Creation of the Inflationary Universe out of a Black Hole

Research Center for the Early Universe (RESCEU)

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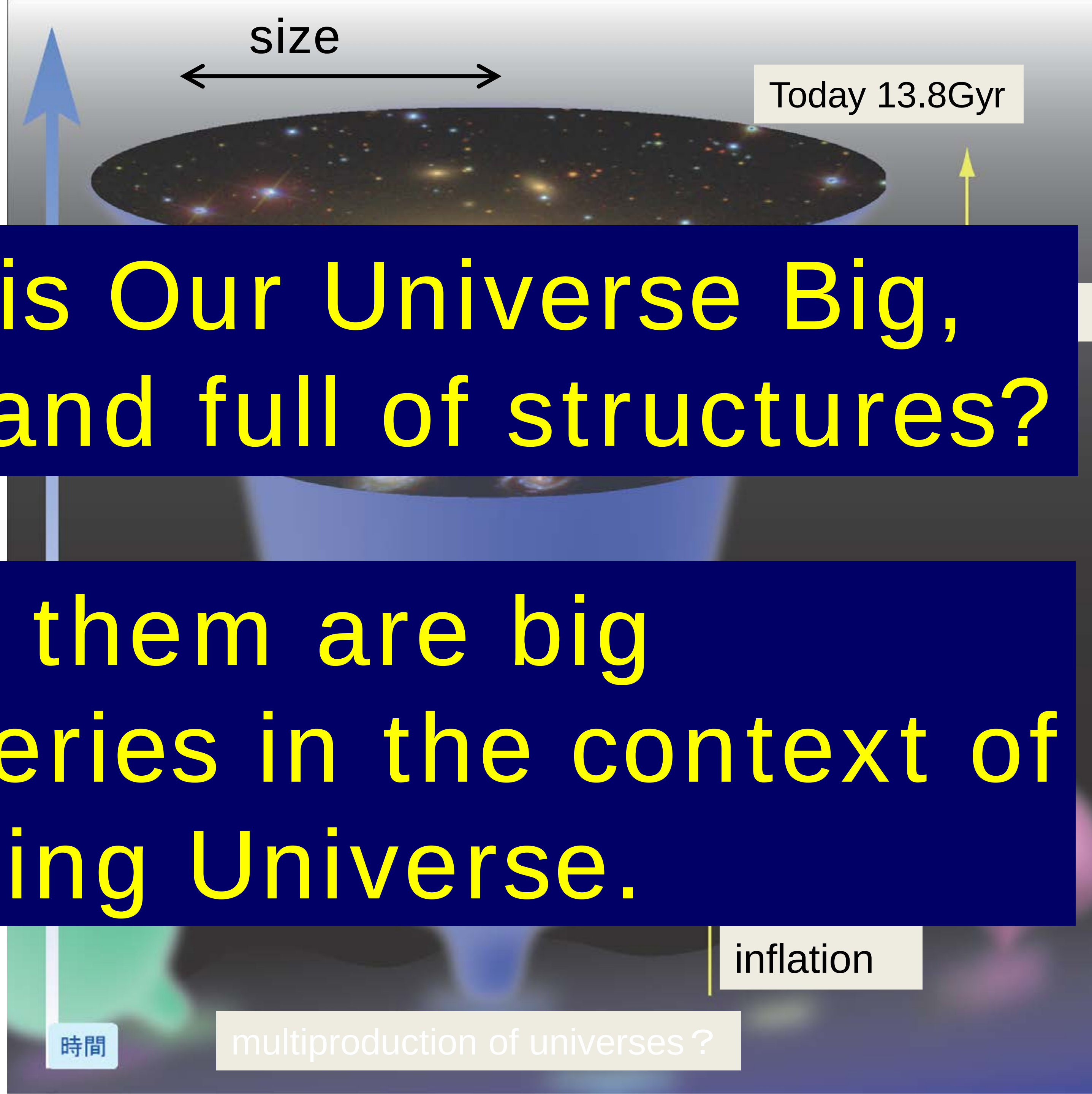
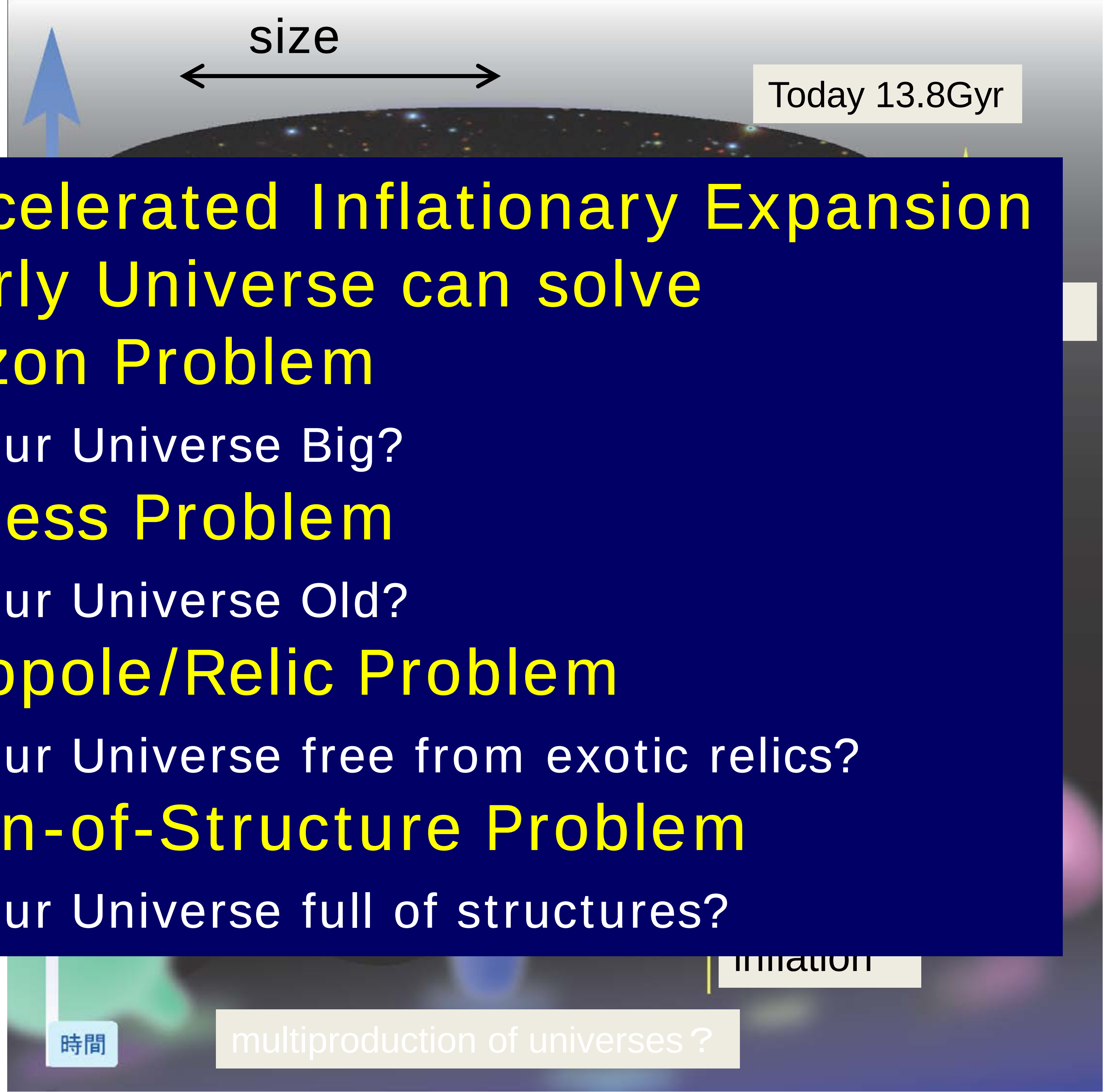
Why is Our Universe Big, Old, and full of structures?

All of them are big mysteries in the context of evolving Universe.





Rapid Accelerated Inflationary Expansion in the early Universe can solve

The Horizon Problem

Why is our Universe Big?

The Flatness Problem

Why is our Universe Old?

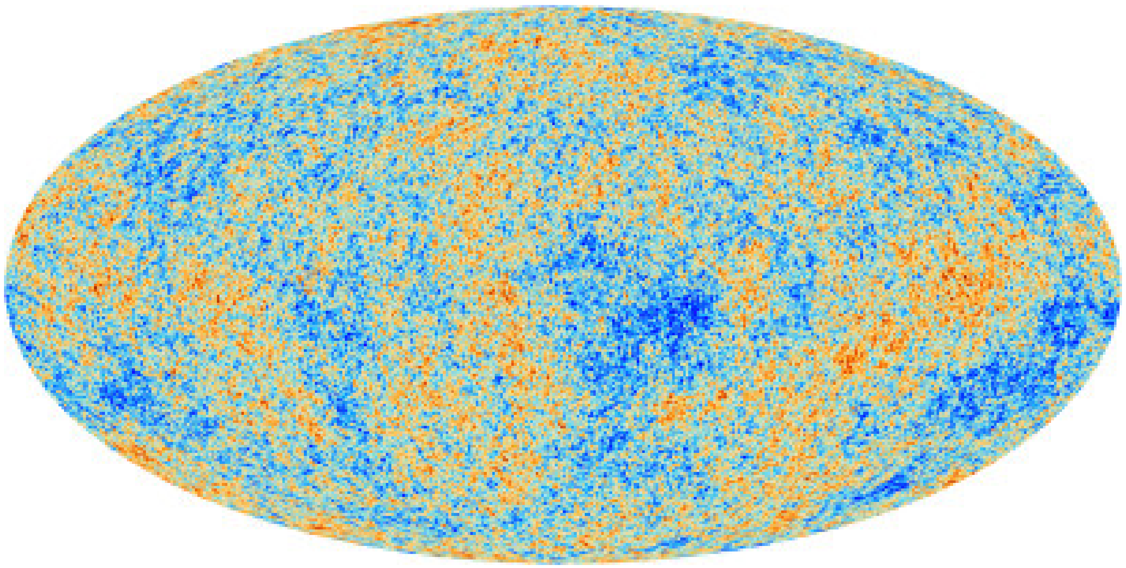
The Monopole/Relic Problem

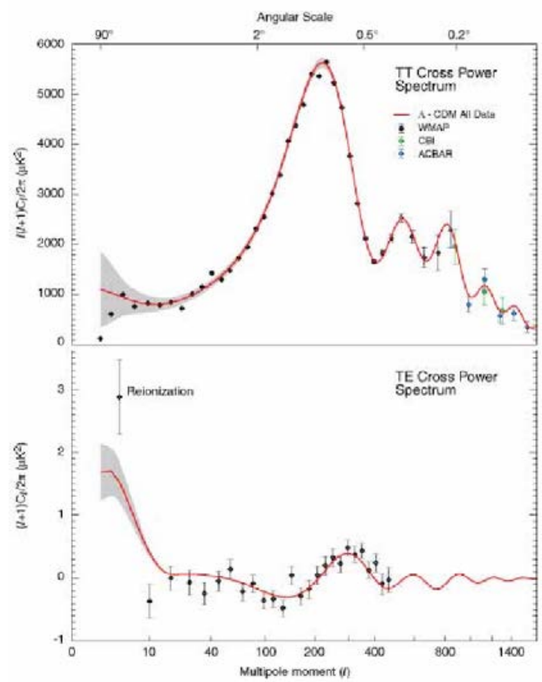
Why is our Universe free from exotic relics?

The Origin-of-Structure Problem

Why is our Universe full of structures?

Quantum fluctuations of the inflaton results in nearly scale-invariant curvature fluctuations which seed formation of large-scale structures as well as CMB temperature and polarization anisotropies.





The last 50～60 e-folds of inflationary expansion has been probed by high precision observations such as those conducted by WMAP, Planck etc., but there is no single observational result that is in contradiction with inflationary cosmology even 35 years after its original proposal.

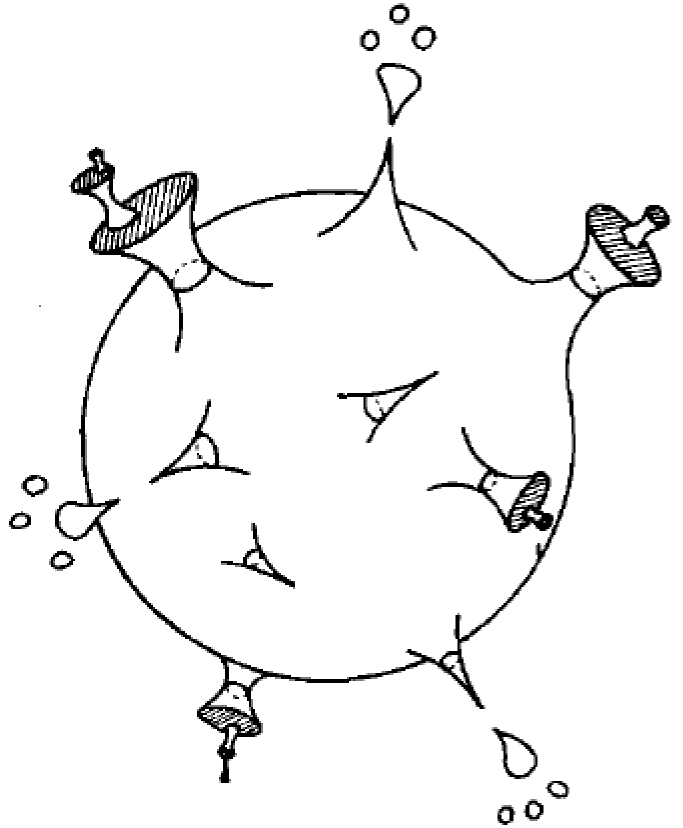
But it is totally unknown how inflation started in our Universe.

Out of quantum gravity regime?

Likely, but not necessarily.

size

Many inflation m
multiproduction



ELSEVIER

Physics Lette

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Multi-production of universes by first-o vacuum

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Then a question arises:

Is our Universe of 1st generation?

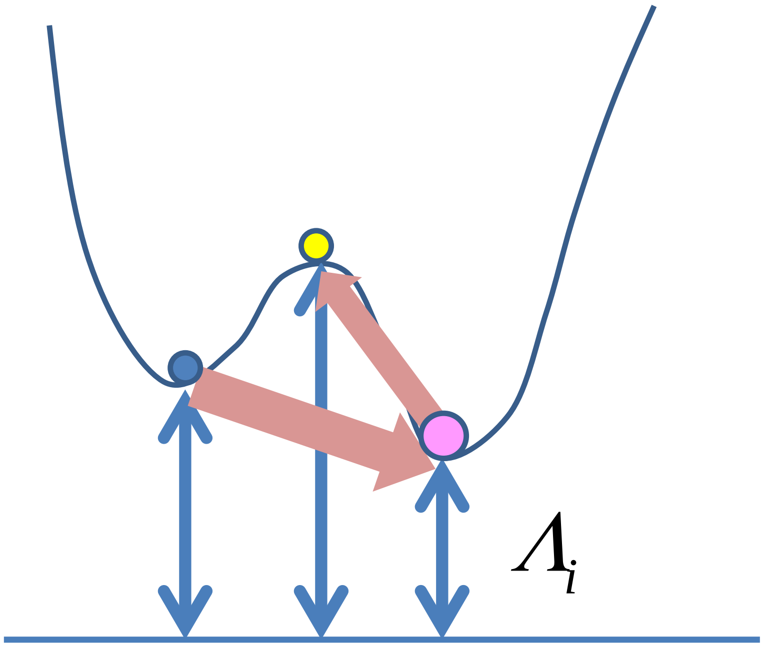
Time

multiproduction of universes？

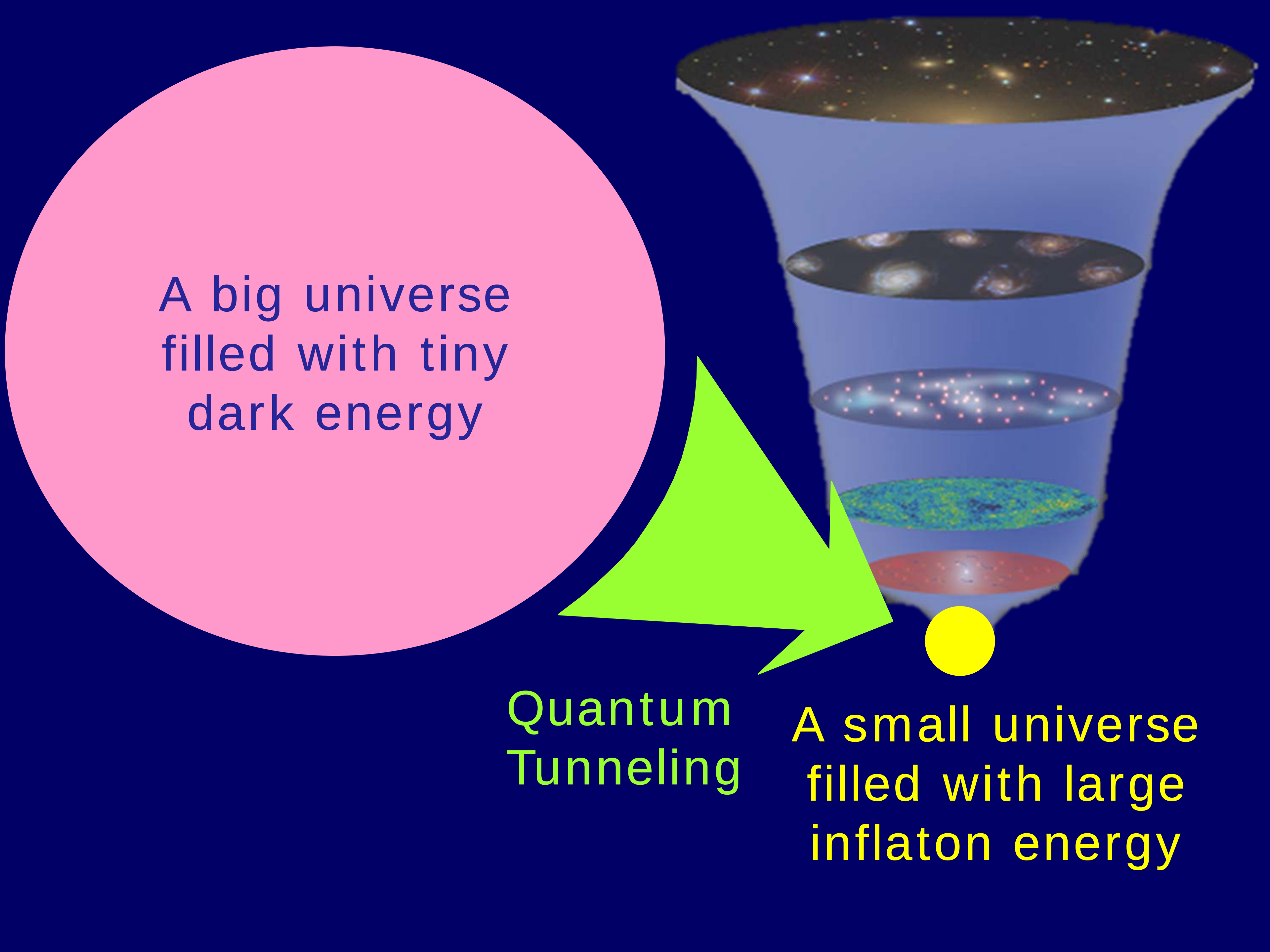
If dark energy is Λ,our Universe will be asymptotically de Sitter.

Inflation ≅ de Sitter expansion

De Sitter space can tunnel to another de Sitter space with a different Λ.



Hawking and Moss (1983)
Lee and Weinberg (1987)
...
multiverse
string landscape...


A big universe filled with tiny dark energy
Quantum Tunneling
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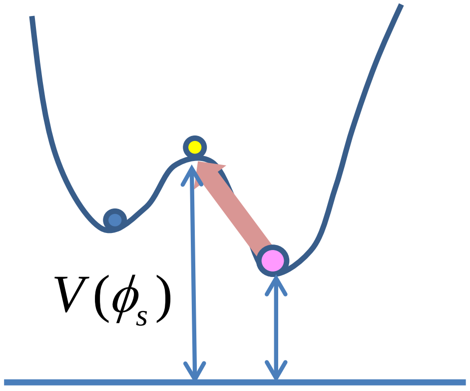
The Hawking Moss Instanton



An O(4) symmetric bounce solution

$$ds^2 = d\xi^2 + \rho(\xi)^2 d\Omega_{\mathrm{III}}^2$$

unit 3 sphere

Euclidean action of the Einstein scalar theory

$$I_{\mathrm{E}} = 2\pi^2 \int d\xi \left[\rho^3 \left(\frac{1}{2}\dot{\phi}^2 + V(\phi) \right) + \frac{3}{8\pi G} \left(\rho^2 \ddot{\rho} + \rho \dot{\rho}^2 - \rho \right) \right] \qquad \dot{\rho} = \frac{d\rho}{d\xi}$$

$$\ddot{\phi} + \frac{3\dot{\rho}}{\rho}\dot{\phi} = \frac{dV}{d\phi},$$

$$\dot{\rho}^2 = 1 + \frac{8\pi G}{3}\rho^2 \left(\frac{1}{2}\dot{\phi}^2 - V \right)$$

Static scalar field configuration →

$$\dot{\phi} = \ddot{\phi} = \frac{dV}{d\phi} = 0$$

$$\dot{\rho}^2 = 1 - \frac{8\pi G}{3}\rho^2 V$$

Solution

$$\rho(\xi) = H_s^{-1} \sin(H_s \xi),$$

$$H_s^2 \equiv \frac{8\pi G}{3} V(\phi_s),$$

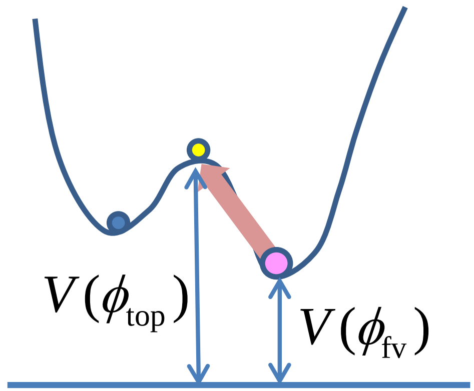
Euclidean action

$$I_{\mathrm{E}}(\phi_s) = -\frac{3}{8G^2 V(\phi_s)}$$

$$\Gamma_{\mathrm{fv}\to\mathrm{top}} = Ae^{-B_{\mathrm{HM}}} = A\exp\left[-I_{\mathrm{E}}(\phi_{\mathrm{top}}) + I_{\mathrm{E}}(\phi_{\mathrm{fv}})\right]$$
$$= A\exp\left[\frac{3}{8G^2}\left(\frac{1}{V(\phi_{\mathrm{top}})} - \frac{1}{V(\phi_{\mathrm{fv}})}\right)\right]$$

If $\Delta V \equiv V(\phi_{top}) - V(\phi_{fv}) \ll V(\phi_{top})$ so that the change of the geometry is negligible, we find

$$B_{HM} \cong \frac{3\Delta V}{8G^2V^2(\phi_{fv})} = \frac{8\pi^2}{3}H_{fv}^{-4}\Delta V = \frac{4\pi}{3}H_{fv}^{-3}\frac{\Delta V}{T_H}$$



with $$T_H = \frac{H(\phi_{fv})}{2\pi}$$

Hawking Temperature of De Sitter space

$$H_{fv}^2 = \frac{8\pi G}{3}V(\phi_{fv})$$

Thus this process can be interpreted as a thermal transition with the Hawking temperature of de Sitter space.

Weinberg (2007)

Here we show that the Euclidean action is determined by the gravitational entropy of de Sitter space using a static metric.

ADM decomposition $ds^2 = -N^2dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$

Wick rotation $d\tilde{s}^2 = N^2 d\tilde{t}^2 + h_{ij}(dx^i + \tilde{N}^i d\tilde{t})(dx^j + \tilde{N}^j d\tilde{t})$

$$t = -i\tilde{t} \qquad \tilde{N}^i \equiv -iN^i.$$

Quantities with a tilde are those in Euclidean space which are multiplied by some power of i upon Wick rotation.
For example, the extrinsic curvature on $\tilde{t} = \mathrm{const}$ 3-space reads

$$\tilde{K}_{ij} = \frac{1}{2N}\left(\frac{\partial h_{ij}}{\partial \tilde{t}} - D_i\tilde{N}_j - D_j\tilde{N}_i\right) = -iK_{ij}$$

D_i : covariant derivative with respect to h_{ij}.

Express Euclidean Einstein action and matter action

$$I_{\mathrm{E}}^{(\mathrm{G})} = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x d\tilde{t}\sqrt{\tilde{g}}\tilde{R} \qquad I_{\mathrm{E}}^{(\mathrm{M})} = \int d^3x d\tilde{t}\sqrt{\tilde{g}}\left[\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

in terms of the Hamiltonian (first-order) formalism

σ_{ij} : induced metric on the boundary surface S

n^i : unit normal vector on S

$$I_{\rm E}^{\rm (G)} = -\frac{1}{16\pi G}\int_{\mathcal{M}} d^3x d\tilde{t}\sqrt{\tilde{g}}\tilde{R}$$

$$= \int d\tilde{t}\left[\int_{\Sigma_{\tilde{t}}} d^3x\left(\tilde{\pi}^{ij}\partial_{\tilde{t}}h_{ij} + N\tilde{\mathcal{H}}^{\rm (G)} - \tilde{N}^i\tilde{\mathcal{H}}_i^{\rm (G)}\right) - \int_{\rm S} d^2x\sqrt{\sigma}\left(\frac{n^i\partial_i N}{8\pi G} - \frac{2}{\sqrt{h}}n_i\tilde{N}_j\tilde{\pi}^{ij}\right)\right]$$

$$I_{\rm E}^{\rm (M)} = \int d^3x d\tilde{t}\sqrt{\tilde{g}}\left[\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right] = \int d\tilde{t}\int_{\Sigma_{\tilde{t}}} d^3x\left(\tilde{\mathcal{P}}_\phi\partial_{\tilde{t}}\phi + N\tilde{\mathcal{H}}^{\rm (M)} - \tilde{N}^i\tilde{\mathcal{H}}_i^{\rm (M)}\right)$$

$$\tilde{\pi}^{ij} = \frac{\sqrt{h}}{16\pi G}\left(\tilde{K}^{ij} - h^{ij}\tilde{K}\right) = -i\pi^{ij} \qquad \tilde{\mathcal{P}}_\phi = \sqrt{h}\left[\frac{1}{N}\partial_{\tilde{t}}\phi - \frac{\tilde{N}^i}{N}\partial_i\phi\right]$$

$$\tilde{\mathcal{H}}^{\rm (G)} = \frac{\sqrt{h}}{16\pi G}\left(-{}^{(3)}R - \tilde{K}_{ij}\tilde{K}^{ij} + \tilde{K}^2\right) \qquad \tilde{\mathcal{H}}^{\rm (M)} = \sqrt{h}\left[-\frac{1}{2}\left(\frac{1}{N}\partial_{\tilde{t}}\phi - \frac{\tilde{N}^i}{N}\partial_i\phi\right)^2 + \frac{1}{2}h^{ij}\partial_i\phi\partial_j\phi + V\right]$$

$$\tilde{\mathcal{H}}_i^{\rm (G)} = -2h_{ij}D_k\tilde{\pi}^{jk} \qquad \tilde{\mathcal{H}}_i^{\rm (M)} = \sqrt{h}\left[\frac{1}{N}\partial_{\tilde{t}}\phi\partial_i\phi - \frac{\tilde{N}^j}{N}\partial_j\phi\partial_i\phi\right]$$

(Euclidean) Classical solution: Hamiltonian and Momentum Constraints

$$\tilde{\mathcal{H}}^{\rm (tot)} \equiv \tilde{\mathcal{H}}^{\rm (G)} + \tilde{\mathcal{H}}^{\rm (M)} = 0, \quad \tilde{\mathcal{H}}_i^{\rm (tot)} \equiv \tilde{\mathcal{H}}_i^{\rm (G)} + \tilde{\mathcal{H}}_i^{\rm (M)} = 0.$$

For static configuration with $\partial_{\tilde{t}}h_{ij} = \partial_{\tilde{t}}\phi = 0$, the classical (Euclidean) action is solely determined by the surface contribution.

$$I_{\mathrm{E\ static}}^{(\mathrm{tot})} = -\int_S d\tilde{t} d^2x \sqrt{\sigma} \left(\frac{n^i \partial_i N}{8\pi G} - \frac{2}{\sqrt{h}} n_i N_j \tilde{\pi}^{ij} \right)$$

(Euclidean) De Sitter space (in static representation)

$$d\tilde{s}^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu = \left(1 - H^2 r^2\right) d\tilde{t}^2 + \frac{dr^2}{1 - H^2 r^2} + r^2 d\Omega_{\mathrm{II}}^2$$

unit 2 sphere

$$N = \sqrt{1 - H^2 r^2}, \ \tilde{N}^i = 0$$

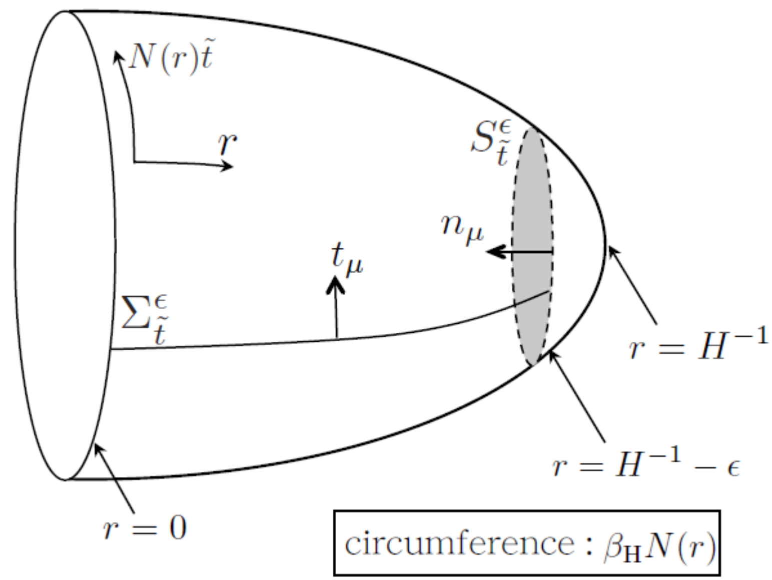
In this Euclidean description, the horizon $r = H^{-1}$ is similar to the origin in the 2D polar coordinate where $\tilde{t}$ behaves like an angular variable θ.

We take the range of $\tilde{t}$ as $0 \le \tilde{t} < \beta$ and take β appropriately to avoid a conical singularity.

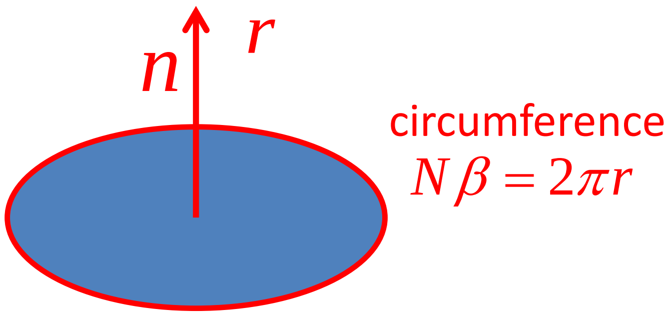
We regularize the coordinate singularity at the horizon by a hypothetical surface at $r = H^{-1} - \epsilon$

To avoid conical singularity we require

$$\lim_{\epsilon \to 0} n^i \partial_i \left[\beta N\right] = \beta H = 2\pi$$



An example of a circle on a plane with no conical singularity



$$n^i \partial_i N\beta = \partial_r N\beta = 2\pi$$

In the case of De Sitter space

$$\lim_{\epsilon \to 0} n^i \partial_i [\beta N] = \beta H = 2\pi \Rightarrow \beta^{-1} = T_H = \frac{H}{2\pi}$$ Hawking Temperature

Contribution of the horizon to the action is proportional to the horizon area

$$\lim_{\epsilon \to 0} \left[-\beta_H \int_{S_t^\epsilon} d^2x \sqrt{\sigma} \frac{n^i \partial_i N}{8\pi G} \right] = -\frac{A}{4G}$$

$$A \equiv \int_{S_t^\epsilon} d^2x \sqrt{\sigma} \Big|_{\epsilon=0} = \frac{4\pi}{H^2}$$

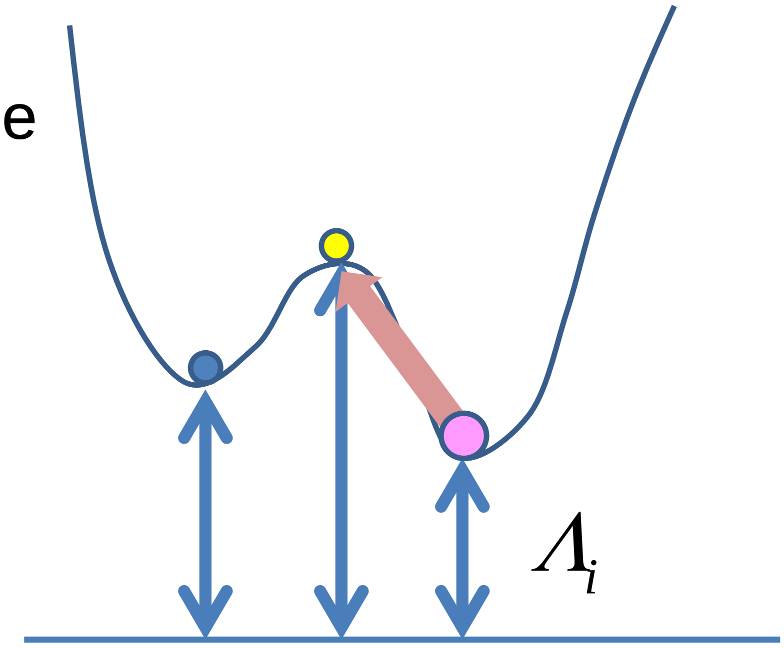
Thus the Hawking Moss action is entirely given by the entropy term.

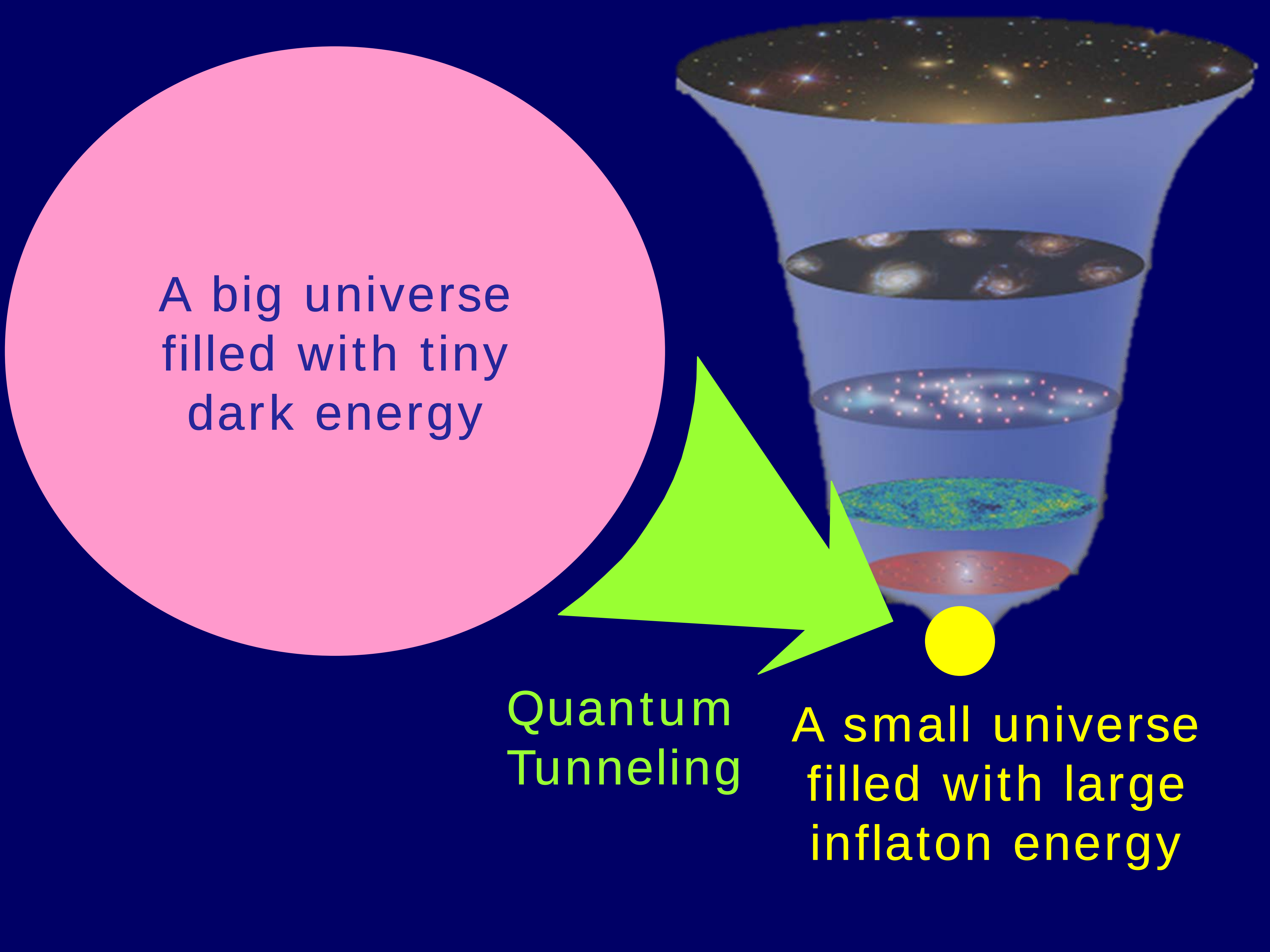
$$I_{\mathrm{E}}^{(\mathrm{tot})}(\phi_{\mathrm{s}}) = -\frac{A(\phi_{\mathrm{s}})}{4G} = \frac{-\pi}{GH^2} = \frac{-3}{8G^2 V(\phi_{\mathrm{s}})}$$

Oshita & JY 1603.06671

$$I_{\mathrm{E}}^{(\mathrm{tot})}(\phi_{\mathrm{s}}) = -\frac{A(\phi_{\mathrm{s}})}{4G} = \frac{-\pi}{GH^2} = \frac{-3}{8G^2V(\phi_{\mathrm{s}})}$$

In this picture transition to the potential top is suppressed NOT because it has larger energy density BUT because it has smaller entropy.

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Reincarnation of the universe

Garriga and Vilenkin 1998

The universe can recycle itself to another universe with possibly different properties.

We may not have to consider the real beginning of the universe.

This scenario prefers a pure cosmological constant/vacuum energy with $w = -1$.

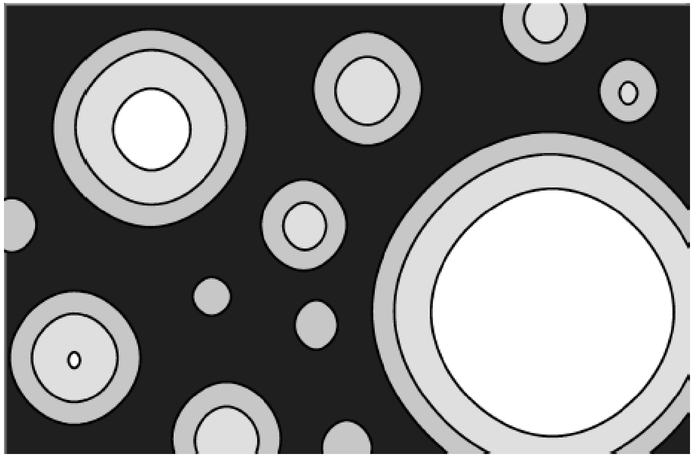
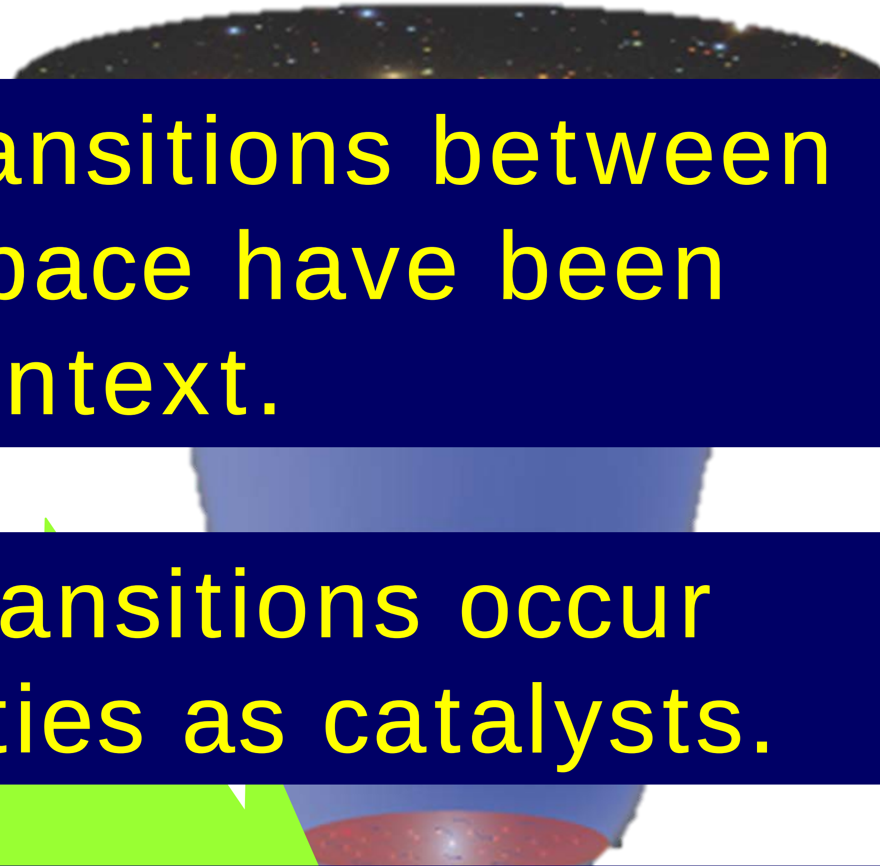


FIG. 1. True vacuum bubbles (white) nucleating in false vacuum (black). The shaded rings represent slow roll regions (external ring) and matter or radiation dominated regions (internal ring).

So far only phase transitions between two pure de Sitter space have been considered in this context.

filled with tiny

But usually, phase transitions occur around some impurities as catalysts.



Black holes may catalyze cosmological phase transitions.

Tunneling

filled with large inflaton energy

Studies on cosmological phase transitions around a black hole was pioneered by Hiscock in 1987. He assumed that the black hole mass does not change during bubble nucleation around a black hole.

More recently, Gregory, Moss, & Withers (2014) revisited the problem. They started with a Schwarzschild de Sitter space and considered nucleation of a thin-wall bubble of true vacuum with a remnant black hole in the center. cf Coleman De Luccia (1980)

They calculated Euclidean actions before and after bubble nucleation, postulating that the nucleation rate is given by

$$\Gamma = Ae^{-B}, \quad B = I_{\square} - I_{SdS}$$

Euclidean action of a true vacuum bubble surrounded by false vacuum with a BH in the center

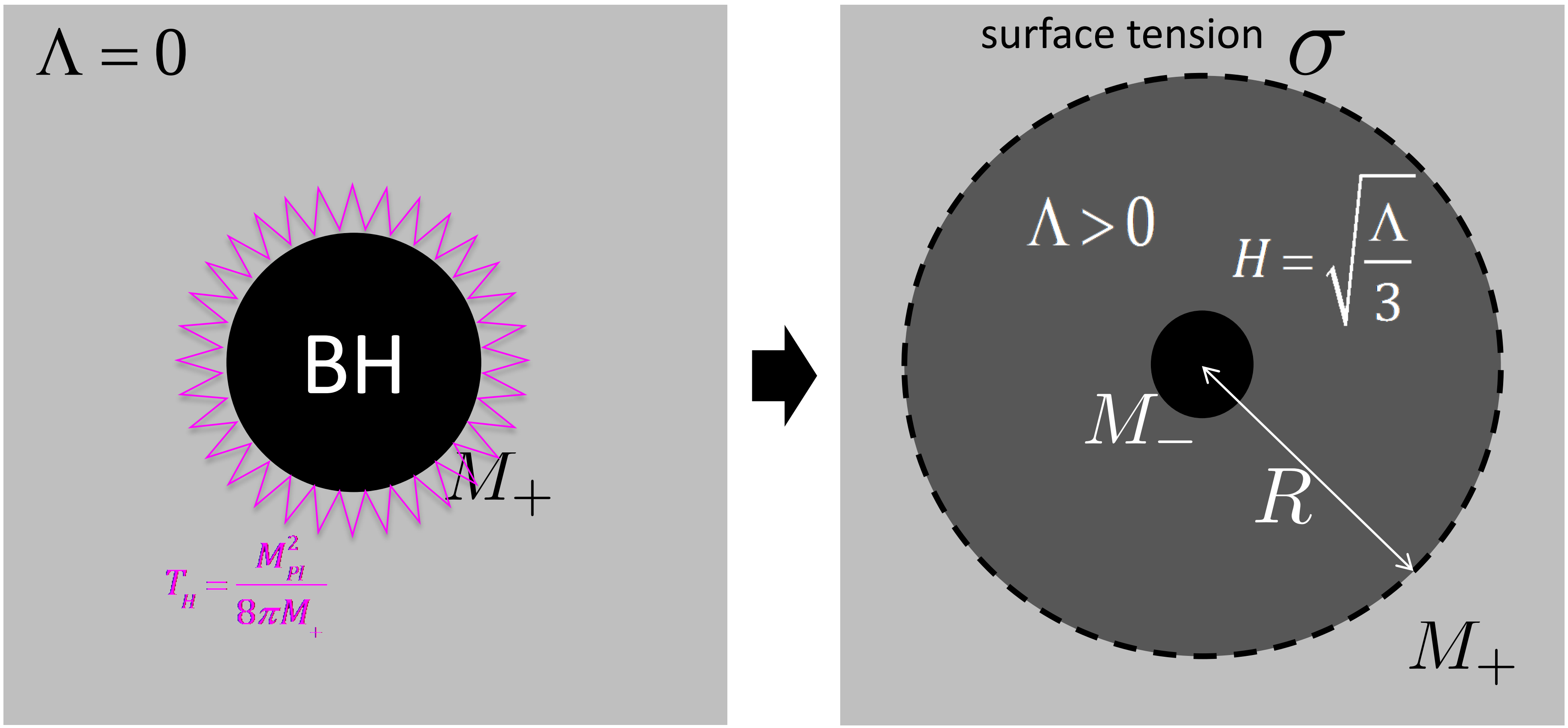
They have taken the effects of conical deficits properly, obtaining a term proportional to the surface area of the horizon.

They have also observed that the BH mass may change.

Here we consider phase transition around a Schwarzschild BH to form a false vacuum bubble separated by a thin wall.

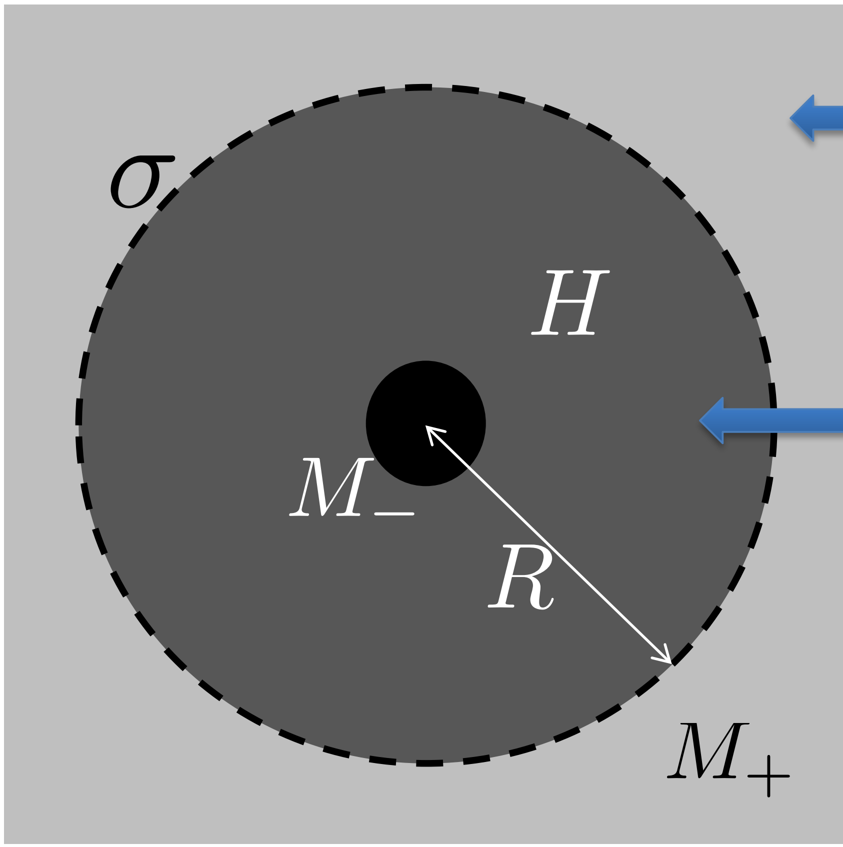
Oshita & JY 1601.03929

thin wall bubble with surface tension σ



The outer geometry remains the same Schwarzschild space even after the phase transition, thanks to the Birkhoff's theorem.

Geometries $$ds^2 = -f_\pm(r)dt^2 + \frac{dr^2}{f_\pm(r)} + r^2 d\Omega^2$$ 3-sphere



Outer Schwarzschild

$$f_+ = 1 - \frac{2GM_+}{r}$$

Inner Schwarzschild de Sitter

$$f_- = 1 - \frac{2GM_-}{r} - H^2 r^2$$

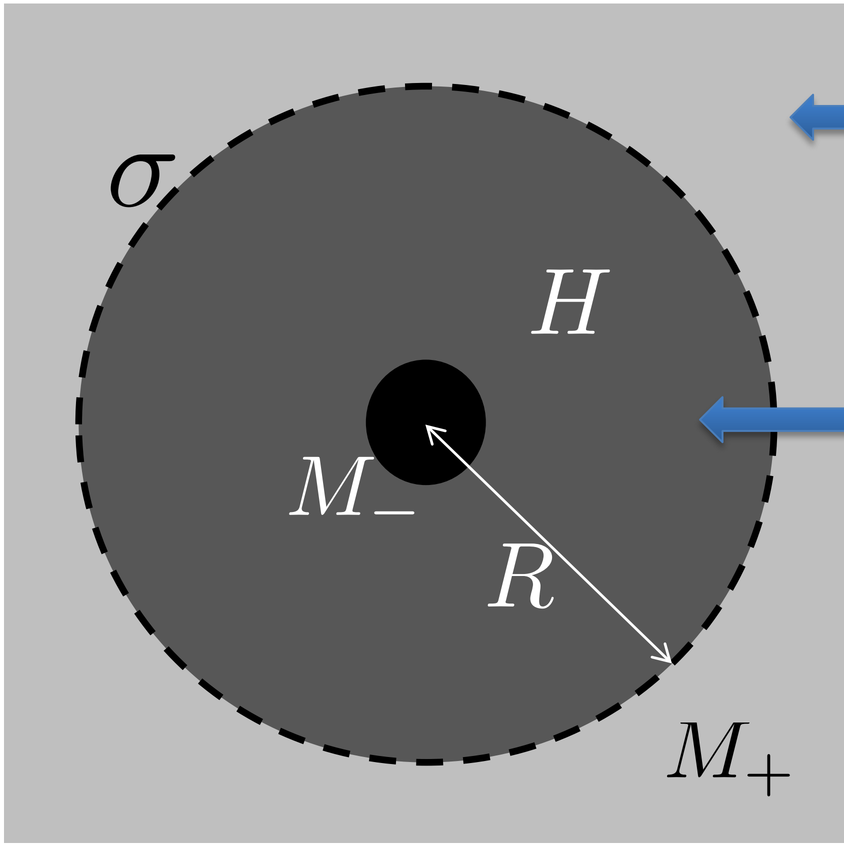
Wall trajectory

$$(t_\pm(\tau), r_\pm(\tau), \theta, \phi)$$

τ : proper time of the wall

$$r_+ = r_- \equiv R \qquad d\tau^2 = f_\pm(R)dt^2 - \frac{dR^2}{f_\pm(R)} - R^2 d\Omega^2 \qquad f_\pm(R)\dot{t}_\pm^2(\tau) - \frac{\dot{R}_\pm^2(\tau)}{f_\pm(R)} = 1$$

Israel's junction condition



Outer Schwarzschild

$$\beta_+ = f_+(R)\dot{t}_+ = \pm\sqrt{f_+^2 + \dot{R}^2}$$

Inner Schwarzschild de Sitter

$$\beta_- = f_-(R)\dot{t}_- = \pm\sqrt{f_-^2 + \dot{R}^2}$$

$$\dot{t} = \frac{\partial t}{\partial \tau} \qquad f_\pm(R)\dot{t}_\pm^2(\tau) - \frac{\dot{R}_\pm^2(\tau)}{f_\pm(R)} = 1$$

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

$$M_+ = M_- + \frac{(H^2 + \Sigma^2)R^3}{2G} + \frac{\Sigma R^2}{G}\beta_+$$

$$M_+ = M_- + \frac{4\pi}{3}R^3\Lambda + \frac{\Sigma R^2}{G}\beta_+$$

$$\frac{8\pi G}{3}\Lambda \equiv H^2 + \Sigma^2 \qquad (\beta_+ < 0)$$

$$\beta_- - \beta_+ = 4\pi G\sigma R \equiv \Sigma R$$

$$\beta_+ = f_+(R)\dot{t}_+ = \pm\sqrt{f_+^2 + \dot{R}^2}$$

$$\beta_- = f_-(R)\dot{t}_- = \pm\sqrt{f_-^2 + \dot{R}^2}$$

square

square

$$\left(\frac{dz}{d\tau'}\right)^2 - \frac{\gamma^2}{1-s}\frac{1}{z} - \left(\frac{1-z^3}{z^2}\right)^2 = -\frac{\gamma^2}{(2GM_+\chi)^{\frac{3}{2}}(1-s)^{\frac{3}{2}}}$$

"potential" V(z) total "energy" E

Thin shell's motion = 1 dimensional system in quantum field theory

$$z \equiv \frac{\chi^{2/3}}{(2GM_+)^{1/3}(1-s)^{1/3}}R \qquad \tau' = \frac{\chi^2}{8\pi G\sigma}\tau \qquad s \equiv \frac{M_-}{M_+}$$

$$\chi \equiv (H^2 + \Sigma^2)^{\frac{1}{2}} \approx \frac{M_X^2}{M_{Pl}}$$

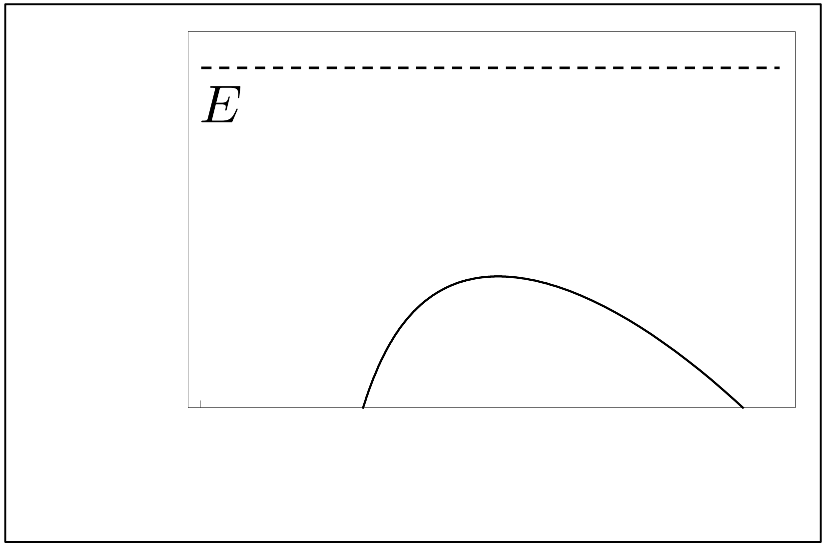
M_X :Energy scale of the theory

$$\gamma \equiv \frac{2\Sigma}{\chi} \approx \frac{M_X}{M_{Pl}} \square 10^{-4} \square 1 \text{ if } M_X \square M_{GUT}$$

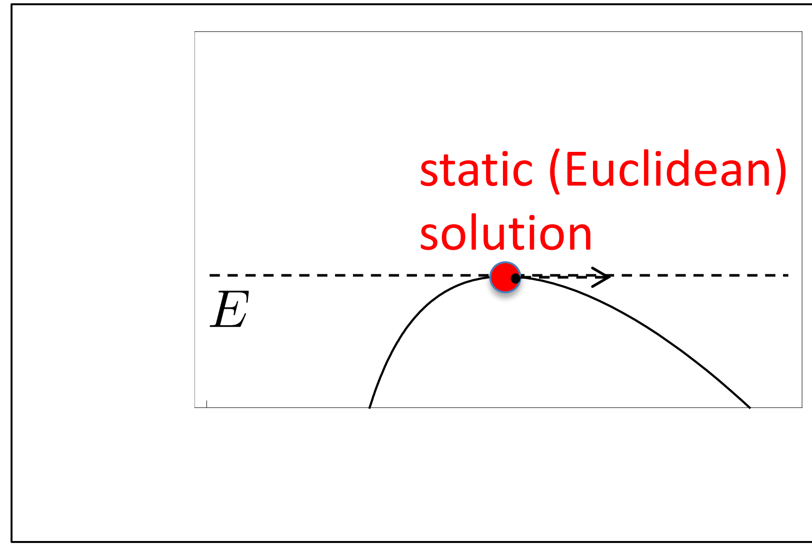
$$\left(\frac{dz}{d\tau'}\right)^2 + V(z) = E = -\frac{\gamma^2}{(2GM_+\chi)^{2/3}(1-s)^{2/3}}$$

$\gamma = 1,\ s = 0.9$ for illustrative purpose

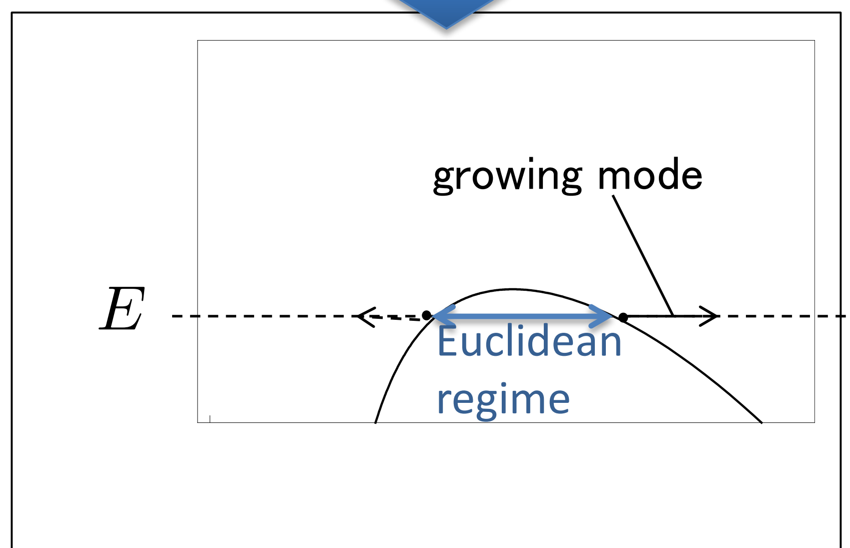
$$M_+ = M_C \sim 1/(G\chi) \sim 10^4 M_{\rm Pl}$$



M_+ □

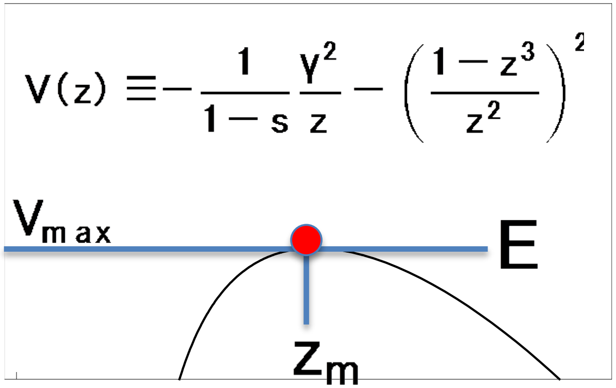


$$M_+ \gg 1/(G\chi)$$



As M_+ decreases due to Hawking radiation, bubble nucleation becomes possible.

Consider the case with the static Euclidean solution



$$V(z_m) \equiv V_{max} = -3\frac{z_m^6 - 1}{z_m^4}$$

$$z_m^3 = \pm\left[2 + \left(\frac{1}{2} - \frac{\gamma^2}{4(1-s)}\right)^2\right]^{\frac{1}{2}} - \left(\frac{1}{2} - \frac{\gamma^2}{4(1-s)}\right)$$

$+ : s < 1, \quad - : s > 1$

For the static Euclidean solution, only the range $s < 1$ is relevant.

Our model has 4 parameters, χ, γ, M_+, and s

determined by high energy theory

From $E = V(z_m)$ and $V'(z_m) = 0$ we can express

$$s = \frac{v^2 + (1 - \frac{\gamma^2}{2})v - 2}{(v-1)(v+2)},$$

$$M_+ = \frac{\gamma v}{3\sqrt{3G\chi}}\frac{v+2}{v+1}(v^2 - 1)^{-\frac{1}{2}}.$$

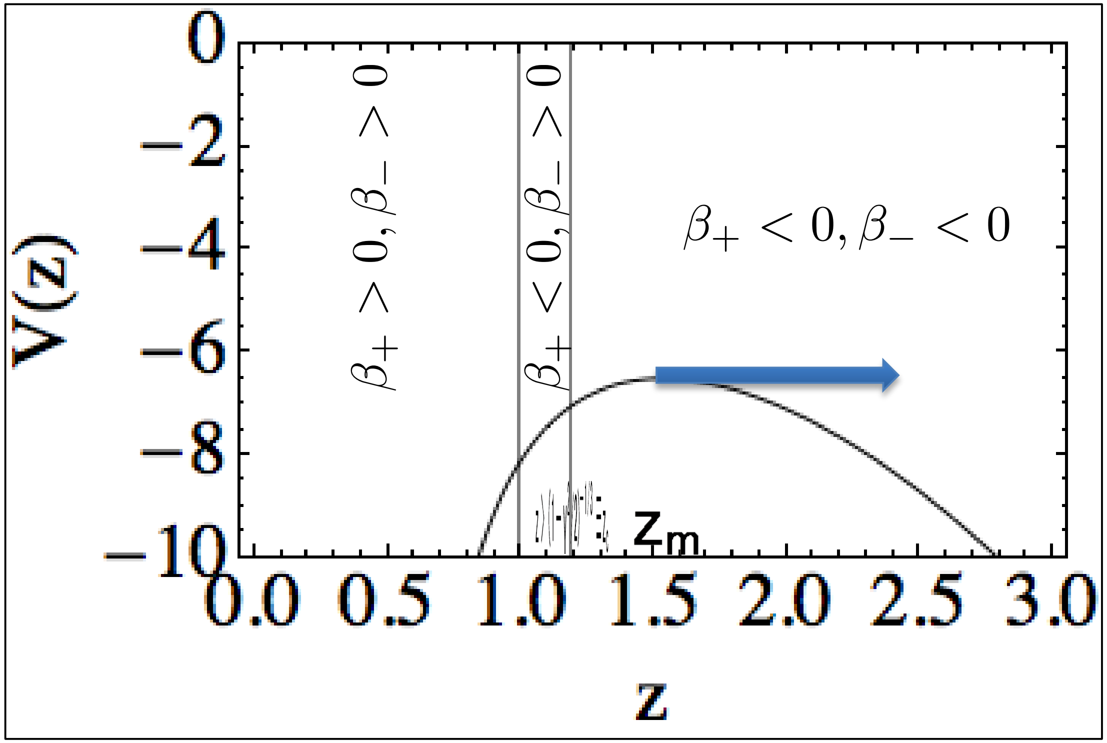
in terms of $v \equiv z_m^3$.

s and M_- may take arbitrary small value down to 0?

NO

On both inside and outside the bubble wall, the time must proceed to the same direction: $\beta_+ > 0, \beta_- > 0$, or $\beta_+ < 0, \beta_- < 0$ $\quad \beta_\pm = f_\pm(R)\dot{t}_\pm$

(Blau, Guendelman, and Guth 1987)



When $s < 1$, we find $\beta_+ < 0$ for $z > 1$.

On the other hand, we find

$$\beta_- = \frac{1}{z^2\sqrt{|E|}} - \left(1 - \frac{\gamma^2}{2}\right)\frac{z}{\sqrt{|E|}}.$$

so only for $z > (1 - \gamma^2/2)^{-1/3} \equiv z_c$ we have $\beta_- < 0$.

$$z_m > z_c \Rightarrow v > (1 - \gamma^2/2)^{-1} > 1$$

The critical mass below which static Euclidean solution exists.

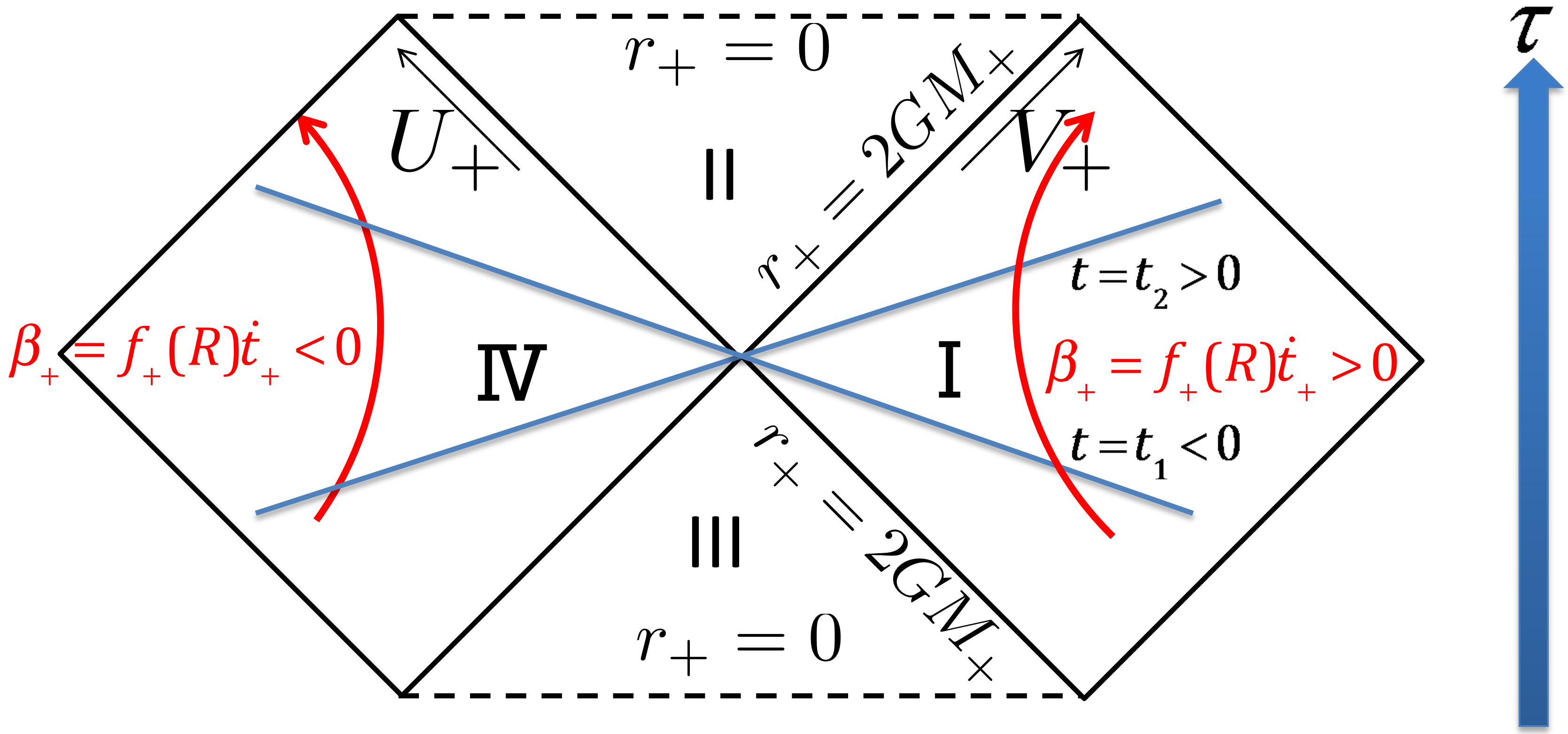
$$M_+ < \frac{1 - \frac{\gamma^2}{3}}{2\sqrt{3}(1 - \frac{\gamma^2}{4})^{\frac{3}{2}} G\chi} \equiv M_c \sim \frac{M_{Pl}^3}{M_X^2}$$

$$s > \frac{2}{3}\frac{1 - \frac{\gamma^2}{4}}{1 - \frac{\gamma^2}{3}} \equiv s_c$$

M_- cannot vanish

For a solution with an expanding bubble wall we must have $\beta_+ < 0, \beta_- < 0$.

Penrose diagram of the Schwarzschild spacetime



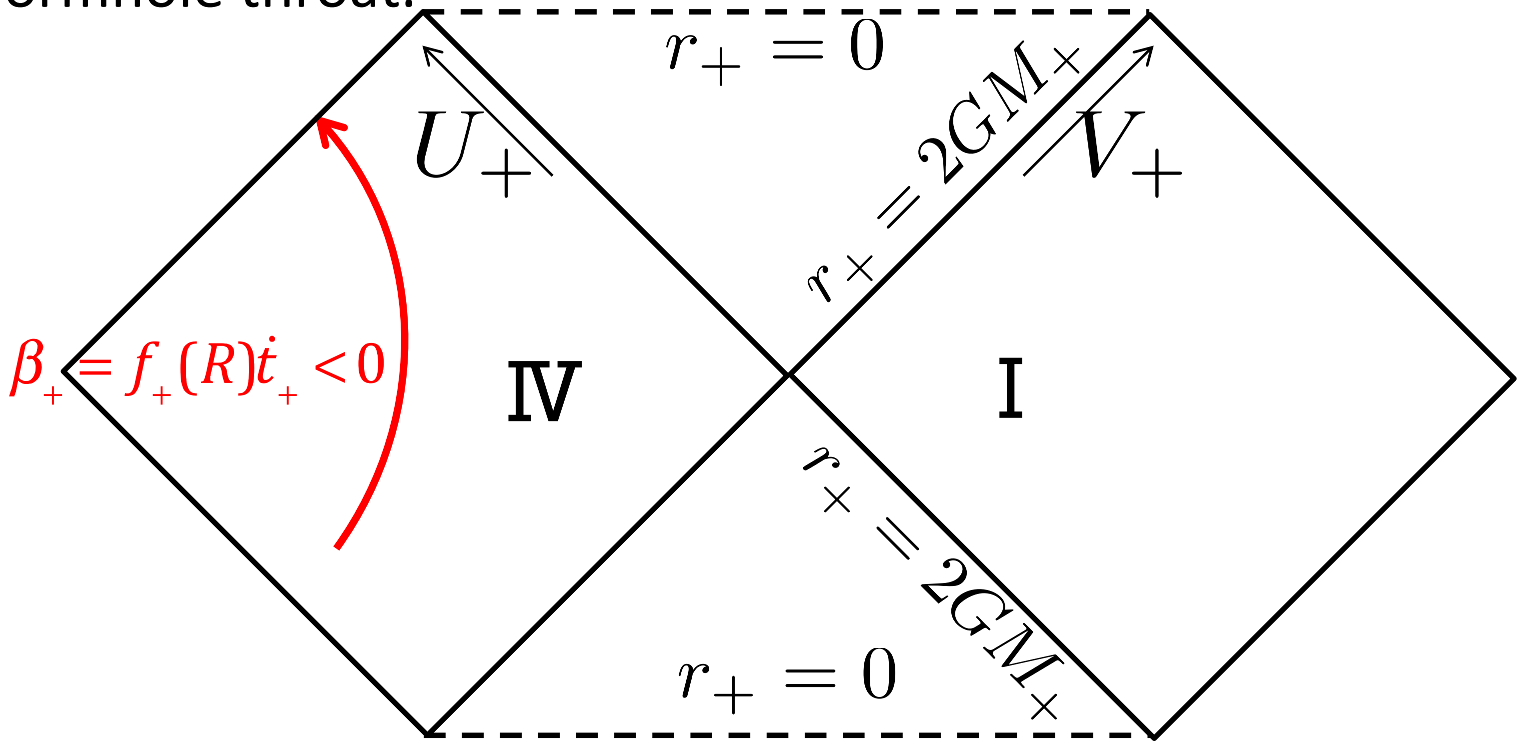
I Our World outside the BH horizon

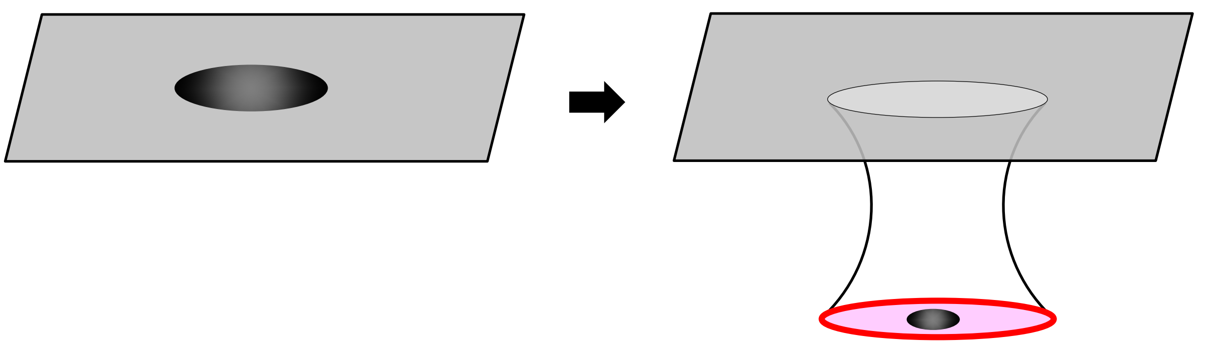
II Inside the Black Hole

III Inside the White Hole

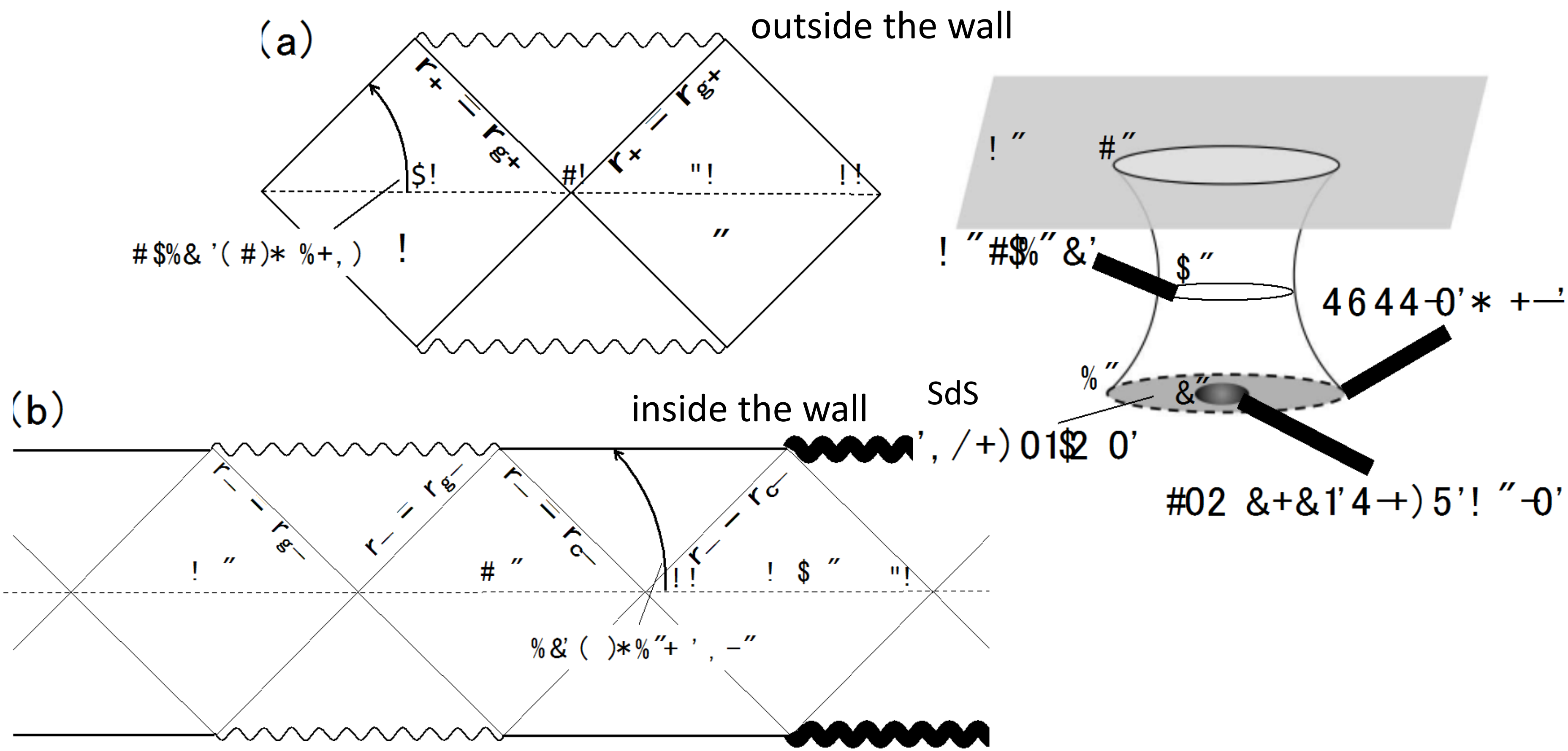
IV Another World causally disconnected from ours

An expanding bubble with $\beta_+ = f_+(R)\dot{t}_+ < 0$ is located beyond the wormhole throat.



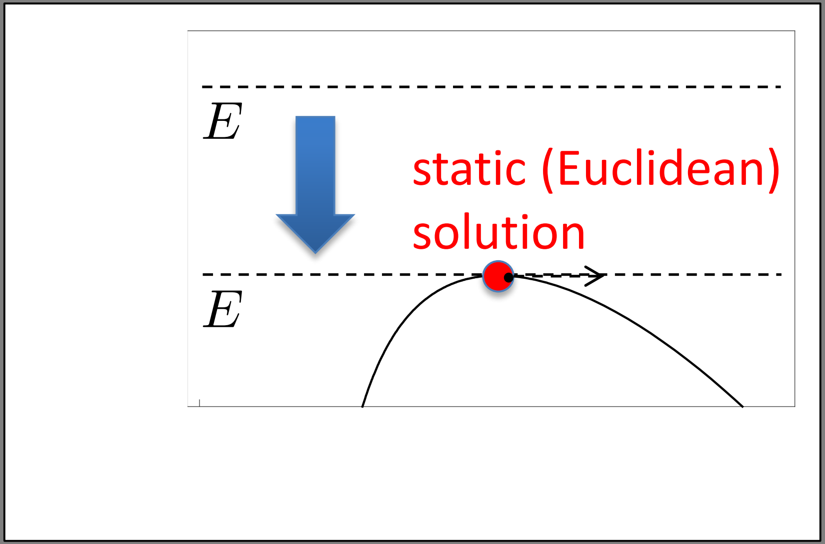


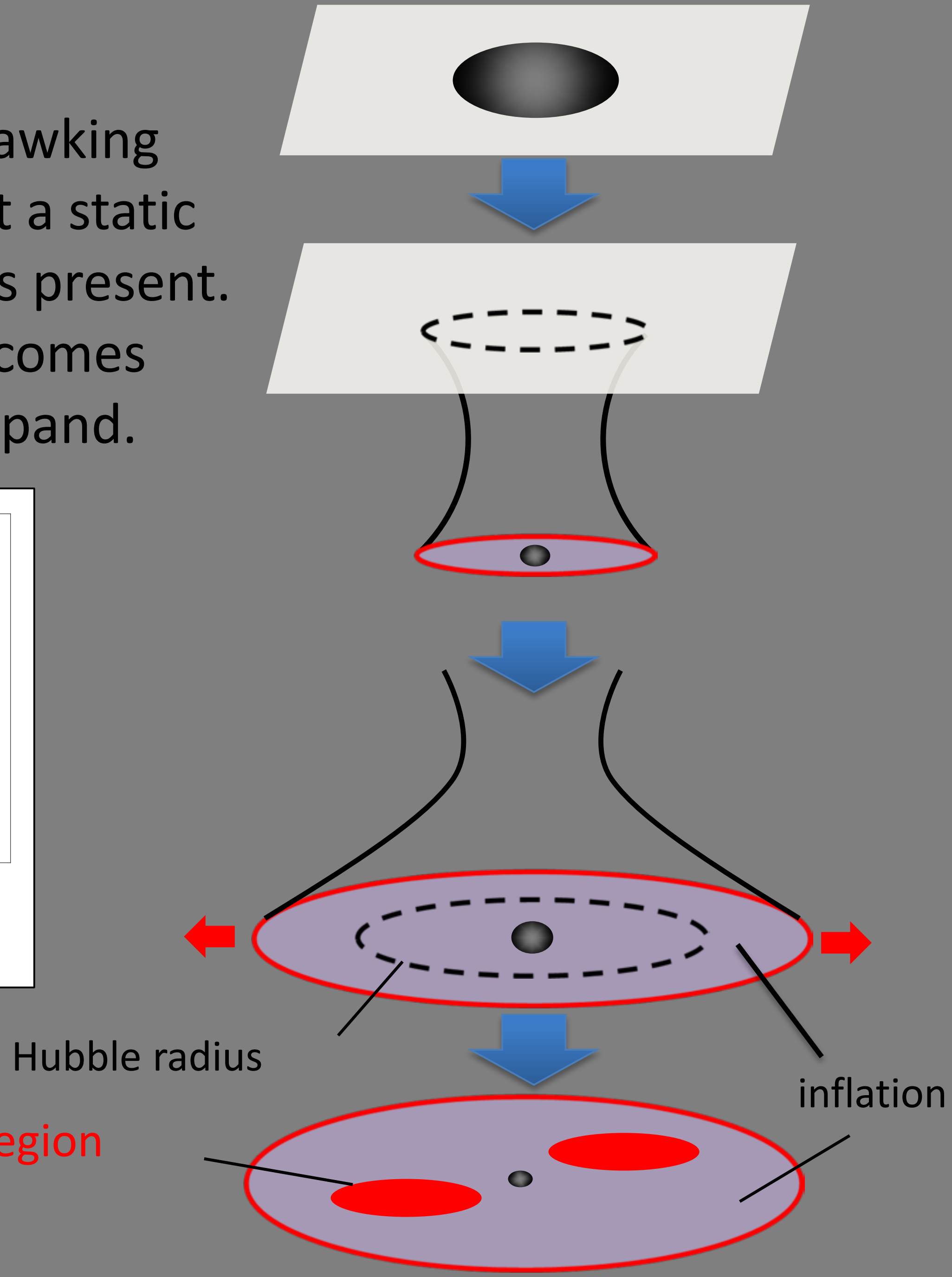
Spacetime structure at the bubble nucleation



Scenario

As BH mass decreases due to Hawking radiation, *E* reaches to the point a static Euclidean solution of a bubble is present. Then nucleation of a bubble becomes possible and the bubble may expand.





Calculate the Euclidean action following Gregory et al (2014).

$$I = I_{\mathcal{B}} + I_- + I_+ + I_{\mathcal{W}},$$

$$I_{\mathcal{W}} = -\int_{\mathcal{W}} \mathcal{L}_m(g,\phi) = \int_{\mathcal{W}} \sigma$$ Wall

$$I_{\pm} = -\frac{1}{16\pi G}\int_{\mathcal{M}_\pm} \mathcal{R} - \int_{\mathcal{M}_\pm} \mathcal{L}_m(g,\phi) + \frac{1}{8\pi G}\int_{\partial\mathcal{M}_\pm} K.$$ Bulk

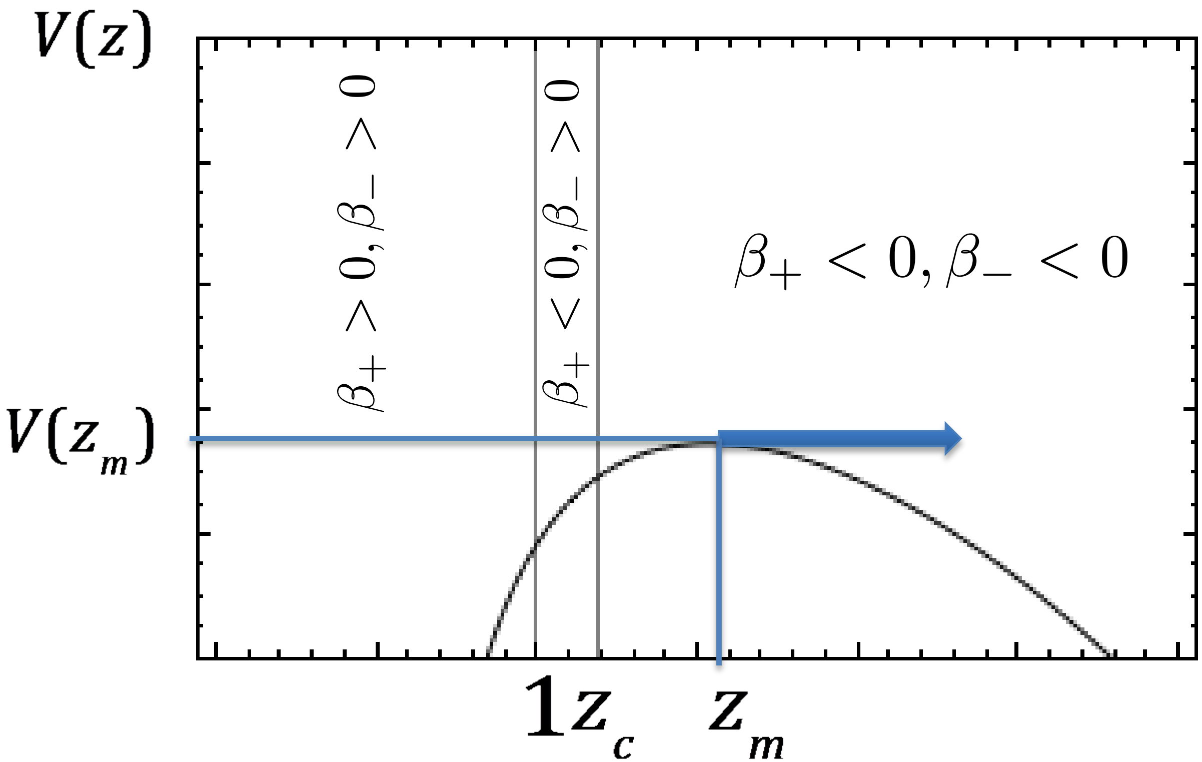
$$I_{\mathcal{B}} = -\frac{1}{16\pi G}\int_{\mathcal{B}} \mathcal{R} + \frac{1}{8\pi G}\int_{\partial\mathcal{B}} K$$ Conical deficits (giving rise to a contribution proportional to the horizon area)

$$I_{bubbly} = -\frac{A_{-horizon}}{4G} - \frac{A_{+horizon}}{4G} + \int d\tau_E \left[(2R - 6GM_+)\dot{t}_{E+} - (2R - 6GM_-)\dot{t}_{E-} \right]$$

In the particular case of the static Euclidean solution, the last integral term vanishes.

$$B \equiv I_{bubbly} - I_{no-bubble} \qquad \Gamma \propto e^{-B}?$$

For γ □ 1 namely, M_X □ M_{Pl}



$$z_c \cong 1 + \frac{\gamma^2}{6},$$

$$z_m \cong 1 + \frac{\gamma^2}{18(1-s)},$$

$$V(z_m) \cong -\frac{\gamma^2}{1-s}$$

$$z_m \geq z_c \Rightarrow s = \frac{M_-}{M_+} \geq \frac{2}{3}$$

$$s = \frac{2}{3}$$

$$E = -\frac{\gamma^2}{(2GM_+\chi)^{2/3}(1-s)^{2/3}} = V(z_m) = -\frac{\gamma^2}{1-s}$$

$$r_{g+} = 2GM_+ = \frac{1}{\sqrt{3}\chi}$$

The horizon of Schwarzschild de Sitter BH is given by

$$f_-(r_{g-}) = 1 - \frac{2GM_-}{r_{g-}} - H^2 r_{g-}^2 = 0 \text{ with } M_- = \frac{2}{3} M_+ \quad \Rightarrow \quad \boxed{r_{g-} \cong \left(1 + \frac{\gamma}{2\sqrt{3}}\right) r_{g+}}$$

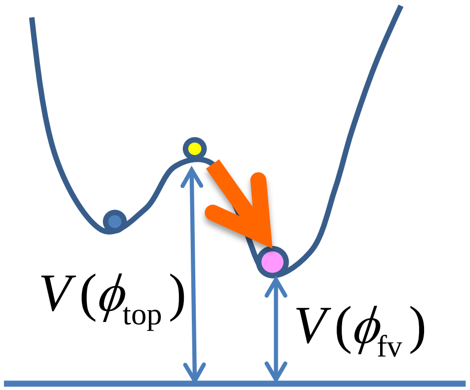
The Euclidean action before transition is simply given by

$$I_{no-bubble} = -\frac{A_{+horizon}}{4G}$$

$$B = I_{bubbly} - I_{no-bubble} = -\frac{A_{-horizon}}{4G}$$

Large and negative !!

Just like a top-to-bottom transition in the Hawking Moss instanton, which would not actually be a tunneling.



The horizon of Schwarzschild de Sitter BH is given by

$$f_{-}(r_{g-}) = 1 - \frac{2GM_{-}}{r_{g-}} - H^2 r_{g-}^2 = 0 \text{ with } M_{-} = \frac{2}{3} M_{+} \quad \Rightarrow \quad \boxed{r_{g-} \cong \left(1 + \frac{\gamma}{2\sqrt{3}}\right) r_{g+}}$$

The Euclidean action before transition is simply given by

$$I_{no-bubble} = -\frac{A_{+horizon}}{4G}$$

$$B = I_{bubbly} - I_{no-bubble} = -\frac{A_{-horizon}}{4G}$$

Large and negative !!

The transition occurs spontaneously as soon as the BH mass reaches $M_{+} = \frac{1}{2\sqrt{3}G\chi}$ determined by the scale of phase tr. !??

Thermodynamic interpretation

$$\Gamma \propto e^{-\beta(F_{bubbly}-F_{no-bubble})} = e^{-\beta(E_{bubbly}-E_{no-bubble})} e^{S_{bubbly}-S_{no-bubble}} = e^{S_{bubbly}-S_{no-bubble}}$$

vanishes due to the Hamiltonian constraint

$$B = I_{bubbly} - I_{no-bubble} = -\frac{A_{-horizon}}{4G}$$

counts the change of -entropy .

Because the final state has much larger entropy, we expect the phase transition to occur.

Conclusion

How did our Universe begin?

?

上州人

上州無智亦無才
剛毅木訥易被欺
唯以正直接萬人
至誠依神期勝利

鑑三

Symmetry restoration of a scalar field around a black hole

$$\left\langle \phi^2(x) \right\rangle = \frac{1}{12}\left(\frac{M_{Pl}^2}{8\pi M_{BH}}\right)^2 \frac{1-(r_g/r)^4}{1-r_g/r} = \frac{1}{12}\left(T_{loc}^2 - T_{acc}^2\right)$$

$$T_{loc}^2 = \left(\frac{M_{Pl}^2}{8\pi M_{BH}}\right)^2 \frac{1}{1-r_g/r}$$

local Hawking temperature

$$T_{acc}^2 = \frac{1}{2\pi}\frac{M_{BH}}{M_{Pl}^2 r^2}\frac{1}{1-r_g/r}$$

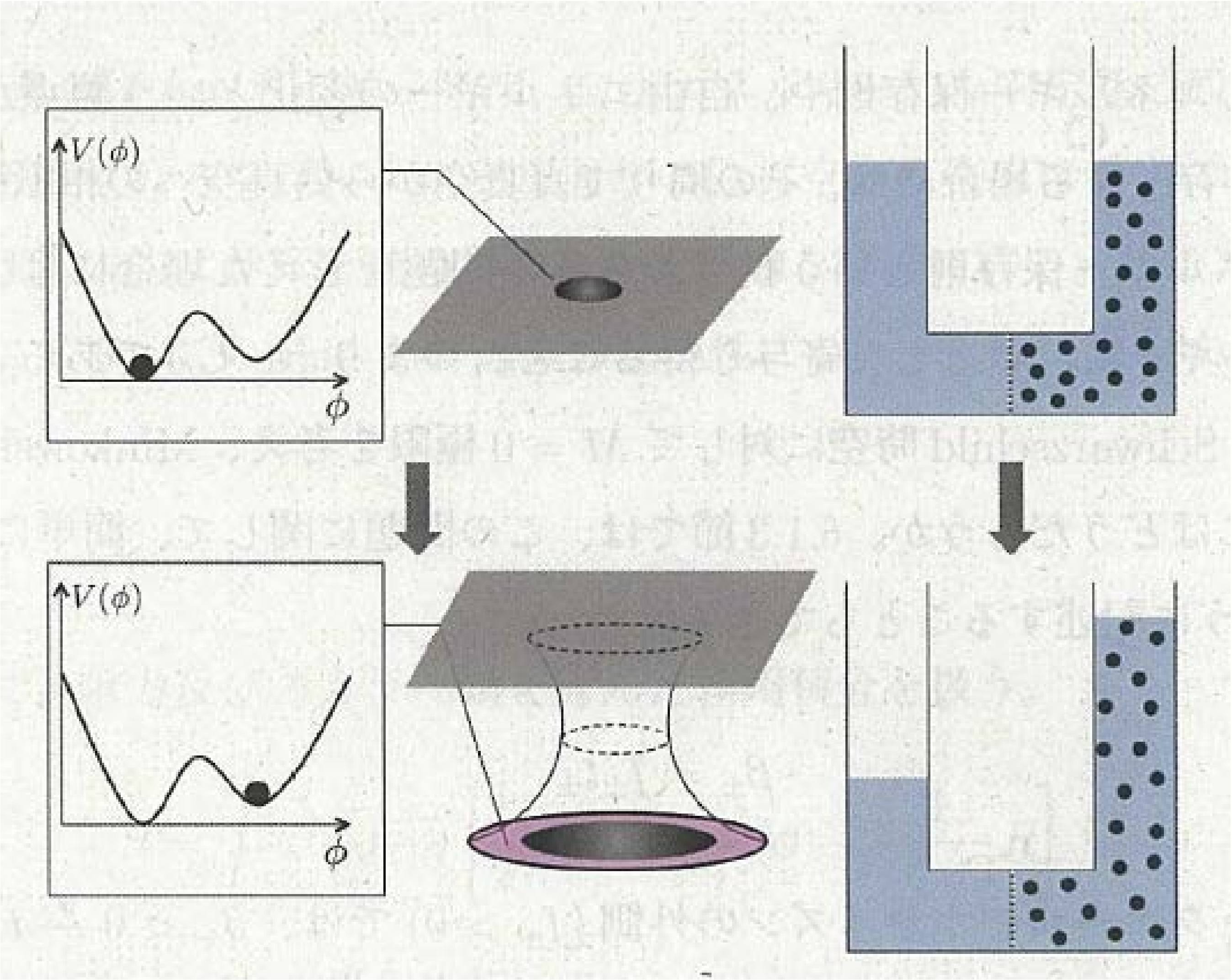
local acceleration (Unruh) temperature

This expression is valid even inside the black-hole horizon.
Symmetry restoration is expected !

(Candelas and Howard 1984,86)

Euclidean Schwarzschild space has a negative mode
Fluctuation around it is unstable.

(Gross, Perry, & Yaffe 1982)



Figures from Oshita (2016)