To the local beginning of inflation – and beyond

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Typically, observational data on present perturbations may give us information about last ~60 e-folds of inflation only.

Is it possible to go further in the past to the local beginning of inflation - e.g. to determine N_{tot}, and even beyond - e.g. to determine local initial conditions at the beginning of inflation?

Locally (on our worldline):
inflation has both the beginning and the end

Globally:
inflation has no beginning and no end in almost all cases
(in the sense that inflating patches always exist somewhere in space)

BEYOND LAST 60 e-folds OF INFLATION TO THE PAST

Only slow-roll inflation is considered.

1. From metric perturbations

Some information can be obtained, though restricted and non-decisive (some classes of models can be excluded only)

Example: intermediate inflation

$V(\varphi) \propto \varphi^{-\alpha}, \quad \varphi > M_{Pl}$

$$n_S - 1 = \frac{2-\alpha}{2N_{beg}}, \quad n_T = -\frac{r}{8} = -\frac{\alpha}{2N_{beg}} = \frac{\alpha}{\alpha-2}(n_S - 1)$$

$n_T < n_S - 1 \rightarrow$ characteristic for such models

2. To obtain decisive information about the previous history of the Universe new measuring devices are required

Light scalar fields

$$\frac{d\langle\varphi^2\rangle}{dt} = \frac{H^3}{4\pi^2} \qquad (1982)$$

! Warning for the de Sitter inflation:

$$\langle R_{ik} R^{ik} \rangle = 16 H^3 t / \pi M_P^2 \qquad (1979)$$

Beyond small perturbations

For $N \gg 1$, $\langle \mathcal{R}^2 \rangle$ becomes $\gtrsim 1$

Stochastic approach to inflation
("stochastic inflation")

$$\hat{R}_i^k - \frac{1}{2}\delta_i^k \hat{R} = 8\pi G \, \hat{T}_i^k$$

with $\hat{T}_i^k = \hat{T}_i^k(\hat{g}_{lm})$ (not $(\langle \hat{g}_{lm} \rangle)$!)

leads to QFT in a stochastic background

1. Can deal with arbitrary large global inhomogeneity
2. Takes backreaction into account
3. Goes beyond any finite order of loop corrections

Another 'time' variables

$$\tau^{(n)} = \int H^n(t, \vec{x})\, dt \qquad H^2 = \frac{8\pi G V(\Phi)}{3}$$

This is not a time reparametrization in GR $t \to f(t)$

$\tau^{(n)}$ describe different stochastic processes and even have different dimensionality

Different 'clocks':

$n = 0$ phase of the wave function of a massive particle $(m \gg H)$

$n = 1$ perturbations

$n = 3$ rms value of a light scalar field generated during inflation

$$\langle \chi^2 \rangle = \frac{1}{4\pi^2} \langle \int H^3 dt \rangle = \frac{\langle \tau^{(3)} \rangle}{4\pi^2}$$

$$\frac{d\Phi}{d\tau} = -\frac{1}{3H^{n+1}} \frac{dV}{d\varphi} + f$$

$$\langle f(\tau_1) f(\tau_2) \rangle = \frac{H^{3-n}}{4\pi^2} \delta(\tau_1 - \tau_2)$$

Einstein-Smolukovsky (Fokker-Planck) equation

$$\frac{\partial \rho}{\partial \tau} = \frac{\partial}{\partial \phi}\left(\frac{V'}{3H^{n+1}}\rho\right) + \frac{\partial^2}{\partial \phi^2}\left(H^{3-n}\rho\right)\cdot\frac{1}{8\pi^2}$$

$\int \rho \, d\phi = 1 \rightarrow$ probability conservation

Remarks

1. More generally: $\ldots + \frac{\partial}{\partial \phi}\left(H^{(3-n)\alpha}\frac{\partial}{\partial \phi}\left(H^{(3-n)(1-\alpha)}\rho\right)\right)\cdot\frac{1}{8\pi^2}$

   $0 \leq \alpha \leq 1$

   $\alpha = 0$ – Ito calculus

   $\alpha = \frac{1}{2}$ – Stratonovich calculus

   However, keeping terms depending on α exceeds the accuracy of the stochastic approach. Thus, α may be put zero.

2. Results are independent on the form of the cutoff in the momentum space as far as it occurs for $k \ll aH$ ($\varepsilon \ll 1$)

3. Backreaction is taken into account

   $$\delta T_\mu^\nu = \delta_\mu^\nu (V - V_{cl})$$

Growth of inflaton fluctuations in the linear regime

$$|\varphi - \varphi_{cl}| \ll \varphi_{cl}$$

Two "time" variables: t, $\ln a = \int H dt$

Since $H \simeq \sqrt{\frac{8\pi G V(\varphi)}{3}}$ is stochastic, transformation from t to $\ln a$ is <u>not</u> simply time-reparametrization.

Different "clocks":

t	$\ln a$
atomic clocks, temporal phase of a massive test particle ($\varphi \propto e^{imt}$, $m \gg H$)	scalar perturbations

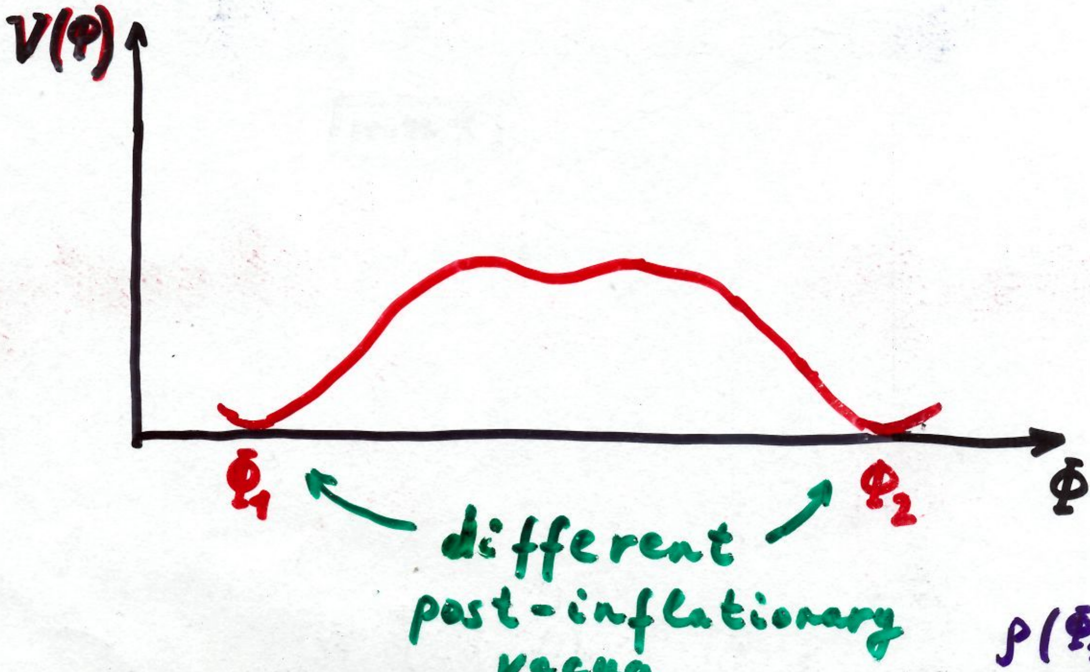
$$\delta\varphi = \varphi - \varphi_{cl}, \quad ' \equiv \frac{d}{d\varphi}$$

$\dot{\delta\varphi} + \frac{H''}{4\pi G}\delta\varphi = f$	$\frac{d\widetilde{\delta\varphi}}{d\ln a} + \frac{1}{4\pi G}\left(\frac{H'}{H}\right)'\widetilde{\delta\varphi} = \tilde{f}$
$\langle f f \rangle = \frac{H^3}{4\pi^2}\delta(t-t')$	$\tilde{f} = \frac{f}{H}$, $\langle f f \rangle = \frac{H^2}{4\pi^2} \cdot \delta(\ln a - \ln a')$
$\langle(\delta\varphi)^2\rangle = \frac{G H'^2}{\pi}\int_{\varphi}^{\varphi_0} d\varphi \cdot \frac{H^3}{H'^3}$	$\langle(\widetilde{\delta\varphi})^2\rangle = \frac{G H'^2}{H^2}\int_{\varphi}^{\varphi_0} d\varphi \cdot \frac{H^5}{H'^3}$

From $\rho(\varphi,\tau)$ during inflation to the distribution $w(\tau)$ over total local duration of inflation.

$$w(\tau) = \lim_{\varphi \to \varphi_{end}} j = \lim_{\varphi \to \varphi_{end}} \frac{|V'|}{3H^{n+1}} \rho(\varphi,\tau)$$

For a smooth transition to a post-inflationary epoch, the stochastic force must be much less than the classical one during last e-folds of inflation



$$\rho(\Phi_1,\tau) = 0$$
$$\rho(\Phi_2,\tau) = 0$$

Let $$Q_m(\Phi) = \int_{\tau_0}^{\infty} (\tau-\tau_0)^m \rho(\Phi,\tau)\, d\tau$$

$$Q_m(\Phi_1) = Q_m(\Phi_2) = 0$$

$\boxed{m=0}$

$$Q_0(\Phi) = \frac{8\pi^2}{H^{3-n}} \exp\left(\frac{\pi}{GH^2}\right) \int_{\varphi_1}^{\varphi} d\tilde{\varphi} \exp\left(-\frac{\pi}{GH^2}\right) \cdot$$

$$\cdot \left(C - \int_{\varphi_1}^{\tilde{\varphi}} p_0(\tilde{\tilde{\varphi}})\, d\tilde{\tilde{\varphi}} \right)$$

$$C = \frac{\int_{\varphi_1}^{\varphi_2} d\varphi \cdot \exp\left(-\frac{\pi}{GH^2}\right) \int_{\varphi_1}^{\varphi} p_0(\tilde{\varphi})\, d\tilde{\varphi}}{\int_{\varphi_1}^{\varphi_2} d\varphi \cdot \exp\left(-\frac{\pi}{GH^2}\right)}$$

$P_1 = C$, $P_2 = 1 - C$

probabilities of post-inflationary vacua

Do not depend on "n"! (branching ratios)

$\boxed{m=1}$

$$Q_1(\Phi) = \frac{8\pi^2}{H^{3-n}} \exp\left(\frac{\pi}{GH^2}\right) \int_{\varphi_1}^{\varphi} d\tilde{\varphi} \cdot \exp\left(-\frac{\pi}{GH^2}\right) \cdot$$

$$\cdot \left(C_1 - \int_{\varphi_1}^{\varphi} Q_0(\tilde{\tilde{\varphi}})\, d\tilde{\tilde{\varphi}} \right)$$

$$C_1 = \frac{\int_{\varphi_1}^{\varphi_2} d\varphi \cdot \exp\left(-\frac{\pi}{GH^2}\right) \int_{\varphi_1}^{\varphi} Q_0(\tilde{\varphi})\, d\tilde{\varphi}}{\int_{\varphi_1}^{\varphi_2} d\varphi \cdot \exp\left(-\frac{\pi}{GH^2}\right)}$$

$\langle \tau_1 \rangle = \frac{C_1}{C}$, $\langle \tau_2 \rangle = \frac{\tilde{C}_1}{1-C}$

$$\langle \tau \rangle_{tot} = C \langle \tau_1 \rangle + (1-C) \langle \tau_2 \rangle = \underline{\int_{\varphi_1}^{\varphi_2} Q_0(\varphi)\, d\varphi}$$

The main problem: choice of the initial condition $\rho_0(\varphi)$ at the local beginning of inflation.

1. Static solutions $\to$ non-normalizable.
2. $\rho_0(\varphi) = \delta(\varphi - \varphi_0)$. why ?
3. "Eternal inflation as the initial condition": $\rho_0(\Phi) = \rho_{E_1}(\varphi)$ $(E_0 = 0)$

   Not possible for the continuum spectrum case.

   In the discrete spectrum case, generically $E_2 - E_1 \sim E_1$ $\to$ not enough time for relaxation.

   'Eternal' inflation is not eternal enough to fix the initial condition uniquely.

Two different types of inflationary models

$n=1$

1. Exponentially decaying

$\int \frac{d\Phi}{H(\Phi)}$ converges

Discrete spectrum of eigenvalues

$\sqrt{-g} \propto a^{3-\lambda_1}$

$\langle \ln \frac{a_f}{a_0} \rangle$ finite

$\rho = \rho_0(\Phi) \cdot \left(\frac{a}{a_0}\right)^{-\lambda_1}$, $\lambda_1 \ll 1$ for $a \to \infty$

$\boxed{\text{Quasi-stationary regime}}$

$w(a_f) \propto \left(\frac{a_f}{a_0}\right)^{-\lambda_1}$

Examples: a) new inflation

b) chaotic inflation $V(\Phi) \propto \Phi^p$, $p > 2$

2. Non-exponentially decaying

$\int \frac{d\Phi}{H(\Phi)}$ diverges

Continuous spectrum of positive eigenvalues

$w \propto (\ln a_f)^{-3/2}$

$\langle \ln \frac{a_f}{a_0} \rangle$ infinite

$\sqrt{-g} \propto a^3/(\ln a)^\gamma$

Examples: a) chaotic inflation $V(\Phi) \propto \Phi^p$, $p \leq 2$

b) $R + R^2$ model

c) $V = V_0 - C\varphi^{-n}$

In both cases: $\langle \frac{a_f}{a_0} \rangle$ infinite

CONCLUSIONS REGARDING BEGINNING OF INFLATION

1. No problems of principle in predicting probability distributions during and after inflation in the original (probability conserving) stochastic approach to inflation, once the initial condition $\rho_0(\varphi)$ is given. No necessity to refer to other universes (they exist but outside our past light cone).
2. No satisfactory principle to fix $\rho_0(\varphi)$ uniquely.
3. Some dependence on $\rho_0(\varphi)$ remains in final answers – a possibility to get some knowledge on it from observations does not seem hopeless.

However, for almost all $\rho_0(\varphi)$ apart from $\rho_0(\varphi) \propto \exp\left(\frac{\pi}{GH^2(\varphi)}\right)$, the main contribution is from the maximum of $V(\varphi)$

No necessity in the tunneling $\rho_0(\varphi)$ specifically