/S_1$$

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RESTRICT $$A|_{\partial\Sigma} = \begin{pmatrix} 0 & L(t+\phi) \\ 1 & 0 \end{pmatrix}$$

MODULI SPACE IS $$T_\Sigma \times Diff(S_1)/S_1$$

THIS SPACE ADMITS A KAHLER STRUCTURE AND A KAHLER FORM: THE WEYL-PETERSSON FORM

THE WEIL-PETERSSON FORM POSSESSES A KAHLER POTENTIAL, WHICH ALLOWS TO QUANTIZE THE TEICHMULLER SPACE IN HOLOMORPHIC QUANTIZATION.

THE RESULT IS JUST A (SUM OF PRODUCT) HILBERT SPACES

$$\mathcal{H} = \sum_L V_L \otimes H_T,$$

$$H_T = \text{holomorphic functions on } T, \qquad \mathrm{tr} \exp D = 2 \cosh L$$

$$V_L = \text{Irrep of Virasoro}$$

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HOW DO STATE VECTORS

$$\psi \in H_T$$

TRANSFORM UNDER THE MODULAR GROUP?

NORM:

$$\langle \psi_I | \psi_J \rangle = \int_T \bar{\psi}_I \exp K \psi_J$$

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SINCE

$$K \to K + f + \bar{f}, \qquad f = \text{holomorphic}$$

THEN

$$\psi_I \to U_I^J e^{-f} \psi_J, \qquad U_I^L U_L^J = \delta_I^J$$

THE KAHLER POTENTIAL FOR THE WEYL PETERSSON FORM OF THE TEICHMULLER SPACE FOR PUNCTURED SURFACE IS KNOWN: ZOGRAF AND TAKHTAJAN PROVED THAT IT EQUALS THE (REGULARIZED) LIOUVILLE ACTION COMPUTED ON SHELL:

$$K_0 = S_L|_{on\ shell} + H(t, \bar{t})$$

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FOR THE PUNCTURED SPHERE THIS IS THE POLYAKOV CONJECTURE (TAKTAJAN, ZOGRAF, MENOTTI,..)

AN EQUIVALENT FORM OF THE KAHLER POTENTIAL USES THE FACTORIZATION OF THE SCALAR 2D LAPLACIAN (QUILLEN, ZOGRAF)

$$K_0 = S_L|_{on\ shell} + H(t, \bar{t})$$

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THE HOLOMORPHIC FUNCTION G TRANSFORMS UNDER M.C.G. AS

$$G \to G' = \det(C\Omega + D)e^{-2f}G$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2g,R)$$

NEW WAVE FUNCTION

$$\Phi^I = G^{c/2}\psi^I$$

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ITS NORM IS THE QUILLEN NORM

$$(\Phi, \Phi') = \int_{T(\Sigma)} \left(\frac{\det \Delta'}{\det \Im\Omega}\right)^{-c/2} \bar{\Phi} \wedge *\Phi'$$

THE NORM IS NOT UNIQUE

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H.VERLINDE (1990):

$$(\Phi, \Phi') = \int_{T(\Sigma)} Z_L^{26-c} Z_{bc} \left(\frac{\det \Delta'}{\det \Im \Omega} \right)^{-c/2} \bar{\Phi} \wedge *\Phi'$$

LIOUVILLE+BC GHOST SYSTEM PARTITION FUNCTIONS, GIVES O(1) CORRECTION TO KAHLER POTENTIAL AT LARGE c

THE SYMPLECTIC FORM FOR A RIEMANN SURFACE WITH A BOUNDARY OF LENGTH L IS (CO-HOMOLOGOUS TO) THE SUM OF THE SYMPLECTIC FORM FOR A SURFACE WITH A PUNCTURE PLUS THE FIRST CHERN CLASS OF THE TANGENT BUNDLE AT THE PUNCTURE (MIRZAKHANI, USING DUISTERMAAT-HECKMAN)

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OUR KAHLER POTENTIAL IS THEN (SEE ALSO DIJKGRAAF, VERLINDE, VERLINDE, 1990)

$$K = K_0 - \frac{k}{2} L^2 \log g_{z\bar{z}} + f(t) + \bar{f}(\bar{t})$$

REABSORB IN REDEFINITION OF W.F.

UNDER M.C.G. TRANSFORMATIONS THE REDEFINED WAVE FUNCTION TRANSFORMS AS

$$\chi^I \to \chi'^I = \omega(z)^{kL^2/2} U^I_J \det{}^{c/2}(C\Omega_0 + D)\chi^J$$

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THIS IS THE TRANSFORMATION LAW FOR THE CONFORMAL BLOCK OF A VIRASORO PRIMARY OF WEIGHT

$$h = \frac{kL^2}{2}$$

QG IN 3D ADS IS DEFINED BY SL(2,R)XSL(2,R) SO WE NEED TO COMBINE TOGETHER THE MODULI SPACES OF BOTH CS

$$\Psi_{QG} = |v\rangle \chi^I(t) \chi^J(\bar{t'})? \qquad |v\rangle \in V_h \otimes V_h'$$

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UNDER GLOBAL DIFFS $\phi : \Sigma \to \Sigma$

$$\mathrm{tr}\, P e^{\oint_\gamma A} \in T \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} A}$$

$$\mathrm{tr}\, P e^{\oint_\gamma \bar{A}} \in T \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} \bar{A}}$$

DIAGONAL ACTION ON $T \times \bar{T}'$

THE BLACK CYCLE MAPS INTO THE RED CYCLE UNDER A GLOBAL DIFFEOMORPHISM

$$\mathrm{tr}\, P e^{\oint_\gamma A} \in T \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} A}$$

$$\mathrm{tr}\, P e^{\oint_\gamma \bar{A}} \in T \to \mathrm{tr}\, P e^{\oint_{\phi^{-1}(\gamma)} \bar{A}}$$

$$T \times T' \to (T \times T')/M$$

SO THE WAVE FUNCTION IS

$$\Psi = |v\rangle \sum_{IJ} \chi^h_I(t)\bar{\chi}^{\bar{h}}_J(\bar{t}')N^{IJ}, \qquad |v\rangle \in V_h \otimes V_{\bar{h}}$$

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ACTUALLY ANALYTIC CONTINUATION TO INDEPENDENT LEFT AND RIGHT MODULI (SEE WITTEN '07, SEGAL)

NEW RESTRICTIONS COME FROM NORMALIZABILITY

$$\Psi = |v\rangle \sum_{IJ} \chi_I^h(t) \bar{\chi}_J^{\bar{h}}(\bar{t}') N^{IJ}, \qquad |v\rangle \in V_h \otimes V_{\bar{h}}$$

$$\langle \sum_{IJ} \chi_I \bar{\chi}_J N^{IJ} | \sum_{KL} \chi_K \bar{\chi}_L N^{KL} \rangle < \infty$$

DANGEROUS REGIONS: DEGENERATING RIEMANN SURFACES

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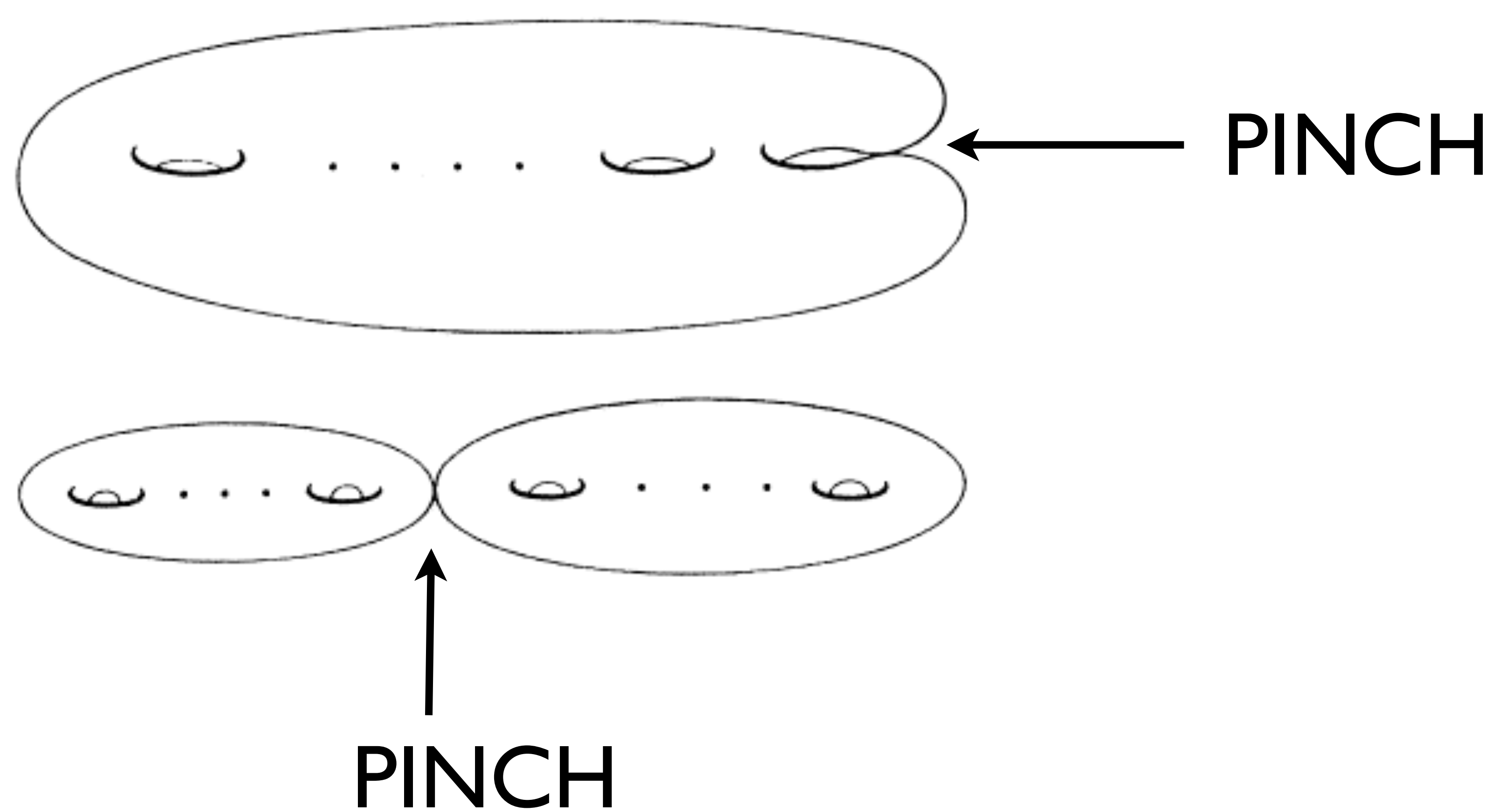
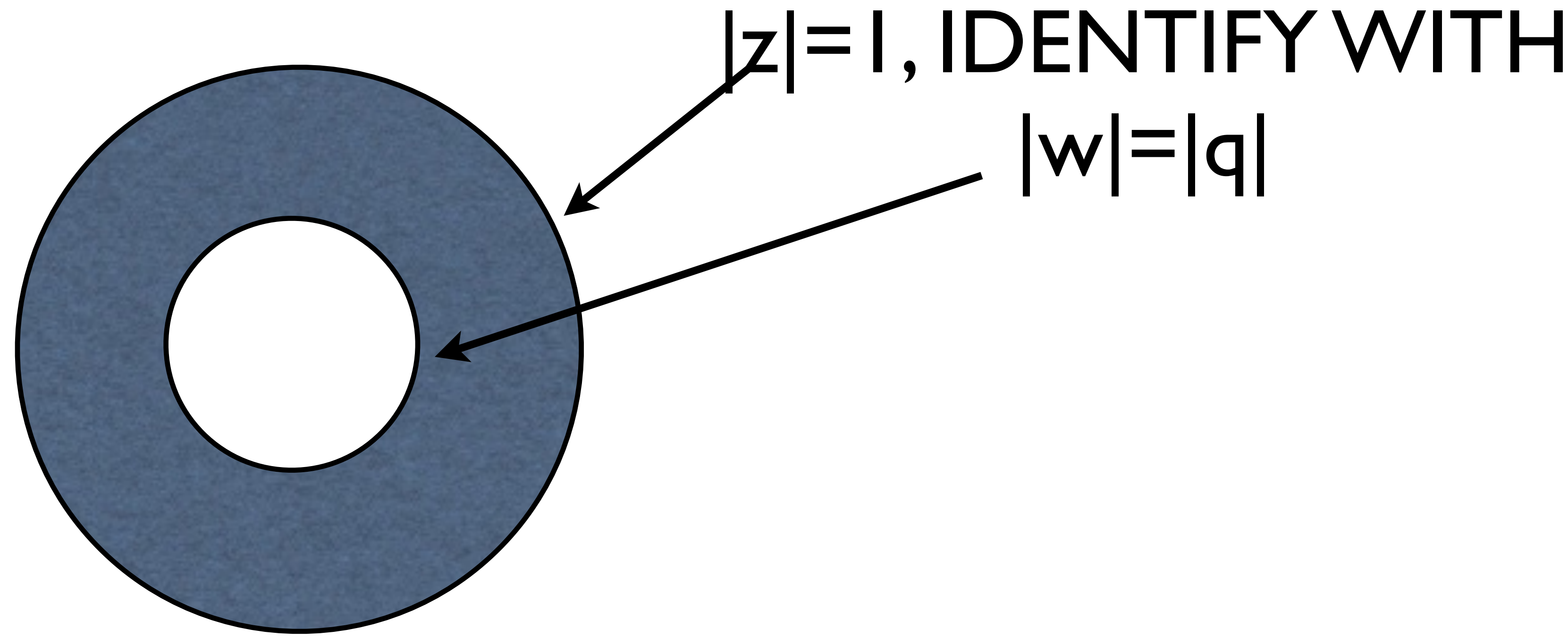
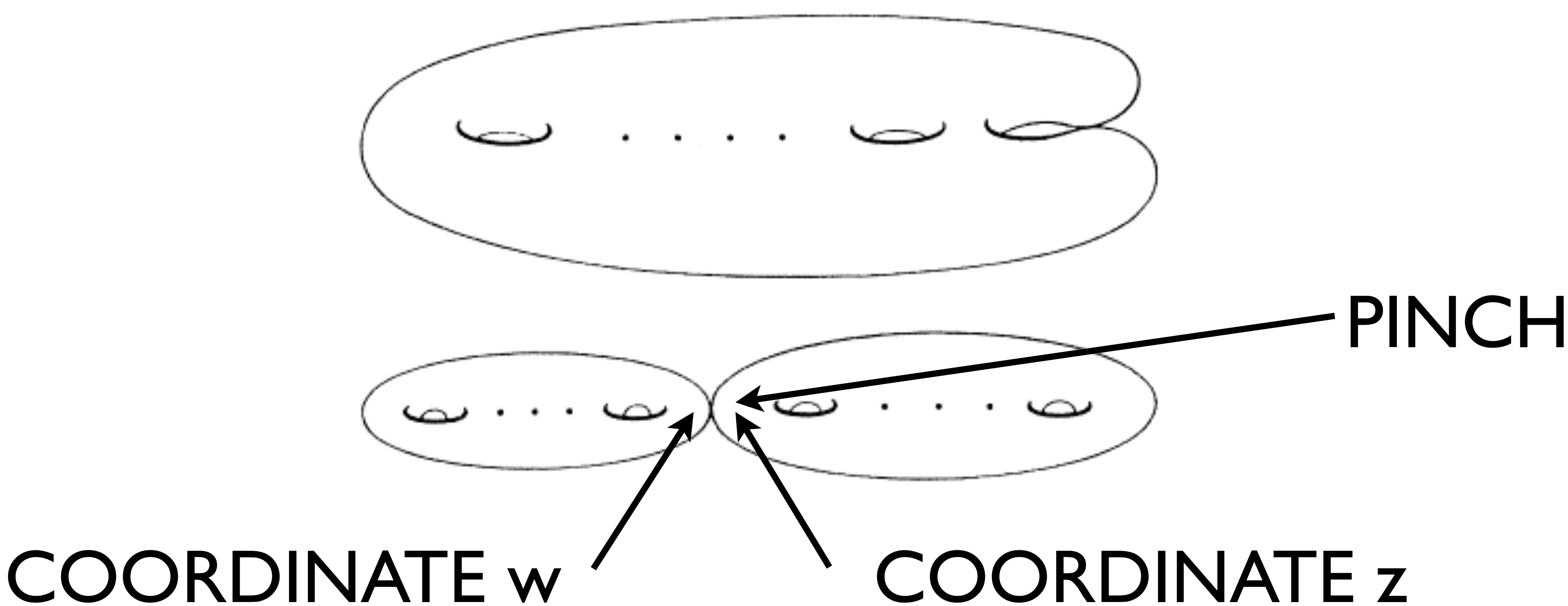




$$z = qw, \qquad |q| < |z| < 1, \qquad |q| < |w| < 1, \qquad q = e^{2\pi i\tau}$$



NEAR A NODE CERTAIN MODULAR TRANSFORMATIONS ARE KNOWN AND SIMPLE

$$(\tau, \tau') \to (\tau + 1, \tau' + 1) \text{ (Dehn Twist)}$$

$$\omega_{WP} \propto \frac{d\tau \wedge d\bar{\tau}}{(\Im\tau)^3}$$

INVARIANCE UNDER THE DIAGONAL DEHN TWIST:

$$\Psi = |v\rangle \times \langle V^{h,\bar{h}}\rangle, \qquad \langle V^{h,\bar{h}}\rangle = \sum_n \int d\mu_n(\Delta) q^{\Delta - c/24} \bar{q}'^{\Delta + n - c/24}$$

NORMALIZABILITY OF THE WAVE FUNCTION IMPOSES NEW CONSTRAINTS

DEFINE:

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$$||\langle V^{h,\bar{h}}\rangle||^2 \approx \sum_n \int^{\infty} d\rho \int^{\infty} d\rho' \int_{-\infty}^{\infty} d\zeta \int d\mu_n(\Delta) d\mu_n(\Delta') e^{-2\pi\rho(\Delta+\Delta'-\frac{c-1}{12})} e^{-2\pi\rho'(\Delta+\Delta'+2n-\frac{c-1}{12})} e^{i\zeta(\Delta-\Delta')}$$

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CONVERGENCE IMPLIES $\quad \Delta, \Delta + n > (c-1)/24$

WE CHOSE THE VERLINDE NORM AND USED:

$$Z_L^{26-c} Z_{bc} \to q\bar{q}'$$

TOGETHER WITH THE FACTORIZATION (WOLPERT)

$$\frac{\det \Delta'}{\det \Im\Omega}(q, \bar{q}' ..) \to C(q\bar{q}')^{1/12}$$

INTEGRAND IN NORM PROPORTIONAL TO DIRAC DELTA: CONTINUOUS SPECTRUM

$$||\langle V^{h,\bar{h}}\rangle||^2 \approx \sum_n \int d\mu_n(\Delta) d\mu_n(\Delta') F(\Delta, n)\delta(\Delta - \Delta')$$

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SO HILBERT SPACE OF PURE 3D ADS GRAVITY
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SPACE OF SUCH CFTs IS NONEMPTY
ONE SOLUTION:
LIOUVILLE CFT

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IN PARTICULAR THEY ARE THE SAME FOR THE UNCOMPACTIFIED BOSON (CONTINUOUS SPECTRUM) AND FOR THE COMPACT BOSON (DISCRETE SPECTRUM)

HILBERT SPACE IS TOO LARGE

$$H_{CQG} = V_h \otimes V_{\bar{h}} \otimes \sum_g H_g$$

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ALSO: TOPOLOGY-CHANGING TRANSITIONS SEEM TO EXIST IN ADS3 PURE GRAVITY

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MAYBE TWO WRONGS DO MAKE A RIGHT!

TOPOLOGY CHANGE ALLOWED WHEN

$$\exists\ M \qquad s.t. \qquad \partial M = \Sigma$$

AND THE SURFACE CAN BE SPLIT BY A HOMOLOGY CYCLE a NON CONTRACTIBLE IN M INTO

$$\Sigma \setminus a = \Sigma_g \cup \Sigma_{g'}$$

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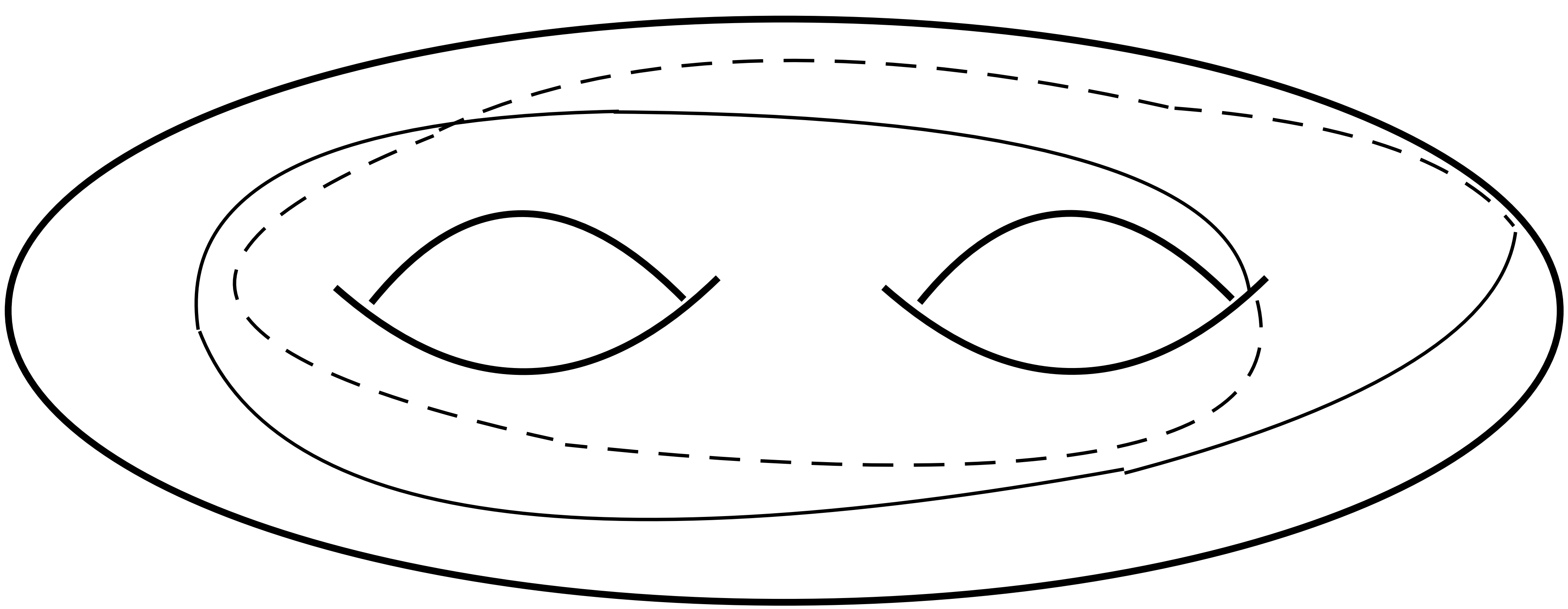
A SIMPLE EXAMPLE WITH g=g'=1 AND M A HANDLEBODY:



ALSO: TOPOLOGY CHANGE DEFINES A PROJECTION:

$$P(A|C) = \sum_B P(A|B)P(B|C)$$

CONJECTURE: EACH EIGENSPACE WITH UNIT EIGENVALUE OF TOPOLOGY CHANGE PROJECTOR DEFINES A CONSISTENT THEORY

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HEURISTIC SEMICLASSICAL CALCULATION SUGGEST THAT THE PROJECTION IS:

$$P = \sum_{g,g'} C(h,g)C(h,g')\langle V^h \rangle_{\Sigma_g} \langle V^h \rangle_{\Sigma'_g}$$

VERTICES OF PRIMARIES IN LIOUVILLE THEORY

SKETCH OF A (FUTURE) CALCULATION

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$$K[B^I|B^F] = \int_R [dA] \exp(iI_{CS}), \qquad R = \{A \in SL(2,R)\}$$

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IF SL(2,C) SADDLE POINT DEFINES A REGULAR EUCLIDEAN METRIC, THEN IT CONTRIBUTES TO FUNCTIONAL INTEGRAL

SO WE PROPOSE THAT THE HILBERT SPACE OF CQG IS ISOMORPHIC TO LIOUVILLE HILBERT SPACE

$$H_{CQG} = \{|v\rangle_h \otimes \sum_g C(g,h)\langle V^h\rangle_{\Sigma_g} : |v\rangle \in V_L,\ h \in R^+ + (c-1)/24\}$$

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ANY DISCRETIZATION GIVES EFFECTIVE CENTRAL CHARGE

$$c_{eff} = c - 24\Delta_{min} = c - 24\frac{c-1}{24} = 1$$

MULTIPLICITY OF STATES NOT ENOUGH TO GIVE BEKENSTEIN-HAWKING ENTROPY

A POSSIBILITY:

$$c_{eff} < c$$

SIGNALS A PHASE OF QG WITHOUT DYNAMICAL BLACK HOLES

ARGUED IN hep-th/0503121 USING LINEAR DILATON BACKGROUND AND STRINGS ON ADS3 AT

$$L_S > L_{AdS} \gg L_{Planck}$$

LASCIATE OGNI SPERANZA?

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$$T/\mathcal{N} = \text{ finite cover of } \mathcal{M}$$

$(T/\mathcal{N} \times \bar{T}'/\mathcal{N})/M$ HAS FINITE VOLUME= DISCRETE SPECTRUM

FOR $c<1$ MINIMAL MODELS WE CAN GET A REASONABLE THEORY, WITH DISCRETE SPECTRUM AND A DUAL TO THE ADS VACUUM IF WE IMPOSE A NEW NON-GEOMETRICAL GAUGE SYMMETRY THAT ACTS ON THE MODULI SPACE AS THE NORMAL, FINITE INDEX KERNEL OF THE MODULAR GROUP

(SEE CASTRO ET AL.)

FLY IN THE OINTMENT: WHEN MORE THAN ONE MODULAR INVARIANT COMBINATION OF CHARACTERS EXISTS, A GENERIC VECTOR IN THE HILBERT SPACE CANNOT BE ASSOCIATED TO ANY GIVEN CFT …. GENERICALLY IT IS:

$$|v\rangle_h \otimes \sum_I \sum_g C_I(g,h) \langle V^h \rangle^I_{\Sigma_g}$$

A LINEAR COMBINATION OF VEVs COMPUTED IN DIFFERENT CFTs

TENTATIVE CONCLUSIONS

- A CANONICAL QUANTIZATION OF PURE GRAVITY IN 3D ADS YIELDS A HILBERT SPACE THAT CAN BE INTERPRETED AS THE TARGET SPACE FOR LIOUVILLE-LIKE CFTs
- LIOUVILLE APPEAR IN THE HILBERT SPACE AS SUPERSELECTION SECTOR DEFINED BY A TOPOLOGICAL PROJECTION
- FOR $c>1$ THERE ARE SEVERAL PROBLEMS WITH THIS PICTURE SUCH AS ABSENCE OF THE DUAL TO ADS SPACE AND CONTINUOUS SPECTRUM
- FOR $c<1$ WE FOUND THAT IMPOSING A NEW, NON GEOMETRIC GAUGE SYMMETRY GIVES A REASONABLE PICTURE (DISCRETE SPECTRUM, VACUUM)

MANY OPEN QUESTIONS

- DOES ALL THIS MAKE SENSE?
- CAN ONE EXTEND THE MINIMAL MODEL PICTURE TO HIGH SPIN 3D ADS THEORIES, OR SL(N,R)XSL(N,R) CS? (ANY RCFT ADMITS A NONTRIVIAL KERNEL OF THE MODULAR GROUP)
- TOPOLOGICAL TRANSITION AMPLITUDES SHOULD BE UNDERSTOOD PROPERLY
- IS $c>1$ IN A DIFFERENT PHASE THAN GRAVITY WITH MATTER? DO PHASE TRANSITIONS EXIST FROM STRINGY ADS3 INTO A PURE GRAVITY PHASE?