

09.07.18

# Spontaneous CP breaking in QCD and the axion potential



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COLLÈGE  
DE FRANCE  
—1530—

based on 1709.00731

(with P. Di Vecchia, G.C. Rossi & S. Yankielowicz)

Related work:

D. Gaiotto, Z. Komargodski, N. Seiberg, 1708.06806  
(as well as older papers by A. Smilga and by M. Creutz)

# Outline

- U(1)-CP problems & their mutually exclusive solutions
- $L_{\text{eff}}$  @ small  $m/\Lambda$  &  $1/N$  (w/  $mN/\Lambda$  fixed)
- CP-viol. @ arbitrary  $\theta$ : two non-viable solutions.

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- Spontaneous CP violation in QCD @  $\theta = \pi$
- $N_f = 1, N_f > 1$

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- $L_{\text{eff}}$  w/ axion & simult. resol. of U(1) & CP problems
- Relevance of QCD @  $\theta \neq 0$  for axion potential
- Relevance of above @  $0 < T < T_{\text{dec}} \sim T_{\text{ch}}?$

# PART 1

## U(1)-CP problems in QCD and their mutually exclusive solutions (a reminder of old stuff)

- Weinberg's 1973 argument for automatic CP
- Weinberg's formulation of U(1) problem
- Instantons may solve U(1) but reintroduce CP

# Weinberg's 1973 argument for automatic CP in QCD

Redefining the quark fields via a  $U(N_f)_L \times U(N_f)_R$  transformation we can rewrite the original mass term (coming from EW breaking and generically CP-violating) in an explicitly **CP-conserving** form:

$$L_{QCD} = \cdots + \bar{\psi}_{iR} m_{ij} \psi_{jL} + \text{h. c.} \Rightarrow \cdots + \sum_{i=1}^{N_f} m_i \bar{\psi}'_i \psi'_i$$

Weinberg knew about the **ABJ anomaly** but at that time a term proportional to  $F^{\wedge}F$  was considered irrelevant.

# Weinberg's formulation of U(1) problem

That same assumption (about the irrelevance of the topological charge density) was at the origin of the **U(1) problem**, since  $U(1)_A$  was spont. broken.

1. Why is the  $\eta$  much heavier than  $\pi$ ?
2. Why is the  $\eta'$  much heavier than the other 8 pseudo NG bosons?
3. Why are the lightest PS (as opposed to V) mesons in approximately **unmixed** SU(3) **reps**?

# Topological charge may solve U(1) but then reintroduces CP problem

Instantons (more generally topological charge fluctuations) may solve the U(1) problem (see below) but then falsify Weinberg's 1973 argument by reintroducing the danger of **CP violation** in the strong interactions (violating bounds on  $D_n$ )

Weinberg's argument still allows to **lump** all the **CP-violation** in a **single** (but now relevant) **term**:

$$L_{QCD} = \dots - \bar{\theta} \frac{g^2}{32\pi^2} F^a \tilde{F}^a \equiv \dots - \bar{\theta} Q$$

$$\bar{\theta} \equiv \theta + \arg \det m$$

Effective Lagrangians  
provide a simple way to  
study strong CPV

$L_{\text{eff}}$  @ small  $m/\Lambda$  &  $1/N$  (w/  $mN/\Lambda$  fixed)

(Di Vecchia and GV, Rosenzweig et al. Nath & Arnowitt, Witten...~1980)

Spontaneous breaking:  $U(N_f)_L \otimes U(N_f)_R \rightarrow U(N_f)_V$

$$U^\dagger U = \frac{F_\pi^2}{2} \quad U = \frac{F_\pi}{\sqrt{2}} e^{i\sqrt{2}\Phi/F_\pi} \quad ; \quad \Phi = \Pi^a T_{ij}^a$$

$$L = \frac{1}{2} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi}{2\sqrt{2}} \text{Tr} [\mu^2 (U + U^\dagger)] \\ + \frac{Q^2}{2\chi_{YM}} + \frac{i}{2} Q \text{Tr} [\log U - \log U^\dagger] - \theta Q.$$

$$\mu_{ij}^2 = \mu_i^2 \delta_{ij} = -2m_i \langle \bar{\psi} \psi \rangle F_\pi^{-2} \delta_{ij} \quad \int d^4x Q(x) = \nu$$

NB: we have put **all CPV** in the  $\theta$  angle



The  $\mu_i^2$  are nothing but the **PNGB masses** in the **absence** of **anomaly** effects (hence in presence of U(1) problem)

$$M_{ij}^2 = \frac{1}{2}(\mu_i^2 + \mu_j^2) ; i, j = 1, 2, \dots, N_f$$

For  $i \neq j$  these are the physical masses (e.g.  $\pi^\pm$ )

The **U(1)<sub>A</sub> anomaly** is implemented in:

$$L = \frac{1}{2}\text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{F_\pi}{2\sqrt{2}}\text{Tr} [\mu^2(U + U^\dagger)] \\ + \frac{Q^2}{2\chi_{YM}} + \frac{i}{2}Q\text{Tr} [\log U - \log U^\dagger] - \theta Q .$$

through the **U(1)<sub>A</sub>** transformation:  $Q \rightarrow Q$  and

$$\frac{i}{2}\text{Tr} (\log U - \log U^\dagger) \rightarrow \frac{i}{2}\text{Tr} (\log U - \log U^\dagger) + 2\alpha N_f$$

The term  $Q^2/(2\chi_{YM})$  corresponds to the crucial assumption that the **topological susceptibility** in pure **YM** theory is **non vanishing** at **large N** (and of order  $\Lambda^4$ : **not** a dilute-instanton effect).

By now lattice calculations have given strong evidence that this is the case.

Furthermore, the **numerical value** of  $\chi_{YM}$  is in (even too good an) agreement with the phenomenologically preferred value (GV-EW '79):  $\chi_{YM} \sim (180 \text{ MeV})^4$

Focusing on the “Cartan” PNCB and introducing

$$\phi_i = -\sqrt{2} \frac{\Phi_{ii}}{F_\pi}$$

we get, after **integrating out** the heavy field **Q**,

$$V(\phi_i) = -\frac{F_\pi^2}{2} \sum_{i=1}^{N_f} \mu_i^2 \cos \phi_i + \frac{\chi_{YM}}{2} \left( \theta - \sum_{i=1}^{N_f} \phi_i \right)^2$$

whose stationary points are the solutions of

$$\mu_i^2 \sin \phi_i - a \left( \theta - \sum_{j=1}^{N_f} \phi_j \right) = 0 \quad ; \quad i = 1, \dots, N_f \quad a = \frac{2\chi_{YM}}{F_\pi^2} .$$

They depend on  $\theta$  and on  $\epsilon_i = \frac{\mu_i^2}{a} = -\frac{m_i \langle \bar{\psi} \psi \rangle}{\chi_{YM}} = O\left(\frac{mN}{\Lambda}\right)$

**Fluctuations** around a given solution are described by the effective action:

$$\begin{aligned}
 L = & -V(\hat{\phi}_i) + \frac{1}{2} \text{Tr} \left( \partial_\mu \hat{U} \partial^\mu \hat{U}^\dagger \right) + \frac{F_\pi^2}{2} \text{Tr} \left[ \mu^2(\theta) \left( \cos \left( \frac{\sqrt{2}}{F_\pi} \hat{\Phi} \right) - 1 \right) \right] - \frac{a}{2} \left[ \text{Tr} \left( \hat{\Phi} \right) \right]^2 \\
 & + \chi_{YM} \left( \theta - \sum_{j=1}^{N_f} \hat{\phi}_j \right) \text{Tr} \left[ \sin \left( \frac{\sqrt{2}}{F_\pi} \hat{\Phi} \right) - \frac{\sqrt{2}}{F_\pi} \hat{\Phi} \right] \\
 & + \frac{1}{2\chi_{YM}} \left[ \hat{Q} - \chi_{YM} \frac{\sqrt{2}}{F_\pi} \text{Tr} \hat{\Phi} \right]^2,
 \end{aligned} \tag{2.14}$$

where  $\mu_{ij}^2(\theta) \equiv \mu_i^2 \cos \hat{\phi}_i \delta_{ij}$ .

$$a = \frac{2\chi_{YM}}{F_\pi^2}.$$

**Solving U(1) problem**

**CP-violating term**

For  $N_f$  massless

quarks **WV formula:**

$$M_s^2 = N_f a$$

CPV @ arbitrary  $\theta$ :  
two non-viable solutions

If we want to kill the  $CP$ -violating term we need

$$\chi_{YM} \left( \theta - \sum_{i=1}^{N_f} \hat{\phi}_i \right) \text{Tr} \left[ \sin \left( \frac{\sqrt{2} \hat{\Phi}}{F_\pi} \right) - \frac{\sqrt{2} \hat{\Phi}}{F_\pi} \right] = 0 \quad \text{where}$$

$$\mu_i^2 \sin \phi_i - a \left( \theta - \sum_{j=1}^{N_f} \phi_j \right) = 0 \quad ; \quad i = 1, \dots \quad a = \frac{2\chi_{YM}}{F_\pi^2} .$$

For generic  $\theta$  only two possibilities

1.  $\chi_{YM} = 0$  , but then we have a  $U(1)$  problem
2.  $\mu_i = 0$  (for at least one  $i$ ) , but it's bad for  $CA$

We are forced to have  $\theta = 0$  or  $\pi$ .

# PART 2

Spontaneous CP violation in QCD @  $\theta = \pi$

$$N_f = 1, N_f > 1$$

$$N_f = 1$$

$$\frac{V}{aF_\pi^2} = -\epsilon \cos \phi + \frac{1}{2}(\theta - \phi)^2 ; \quad \epsilon \equiv \frac{\mu^2}{a}$$

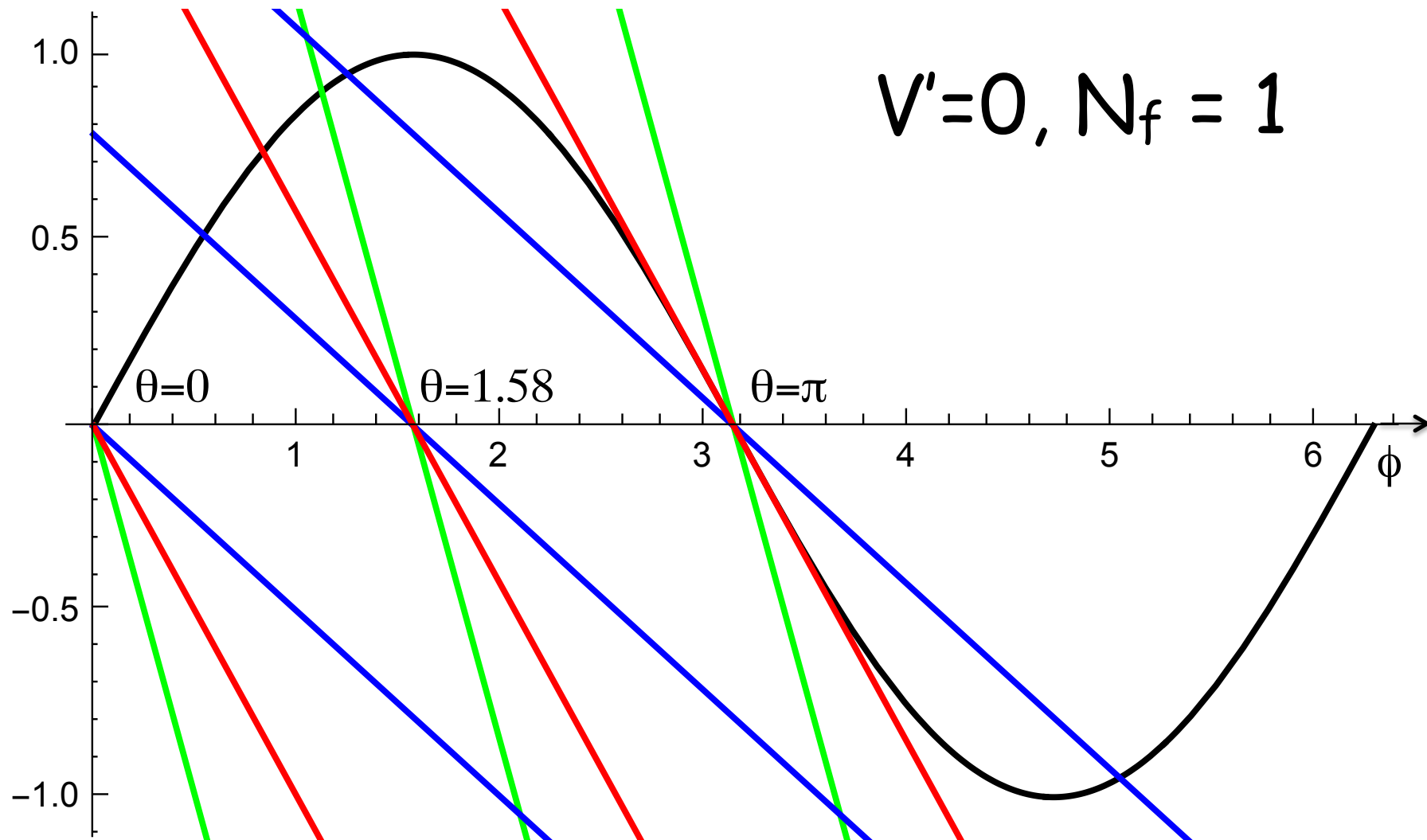
$$\frac{V'}{aF_\pi^2} = \epsilon \sin \phi - \theta + \phi ; \quad \frac{V''}{aF_\pi^2} = \epsilon \cos \phi + 1$$

The structure of the solutions depends crucially on the value of  $\epsilon$ .

Remember:

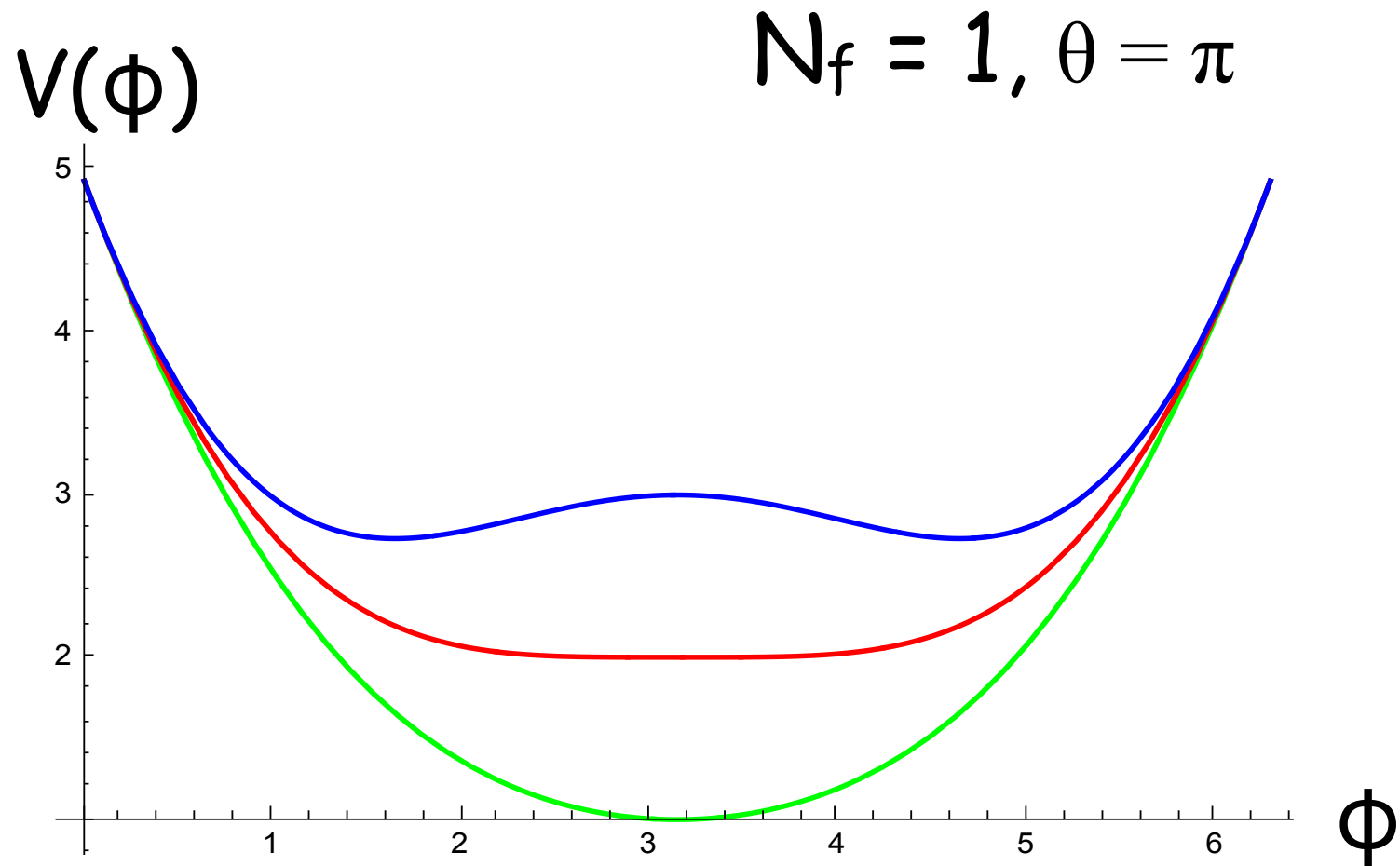
$$\epsilon_i = \frac{\mu_i^2}{a} = -\frac{m_i \langle \bar{\psi} \psi \rangle}{\chi_{YM}} = O\left(\frac{mN}{\Lambda}\right)$$





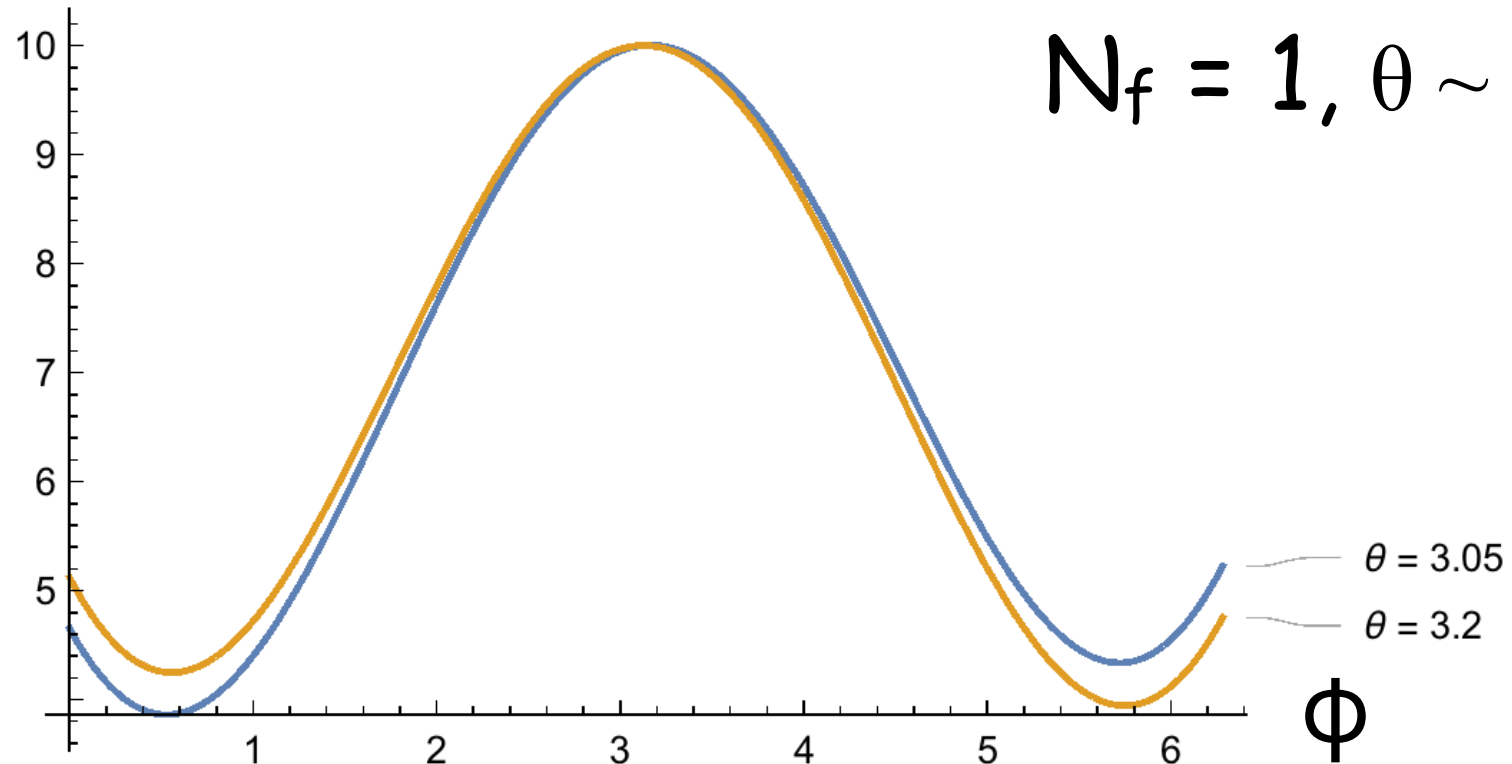
$\varepsilon < 1$ :green lines ;  $\varepsilon > 1$ :blue lines ;  $\varepsilon = 1$ :red lines

Alternatively, we can look at  $V(\phi)$  for  $\theta \sim \pi$



$\varepsilon < 1$ : green curve ;  $\varepsilon > 1$ : blue curve ;  
 $\varepsilon = 1$ : red curve

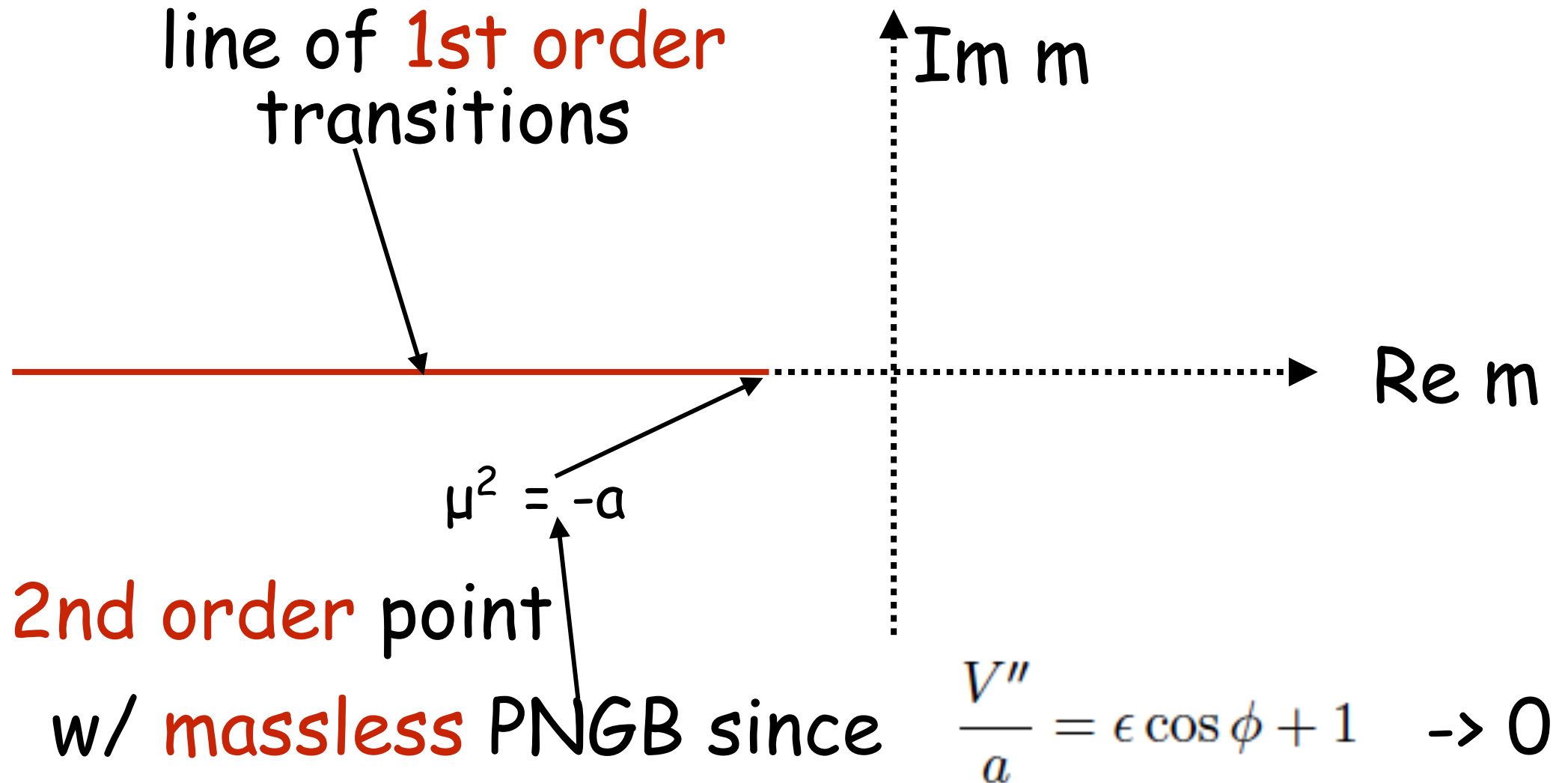
$V(\phi)$



ground state **jumps** by a finite amount as one goes through  $\theta = \pi$

# Complex-m plane

( $\theta = 0$ , CPV lumped in  $\arg m$ )  
as emphasized by GKS



A related phenomenon: @ 2nd-order point,  $\chi_{QCD}$  diverges. In general

$$\chi_{QCD} = \frac{\chi_{YM}}{1 + a \sum_{i=1}^{N_f} \frac{1}{\mu_i^2(\theta)}} = \left( \chi_{YM}^{-1} - \sum_{k=1}^{N_f} (m_k \langle \bar{\psi} \psi \rangle) \right)^{-1}$$

showing that  $\chi_{QCD} \rightarrow 0$  if any one of the quarks is massless. Here, instead, no quark is massless but a physical PNCB is.

$$N_f > 1$$

Only slightly more complicated.

The **2nd-order** critical point is given by

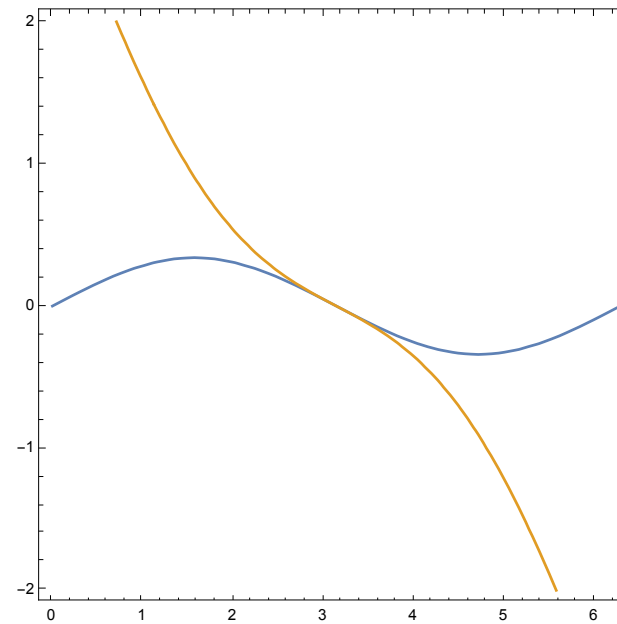
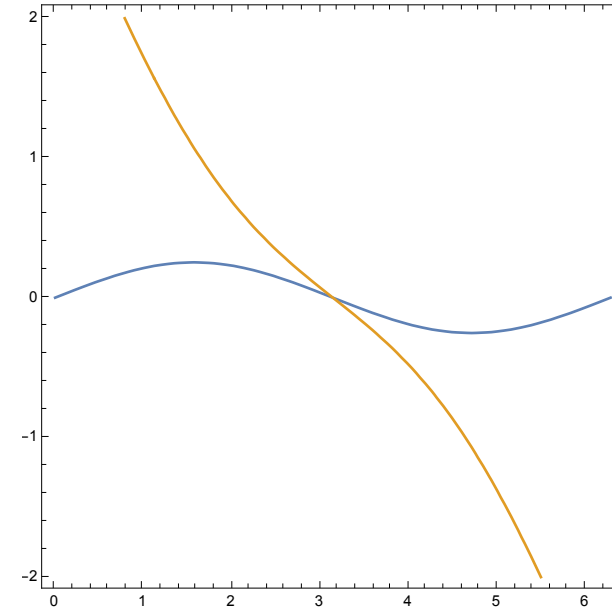
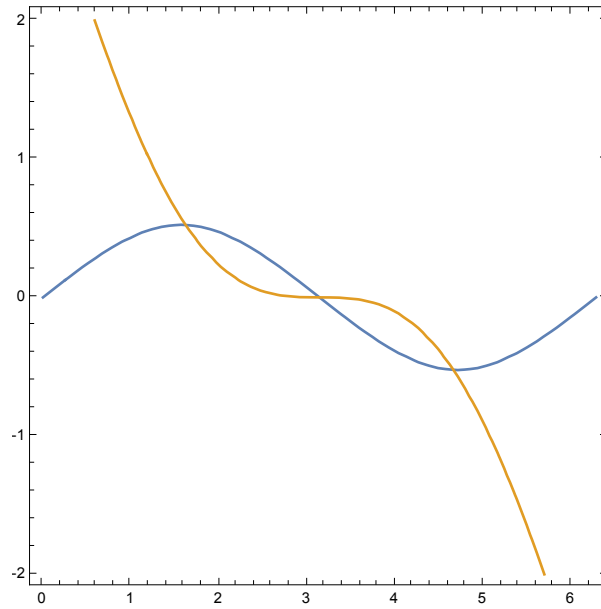
$$\sum_i \frac{1}{\mu_i^2(\theta = \pi)} + \frac{1}{a} = 0 \quad \text{i.e. in practice}$$

$$\sum_{i \neq 1} \frac{1}{\mu_i^2} + \frac{1}{a} = \frac{1}{\mu_1^2} ; \mu_1^2 \leq \mu_i^2 \quad \begin{array}{l} \text{noticed by} \\ \text{Creutz ('04) for} \\ N_f=3, a \gg \mu_i^2 \end{array}$$

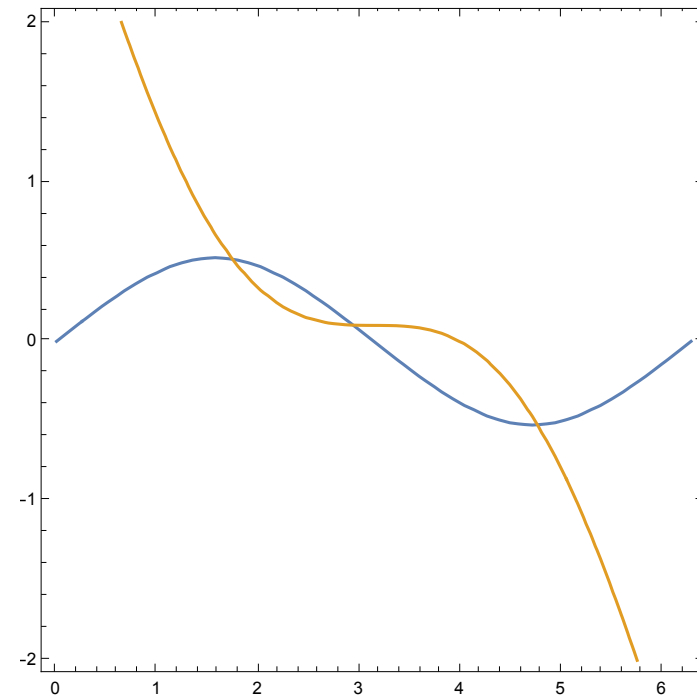
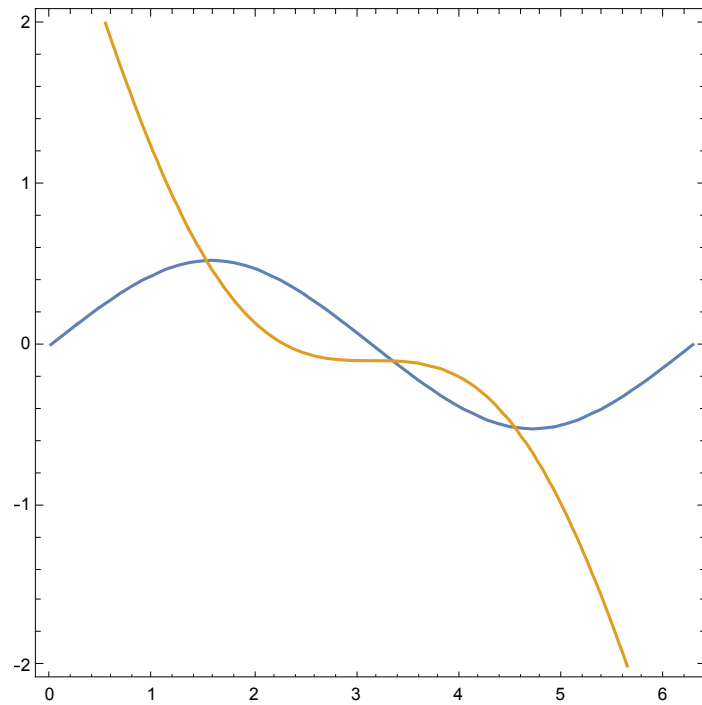
CP (un)broken if l.h.s. (<) > r.h.s.

In real QCD at  $T=0$ , CP is unbroken at  
 $\theta = \pi$  (but excluded by CA)

$N_f = 2, \theta = \pi, m_d = 2 m_u$   
(for different values of  $\mu_1^2/a$ )



$$N_f = 2, \theta \sim \pi, m_d = 2 m_u$$





A very special case:  $N_f = 2$ ,  $\theta = \pi$ ,  $m_d = m_u$

Looks straightforward: we are always in the **CP-broken** situation except if we send  $\varepsilon_1 = \varepsilon_2$  to zero.

However, as first noted by **Smilga ('99)**, at  $\theta = \pi$  the potential develops a flat direction @  $O(\varepsilon)$  (with  $\phi_1 + \phi_2 = \pi$ )

$$V(\phi_i) = -\frac{F_\pi^2}{2} \sum_{i=1}^{N_f} \mu_i^2 \cos \phi_i + \frac{\chi_{YM}}{2} \left( \theta - \sum_{i=1}^{N_f} \phi_i \right)^2$$

In order to lift the flat direction we need to go to  $O(\varepsilon^2)$  which looks beyond control.

Smilga does it within a Skyrmon model and argues that  $CP$  is broken.

We can do better(?) since the terms we neglect are down by at least a  $1/N$  factor.

We conclude in favor of  $CP$  breaking at sufficiently large  $N$ .

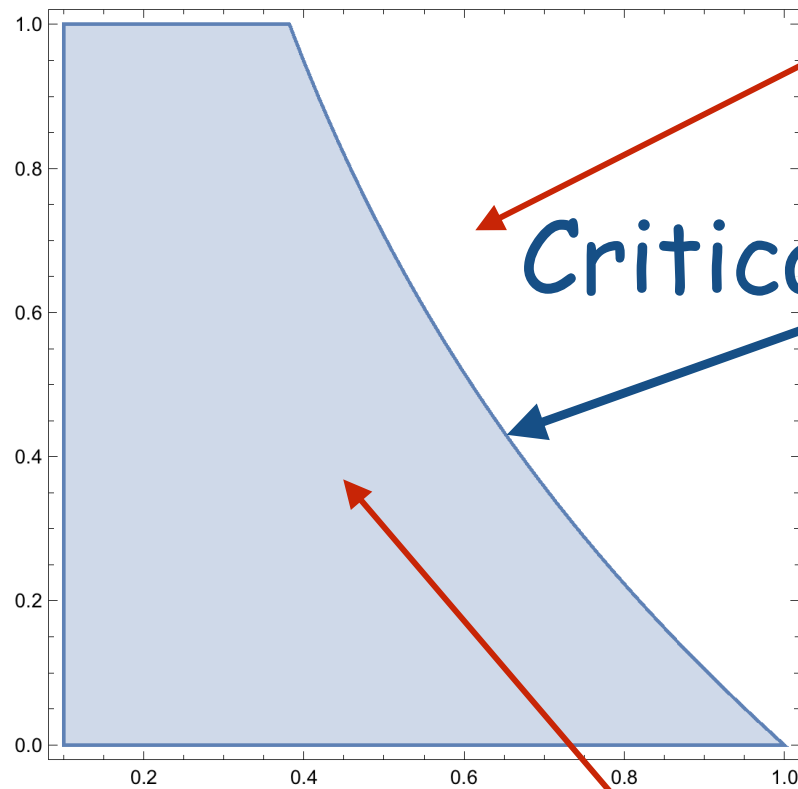
# General condition for SCPV

$$|D| > |D_c| \quad ; \quad |D_c^{1/N_f}| = (1 - \Sigma)\Pi^{-1/N_f} \quad ; \quad \Pi = \prod_{i=2}^{N_f} \rho_i \quad \Sigma \equiv \sum_{i=2}^{N_f} \rho_i$$

$$D \equiv \det(\mu^2/a^2)$$

broken CP

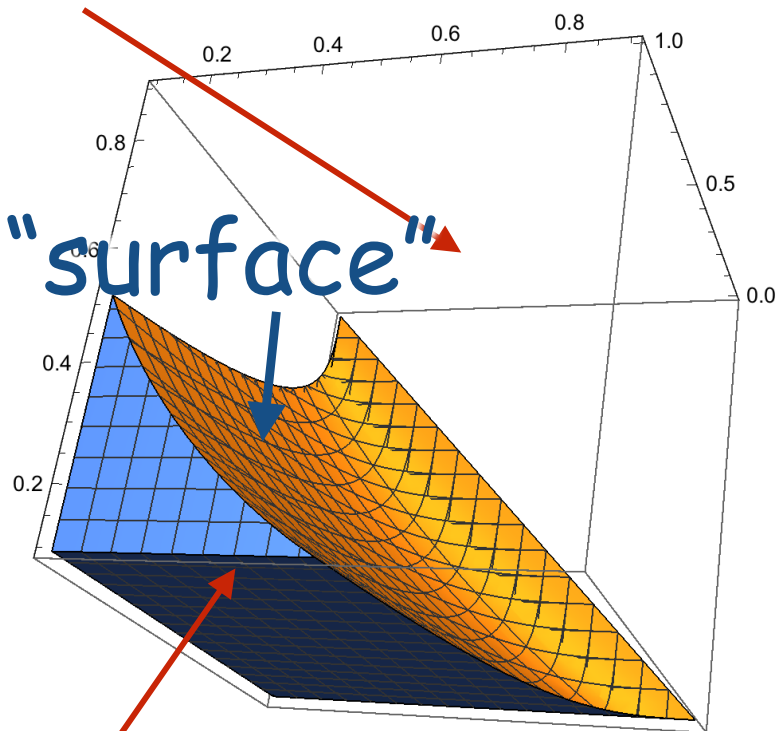
$$\rho_i \equiv \mu_1^2 / \mu_i^2$$



$N_f = 2$

$N_f=2$

unbroken CP



$N_f = 3$

$N_f=3$

Our results satisfy "**AC-decoupling**" in a stronger-than-usual sense:

if a quark is much **heavier than  $\Lambda/N$**  the problem reduces to the one in which the "heavy" quark is removed.

=> smooth **connection with YM** (where there are interesting results by GKKS).

# PART 3

Adding a generic axion to QCD

An **axion** solves the strong CP problem exactly **like a massless quark** would do.

The NGB corresponding to the  **$U(1)_{PQ}$**  would be exactly massless if the anomaly were ineffective.

Phenomenological problems force **scale** of  $U(1)_{PQ}$  breaking to be **sufficiently large**

Simultaneous **resolution of  $U(1)$  & CP** problems with axion in  $L_{\text{eff}}$  language

(see e.g. **Di Vecchia & Sannino**, 1310.0954)

$$L = \frac{1}{2} \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{2} \partial_\mu N \partial^\mu N^\dagger + \frac{F_\pi}{2\sqrt{2}} \text{Tr} (\mu^2 (U + U^\dagger)) + \frac{Q^2}{2\chi_{YM}} \\ + \frac{i}{2} Q [\log U - \log U^\dagger + \alpha_{PQ} (\log N - \log N^\dagger)] - \theta Q .$$

$$L = \frac{1}{2} \sum_{i=1}^{N_f} \partial_\mu v_i \partial^\mu v_i + \frac{F_\pi^2}{2} \sum_{i=1}^{N_f} \mu_i^2 \cos \left( -\phi_i + \frac{\sqrt{2}}{F_\pi} v_i \right) + \frac{Q^2}{2\chi_{YM}} \\ + \frac{1}{2} (\partial_\mu \alpha)^2 - Q \left( \theta - \sum_{i=1}^{N_f} \phi_i - \beta + \frac{\sqrt{2}}{F_\pi} \sum_{i=1}^{N_f} v_i + \frac{\alpha_{PQ} \sqrt{2}}{F_\alpha} \sigma \right) ,$$

$$N = \frac{F_\alpha}{\sqrt{2}} e^{i \frac{\sqrt{2}}{F_\alpha} \alpha} \quad \alpha = \sigma - \frac{\sqrt{2} \alpha_{PQ}}{F_\alpha} \beta$$

NB:  $U(1)_{PQ}$  is **only broken** expl. by the **anomaly**!

Upon **integrating out Q** we get:

$$V(\phi_i, \beta) = -\frac{F_\pi^2}{2} \sum_{i=1}^{N_f} \mu_i^2 \cos \phi_i + \frac{\chi_{YM}}{2} \left( \theta - \sum_{i=1}^{N_f} \phi_i - \beta \right)^2$$

and therefore:

$$-\frac{F_\pi^2}{2} \mu_i^2 \sin \phi_i + \chi_{YM} (\theta - \sum_i \phi_i - \beta) = 0$$

$$\theta - \sum_i \phi_i - \beta = 0 \qquad \beta = \theta, \quad \phi_i = 0$$

The mass<sup>2</sup>-matrix  
of fluctuations  
reads

$$b \equiv \frac{\alpha_{PQ} F_\pi}{F_\alpha} \ll 1$$

$$A = \begin{pmatrix} b^2 a & ba & ba & ba & \dots & ba \\ ba & \mu_1^2 + a & a & a & \dots & a \\ ba & a & \mu_2^2 + a & a & \dots & a \\ \dots & \dots & \dots & \dots & \dots & \dots \\ ba & a & a & a & \dots & \mu_{N_f}^2 + a \end{pmatrix}$$



from which we get:

$$\det (p^2 \delta_{ij} - A_{ij}) = p^2 \prod_{i=1}^{N_f} (p^2 - \mu_i^2) \left[ 1 - a \left( \sum_{i=1}^{N_f} \frac{1}{p^2 - \mu_i^2} + \frac{b^2}{p^2} \right) \right]$$

$$= \prod_{i=1}^{N_f+1} (p^2 - M_i^2) , \quad \det A = ab^2 \prod_{i=1}^{N_f} \mu_i^2 = \prod_{j=1}^{N_f+1} M_j^2$$

$$M_{axion}^2 = \frac{b^2}{\frac{1}{a} + \sum_{i=1}^{N_f} \frac{1}{\mu_i^2}}$$

=> standard expression if  $a \gg \mu_1^2, \mu_2^2$

# Why is QCD @ $\theta \sim \pi$ relevant for axion potential?

$\beta$  is like a dynamical  $\theta$ .

There is periodicity  $2\pi$  in both.

Considering the axion potential near the boundary of its periodicity range is analogous to studying QCD for  $\theta \sim \pi$

Standard axion potential (PdV&GV) is obtained by integrating out the PNCB in the chiral limit ( $\epsilon_i \ll 1$ )

For the realistic case of two non-degenerate light quarks it takes the (periodic and smooth) form:

$$V_{axion}(\sigma) = -F_\pi^2 \sqrt{(\mu_1^2 + \mu_2^2)^2 - 4\mu_1^2\mu_2^2 \sin^2 \left( \frac{\alpha_{PQ}\sigma}{\sqrt{2}F_\alpha} \right)} + O(\mu_i^2/a)$$

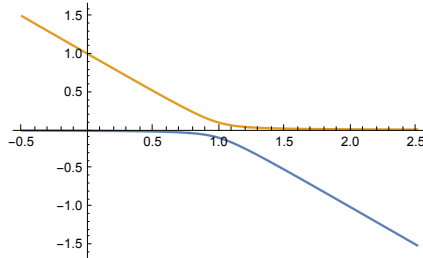
However we have seen that, at (near) the 2nd-order phase trans. points, a **PNGB becomes massless** (very light) at  $\theta = \pi$ .

Integrating it out becomes tricky. There is also **strong mixing** between the (bare) axion and the light PNGB...

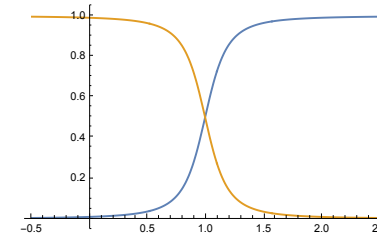
Eigenvalues

$N_f = 1$

Eigenvectors



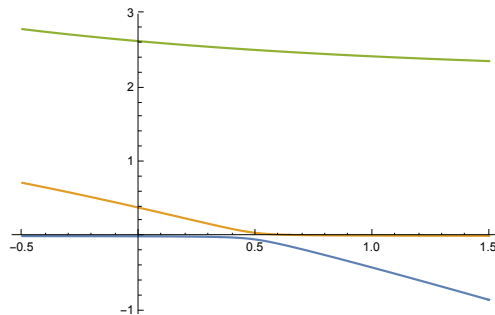
$\mu^2/a$



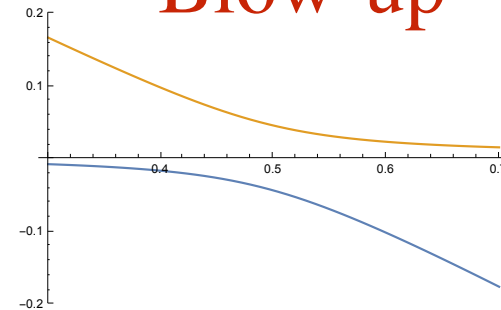
$\mu^2/a$

$N_f = 2$

Eigenvalues

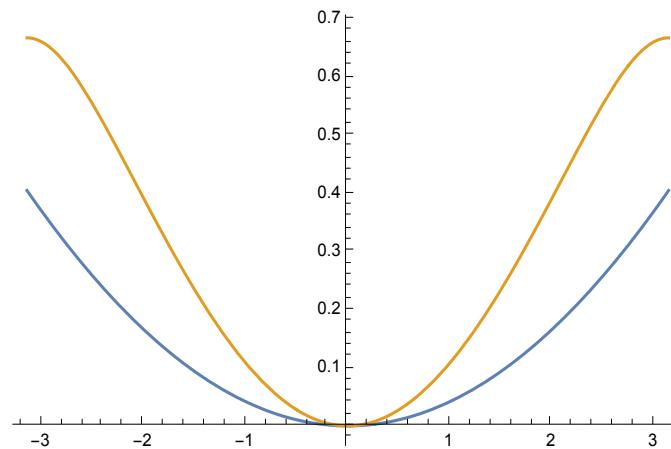
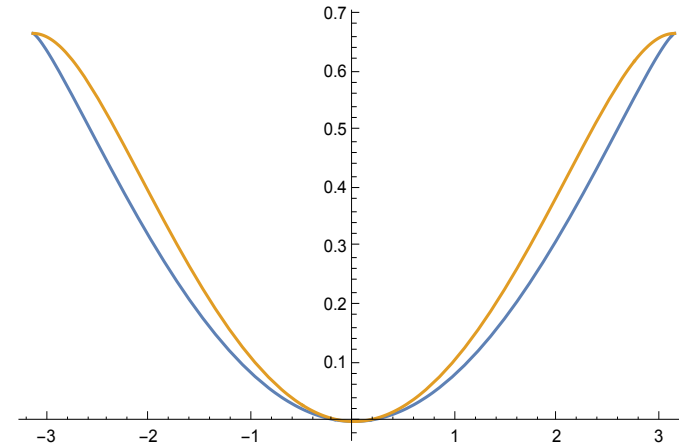
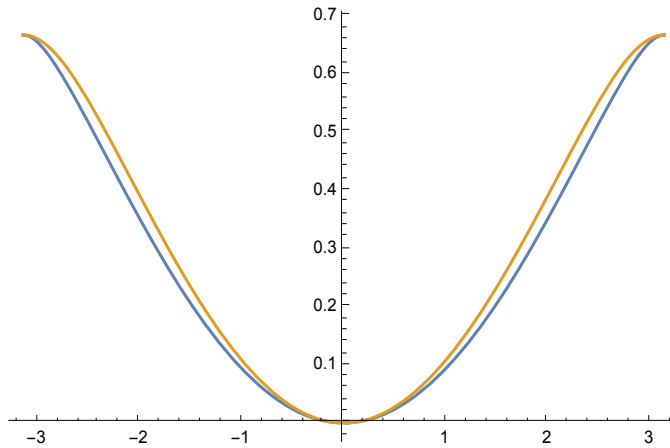


Blow-up



The nature of the **tachyonic state changes** abruptly along the critical point

In the vicinity and beyond such critical point the axion potential is **strongly modified at its boundary**  
(develops kink)



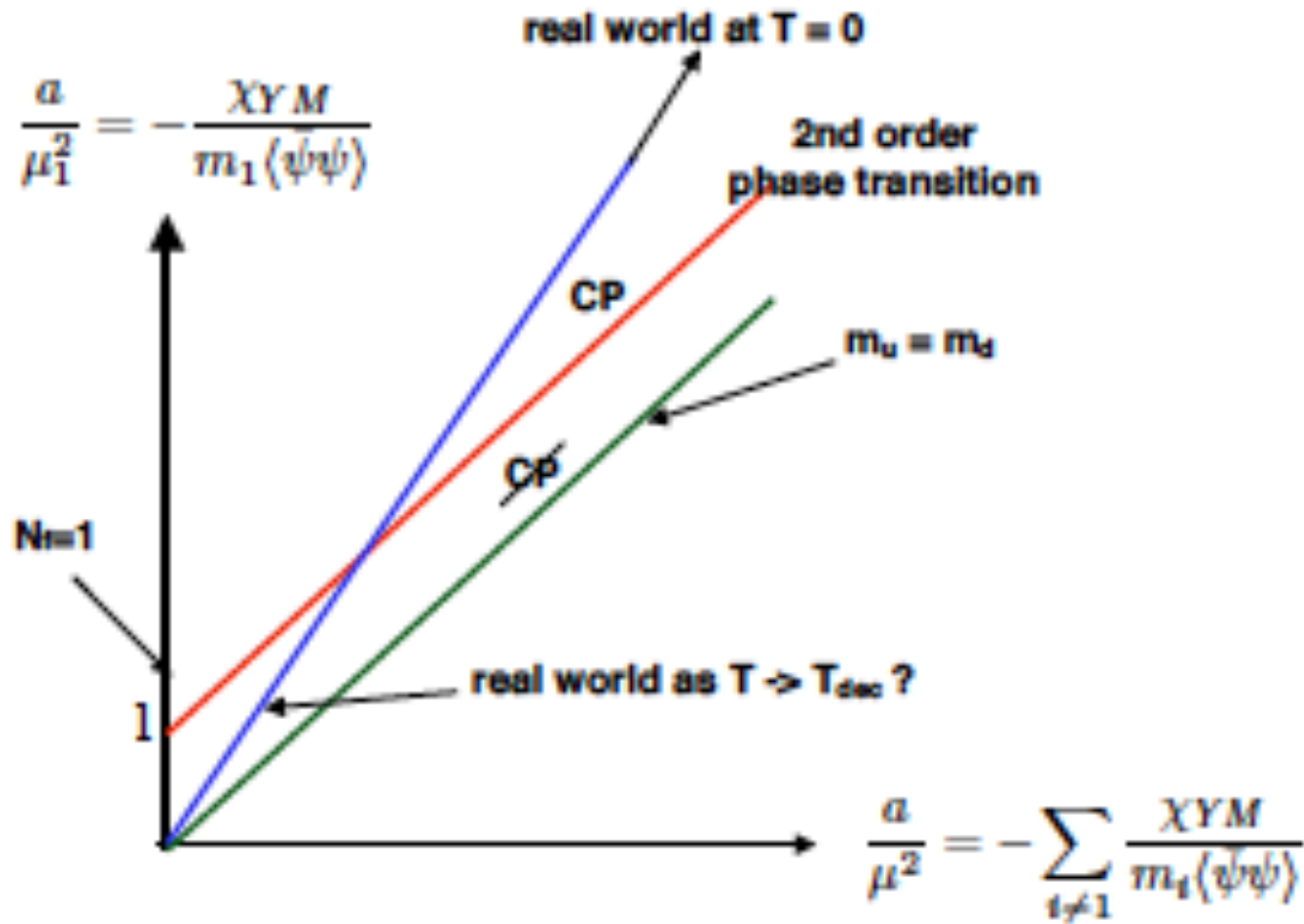
Actually, in the vicinity of the critical point, one cannot really integrate out all the PNCBs and the description in terms of  $V(\sigma)$  should be replaced by one using a  $V(\sigma, v_{\text{light}})$  where  $v_{\text{light}}$  is the combination of PNCBs that becomes massless @ critical point.

Physics-wise the relevant question is: can the critical point be approached/reached in QCD **at some finite T**? Since quark-mass ratios are T-independent the question becomes: can

$$\frac{m\langle\bar{\psi}\psi\rangle}{\chi_{YM}}$$

grow (by  $\sim O(10)$ ) as we approach  $T_c$ ?

Relevance of above @  $0 < T < T_{\text{dec}} \sim T_{\text{ch}} ?$



Need dedicated lattice calculations measuring, at the same time, the **quenched quark condensate** and  **$\chi_{YM}$**  as a function of  **$T$  near  $T_c$**

# Summary

- There is **either** a  **$U(1)$**  or a strong  **$CP$**  problem in QCD. All evidence is in favor of the latter.
- QCD at  $\theta \sim \pi$  shows a very **rich structure** of possible **phase transitions**. These can be studied explicitly in the **small- $m$ , large- $N$**  limit (with  **$mN/\Lambda$**  fixed) by effective Lagrangian techniques.



- When the strong-CP problem is solved a la PQ the properties of QCD near  $\theta \sim \pi$  have a bearing on the properties of the axion potential near the boundary of its periodicity interval.
- More lattice results are necessary before deciding whether important modifications of the standard axion potential are needed as one approaches  $T_c$