$\frac{M}{N}$ and $\frac{1}{N}$ Anomalous Dimensions in Chern-Simons Theories

V. Guru Charan

Dayalbagh Educational Institute

Based on arXiv:1404.7849, arXiv:1610.8472 and arXiv:1711.11300

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Outline

Chern-Simons Theories

- In three dimensions apart from Maxwell theory, we have Chern-Simons action as follows:

$$S_{\mathrm{CS}} = \frac{k}{4\pi} \int \mathrm{tr}(A \wedge dA + \frac{2}{3} A^3)$$

- For topological reasons, the strength of interactions in Chern-Simons theories is parameterized by an integer k, which cannot flow under RG because it is an integer.
- Chern-Simons interactions give rise to a large number of conformal field theories in three dimensions (without supersymmetry).

Level-Rank (Strong-Weak) Duality

- Pure Chern-Simons is an exactly solvable theory.
  Level-Rank Duality in Pure CS: $SU(N)_k \longleftrightarrow U(K)_{-N,-N}$
- Supersymmetric generalization of CS coupled to one boson and one fermion enjoys the following duality (Giveon, Kutasov,'08):
  $SU(N)_k \longleftrightarrow U(k+F/2)_{-(-N-F/2),-(-N-F/2)}$
- Another useful level-rank duality relates:
  $U(N)_{k,k+N} \longleftrightarrow U(k)_{-N,-k-N}$

Bosonization Duality

Another interesting duality arising in CS theories coupled to matter is Bosonization duality (Aharony, Gur-Ari, Yacoby, 2012):

$$\begin{array}{ccc}
S = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(D_\mu \bar{\phi} D^\mu \phi + \frac{\lambda_6}{N^2} (\bar{\phi}\phi)^3 \right) & \xleftrightarrow[\text{Bosonization Duality}]{} & S_{\mathrm{crit}} = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(\bar{\psi} \not{D} \psi + \frac{1}{N} \sigma_f \bar{\psi}\psi \right) \\
\Big\downarrow \text{RG flow} & & \Big\downarrow \text{RG flow} \\
S_{\mathrm{crit}} = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \left(D_\mu \bar{\phi} D^\mu \phi + \frac{1}{N} \sigma_b \bar{\phi}\phi \right) & \xleftrightarrow[\text{Bosonization Duality}]{} & S = \frac{ik}{4\pi} S_{\mathrm{CS}} + \int d^3x \, \bar{\psi} \not{D} \psi
\end{array}$$

In the large N limit the regular/critical bosonic theory at rank N_B and level k_B is conjectured to be dual to the critical/regular fermionic theory at rank N_F and level k_F with $k_F = -k_B$, $N_F = |k_B| - N_B$. In terms of the t Hooft coupling, the duality map takes the form $\lambda_F = \lambda_B - sgn(\lambda_B)$.

Gauge-Gravity Duality

$U(M) \times U(N)$ ABJ theory is dual to Vasiliev gravity when $M \ll N$.

$U(M) \times U(M)$ ABJM theory is dual to supersymmetric Einstein gravity.

It is natural to ask whether a similar duality holds when the Chern-Simons theory is non-supersymmetric.

This question motivated the study presented in this work.

Chern-Simons Theories

Chern-Simons action takes the following form in Euclidean theory:

$$
\begin{aligned}
S_{\text{CS}}^{\text{euc}} &= \frac{i}{4\pi}\int d^3x\ \epsilon^{\mu\nu\lambda}(k_M \text{tr}(A_\mu\partial_\nu A_\lambda + \frac{2}{3}A_\mu A_\nu A_\lambda) \\
&\quad - k_N \text{tr}(B_\mu\partial_\nu B_\lambda + \frac{2}{3}B_\mu B_\nu B_\lambda))
\end{aligned}
$$

The fermion action is:

$$\int d^3x\ \text{tr}\ \bar{\psi}\gamma^\mu D_\mu\psi$$

where covariant derivative is defined as:

$$D_\mu\psi = \partial_\mu\psi - iA_\mu\psi + i\psi B_\mu$$

We choose to work in light cone gauge as in this gauge ghosts decouple and cubic interactions are not present.

Large N Limit

There are two very different Large N expansions:

1. Large N vector models : Eg. Critical $O(N)$ vector model, Gross Neveu model
2. Large N matrix models : Eg. $\mathcal{N} = 4$ SYM, Large N QCD

Large N matrix models are not exactly solvable in Large N limit and their dynamics is much more interesting at strong coupling where there is a possibility of dual gravity description.

M/N expansion

Bifundamental theories are large-N matrix models, which are intractable at strong coupling.

The crucial (indirect) way of obtaining information about the large N limit of $U(N) \times U(N)$ theories at strong coupling is to instead consider a $U(N) \times U(M)$ theory, with $M \ll N$, in an M/N expansion.

M/N expansion

There are two special cases in M/N expansion:

- $M \ll N$ ($\alpha = 0$) which is an exactly solvable large N vector model.
- $M = N$ ($\alpha = 1$), which is a large-N field theory with matrix degrees of freedom (that preserves parity if $k_M = k_N$).

Perturbation theory in $\alpha = M/N$ takes us from the exactly solvable large N vector model to the unsolvable, but more interesting, large N matrix model.

M/N expansion

Let us define the following 't Hooft couplings of our $U(M) \times U(N)$ Chern-Simons theory coupled to bifundamental matter with levels k_M and k_N respectively, which will be held fixed in the large N limit.

$$\lambda_M = \frac{N}{k_M}, \ \lambda_N = \frac{N}{k_N}, \ \alpha = \frac{M}{N}$$

Various limits

We can consider the following three limits:

1. $\lambda_M = 0$, and calculate to all orders in λ_N
2. $\lambda_N = 0$, and calculate to all orders in λ_M
3. To all orders in λ_N and λ_M

Conformal Algebra

The conformal group is generated by : $K_\mu, P_\mu, D, M_{\mu\nu}$
Conformal group is represented by $|\Delta, s\rangle$
There are two representations of conformal group (if representation is unitary), for $s \geqslant 1$:

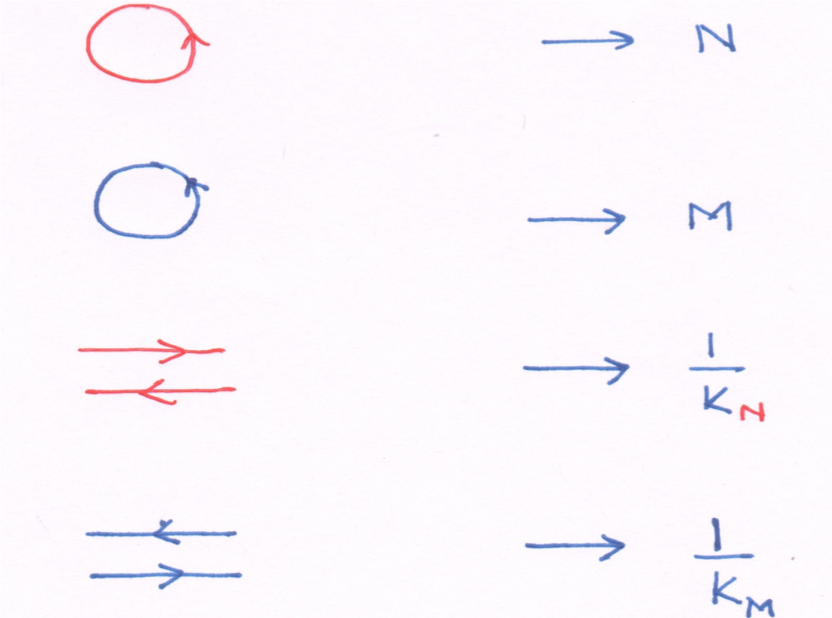
Long: $\Delta > s + 1$
Short: $\Delta = s + 1$

- Via AdS/CFT conjecture, spin of operator in CFT is equal to spin of the dual bulk field.
- And mass of the bulk field is proportional to the twist.

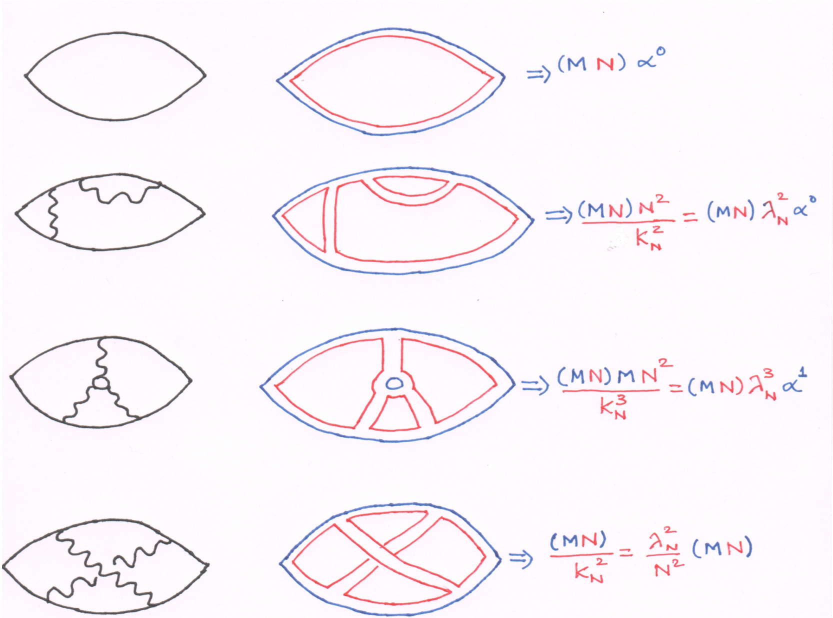
Problem statement: M/N expansion

Goal: To calculate anomalous dimensions of simplest unprotected single-trace primary operator, i.e., $\mathrm{Tr}[\bar{\psi}\psi]$ in limit $\lambda_N = 0$ and to first order in $\frac{M}{N}$, but to all orders in λ_M

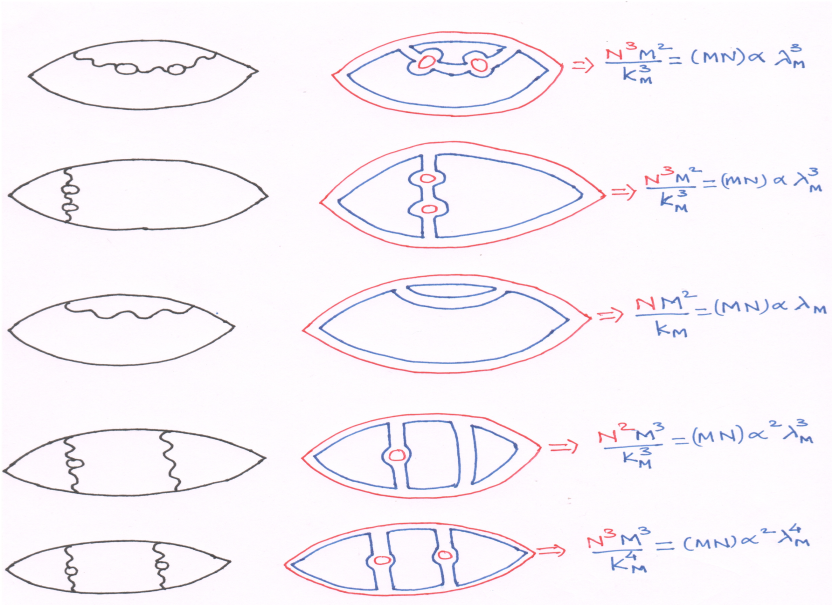
M/N expansion



M/N expansion



M/N expansion



Exact gauge propagator

G is the gauge propagator with an infinite series of self energy corrections.

$$\begin{pmatrix} G_{33} & G_{3+} \\ G_{+3} & G_{++} \end{pmatrix} = \frac{1}{N}\frac{2\pi^2 q_+^2}{q q_s^4}\frac{64}{64+\pi^2\lambda^2}\begin{pmatrix} \lambda^2 q_-^2 & \frac{8i\lambda}{\pi} q_- q - \lambda^2 q_- q_3 \\ -\frac{8i\lambda}{\pi} q_- q - \lambda^2 q_- q_3 & -q_s^2\lambda^2 \end{pmatrix}$$

Calculation of Anomalous dimensions

The following three Feynman diagrams contribute to first order in $\frac{M}{N}$:

Figure 1. Correction to the scalar vertex due to the $O(M/N)$ fermion self-energy.

Calculation of Anomalous dimensions

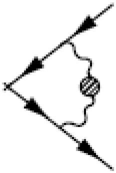


Figure 2. Another correction to the vertex.

Calculation of Anomalous dimensions

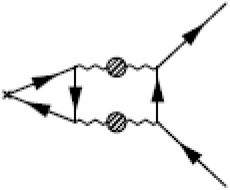


Figure 3. Additional contribution to the two-point function.

Anomalous dimensions

$$\delta = \alpha \frac{128\lambda^2(\pi^2\lambda^2 - 128)}{3(\pi^2\lambda^2 + 64)^2}$$

where δ is anomalous dimension of $\mathrm{Tr}[\bar{\psi}\psi]$.

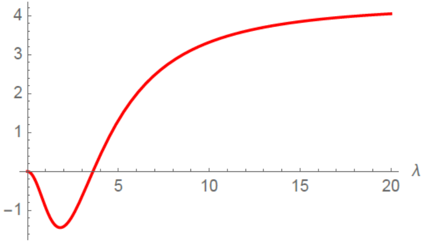


Figure 4: The anomalous dimension $\delta/(\frac{M}{N})$ of tr $\bar{\psi}\psi$, as a function of λ. It reaches a minimum of $\delta = -\frac{128}{9\pi^2}\frac{M}{N}$ at $\lambda\pi = 4\sqrt{2}$. The limiting value as $\lambda \to \infty$ is $\delta \to \frac{128}{3\pi^2}\frac{M}{N}$.

Anomalous dimensions of higher spin operators

A similar calculation for higher spin operators in light cone gauge yields the following results.

$$\delta_s(\lambda_M,0) = \frac{16}{3\left(\pi^2\lambda_M^2+64\right)}\left(3H_{s-\frac{1}{2}}+6\log(2)-\frac{2\left(11s^4+3s^3-13s^2+15s+2\right)}{4s^4-5s^2+1}\right) + \frac{64^2}{\left(\pi^2\lambda_M^2+64\right)^2}\frac{(s-2)(s-1)^2}{2\left(4s^4-5s^2+1\right)},\ \text{for even } s, \tag{1}$$

and

$$\delta_s(\lambda_M,0) = \frac{16\lambda_M^2\left(3H_{s-\frac{1}{2}}+6\log(2)+\frac{4-22s^2}{4s^2-1}\right)}{3\left(\pi^2\lambda_M^2+64\right)},\ \text{for odd } s. \tag{2}$$

$\frac{M}{N}$ Anomalous dimensions

At large s, we have:

$$\delta = \alpha \frac{16\lambda^2}{\pi^2\lambda^2 + 64} \log s$$

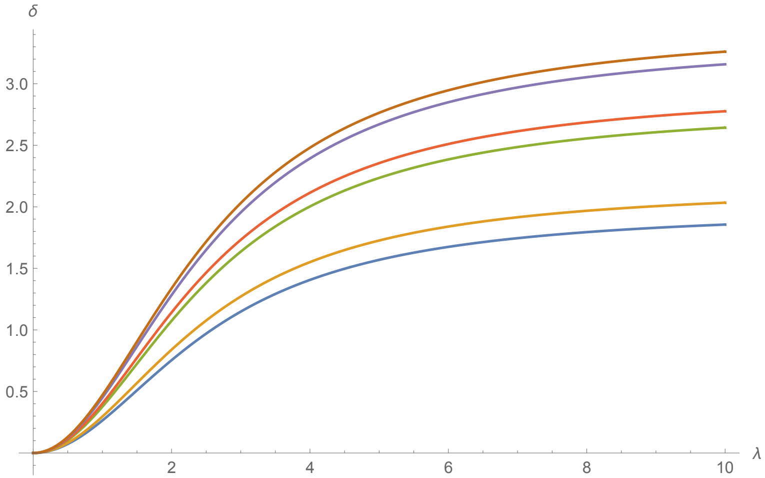


Figure 1: Anomalous dimension of the spin s current, for $s = 3, 4, 5, 6, 7, 8$ as a function of λ_M (to be multiplied by M/N).

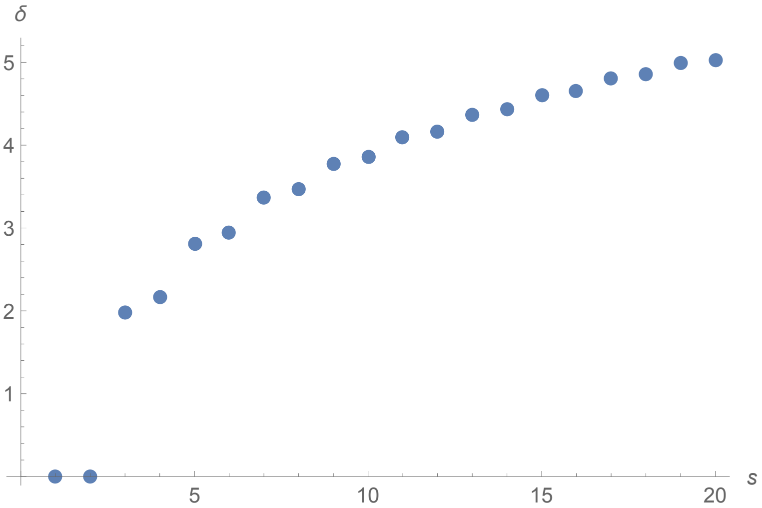


Figure 2: Anomalous dimensions of the current as a function of spin, in the strong coupling limit, (to be multiplied by M/N).

Large s limit

$$
\begin{aligned}
\delta_s =& \frac{M}{N}\left(\frac{8\lambda_M^2}{3\pi^2\lambda_M^2+192}\right)\left(6\log(4s)+6\gamma-11\right) \\
&+\frac{M}{N}\left(\frac{\lambda_M^2}{3\pi^2\lambda_M^2+192}\right)\left(-\frac{4}{s^2}-\frac{37}{20s^4}-\frac{4}{21s^6}+O\left(\frac{1}{s^7}\right)\right), \\
&\text{for odd } s,
\end{aligned}
\tag{3}
$$

and

$$
\begin{aligned}
\delta_s =& \frac{M}{N}\left(\frac{8\lambda_M^2}{3\pi^2\lambda_M^2+192}\right)\left(6\log(4s)+6\gamma-11\right) \\
&+\frac{M}{N}\left(\frac{512\lambda_M^2}{3\left(\pi^2\lambda_M^2+64\right)^2}\right)\left(-\frac{25}{2s^2}-\frac{3397}{160s^4}-\frac{1955}{84s^6}+O\left(\frac{1}{s^7}\right)\right) \\
&+\frac{M}{N}\left(\frac{8\pi^2\lambda_M^4}{3\left(\pi^2\lambda_M^2+64\right)^2}\right)\left(-\frac{3}{s}-\frac{1}{2s^2}-\frac{75}{4s^3}-\frac{37}{160s^4}+O\left(\frac{1}{s^5}\right)\right),
\end{aligned}
\tag{4}
$$

for even s.

$1/N$ Anomalous Dimensions

Chern-Simons theories coupled to vector matter are believed to be dual to higher spin gauge theories. (Giombi, Yin, '09,'10,'11)

Let us calculate the $\frac{1}{N}$ anomalous dimensions of single-trace primary operators.

Via conformal algebra the anomalous dimensions are determined by the divergence of the currents. (Giombi, Kirilin '16)

Higher Spin Spectrum

The single trace operators in the theory are easy to enumerate: there is one single trace primary operator for each spin $s > 0$, with scaling dimension $\Delta = s + 1$, and a scalar with $\Delta = 2$. These take the following schematic form:

$$j_0 = \bar{\psi}\psi$$

$$j_s \sim \bar{\psi}\gamma\partial^{s-1}\psi$$

They can be packaged into a generating function, where z is a null polarization vector:

$$\begin{aligned} J_{\mathrm{f}} &= \bar{\psi}(x) z \cdot \gamma f_{\mathrm{f}}(z \cdot \overleftarrow{\partial}, z \cdot \overrightarrow{\partial})\psi(x) \\ = f_{\mathrm{f}}(\hat{\partial}_1, \hat{\partial}_2)\bar{\psi}(x_1)\hat{\gamma}\psi(x_2)|_{x_1, x_2 \to x}, \; f_{\mathrm{f}}(u, v) &= e^{u-v}\frac{\sin(2\sqrt{uv})}{2\sqrt{uv}} \end{aligned}$$

In interacting theory derivatives are replaced by covariant derivatives:

$$J_{\mathrm{f}} = \bar{\psi}(x)\hat{\gamma} f_{\mathrm{f}}(\overleftarrow{\hat{D}}, \overrightarrow{\hat{D}})\psi(x) = \sum_{s=1}^{\infty} j_s^{\mathrm{f}}(x,z)\,,$$

$$\mathrm{j}_s^{\mathrm{f}}(x,z) = \textstyle\sum_{k=0}^{s} \frac{(-1)^{k+s+1}}{2s!} \begin{pmatrix} 2s \\ 2k+1 \end{pmatrix} \hat{D}_1^k \hat{D}_2^{s-k} \bar{\psi}(x_1)\hat{\gamma}\psi(x_2)|_{x_1,x_2\to x}\,.$$

Divergence of Higher Spin Currents

Currents with $s = 1$ and $s = 2$ are conserved in the interacting theory, but higher spin currents are conserved only in the free theory.

Matching scaling dimensions and spins, the divergence of the higher spin currents must take the following form, where $[j_s]$ denotes a conformal descendent of j_s.

$$\begin{aligned} \partial \cdot j_s &\equiv \sum_{s_1, s_2} \partial \cdot j_s \Big|_{s_1, s_2} + \sum_{s_1, s_2, s_3} \partial \cdot j_s \Big|_{s_1, s_2, s_3} \\ &= \sum_{s_1, s_2} \left(C_{s_1, s_2, s}(\tilde{\lambda}) \frac{1}{\tilde{N}} [j_{s_1}][j_{s_2}] \right) + \sum_{s_1, s_2, s_3} \left(C_{s_1, s_2, s_3, s}(\tilde{\lambda}) \frac{1}{\tilde{N}^2} [j_{s_1}][j_{s_2}][j_{s_3}] \right) \end{aligned}$$

We can fix the N and λ dependence of the functions above, using planar three-point functions.

Divergence of Higher Spin Currents

Let us focus on the double-trace term:

$$\langle j_{s_1} j_{s_2} \partial \cdot j_s \rangle \quad \sim \quad \frac{1}{\tilde{N}} C_{s_1,s_2,s}(\tilde{\lambda}) \langle j_{s_1} j_{s_1} \rangle \langle j_{s_2} j_{s_2} \rangle \,.$$

From the results of Maldacena-Zhiboedov (or, for lower spins, from summing planar diagrams (Gur-Ari, Yacoby), we have:

$$\langle j_{s_1} j_{s_2} \partial \cdot j_s \rangle \sim \tilde{N} \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} \,,$$

$$\langle j_{s_1} j_0 \partial \cdot j_s \rangle \sim \tilde{N} \tilde{\lambda} \,, \qquad \langle j_{s_1} j_{s_1} \rangle \sim \tilde{N} \,, \qquad s_1 \neq 0 \,,$$

$$\langle j_{s_1} \tilde{j}_0 \partial \cdot j_s \rangle \sim \tilde{N} \tilde{\lambda} \,, \qquad \langle j_0 j_0 \rangle \sim \tilde{N} \left(1+\tilde{\lambda}^2\right) \,, \qquad \langle \tilde{j}_0 \tilde{j}_0 \rangle \sim \tilde{N} \left(1+\tilde{\lambda}^2\right)$$

Putting these together:

$$C_{s_1,s_2,s}(\tilde{\lambda}) \sim \frac{\tilde{\lambda}}{1+\tilde{\lambda}^2} = \left(\tilde{\lambda} + \frac{1}{\tilde{\lambda}}\right)^{-1}$$

Constraining the divergence

We can also uniquely constrain the particular combination of descendents represented by $[j_{s1}][j_{s2}]$ up to an overall constant, by requiring that the divergence of j_s is a conformal primary.

$$\partial \cdot j_s \Big|_{s_1,s_2} = C_{s_1,s_2,s}[j_{s_1}][j_{s_2}]\,,$$

We illustrate this with combinations involving the scalar operator j_0. which is parity odd. Matching scaling dimensions and spin requires it to be of the following form:

$$\partial \cdot j_s \Big|_{s_1,0} = \sum_{m=0}^{p} c_m \partial_-^m j_{s_1} \partial_-^{p-m} \tilde{j}_0\,.$$

Constraining the divergence

We can rewrite this using radial quantization as:

$$\partial \cdot j_s\Big|_{s_1,0} \sim \sum_{m=0}^{p} c_m P_-^m \left|j_{s_1}\right\rangle P_-^{p-m} \left|\tilde{j}_0\right\rangle .$$

Acting on this with K_+ and using the conformal algebra:

$$\begin{aligned} 0 &= K_+ \partial \cdot j_s\Big|_{s_1,0} \\ &= \sum_{m=0}^{p} c_m \left(\left(K_+ P_-^m \left|j_{s_1}\right\rangle \right) P_-^{p-m} \left|\tilde{j}_0\right\rangle + P_-^m \left|j_{s_1}\right\rangle \left(K_+ P_-^{p-m} \left|\tilde{j}_0\right\rangle \right) \right) \\ &= \sum_{m=0}^{p} c_m \left(\left([K_+, P_-^m] \left|j_{s_1}\right\rangle \right) P_-^{p-m} \left|\tilde{j}_0\right\rangle + P_-^m \left|j_{s_1}\right\rangle \left([K_+, P_-^{p-m}] \left|\tilde{j}_0\right\rangle \right) \right) . \end{aligned}$$

$$[K_\delta, P_-^n] = 2inP_-^{n-1} \left(\eta_{-\delta} D + M_{-\delta} \right) + 2n(n-1)\eta_{-\delta} P_-^{n-1}$$

Constraining the divergence

We obtain:

$$\sum_{m=1}^{p} (m(m+2s_1)c_m)\, P_-^{m-1} \left| j_{s_1} \right\rangle P_-^{p-m} \left| \tilde{j}_0 \right\rangle + \sum_{m=0}^{p-1} (m-p)(m-p-1) c_m P_-^{m} \left| j_{s_1} \right\rangle P_-^{p-m-1} \left| \tilde{j}_0 \right\rangle = 0$$

This translates into a recurrence relation for the c_m:

$$c_m = \frac{-(m-p-1)(m-p-2)}{m(m+2s_1)} c_{m-1},$$

That can be solved to give:

$$c_m = (-1)^m \binom{s-s_1}{m} \binom{s+s_1-1}{m+2s_1} C_{s_1,\tilde{0},s}.$$

Constraining the divergence

The coefficients $C_{s_1 s_2 s}$ are structure constants that are nonzero only when $s_1 + s_2 < s$, and are closely related to the parity odd forms for the three point functions.

While we expect that they can be determined from first principles using the slightly broken higher spin symmetry, we fixed them using the classical equations of motion.

General Form for anomalous dimensions

Inserting this into the formula relating divergence and anomalous dimension, we find that they must take the following form:

$$\tau_s - 1 = \sum_{s_1 \neq 0} \left(C_{s_1,0,s}(\tilde{\lambda}) \right)^2 \alpha_{s_1,0,s} \frac{n_{s_1} n_0 (1+\tilde{\lambda}^2)}{\tilde{N} n_s} + \sum_{s_1, s_2 \neq 0} \left(C_{s_1,s_2,s}(\tilde{\lambda}) \right)^2 \alpha_{s_1,s_2,s} \frac{n_{s_1} n_{s_2}}{\tilde{N} n_s}$$

Simplifying, we find the anomalous dimensions take the form, where a_s and b_s are spin dependent constants:

$$\tau_s - 1 = \frac{1}{\tilde{N}} \left(a_s^{(F/B)} \frac{\tilde{\lambda}^2}{1+\tilde{\lambda}^2} + b_s^{(F/B)} \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2} \right)$$

Spin dependent constants

The spin dependent constants can be determined by summing contributions from all the double trace operators in the divergence. We find:

$$a_s = \sum_{s_1=s-2,s-4,\ldots} \frac{32(s^2-s_1^2)}{\pi^2 s\,(4s^2-1)} = \begin{cases} \frac{16(s-2)}{3\pi^2(2s-1)}\,, & s \ \text{even}\,, \\ \frac{32(s^2-1)}{3\pi^2(4s^2-1)}\,, & s \ \text{odd}\,, \end{cases}$$

$$b_s = \sum_{s_1+s_2=s-2,s-4,\ldots} \frac{64(s+s_1-s_2)!(s-s_1+s_2)!}{\pi^2 s\,(4s^2-1)\,(s-s_1-s_2-1)!(s+s_1+s_2-1)!}$$

$$= \begin{cases} \frac{2}{3\pi^2}\left(3\sum_{n=1}^{s}\frac{1}{n-1/2} + \frac{-38s^4+24s^3+34s^2-24s-32}{(4s^2-1)(s^2-1)}\right)\,, & s \ \text{even}\,, \\ \frac{2}{3\pi^2}\left(3\sum_{n=1}^{s}\frac{1}{n-1/2} + \frac{-38s^2+20}{4s^2-1}\right)\,, & s \ \text{odd}\,. \end{cases}$$

Conclusion

At large spin, logarithmic behaviour is observed due to the harmonic sum and the anomalous dimension is:

$$\tau_s - 1 \simeq \frac{1}{\tilde{N}} \frac{\tilde{\lambda}^2}{(1+\tilde{\lambda}^2)^2} \frac{2}{\pi^2} \log s = \frac{\lambda \sin(\pi\lambda)}{4\pi N} \log s \,.$$

This logarithmic behavior is a hallmark of gauge theory, and is expected from the recent bootstrap analysis in Alday and Zhiboedov, '16. (The logarithmic behaviour disappears at strong and weak coupling.)

These should correspond to classically massless higher-spin gauge fields that acquire a mass due to loop effects in a holographic description.

Short Summary

- We calculated $\frac{M}{N}$ anomalous dimensions of all higher spin operators in non-supersymmetric Chern-Simons theory coupled to bifundamental matter under different limits under two different limits $\lambda_M = 0$ and $\lambda_N = 0$ respectively.
- We calculated $\frac{1}{N}$ anomalous dimensions of all higher spin currents in non-supersymmetric Chern-Simons theory coupled to fundamental fermions.

Future Directions

- Explore the relevance of non-supersymmetric Chern-Simons theories in Condensed matter such as Quantum Hall physics.
- Study the higher spin spectrum at order $\frac{M^2}{N^2}$ and so on.
- Determine the M/N anomalous dimensions in the bifundamental theory to all orders in both λ_M and λ_N.
- Calculate mass of higher spin Vasiliev fields in four dimensions.
- Explore relation between the bootstrap calculations and the calculations presented here.
- Generalize bosonization duality to bifundamental Chern-Simons theories when both couplings are non-zero.

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Thank You

ありがとうございます