Quantum Nature of D-branes

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JHEP 1509 (2015) 067 + work in progress

1. Introduction

One of important directions in string theory is to reveal quantum nature of the gravity.

In this talk, we focus on D0-branes in type IIA superstring theory.

We take into account

- Higher derivative corrections
- Hawking radiation
- dual Matrix model analysis

to reveal quantum nature of D0-branes.

Plan of talk

1. Introduction
2. Higher Derivative Corrections in M-theory
3. Smeared black 0-brane
4. Hawking Radiation of a D0-brane from Smeared black 0-brane
5. Matrix Model Analysis of Smeared Black 0-brane
6. Summary

2. Higher Derivative Corrections in M-theory

$$\delta_{\mathrm{SUSY}} S_{\mathrm{SUGRA}} = 0$$

⬇

$$\delta_{\mathrm{SUSY}} (S_{\mathrm{SUGRA}} + S_{\mathrm{Quantum}}) = 0$$

↑ want to know

First we review the construction of 11 dimensional supergravity.
The massless fields consists of vielbein ${e^a}_\mu$, Majorana gravitino ψ_μ,
and 3-form field $A_{\mu\nu\rho}$.

The building blocks of the Lagrangian are

$$R^{ab}{}_{\mu\nu} : [L]^{-2}, \quad F_{\mu\nu\rho\sigma} : [L]^{-1}, \quad D_\mu : [L]^{-1}, \quad \bar{\psi}_\mu \gamma^{\rho_1 \cdots \rho_n} \psi_\nu = [L]^{-1}, \quad \psi_{\mu\nu} = 2D_{[\mu}\psi_{\nu]} : [L]^{-\frac{3}{2}}$$

Then generic form of the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = & [eR] + [eF^2] + [e\epsilon_{11}AF^2]_1 \\ & + [e\psi\psi_2] + [eF\psi\psi]_4 + \mathcal{O}(\psi^4) \end{aligned}$$

Here we used following abbreviations.

$$\begin{aligned} [eF\psi\psi]_4 = & c_1 e F^{\mu\nu\rho\sigma} \bar{\psi}_\mu \gamma_{\rho\sigma} \psi_\nu + c_2 e F_{\alpha\beta\gamma\delta} \bar{\psi}_\mu \gamma^{\mu\nu\alpha\beta\gamma\delta} \psi_\nu \\ & + c_3 e F_{\alpha\beta\gamma\delta} \bar{\psi}_\mu \gamma^{\alpha\beta\gamma\delta} \psi^\mu + c_4 e {F_{\alpha\beta\gamma}}^\mu \bar{\psi}_{(\mu} \gamma^{\nu\alpha\beta\gamma} \psi_{\nu)} \end{aligned}$$

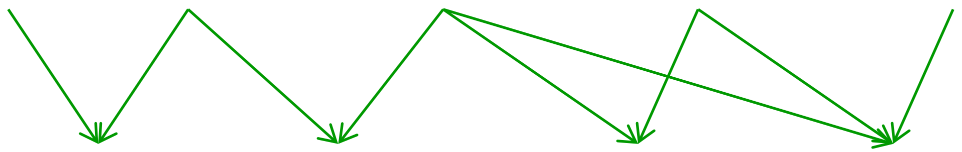
$$[e\epsilon_{11}AF^2]_1 = c_5 e \epsilon_{11}^{\mu_1 \cdots \mu_{11}} A_{\mu_1\mu_2\mu_3} F_{\mu_4 \cdots \mu_7} F_{\mu_8 \cdots \mu_{11}}$$

Generic form of the local supersymmetry transformation is given by

$$\delta e^{\mu}{}_{a} = -\bar{\epsilon}\gamma^{\mu}\psi_{a},$$
$$\delta\psi_{\mu} = d_1 D_{\mu}\epsilon + d_2 F_{\mu jkl}\gamma^{jkl}\epsilon + d_3 F_{ijkl}\gamma^{ijkl}{}_{\mu}\epsilon + \mathcal{O}(\psi^2),$$
$$\delta A_{\mu\nu\rho} = d_4\bar{\epsilon}\gamma_{[\mu\nu}\psi_{\rho]} + d_5\bar{\epsilon}\gamma_{\mu\nu\rho\sigma}\psi^{\sigma}$$

By imposing local supersymmetry, 10 unknown coefficients c_i and d_i are uniquely determined.

$$[eR] \quad [e\psi\psi_2] \quad [eF\psi\psi]_4 \quad [eF^2] \quad [e\epsilon_{11}AF^2]_1$$



$$[eR\bar{\epsilon}\psi] \quad [eF\bar{\epsilon}D\psi] \quad [eDF\bar{\epsilon}\psi] \quad [eF^2\bar{\epsilon}\psi] \quad = 0$$

⇨

$$c_1 = -\tfrac{1}{8}, \quad c_2 = -\tfrac{1}{96}, \quad c_3 = 0, \quad c_4 = 0, \quad c_5 = -\tfrac{1}{(144)^2},$$
$$d_1 = 2, \quad d_2 = -\tfrac{1}{18}, \quad d_3 = \tfrac{1}{144}, \quad d_4 = -3, \quad d_5 = 0$$

Higher derivative corrections in string theory are investigated in various ways

- String scattering amplitude — Gross, Witten; Gross, Sloan
- Non linear sigma model — Grisaru, Zanon
- Superfield method
- Duality
- Noether's method … and so on

By combining all these results, we find that corrections start from α'^3 order, and a part of bosonic terms in type IIA is written as

$$\begin{aligned}\mathcal{L} = & \, e^{-2\phi} R + \cdots && \text{SUGRA} \\ & + \frac{\zeta(3)\alpha'^3}{2^8 \cdot 4!} e^{-2\phi}\Big(t_8 t_8 R^4 + \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} R^4\Big) + \cdots && \text{tree} \\ & + \frac{\pi^2\alpha'^3}{3\cdot 2^8 \cdot 4!} g_s^2 \Big(t_8 t_8 R^4 - \frac{1}{4\cdot 2!}\epsilon_{10}\epsilon_{10} R^4 - \frac{1}{6}\epsilon_{10} t_8 B R^4\Big) + \cdots && \text{1-loop}\end{aligned}$$

t_8 : tensor with 8 indices , ϵ_{10} : 10D antisymmetric tensor, B : NS 2-form field

Let us discuss how to obtain higher derivative corrections via supersymmetry.

The complete structure of higher derivative terms can be determined by local supersymmetry.

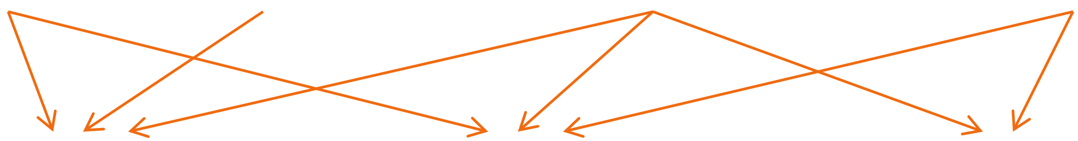
Hyakutake, Ogushi (2005)

Local supersymmetry transformation (neglect flux dependence):

$$\delta e^{a}{}_{\mu} = \bar{\epsilon}\gamma^{a}\psi_{\mu}, \qquad \delta\psi_{\mu} = 2D_{\mu}\epsilon, \qquad \delta A_{\mu\nu\rho} = -3\bar{\epsilon}\gamma_{[\mu\nu}\psi_{\rho]}$$

Cancellation (neglect flux dependence):

$$[R^4]_7, \qquad [AR^4]_2, \qquad [R^3\psi\psi_2]_{92}, \qquad [R^2\psi_2 D\psi_2]_{25}$$



$$[R^4\epsilon\psi]_{116}, \qquad [R^2 DR\epsilon\psi_2]_{88}, \qquad [R^3\epsilon D\psi_2]_{40} \quad = 0$$

Solution is given by

$$a\Big(t_8t_8R^4 + \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4\Big) + b\Big(t_8t_8R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4 - \tfrac{1}{6}\epsilon_{11}t_8AR^4\Big)$$

So far we have just considered the cancellation of variations which do not depend on the 4-form field strength F . Then next step is to examine the cancellation of variations which linearly depend on F .

To do this we add following terms,

$$[eR^3F^2]_{30}, \quad [eR^2(DF)^2]_{24},$$
$$[eR^3F\bar{\psi}\psi]_{447}, \quad [eR^2F\bar{\psi}_2\psi_2]_{190}, \quad [eR^2DF\bar{\psi}\psi_2]_{614}, \quad [eRDF\bar{\psi}_2D\psi_2]_{113}$$

Under the local supersymmetry, these transform into

$$\boxed{\begin{array}{l}[eR^2DRF\bar{\epsilon}\psi]_{1563}, \quad [eR^3F\bar{\epsilon}\psi_2]_{513}, \quad [eR^3DF\bar{\epsilon}\psi]_{995}, \\ [eRDRDF\bar{\epsilon}\psi_2]_{371}, \quad [eR^2DF\bar{\epsilon}D\psi_2]_{332}, \quad [eR^2DDF\bar{\epsilon}\psi_2]_{151}\end{array}} = 0$$

Then we have 4169 equations among 1544 variables. Solution becomes

$$b\Big(t_8t_8R^4 - \tfrac{1}{4!}\epsilon_{11}\epsilon_{11}R^4 - \tfrac{1}{6}\epsilon_{11}t_8AR^4\Big) + ([R^3F^2] \oplus [R^2(DF)^2])$$

↑

uniquely determined!

Hyakutake (2007)

In summary, the higher derivative corrections in M-theory becomes

$$S_{11} = \frac{1}{2\kappa_{11}^2}\int d^{11}x\ e\Big\{R + \gamma\Big(t_8 t_8 R^4 - \frac{1}{4!}\epsilon_{11}\epsilon_{11}R^4\Big)\Big\} + \cdots$$

$$= \frac{1}{2\kappa_{11}^2}\int d^{11}x\ e\Big\{R + 24\gamma\big(R_{abcd}R_{abcd}R_{efgh}R_{efgh} - 64R_{abcd}R_{aefg}R_{bcdh}R_{efgh}$$

$$+ 2R_{abcd}R_{abef}R_{cdgh}R_{efgh} + 16R_{acbd}R_{aebf}R_{cgdh}R_{egfh}$$

$$- 16R_{abcd}R_{aefg}R_{befh}R_{cdgh} - 16R_{abcd}R_{aefg}R_{bfeh}R_{cdgh}\big)\Big\} + \cdots$$

$$\gamma \sim \ell_p^6$$

This action contains enough information to deal with quantum corrections to geometrical solution, such as M-wave in 11 dimensions, or black 0-brane in 10 dimensions.

3. Smeared Black 0-brane

uplift

4 dim. BH → 11dim. Smeared BH

boost

11dim. Smeared M-wave

reduction

10 dim. Smeared black 0-brane

Let us review the construction of smeared black 0-brane.

First we uplift 4 dim. BH solution into 11 dimensions.

$$ds_{11}^2 = \underbrace{-Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_2^2}_{\text{4 dim. part}} + \underbrace{dy_m^2 + dz^2}_{\text{add 7 dim. directions}}, \qquad F = 1 - \frac{r_{\rm h}}{r}$$

4 dim. part

add 7 dim. directions $m = 4, \cdots, 9$

Next we boost the smeared BH along z direction.

$$\begin{aligned} ds_{11}^2 &= -F(\cosh\beta dt + \sinh\beta dz)^2 + (\sinh\beta dt + \cosh\beta dz)^2 + F^{-1}dr^2 + r^2 d\Omega_2^2 + dy_m^2 \\ &= -H^{-1}Fdt^2 + F^{-1}dr^2 + r^2 d\Omega_2^2 + dy_m^2 + H\left(dz + (1 - H^{-1})\frac{\cosh\beta}{\sinh\beta}dt\right)^2 \end{aligned}$$

$$H = 1 + \frac{r_{\rm h}\sinh^2\beta}{r}$$

Finally reduce z direction.

$$ds_{10}^2 = -H^{-\frac{1}{2}}Fdt^2 + H^{\frac{1}{2}}\left(F^{-1}dr^2 + r^2 d\Omega_2^2 + dy_m^2\right),$$

$$e^{\phi} = H^{\frac{3}{4}}, \qquad C^{(1)} = (1 - H^{-1})\frac{\cosh\beta}{\sinh\beta}dt$$

Smeared black 0-brane

11dim. Smeared
Quantum BH

↓ boost

11dim. Smeared
Quantum M-wave

← reduction

10 dim. Smeared
Quantum black 0-brane

Hyakutake (2015)

Let us construct a smeared quantum BH solution in 11 dimensions.

The ansatz is given by

$$ds_{11}^2 = -G_1^{-1} F_1 dt^2 + F_1^{-1} dr^2 + r^2 d\Omega_2^2 + G_2 (dy_m^2 + dz^2)$$

EOMs are messy but, up to linear order of γ , we can solve them as

$$F_1(x) = 1 - \frac{1}{x} + \frac{\gamma}{r_\mathrm{h}^6}\left[\frac{448}{x}\left\{c + \sum_{n=1}^{9}\frac{1}{nx^n} + \frac{216}{7x^8} - \frac{215}{7x^9}\right\}\right], \qquad x = \frac{r}{r_\mathrm{h}}$$

$$G_1(x) = 1 + \frac{\gamma}{r_\mathrm{h}^6}\left[896\left\{\sum_{n=1}^{9}\frac{n+1}{nx^n} + \frac{12}{x^9}\right\}\right],$$

$$G_2(x) = 1 + \frac{\gamma}{r_\mathrm{h}^6}\left[256\sum_{n=1}^{9}\frac{1}{nx^n}\right]$$

Here c is an integral constant, but it can be absorbed by redefinition of r_h .

There are two more integral constants, but they are fixed by demanding asymptotic flatness and no singularity at $r = r_\mathrm{h}$.

Now we boost the smeared BH along z direction.

$$
\begin{aligned}
ds_{11}^2 &= -G_1^{-1}F_1(\cosh\beta dt + \sinh\beta dz)^2 + G_2(\sinh\beta dt + \cosh\beta dz)^2 \\
&\quad + F_1^{-1}dr^2 + r^2 d\Omega_8^2 + G_2 dy_m^2 \\
&= -H_1^{-1}F_1 dt^2 + F_1^{-1}dr^2 + r^2 d\Omega_8^2 + G_2 dy_m^2 \\
&\quad + H_2\left(dz + \left(1 - H_2^{-\frac{1}{2}}H_3^{-\frac{1}{2}}\right)\frac{\cosh\beta}{\sinh\beta}dt\right)^2
\end{aligned}
$$

$$
H_1 = G_1 G_2^{-1} H_2, \qquad H_2 = G_2 + (G_2 - G_1^{-1}F_1)\sinh^2\beta, \qquad H_3 = G_2^{-2}H_2
$$

And reduce z direction. We obtain smeared quantum black 0-brane solution.

$$
ds_{10}^2 = -H_1^{-1}H_2^{\frac{1}{2}}F_1 dt^2 + H_2^{\frac{1}{2}}\left(F_1^{-1}dr^2 + r^2 d\Omega_2^2 + G_2 dy_m^2\right),
$$

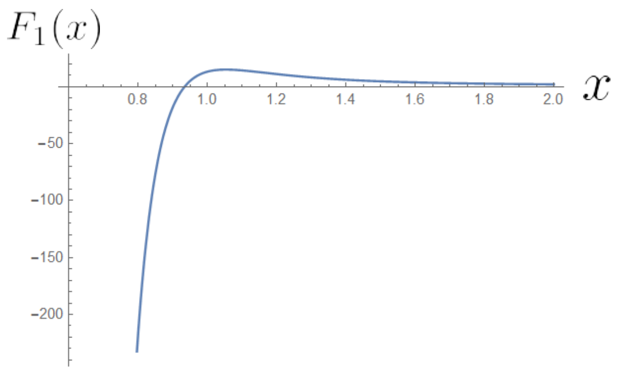
$$
e^{\phi} = H_2^{\frac{3}{4}}, \qquad C^{(1)} = \left(1 - H_2^{-\frac{1}{2}}H_3^{-\frac{1}{2}}\right)\frac{\cosh\beta}{\sinh\beta}dt
$$

Thermodynamics of the smeared quantum black 0-brane

Horizon

$$F_1\left(\frac{r_{\rm horizon}}{r_{\rm h}}\right) = 0$$ at the horizon

$$r_{\rm horizon} = r_{\rm h} - \frac{\gamma}{r_{\rm h}^5} f_1(1)$$



Temperature

$$T = \frac{1}{4\pi} H_1^{-1/2} \frac{dF_1}{dr}\Big|_{r_{\rm horizon}} = \frac{1}{4\pi r_{\rm h}} \sqrt{\frac{\alpha}{1+\alpha}} \left\{ 1 - \gamma \frac{128 - c}{r_{\rm h}^6} \right\} \qquad \alpha = \frac{1}{\sinh^2 \beta}$$

Entropy

Black hole entropy is evaluated by using Wald's formula.

$$S = -2\pi V_6 \int_{\rm horizon} d\Omega_2 dz \sqrt{h} \frac{\partial \mathcal{S}}{\partial R_{\mu\nu\rho\sigma}} N_{\mu\nu} N_{\rho\sigma}$$

By inserting the solution obtained so far, the entropy is calculated as

$$S = \frac{4\pi V_6}{2\kappa_{10}^2} 4\pi r_{\rm h}^2 \sqrt{\frac{1+\alpha}{\alpha}} \left\{ 1 + \gamma \frac{2(1088 - c)}{r_{\rm h}^6} \right\}$$

Mass and Charge

Mass is calculated by using ADM mass formula.

$$M = \frac{4\pi V_6}{2\kappa_{10}^2} r_{\rm h} \left\{ 2 + \frac{1}{\alpha} - \gamma \frac{2(-896+c) + \alpha^{-1}(-2048+c)}{r_{\rm h}^6} \right\},$$

$$Q = \frac{4\pi V_6}{2\kappa_{10}^2} \frac{\sqrt{1+\alpha}}{\alpha} r_{\rm h} \left\{ 1 + \gamma \frac{2048-c}{r_{\rm h}^6} \right\}$$

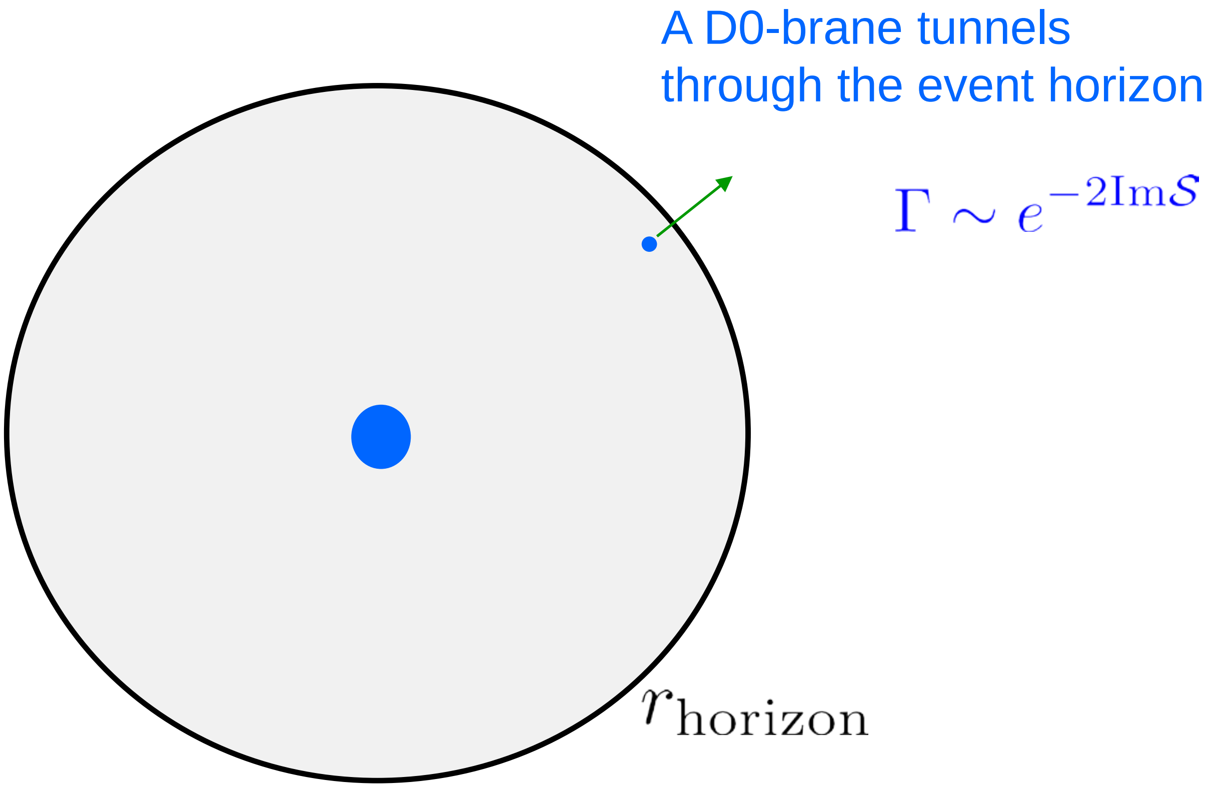
Charge is not renormalized if we choose $c = 2048$.

1st Law of thermodynamics

$$\delta M = T\delta S + \Phi \delta Q$$

Electric potential : $\Phi = C_t^{(1)}\big|_{\rm horizon} = 1/\sqrt{1+\alpha}$

4. Hawking Radiation of a D0-brane from Smeared Black 0-brane



Smeared quantum black 0-brane

Tunneling effect can be evaluated by using WKB approximation.

Parikh-Wilczek(1998)

Thus we want to evaluate the imaginary part of the action in the background of smeared quantum black 0-brane.

Lagrangian for a D0-brane, moving along the radial direction, is given by

$$\mathcal{S} = -T_0 \int dt\, e^{-\phi} \sqrt{-\det\left(g_{\mu\nu}\partial_t X^\mu \partial_t X^\nu\right)} - T_0 \int dt C_t^{(1)}$$

$$\mathcal{L} = -T_0 e^{-\phi} H_1^{-\frac{1}{2}} H_2^{\frac{1}{4}} F_1^{\frac{1}{2}} \sqrt{1 - H_1 F_1^{-2} \dot{r}^2} - T_0 C_t^{(1)}$$

Then conjugate momentum and Hamiltonian are written as

$$p_r = \frac{\partial \mathcal{L}}{\partial \dot{r}} = T_0 H_1^{-\frac{1}{2}} H_2^{-\frac{1}{2}} F_1^{\frac{1}{2}} \frac{H_1 F_1^{-2} \dot{r}}{\sqrt{1 - H_1 F_1^{-2} \dot{r}^2}}$$

$$\mathcal{H}(r, p_r) = H_1^{-\frac{1}{2}} H_2^{-\frac{1}{2}} F_1^{\frac{1}{2}} \sqrt{T_0^2 + H_2 F_1 p_r^2} + T_0 C_t^{(1)}$$

Next we consider Hamilton-Jacobi equation.

$$-\frac{\partial \mathcal{S}}{\partial t} = \mathcal{H}(r, p_r), \qquad p_r = \frac{\partial \mathcal{S}}{\partial r}$$

To solve this, we assume

$$\mathcal{S} = -\underline{\delta E}\, t + W(r)$$

energy of test D0-brane

Then we obtain

$$W(r) = \int dr\, H_1^{\frac{1}{2}} F_1^{-1} \sqrt{\left(\delta E - T_0 C_t^{(1)}\right)^2 - T_0^2 H_1^{-1} H_2^{-1} F_1}$$

$$\sim \frac{1}{2\pi T} \int \frac{d\rho}{\rho} \sqrt{\left(\delta E - T_0 C_t^{(1)}\right)^2 - T_0^2 H_1^{-1} H_2^{-1} F_1}$$

pole at horizon

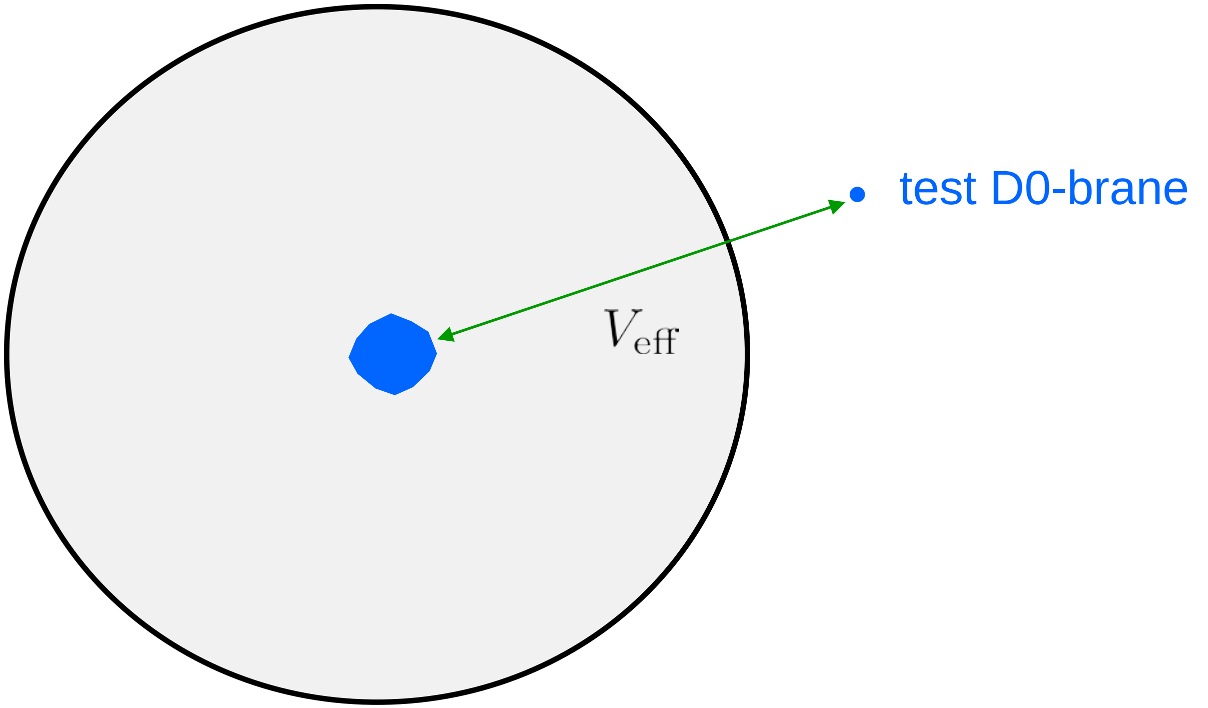
$$d\rho = H_2^{\frac{1}{4}} F_1^{-\frac{1}{2}} dr, \qquad H_1^{-1} H_2^{\frac{1}{2}} F_1 \sim \frac{1}{4} H_1^{-1} \left(\frac{dF_1}{dr}\right)^2 \Big|_{r_{\mathrm{horizon}}} \rho^2$$

The emission rate for a D0-brane is estimated as

$$P = e^{-2\,\mathrm{Im}\,W} = e^{-\frac{\delta E - T_0 \Phi}{T}} = e^{-\delta S}$$

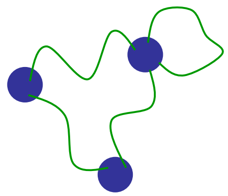
5. Matrix Model Analysis of Smeared Fuzzy Sphere

fuzzy sphere = Non-commutative configuration of D0-branes



Smeared fuzzy sphere via D0-branes

N D0-branes are dynamical due to oscillations of open strings



massless modes : $N \times N$ matrices

$$\left(A_t(t),\ \Phi_i(t),\ \theta(t)\right) \qquad i = 1, \cdots, 9$$

Action for N D0-branes is obtained by requiring global supersymmetry with 16 supercharges.

➡ (1+0) dimensional supersymmetric $U(N)$ gauge theory

$$\mathcal{S}_0 = \frac{1}{g_{\rm YM}^2}\int dt\, {\rm tr}\left(\frac{1}{2}D_t\Phi_i D_t\Phi^i + \frac{1}{4}[\Phi_i, \Phi_j]^2 + \frac{i}{2}\theta^T D_t\theta + \frac{1}{2}\theta^T\gamma^i[\Phi_i, \theta]\right)$$

$$D_t\Phi_i = \partial_t\Phi_i - i[A_t, \Phi_i]$$

EOMs become

$$\ddot{\Phi}_i = [\Phi^j, [\Phi_i, \Phi_j]], \qquad [\Phi^i, D_t\Phi_i] = 0 \qquad \theta = 0$$

Ansatz for the fuzzy sphere is written as

$$\Phi_a = \frac{f(t)}{2\pi\ell_s^2}\frac{\Sigma_a}{2}, \qquad \left[\frac{\Sigma_a}{2}, \frac{\Sigma_b}{2}\right] = i\epsilon_{abc}\frac{\Sigma^c}{2}$$

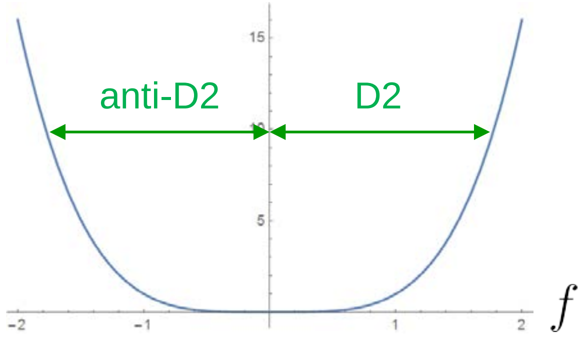
$$a, b, c = 1, 2, 3$$

Then EOMs become

$$\ddot{f} = -\frac{2}{(2\pi\ell_s^2)^2} f^3$$

➡ Oscillation in f^4 potential

Collins, Tucker (1976)



Radius of the fuzzy sphere is estimated as

$$R(t) = \sqrt{\frac{1}{N}\mathrm{tr}\left(X_1^2 + X_2^2 + X_3^2\right)} = \frac{f(t)}{2}\sqrt{N^2 - 1}$$

$$X_i = 2\pi\ell_s^2 \Phi_i$$

Now we consider the fuzzy sphere smeared into $(x_4, \cdots, x_9)$ directions, which are represented as

$$\Phi_a = \frac{f(t)}{2\pi\ell_s^2}\frac{\Sigma_a}{2} \otimes \mathbf{1}_Z, \qquad \Phi_m = \mathbf{1}_N \otimes P_m$$

$$a = 1,2,3 \qquad\qquad m = 4,\cdots,9$$

We want to evaluate an effective potential between smeared fuzzy sphere and a test D0-brane.

Kabat, Taylor (1997)

Euclidean action is given by

$$\mathcal{S}_{\rm E} = \frac{1}{2g_{\rm YM}^2}\int dt\,{\rm tr}\Big(D_\tau\Phi_i D_\tau\Phi^i - \frac{1}{2}[\Phi_i,\Phi_j]^2 + (\dot{A}_\tau - i[B_i,\Phi^i])^2 + \theta^T D_\tau\theta - \theta^T\gamma^i[\Phi_i,\theta] - D_\mu\bar{C}D^\mu C\Big)$$

Let us consider the fluctuation around fuzzy sphere background and evaluate 1-loop effective potential.

$$B_i = \begin{pmatrix} b_i & 0 \\ 0 & x_i \end{pmatrix}$$

b_i: smeared fuzzy sphere

x_i: test D0

$$A_\tau = \begin{pmatrix} 0 & a \\ a^\dagger & 0 \end{pmatrix}, \quad \Phi_i = B_i + \begin{pmatrix} 0 & \phi_i \\ \phi_i^\dagger & 0 \end{pmatrix}, \quad \theta = \begin{pmatrix} 0 & \psi \\ \psi^\dagger & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & c \\ c^\dagger & 0 \end{pmatrix}$$

Mass squared for fluctuations : $\Omega^2 = M_0 + M_1$

$$K_k = b_k - x_k \mathbf{1}_{N'} \qquad k = 1, \cdots, 9$$

$$K^2 = K_k^2$$

boson

$$\Omega_b^2 = K^2 \mathbf{1}_{10} + M_{1b}$$

$$M_{1b} = 2i \begin{pmatrix} 0 & \dot{K}_j \\ -\dot{K}_i & -i[K_i, K_j] \end{pmatrix} \equiv 2i \begin{pmatrix} 0 & F_{\tau j} \\ F_{i\tau} & F_{ij} \end{pmatrix}$$

fermion

$$\Omega_f^2 = K^2 \mathbf{1}_{16} + M_{1f}$$

$$M_{1f} = \gamma^i \dot{K}_i + \frac{1}{2}\gamma^{ij}[K_i, K_j] \equiv \frac{i}{2}\gamma^{\mu\nu} F_{\mu\nu}$$

ghost

$$\Omega_g^2 = K^2$$

The effective potential is given by

$$V_{\rm eff} = {\rm tr}(\Omega_b) - \frac{1}{2}{\rm tr}(\Omega_f) - 2{\rm tr}(\Omega_g)$$
$$= -\frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{d\tau}{\tau^{3/2}}{\rm tr}\left(e^{-\tau\Omega_b^2}\right) + \frac{1}{4\pi}\int_0^\infty \frac{d\tau}{\tau^{3/2}}{\rm tr}\left(e^{-\tau\Omega_f^2}\right) + \frac{1}{\pi}\int_0^\infty \frac{d\tau}{\tau^{3/2}}{\rm tr}\left(e^{-\tau\Omega_g^2}\right)$$

Each term can be evaluated perturbatively like

$${\rm tr}\left(e^{-\tau(M_0+M_1)}\right) = {\rm tr}\left(e^{-\tau M_0}U(\tau)\right)$$

$$U(\tau) \equiv e^{\tau M_0}e^{-\tau(M_0+M_1)} = \mathbf{1} - \int_0^\tau dz_1 M_1(z_1) + \int_0^\tau dz_1 M_1(z_1)\int_0^{z_1} dz_2 M_1(z_2) - \cdots$$

$$M_1(\tau) \equiv e^{\tau M_0}M_1 e^{-\tau M_0}$$

$$= {\rm tr}\left(e^{-\tau M_0}\right) - \int_0^\tau dz_1 {\rm tr}\left(e^{-\tau M_0}M_1(z_1)\right) + \cdots$$

Non-trivial contributions arises at M_1^4 .

The effective action of $\mathcal{O}(F^4)$ is expressed as

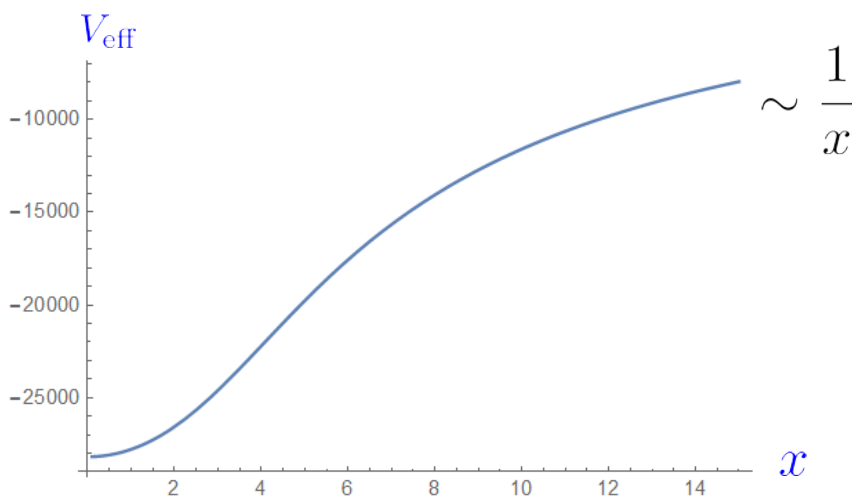
$$V_{\rm eff} = -\frac{1}{2\sqrt{\pi}}\int_0^\infty \frac{d\tau}{\tau^{3/2}}\int_0^\tau dz_1\int_0^{z_1} dz_2\int_0^{z_2} dz_3\int_0^{z_3} dz_4$$

$$\mathrm{tr}_{N'}\Big[e^{-\tau K^2}\Big\{8F^{\mu}{}_{\nu}(z_1)F^{\nu}{}_{\rho}(z_2)F^{\rho}{}_{\sigma}(z_3)F^{\sigma}{}_{\mu}(z_4)+16F_{\mu\nu}(z_1)F^{\mu\lambda}(z_2)F^{\nu\sigma}(z_3)F_{\lambda\sigma}(z_4)$$

$$-4F_{\mu\nu}(z_1)F^{\mu\nu}(z_2)F_{\rho\sigma}(z_3)F^{\rho\sigma}(z_4)-2F_{\mu\nu}(z_1)F_{\rho\sigma}(z_2)F^{\mu\nu}(z_3)F^{\rho\sigma}(z_4)\Big\}\Big]$$

It is possible to evaluate the above potential by inserting the background.

The result for $N=2$ smeared fuzzy sphere with $R\sim 5$ becomes :



6. Summary

Summary :

- We have constructed the smeared black 0-brane solution including quantum corrections.
- The radiation of a D0-brane from smeared black 0-brane is estimated.
- Dual matrix analysis for the smeared fuzzy sphere has been done.

Future directions :

- Numerical analysis for the smeared fuzzy sphere
- Test of the gauge/gravity duality