The ϵ -expansion of the codimension two twist defect from CFT (conformal field theory)

Satoshi Yamaguchi

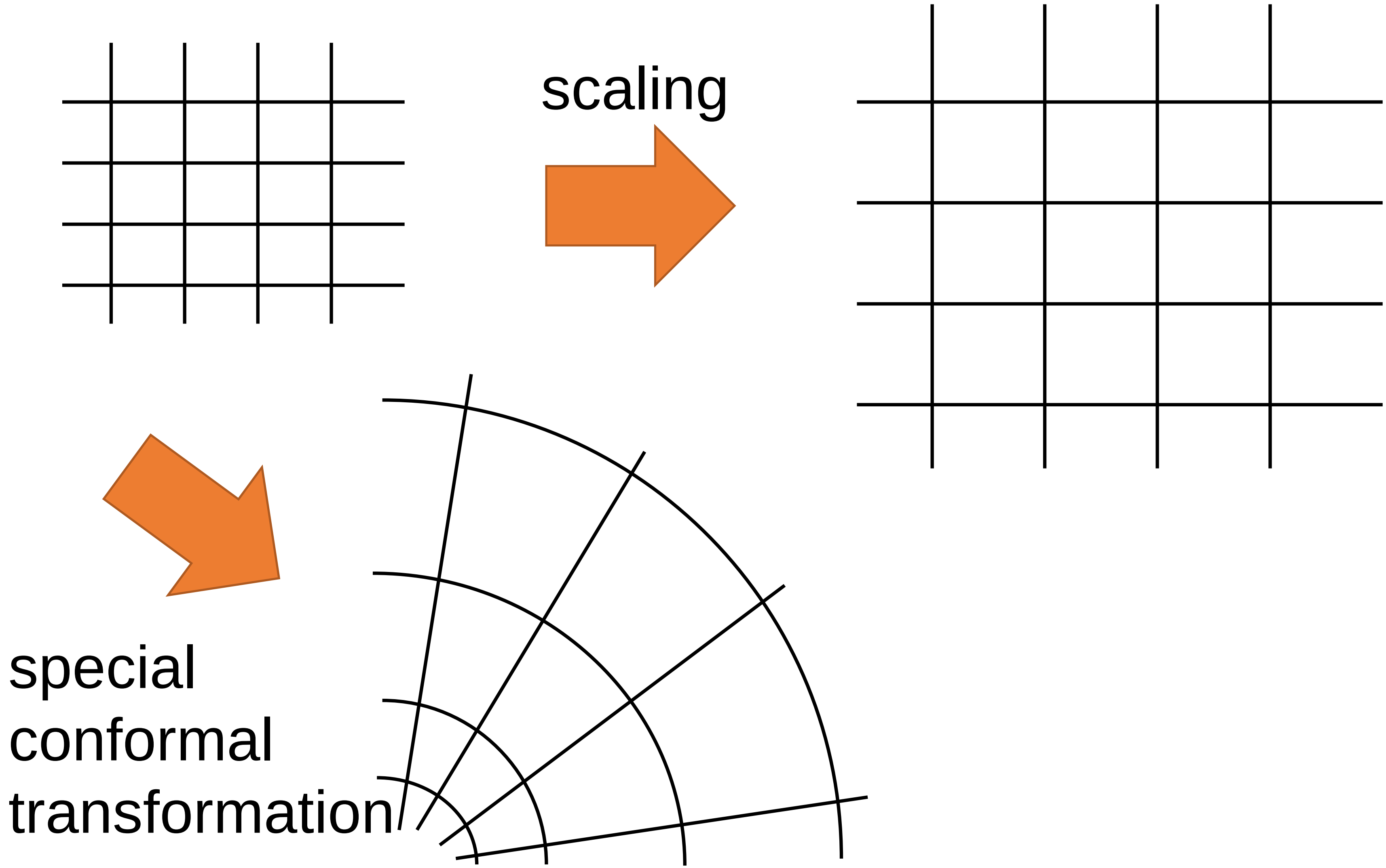
Osaka University

Conformal field theory (CFT)

Quantum field theory with
Conformal symmetry
(scaling symmetry)

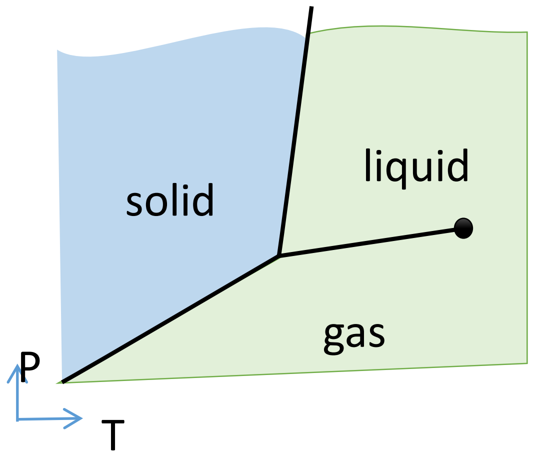
Conformal symmetry: invariance under

conformal transformation

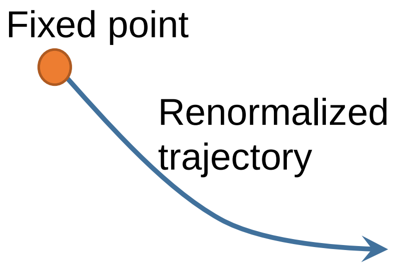


Motivation to study CFT

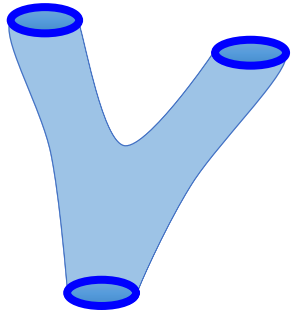
- Critical phenomena



- UV complete QFT



- Worldsheet of string theory



- AdS/CFT correspondence

Eg. 3 dim free massless scalar theory

$$S = \int d^3x \frac{1}{2} \partial_\mu \phi \partial^\mu \phi$$

Eg. 3 dim Ising CFT (Wilson-Fisher CFT)

A description (there are others)

$$S = \int d^3x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

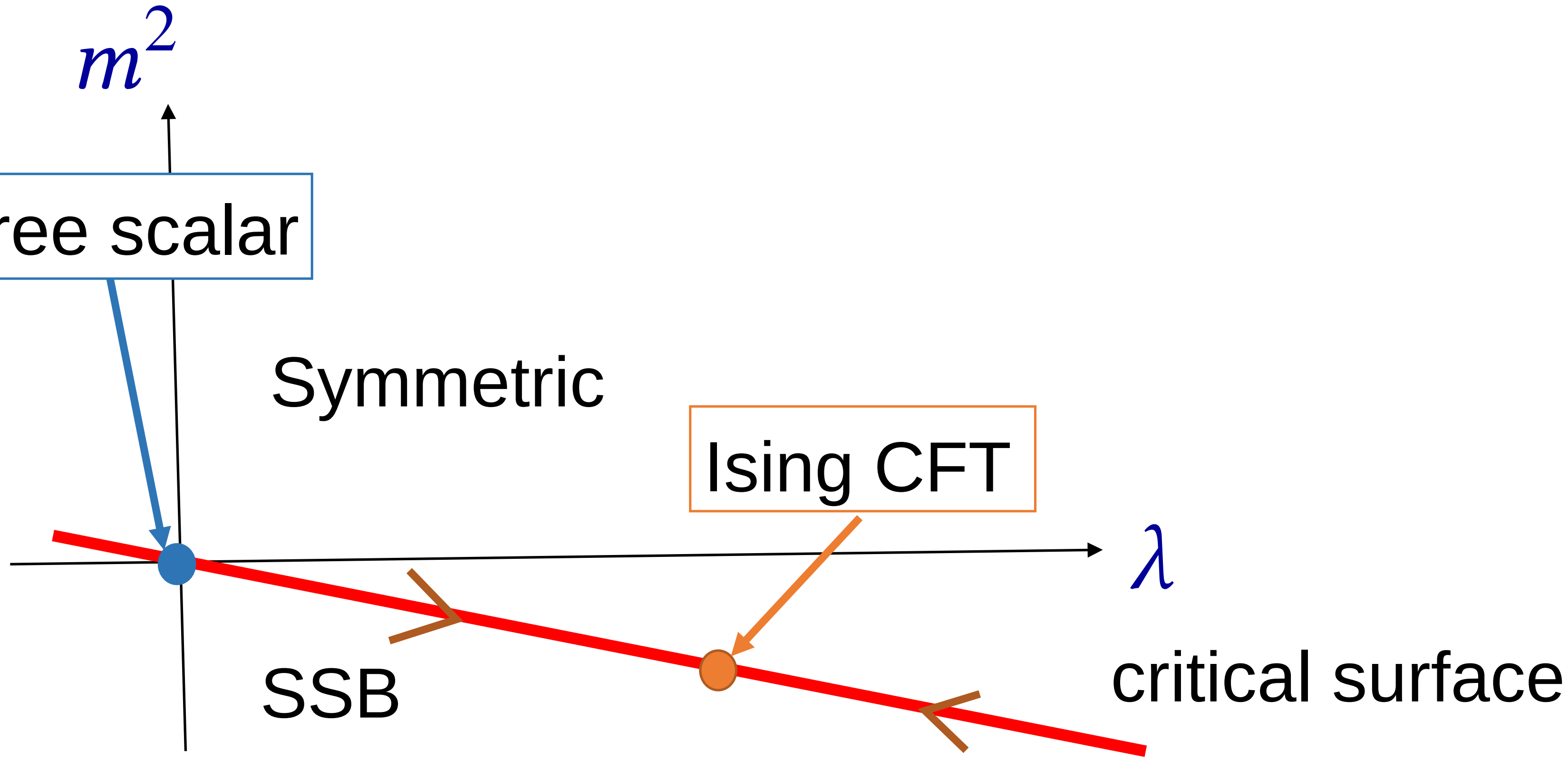
Euclidean

Choose $m^2 = m^2_{\text{critical}}$ critical value

Symmetric phase $m^2 > m^2_{\text{critical}}$

Sponteneous symmetry breaking phase $m^2 < m^2_{\text{critical}}$

Low energy limit ➡ Ising CFT



Ising CFT is more **universal** than free scalar

(important)

$m^2 = m^2_{\text{critical}}$

tuning one parameter

$\lambda = 0$

$m^2 = 0$

tuning two params

But Ising CFT is difficult

Perturbation is not justified well.

In 2 dim, constraint from the conformal symmetry completely solve the Ising CFT.

Does it work in 3dim ?

Naively it does not work in 3 dim

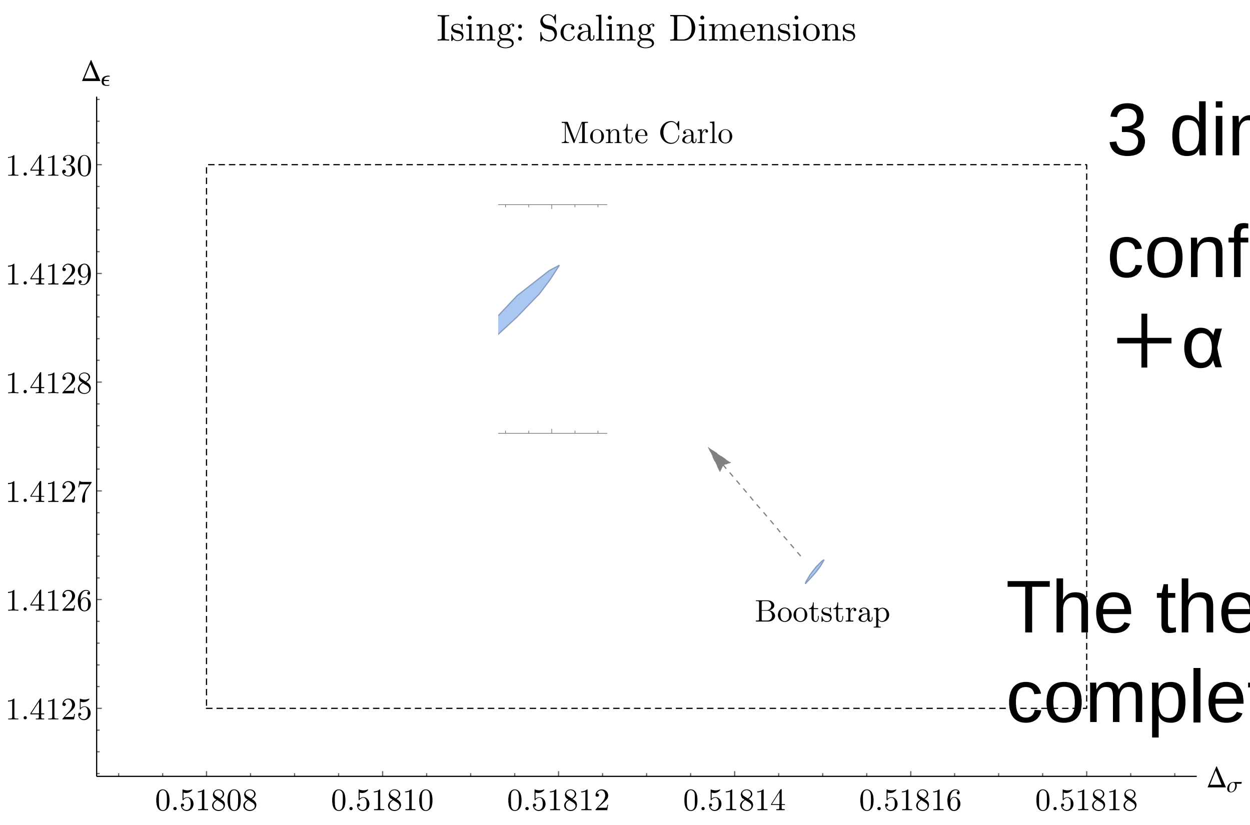
2 dim: infinite dim Virasoro symmetry
3 dim: finite dim symmetry

It was too naive

Recent progress in CFT

Numerical Bootstrap

[F. Kos, D. Poland, D. Simmons-Duffin and A. Vichi, arXiv:1603.04436]



3 dim Ising CFT

conformal symmetry
+α

The theory is determined completely?

question

By conformal symmetry +α

- How completely is the theory solved ?
- Why it is so strong ?

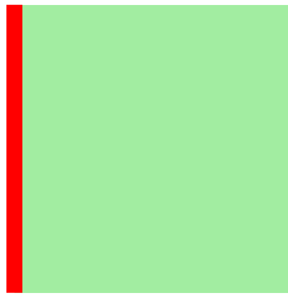
↓

Need analytic method

An interesting approach :
[Rychkov,Tan ’15] ε-expansion

Will be reviewed later

Defect: Generalization of boundary



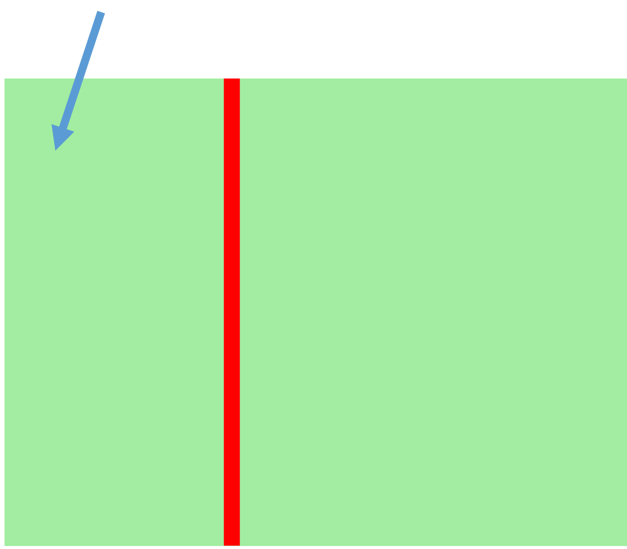
2 dim Boundary CFT

(open string)

D-brane

Generalization

We also have a theory here



1 dim defect in 2 dim

generalization

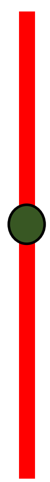
D dim defect in d dim

Summary of the result

Analogue of **spectrum of open string**

4- ε dim O(N) model Wilson-Fisher(WF) fixed point
(CFT)

Twist defect



local operators on the defect

ψ_s

Obtained the scaling dimensions in Rychkov-Tan’ s framework.

Plan

- Conformal field theory (CFT)
- Review of Rychkov-Tan
- Twist defect
- Discussion

Conformal Field Theory (CFT)

Conformal symmetry

dimension

+1	P_μ (translation)		
0	$M_{\mu\nu}$ (rotation)	H (dilatation)	
-1	K_μ (special conformal transformation)		

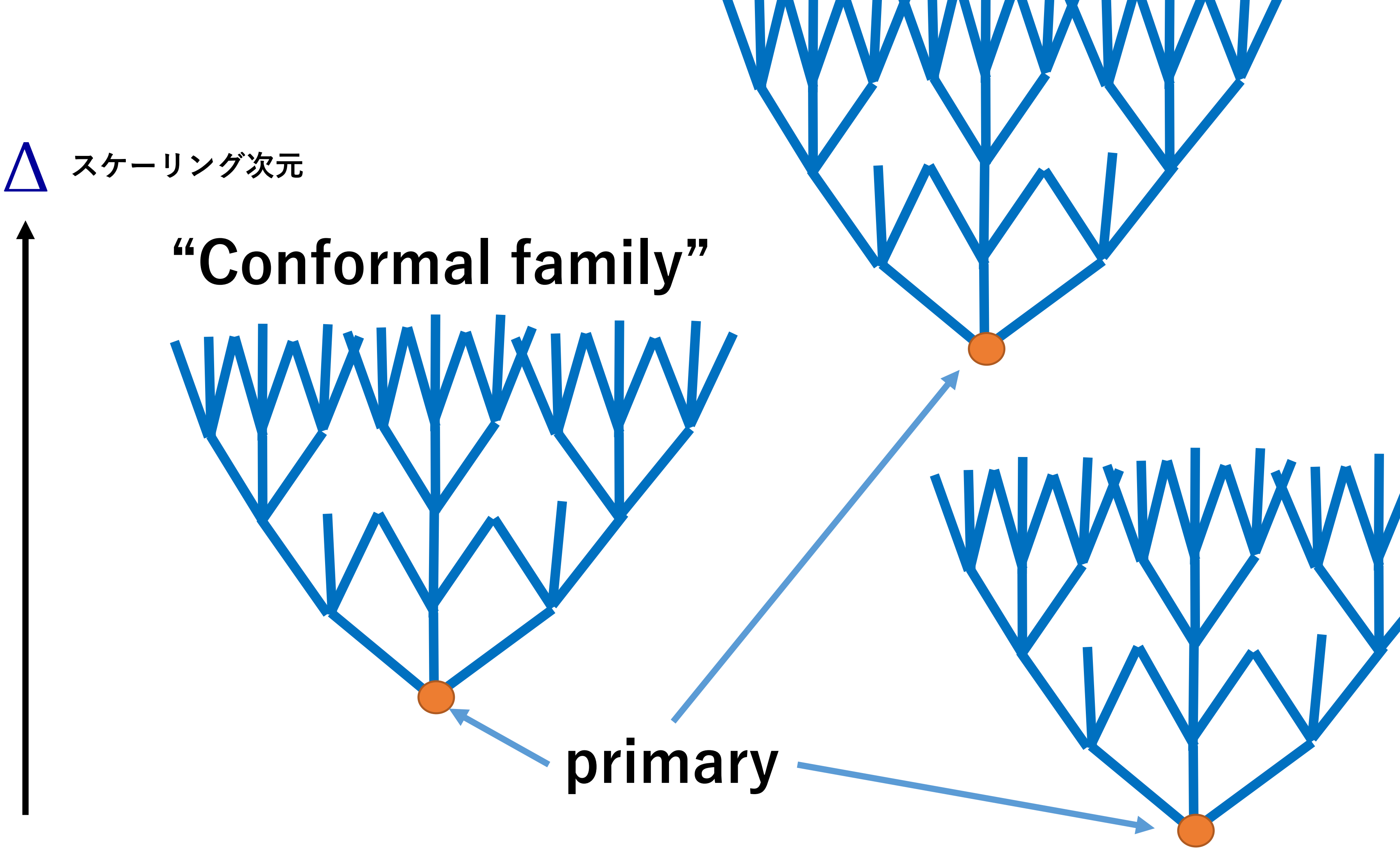
Local operators $\mathcal{O}_a(x)$

Scaling dimension

H $$[H, \mathcal{O}_a(0)] = \Delta_a \mathcal{O}_a(0)$$

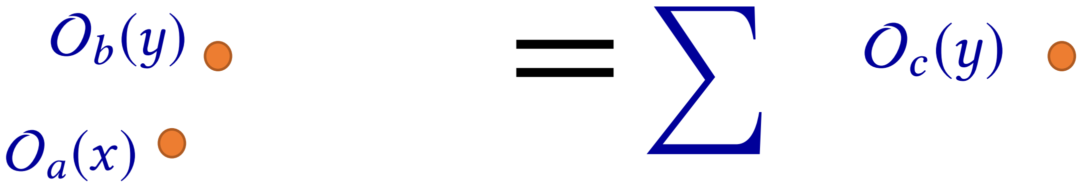
"primary" $$[K_\mu, \mathcal{O}_a(0)] = 0$$

"descendant" $$\mathcal{O}_a(0) = \partial_\mu \mathcal{O}'_a(0)$$



If you find the correlation functions of primary fields, you will find all the correlation function

Operator product expansion (OPE)



$$O_a(x)O_b(y) = \sum_c C^c_{ab}(x-y)O_c(y)$$

You can replace this by this in correlation functions

CFT

- Spectrum of local operators (spin , scaling dimension)
- OPE

} All correlation functions

primary and OPE

Primary

$$\mathcal{O}_A(x)\mathcal{O}_B(y) = \sum_{C:\text{primary}} C^C_{AB}(x-y)\left(\mathcal{O}_C(y) + \bullet\ \partial^n \mathcal{O}_C(y) + \cdots\right)$$

This coefficient is determined by spins and scaling dimensions of $\mathcal{O}_A, \mathcal{O}_B, \mathcal{O}_C$ and the form of ∂^n

Independent of the theory

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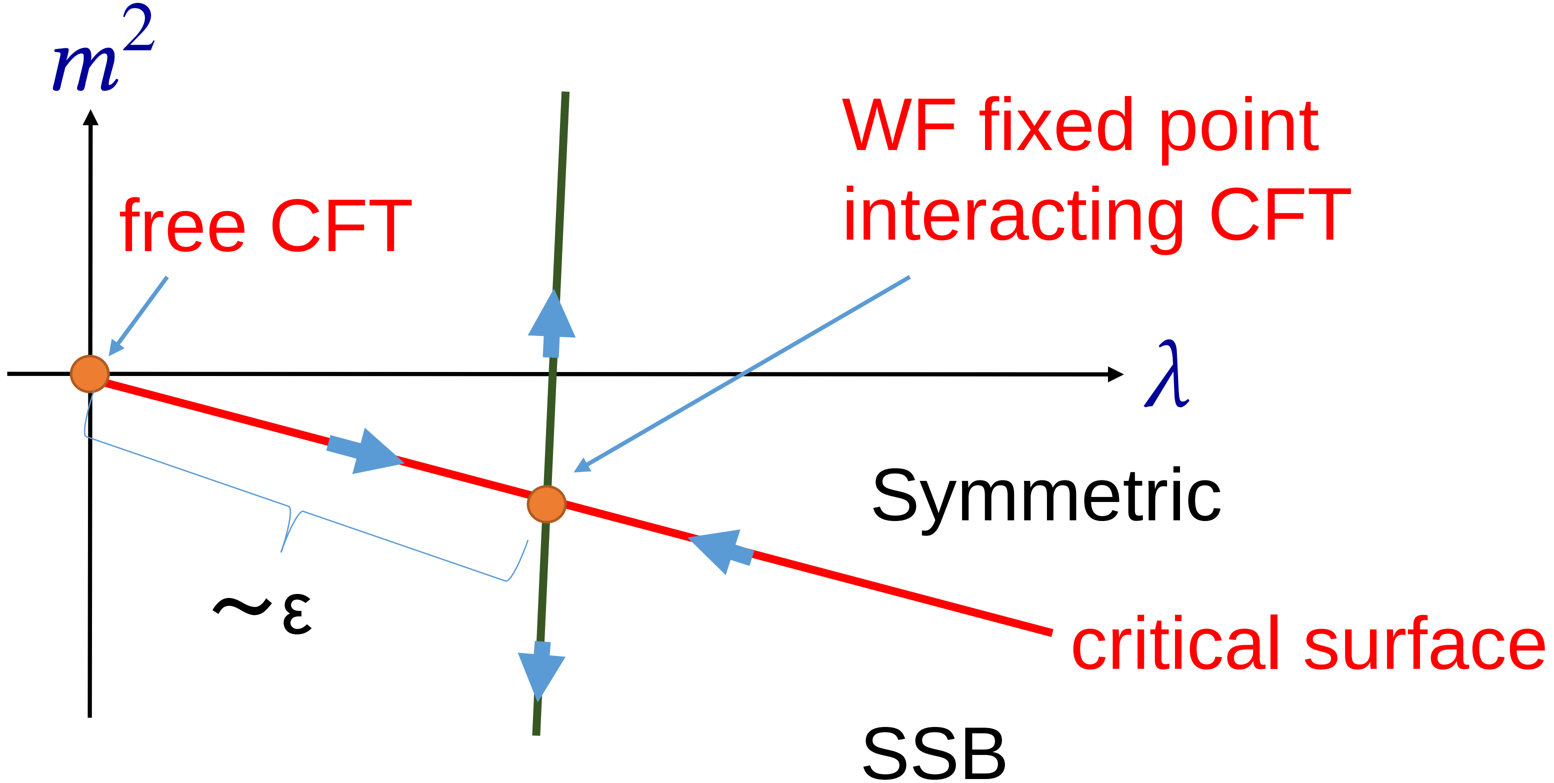
ε-expansion by Rychkov-Tan

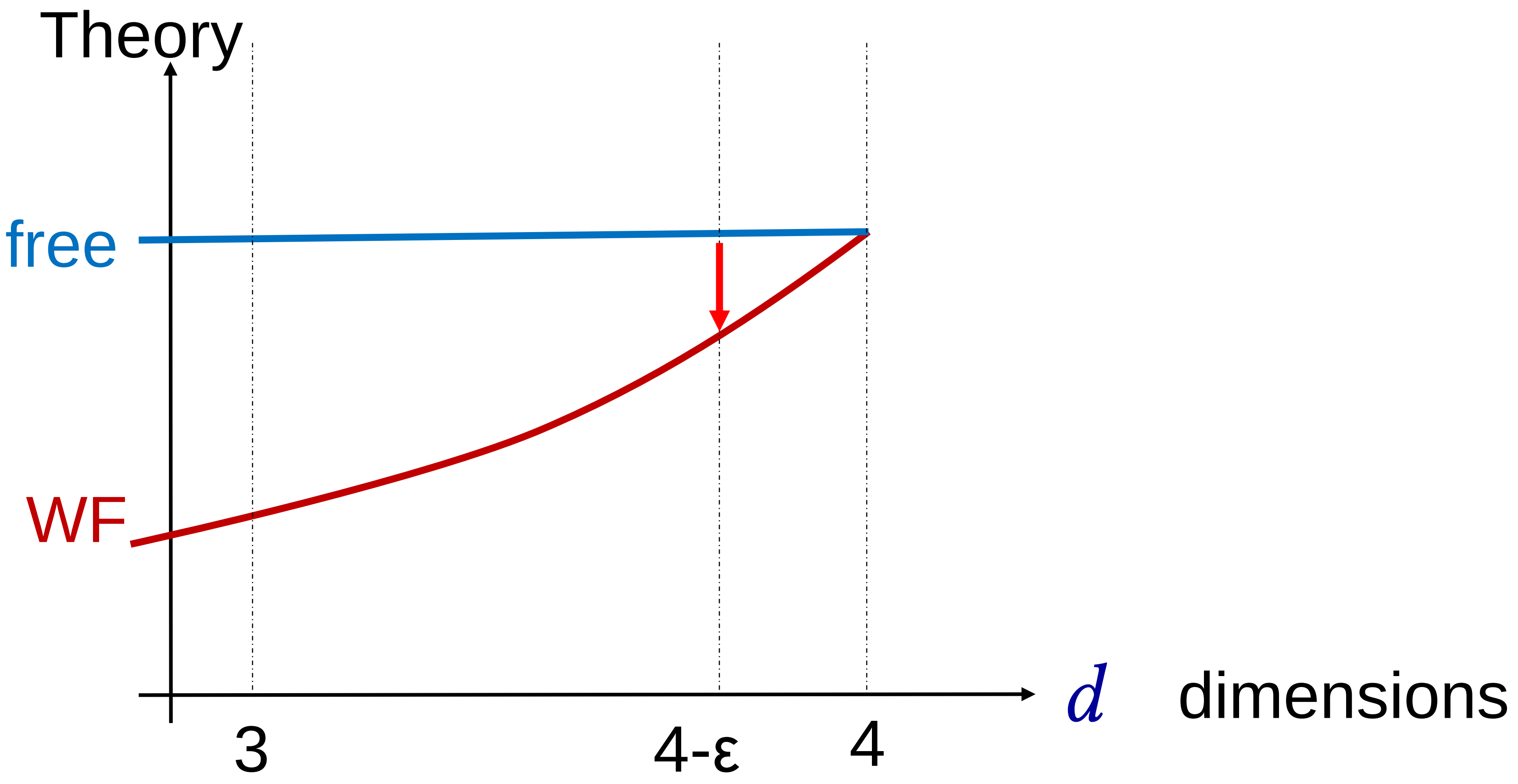
Theory

4-ε dim φ^4 theory [Wilson, Fisher]

$$S = \int d^d x \left(\frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}m^2\varphi^2 + \frac{\lambda}{4!}\varphi^4 \right)$$

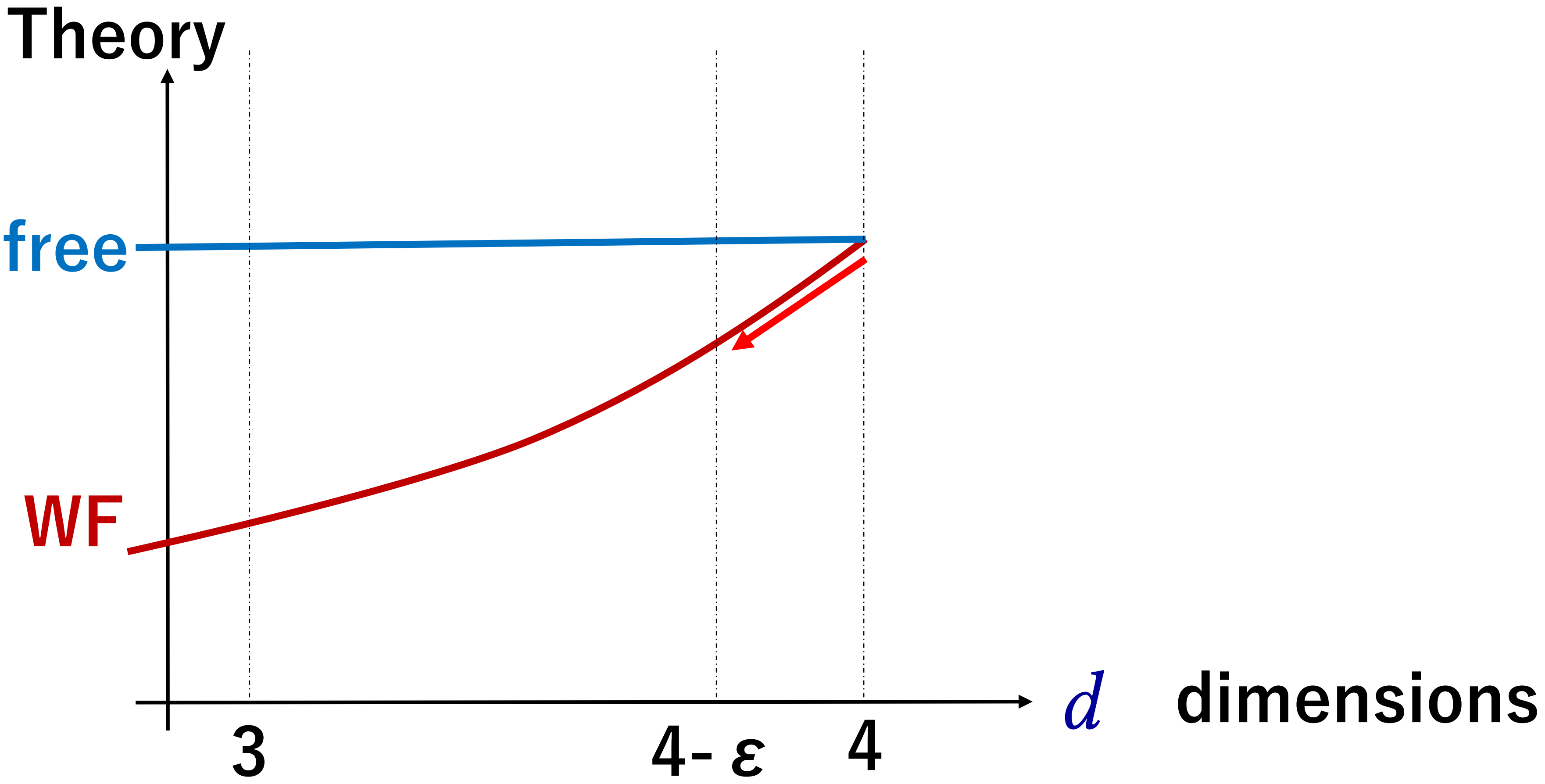
(Euclidean)





In 4-εdim, WF is close to free

Perturbation is good
Old ε-expansion



RT: Do not want to use Lagrangian as far as possible.

Conformal symmetry + α

What you should take

Strategy:

Put a few axioms and find the consequence of them.

We will use conformal symmetry

Axiom I:
Theory at WF fixed point is CFT

$+\alpha$ characterize WF CFT

Axiom II

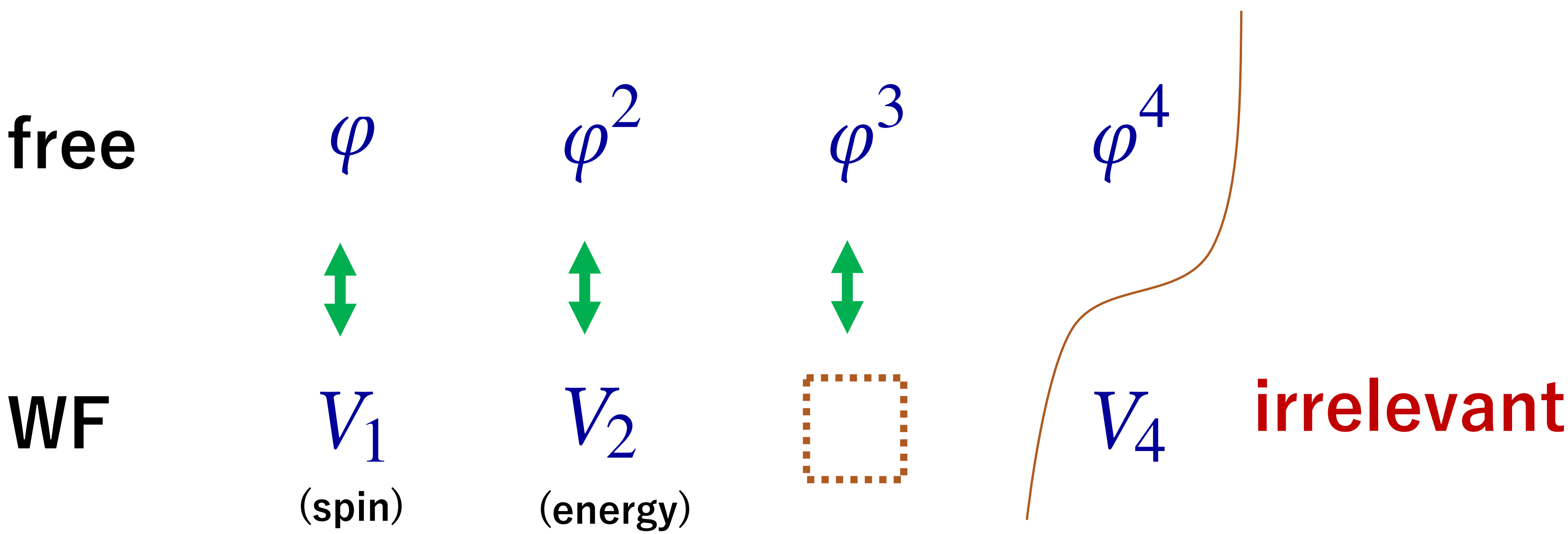
In $\varepsilon \to 0$

$4-\varepsilon$ dim WF CFT $\to$ free

Free CFT also satisfy this axiom.
We need more characterization.

relevant operators $\Delta < d$

In $4-\varepsilon$ dim, free is close to WF $\cdots$



$\varphi^3 \leftrightarrow V_3$ **is a descendant of** V_1 **in WF**

Equation of motion

$$\varphi^3 = \frac{3!}{\lambda}(-m^2 + \Box)\varphi$$

notation $V_n(x)$ local operator

In $\varepsilon \to 0$ $V_n(x) \to \varphi^n(x)$

（They exists from Axiom II）

Axiom III a constant α

$$\Box V_1 = \alpha V_3$$

Idea

Determined by $\Delta_{n+1}, \Delta_n, \Delta_1$

$$\frac{1}{\alpha}\Box V_1(0)$$

$$V_{n+1}(x)V_n(0) = \cdots + \bullet\,(V_1(0) + \bullet\, V_3(0) + \cdots) + \cdots$$

Compare

Relation between $\Delta_{n+1}, \Delta_n, \Delta_1$

free in $\varepsilon \to 0$

$$\varphi^{n+1}(x)\varphi^n(0) = \cdots + \bullet\,(\varphi(0) + \bullet\, \varphi^3(0) + \cdots) + \cdots$$

Calculated by Wick's theorem

Results

$$\Delta_1 = 1 - \frac{\epsilon}{2} + \frac{\epsilon^2}{108} + O(\epsilon^3)$$

$$\Delta_n = n - n\frac{\epsilon}{2} + \frac{1}{6}n(n-1)\epsilon + O(\epsilon^2), \quad n = 2, 3, 4, \ldots$$

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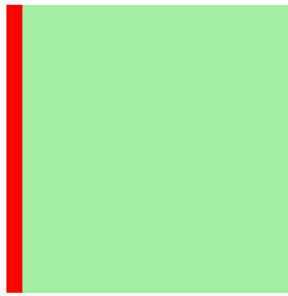
Twist defect

Defect: Generalization of boundary

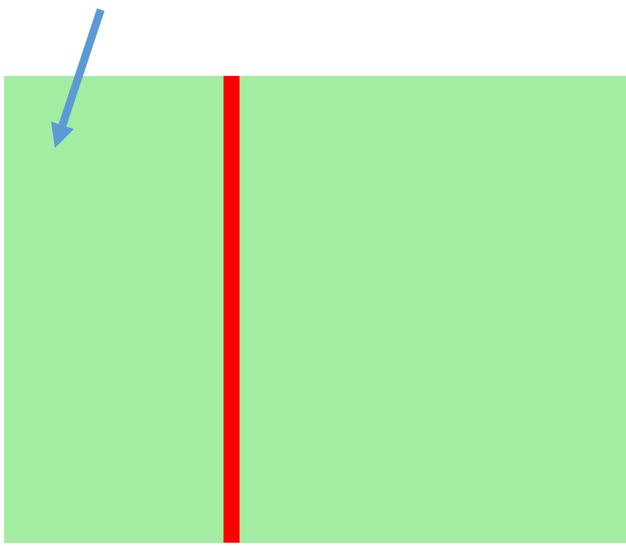


2 dim Boundary CFT
(open string)

D-brane

Generalization

We also have a theory here

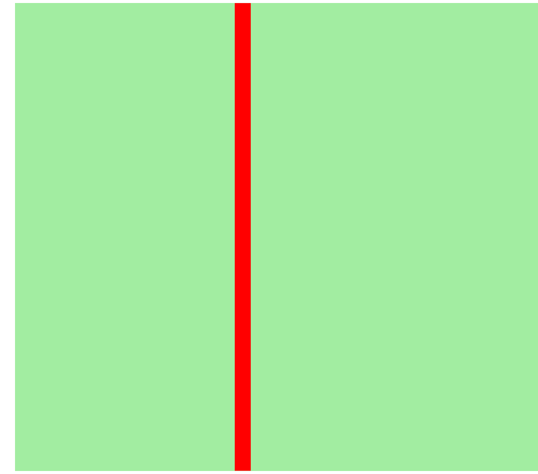


1 dim defect in 2 dim

generalization

D dim defect in d dim

Defect: generalization of Wilson line



Wilson line: Introduce a test particle which couples to the gauge field electorically.

generalization

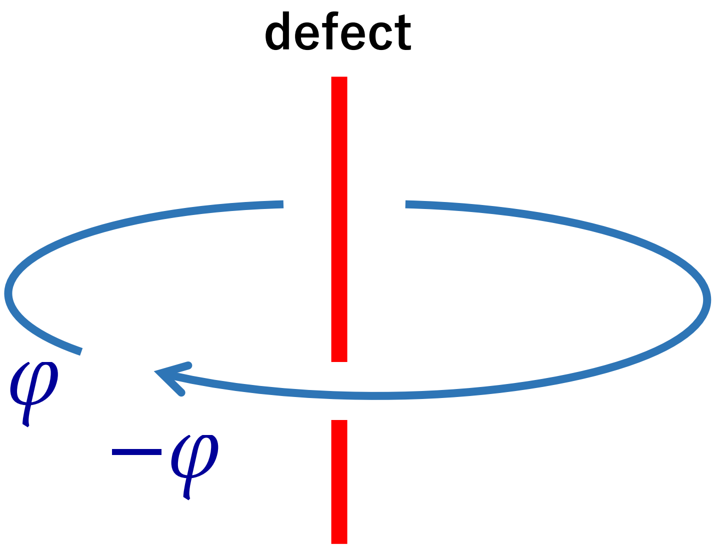
Introduce external test object

Example of defect: “Twist defect”

[Billo, Caselle, Gaiotto, Gliozzi, Meineri], [Gaiotto, Mazac, Paulos]

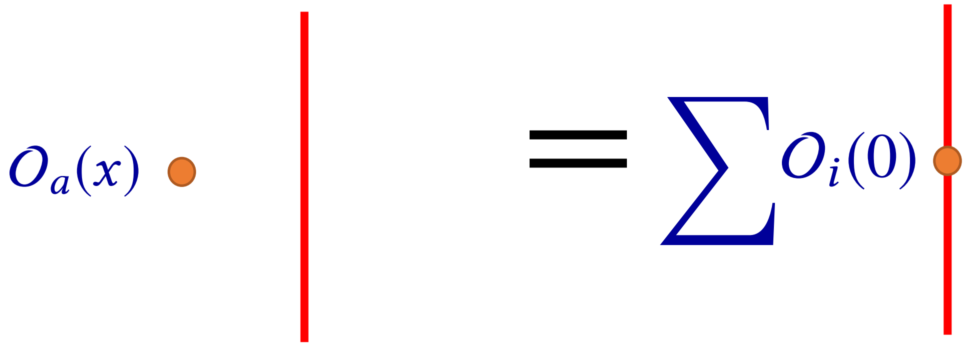
codimension 2, monodromy

Eg. : 3dim Ising
spin operator φ



cf 2 dim orbifold CFT vertex operators in the twisted sector.

bulk-defect OPE

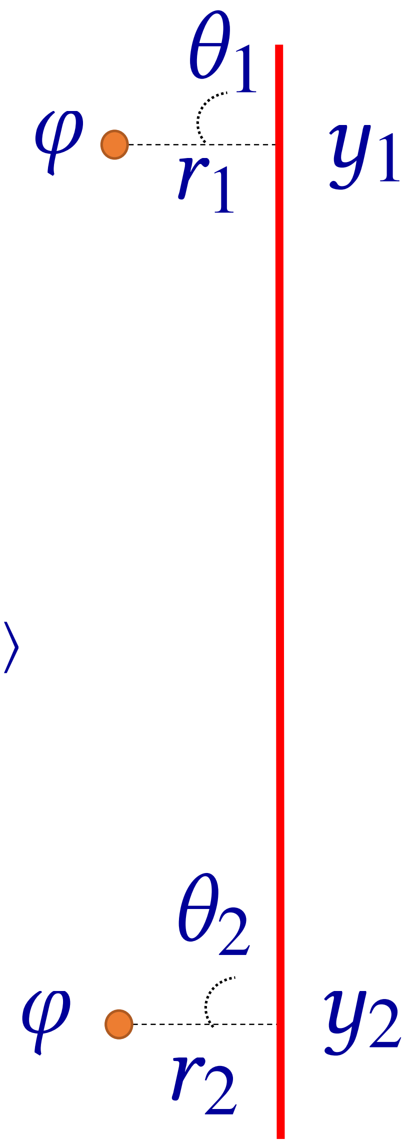


$$\mathcal{O}_a(x) = \sum_i C_{ai}(x)\mathcal{O}_i(0)$$

Local operators on the defect

Bulk 2 point function includes some information on the operators on the defect.

codim 2 defect



$$\varphi(x_1) = \sum_i \frac{e^{is_i\theta_1}}{r_1^{\Delta_\varphi - \Delta_i}} O_i(y_1)$$

$$\langle \varphi(x_1)\varphi(x_2)\rangle = \sum_{i,j} C_{\varphi i} \frac{e^{is_i\theta_1}}{r_1^{\Delta_\varphi - \Delta_i}} C_{\varphi j} \frac{e^{is_j\theta_2}}{r_2^{\Delta_\varphi - \Delta_j}} \langle O_i(y_1) O_j(y_2)\rangle$$

$$= \sum_i |C_{\varphi i}|^2 \frac{e^{is_i(\theta_1-\theta_2)}}{r_1^{\Delta_\varphi - \Delta_i} r_2^{\Delta_\varphi - \Delta_j}} \left(\frac{1}{|y_1 - y_2|^{2\Delta_i}} + (\text{descendants}) \right)$$

4 dim free theory [Gaiotto, Mazac, Paulos]

Calculate

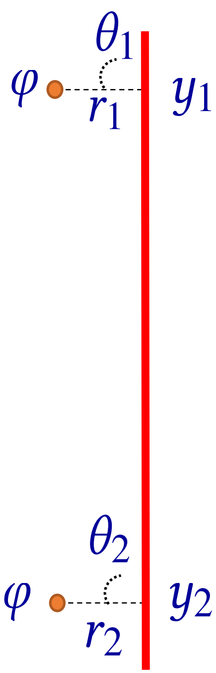
$$\langle \varphi(x_1)\varphi(x_2)\rangle_{\text{defect}}$$

cf bulk-to-bulk propagator in AdS

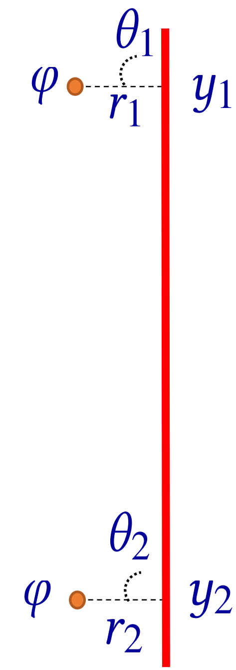
$$\langle \varphi(x_1)\varphi(x_2)\rangle_{\text{defect}} = \sum_{s\in\mathbb{Z}+1/2} G_0(x_1,x_2,s)$$

$$G_0(x_1,x_2,s) = \frac{e^{is(\theta_1-\theta_2)}}{4r_1r_2}\frac{\xi^{-\frac{1}{2}}}{\sqrt{1+\xi}(\sqrt{\xi}+\sqrt{1+\xi})^{2|s|}}$$

$$\xi = \frac{(y_1-y_2)^2+(r_1-r_2)^2}{4r_1r_2}$$



Read the information of local operators on the defect



$$|y_1 - y_2| \to \infty \qquad (\xi \to \infty)$$

$$G_0(x_1, x_2, s) \to \frac{e^{is(\theta_1 - \theta_2)}}{(r_1 r_2)^{-|s|}} \frac{1}{|y_1 - y_2|^{2(|s|+1)}}$$

local operators on the defect

$\psi_s(y) \quad s \in \mathbb{Z} + 1/2$ exist

scaling dimensions $|s| + 1$

bulk-defect OPE $\varphi(x) = \cdots + \frac{e^{is\theta}}{r^{-|s|}} \psi_s(0) + \cdots$

local operators on the defect

$\psi_s(y)$ $s \in \mathbb{Z} + 1/2$ exist

scaling dimensions $|s| + 1$

How about WF CFT ?

Employ Rychkov-Tan's framework.

4 dim free CFT bulk-defect OPE

$$\varphi^3(x) = \cdots - \frac{3}{8}\frac{e^{is\theta}}{r^{2-|s|}}\psi_s(0) + \cdots$$

4- ε dim WF theoy, bulk-defect OPE

$$V_1(x) = \cdots + C_{1s} \frac{e^{is\theta}}{r^{\Delta_1 - \Delta_s}} \psi_s(0) + \cdots$$

$$\begin{aligned} V_3(x) =& \frac{1}{\alpha} \Box V_1(x) \\ =& \cdots + \frac{1}{\alpha} C_{1s} \Box \frac{e^{is\theta}}{r^{\Delta_1 - \Delta_s}} \psi_s(0) + \cdots \ , \\ =& \cdots + \frac{-s^2 + (\Delta_1 - \Delta_s)^2}{\alpha} C_{1s} \frac{e^{is\theta}}{r^{\Delta_1 + 2 - \Delta_s}} \psi_s(0) + \cdots \ . \end{aligned}$$

Compare this in ε →0 with free CFT

➡ Δ_s

Result

$$\Delta_s = |s| + 1 + \left(-\frac{1}{2} - \frac{1}{24|s|} \right) \epsilon + O(\epsilon^2)$$

※Agree with Feynman diagrammatic calculation

[Gaiotto, Mazac, Paulos]

O(N) mode

$$\Delta_s = |s| + 1 + \left(-\frac{1}{2} - \frac{N+2}{8(N+8)|s|}\right)\epsilon + O(\epsilon^2)$$

※Agree with Feynman diagrammatic calculation

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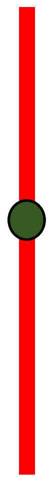
Summary of the result

4- ε dim O(N) model Wilson-Fisher(WF) fixed point
(CFT)

Twist defect



local operators on the defect

ψ_s

Obtained the scaling dimensions
in Rychkov-Tan’ s framework.

Prospects: validity of various methods

Study O(N) model by

- **Large N**
- **Numerical bootstrap**
- **Monte Carlo**
- **Large s ?**
- **Experiments ?**

Question： Free from Lagrangian?

Axiom II is a “equation of motion” though it is scheme independent.

We can derive Feynman rule from equation of motion.

The framework of Rychkov-Tan is at least a nice short-cut.

Non-unitarity of the CFT in non-integer dimensions

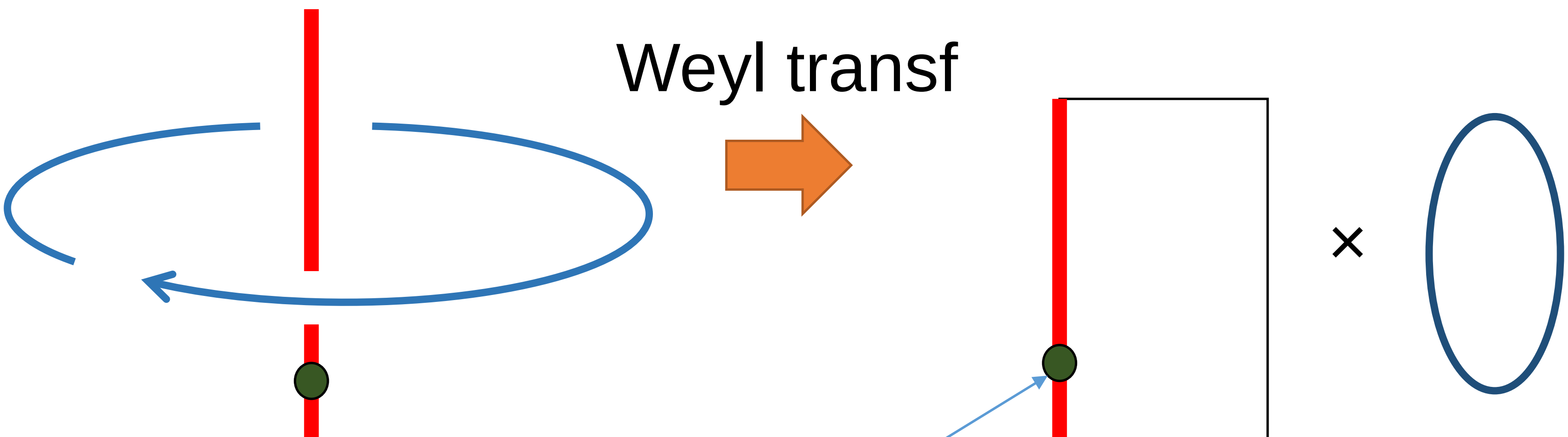
Extra null state in d=integer.

Can it be a hint to solve?

Analogy to 2 dim CFT

Is the defect CFT a miniature of AdS/CFT?

flat space + defect $\qquad AdS_{d-1} \times S^1$



Local operator on the defect ψ_s $\qquad \varphi_s$ Each KK mode

$$\Delta_s = \frac{d-2}{2} + \sqrt{\frac{d-2}{2} + m_s^2}$$

In the large N limit

Same as AdS/CFT correspondence