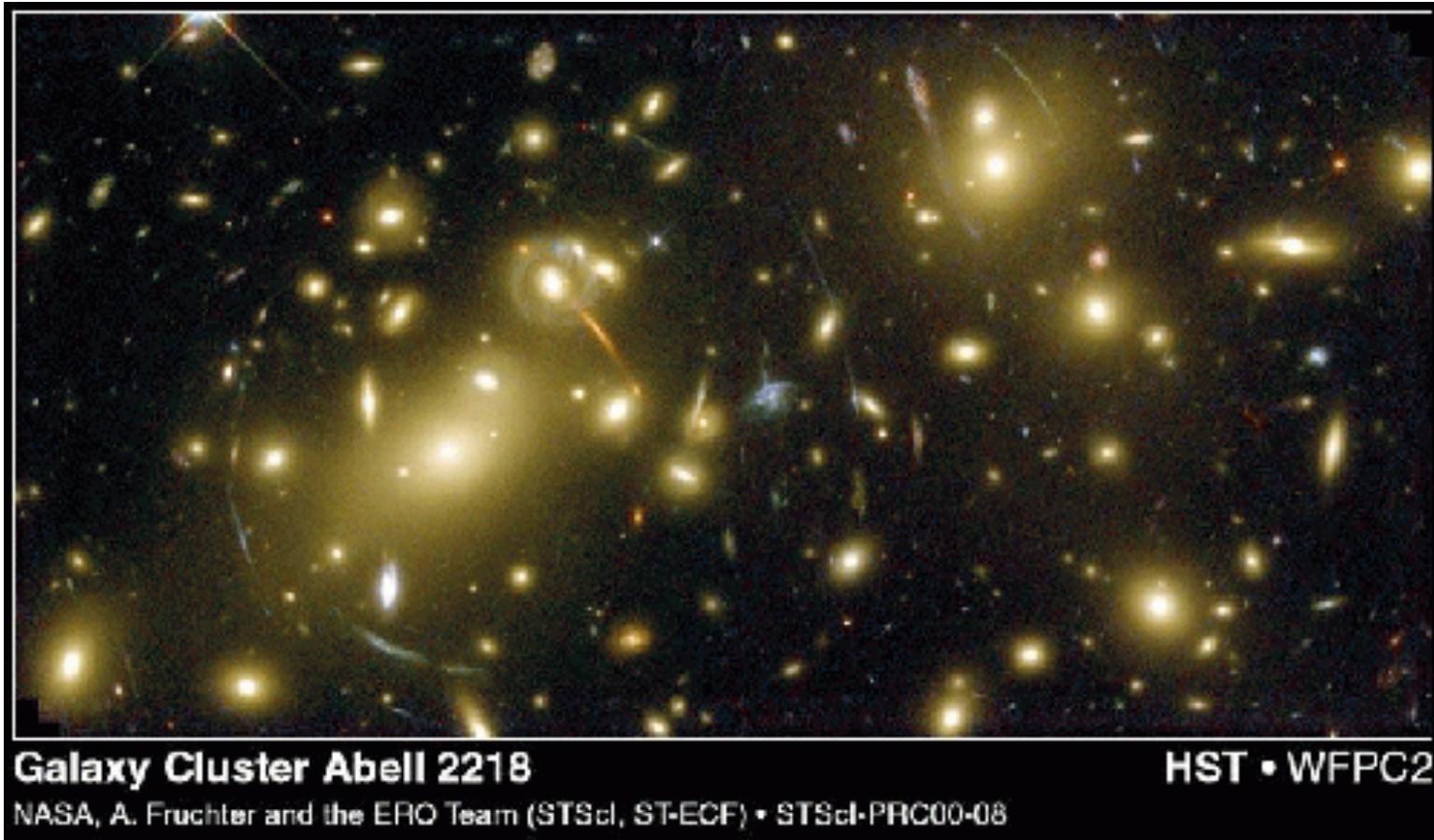


The Physics of Gravitational Redshifts in Clusters of Galaxies

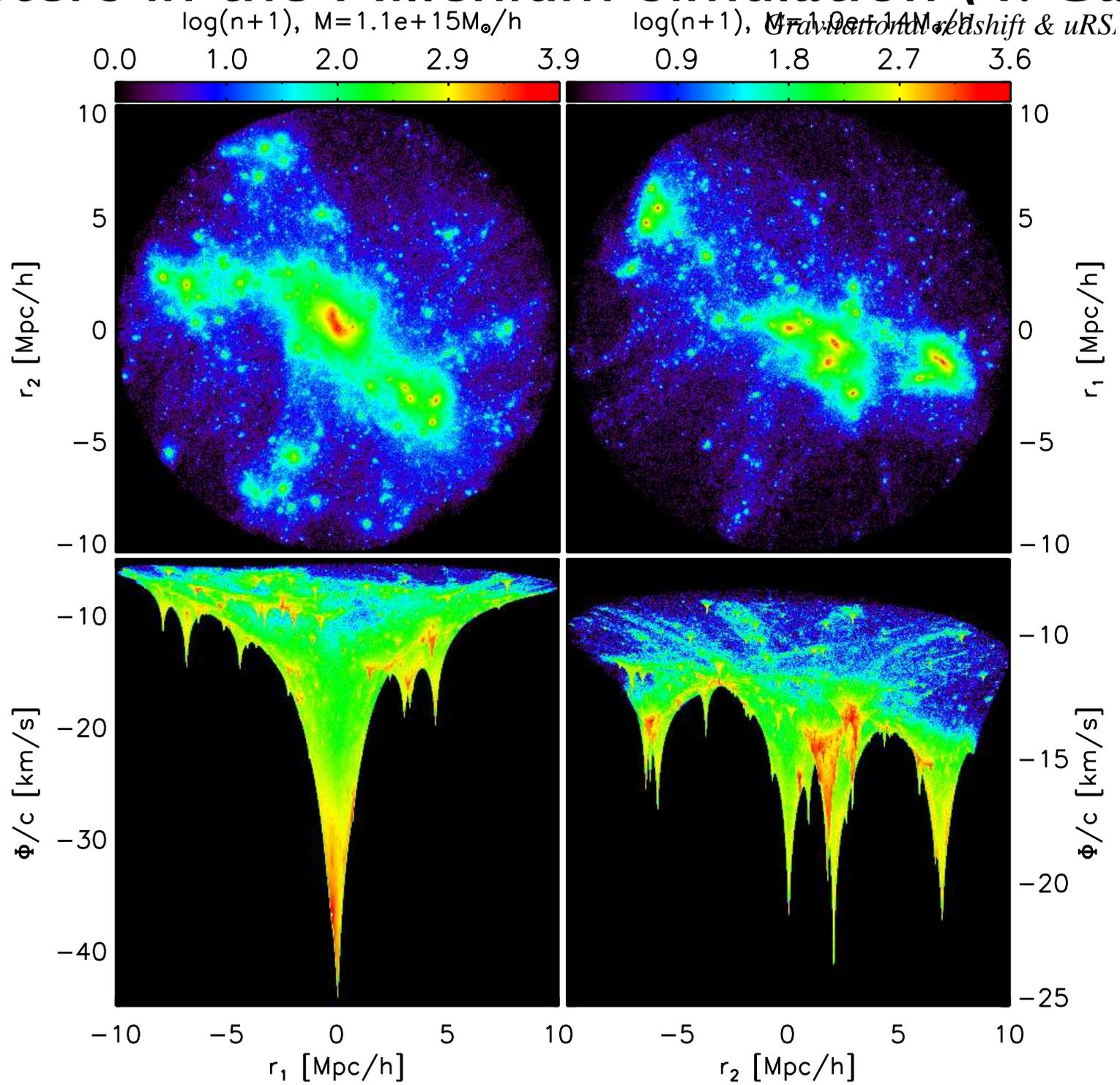
Nick Kaiser
Institute for Astronomy, U. Hawaii

Clusters of galaxies



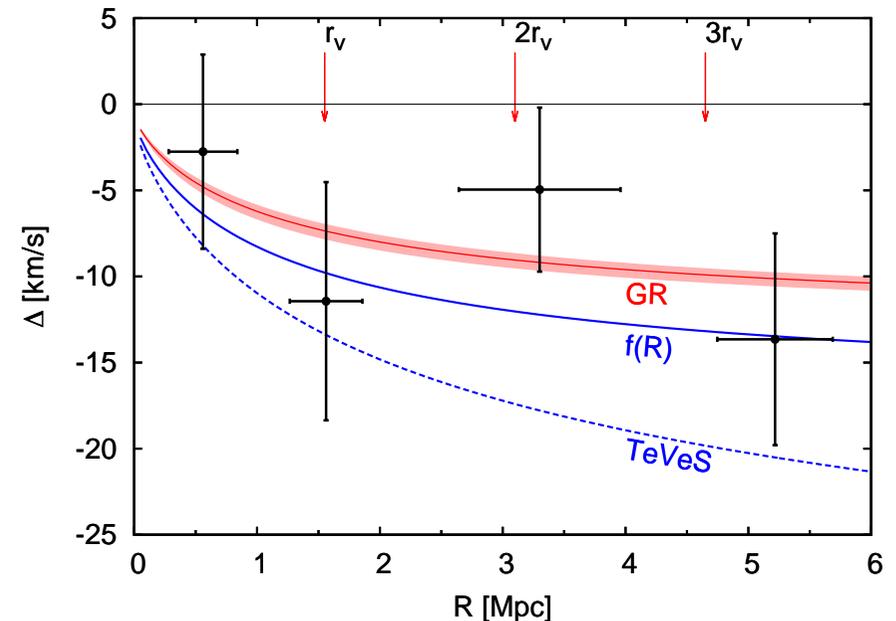
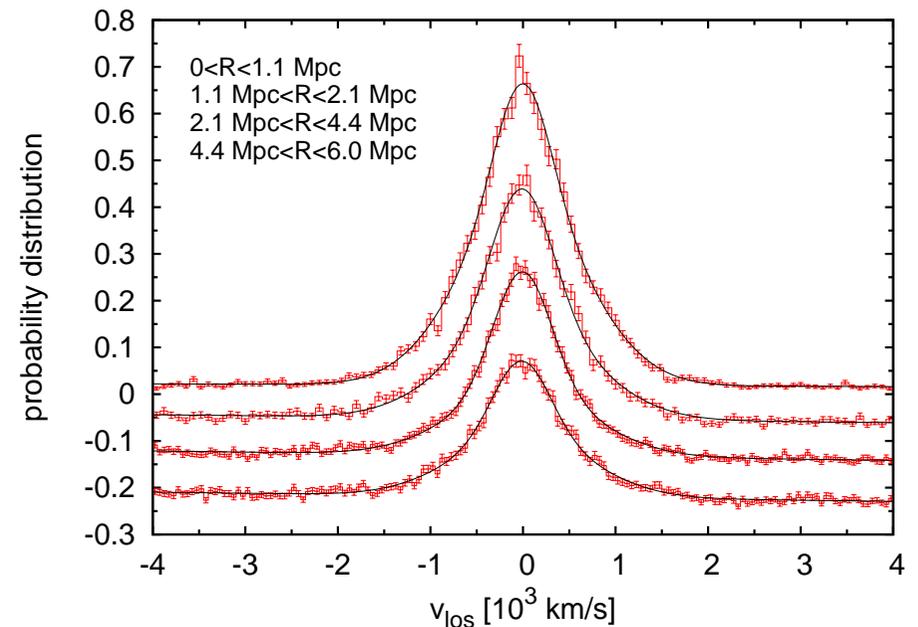
- Largest bound virialised systems $\sim 10^{14}-10^{15} M_{\text{sun}}$
- Velocity dispersion $\sigma_v \sim 1000 \text{ km/s}$ ($\sim 0.003c$)
 - so grav. potential is $\varphi \sim \sigma_v^2 \sim 10^{-5} c^2$
- Centres - often defined by the brightest galaxy (BCG)
 - Usually very close to peak of light, X-rays, DM

Clusters in the Millenium Simulation (Y. Cai)



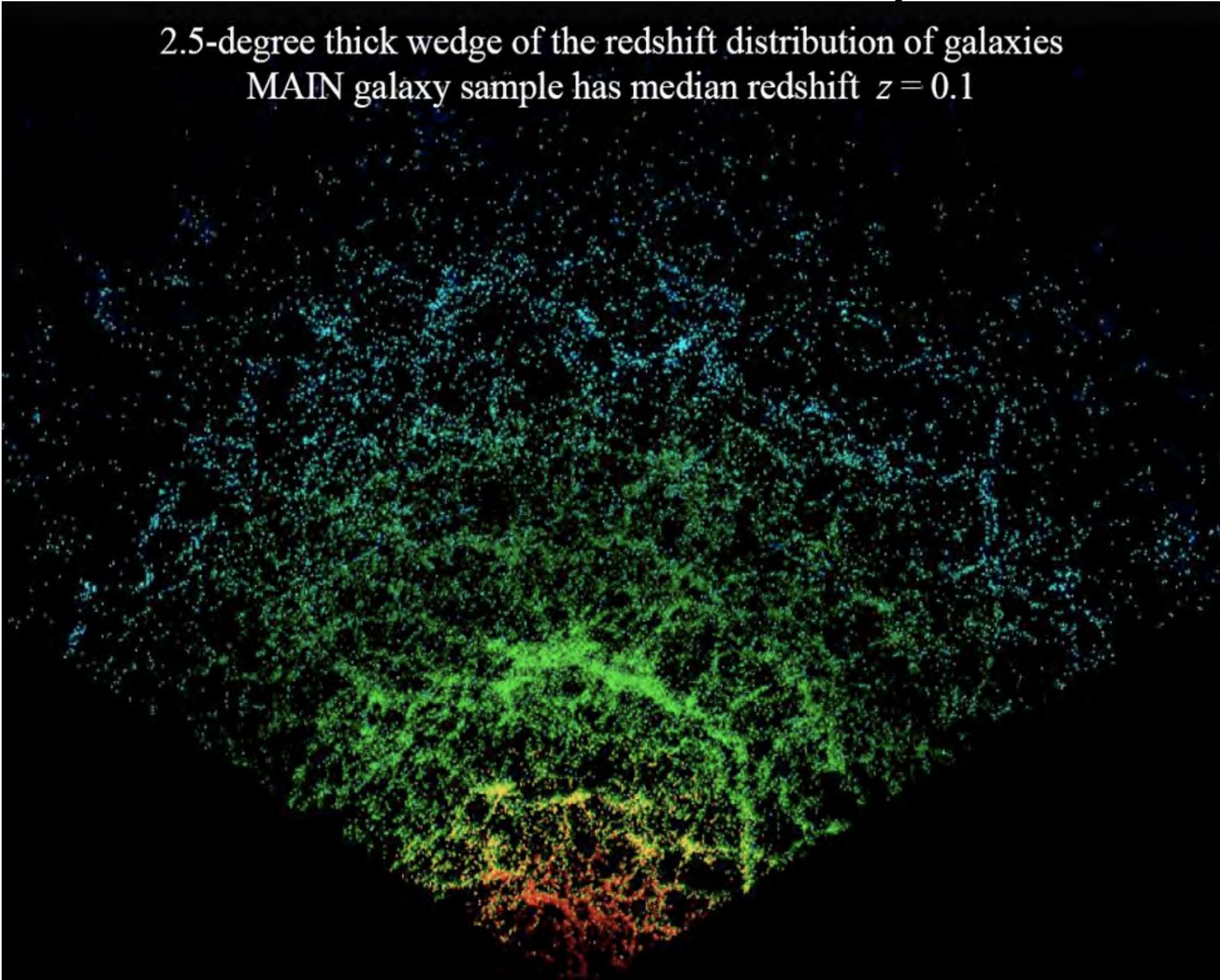
Wojtak, Hansen & Hjorth (Nature 2011)

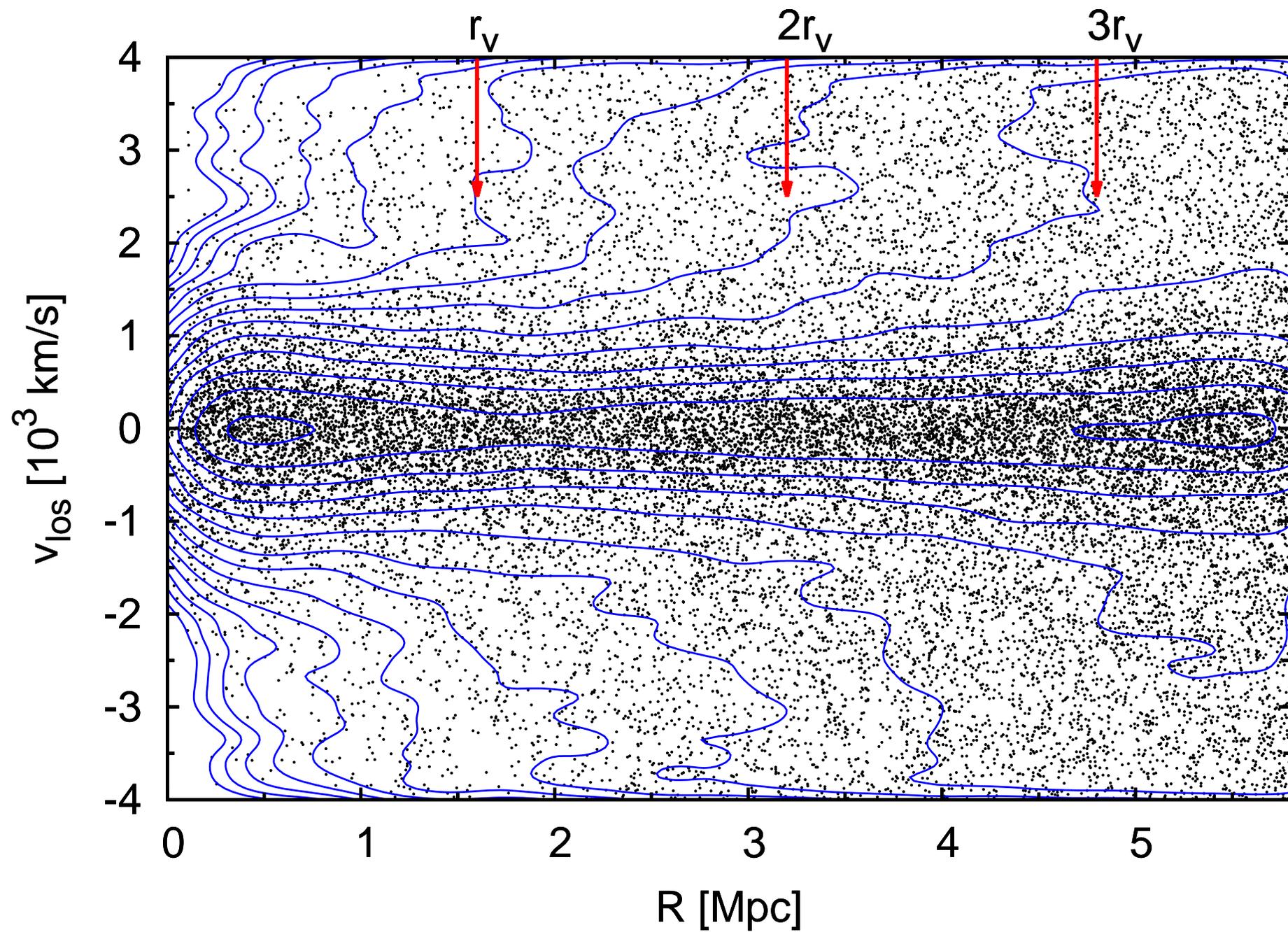
- Wojtek, Hansen & Hjorth stacked 7,800 galaxy clusters from SDSS DR7 in redshift space
- centres defined by the brightest cluster galaxies (BCGs)
- approx 10 redshifts per cluster
- They found a net offset (blue-shift) corresponding to $v = -10$ km/s
- c.f. ~ 600 km/s l.o.s velocity dispersion
- Interpreted as gravitational redshift effect
- right order of magnitude, sign
- “Confirms GR, rules out TeVeS”
- Had been suggested before (Cappi 1995; Broadhurst+Scannapiaco,)



SDSS Redshift Survey

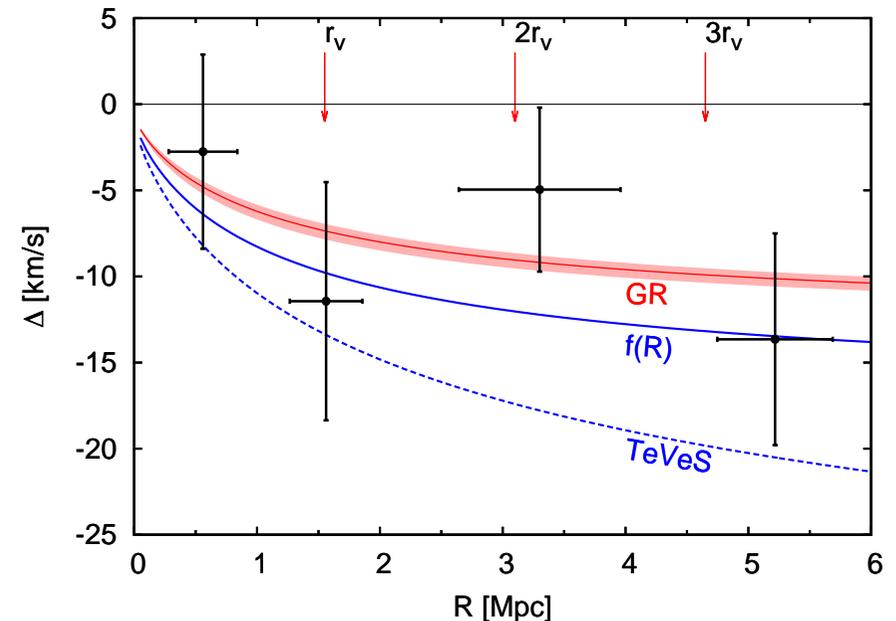
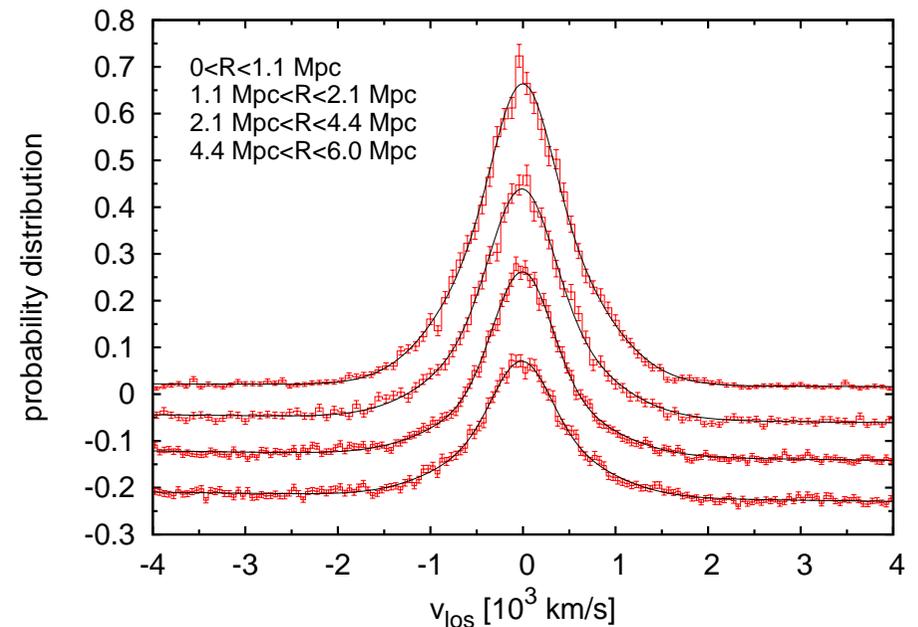
2.5-degree thick wedge of the redshift distribution of galaxies
MAIN galaxy sample has median redshift $z = 0.1$





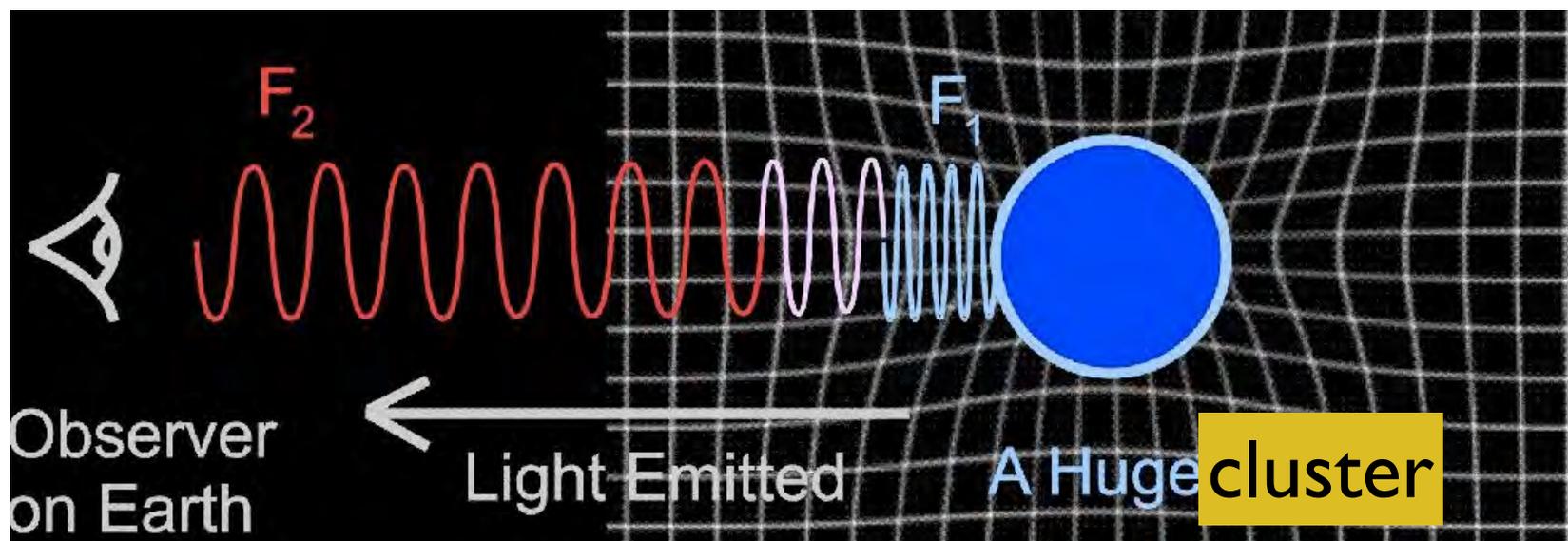
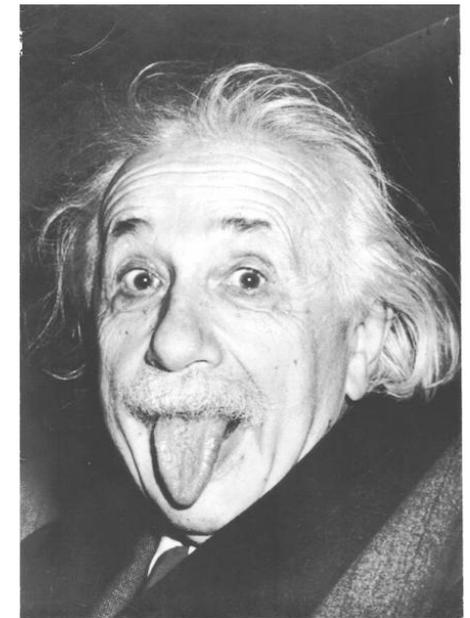
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- Interpreted as gravitational redshift effect
- right order of magnitude, sign
- “Confirms GR, rules out TeVeS”
- Had been suggested before (Cappi 1995; Broadhurst+Scannapiaco,)



The physics of cluster gravitational redshifts

- Einstein gravity
 - gravitational "time dilation"
- Weak field limit
 - $\delta v/v = -\Phi/c^2$
- Measured by Pound & Rebka (Harvard '59)



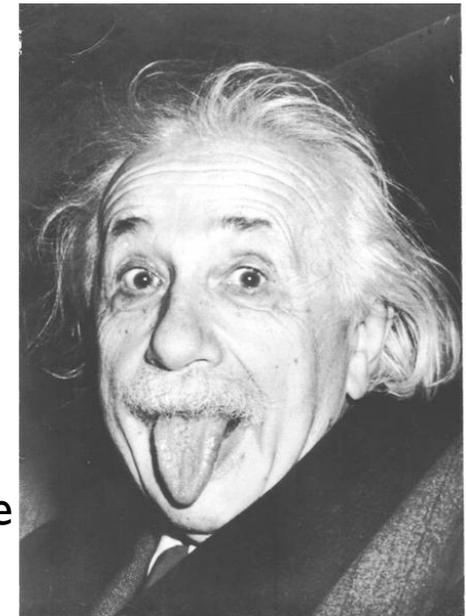
Is that it?

Equivalence principle & the Pound + Rebka experiment



$$\left(\frac{\Delta E}{E}\right)_{down} - \left(\frac{\Delta E}{E}\right)_{up} = (5.1 \pm 0.5) \times 10^{-15}$$

- Einstein's Equivalence Principle: Observers on earth (being accelerated by the stress in the ground under them imparting momentum to them) will see light being red-shifted (and all other local physics being modified) exactly as would a pair of astronauts in empty space being accelerated by a rocket motor.
- Pound and Rebka (1959, 1960): He was right.
- But if you replace non-inertial apparatus by freely falling kit with same instantaneous velocities then B&H say Doppler formula will apply. They are right too - almost exactly...



GRAVITATIONAL RED-SHIFT IN NUCLEAR RESONANCE

R. V. Pound and G. A. Rebka, Jr.

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received October 15, 1959)

It is widely considered desirable to check experimentally the view that the frequencies of electromagnetic spectral lines are sensitive to the gravitational potential at the position of the emitting system. The several theories of relativity predict the frequency to be proportional to the gravitational potential. Experiments are proposed to observe the timekeeping of a "clock" based on an atomic or molecular transition, when held aloft in a rocket-launched satellite, relative to a similar one kept on the ground. The frequency ν_h and thus the timekeeping at height h is related to that at the earth's surface ν_0 according to

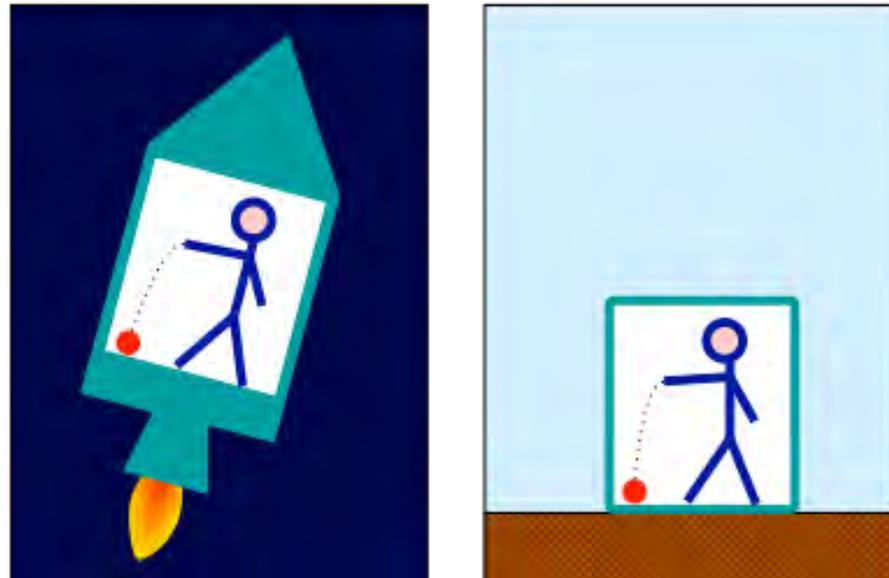
$$\begin{aligned}\Delta\nu_h = \nu_h - \nu_0 &= \nu_0 gh/c^2(1 + h/R) \\ &\approx \nu_0 h \times (1.09 \times 10^{-18}),\end{aligned}$$

where R is the radius of the earth and h is the altitude measured in cm. Very high accuracy is required of the clocks even with the altitudes available with artificial satellites. Although several ways of obtaining the necessary frequency stability look promising, it would be simpler if a way could be found to do the experiment between fixed terrestrial points. In particular, if an accuracy could be obtained allowing the measurement of the shift between points differing as little as one to ten kilometers in altitude, the experiment could be performed between a mountain and a valley, in a mineshaft, or in a borehole.

Recently Mössbauer has discovered¹ a new aspect of the emission and scattering of γ rays by nuclei in solids. A certain fraction f of γ rays of the nuclei of a solid are emitted without

Is there any more to the physics of cluster grav-z?

- Is the Pound & Rebka (P&R) experiment relevant here?
 - equipment fixed to the tower in Harvard phys. dept
- Einstein rocket thought experiment:
 - observers on surface of earth (e.g. P&R) *being accelerated by the earth* see the same physics as accelerated observers in a rocket
 - but *there is no gravity in the rocket*
- P&R measured effect of *non-gravitational* acceleration



Non-GRAVITATIONAL RED-SHIFT IN NUCLEAR RESONANCE

R. V. Pound and G. A. Rebka, Jr.

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

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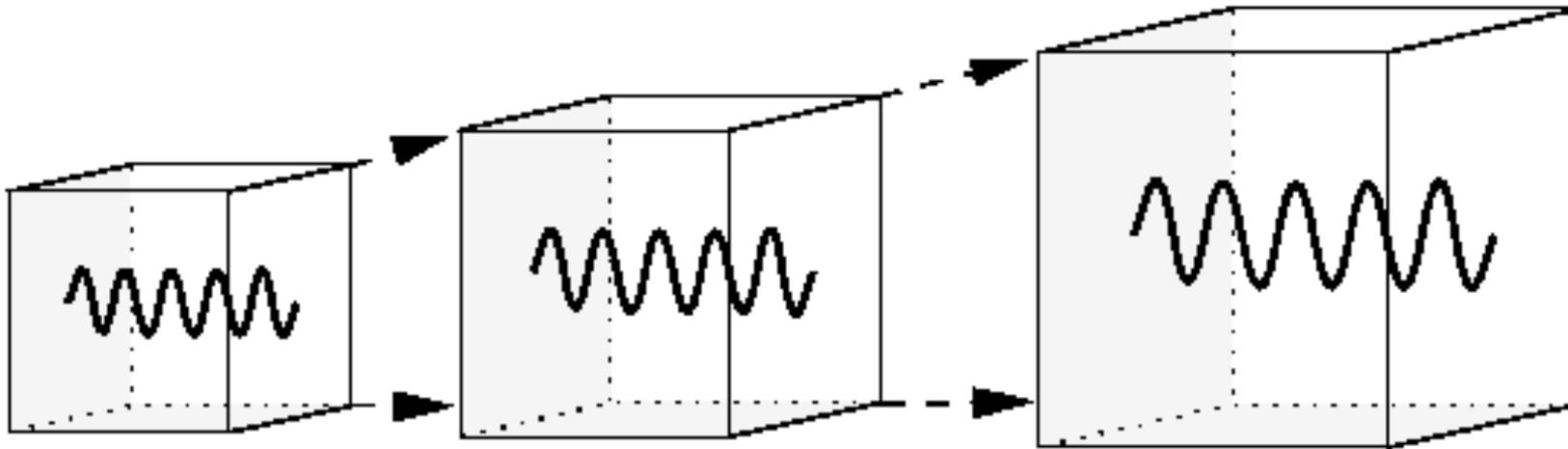
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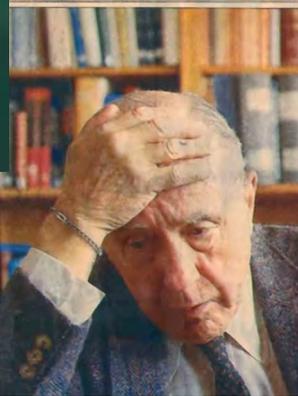
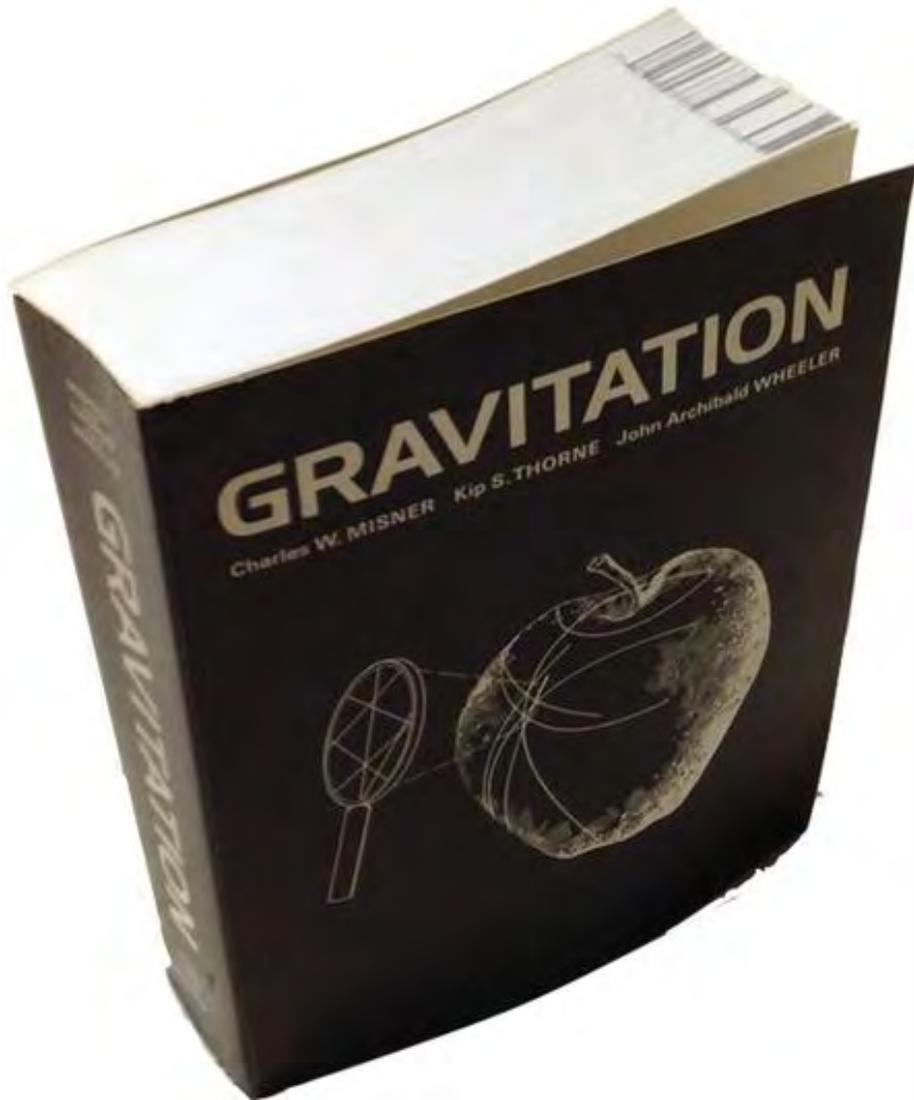
How do we understand the grav-z in clusters?

- Cluster gravitational redshift is difference between redshift for centre galaxy and general cluster population
- Equivalently, what is redshift of centre as seen by others?
- But these are objects in free-fall
 - P&R analogy is questionable at best
 - GR: gravity is "transformed away" for freely-falling observer
- How should we understand redshifts in astronomy?
 - Digression:
 - redshifts in cosmology
 - redshifts in general

Redshift in homogeneous FLRW cosmology...

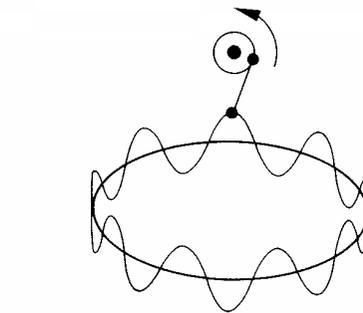
- Wavelength scales as $a(t)$ - but why?
 - Analogy with expanding reflecting cavity
 - a) lots of little redshifts as photons bounce off walls
 - b) symmetry - standing waves - fixed # of nodes
 - either way: accumulated effect: $\lambda \sim a(t)$



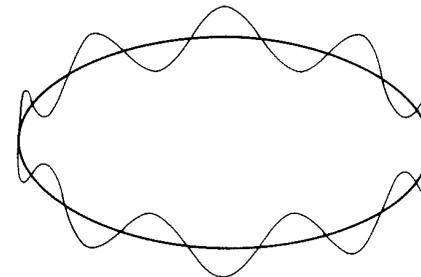


776

29. PRESENT STATE AND FUTURE EVOLUTION OF THE UNIVERSE



Emission:
atom excites n -node standing wave;
universe small, $a(t_e) = a_{em}$;
wavelengths small, $\lambda(t_e) = \lambda_{em}$.



Reception:
universe larger, $a(t_r) = a_{rec}$;
wavelengths larger, $\lambda(t_r) = \lambda_{rec}$;
number of nodes in standing
wave unchanged:

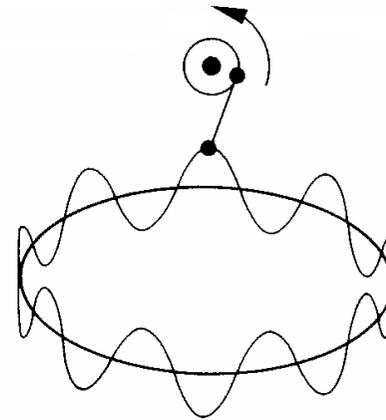
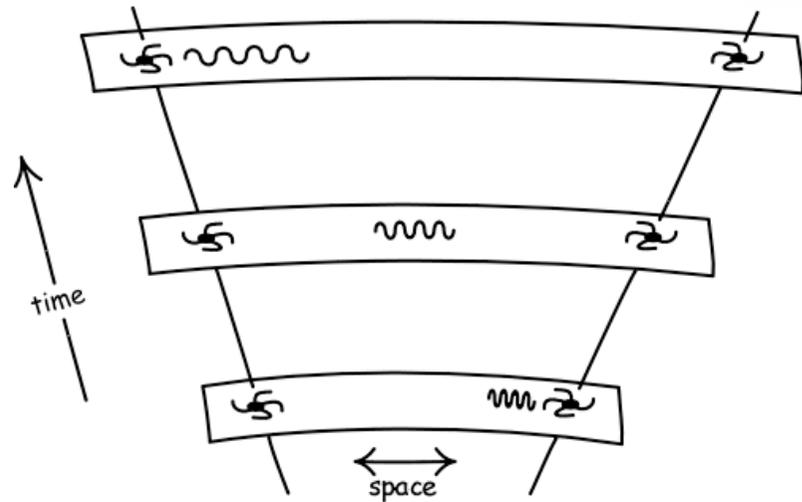
$$n = \text{constant} = \frac{2\pi a_{rec}}{2\pi \lambda_{rec}} = \frac{a_{em}}{\lambda_{em}}$$

Figure 29.1.

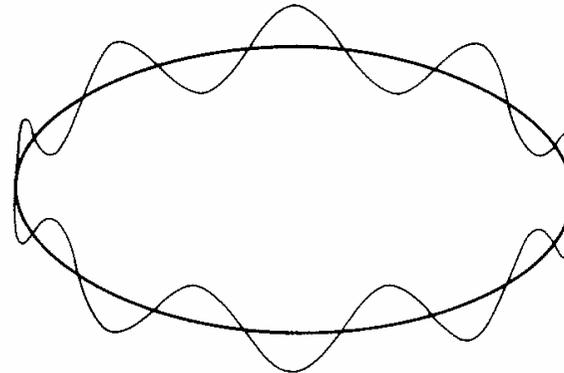
Redshift as an effect of standing waves. The ratio of wavelengths, $\lambda_{rec}/\lambda_{em}$, is identical with the ratio of dimensions, a_{rec}/a_{em} in any closed spherically symmetrical (Friedmann) model universe. The atom excites an n -node standing wave in the universe. The number n stays constant during the expansion.

Misner, Thorne and Wheeler

redshift as an effect on standing waves....



Emission:
 atom excites n -node standing wave;
 universe small, $a(t_e) = a_{em}$;
 wavelengths small, $\lambda(t_e) = \lambda_{em}$.



Reception:
 universe larger, $a(t_r) = a_{rec}$;
 wavelengths larger, $\lambda(t_r) = \lambda_{rec}$;
 number of nodes in standing wave unchanged:

$$n = \text{constant} = \frac{2\pi a_{rec}}{2\pi \lambda_{rec}} = \frac{a_{em}}{\lambda_{em}}$$

Figure 29.1.

Redshift as an effect of standing waves. The ratio of wavelengths, $\lambda_{rec}/\lambda_{em}$, is identical with the ratio of dimensions, a_{rec}/a_{em} in any closed spherically symmetrical (Friedmann) model universe. The atom excites an n -node standing wave in the universe. The number n stays constant during the expansion. Therefore wavelengths increase in the same proportion as the dimensions of the universe. One sees immediately in this way that the redshift is independent of all such details as (1) why the expansion came about (spherical symmetry, but arbitrary equation of state); (2) the rate—uniform or nonuniform—at which it came about; and (3) the distance between source and receptor at emission, at reception, or at any time in-between. The reasoning in the diagram appears to depend on the closure of the universe (standing waves; $k = +1$ rather than 0 or -1). That closure is not required for this simple result is seen from the further analysis given in the text.

But is this a standing wave?

Expanding space and redshifts in textbooks.....

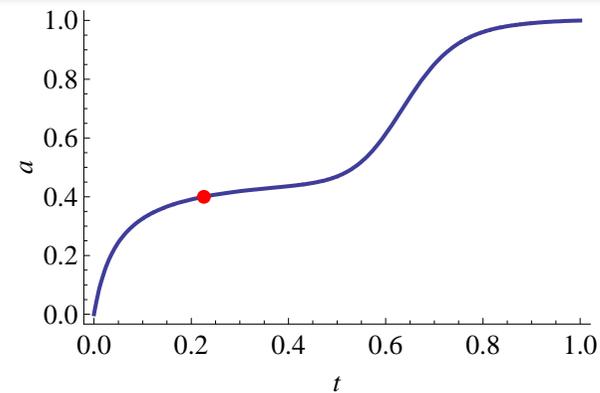
- E.R. Harrison (2000)

We suppose that all galaxies are comoving and that their light is received by observers who are also comoving. Light leaves a galaxy, which is stationary in its own local region of space, and is received by observers who are also stationary in their own local region of space. Between the galaxy and the observers light travels through vast regions of expanding space. What happens is immediately obvious: All wavelengths of the light are stretched by the expansion of space (see Figure 11.1). It is as simple as that.

- Wolfgang Rindler (1970)

Note that the cosmological red shift is really an *expansion* effect rather than a *velocity* effect.

Expansion redshifts are produced by the expansion of space between bodies that are stationary in space: They depend on the increase of distance between the emitter and the receiver during the time of propagation; they are the result of recession velocities and not peculiar velocities; and they are not governed by the rules of special relativity.



MISCONCEPTIONS ABOUT THE BIG BANG

Baffled by the expansion of the universe? You're not alone. Even astronomers frequently get it wrong

By Charles H. Lineweaver and
Tamara M. Davis



Stretching and Cooling

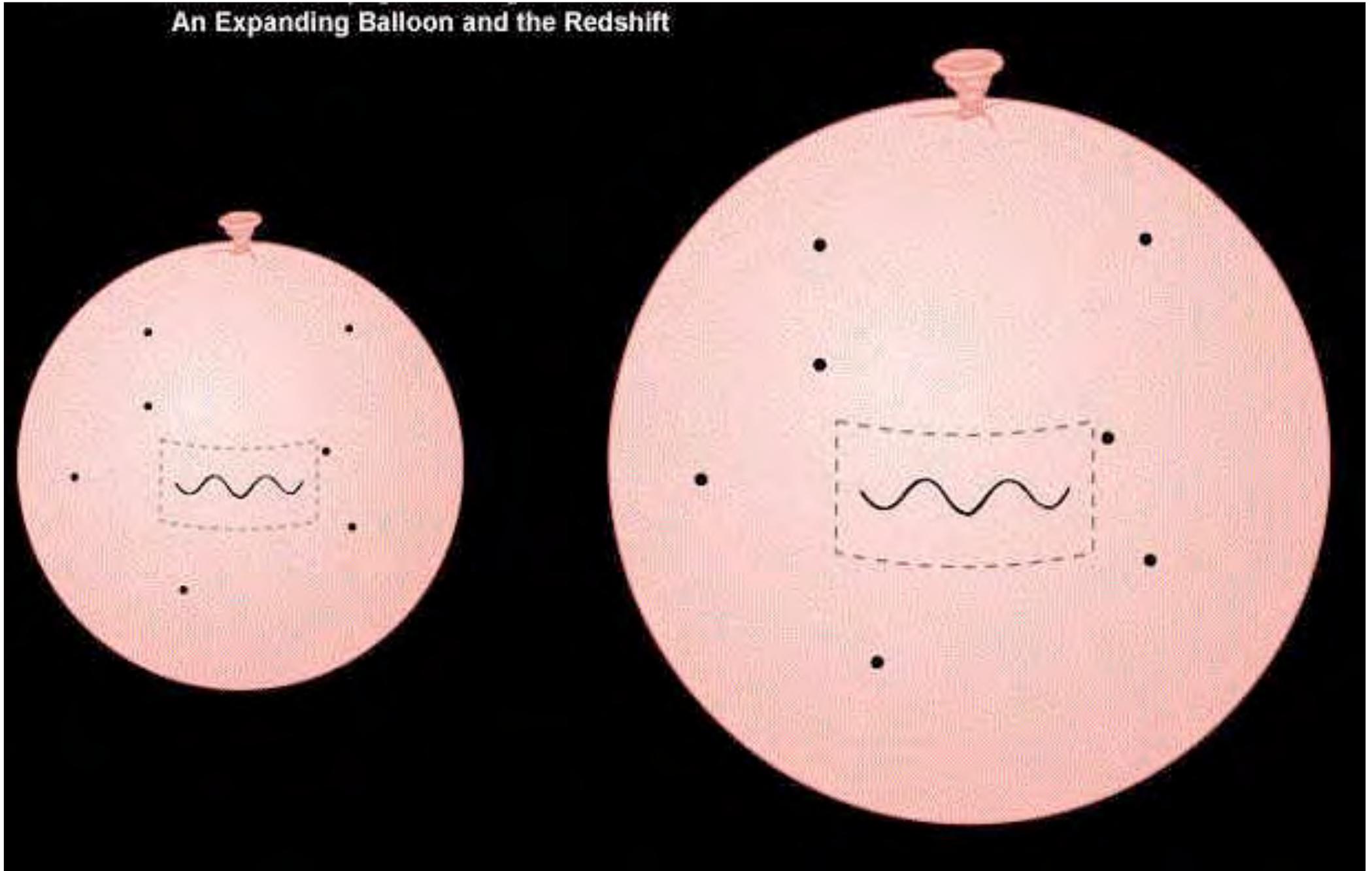
THE PRIMARY OBSERVATION that the universe is expanding emerged between 1910 and 1930. Atoms emit and absorb light of specific wavelengths, as measured in laboratory experiments. The same patterns show up in the light from distant galaxies, except that the patterns have been shifted to longer wavelengths. Astronomers say that the galactic light has been redshifted. The explanation is straightforward: As space expands, light waves get stretched. If the universe doubles in size during the waves' journey, their wavelengths double and their energy is halved.

This process can be described in terms of temperature. The photons emitted by a body collectively have a temperature—a certain distribution of energy that reflects how hot the body is. As the photons travel through expanding space, they lose energy and their temperature decreases. In this way, the universe cools as it expands, much as compressed air in a scuba tank cools when it is released and allowed to expand. For example, the microwave background radiation currently has a temperature of about three kelvins, whereas the process that released the radiation occurred at a temperature of about 3,000 kelvins. Since the time of the emission of this radiation, the universe has increased in size by a factor of 1,000, so the temperature of the photons has decreased by the same factor. By observing the gas in distant galaxies, astronomers have directly measured the temperature of the radiation in the distant past. These measurements confirm that the universe has been cooling with time.

Misunderstandings about the relation between redshift and velocity abound. The redshift caused by the expansion is often confused with the more familiar redshift generated by the Doppler effect. The normal Doppler effect causes sound

The rubber balloon analogy

An Expanding Balloon and the Redshift



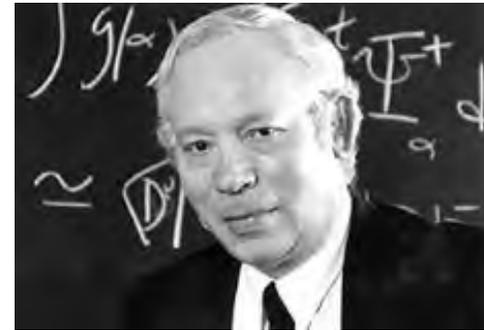
redshift caused by expansion of space?

- Textbooks are correct
 - λ does increase with $a(t)$
- But is it reasonable to say expansion causes the shift?
- And is it obvious?
 - what is the mechanism by which space stretches light?
 - is space expanding in this room?
 - is space expanding in a cluster of galaxies?

Expanding Space: the Root of all Evil?*

Matthew J. Francis^{1,4}, Luke A. Barnes^{1,2}, J. Berian James^{1,3} & Geraint F. Lewis¹

However, the academic argument surrounding the expansion of space is not as clear as standard explanations suggest; an interested student and reader of New Scientist may have seen Martin Rees & Steven Weinberg (1993) state



...how is it possible for space, which is utterly empty, to expand? How can nothing expand? The answer is: space does not expand. Cosmologists sometimes talk about expanding space, but they should know better.

But see also Weinberg, 1st 3 Minutes, p31: "One can think of the wave crests being pulled farther and farther apart by the expansion of the universe."

while being told by Harrison (2000) that

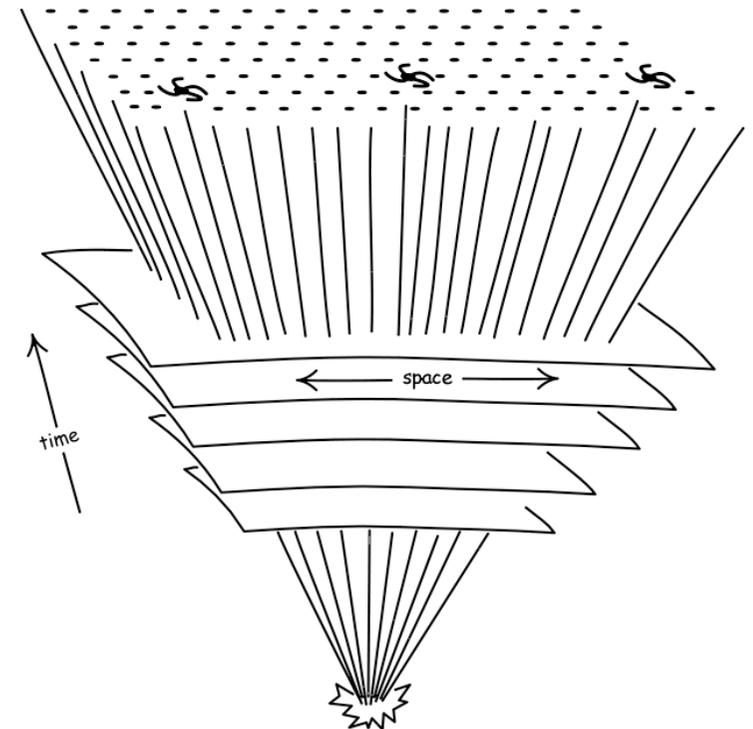
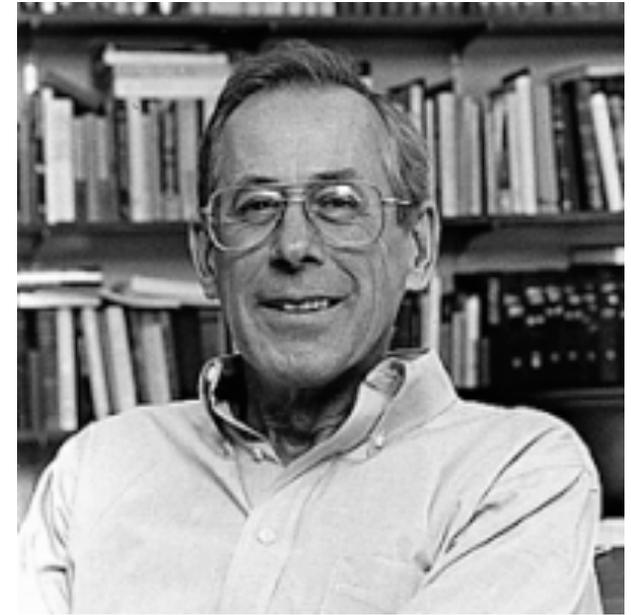
expansion redshifts are produced by the expansion of space between bodies that are stationary in space

What is a lay-person or proto-cosmologist to make of this apparently contradictory situation?



Peebles ('71) explanation of cosmological redshift

- The redshift $\lambda_{\text{rec}}/\lambda_{\text{em}}$ is the product of a lot of small shifts between a set of FOs along the look-back path
- In the vicinity of a neighbouring pair of FOs
 - space-time is locally flat, so
 - incremental redshifts are Doppler shifts
- Yields differential equation
 - $d\lambda/\lambda = da/a$ with solution $\lambda \propto a(t)$
- So fractional change in proper separation is the same as the fractional change in λ
- i.e. $\delta \log(\lambda/D) = 0$

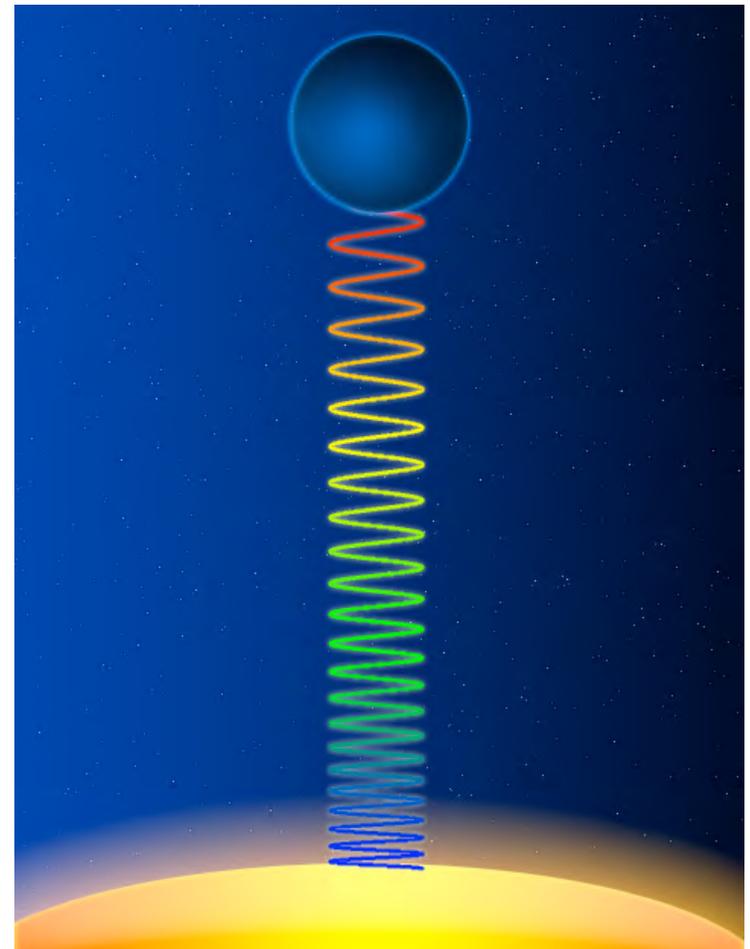
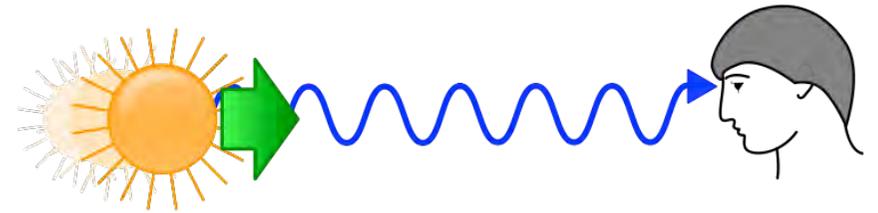
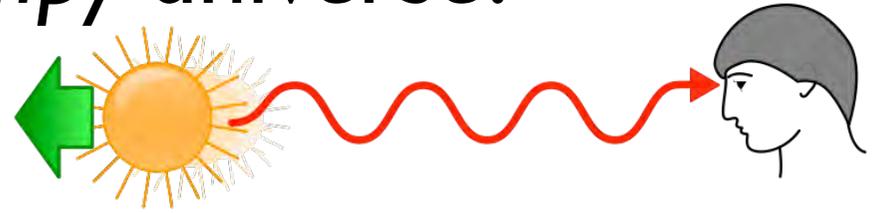


Redshift and expansion in cosmology

- So in cosmology wavelength λ is tied to expansion $a(t)$
 - may or may not be caused by it
- If observer/source are moving apart then λ increases
- Exactly as for Doppler shift in empty space
- So any gravitational component to redshift is somehow hidden
- Mathematically: $\Delta \ln(\lambda/D) = 0$
- Is this a general principle?

What about our *lumpy* universe?

- Bondi (1947): Spherical models:
 - for low- Z , redshift is product of Doppler and gravitational redshift
- But Synge (1960) argued that *all* redshifts are Doppler shifts
 - “In attributing a *cause* to this spectral shift, one would say that the spectral shift was caused by the relative velocity of the source and the observer”.



Synge, 1960; General Relativity

- Observed (or emitted) energy is dot product of observer 4-velocity and the photon 4-momentum.
- → wavelength shift is given by Doppler's formula with "relative velocity" being the l.o.s. component of the difference of the receiver 4-velocity and a parallel transported version of the emitter 4-velocity
- "Not a gravitational redshift as the Riemann tensor does not appear in formula"

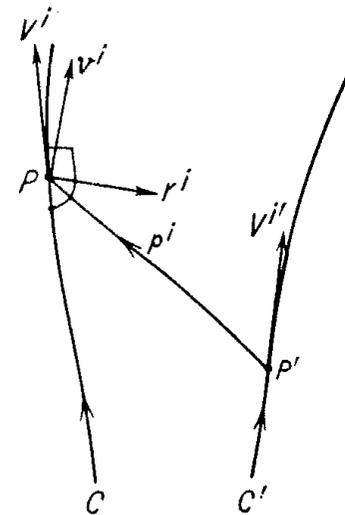


Fig. 10—Relative velocity and Doppler effect (mechanical)

some luminous object, such as a star or planet. We connect them with null geodesics such as $P'P$. We cannot immediately compare the 4-velocity V^t of C at P with the 4-velocity V'^t of C' at P' , because they are vectors at different events. The obvious plan is to bring them to a common event by subjecting V'^t to parallel transport along $P'P$; this gives us at P the vector

$$v_i = g_{ij} V'^j, \quad (32)$$

where g_{ij} is the parallel propagator [II-(71)]. Let $\lambda^i_{(\alpha)}$ be a frame of reference on C with $\lambda^i_{(4)} = V^t$. This might be a Fermi frame, but the question does not arise at the moment, because we are

We now define the 3-velocity components

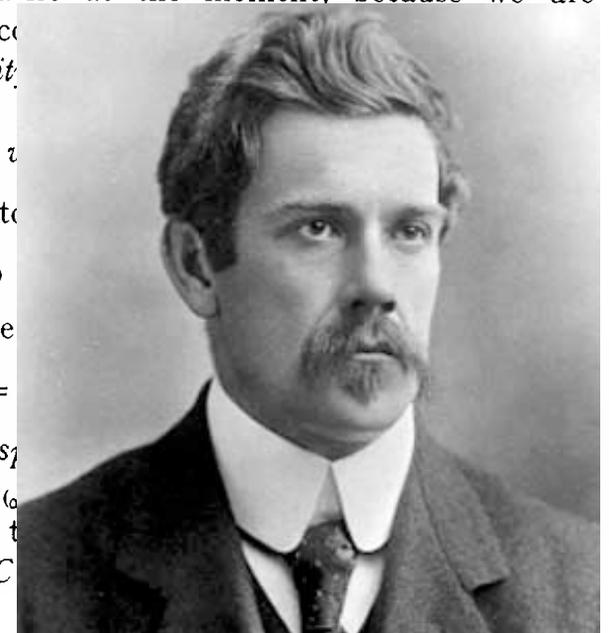
Since v^t and V^t are unit vectors

$$v_{(4)}$$

is expressible in terms of the

$$v^{(4)} = -v_{(4)} =$$

We may call v the relative speed all the three components $v_{(\alpha)}$ parallel for transport along $P'P$ that C' is at rest relative to C



Bunn & Hogg, 2009

- Like Peebles they break photon path into a set of intervals
 - set of intervening observers along line of sight
 - local flatness → product of Doppler shifts
- But intervening observers need not be freely falling
- Claim: Any incremental shift can be *considered* to be either *Doppler* or *gravitational*
- "gravitational redshifts are just Doppler shifts viewed from an unnatural coordinate system"
- "an enlightened cosmologist would never try to draw any distinction"
- All redshifts can (and should!) be considered to be Doppler, or 'kinematic' in nature. (much like Synge)
- Again suggests $\Delta \ln(\lambda/D) = 0$ is *universal*?



ideas about redshifts in astronomy - summary

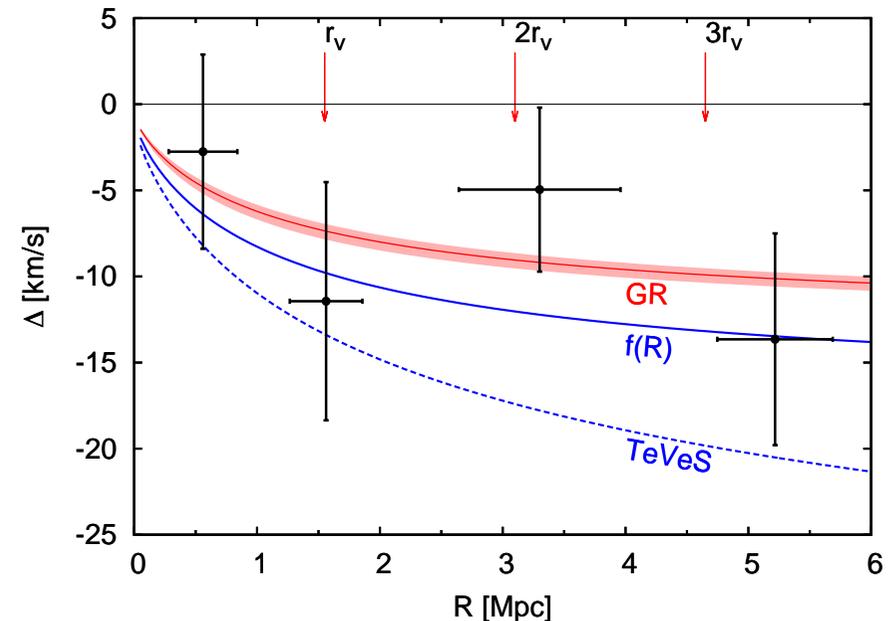
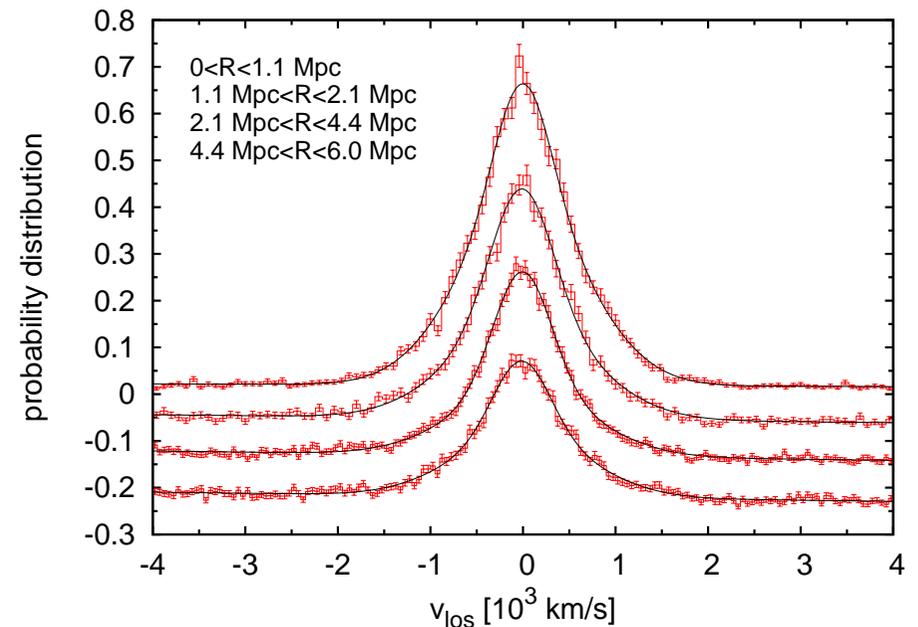
- The redshift of light in cosmology
 - redshift is caused by the expansion of space?
 - standing waves in a cavity
 - Maxwell's equations in expanding space:
 - "Hubble damping" + the adiabatic invariant
 - Thermodynamics & photons as particles
 - Peebles' picture - lots of little Doppler shifts
- The redshift of light in general
 - Synge ('60): redshifts "caused by the relative velocity..."
 - Bunn & Hogg ('09): "gravitational redshifts are just Doppler shifts viewed from an unnatural coordinate system"
 - 1st order "relativistic" redshift space distortion (Yoo+09)
 - $\Delta_z = \Delta_r + \dots$ is also purely a "Doppler" effect

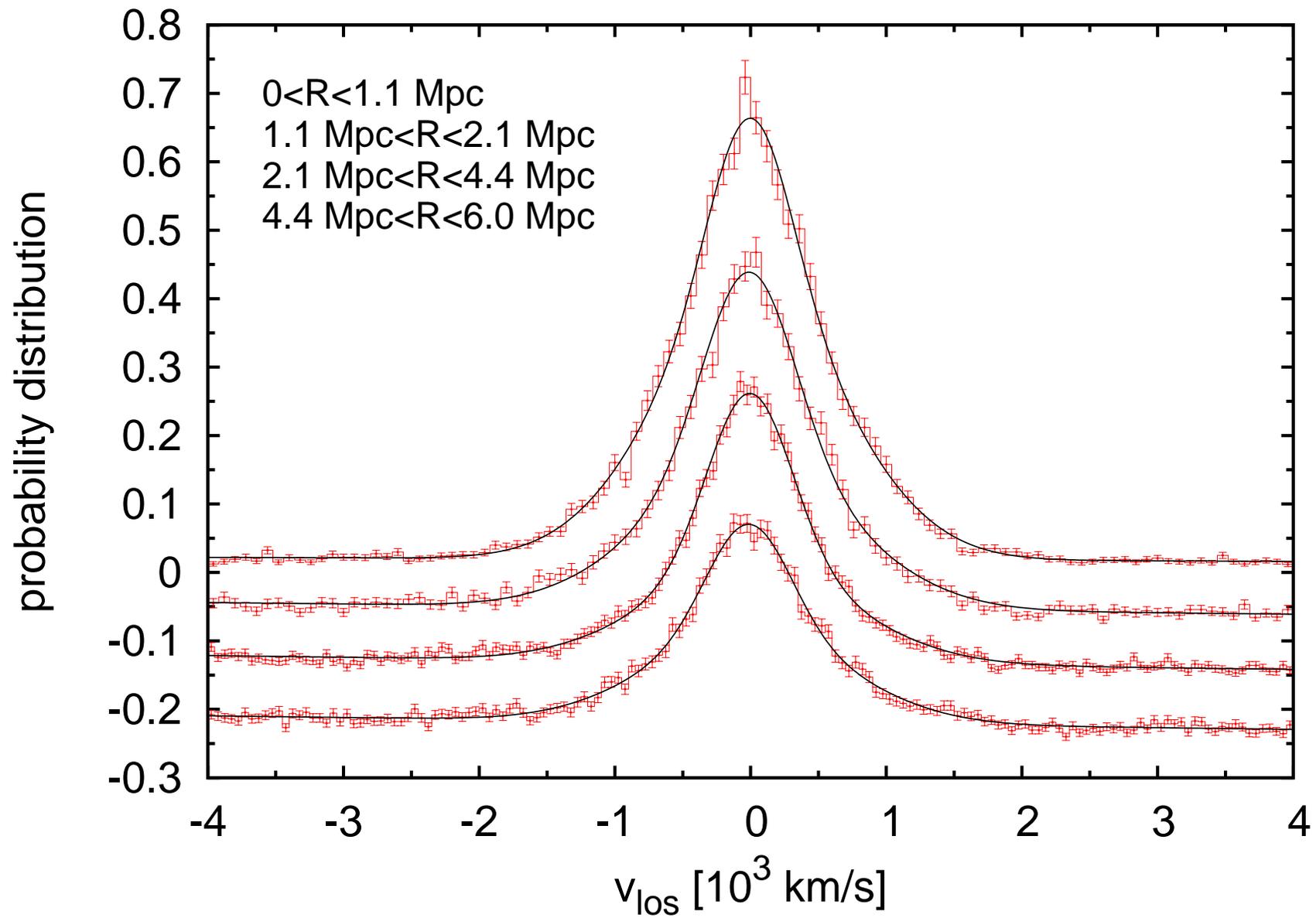
"what causes the redshift?" and why do we care?

- All the foregoing support the "kinematic picture" for astronomical redshifts.
 - redshifts come entirely from motions
 - in nice accord with Equivalence Principle
- But clusters are not expanding!
 - and observers, sources are freely falling
 - *so why would we see any gravitational redshift?*
- At the very least one might have doubts about the Einstein/Newton/Pound+Rebka picture
- What additional physics might there be?

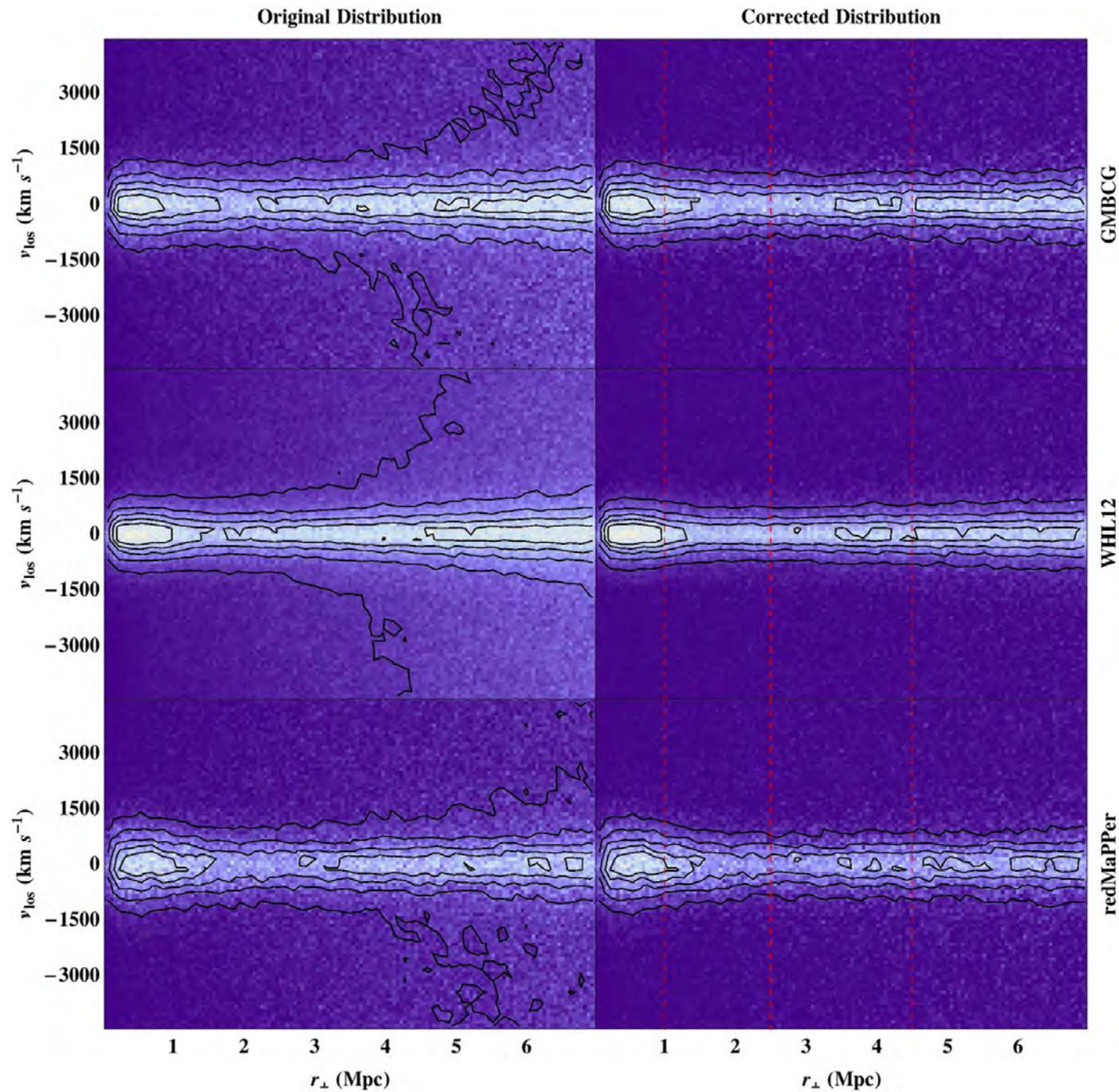
Back to the Wojtak *et al.* measurement

- Gravitational redshift for light climbing out of potential wells of clusters of galaxies
- Long predicted by theorists
- perhaps a bit oversimplified
- Now finally measured
- at ~ 2.5 sigma level
- Claimed to conflict with TeVeS modified gravity
- descendent of Milgrom theory
- But OK with GR or e.g. $f(R)$ modifications

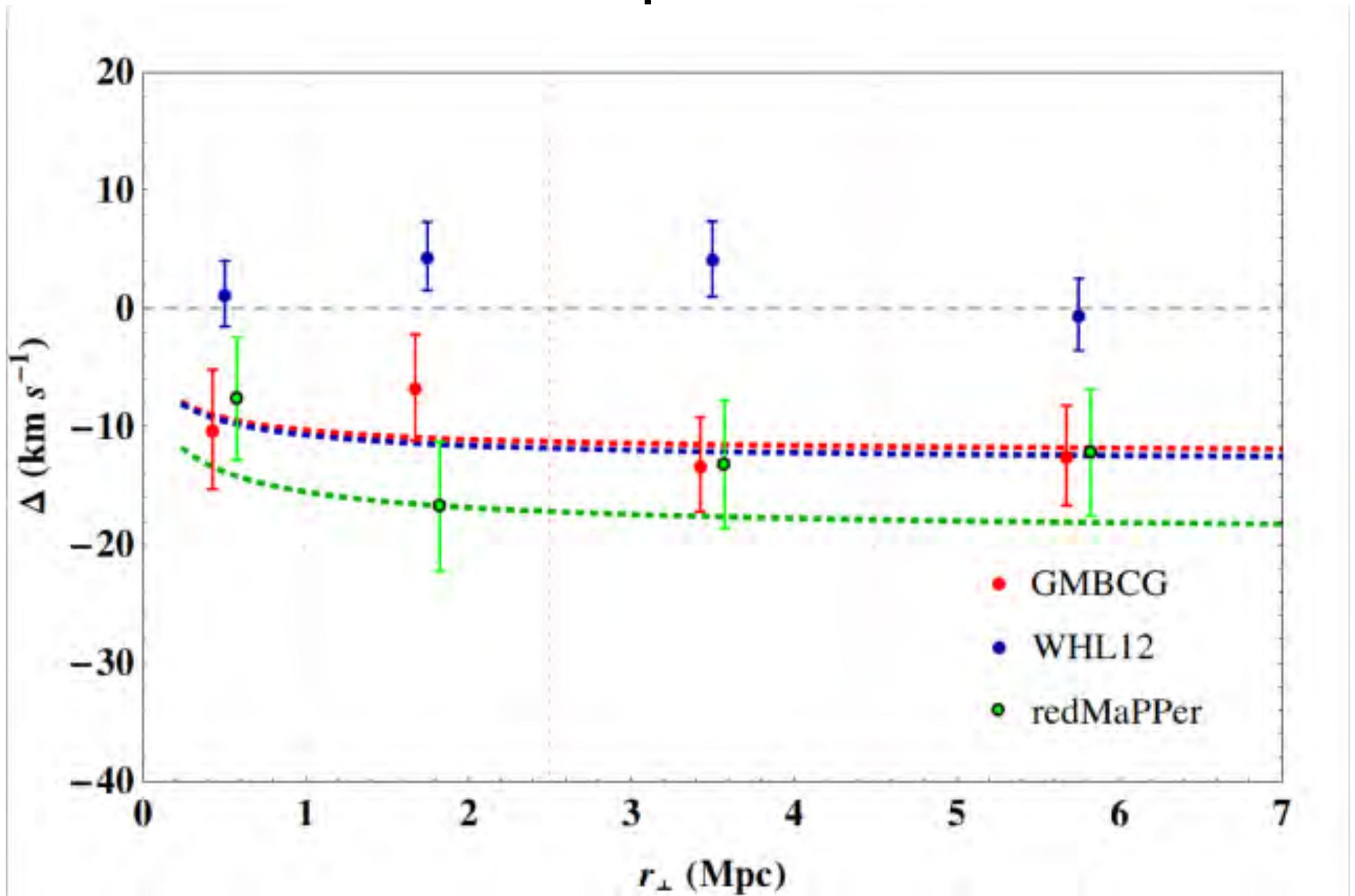




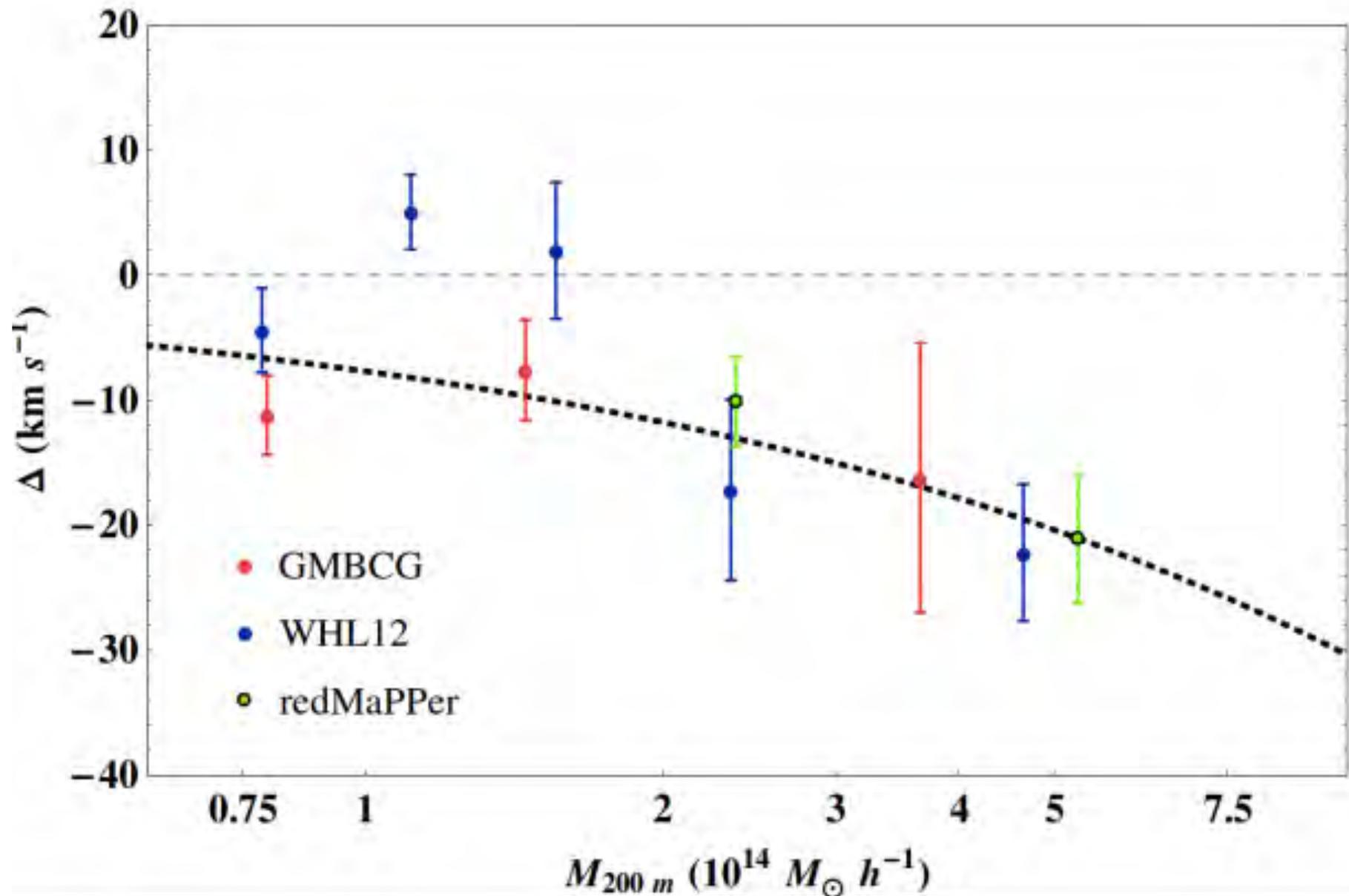
Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015



Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015

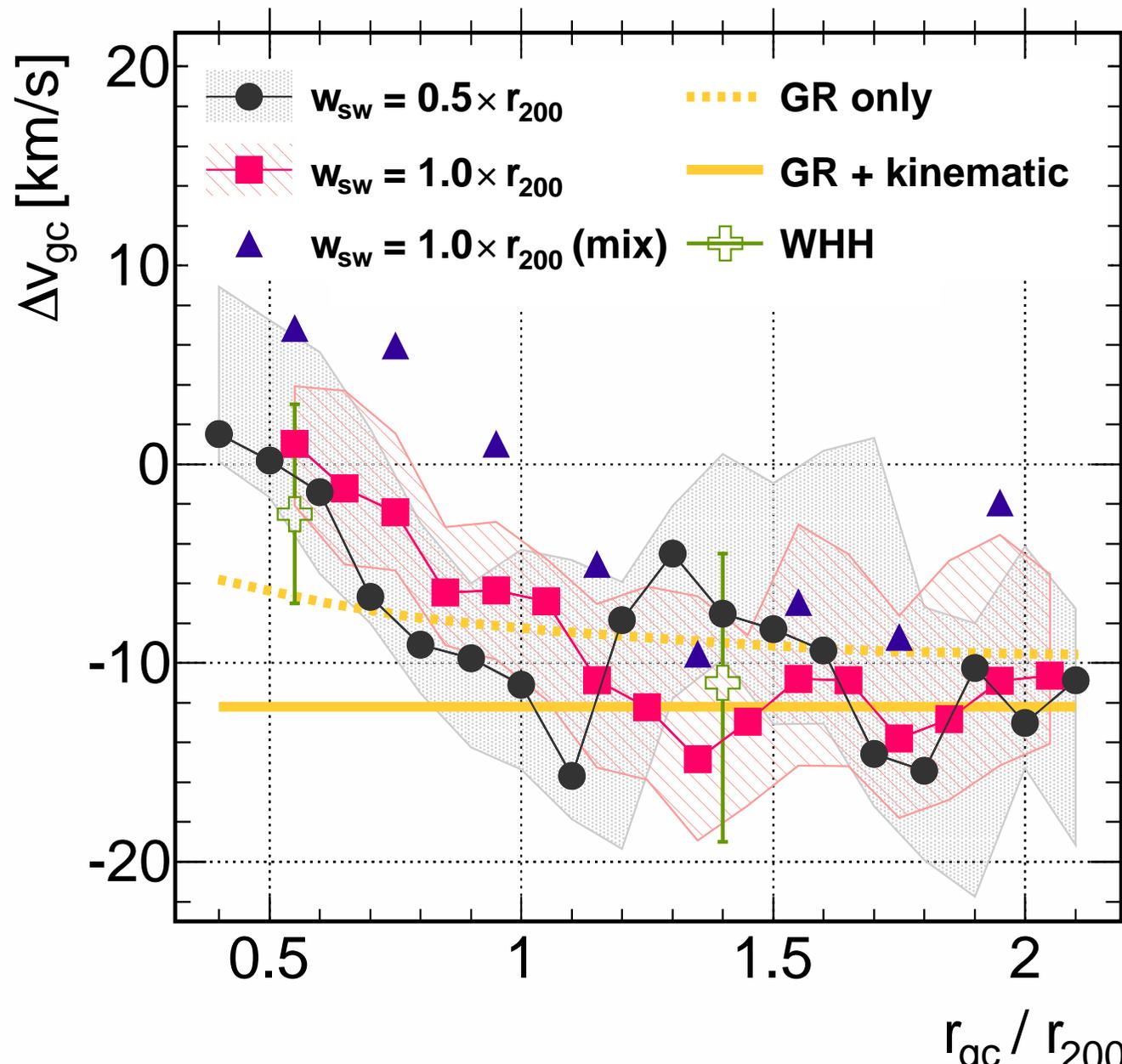


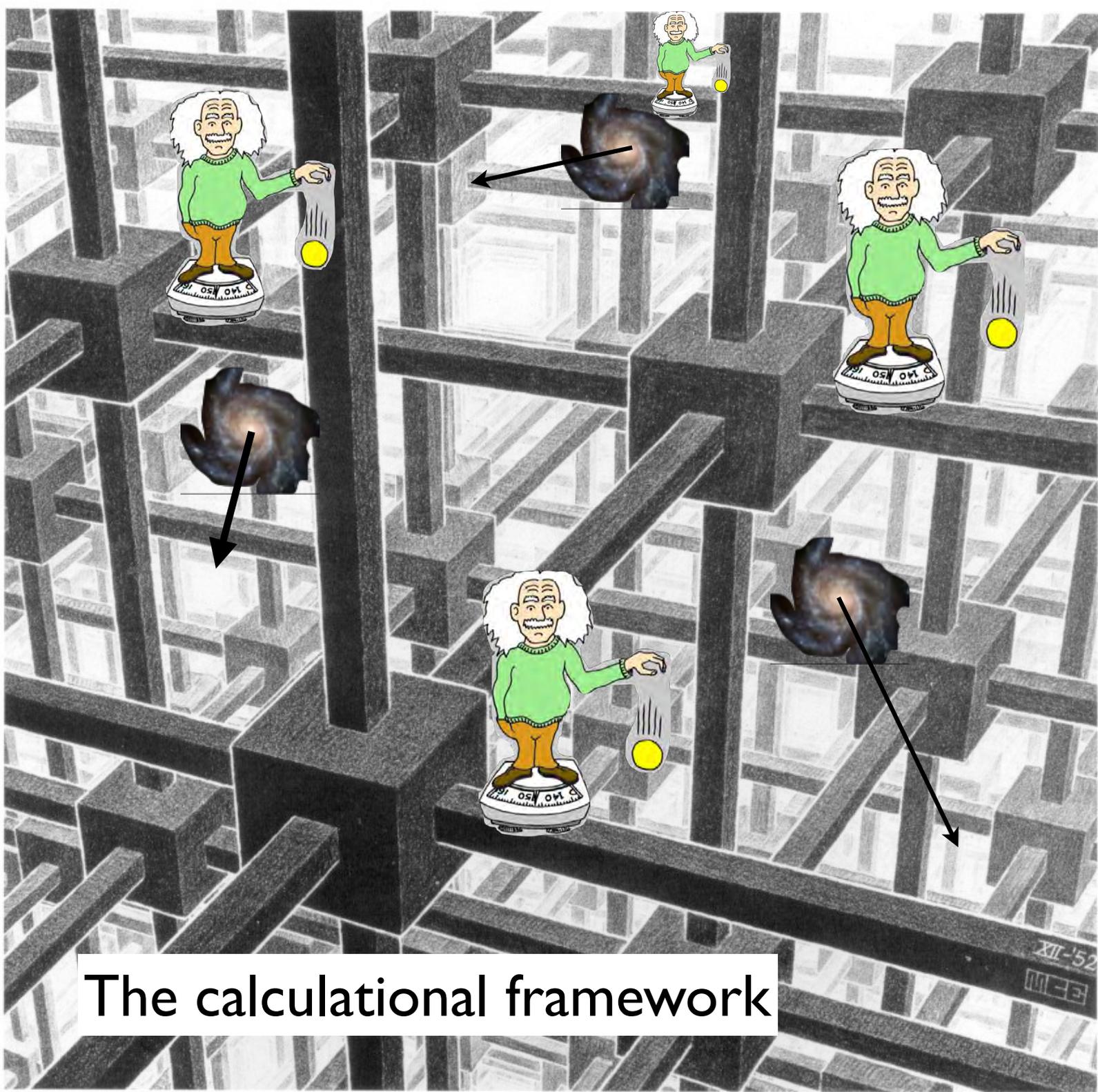
Jimeno, Broadhurst, Coupon, Imetzu, Lazkos 2015



Sadeh et al 2014

r_{gc} [Mpc] : $0.5^{+0.2}_{-0.2}$ $1.0^{+0.3}_{-0.2}$ $1.6^{+0.4}_{-0.3}$ $2.1^{+0.5}_{-0.3}$

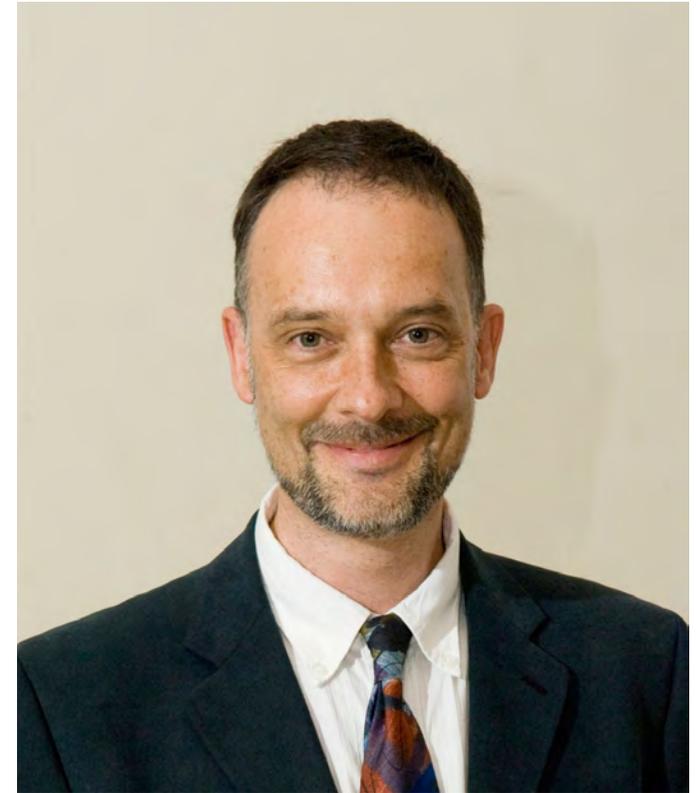




The calculational framework

Zhao, Peacock & Li, 2012

- δz is not just a gravitational redshift
- Sources are moving, so we also see
 - *transverse Doppler effect*:
 - 1st order Doppler effect averages to zero, but....
 - to 2nd order $\langle \delta z \rangle = \langle v^2/c^2 \rangle / 2$
 - can be understood as time dilation - moving clocks run slow
- Generally of same order of magnitude as gravitational redshift from virial theorem, Jeans eq...
- (And this doesn't really test GR
 - see also Bekenstein & Sanders, 2012
 - more later.....)
- Is that the full story?



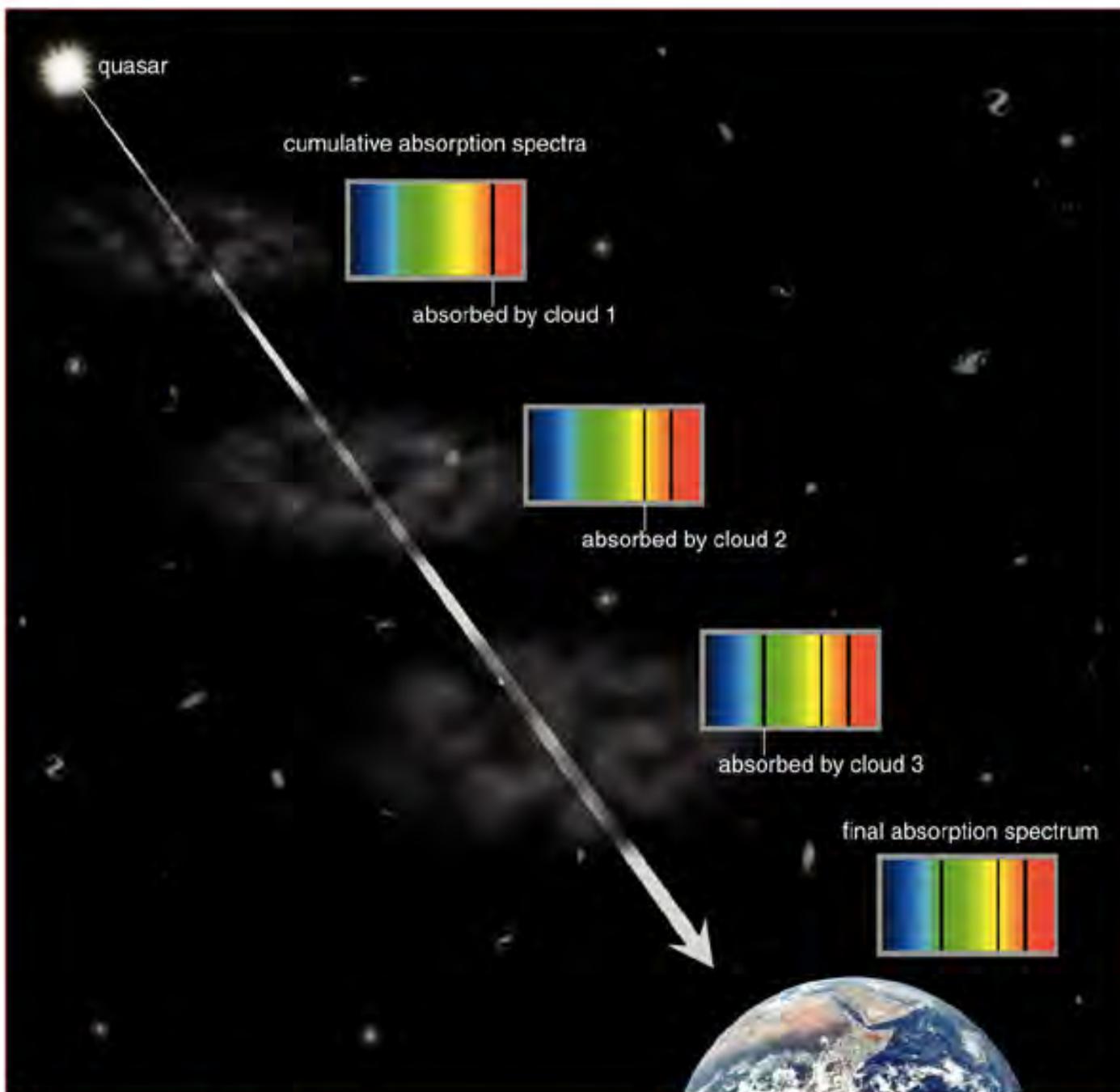
No - there is another effect of same order

- Light cone effect
 - we will tend to see more objects moving away from us than towards us in any observation made using light as a messenger
 - this gives an extra red-shift effect
 - again of the same order of magnitude as the gravitational redshift

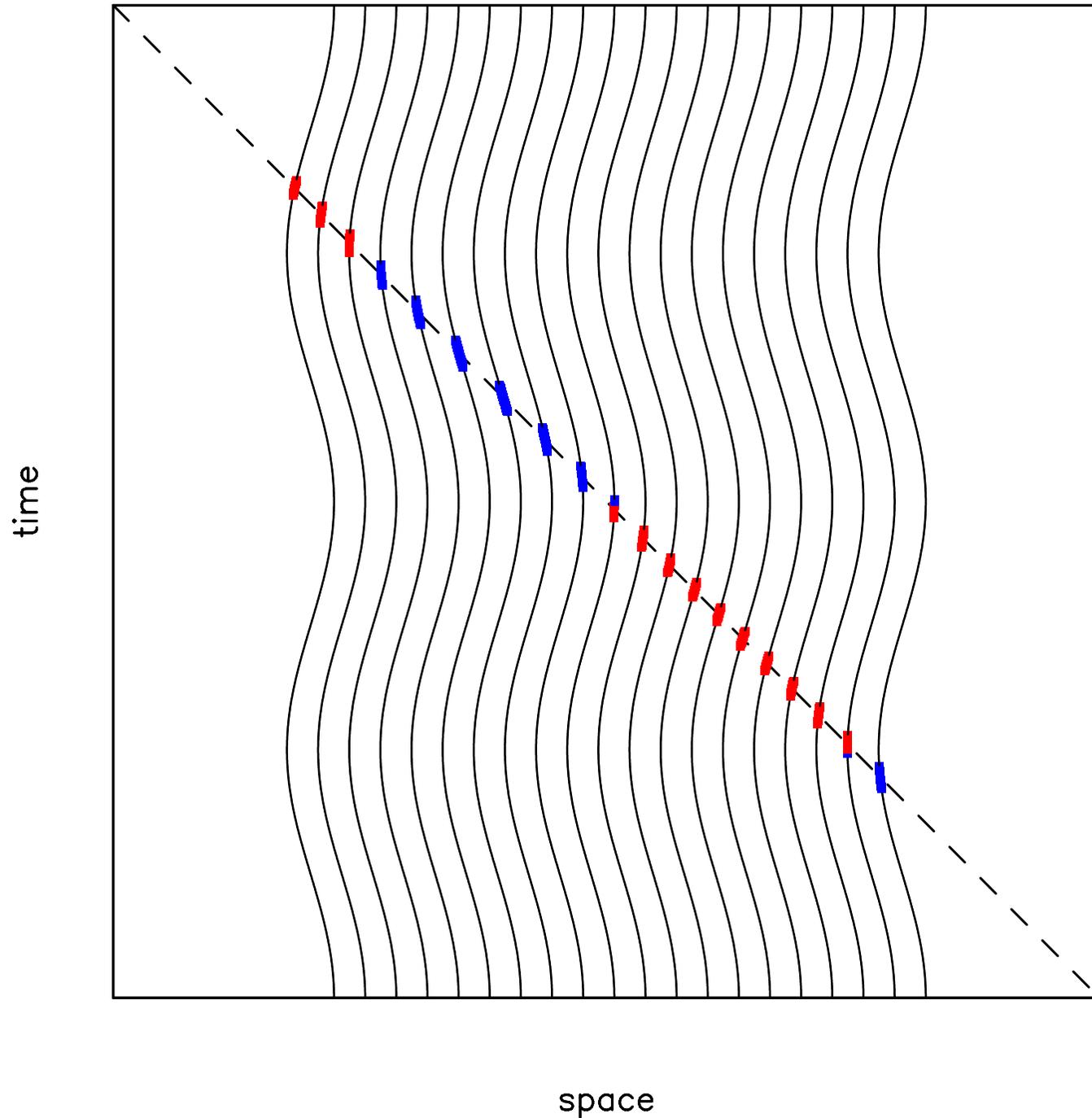
Light-cone effect

- Light cone effect
 - we will see more particles moving away from us in a photograph of a swarm of particles
 - past light cone of event of our observation overtakes more galaxies moving away than coming towards us
 - just as a runner on a trail sees more hikers going the other way...
 - So *not* Lorentz-Fitzgerald contraction effect
 - phase space density contains a factor $(1-v/c)$
 - $\langle \delta z \rangle = \langle (v_{los}/c)^2 \rangle$
 - same sign as TD effect
 - 2/3 magnitude (for isotropic orbits)

Quasar absorption lines



Light-cone effect - more particles moving away!

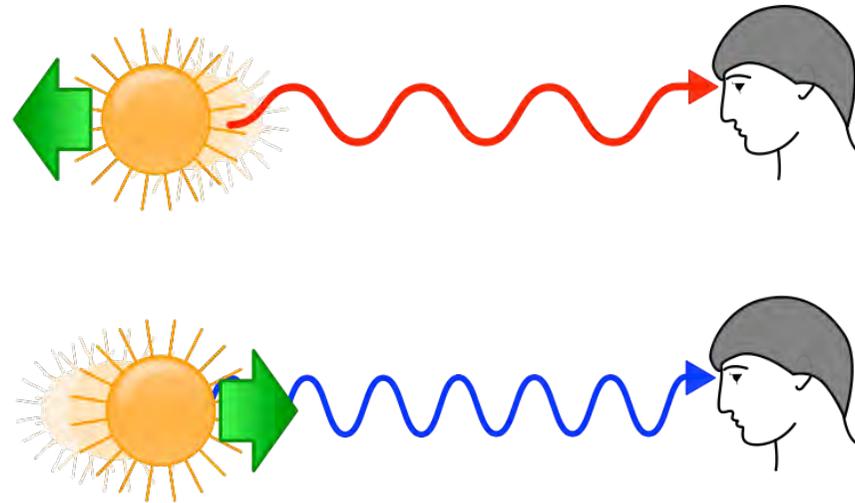


Another way to look at LC effect

- Particle oscillating in a pig-trough
 - $r(t) = a \cos(\omega t + \varphi)$
 - $v(t)/c = -(a\omega/c) \sin(\omega t + \varphi)$
 - $v(t)$ averages to zero
 - average could be over phase or time
- but $v_{\text{obs}} = v + (r/c) dv/dt + \dots$
 - where r/c is the look-back time
 - and the extra term does *not* average to zero
- ~ same as Einstein prediction for Pound & Rebka
 - $\delta z \approx \langle r dv/dt \rangle / c^2$.



Yet another view of the light-cone effect



- Consider a particle oscillating in a square well potential and emitting pulses at a steady rate ($2N$ per period)
- Observer sees intervals between pulses red- or blue-shifted
 - N short intervals followed by N long intervals
- In observation taken at a random time there is a greater chance to catch the particle when it is moving away
- In an observation of an ensemble of particles more particles will be seen going away from the observer

Why is the transverse Doppler effect a redshift?

- Transverse Doppler redshift effect:
 - first order Doppler shift $\sim v/c$ is large but averages to zero
 - residual is a quadratic $\sim (v/c)^2$ effect which caused randomly moving objects appear redshifted on average
 - can also be understood as a time dilation effect
- But moving objects have more energy per unit mass (in the observer frame)
- So if they convert their rest mass to photons we should see a blue-shift on average

a thought experiment

- bake cake, light candles, spin the cake up on a turntable and measure the energy of the photons in the lab frame
- $\langle \text{1st order Doppler} \rangle = 0$
- 2nd order transverse Doppler effect gives a redshift
- but the candles are moving....
- so they have more energy (in our frame) per unit rest mass...
- so should there not be, on average, a transverse Doppler blueshift?



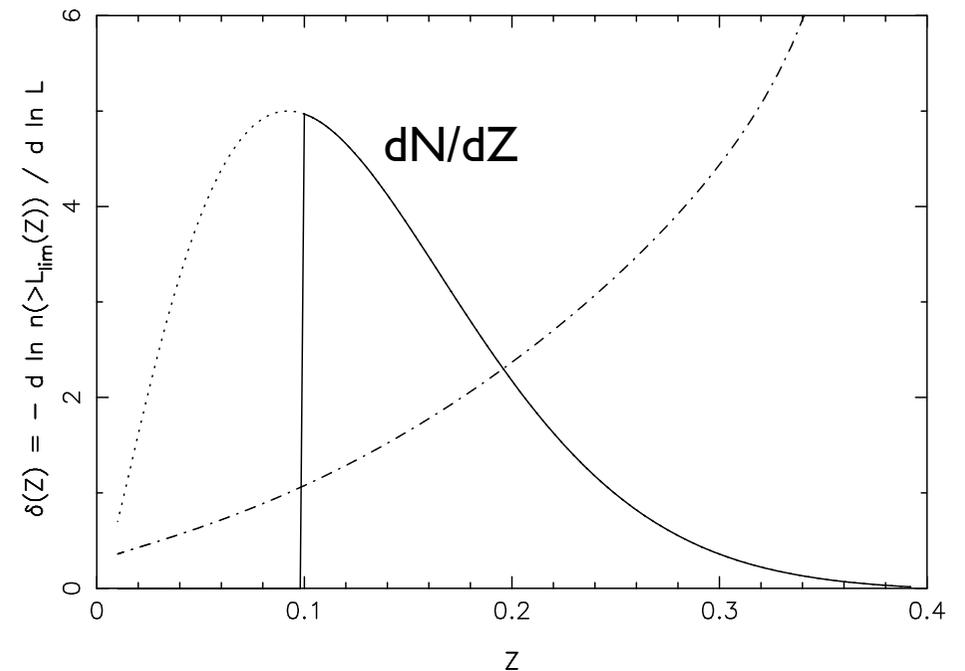
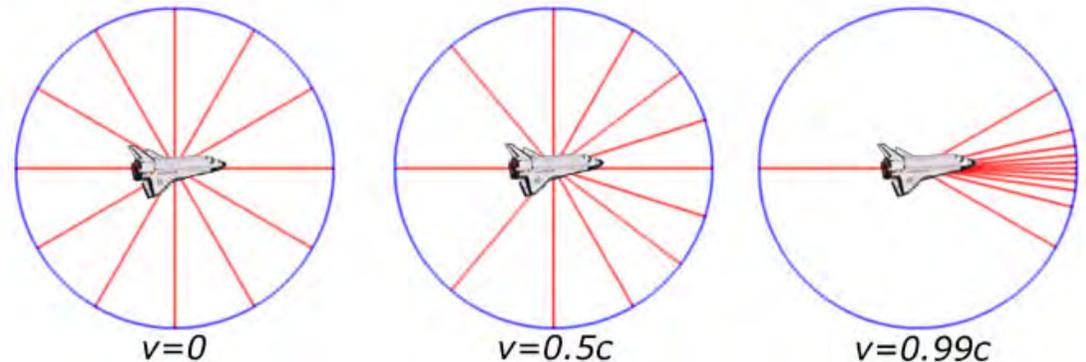
How do we resolve this?

Transverse Doppler Effect: Redshift or Blueshift?

- Averaging over objects vs averaging over photons
 - averaging over objects we will see a redshift
 - but objects emitting isotropically in their rest frame do not emit isotropically in the lab frame - more photons come out in the forward direction - and these have a blue shift on average in the lab frame
 - this flips the sign of the effect
 - e.g. unresolved objects show blue shift (e.g. stars in the BCG or low resolution 21cm radio for integrated cluster z)
 - here we have a hybrid situation:
 - redshifts measured for objects
 - but objects are selected according to flux density

Surface brightness modulation

- Line of sight velocity changes surface brightness
- relativistic beaming (aberration) plus change of frequency
- but doesn't change the surface area
- so velocities modulate luminosity
 - depends on SED: $\delta L/L = (3 + \alpha)v/c$
 - $\alpha \approx 2$, so big amplification
 - spectroscopic sample is flux limited at $r=17.8$
 - $\Delta n/n = -d \ln n(>L_{\text{lim}}(Z))/d \ln L * \Delta L/L$
- opposite sign to LC, TD effects, but larger because the sample here is limited to bright end of the luminosity function

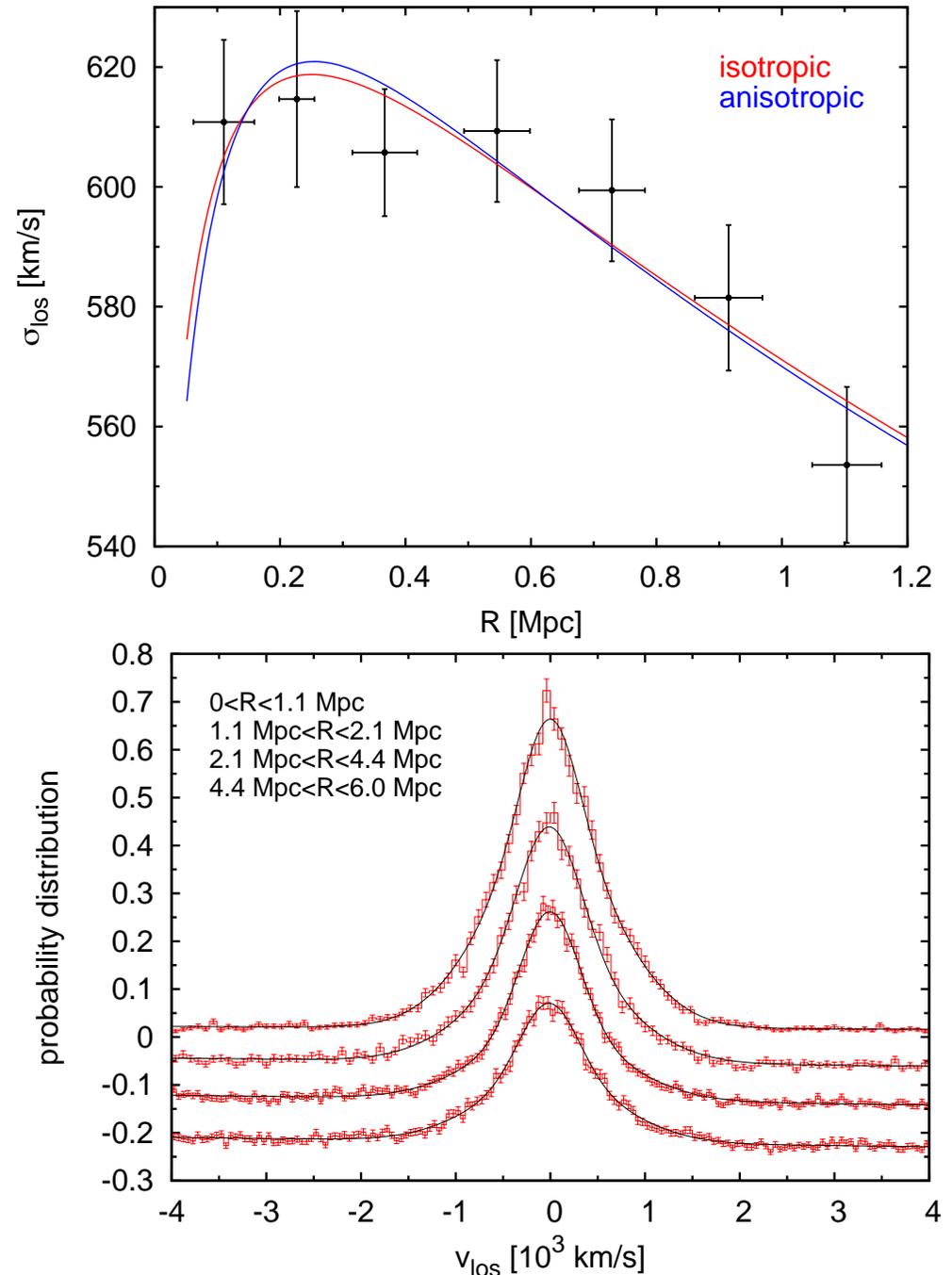


More implications of the transverse Doppler red/blue-shift dichotomy

- Contribution to cluster grav- Z from motions of stars in the BCG
 - velocity dispersions are smaller than in cluster, but not negligible
 - stars are unresolved so we get a transverse Doppler blue-shift
- 21 cm radio observations of galaxies
 - sees mostly galaxies falling into cluster for first time as gas is stripped within virial region
 - should have a large potential difference relative to BCG
 - but the prediction for δZ is highly dependent on whether one makes unresolved single dish (e.g. Aricebo) measurements or resolved (e.g. Westerbork, ASKAP)

Corrected grav-z measurement

- Fairly easy to correct for TD +LC+SB effects
- TD depends on vel. disp. anisotropy
- LC+SB directly measured
- net effect is a blue-shift
 - $\sim -9\text{km/s}$ in centre, falling to $\sim -6\text{km/s}$ at larger r
- minor effects from infall/outflow velocity
- Substantial change in measured grav-z term
 - but still consistent with dynamical mass estimate



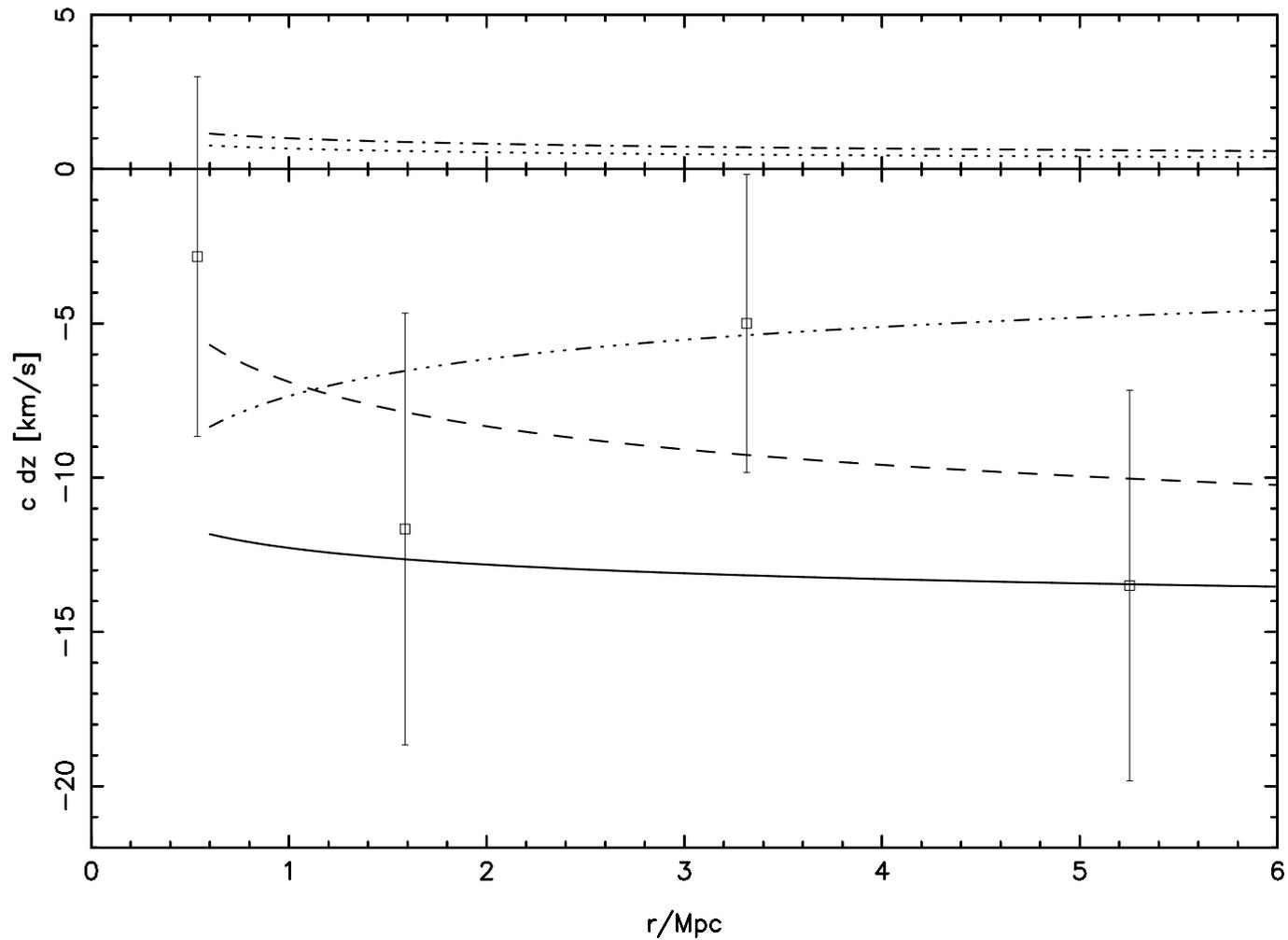


Figure 3. Data points from figure 2 of WHH and prediction based on mass-traces-light cluster halo profile and measured velocity dispersions as described in the main text. The dashed line is the gravitational redshift prediction, which is similar to the WHH model prediction. The dot-dash line is the transverse Doppler effect. The dotted line is the LC effect. The triple dot-dash line is the surface brightness effect. The solid curve is the combined effect.

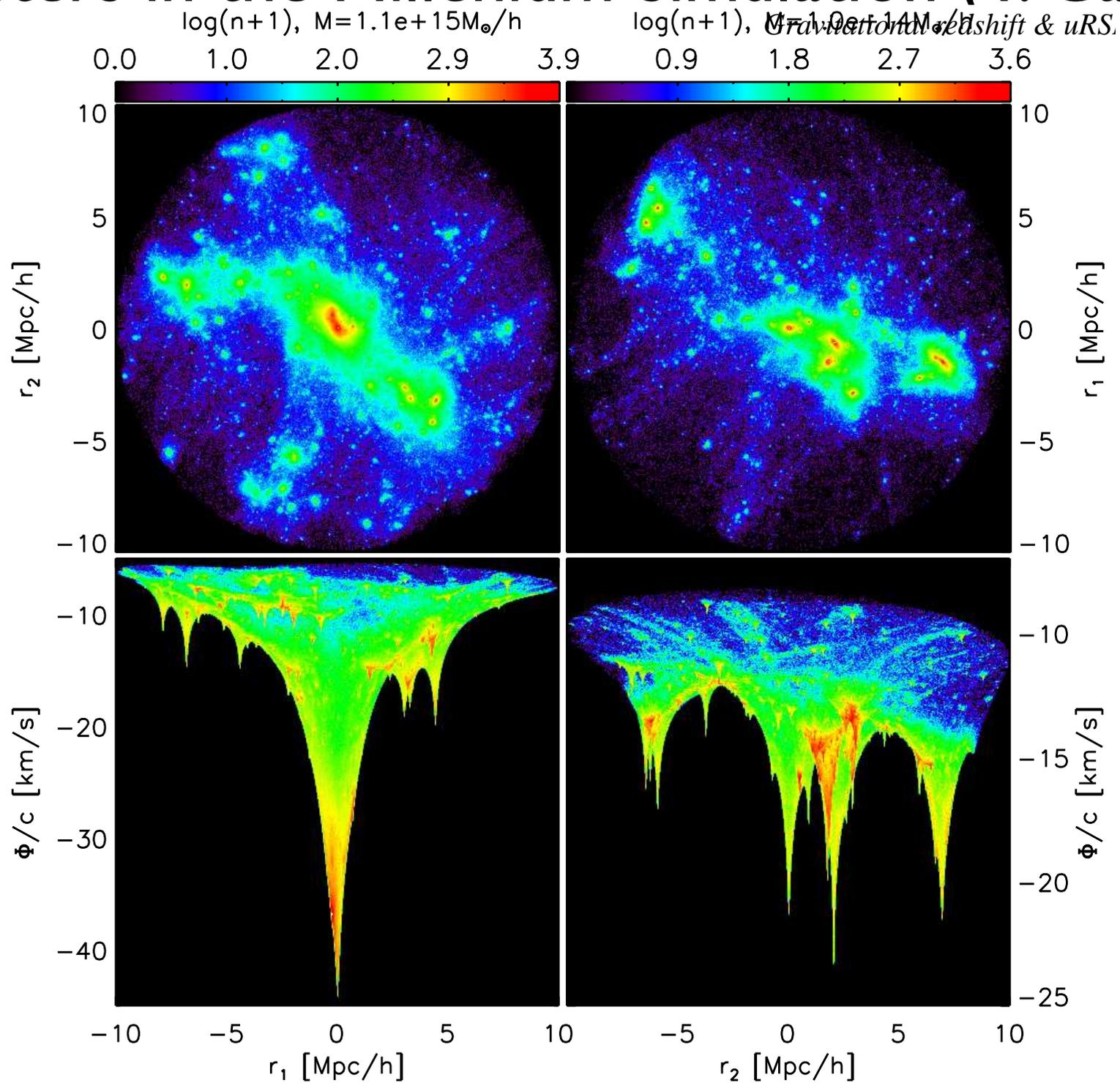
Modelling gravitational-z in simulations (Cai+'06)

- NK'13 modelling assumed virialised (non-expanding) clusters
 - this breaks down at large r
 - need to allow for infall
 - asymmetry gives other biases
- Cai+2016 have used Millennium simulation to quantify this
 - formalism for extracting observables from "snapshots":

$$cz = Hx + v_x + v^2/2c - \Phi/c \\ - xg_x + Hxv_x/c + [H^2 - \ddot{a}/(2a^2)] x^2/c,$$

- includes light-cone effects
- valid to 2nd order in velocity (Hubble and/or peculiar)

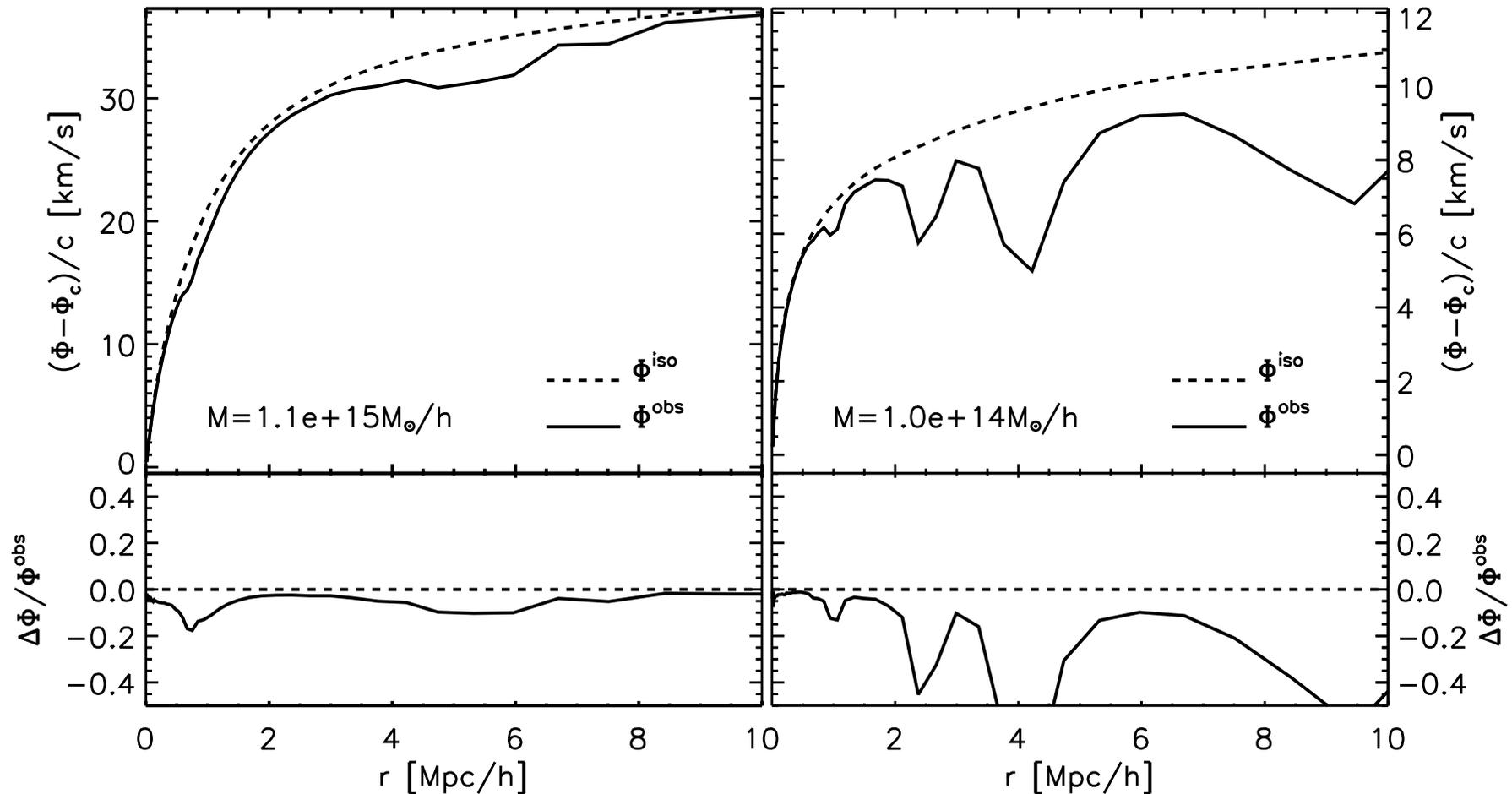
Clusters in the Millenium Simulation (Y. Cai)



Modelling gravitational-z in simulations (Cai+'06)

Formalism: mapping from x to v (2nd order)

$$cz = Hx + v_x + v^2/2c - \Phi/c \\ - xg_x + Hxv_x/c + [H^2 - \ddot{a}/(2a^2)] x^2/c,$$

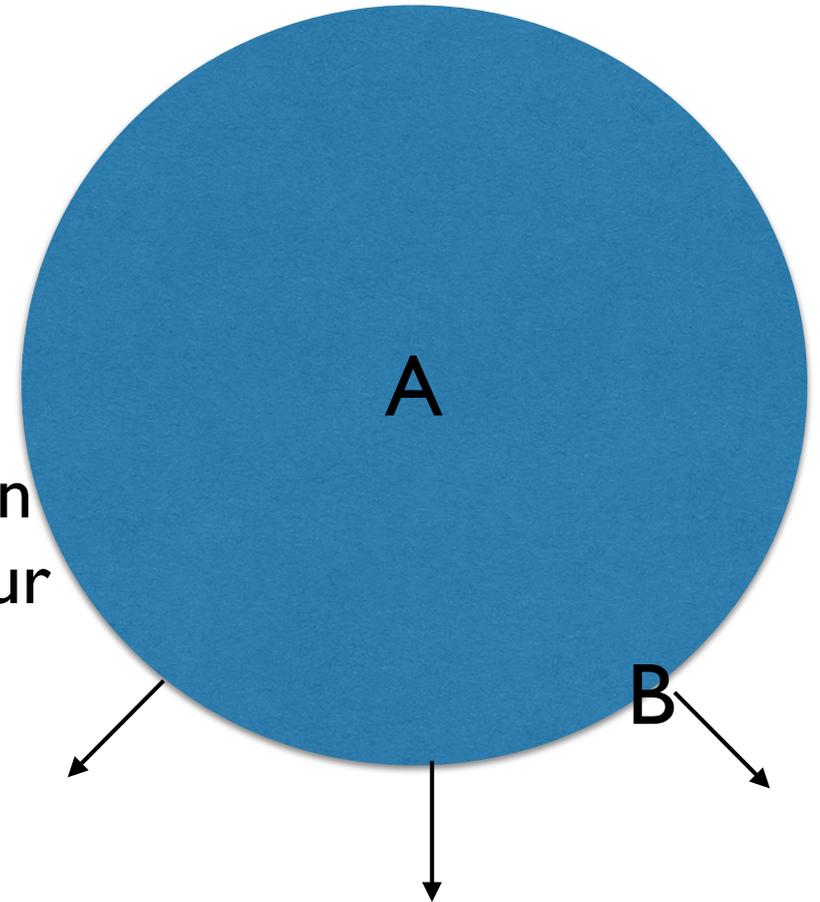


What was wrong with the "kinematic picture"?

- "A gravitational redshift is just a Doppler shift viewed from an unnatural coordinate system"?
- This confuses gravity and acceleration
- In GR the gravitational field is the Riemann (curvature) tensor
 - just the tidal field in the Newtonian limit
 - can be measured from relative motion of test particles
- So is there a truly gravitational component to the redshift?
 - and why does e.g. cosmological z appear kinematical?

Why is the gravitational-z hidden in cosmology?

- Consider expanding sphere of dust and source A sending photon to receiver B
- Photon suffers **gravitational red-shift** climbing up the potential and then a **Doppler red-shift** on reception
- For source B sending to A the photon has a **Doppler red-shift** (as seen in our frame) then enjoys a **gravitational blue-shift**
- But the net effect is the same.
- The opposite gravitational shifts are cancelled by the Doppler shift change
- But this is a special situation



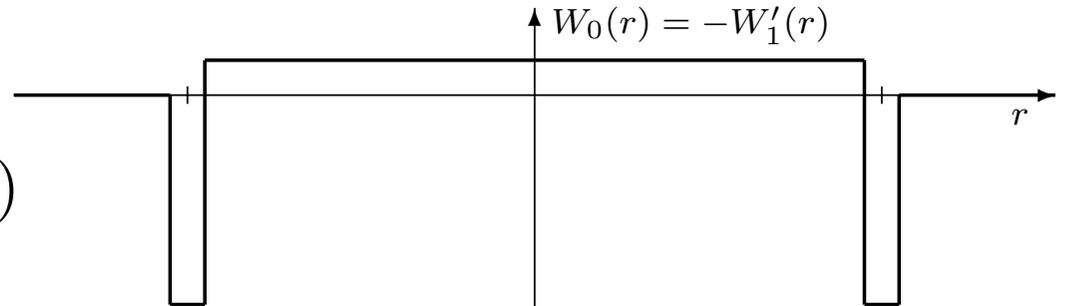
The non-kinematic part of the redshift

- Consider pair of freely-falling observers 1,2 in arbitrary gravitational field who exchange a photon.
- Use rigid, non-rotating lattice picture to calculate changes in wavelength and proper separation (work in CoM frame)
 - work to 2nd order in v/c and 1st order in φ/c^2
- $\Delta\lambda/\lambda = \mathbf{n} \cdot (\mathbf{v}_1 - \mathbf{v}_2)_{t1} / c + \int d\mathbf{r} \cdot (\mathbf{g}_2 - \mathbf{g}(\mathbf{r})) / c^2 \quad (1)$
- $\Delta D/D = \mathbf{n} \cdot (\mathbf{v}_1 - \mathbf{v}_2)_{t1} / c + \Delta\mathbf{r} \cdot (\mathbf{g}_2 - \mathbf{g}_1) / 2c^2 \quad (2)$
- Both are 1st order Doppler (with initial Δv) plus 'tidal' term
- Spatially constant tidal field stretches λ just like D
 - includes Minkowski spacetime and FRW
 - but that's because of special *symmetry* of FRW
 - does not apply for a galaxy cluster
- extra intrinsically gravitational term (gradient of tide)

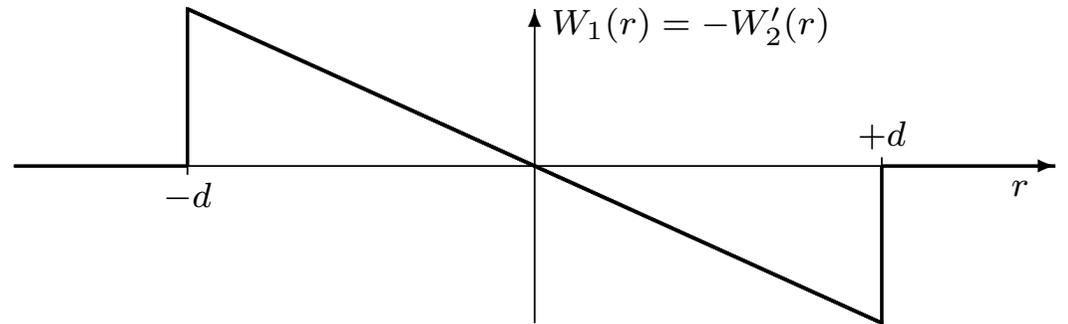
Subtracting (1) - (2) gives

$$\Delta \log(\lambda/D) = \frac{1}{c^2} \left(d \mathbf{n} \cdot (\mathbf{g}_1 + \mathbf{g}_2) - \int d\mathbf{r} \cdot \mathbf{g} \right) \quad \text{or}$$

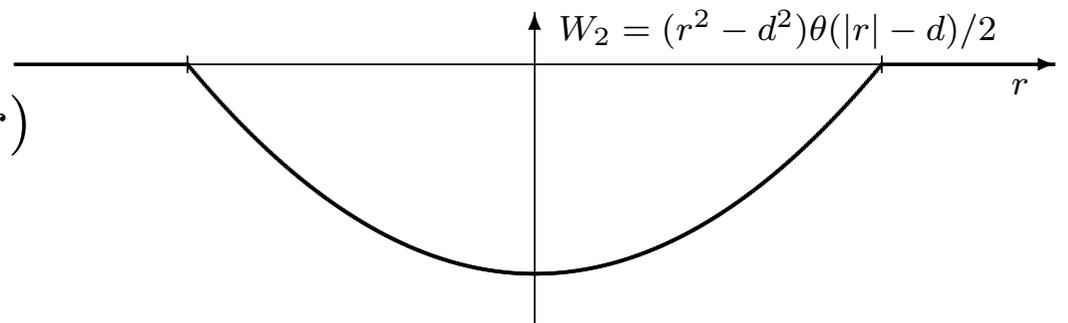
$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_0(r) \phi'(r)$$



$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_1(r) \phi''(r)$$



$$\Delta \log(\lambda/D) = \frac{1}{c^2} \int dr W_2(r) \phi'''(r)$$



There is a non-kinematic component of the redshift: it is a measurement of the gradient of the tide

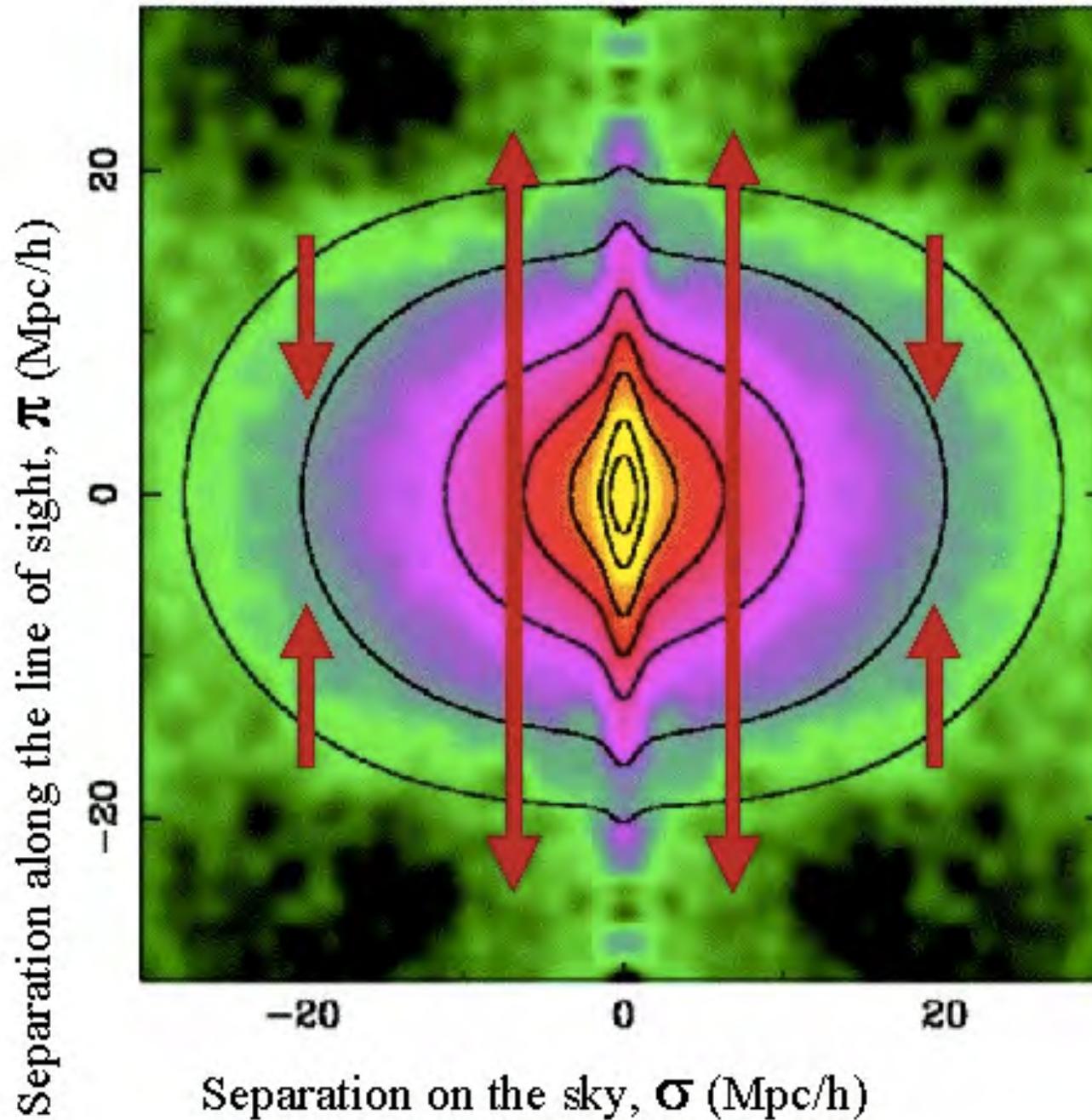
Why we observe a gravitational z in clusters

- The "kinematic picture" is wrong
 - redshifts are not solely determined by change of separation of observer, source
 - there is an additional, intrinsically gravitational, effect
 - but the gravitational-z comes from gradients of the tide
 - that's why it's not seen in FRW cosmology
 - a consequence of symmetry
- Total z is kinematic plus an integral involving $\text{grad}(\text{tide})$
 - sums to give naive (P&R) gravitational redshift
- but we also need TD, LC and SB effects..

Future prospects...

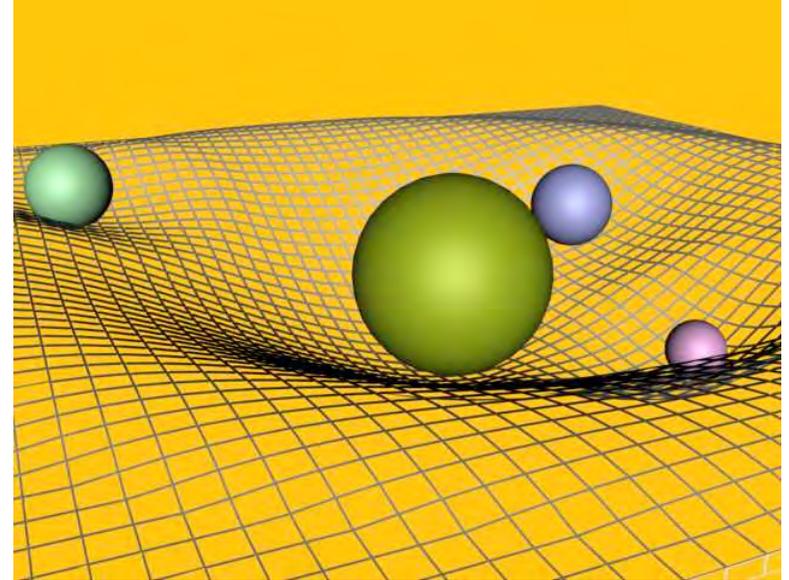
- Can expect immediate improvements in measurement
 - 3x increase in number of redshifts available (BOSS)
 - and more to come:
 - optical: big-BOSS
 - radio: FAST, ASKAP-Wallaby+WNSHS
 - interesting to compare unresolved radio and optical
- Extension to larger scales. Bright-faint cross correlation Gaztanaga++2015, Alam++2016..
- Lots of rich material in the front-back asymmetry of the galaxy correlation function.
- Lots of interesting scope for modelling:

Redshift space distortions (symmetric)



What else does it mean?

- Probe of curvature of space in GR?
 - *matter tells space how to curve*
 - *space tells matter how to move....*
- Like how lensing tests gravity?
- Not quite:
 - motion of galaxies & grav-z are determined only by g_{tt}
- It is really a test of the equivalence principle
- Provides a test of theories with long-range non-gravitational forces in the “dark sector”
 - e.g. Gradwohl & Frieman 1992; Farrar & Peebles 2004; Farrar & Rosen 2007; Keselman, Nusser & Peebles 2010; and many, many more.... and (maybe) $f(R)$ gravity.
- though such theories are already constrained by X-ray temp. vs galaxy motions in clusters....



Scalar fields, "Fifth forces" & Violation of the EP

- a common feature of modified gravity theories
 - string theory inspired: dilaton field - couples to matter
 - also interacting DE & DM models where $m = m(\phi)$
 - $f(R)$ gravity etc. etc.
- extra long-range ($1/r$ potential) force augmenting gravity
 - must be suppressed/small on solar system scale
 - or only coupling to DM
- Violations of the Equivalence Principle (foundation of GR)
 - interesting - and testable - consequences
 - lensing - galaxy clustering - gravitational redshifts - BHs see different \mathbf{g} - dynamics in clusters (gas vs \ast s vs DM)

Conclusions

- Gravitational redshifts in clusters of galaxies have been measured!
- Technically challenging but apparently real and prospects for better measurements and extension to larger scales is promising.
- Potentially useful test of alternatives to GR & 5th forces
- But also interesting as a "sand-box" that illustrates some subtleties of simple special relativity + Newtonian gravity
- Effect raises some questions of principle about how to think about redshifts in cosmology and astronomy in general.
- Redshifts are not purely kinematic - there is an truly gravitational component - but it is hidden in cosmology