Dark Matter Primordial Black Holes and their Formation

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Based on

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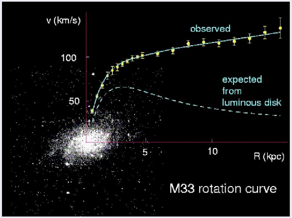
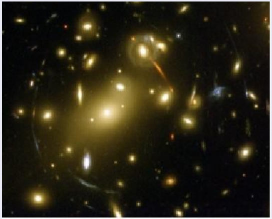
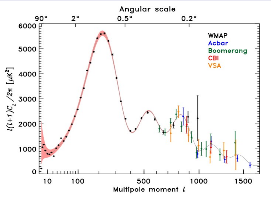
Outline

Dark Matter

Evidences







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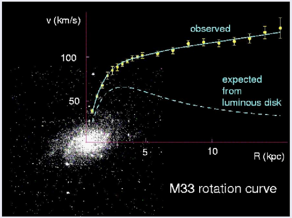

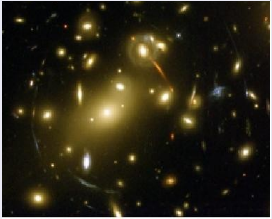

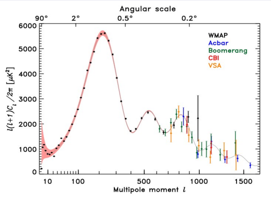

How much DM?

Planck.XIII, [arXiv: 1502.01589]

$$\Omega_{\rm DM} h^2 = 0.1188 \pm 0.0010$$

Properties

- stable
- neutral
- weakly interacting
- right relic density

Candidates

- Axions
- Sterile neutrinos
- WIMPs
- ...

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Any candidate in Standard Model?

Primordial Black Holes (PBHs)

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Definition

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RD era $\quad t \propto T^{-2} \longrightarrow M_{\rm PBH} = M_{\rm P}\left(\frac{T}{T_{\rm P}}\right)^{-2} \xrightarrow{T_{\rm RH}\simeq 10^{16}{\rm GeV}} M_{\rm min} = 1\,{\rm g}$

Hawking radiation

Temperature: $T_{\rm BH} \approx 10^{-7} \left(\frac{M}{M_\odot}\right)^{-1}$ K

$M > 10^{17}\,\mathrm{g}$	massless particles
$10^{15}\,\mathrm{g} \lesssim M \lesssim 10^{17}\,\mathrm{g}$	electrons
$10^{14}\,\mathrm{g} \lesssim M \lesssim 10^{15}\,\mathrm{g}$	muons
$M < 10^{14}\,\mathrm{g}$	hadrons

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Lifetime: $\tau_{\rm BH} \approx 10^{64} \left(\frac{M}{M_\odot}\right)^{3}$ y

$M_{\rm BH}$	$\tau_{\rm BH}$
A man	10^{-12} s
A building	1 s
10^{15} g	10^{10} y
The Earth	10^{49} y
The Sun	10^{66} y
The Galaxy	10^{99} y

Why PBHs are useful?

- PBHs as a probe of the early Universe $(M < 10^{15}\,\mathrm{g})$
- PBHs as a probe of gravitational collapse $(M > 10^{15}\,\mathrm{g})$ ✓
  DM candidates $\Omega^0_{\mathrm{PBH}} \lesssim \Omega^0_{\mathrm{CDM}} (= 0.23)$
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How PBHs form?

- Soft equation of state
- Bubble collisions
- Collapse of cosmic loops
- Fluctuations by inflation ✓

Inflation

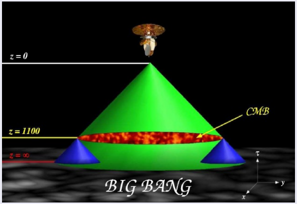
Accelerated expansion of the Universe $\ddot{a} > 0$

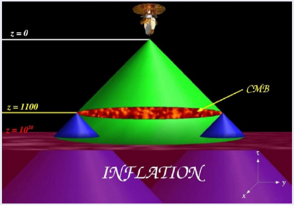
Why inflation?

- Flatness problem

$$\Omega_0 - 1 = (\Omega_{\rm i} - 1)\left(\frac{\dot{a}_{\rm i}}{\dot{a}_0}\right)^2$$

- Horizon problem





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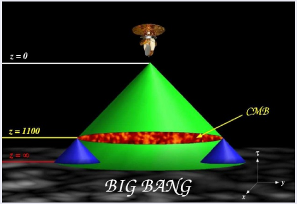

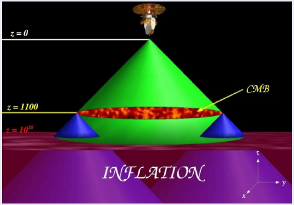

$$\frac{\ddot{a}}{a} = -\frac{4\pi\, G}{3}\,(\rho + 3\,p) \xrightarrow{\ddot{a}>0} w < -\frac{1}{3}$$

Scenario

Equation of motion $\quad \ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \xrightarrow{V(\phi) \gg \dot{\phi}^2} 3H\dot{\phi} \simeq -V'(\phi)$

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Slow-roll parameters

$$\epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2, \quad \eta \equiv M_P^2 \frac{V''}{V}, \quad \xi \equiv M_P^4 \frac{V' V'''}{V^2}, \quad \sigma \equiv M_P^6 \frac{V'^2 V''''}{V^3}.$$

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Power Spectrum

$$\mathcal{P}_{\mathcal{R}_c}(k) = \mathcal{P}_{\mathcal{R}_c}(k_0)\left(\frac{k}{k_0}\right)^{n_s(k)-1}$$

$$n_s(k) = n_s(k_0) + \frac{1}{2!}\alpha_s(k_0)\ln\left(\frac{k}{k_0}\right) + \frac{1}{3!}\beta_s(k_0)\ln^2\left(\frac{k}{k_0}\right) + \dots$$

$$n_s(k_0) \equiv \left.\frac{d\ln\mathcal{P}_{\mathcal{R}_c}}{d\ln k}\right|_{k=k_0}, \ \alpha_s(k_0) \equiv \left.\frac{dn_s}{d\ln k}\right|_{k=k_0}, \ \beta_s(k_0) \equiv \left.\frac{d^2 n_s}{d\ln^2 k}\right|_{k=k_0}$$

Inflation parameters

$$
\begin{aligned}
\mathcal{P}_{\mathcal{R}_c} &= \frac{1}{12\pi^2 M_{\rm P}^6}\frac{V^3}{V'^2} \\
n_s &= 1 - 6\epsilon + 2\eta \\
\alpha_s &= -24\epsilon^2 + 16\epsilon\eta - 2\xi \\
\beta_s &= -192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 - 24\epsilon\xi + 2\eta\xi + 2\sigma \\
r &\simeq 16\epsilon \\
B_\zeta(k_1,\, k_2,\, k_3) &= f_{\rm NL} F(k_1,\, k_2,\, k_3)
\end{aligned}
$$

Observation

Planck XX, arXiv: 1502.01592

$\ln(10^{10}\mathcal{P}_{\zeta,\,{\rm vac.}}(k_0)) = 3.094 \pm 0.034$ $\qquad k_0 = 0.05\ {\rm Mpc}^{-1}$

$n_{\rm s} = 0.9645 \pm 0.0049$

$\alpha_{\rm s} = -0.0065 \pm 0.0076$

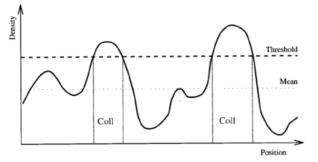
$\beta_{\rm s} = 0.025 \pm 0.013$

$r_{0.002} < 0.10 \qquad (95\,\%\,{\rm CL})$

$f_{\rm NL} = 22.7 \pm 25.5$

Press-Schechter Formalism

The Press-Schechter formalism is a model for predicting the number density of bound objects of a certain mass.



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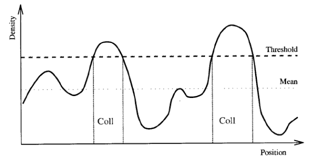

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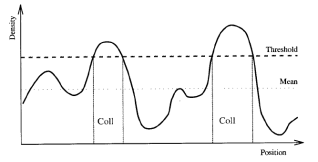

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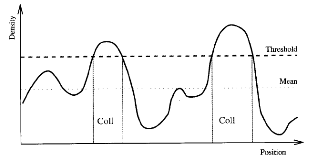

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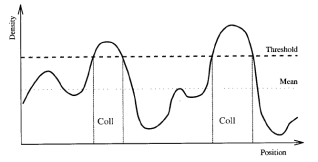

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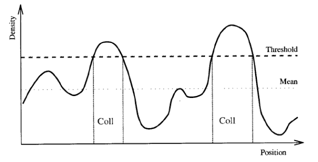

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$$\sigma_\delta^2(R) = \int_0^\infty W^2(kR)\mathcal{P}_\delta(k)\frac{{\rm d}k}{k} \qquad W(kR) = \exp\left(-k^2R^2/2\right)$$

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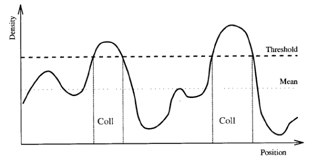

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$M_{\rm PBH} = \gamma M_{\rm PH} \xrightarrow{\gamma = w^{3/2}} \dfrac{R}{1\,{\rm Mpc}} = 5.5 \times 10^{-24}\gamma^{-\frac{1}{2}}\left(\dfrac{M_{\rm PBH}}{1\,{\rm g}}\right)^{1/2}\left(\dfrac{g_*}{3.36}\right)^{1/6}$

Gaussian PDF:

$$P_{\rm G}(\delta;\, R) = \frac{1}{\sqrt{2\pi}\sigma_\delta(R)} \exp\left(-\frac{\delta^2(R)}{2\sigma_\delta^2(R)}\right)$$

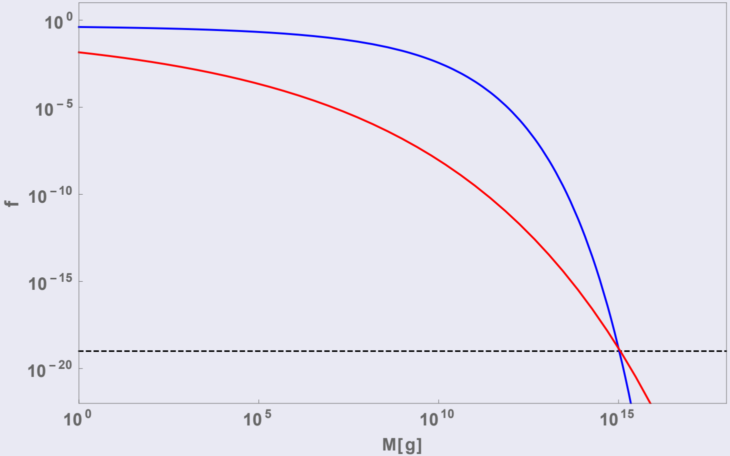
$$f_{\rm G} = \frac{1}{2}\,\text{erfc}\left(\delta_{\rm th}/\sqrt{2\sigma_\delta^2(R)}\right)$$

non-Gaussian PDF:

$$P_{\rm NG}(\delta;\, R) = \frac{1}{\sqrt{2\pi\left(\delta+\sigma_g^2(R)\right)}\,\sigma_g(R)} \exp\left(-\frac{\delta+\sigma_g^2(R)}{2\sigma_g^2(R)}\right)$$

$$f_{\rm NG} = \text{erfc}\left(\sqrt{\delta_{\rm th}+\sigma_g^2(R)}/\sqrt{2\sigma_g^2(R)}\right)$$

$f(\geq M)$ diagram for the mass range $10^0 - 10^{20}\,\mathrm{g}$



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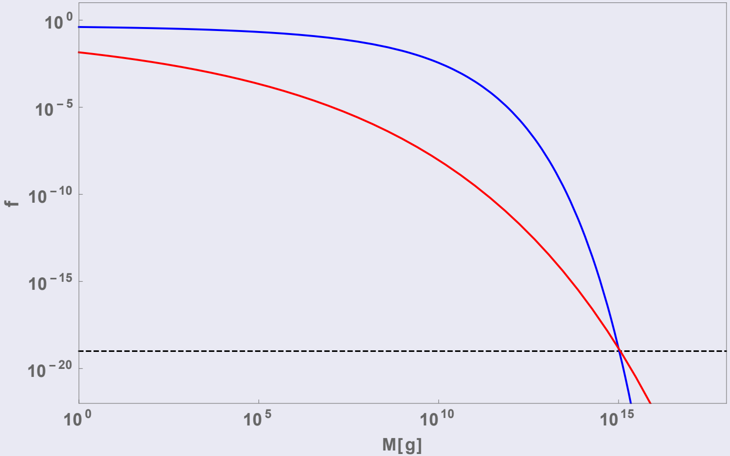
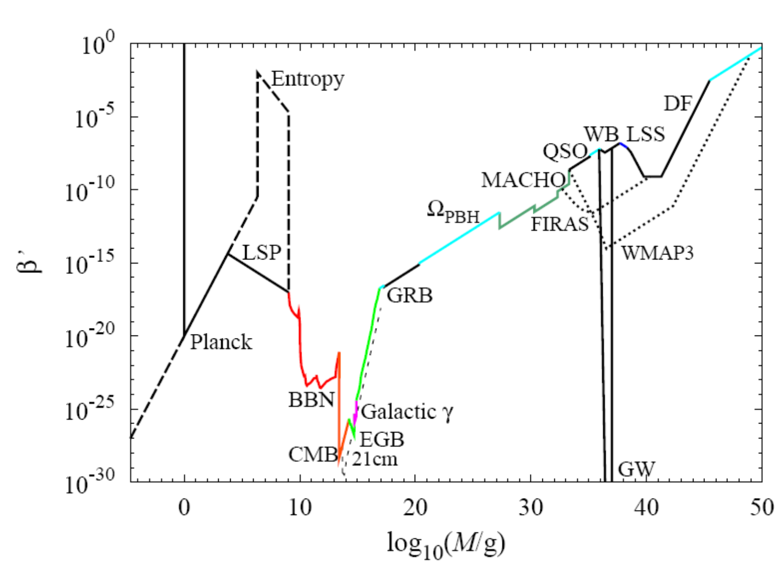


Result

$n_s(k_{\mathrm{PBH}}) \geq 1.418 \quad \Rightarrow \quad \mathcal{P}_\zeta \simeq 2 \times 10^{-2}$ for Gaussian PDF

$n_s(k_{\mathrm{PBH}}) \geq 1.322 \quad \Rightarrow \quad \mathcal{P}_\zeta \simeq 4 \times 10^{-4}$ for non-Gaussian PDF

Carr *et.al*, PRD 81 (2010) 104019



Result

$$n_s(k) = n_s(k_0) + \frac{1}{2!}\,\alpha_s(k_0)\,\ln\left(\frac{k}{k_0}\right) + \frac{1}{3!}\,\beta_s(k_0)\,\ln^2\left(\frac{k}{k_0}\right)$$

$$\begin{aligned} \beta_s(k_0) &\leq 0.0025 \qquad \Lambda CDM + dn_s/d\ln\, k \\ \beta_s(k_0) &\leq 0.0017 \qquad \Lambda CDM + dn_s/d\ln\, k + d^2 n_s/d\ln^2 k \end{aligned}$$

Non-production of DM PBHs put stronger upper bound on β_s.

Hilltop/inflection point inflation

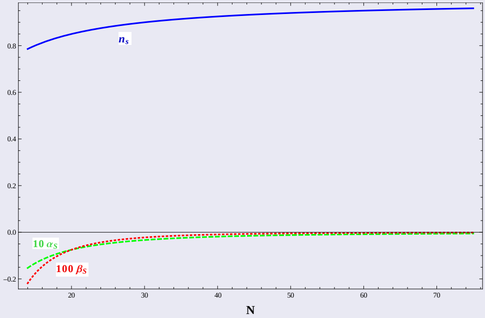
$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$ ☒

for $p > 2$

$$n_{\rm s} - 1 \simeq -\frac{p-1}{p-2}\frac{2}{N} < 0$$

$$\alpha_{\rm s} \simeq -\frac{p-1}{p-2}\frac{2}{N^2} < 0$$

$$\beta_{\rm s} \simeq -\frac{p-1}{p-2}\frac{2}{N^3} < 0$$



This model is not ruled out by *Planck*.

Inverse power low inflation

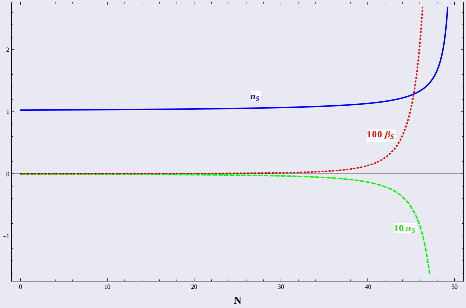
$$V(\phi) = V_0 + \frac{\Lambda_3^{p+4}}{\phi^p} \quad \boxtimes$$

$$n_s - 1 \simeq \frac{p+1}{p+2} \frac{2}{N_{tot}\left(1 - \frac{N}{N_{tot}}\right)}$$

$$\alpha_s \simeq -\frac{p+2}{p+1} \frac{(n_s - 1)^2}{2}$$

$$\beta_s \simeq \left(\frac{p+2}{p+1}\right)^2 \frac{(n_s - 1)^3}{2}$$

This model is disfavored by *Planck* for any p.



Running-mass inflation

The inflation potential is dominated by the soft SUSY breaking mass term generated by V_0 and its radiative corrections

$$V(\phi) = V_0 + \frac{1}{2} m_\phi^2(\phi)\phi^2 + \ldots$$

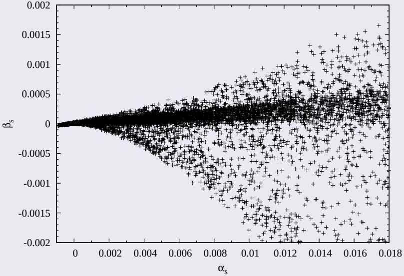
RGE $\quad \dfrac{dm^2}{d\ln\phi} \equiv \beta_m \qquad$ with $\quad \beta_m = -\dfrac{2C}{\pi}\alpha\, \tilde{m}^2 + \dfrac{D}{16\pi^2}|\lambda_Y|^2 m_{\mathrm{s}}^2$

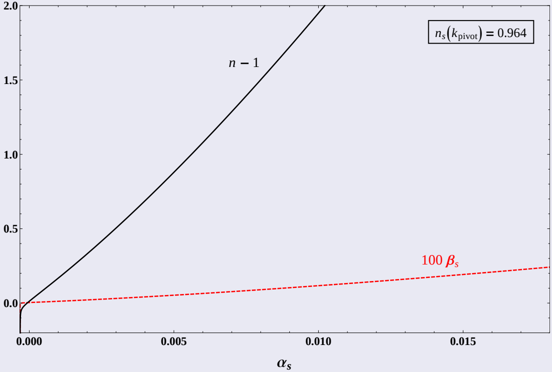
Over a sufficiently small range of ϕ, or small inflaton coupling, we can do the Taylor expansion:

$$V = V_0 + \frac{1}{2} m_\phi^2(\phi_*)\phi^2 + \frac{1}{2}\left.\frac{dm_\phi^2}{d\ln\phi}\right|_{\phi_*} \ln\left(\frac{\phi}{\phi_*}\right) + \frac{1}{4}\left.\frac{d^2 m_\phi^2}{d(\ln\phi)^2}\right|_{\phi_*} \ln^2\left(\frac{\phi}{\phi_*}\right)$$

where ϕ_* is the local extremum of the potential.

$$\alpha_s \geq -\frac{(n_s - 1)^2}{4}$$





Large Field Models $\quad |\triangle\phi| \gtrsim M_{\rm P}\,, \quad r \neq 0$

Chaotic inflation

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{\mu}\right)^p \quad \boxtimes$$

$$n_{\rm s} - 1 = -\frac{2(p+2)}{4N+p}$$

$$\alpha_{\rm s} = -\frac{2}{p+2}(n_{\rm s}-1)^2$$

$$\beta_{\rm s} = \frac{8}{(p+2)^2}(n_{\rm s}-1)^3$$

$$r = -\frac{7p}{p+2}(n_{\rm s}-1)$$

This model is ruled out with *Planck* data for $p = 4$.

Natural inflation

$$V(\phi) = \Lambda^4 \left[1 + \cos\left(\frac{\phi}{f}\right)\right] \quad \boxtimes$$

$$n_{\rm s} - 1 \propto -\frac{2}{N} < 0$$

$$\alpha_{\rm s} \propto -\frac{2}{N^2} < 0$$

$$\beta_{\rm s} \propto -\frac{2}{N^3} < 0$$

This model agrees with *Planck*+WP data for $f \gtrsim 5\,M_{\rm P}$.

Negative exponential inflation

$$V(\phi) = V_0 \left[1 - \exp\left(\frac{-q\phi}{M_\mathrm{P}}\right)\right], \quad q > 0 \qquad \boxtimes$$

$$\begin{aligned} n_\mathrm{s} - 1 &\simeq -2/(N+1) < 0 \\ \alpha_\mathrm{s} &\simeq -2/(N+1)^2 < 0 \\ \beta_\mathrm{s} &\simeq -4/(N+1)^3 < 0 \end{aligned}$$

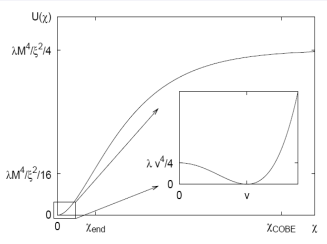
This model is ruled out by *Planck*.

- Higgs Inflation

$$S_\mathrm{J} = \int \mathrm{d}^4x\sqrt{-g}\left\{-\frac{M^2 + \xi\, h^2}{2}R + \frac{\partial_\mu h\,\partial^\mu h}{2} - \frac{\lambda}{4}(h^2 - h_0^2)^2\right\}$$

$$U(\chi) = \frac{\lambda M_\mathrm{P}^4}{4\xi^2}\left[1 - \exp\left(-\frac{2\chi}{\sqrt{6}M_\mathrm{P}}\right)\right]^2 \quad \text{where } h \simeq \frac{M_\mathrm{P}}{\sqrt{\xi}}\exp\left(\frac{\chi}{\sqrt{6}M_\mathrm{P}}\right) \qquad \boxtimes$$

$$\begin{aligned} n_\mathrm{s} - 1 &\simeq -\frac{8}{3}\frac{M_\mathrm{P}^2}{\xi\, h^2} < 0 \\ \alpha_\mathrm{s} &= -(n_\mathrm{s} - 1)^2/2 < 0 \\ \beta_\mathrm{s} &= (n_\mathrm{s} - 1)^3/2 < 0 \end{aligned}$$



This model is fully consistent with *Planck* constraints.

Modulated Inflation Model

Idea

$$V(\phi) = V_0(\phi) + V_{\rm mod}(\phi)$$

Conditions:

$$|V_0(\phi)| \gg |V_{\rm mod}(\phi)|$$

Hierarchy:

$$\left|\frac{V_{\rm mod}}{V_0}\right| \ll \left|\frac{V'_{\rm mod}}{V'_0}\right| \ll \left|\frac{V''_{\rm mod}}{V''_0}\right| \ll \left|\frac{V'''_{\rm mod}}{V'''_0}\right| \ll \left|\frac{V''''_{\rm mod}}{V''''_0}\right|$$

Inflation parameters

$$
\begin{aligned}
n_s &\simeq 1 - 6\epsilon_0 + 2\eta_0 - M_{\rm P}^2\left[6\frac{V' V'_{\rm mod}}{V_0^2} + 2\frac{V''_{\rm mod}}{V_0}\right] \\
\alpha_s &\simeq -24\epsilon_0^2 + 16\epsilon_0\eta_0 - 2\xi_0^2 \\
&- M_{\rm P}^4\left[\frac{24\,V'^3\,V'_{\rm mod}}{V_0^4} + \frac{8\,V'^2\,V''_{\rm mod}}{V_0^3} - \frac{2\,V'\,V'''_{\rm mod}}{V_0^2}\right] \\
\beta_s &\simeq -192\epsilon_0^3 + 192\epsilon_0^2\eta_0 - 32\epsilon_0\eta_0^2 - 24\epsilon_0\xi_0^2 + 2\eta_0\xi_o^2 + 2\sigma_0^3 \\
&- M_{\rm P}^6\left[24\frac{V'^5\,V'_{\rm mod}}{V_0^6} + \frac{48\,V'^3\,V'_{\rm mod}\,V''_{\rm mod}}{V_0^5} - \frac{16\,V'\,V'_{\rm mod}\,V''^2_{\rm mod}}{V_0^4}\right. \\
&- \left.\frac{12\,V'^2\,V'_{\rm mod}\,V'''_{\rm mod}}{V_0^4} + \frac{2\,V'\,V''_{\rm mod}\,V'''_{\rm mod}}{V_0^3} + \frac{2\,V'^2\,V''''_{\rm mod}}{V_0^3}\right]
\end{aligned}
$$

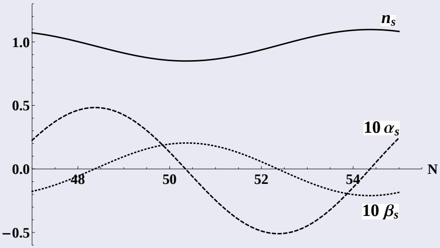
Modulated Power-law Inflation

$$V(\phi) = \lambda\phi^p + \Lambda^4 \cos\left(\frac{\phi}{f} + \theta\right)$$

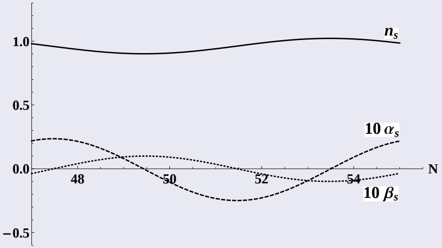
for Planck+WP+highL data

$$\begin{cases} \text{p=2/3} & \text{on the boundary of the joint 95\% CL region} \\ \text{p=1} & \text{within the 95\% CL region in the } n_s - r \text{ plane} \\ \text{p=2} & \text{outside the joint 95\% CL region} \\ \text{p=4} & \text{outside of the joint 99.7\% CL region} \end{cases}$$

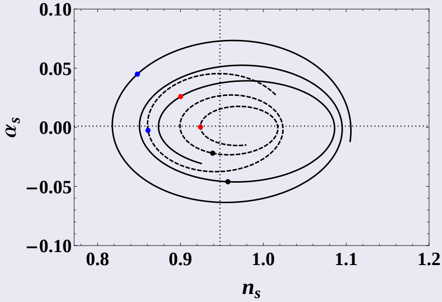
p=1

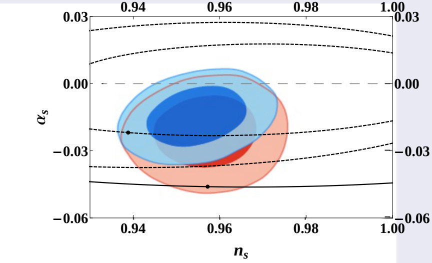


p=2

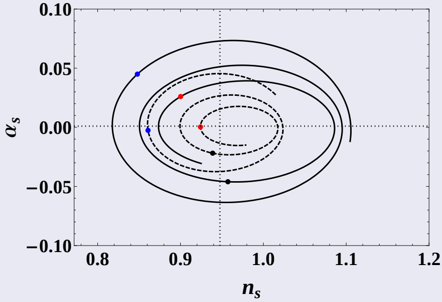


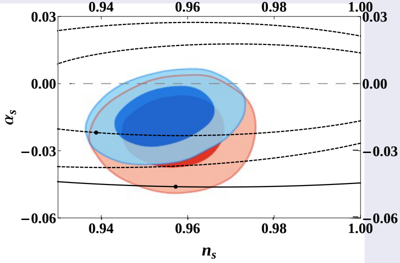
$n_s - \alpha_s$ plane



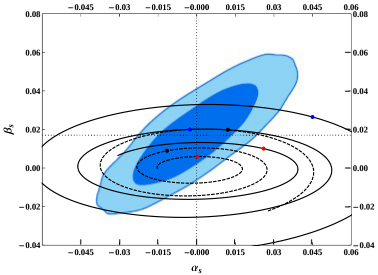


$n_s - \alpha_s$ plane



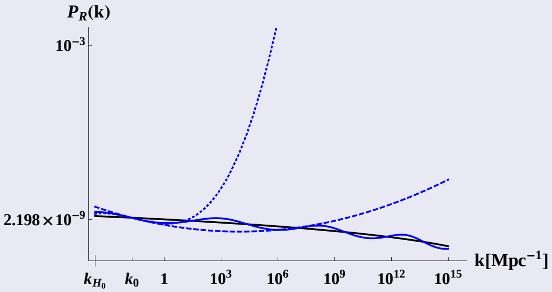


$\alpha_s - \beta_s$ plane

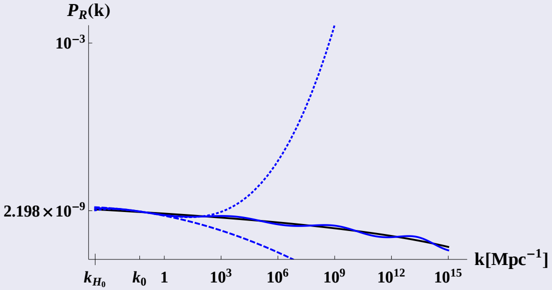


Power spectrum

$p = 1$



$p = 2$



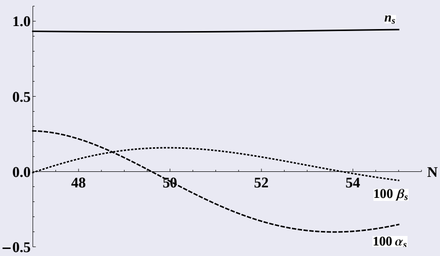
Hilltop/inflection point Inflation

$$V(\phi) \approx \lambda^4 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] + \Lambda^4 \cos\left(\frac{\phi}{f} + \theta\right)$$

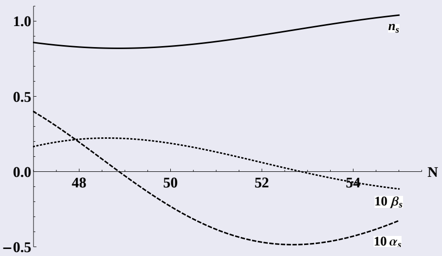
for Planck+WP+BAO data

$$\begin{cases} \text{p=2} & \text{within the 95\% CL region for } \mu \geq 9M_{\rm P} \\ \text{p=3} & \text{outside the joint 95\% CL region} \\ \text{p=4} & \text{within the joint 95\% CL region for } N \geq 50 \end{cases}$$

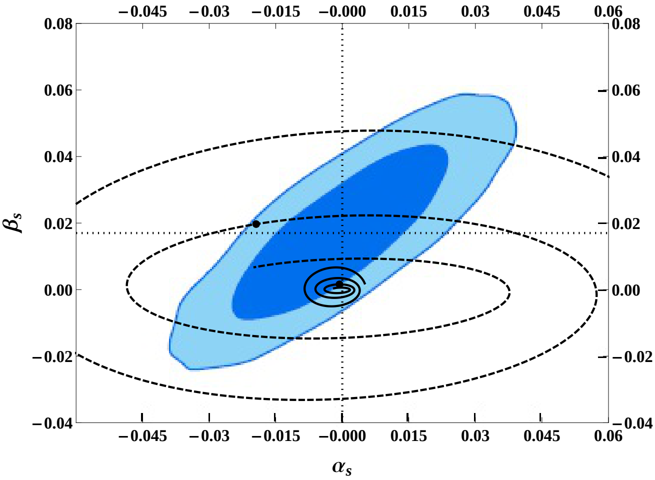
p=3



p=4

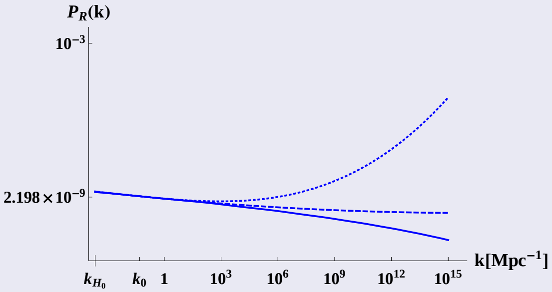


$\alpha_s - \beta_s$ plane

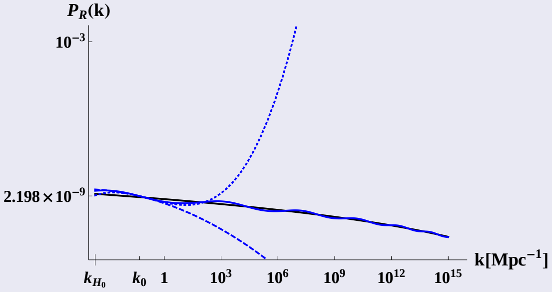


Power spectrum

$p = 3$



$p = 4$



PBHs formation from Particle Production

direct or gravitational coupling of the inflaton (ϕ) to another field (χ)

$$\mathcal{L}(\phi,\, \chi) = -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) - \frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - U(\chi) + \mathcal{L}_{\rm int}(\phi, \chi)$$

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The equations of motion for the inflaton field:

$$H^2 = \frac{1}{3M_{\rm P}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi) + \rho_\chi\right)$$

$$\ddot{\phi} + 3H\,\dot{\phi} + V'(\phi) = \frac{\partial\mathcal{L}_{\rm int}}{\partial\phi}$$

The inflaton fluctuations satisfy

$$\ddot{\delta\phi} + 3H\,\dot{\delta\phi} - \frac{\nabla^2}{a^2}\delta\phi + V''(\phi)\,\delta\phi = \delta\left(\frac{\partial\mathcal{L}_{\rm int}}{\partial\phi}\right)$$

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Result

$$\mathcal{P}_\zeta(k) = \mathcal{P}_{\zeta,\,\rm vac.}(k) + \mathcal{P}_{\zeta,\,\rm src.}(k)$$

$$\mathcal{P}_{\rm t}(k) = \mathcal{P}_{\rm t,\,vac.}(k) + \mathcal{P}_{\rm t,\,src.}(k)$$

Gauge Production

$$\mathcal{L}_{\rm int} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \Phi \, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Gauge Production

$$\mathcal{L}_{\rm int} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\alpha}{4f} \Phi \, F_{\mu\nu} \tilde{F}^{\mu\nu}$$

direct coupling

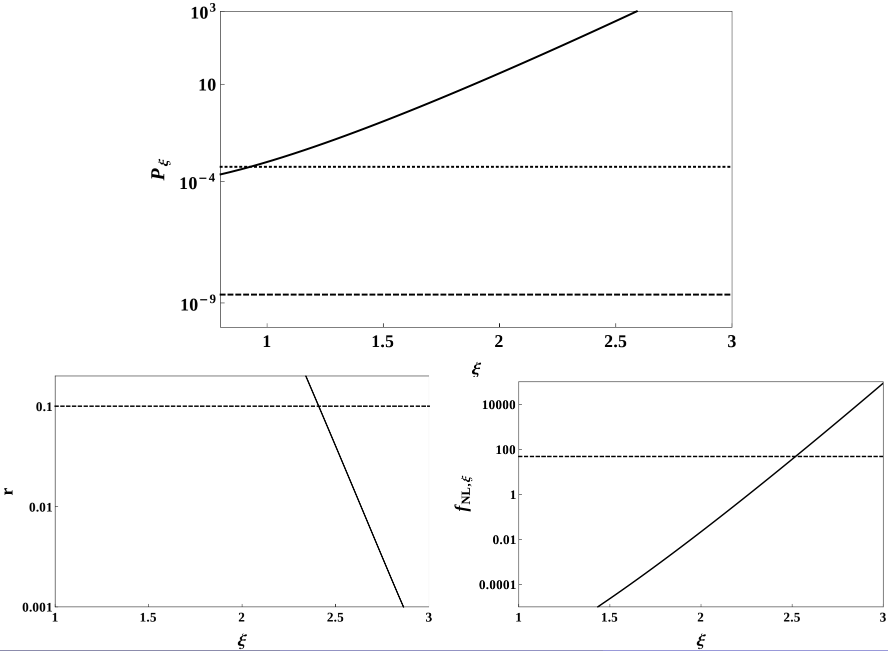
$$\mathcal{P}_\zeta = \mathcal{P}_{\zeta,\,\rm vac.} \left(1 + 7.5 \times 10^{-5}\, \epsilon^2\, \mathcal{P}_{\zeta,\,\rm vac.} X^2\right)$$

$$r = 16\epsilon \, \frac{1 + 2.2 \times 10^{-7}\, \mathcal{P}_{\rm t,\,vac.}\, X^2}{1 + 7.5 \times 10^{-5}\, \epsilon^2\, \mathcal{P}_{\zeta,\,\rm vac.} X^2}$$

$$f_{\rm NL,\,\zeta}^{\rm equil.} \approx 4.4 \times 10^{10}\, \epsilon^3\, \mathcal{P}_{\zeta,\,\rm vac.}^3\, X^3$$

where

$$X \equiv \frac{e^{2\pi\xi}}{\xi^3} \qquad \xi \equiv \frac{\alpha}{2fH} \dot{\Phi}$$

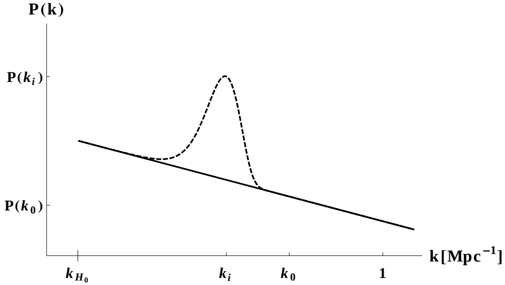

10^3
10
10^{-4}
10^{-9}
P_ξ
1
1.5
2
2.5
3
ξ
0.1
0.01
0.001
r
1
1.5
2
2.5
3
ξ
10000
100
1
0.01
0.0001
$f_{\mathrm{NL},\xi}$
1
1.5
2
2.5
3
ξ

Scalar Production

$$\mathcal{L}_{\rm int}(\phi,\,\chi) = -\frac{g^2}{2}\,(\phi-\phi_0)^2\,\chi^2$$

$$\mathcal{P}_{\zeta,\,\rm src.}(k) \sim A\,k^3 e^{-\frac{\pi}{2}\left(\frac{k}{k_i}\right)^2}$$

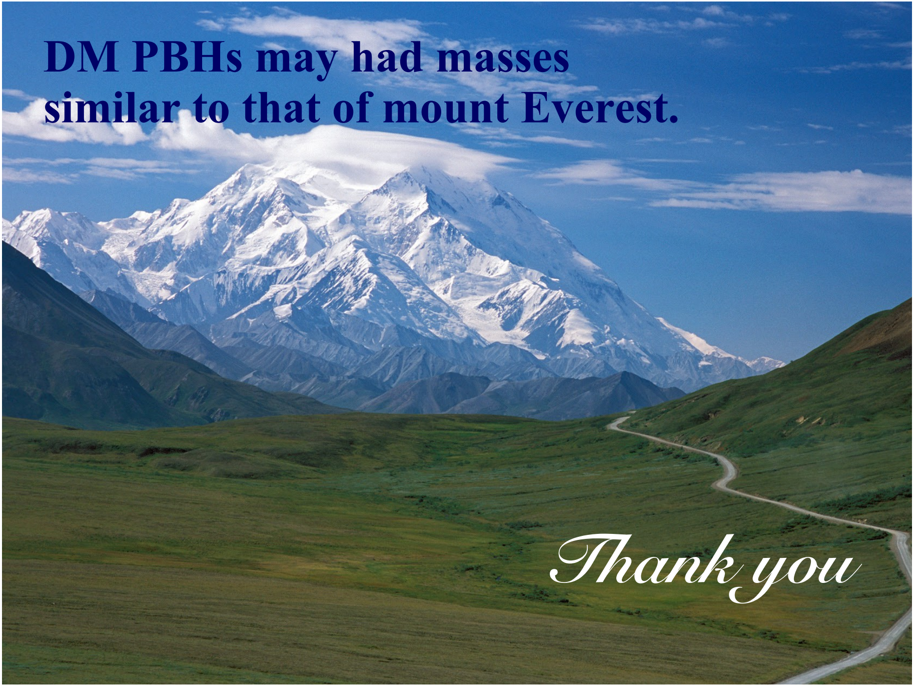
Result



$$A \lesssim 4\times 10^{-4}$$

Conclusions

- The fluctuation which arise at inflation are the most likely source of PBHs formation.
- The spectral index at scale of PBHs formation should be at least 1.418 (1.322) for Gaussian (non-Gaussian) PDF.
- Non-production of DM PBHs put stronger upper bound on the value of the running of the running of the spectral index, β_s.
- Except running-mass inflation model, most of the single field inflation models can not accommodate long–lived PBHs formation.
- PBHs' formation is possible in the modulated chaotic and hilltop inflationary models.
- The most stringent constraints on the gauge production parameter is derived from the non-production of DM PBHs at the end of inflation.
- In the scenario where the inflaton field coupled to a scalar field, the model is free of DM PBHs overproduction in the CMB observational range if the amplitude of the generated bump in the scalar power spectrum, A is less than 4×10^{-4}.


DM PBHs may had masses
similar to that of mount Everest.
Thank you