Towards understanding the Large-Scale Structure in the Universe using perturbation theory

Zvonimir Vlah

Stanford University & SLAC

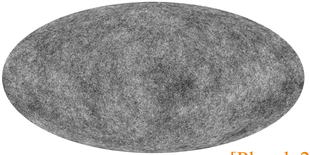
with:

Raul Angulo (CEFCA), Alejandro Aviles (ABACUS), Emanuele Castorina (Berkeley), Matteo Fasiello (Stanford), Yu Feng (Berkeley), Patrick McDonald (Berkeley), Marcel Schmittfull (Berkeley), Uros Seljak (Berkeley), Leonardo Senatore (Stanford), Martin White (Berkeley),

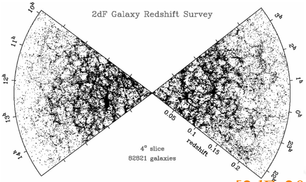
CMB: $\Delta\rho/\rho \sim 10^{-6}$ — z=1100

LSS: $\Delta\rho/\rho \sim 10^{0}$ — z=2

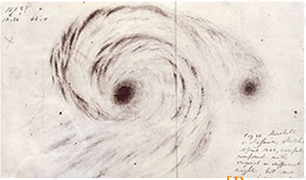
Galaxies: $\Delta\rho/\rho \sim 10^{6}$ — z=0



[Planck,2013]



[2dF, 2002]



[Parsons, 1845]

LSS: motivations and observations

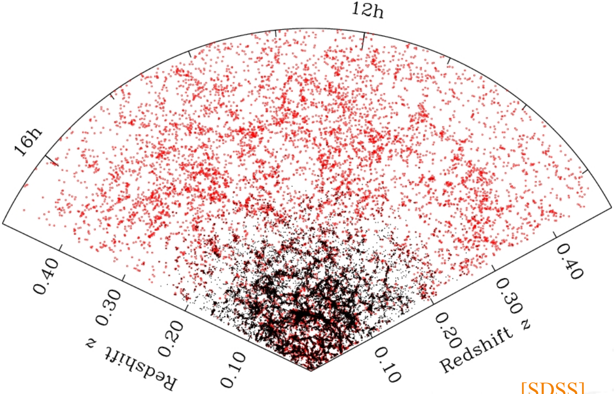
Theoretical motivations:

- Inflation - origin of structures
- Expansion history
- Composition of the universe
- Nature of dark energy and dark matter
- Neutrino mass and number of species
- Test of GR and modifications of gravity

Current and future observations:

- SDSS and SDSS3/4: Sloan Digital Sky Survey
- BOSS: the Baryon Oscillation Spectroscopic Survey
- DES: the Dark Energy Survey
- LSST: the large synoptic survey telescope.
- Euclid: the ESA mission to map the geometry of the dark Universe
- DESI: Dark Energy Spectroscopic Instrument
- SPHEREx: An All-Sky Spectral Survey

Galaxy clustering



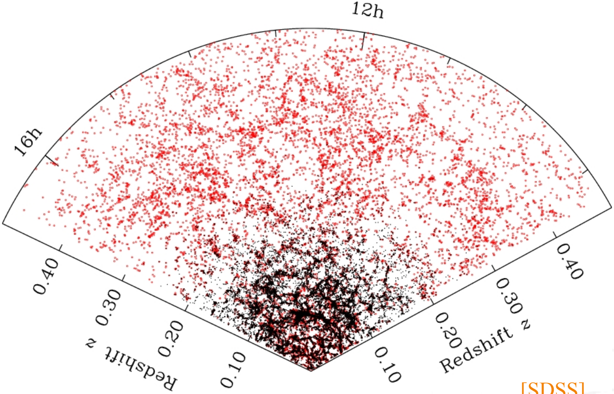
[SDSS]

- Measured $3D$ distribution $\Rightarrow$ much more modes than projected quantities (shear from weak lensing, etc.)
- Redshift surveys measure: θ, ϕ, redshift z

overdensity: $\delta = (n - \bar{n})/\bar{n}$,

power spectrum: $P(k) \sim \langle \delta(\mathbf{k}) | \delta(\mathbf{k}) \rangle$

Galaxy clustering



[SDSS]

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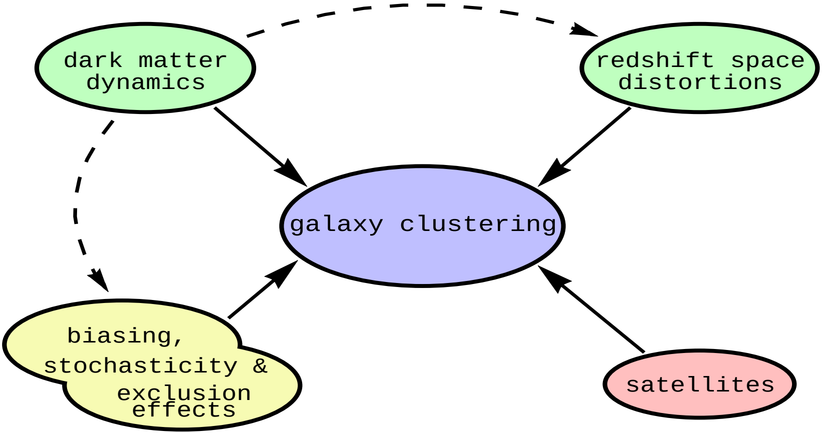
Generalization is the multi-spectra:

$$\langle \delta(\mathbf{k}_1) \ldots \delta(\mathbf{k}_N) \rangle_c \sim P_N(\mathbf{k}_1, \ldots, \mathbf{k}_N)$$

Galaxy clustering scheme

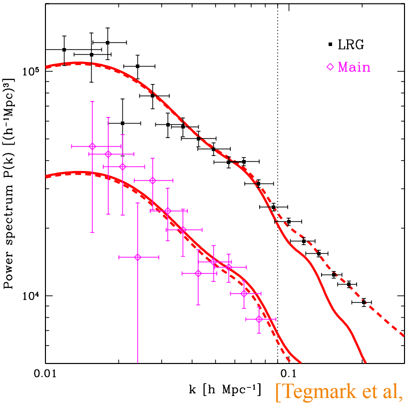


+ others: baryons, assembly bias, neutrinos, (clustering) dark energy, GR effects, multiple d.m. species ...

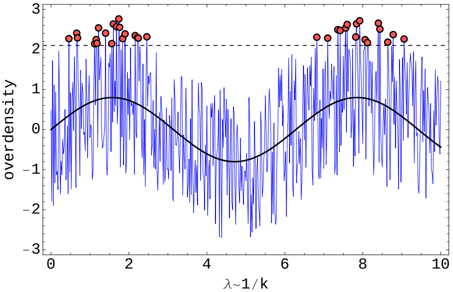
Galaxies and biasing of dark matter halos

Galaxies form at high density peaks of initial matter density:

rare peaks exhibit higher clustering!



[Tegmark et al, 2006]

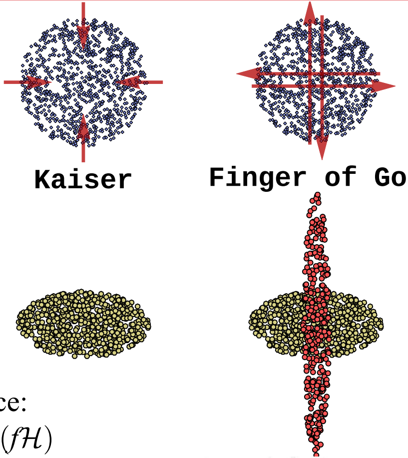


- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \dots$
- Understanding bias is crucial for understanding the galaxy clustering

Redshift space distortions (RSD)

Real space:

Redshift space:



Object position in redshift-space:

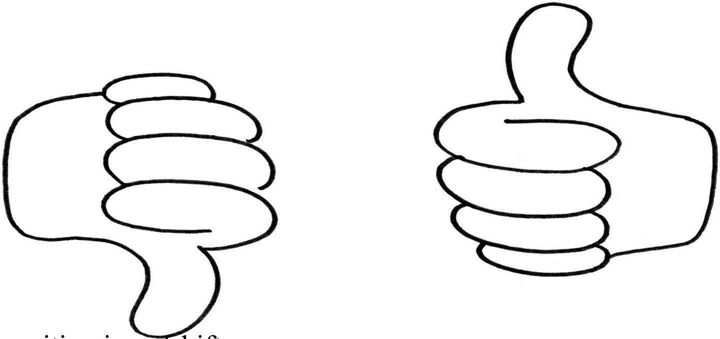
$$\mathbf{s} = \mathbf{x} - f u_z(\mathbf{x})\hat{z}, \quad u_z \equiv -v_z/(f\mathcal{H})$$

Density in redshift-space:

$$\delta_s(\mathbf{k}) = \int_x e^{i\boldsymbol{k}\cdot\boldsymbol{x}} e^{-ifk_z u_z(\mathbf{x})} \Big(\delta(\boldsymbol{x}) + f\nabla_z u_z(\mathbf{x})\Big), \quad f\nabla_z u_z(\mathbf{x}) < 1.$$

Redshift space distortions (RSD)

Fingers of God (FoG)



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Why perturbative approach?

- This problem is in principle amenable to direct simulation.
  - Though the combination of volume, mass and force resolution and numerical accuracy is actually extremely demanding - especially for next gen. surveys.
  - PT guides what range of k, M_h, etc. scales are necessary and what statistics need to be best converged.
  - N-body can be used to test PT for `fiducial' models.
- However PT can be used to search a large parameter space efficiently, and find what kinds of effects are most important.
  - Can be much more flexible/inclusive, especially for biasing schemes.
  - It much easier to add new physics, especially if the effects are small (e.g. neutrinos, clustering dark energy, non-Gaussianity)
- Hopefully we gain some insight, not just numbers!
- Our goal is to do highly precise computations at large scales, in preparation for next gen. surveys, not to push to very small scales.
- For complementarity; because we can, we should.

Gravitational clustering of dark matter

Evolution of collisionless particles - Vlasov equation:

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{1}{m}\mathbf{p} \cdot \nabla f - am\nabla\phi \cdot \nabla_p f = 0,$$

and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Integral moments of the distribution function:

mass density field & mean streaming velocity field

$$\rho(\mathbf{x}) = ma^{-3}\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p}), \qquad v_i(\mathbf{x}) = \frac{\int d^3p\, \frac{p_i}{am} f(\boldsymbol{x}, \boldsymbol{p})}{\int d^3p\, f(\boldsymbol{x}, \boldsymbol{p})},$$

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Eulerian framework - fluid approximation:

$$\begin{aligned} &\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0 \\ &\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \mathbf{v} \cdot \nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_i(\rho\sigma_{ij}), \end{aligned}$$

where σ_{ij} is the velocity dispersion.

Gravitational clustering of dark matter

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and $\nabla^2\phi = 3/2\mathcal{H}\Omega_m\delta$.

Eulerian framework - **pressureless perfect fluid** approximation:

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot [(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H} v_i + \mathbf{v} \cdot \nabla v_i = -\nabla_i \phi.$$

Irrotational fluid: $\theta = \nabla \cdot \mathbf{v}$.

Gravitational clustering of dark matter

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EFT approach introduces a tress tensor for the long-distance fluid:

$$\frac{\partial \delta}{\partial \tau} + \nabla\cdot[(1+\delta)\mathbf{v}] = 0$$
$$\frac{\partial v_i}{\partial \tau} + \mathcal{H}v_i + \mathbf{v}\cdot\nabla v_i = -\nabla_i\phi - \frac{1}{\rho}\nabla_j(\tau_{ij}),$$

with given as $\tau_{ij} = p_0\delta_{ij} + c_s^2\delta\rho\delta_{ij} + O(\partial^2\delta,\ldots)$
-derived by smoothing the short scales in the fluid with
the smoothing filter $W(\Lambda)$, where $\Lambda \propto 1/k_{\mathrm{NL}}$.

[Baumann et al 2010, Carrasco et al 2012]

Lagrangian vs Eulerian framework

Eulerian:

Lagrangian:

Coordinate of a (t)racer particle at a given moment in time $\mathbf{r}$

$$\mathbf{r}(\mathbf{q},\tau) = \mathbf{q} + \Psi(\mathbf{q},\tau),$$

is given in terms of Lagrangian displacement.
Continuity equation:

$$(1+\delta(\mathbf{r}))\, d^3r = d^3q \quad \to \quad 1+\delta(\mathbf{r}) = \int_q \delta^D\left(\mathbf{r}-\mathbf{q}-\Psi(\mathbf{q})\right),$$

Fourier space

$$(2\pi)^3\delta^D(\mathbf{k}) + \delta(\mathbf{k}) = \int_q e^{i\mathbf{k}\cdot\mathbf{q}}\ \exp\left(i\mathbf{k}\cdot\Psi\right),$$

Lagrangian dynamics and EFT

Fluid element at position $\boldsymbol{q}$ at time t_0, moves due to gravity:
Lagrangian displacement field; $\boldsymbol{x}(\boldsymbol{q},t) = \boldsymbol{q} + \psi(\boldsymbol{q},t)$.
Density field at any time is given by

$$1 + \delta(\boldsymbol{x}) = \int_q \delta_D\left[\boldsymbol{x} - \boldsymbol{q} - \psi(\boldsymbol{q})\right] \quad \Rightarrow \quad \delta(\boldsymbol{k}) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}}\left(e^{i\boldsymbol{k}\cdot\psi(\boldsymbol{q})} - 1\right)$$

The evolution of ψ is governed by

$$\partial_t^2\psi + 2H\partial_t\psi = -\nabla\phi(\boldsymbol{q}+\psi).$$

Integrating out short modes (using filter $W_R(\boldsymbol{q},\boldsymbol{q}'))$ system is splitting tnto $L-$long and $S-$short wavelength modes, e.g.

$$\psi_L(\boldsymbol{q}) = \int_q W_R(\boldsymbol{q},\boldsymbol{q}')\psi(\boldsymbol{q}'), \quad \psi_S(\boldsymbol{q},\boldsymbol{q}') = \psi(\boldsymbol{q}') - \psi_L(\boldsymbol{q}).$$

This defines δ_L as the long-scale component of the density perturbation corresponding to ψ_L and also Φ_L as the gravitational potential $\nabla^2\Phi_L \sim \delta_L$.
E.o.m. for long displacement:

$$\ddot{\psi}_L + \mathcal{H}\dot{\psi}_L = -\nabla\Phi_L(\boldsymbol{q}+\psi_L(\boldsymbol{q})) + \boldsymbol{a}_S\big(\boldsymbol{q},\psi_L(\boldsymbol{q})\big), \qquad \text{[Vlah et al, '15]}$$

and $\boldsymbol{a}_S(\boldsymbol{q}) = -\nabla\Phi_S(\boldsymbol{q}+\psi_L(\boldsymbol{q})) - \frac{1}{2}Q_L^{ij}(\boldsymbol{q})\nabla\nabla_i\nabla_j\Phi_L(\boldsymbol{q}+\psi_L(\boldsymbol{q})) + \ldots,$
Similar formalism was also derived in [Porto et al, '14].

Lagrangian dynamics and EFT

The correlation function and power spectrum can now be defined through the cumulants of the displacement, e.g.

$$P(k) = \int_q e^{i\boldsymbol{q}\cdot\boldsymbol{k}} \left[\left\langle e^{i\boldsymbol{k}\cdot\boldsymbol{\Delta}(\boldsymbol{q})} \right\rangle - 1 \right].$$

For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q}} \exp\left[-\frac{1}{2} k_i k_j \left\langle \Delta_i \Delta_j \right\rangle_c + \frac{i}{6} k_i k_j k_k \left\langle \Delta_i \Delta_j \Delta_k \right\rangle_c + \cdots \right]$$

Final results equivalent to the Eulerian scheme. [Sugiyama '14, Vlah et al, '14 & '15]
Allows for the insight in the counter term structure and IR resummation schemes (in particular one leads to the scheme in [Senatore&Zaldarriaga, '14]).
Simple IR scheme was suggested also in [Baldauf et al, '15].

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For one loop power spectrum results, keeping linear modes resumed:

$$P(k) = \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q} - (1/2)k_i k_j A^{\rm lin}_{ij}} \left[1 - \frac{1}{2} k_i k_j A^{\rm lpt+eft}_{ij} + \frac{i}{6} k_i k_j k_k W^{\rm lpt+eft}_{ijk} + \cdots \right]$$

where $A_{ij}(\boldsymbol{q}) = 2 \left\langle \Psi_i(\boldsymbol{0}) \Psi_j(\boldsymbol{0}) \right\rangle - 2 \left\langle \Psi_i(\boldsymbol{q}_1) \Psi_j(\boldsymbol{q}_2) \right\rangle$.

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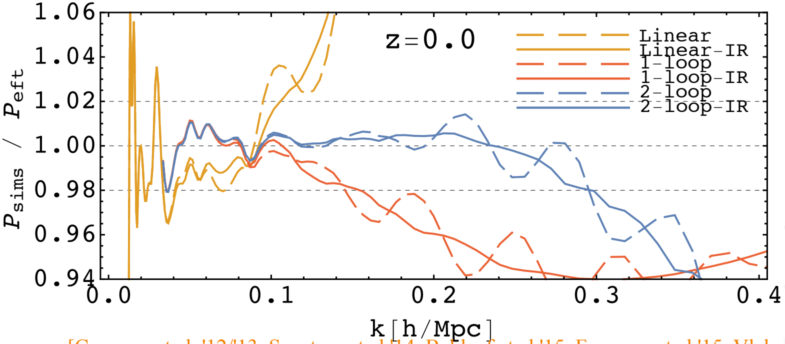
Linear power spectrum, correlation function & BAO

$$P_{\text{EFT-1-loop}} = P_{11} + P_{\text{1-loop}} - 2(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}P_{11}$$

$$P_{\text{EFT-2-loop}} = P_{11} + P_{\text{1-loop}} + P_{\text{2-loop}} - 2(2\pi)c^2_{s(1)}\frac{k^2}{k^2_{\text{NL}}}(P_{11} + P_{\text{1-loop}}) + \text{c.t.}$$

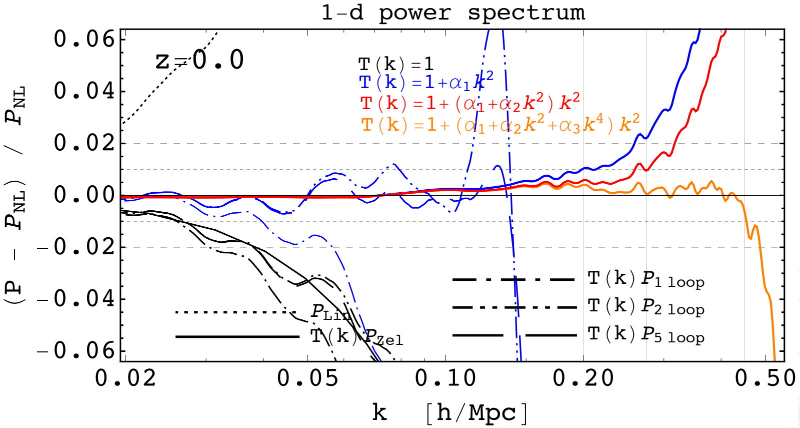


[Carrasco et al, '12/'13, Senatore et al '14, Baldauf et al '15, Foreman et al '15, Vlah et al '15]

- Well defined/convergent expansion in k/k_{NL} (one parameter).
- IR resummation (Lagrangian approach) - BAO peak! [Vlah et al '15]
- Six c. t. for two-loop - approximate degeneracy! [Zaldarriaga et al, '15]

Clustering in 1D

1D case studied recently in: [McQuinn&White, '15, Vlah et al, '15]



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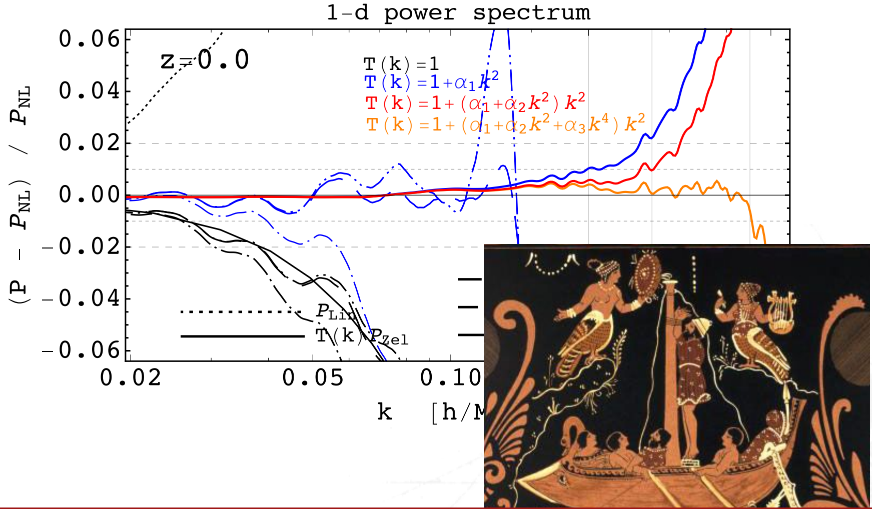
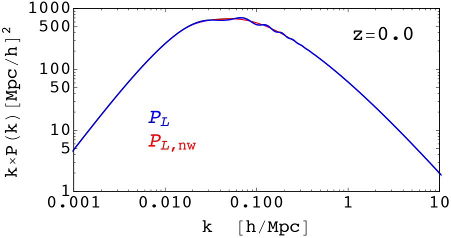
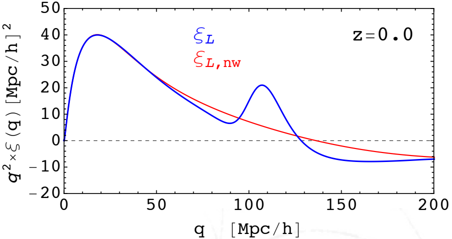


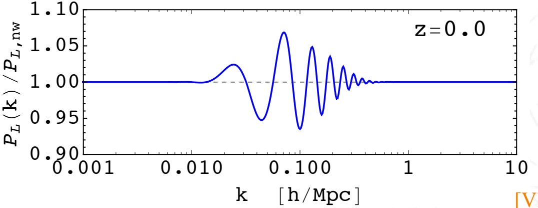
Linear power spectrum, correlation function & BAO

Linear power spectrum P_L: obtained form Boltzmann codes (CAMB, Class). Formally we can divide it into smooth part $P_\mathrm{L,nw}$ and wiggle part $P_\mathrm{L,w}$ so that:
$$P_\mathrm{L} = P_\mathrm{L,nw} + P_\mathrm{L,w}$$





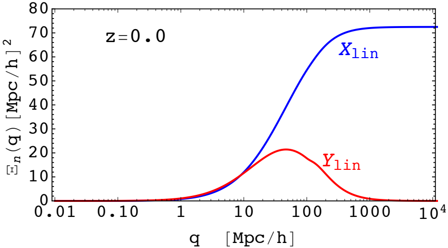
Wiggle power spectrum: $P_\mathrm{L,w} \quad \rightarrow \quad \sigma_n = \int_q q^{-n} P_\mathrm{L,w}(q) = 0$ for $n = \{0, 2\}$.

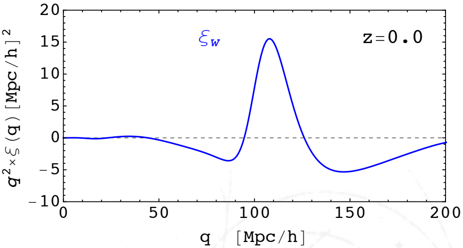


[Vlah et al, '14 & '15]

Separating the wiggle and non-wiggle part $A^{ij}_{\rm L}(\boldsymbol{q}) = A^{ij}_{\rm L,nw}(\boldsymbol{q}) + A^{ij}_{\rm L,w}(\boldsymbol{q})$;

$$P = P_{\rm nw} + \int_q e^{i\boldsymbol{k}\cdot\boldsymbol{q} - (1/2)k_ik_jA^{ij}_{\rm L,nw}} \left[-\frac{k_ik_j}{2} A^{ij}_{\rm L,w} + \cdots \right] \simeq P_{\rm nw} + e^{-k^2\Sigma^2} P_{\rm L,w} + \ldots$$



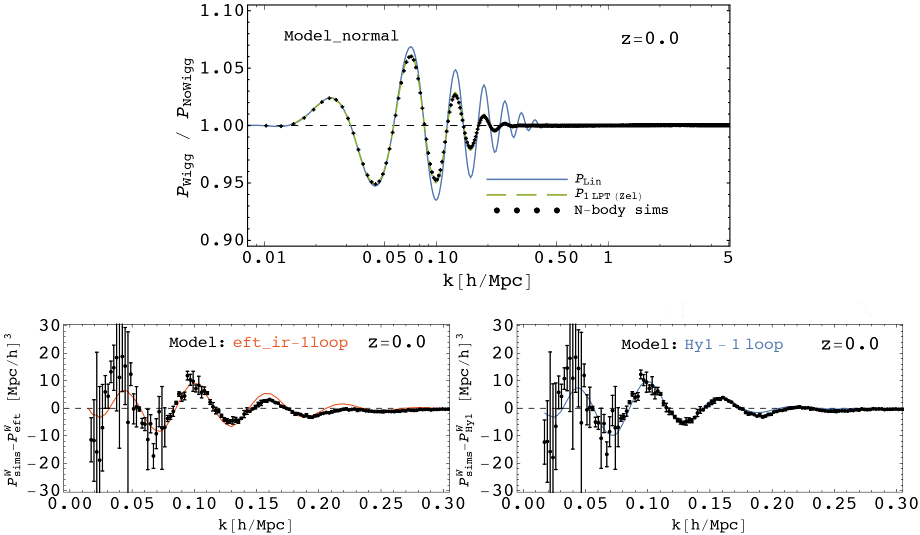


IR-SPT resummation model:

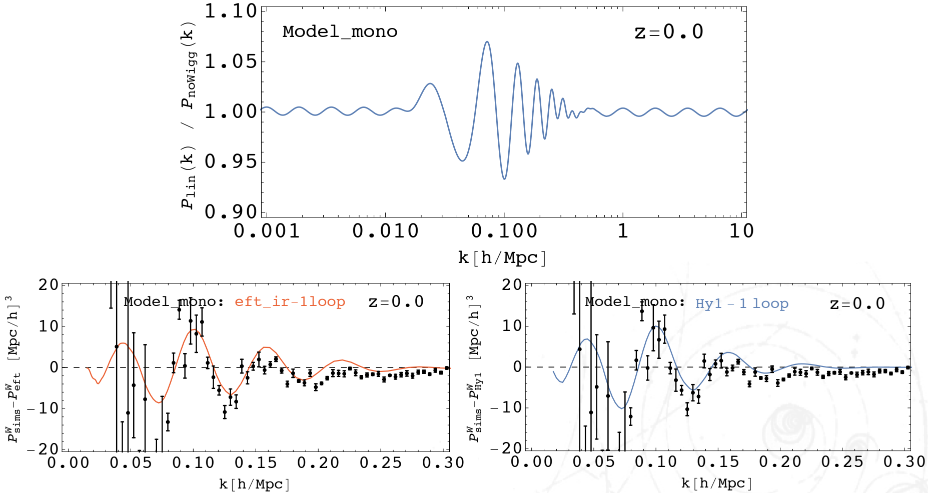
$$P_{\rm dm}(k) = P_{\rm nw,L}(k) + P_{\rm nw,SPT,1-loop}(k) + \alpha_{\rm SPT,1-loop,IR}(k)k^2P_{\rm nw,L}(k)$$
$$+ e^{-k^2\Sigma^2}\Big(\Delta P_{\rm w,SPT,1-loop}(k) + \left(1 + (\alpha_{\rm SPT,1-loop,IR} + \Sigma^2)k^2\right)\Delta P_{\rm w,L}(k)\Big).$$

Alternative derivation in: [Baldauf et al, 2015]

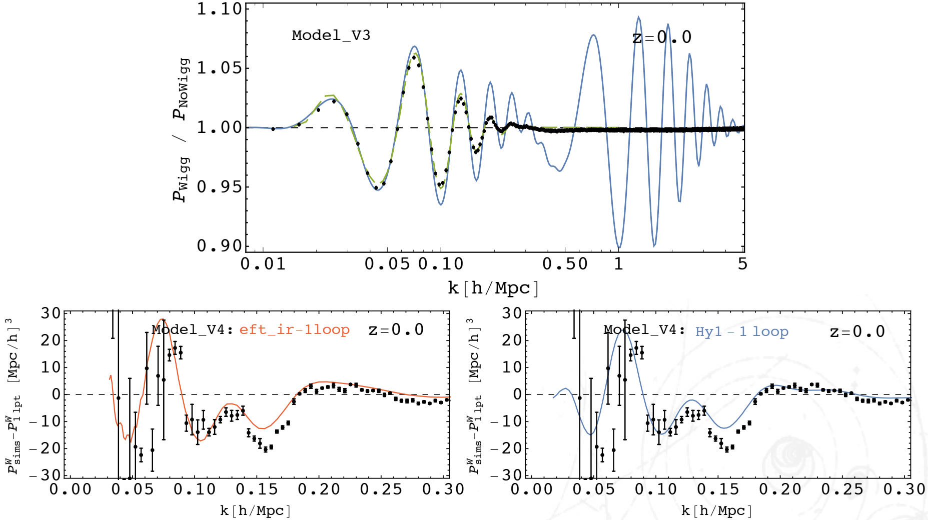
Wiggle residuals in our schemes: BAO



BAO+: Monodromy



BAO++: Small scale wiggles



Earlier approaches to halo biasing

Local biasing model: halo field is a function of just DM density field

$$\delta_{\rm h} = c_\delta \delta + c_{\delta^2}\left(\delta^2 - \left\langle \delta^2 \right\rangle\right) + c_{\delta^3}\delta^3 + \dots$$

[Fry & Gaztanaga, 1993]

Quasi-local (in space) relation of the halo density field to the dark matter

[McDonald & Roy 2008, Assassi et al, 2014]

$$\begin{aligned}\delta_{\rm h}(\mathbf{x}) &= c_\delta \delta(\mathbf{x}) + c_{\delta^2}\delta^2(\mathbf{x}) + c_{\delta^3}\delta^3(\mathbf{x}) \\ &\quad + c_{s^2}s^2(\mathbf{x}) + c_{\delta s^2}\delta(\mathbf{x})s^2(\mathbf{x}) + c_\psi \psi(\mathbf{x}) + c_{st}s(\mathbf{x})t(\mathbf{x}) + c_{s^3}s^3(\mathbf{x}) \\ &\quad + c_\epsilon \epsilon + \dots,\end{aligned}$$

with effective ('Wilson') coefficients c_l and variables:

$$s_{ij}(\mathbf{x}) = \partial_i\partial_j\phi(\mathbf{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta(\mathbf{x}), \qquad t_{ij}(\mathbf{x}) = \partial_i v_j - \frac{1}{3}\delta^{\rm K}_{ij}\theta(\mathbf{x}) - s_{ij}(\mathbf{x}),$$

$$\psi(\mathbf{x}) = [\theta(\mathbf{x}) - \delta(\mathbf{x})] - \frac{2}{7}s(\mathbf{x})^2 + \frac{4}{21}\delta(\mathbf{x})^2,$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

Effective field theory of biasing

Non-local (time) and quasi-local (spece) relation of the halo density field to the dark matter

[Senatore 2014, Mirbabayi et al, 2014]

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\; H(t')\; \Big[\bar{c}_\delta(t,t')\; :\delta(\boldsymbol{x}_{\rm fl},t'):$$
$$+\,\bar{c}_{\delta^2}(t,t')\; :\delta(\boldsymbol{x}_{\rm fl},t')^2: +\bar{c}_{s^2}(t,t')\; :s^2(\boldsymbol{x}_{\rm fl},t'):$$
$$+\,\bar{c}_{\delta^3}(t,t')\; :\delta(\boldsymbol{x}_{\rm fl},t')^3: +\bar{c}_{\delta s^2}(t,t')\; :\delta(\boldsymbol{x}_{\rm fl},t')s^2(\boldsymbol{x}_{\rm fl},t'): + \dots$$
$$+\,\bar{c}_\epsilon(t,t')\;\epsilon(\boldsymbol{x}_{\rm fl},t') + \bar{c}_{\epsilon\delta}(t,t')\; :\epsilon(\boldsymbol{x}_{\rm fl},t')\delta(\boldsymbol{x}_{\rm fl},t'): + \dots$$
$$+\bar{c}_{\partial^2\delta}(t,t')\;\frac{\partial^2_{x_{\rm fl}}}{k_M^2}\delta(\boldsymbol{x}_{\rm fl},t') + \dots\Big]$$

Novice consideration of non-local in time formation, which depends on fields evaluated on past history on past path:

$$\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\; \boldsymbol{v}(\tau'', \boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

Alternative - all effects chaptered in Lagrangian approach.

Note: Assembly bias effects captured in the scheme.

Effective field theory of biasing

New physical scale $k_M \sim 2\pi \left(\frac{4\pi}{3}\frac{\rho_0}{M}\right)^{1/3}$, which can be different then k_{NL}.

Interesting case $k_{NL} \gg k_M$!

We look at the correlations at $k \ll k_M$.

Each order in perturbation theory we get new bias coefficients:

$$\delta_{\rm h}(k,t) = \int_t \tilde{c}_{\delta,1} \left[D_t \delta^{(1)}(k) + \text{flow terms}\right] + \int_t \tilde{c}_{\delta,2} \left[D_t^2 \delta^{(2)}(k) + \text{flow terms}\right] + \dots$$
$$= c_{\delta,1} \left[\delta^{(1)}(k) + \text{flow terms}\right] + c_{\delta,2} \left[\delta^{(2)}(k) + \text{flow terms}\right] + \dots$$

Emergence of degeneracy: choice of most convenient basis

Renormalization! (takes care of short distance effects at long distances)

In practice, $\tilde{c}_{\delta,1}$ is a bare parameter, the sum of a finite part and a counterterm:

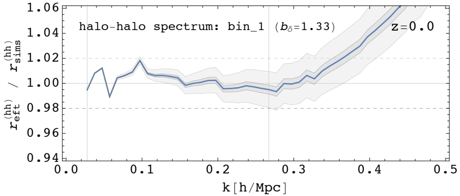
$$\tilde{c}_{\delta,1} = \tilde{c}_{\delta,1,\,\text{finite}} + \tilde{c}_{\delta,1,\,\text{counter}},$$

After renormalization we end up with using 7 finite bias parameters b_i.

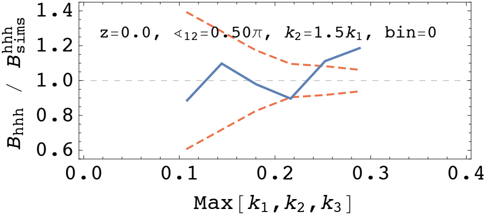
Observables: $P_{\rm hm}$, $P_{\rm hh}$, $B_{\rm hmm}$, $B_{\rm hhm}$, $B_{\rm hhh}$

Effective field theory of biasing

Consistency with N-body simulations achieved up to the $k < 0.3\text{Mpc}/h$ for the Power Spectra, similar for the Bispectrum $k < 0.15\text{Mpc}/h$



nLIT: k_{min}=0.04, k_{max}=0.15						
hm	hh	hmm	hhm	hhh	chi2	p
+	+	−	−	−	0.0804	1.000
+	+	+	−	−	0.719	0.9963
+	+	−	+	−	0.645	0.9998
+	+	−	−	+	0.747	0.9915
+	+	+	+	−	0.835	0.9746
+	+	+	−	+	1.08	0.1685
+	+	−	+	+	0.990	0.5345
+	+	+	+	+	1.08	0.1335



Most of the constraint comes form the 3-pt function

If we had the simulations for the 4-pt function 2-pt function would be fully predicted.

EFT of biased tracers: bias fits

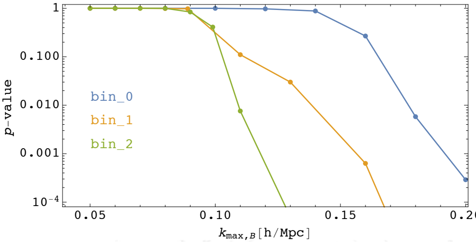
Error bars of the theory are given by the higher loop estimates:

e.g. $\Delta P_{hm} \sim (2\pi)\, b_1 \left(\frac{k}{k_{\rm NL}}\right)^3 P_{11}(k)$.

This determines the theory reach $k_{\rm max}$.

	bin0	bin1
$b_{\delta,1}$	1.00 ± 0.01	1.32 ± 0.01
$b_{\delta,2}$	0.23 ± 0.01	0.52 ± 0.01
$b_{\delta,3}$	0.48 ± 0.12	0.66 ± 0.13
b_{δ^2}	0.28 ± 0.01	0.30 ± 0.01
b_{c_s}	0.72 ± 0.16	0.27 ± 0.17
$b_{\delta\epsilon}$	0.31 ± 0.08	0.76 ± 0.17
Const_ϵ	5697 ± 108	10821 ± 169

Characteristic sharp drop in the p-value after the maximal Bispectrum scale $k_{{\rm max},B}$



Within these scales EFT results fit the data well, and then fail after crossing this scales.

Adding baryonic effects

- baryons at large distances described as additional fluid component (short distance physics is encoded in an effective stress tensor)

$$\delta_h(\boldsymbol{x},t) \simeq \int^t dt'\ H(t')\ \Big[\bar{c}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\delta_b}(t,t')\, w_b\, \delta_b(\boldsymbol{x}_{{\rm fl}\,b})$$
$$+ \bar{c}_{\partial_i v^i_c}(t,t')\ w_c\ \frac{\partial_i v^i_c(\boldsymbol{x}_{{\rm fl}\,c},t')}{H(t')} + \bar{c}_{\partial_i v^i_b}(t,t')\ w_b\ \frac{\partial_i v^i_b(\boldsymbol{x}_{{\rm fl}\,b},t')}{H(t')}$$
$$+ \bar{c}_{\partial_i\partial_j\phi\partial^i\partial^j\phi}(t,t')\ \frac{\partial_i\partial_j\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2}\,\frac{\partial^i\partial^j\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \dots$$
$$+ \bar{c}_{\epsilon_c}(t,t')\ w_c\ \epsilon_c(\boldsymbol{x}_{{\rm fl}\,c},t') + \bar{c}_{\epsilon_b}(t,t')\ w_b\ \epsilon_b(\boldsymbol{x}_{{\rm fl}\,b},t')$$
$$+ \bar{c}_{\epsilon_c\partial^2\phi}(t,t')\ w_c\ \epsilon_c(\boldsymbol{x}_{{\rm fl}\,c},t')\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \bar{c}_{\epsilon_b\partial^2\phi}(t,t')\ w_b\ \epsilon_b(\boldsymbol{x}_{{\rm fl}\,b},t')\frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} \dots \Big]$$

where $\boldsymbol{x}_{\rm fl}$ is defined by Poisson equation and:

$$\boldsymbol{x}_{{\rm fl}\,b}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_b(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))\,, \quad \boldsymbol{x}_{{\rm fl}\,c}(\boldsymbol{x},\tau,\tau') = \boldsymbol{x} - \int_{\tau'}^{\tau} d\tau''\ \boldsymbol{v}_c(\tau'',\boldsymbol{x}_{\rm fl}(\boldsymbol{x},\tau,\tau''))$$

- similar expressions valid when including neutrinos, clustering dark energy …

Adding Non-Gaussianities

We assume that non-G. correlations are present only in the initial conditions and effect can be described by the squeezed limit, $k_L \ll k_S$ of correlation functions.

After horizon re-rentry, but still early enough to neglect all gravitational non-linearities, the primordial density fluctuation are given by

$$\delta^{(1)}(\boldsymbol{k}_S, t_{\rm in}) \simeq \delta_g(\boldsymbol{k}_S) + f_{\rm NL}\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in})\delta_g(\boldsymbol{k}_S - \boldsymbol{k}_L, t_{\rm in}) \ ,$$

where $\tilde{\phi}(\boldsymbol{k}_L, t_{\rm in}) = \frac{3}{2}\frac{H_0^2\,\Omega_m}{D(t_{\rm in})}\frac{1}{k_S^2\,T(k)}\left(\frac{k_L}{k_S}\right)^{\alpha}\delta_g(\boldsymbol{k}_L, t_{\rm in})$ and where $T(k)$ is the transfer function.

In the presence of primordial non-Gaussianities, additional components:

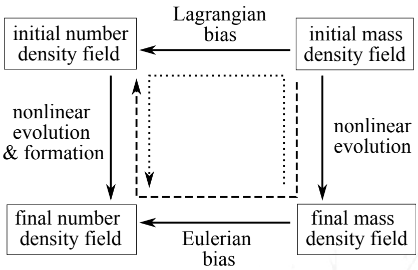
$$\delta_h(\boldsymbol{x},t) \simeq f_{\rm nl}\ \tilde{\phi}(\boldsymbol{x}_{\rm fl}(t,t_{\rm in}),t_{\rm in}) \int^t dt'\ H(t')\ \left[\bar{c}^{\ \tilde{\phi}}(t,t') + \bar{c}^{\ \tilde{\phi}}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \ldots\right]$$
$$+ f_{\rm nl}^2\ \tilde{\phi}(\boldsymbol{x}_{\rm fl}(t,t_{\rm in}),t_{\rm in})^2 \int^t dt'\ H(t')\ \left[\bar{c}^{\ \tilde{\phi}^2}(t,t') + \bar{c}^{\ \tilde{\phi}^2}_{\partial^2\phi}(t,t')\ \frac{\partial^2\phi(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \ldots\right] + \ldots$$

Recently also studied in: [Assassi et al, 2015]

Bias in Lagrangian space

- Eulerian bias: relation between the final mass density field and the final halo density field
- Lagrangian bias: relation between the initial mass density field and the initial halo density field

[Matsubara, 2011]



$$\delta_X(\boldsymbol{q}) = c_\delta \delta_L(\boldsymbol{q}) + c_{\delta^2}\delta_L^2(\boldsymbol{q}) + c_{s^2} s_L^2(\boldsymbol{q}) + c_{\delta^3}\delta_L^3(\boldsymbol{q}) + c_{\delta s^2}\delta_L s_L^2(\boldsymbol{q}) + c_{s^3} s_L^3(\boldsymbol{q})$$
$$+ c_{\partial^2\delta}\frac{\partial_q^2}{k_L^2}\delta_L(\boldsymbol{q}) + \text{"stochastic"} + \ldots,$$

- Tracer defined in Lagrangian space need to be displaced to the final time.

Bias in Lagrangian space in redshift space

Final and initial density in real space (Lagrangian mapping):

$$(1+\delta_X(\boldsymbol{x},\tau))d^3x = (1+\delta_X(\boldsymbol{q},\tau_{\text{in}}))d^3q,$$

Density $\delta_s(\boldsymbol{s})$ can be obtained from $\delta(\boldsymbol{x})$ requiring that the redshift-space mapping conserves mass:

$$(1+\delta(\boldsymbol{s}))d^3s = (1+\delta(\boldsymbol{x}))d^3x$$

Power spectrum in redshift space: Exact expression! [Vlah et al, '16]

$$P_s(\boldsymbol{k}) = \int_{\boldsymbol{r}} e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\left[1+\xi(\boldsymbol{r})\right]\exp\left(ik_\parallel v_{12}^\parallel(\boldsymbol{r}) - \frac{1}{2}k_\parallel^2\sigma_{12}^\parallel(\boldsymbol{r}) + \dots\right),$$

- sometimes called as Gaussian streaming model (GSM) if cumulants beyond σ_{12} are neglected,

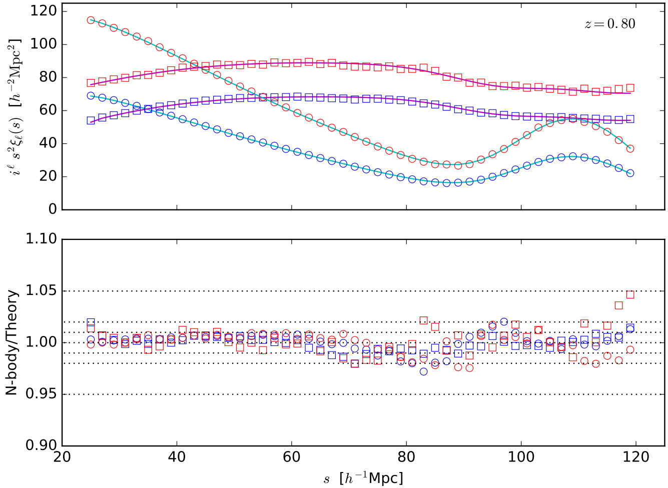
Mass	b_1	b_2	b_{s^2}	α_ξ	α_v	α_σ
$12.5 < \lg M < 13.0$	0.68	-1.01	-0.92	-24	-52	-18
$13.0 < \lg M < 13.5$	1.28	-1.34	-0.14	-9	25	-3

Bias in Lagrangian space in redshift space

Results for a sample of DM halo multipoles in configurations space

○ ○ ○ $\ell = 0,\ 12.5 < \lg M < 13.0$ □ □ □ $\ell = 2,\ 12.5 < \lg M < 13.0$

○ ○ ○ $\ell = 0,\ 13.0 < \lg M < 13.5$ □ □ □ $\ell = 2,\ 13.0 < \lg M < 13.5$



Beyond the EdS-like approximations

standard Eularian fluid solution: [Fasiello, Vlah 2016]

$$\delta(\boldsymbol{k}, a) = \sum_n F_n(\boldsymbol{q}_1..\boldsymbol{q}_n, a)\delta_L(\boldsymbol{q}_1, a)\ldots\delta_L(\boldsymbol{q}_n, a)$$

$$\theta(\boldsymbol{k}, a) = \sum_n G_n(\boldsymbol{q}_1..\boldsymbol{q}_n, a)\delta_L(\boldsymbol{q}_1, a)\ldots\delta_L(\boldsymbol{q}_n, a)$$

where:

$$F_n(\eta) = \int_{-\infty}^{\eta} \frac{d\tilde{\eta}}{C(\tilde{\eta})} \left\{ e^{(n-1)(\tilde{\eta}-\eta)} \frac{\tilde{f}_+}{\tilde{f}_+ - \tilde{f}_-} \left[\left(\tilde{h}_\beta^{(n)} - \frac{\tilde{f}_-}{\tilde{f}_+} \tilde{h}_\alpha^{(n)} \right) + e^{\tilde{\eta}-\eta} \frac{D_-(\eta)}{\tilde{D}_-(\eta)} \left(\tilde{h}_\alpha^{(n)} - \tilde{h}_\beta^{(n)} \right) \right] \right\}$$

similar for G_n, D_+ is linear growth rate and f_+ logarithmic growth rate.

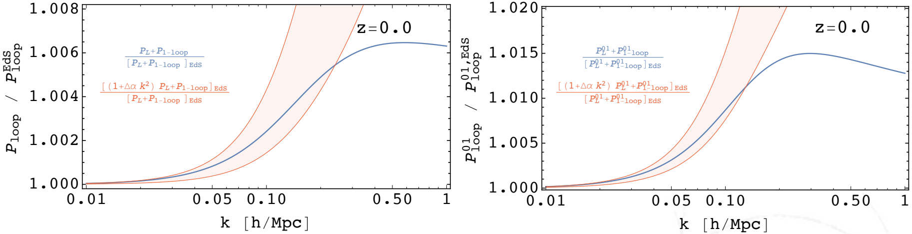
- integral and differential formulation: [Bernardeau, 1994]

$$F_n(\boldsymbol{q}_1..\boldsymbol{q}_n, a) = \sum_i I_i(a)\mathcal{F}_i(\boldsymbol{q}_1..\boldsymbol{q}_n).$$

Fast! Both in time and momentum aspect! [Schmittfull, Z.V., McDonald 2016] [McEwen et al. 2016]

Beyond the EdS-like approximations

$$P_{1-\text{loop}} = P_{\text{lin}} + P_{22} + 2P_{13} + P_{\text{c.t.}} \qquad \text{and} \qquad P_{01} = \frac{dP_{00}}{d\ln a}$$



- important for RSD!
- biasing models of galaxy clustering (brake some of the degeneracies?)
- fast to evaluate - in differential form!

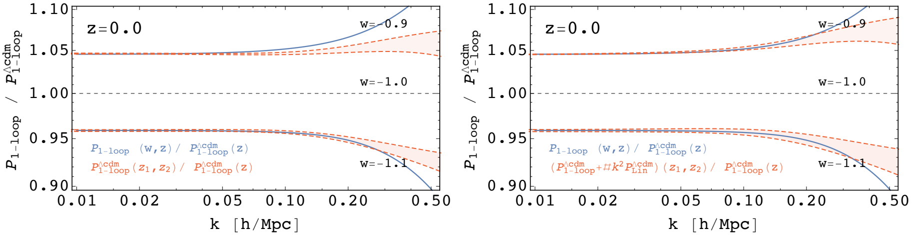
Clustering Quintessence System

system of clustering dark matter and Quintessence [Fasiello, Vlah 2016]

$$
\begin{aligned}
\frac{\partial \delta_m}{\partial \tau} + \partial_i[(1+\delta_m)v^i_m] &= 0 \, , \\
\frac{\partial \delta_Q}{\partial \tau} - 3(w - c_s^2)\mathcal{H}\delta_Q + \partial_i\{[(1+\omega) + (1+c_s^2)\delta_Q]v^i_Q\} &= 0, \\
\frac{\partial v^i_m}{\partial \tau} + \mathcal{H}v^i_m + v^j_m \partial_j v^i_m &= -\nabla^i \Phi, \\
\frac{\partial v^i_Q}{\partial \tau} + \mathcal{H}(1-3w)v^i_Q + v^j_Q \partial_j v^i_Q &= -\partial_i \Phi - \frac{c_s^2 \, \partial_i \delta_Q}{1+w}, \\
\nabla^2 \Phi = \frac{3}{2}\mathcal{H}^2 \Omega_m \left(\delta_m + \frac{\Omega_q}{\Omega_m}\delta_Q \right) &\equiv \frac{3}{2}\mathcal{H}^2 \Omega_m \delta_T,
\end{aligned}
$$

- consistency conditions, for $c_s = w$, conserved outside the horizon but generically not inside
- P.S. enhanced in the IR with respect to dark matter case (similar to the non-equal time pure (DM) correlator)

Clustering Quintessence System



These effects may propagate all the way to biased tracers observables.

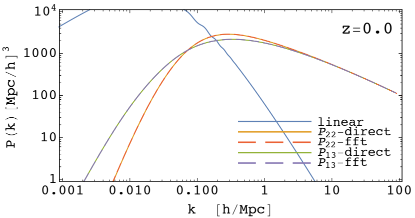
$$
\begin{aligned}
\delta_h(\boldsymbol{x},t) \simeq \int^t H(t')\ \Bigg[& c_{\delta_T}(t')\ \frac{\delta_T(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + c_{\delta_{\rm d.e.}}(t')\,\delta_{\rm d.e.}(\boldsymbol{x}_{\rm fl}) \\
& + c_{\partial v_c}(t')\ \frac{\partial_i v_c^i(\boldsymbol{x}_{\rm fl},t')}{H(t')} + c_{\partial v_{\rm d.e.}}(t')\,\frac{\partial_i v^i_{\rm d.e.}(\boldsymbol{x}_{\rm fl},t')}{H(t')} \\
& + c_{\epsilon_c}(t')\,\epsilon_c(\boldsymbol{x}_{\rm fl},t') + c_{\epsilon_{\rm d.e.}}(t')\,\epsilon_{\rm d.e.}(\boldsymbol{x}_{\rm fl},t') \\
& + c_{\partial^2\delta_T}(t')\ \frac{\partial^2_{x_{\rm fl}}}{k_{\rm M}^2}\,\frac{\delta_T(\boldsymbol{x}_{\rm fl},t')}{H(t')^2} + \dots \Bigg]\ .
\end{aligned}
$$

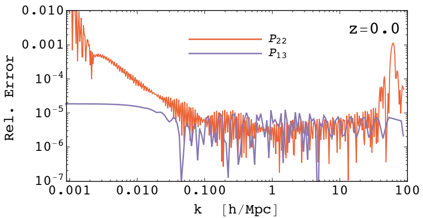
- time evolution can brake degeneracies in bias operators (at third order)

Efficient Evolution of Loops

$$P_{1-\text{loop}} = P_{\text{lin}} + P_{22} + 2P_{13} + P_{\text{c.t.}} \text{ where e.g.}$$

$$P_{22} \sim \int_q f(\boldsymbol{q}) g(\boldsymbol{k}-\boldsymbol{q}) P_q^{\text{lin}} P_{k-q}^{\text{lin}} = \int_0^\infty r^2 j_0(rk) \left[\int_0^\infty q^2 f(q) P_q^{\text{lin}} j_0(qr) \int_0^\infty p^2 g(p) P_p^{\text{lin}} j_0(rp) \right]$$



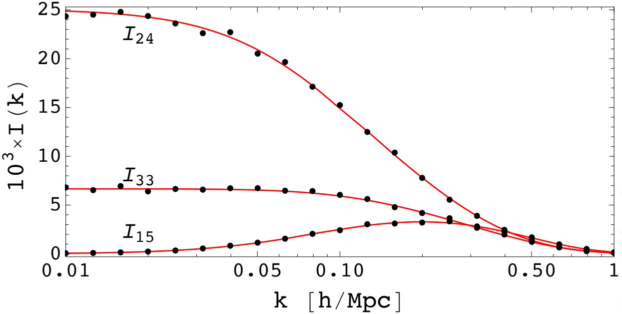


- nonlinear corrections are products correlations of field derivatives
- useful for variation of IC paremeters
- very fast to evaluate - useful is FFTLog (public code) [Hamilton, 2000]

Efficient Evolution of Loops

$P_{2-\text{loop}} = P_{\text{lin}} + P_{33} + 2P_{24} + 2P_{15} + P_{\text{c.t.}}$ where

$$I_{24}(k,\alpha,\beta) = \int_{q_1 q_2} \frac{e^{i\alpha\cdot q_1} e^{i\beta\cdot q_2}}{q_1^{2n_1}|k+q_1|^{2n_1'} q_2^{2n_2}|k+q_2|^{2n_2'}} \frac{P_L(q_1)P_L(q_2)P_L(|k+q_2|)}{|q_1+q_2|^{2n_3}|k+q_1+q_2|^{2n_3'}}$$



- $P_{2-\text{loop}}$ given by taking derivatives of generating function I_N.
- much more efficient for evaluate the using M.C. integration.
- simpler way to obtained asymptotic solutions

Summary

- Large redshift surveys can be used for precision tests of the ΛCDM model.
  - Expansion history (BAO), Growth of structure (RSD), …
- Analytic models can shed light on the relevant physics and we hope they can be made accurate enough to fit next-generation data (on large scales).
- Modeling BAO+RSD requires beyond-linear modeling.
- Lagrangian framework offers a nice physical insight in LSS, application is e.g. IR resummation (BAO+)
- EFT gives a consistent expansion in $(k/k_{\rm NL})^2$, and for halos also in $(k/k_{\rm M})^2$, nonlocal effect in time and space included
- EFT approach is well suited for galaxy clustering (one-loop power spectra $k \sim 0.25h/{\rm Mpc}$, tree level bispectra $k \sim 0.1 - 0.15h/{\rm Mpc}$)
- Consistent description of five different observables ($P_{\rm hm}$, $P_{\rm hh}$, $B_{\rm hmm}$, $B_{\rm hhm}$, $B_{\rm hhh}$) with seven bias parameters.
- Exact time evolution can be important!
- All integrals can be evaluated in a efficient way (FFTLog).

Summary

Outlook:

- Higher loops calculations in order to extend the k_{max} on one hand and improve precision on large scales,
- Higher statistics (e.g. 4-pt function - great potential),
- Calculation of observables taking into account baryons, non-Gaussianities …,
- Generalisation of the formalism in order include GR effects (become important as surveys grow).
- How truly effective are effective approaches (degeneracies etc.)?