4d N=1 from 6d (1,0)

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Motivation

- Compactification of 6d SCFT's can be used to better understand dynamics of 4d SCFT's.
- Has been successfully carried out for the case of the 6d SCFT living on N M5-branes, for 4d N=2 and N=1 SCFT's (class S).
- Recently extended also to the 6d SCFT living on N M5-branes probing a C^2/Z_k singularity, and 4d N=1 SCFT's (class S_k).
- In this talk we will concentrate on better understanding the simpler case of $N = k = 2$. Particularly we shall present various expectations from 6d and compare against the 4d result.

Outline

1. Introduction
    - N=2 SCFT's and Class S theories
    - N=1 SCFT's and Class S theories
    - Class S_k theories
2. 6d perspective
3. 4d perspective
4. Conclusions

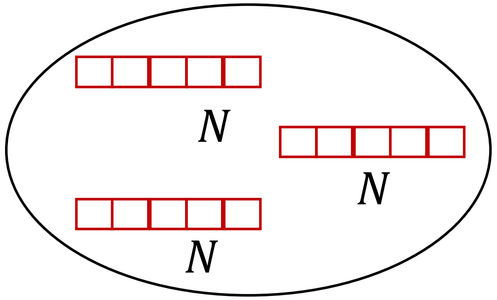
Class S theories

- Provide a systematic construction of 4d N=2 SCFT's by compactification on a Riemann surface of the 6d SCFT living on N M5-branes.
- Different realizations of the same Riemann surface imply duality relations for the 4d SCFT.
- For example: take the Riemann surface to be a torus. Leads to 4d maximally supersymmetric Yang-Mills. The complex structure moduli of the torus becomes the 4d complexified coupling constant.
- Modular invariance of the torus then leads to the $SL(2, Z)$ symmetry of 4d maximally supersymmetric Yang-Mills theory.

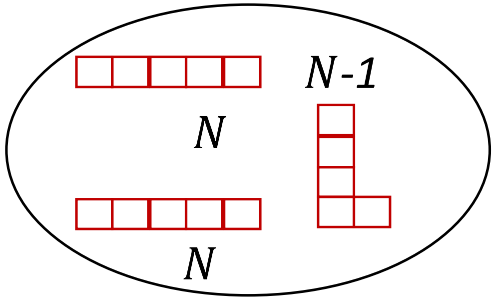
Class S theories: three punctured spheres

- Three punctured spheres play an important role in class S constructions.
- There are several types of punctures which are conveniently represented by Young diagrams.
- T_N theory: corresponds to three maximal punctures. It has an $SU(N)^3$ global symmetry.
- Bifundamental: corresponds to two maximal punctures and a minimal one.
- Other theories can be generated from T_N by Higgsing.

T_N

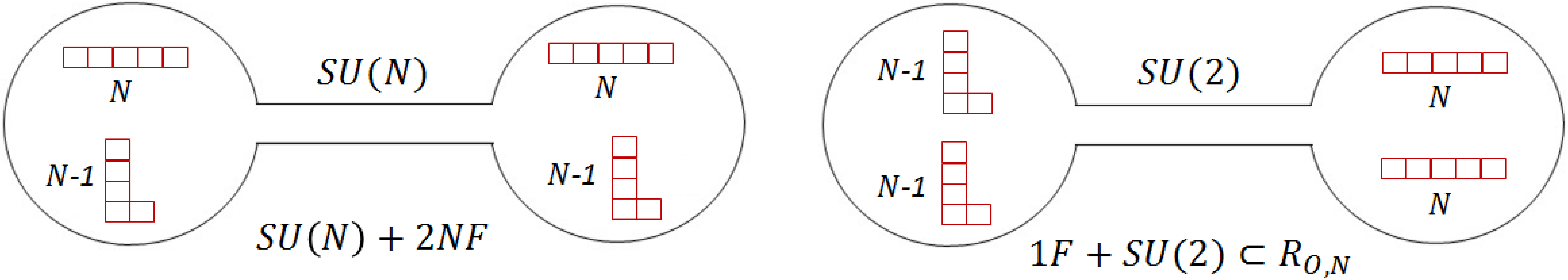


$SU(N) \times SU(N)$
Bifundamental



[Gaiotto, 2009]

Class S theories: general surfaces



- One can connect three punctured spheres with tubes to construct more general theories. Correspond to gauging the global symmetry associated to the punctures.
- The resulting 4d theory is the one associated with the surface.
- There may be more than one way to build a given surface from three punctured spheres. These different “pair of pants” decompositions correspond to dual descriptions of the same SCFT. [Gaiotto, 2009]

N=1 Class S theories

- Can construct N=1 SCFT's using Class S building blocks.
- Done by giving an N=1 preserving deformation to an N=2 SCFT.
- Example: mass term for the adjoint chiral in the N=2 vector multiplet.
- Seiberg duality [Seiberg, 1995]:

$$SU(N) + 2NF,\ W = \rho(Q\bar{Q})^2 \qquad \leftrightarrow \qquad SU(N) + 2NF,\ W = q\bar{q}M + \rho M^2$$

- In 6d: corresponds to N=1 preserving compactifications of the (2,0) theory.

6d construction

- The curvature of the Riemann surface breaks SUSY.
- In order to preserve SUSY we must perform a twist $SO(2)_S \rightarrow SO(2)_S - U(1)_R$ for $U(1)_R$ a subgroup of $U(1)_R \times SU(2)_J \subset SO(4) \subset SO(5)_R$.
- The twist makes some of the spinors covariantly constant and preserves N=1 SUSY.
- This breaks $SO(5)_R$ to $U(1)_R \times SU(2)_J$.
- We can also give flux for the $U(1)_J$ Cartan of the $SU(2)_J$.
- This is necessary to get N=2 SUSY, but for N=1 we have a choice for the total flux.

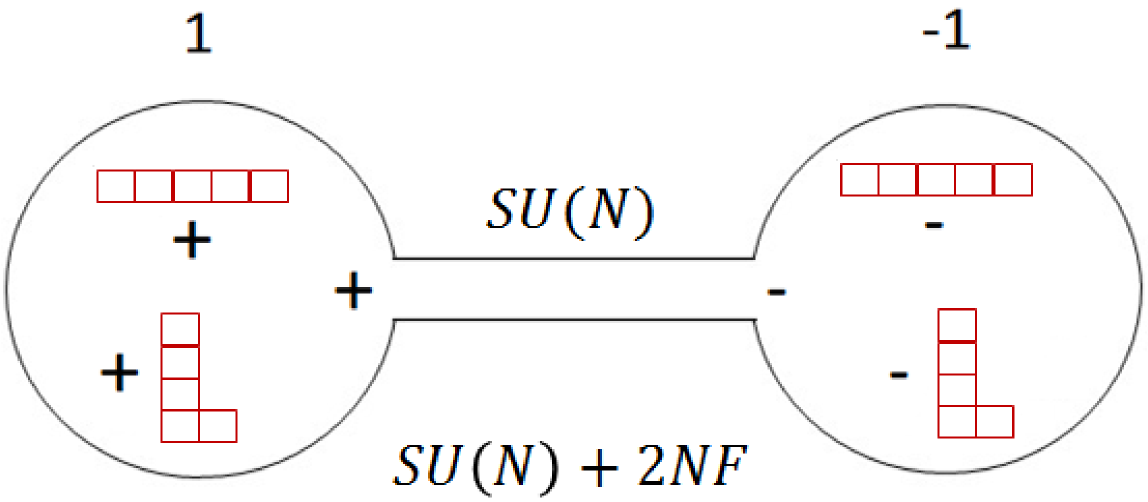
6d construction of N=1 class S

- The resulting theories are labeled by the Riemann surface and flux value.
- Gaiotto case: flux chosen to preserve N=2. Gives the N=2 class S theories.
- Sicillian case: flux is zero. Have an extra $SU(2)_J$ global symmetry [Benini, Tachikawa, Wecht, 2010].
- BBBW case: generic case. Have an extra $U(1)_J$ global symmetry [Bah, Beem, Bobev, Wecht, 2012].

6d construction of N=1 class S

- Theories on a general Riemann surface can be constructed from three punctured spheres with fluxes.
- When connecting components with tubes the total flux is the sum of fluxes.
- Punctures have a discrete label: sign. Determines the charges of operators under $U(1)_J$. Can be either positive or negative.
- Gluing punctures with the same sign is done with N=2 vector multiplets. While gluing punctures with opposite signs is done with N=1 vector multiplets.

N=1 Class S theories



- Using these rules we can construct N=1 SCFT's.
- Example: N=1 $SU(N) + 2NF$ SQCD correspond to a limit of a 4 punctured sphere with vanishing flux.
- Other degeneration limits give dual descriptions.

[Gadde, Maruyoshi, Tachikawa, Yan, 2013]
[Agarwal, Intriligator, Song, 2015]

N=1 class S: conformal manifold

- The conformal manifold is given by the complex structure moduli of the Riemann surface.
- In addition one can turn on flat connections, holonomies, for $SU(2)_J$ or $U(1)_J$ symmetry. These preserve only N=1 SUSY.
- For a genus $g > 1$ Riemann surface with no punctures, we can turn on such holonomies on each of the $2g$ cycles A_i, B_j. These are subject to one relation and the action of global flavor rotation.

$$\prod_{i=1}^{g}[A_i,\, B_i] = 1$$

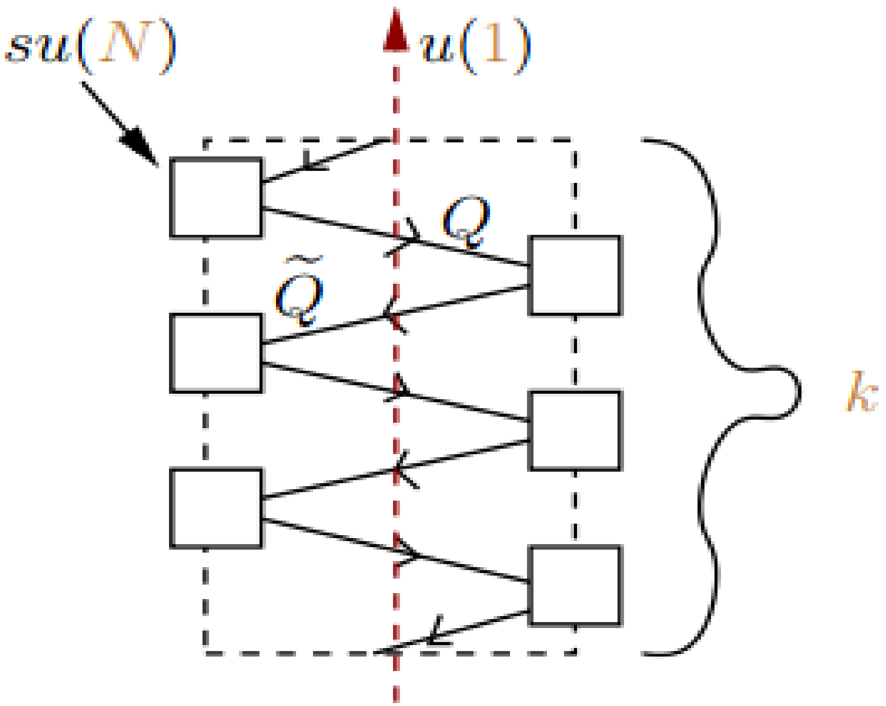
- For Sicilian case: $dim\mathcal{M}_g = (3g-3) + (g-1)\cdot 3 = 6g-6$
- For BBBW case: $dim\mathcal{M}_g = (3g-3) + g = 4g-3$

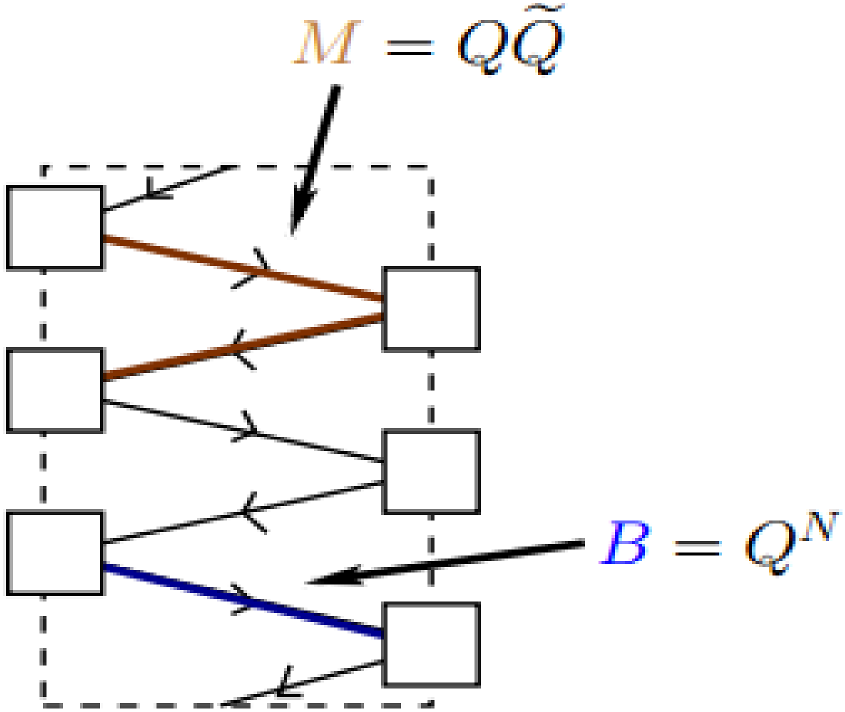
[Benini, Tachikawa, Wecht, 2010]
[Bah, Beem, Bobev, Wecht, 2012]

Class S_k theories

- 4d SCFT's can be constructed by the compactification of the 6d SCFT living on N M5-branes probing a C^2/Z_k singularity. This in general produces N=1 theories.
- A class of field theories arising from this construction was conjectured in [Gaiotto, Razamat, 2015].
- Lagrangian cases can be constructed using the free trinion: corresponding to a sphere with two maximal punctures and one minimal.
- Maximal punctures have an $SU(N)^k$ global symmetry.
- Minimal punctures have a $U(1)$ global symmetry.
- In addition there are also $2k-1$ internal symmetries $U(1)_{\beta_i} \times U(1)_{\gamma_i} \times U(1)_t$. These are conjectured to come from the Cartan of the $SU(k) \times SU(k) \times U(1)$ global symmetry of the 6d theory.

Free trinion





- Contains $2kN^2$ free chiral fields $Q, \tilde{Q}$.

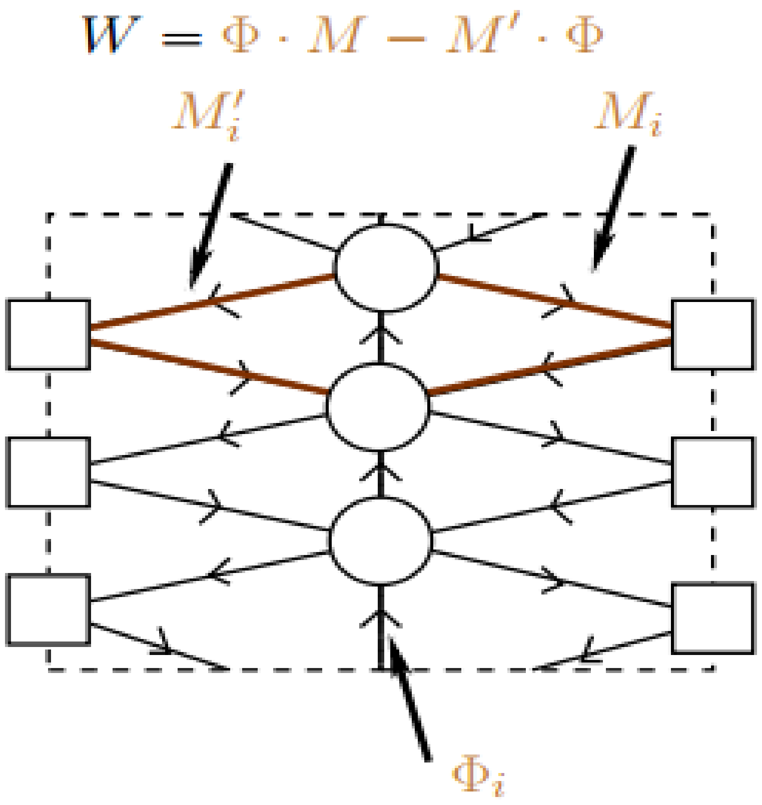
Properties of class S_k theories

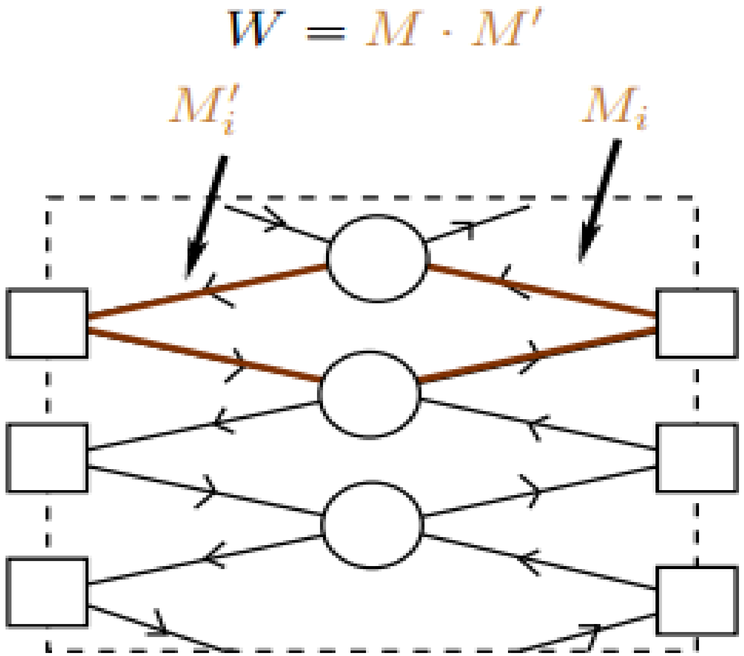
- Can construct general theories corresponding to a Riemann surface and fluxes for the internal symmetries.
- Punctures have additional discrete labels: sign, color.
  - Sign: determine the sign of the charges of the baryons and mesons under $U(1)_t \times U(1)_{\beta_i} \times U(1)_{\gamma_i}$. Can be positive or negative.
  - Color: determine the charge ordering of the baryons and mesons under $U(1)_{\beta_i} \times U(1)_{\gamma_i}$.

Properties of class S_k theories

- Theories corresponding to a Riemann surface can be build by gluing three punctured spheres with fluxes.
- The fluxes of the different pieces are summed.
- Gluing punctures of different color breaks some of the internal symmetry.
- Gluing punctures of different signs does not break any symmetry, but the form of the gluing is different.

Gluing





Φ gluing: punctures of the same sign

S gluing: punctures of different signs

Introduction summary

- We have seen that compactifying 6d theories to 4d helps understanding 4d dynamics.
- Examples: N=2 and N=1 theories in class S, N=1 theories in class S_k.
- Class S_k: construction is 4d. Interesting if:
  - Relate the aspects observed in 4d to 6d.
  - Test the suggested correspondence by direct computation of objects in 6d and 4d.

Plan

- Consider the reduction of the 6d SCFT on N M5-branes probing a C^2/Z_k singularity, to 4d on a Riemann surface. This leads to various expectations for the resulting 4d theory.
- Consider the expected 4d theory in class S_k.
- Compare the two expectations.
- Limitations:
  - Consider only the $N = k = 2$ case.
  - Riemann surface with no continuous isometries.
  - In this talk we concentrate on Riemann surfaces with no punctures.

6d perspective

- Take the 6d SCFT on 2 M5-branes probing a C^2/Z_2 singularity and compactify it to 4d on a genus $g > 1$ Riemann surface.
- To preserve SUSY must twist: $SO(2)_S \rightarrow SO(2)_S - U(1)_R$, for $U(1)_R \subset SU(2)_R$.
- The $SU(k) \times SU(k) \times U(1)$ global symmetry is thought to enhance to $SO(7)$ for the $N = k = 2$ case [Ohmori, Shimizu, Tachikawa, Yonekura, 2014].
- Can also have non-zero flux in an abelian subgroup of $SO(7)$.

6d perspective: fluxes

- Decompose the flavor symmetry: $SO(7) \to SO(3) \times SO(4) \to SO(3)_t \times SU(2)_\beta \times SU(2)_\gamma$
- Define a flux vector $\mathcal{F} = (\beta, \gamma, t)$.
- The flux breaks $SO(7)$ to a subgroup G^{max} with abelian part L.
- The possible values for G^{max} are:

G^{max}	$u(1)^3$	$su(2)u(1)^2$	$su(2)_{diag}u(1)^2$	$su(2)su(2)u(1)$
L	$u(1)^3$	$u(1)^2$	$u(1)^2$	$u(1)$
$\mathcal{F}$	(a,b,c)	$(a,0,b)/(0,a,b)$	$(a,\pm a,b)$	$(a,0,0)/(0,a,0)$

G^{max}	$so(5)u(1)$	$su(3)u(1)$	$so(7)$
L	$u(1)$	$u(1)$	$\emptyset$
$\mathcal{F}$	$(0,0,a)$	$(a,0,\pm a)/(0,a,\pm a)$	$(0,0,0)$

6d perspective: conformal manifold

- Complex structure moduli.
- Can also turn on flat connections for global symmetries. Conformal manifold:

$$dim\mathcal{M}_{g,0} = \quad 3g-3+(g-1)dim\ G^{max}+dim\ L$$

- G^{max} is the global symmetry preserved by the flux. In 4d should be the maximal global symmetry on the conformal manifold.
- L is the abelian part of G^{max}. In 4d should be the global symmetry on a generic point on the conformal manifold.

6d perspective: anomalies

- Can calculate the 4d anomaly polynomial by integrating the 6d one.
- 6d anomaly polynomial [Ohmori, Shimizu, Tachikawa, Yonekura, 2014]:

$$I_8^{so(7)} = \frac{11C_2^2(R)}{12} - \frac{C_2(R)p_1(T)}{24} + \frac{C_2(so(7))_{\mathbf{8}}p_1(T)}{24} - \frac{C_2(R)C_2(so(7))_{\mathbf{8}}}{2} + \frac{7C_2^2(so(7))_{\mathbf{8}}}{48} - \frac{C_4(so(7))_{\mathbf{8}}}{6} + \frac{29p_1^2(T) - 68p_2(T)}{2880}$$

- Decompose the characteristic classes:
  - Space-time parts broken to parallel t, or orthogonal to the surface.
  - The $SU(2)_R$ bundle is broken to $U(1)_R$ by the twist. Decompose it to its Chern roots n_1, n_2.

6d perspective: anomalies

- Without the twist we take $n_1 = -n_2$. Twist changes this by shifting: $n_2 \to n_2 - t$.
- If flux for $\mathrm{U}(1) \subset SO(7)$ is turned on, must also decompose the $SO(7)$ characteristic classes. Need to perform a-maximization in 4d.
- Integrate the anomaly polynomial over the Riemann surface using:

$$\int t = 2(1-g)$$

- Example for $SO(7)$ compactification:

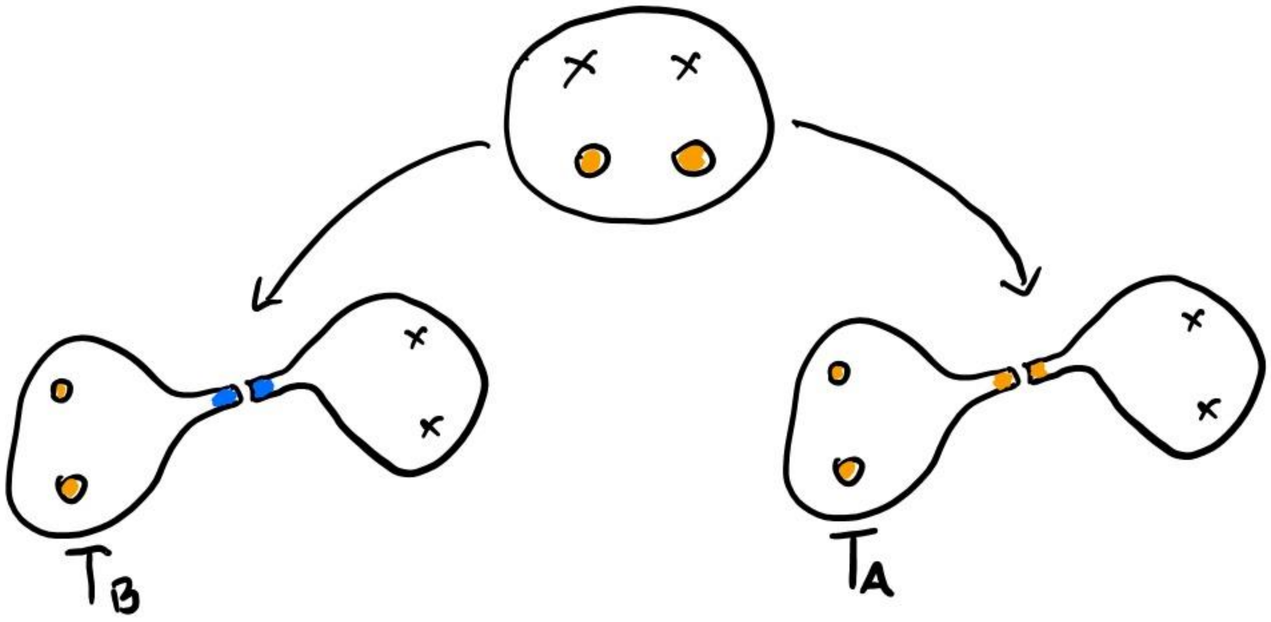
$$Tr(R^3) = 22(g-1), Tr(R) = -2(g-1)$$

$$a = \frac{3}{32}(3TrR^3 - TrR) = \frac{51}{8}(g-1)\,, \qquad c = \frac{1}{32}(9TrR^3 - 5TrR) = \frac{13}{2}(g-1)$$

4d perspective: plan

- To build the analogous theories in class S_2 we need the theories corresponding to a three punctured sphere. These are strongly interacting theories.
- We can learn about these theories using duality relations.
- Consider a duality relating a gauging of these theories to a Lagrangian theory.
- The duality can then be “inverted” leading to information on the strongly coupled theory.

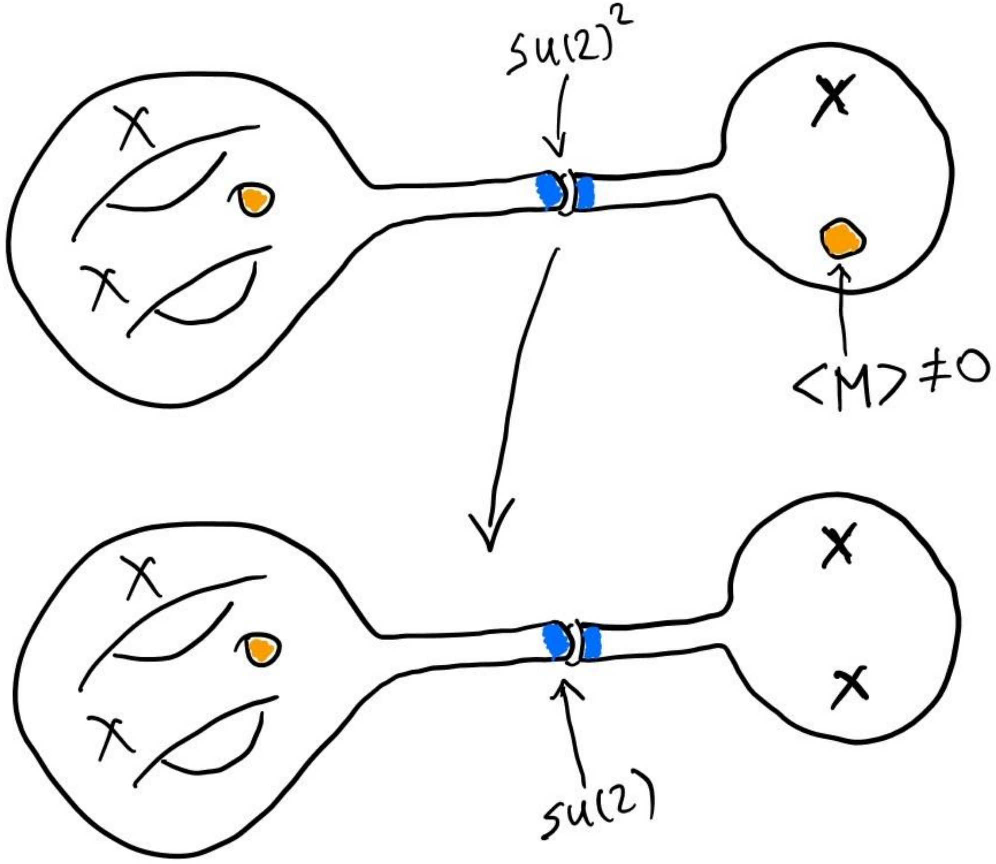
4d perspective: duality



- Connecting two free trinions leads to a Lagrangian SCFT.
- This SCFT has at least two dual descriptions involving different strongly coupled three punctured spheres.

4d perspective: duality

- To understand the second dual we can consider the analogue gauging but with the free trinion.
- Can close the maximal puncture down to a minimal one by giving a vev to a meson.
- The resulting theory is the one corresponding to the connecting the surface with two minimal punctures.



4d perspective: duality

- We find that the Lagrangian theory is dual to an $SU(2)+2F$ gauging of the strongly interacting SCFT with 3 singlets connected via a cubic superpotential.
- The duality implies that the indices of the two theories are related as:

$$I^{Lagr} = \int d\Delta_{Haar SU(2)} I^{\text{Vector + Flavor + Singlets}} I^{\text{Interacting SCFT}}$$

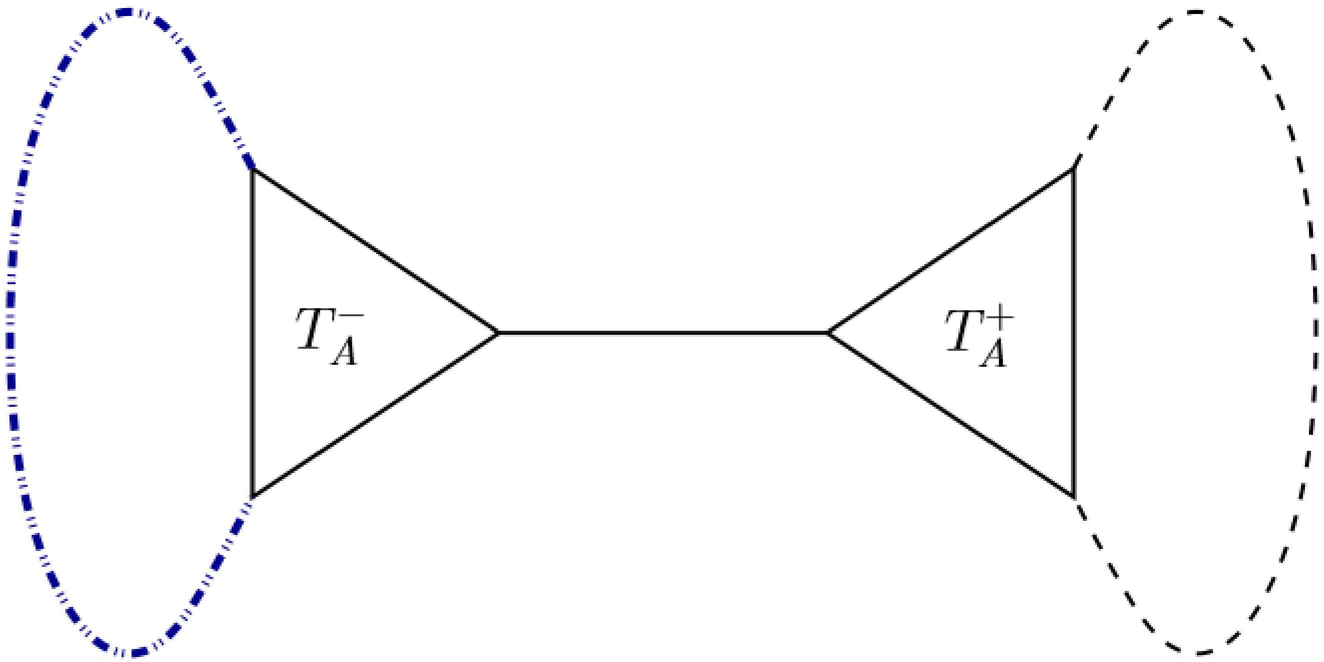
4d perspective: inversion

- There is a mathematical identity that allows to invert the relation: express the index of the strongly interacting theory as an $SU(2)+1F$ gauging with singlets of the Lagrangian theory [Spiridonov, Warnaar, 2006].

$$I^{\text{Interacting SCFT}} = \int d\Delta_{HaarSU(2)} I'^{\text{Vector + Flavor + Singlets}} I^{Lagr}$$

- Take the further step and interpret the gauging as a physical process: duality. This gives a Lagrangian for the strongly interacting theory.
- Unfortunately Lagrangian is strongly coupled and only useful for the calculation of protected quantities: anomalies, index.

4d perspective: $SO(7)$



- Simple to generate a theory with SO(7) global symmetry by connecting a trinion to the conjugate trinion.

4d perspective: $SO(7)$

- No mixing of $U(1)_\beta$, $U(1)_\gamma$ or $U(1)_t$ in $U(1)_R$.
- Can evaluate the index of this theory:

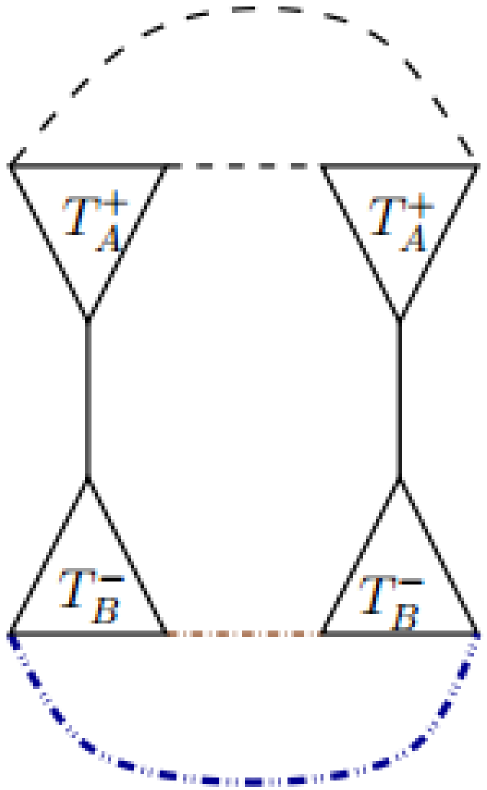
$$\mathcal{I}_{g,0}^{so(7)} = 1 + \left((3g - 3 + (g-1)\mathbf{21}_{so(7)})\big|_{g=2} \right) pq + \cdots .$$

- Can evaluate the conformal anomalies of this theory:

$$a = \frac{51}{8}(g-1)\,, \quad c = \frac{13}{2}(g-1)$$

- Same result also using T_B.

4d perspective



- Can construct and match other theories. Build examples of all the G^{max} cases.
- Example: $U(1) \times SU(2)^2$.
- Global symmetry, anomalies and number of marginal deformations agree with the 6d expectation.
- From matching we find: $\mathcal{F}_{T_A} = (\frac{1}{4}, \frac{1}{4}, 1), \mathcal{F}_{T_B} = (-\frac{1}{4}, \frac{1}{4}, 1), \mathcal{F}_{Free} = (0,0,\frac{1}{2})$.

Conclusions

- Matched global symmetries, dimension of conformal manifold and anomalies for selected $N = 2$ class S_2 theories against those from the 6d construction.
- Provide a non-trivial test on class S_k theories.
- Provide a bridge between field theory objects and properties of the compactification.

Open questions

- General N and k:
  - Concentrate on tori.
  - Understanding the index of class S_k theories.
  - Generalizing the inversion procedure.

Thank you