

# New applications of gravitational lensing to probe dark matter substructure

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(IAS; Einstein Fellow)

Astro Seminar @ Kavli IPMU  
June 2018

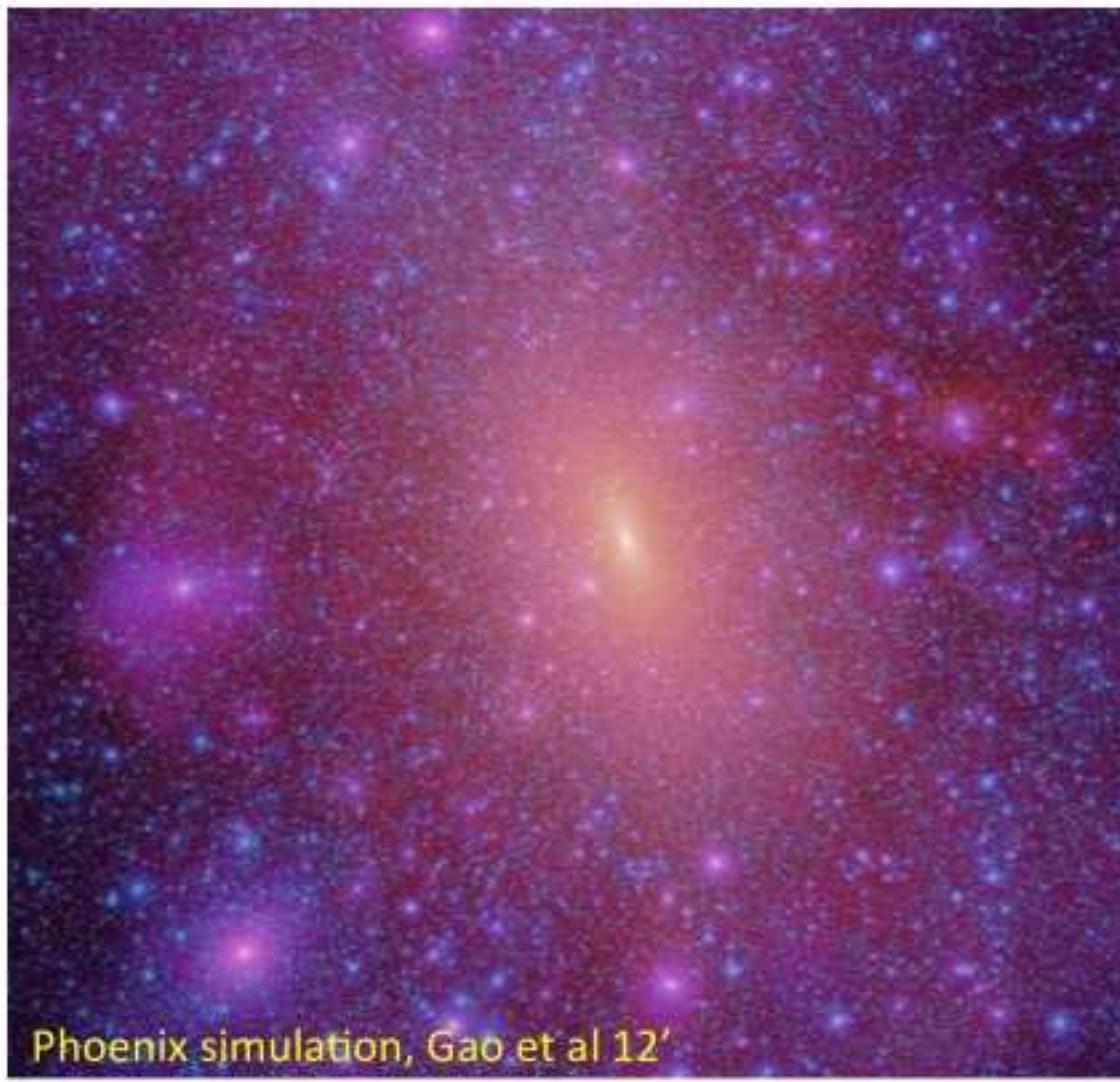


# The physical nature of the dark matter?



A galaxy cluster is held together by a lot dark matter

# The physical nature of the dark matter?



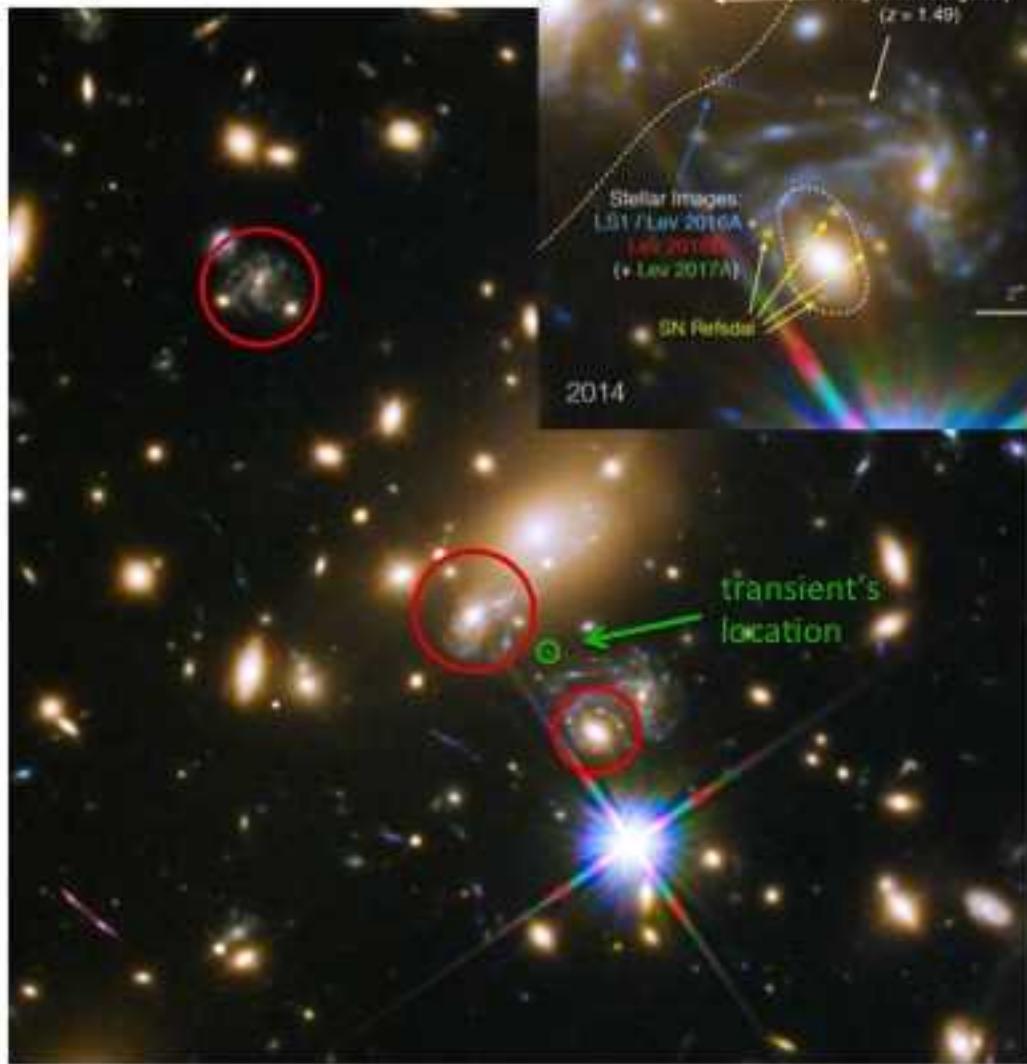
# Topics

- ◆ Caustic crossing stars in lensing clusters  
with Tejaswi Venumadhav (IAS), Jordi Miralda-Escudé (Barcelona)
- ◆ Probing dark matter substructure with highly magnified stars  
with Tejaswi Venumadhav (IAS), Jordi Miralda-Escudé (Barcelona), Alexander Kaurov (IAS)
- ◆ Diffractive lensing of gravitational waves by substructure  
With Shun-Sheng Li (NAOC), Barak Zackay (IAS), Shude Mao (Tsinghua), You-jun Lu (NAOC)

# Part I: Caustic crossing stars in strong lensing clusters

# An Intriguing Transient in MACS J1149

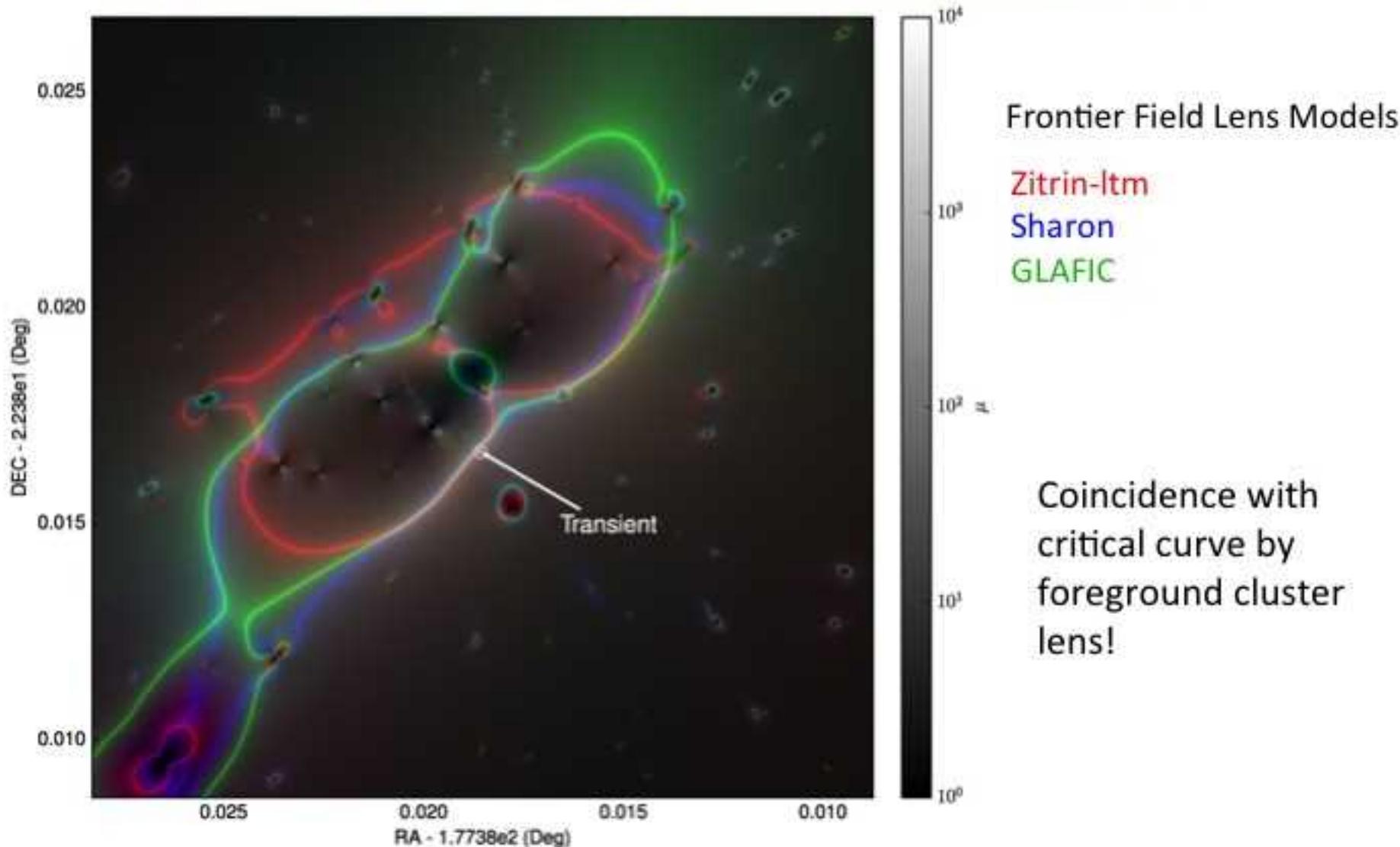
Kelly+ 2017



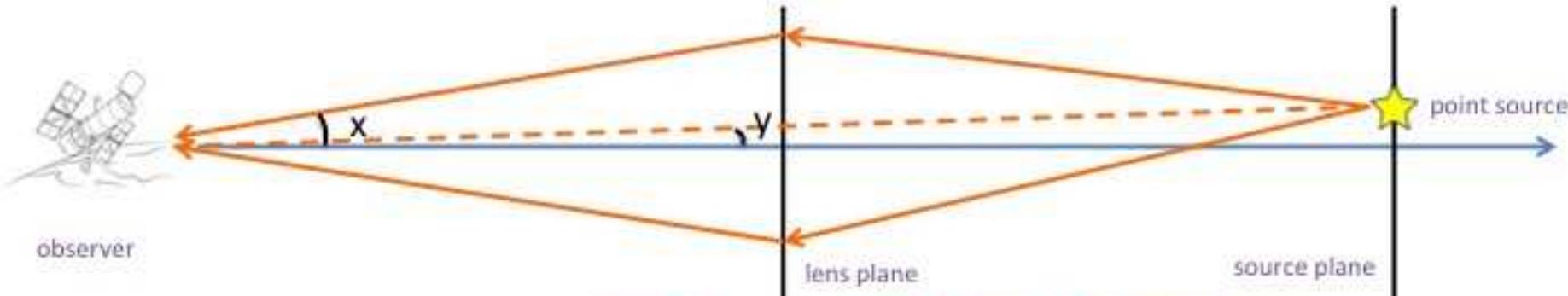
- Strong lensing cluster at  $z \sim 0.54$
- Host galaxy of SN Refdal at  $z \sim 1.5$
- Point-like slowly brightened to peak mag.  $< 26$
- Color resembles a **B-type star**; Balmer break detected;

# Gravitational lensing phenomenon?

magnification map predicted by mass reconstruction for source at  $z = 1.5$



# Caustic & Critical Curve



A pair of images

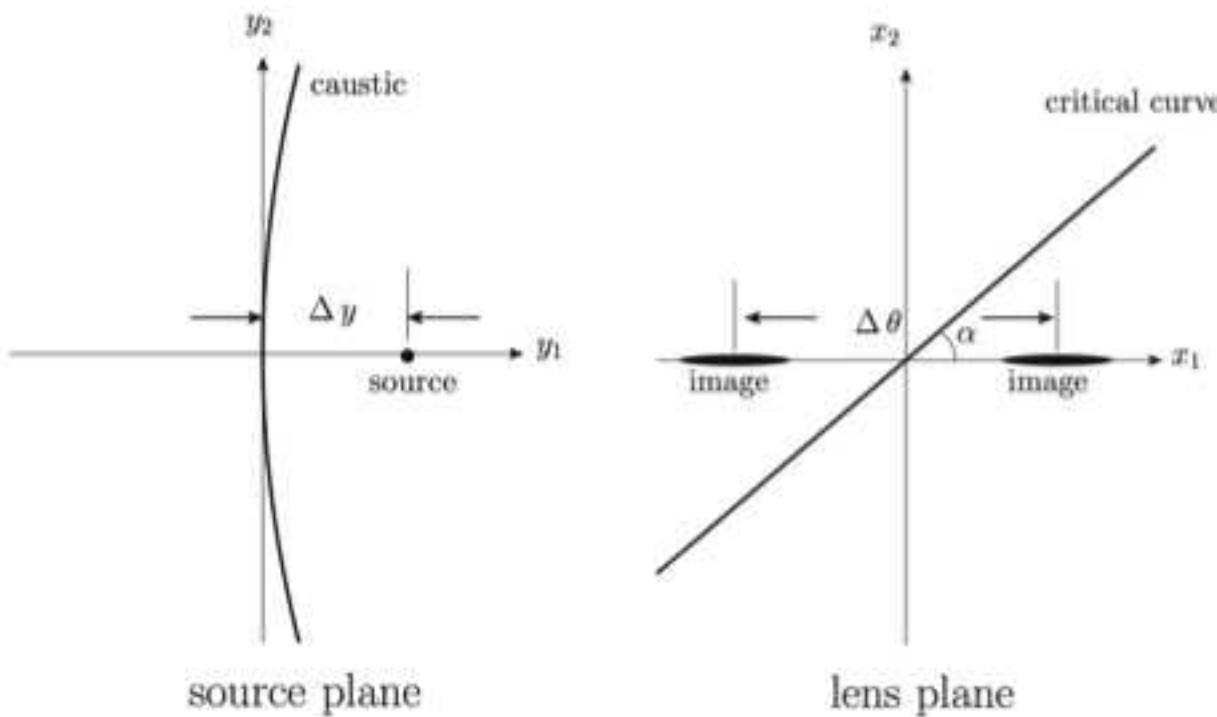
$$\delta\theta \sim 10 - 100 \text{ mas}$$

Extreme magnification factor

$$\mu \sim \frac{\theta_C}{\theta} \sim \text{few} \times 10^{2-3}$$

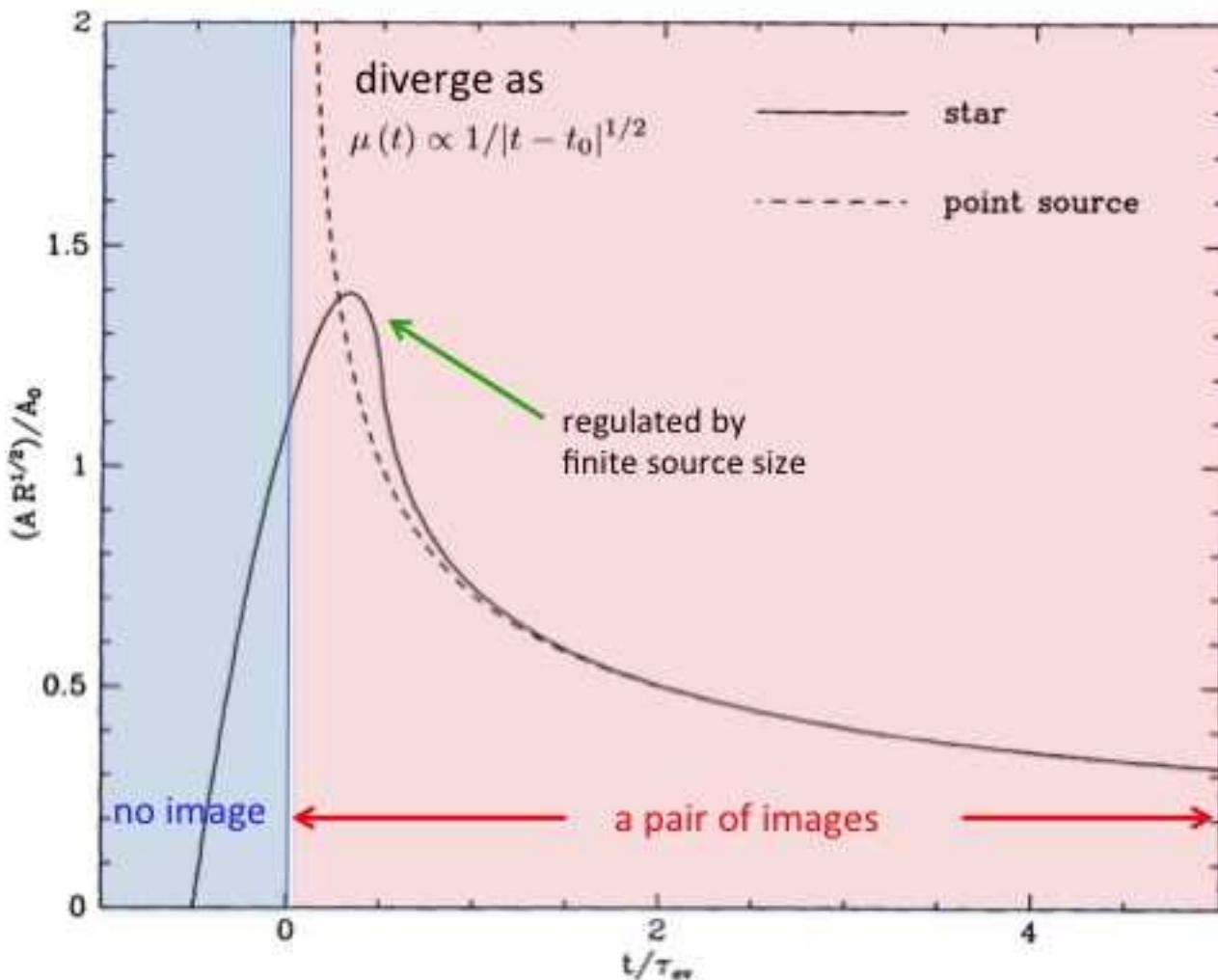
Individual star visible

$$L \simeq 10^{4-6} L_{\odot}$$



# (Unresolved) light curve

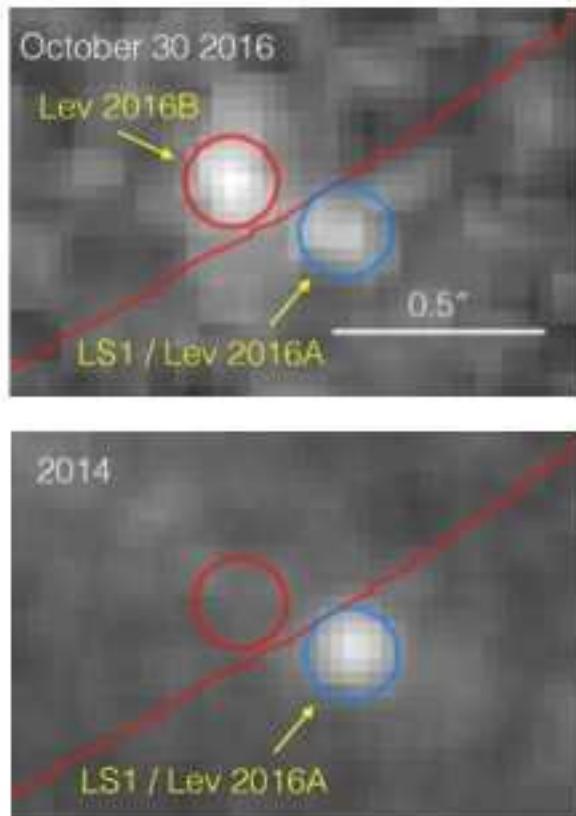
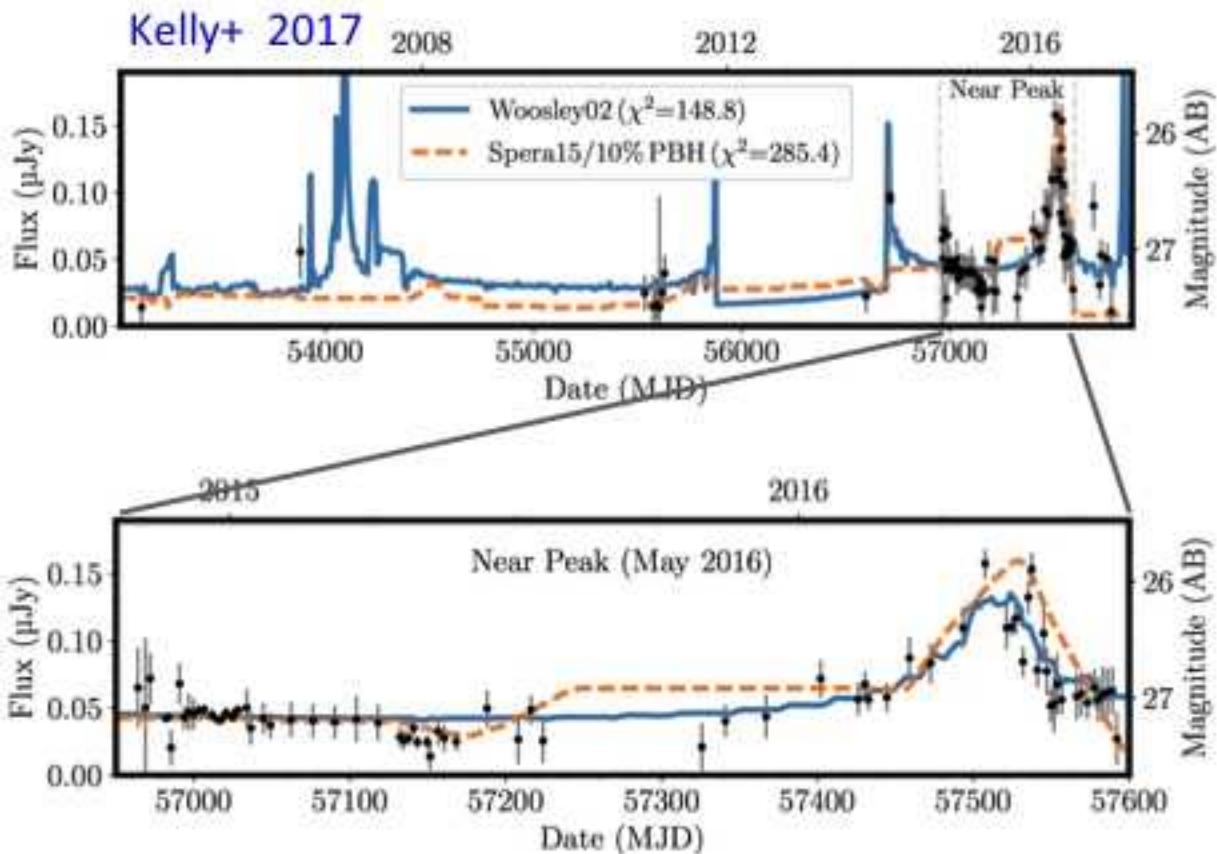
Miralda-Escudé 1991



infinite magnification factor regulated for two reasons:

- (1) Finite source size <  $10^7$  for one solar radius
- (2) Diffraction <  $10^9$  at  $\sim 10^{15}$  Hz

# Observed light curve



**What was observed looked very different:**

- Multiple brightening peaks observed; object persistently visible.
- An pair of images resolved; far from merging together

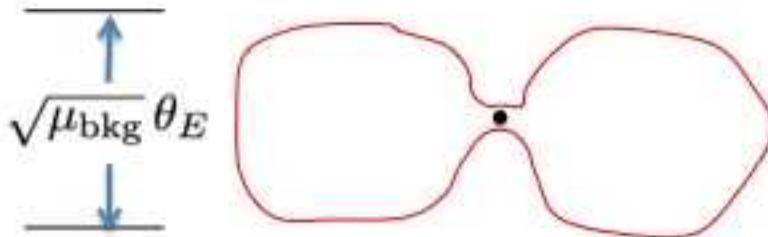
# Importance of micro-lensing

An isolated star: tiny Einstein angular scale,  
very improbable to cause micro-lensing

- $\theta_E = \left[ 4GM \frac{D_{LS}}{D_L D_S} \right]^{1/2} \sim 10^{-6} \text{ arcsec}$

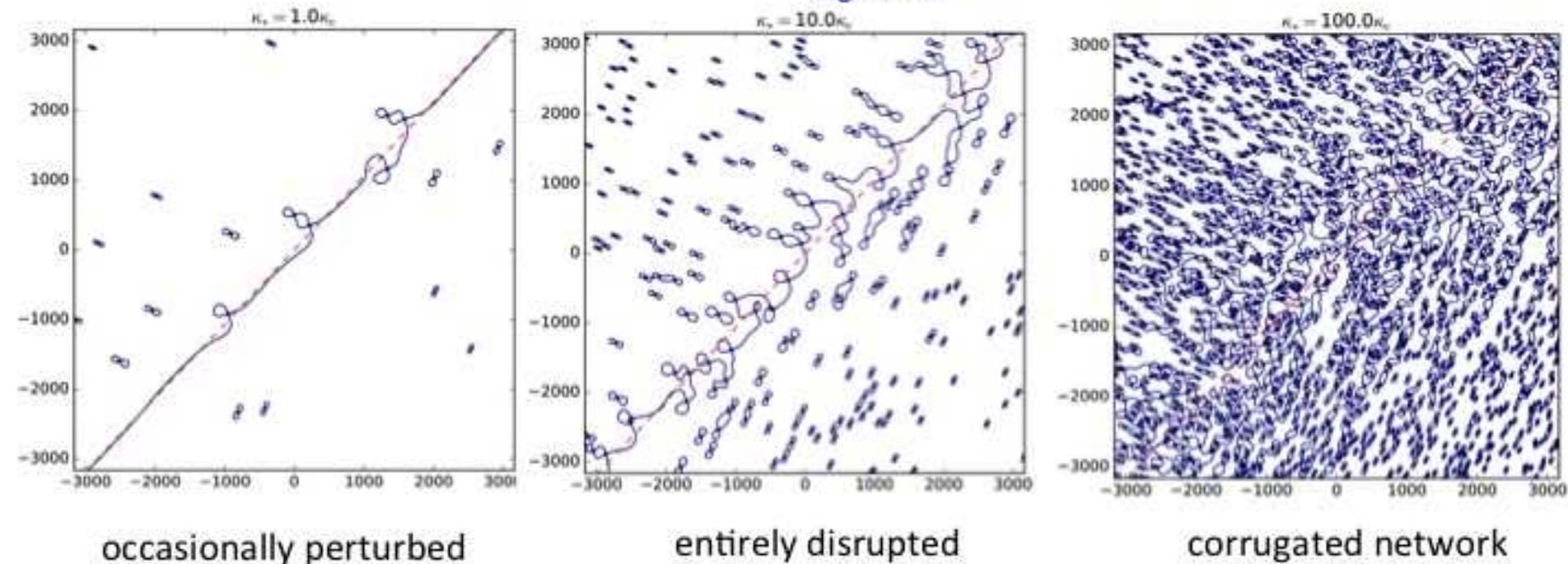
By contrast, cluster lensing has typical  
angular scale  $\sim \text{arcmin}$

Under the influence of a massive cluster:  
Bkg. Magnification and shear  $\sim 10^3 - 10^6$  near  
macro-critical curve!



# Effect of many microlenses

Diego+ 17  
 Venumadhav Dai & Mirada-Escudé 17  
 Oguri+ 17



**Critical surface density:**

$$\kappa_c \sim (\theta_E/\theta_C)^{2/3} \sim 10^{-5} \left( \frac{\theta_E}{10^{-6} \text{arcsec}} \right)^{2/3} \left( \frac{\text{arcmin}}{\theta_C} \right)^{2/3} \ll 1$$

**Surface density:**

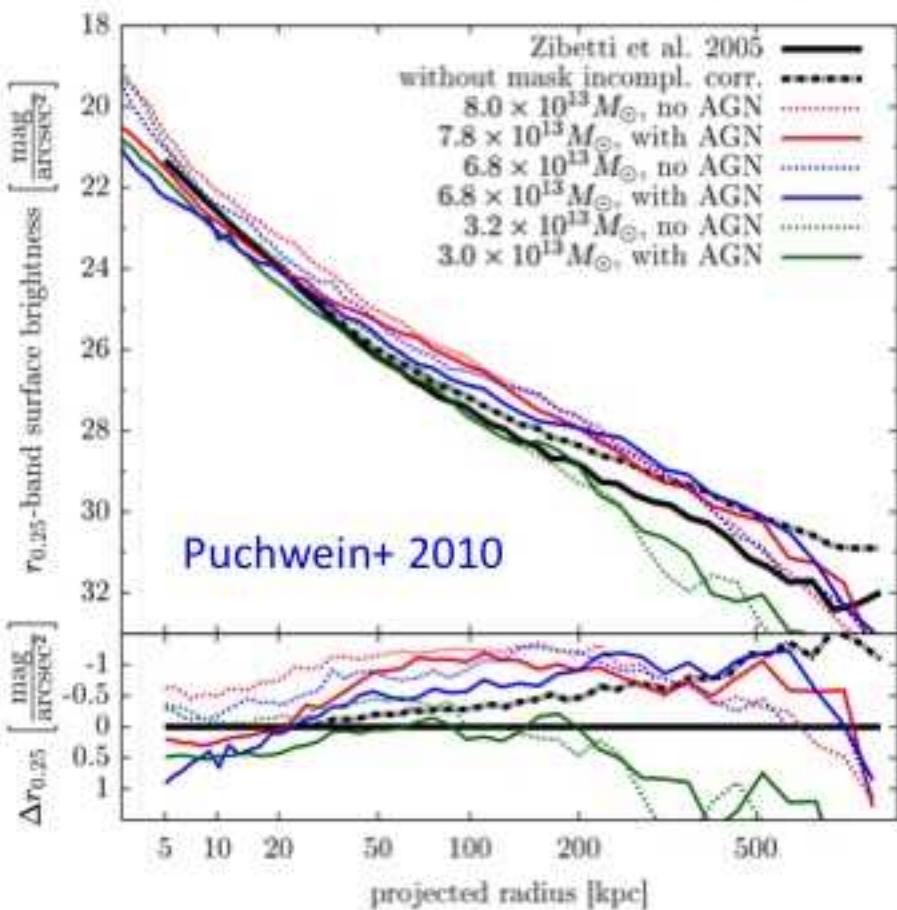
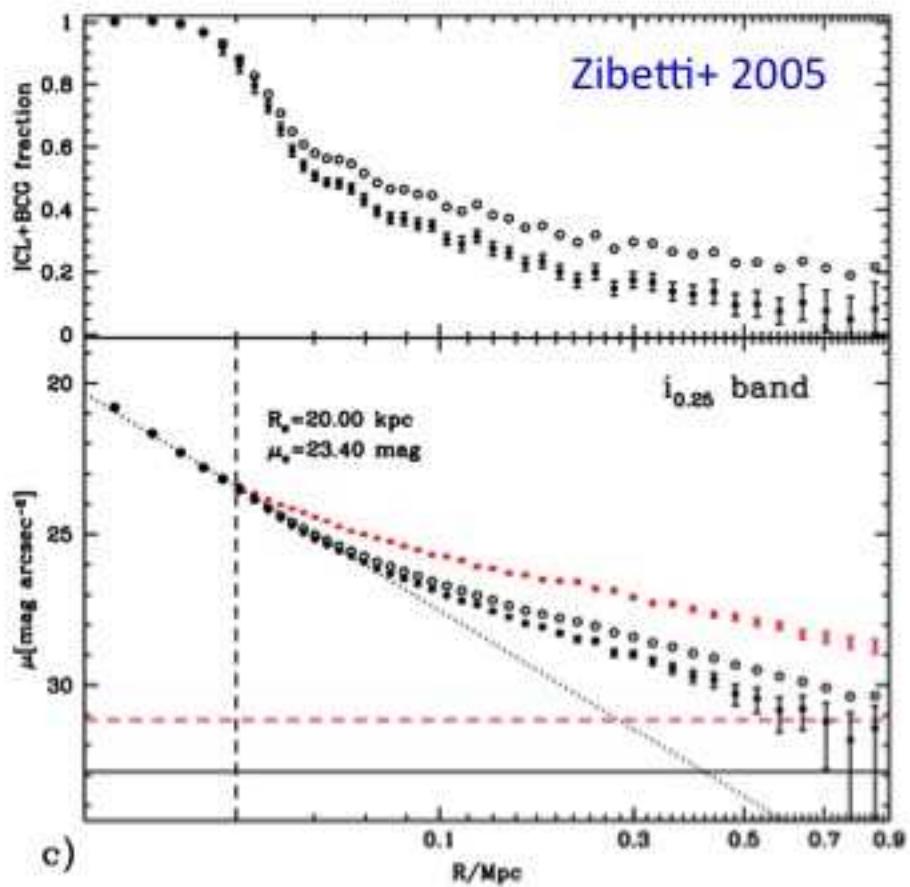
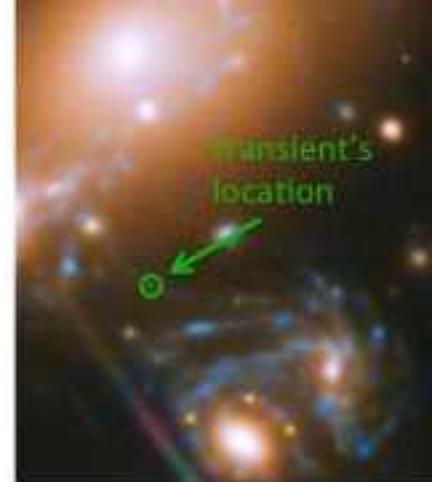
$$\kappa_* = \frac{\Sigma_F}{\Sigma_{\text{crit}}} = 4\pi G \Sigma_F \frac{D_{LS}}{D_L D_S} = 0.01 \left( \frac{\Sigma_F}{10^9 M_\odot/\text{arcsec}^2} \right)$$

# Intracluster stars as microlenses

Line of sight  $\sim 60\text{ kpc}$  away from the BCG center

Traverses a halo of **intracluster stars**!

Formation: merger assembly and tidal disruption



# Corrugated network of micro critical curves

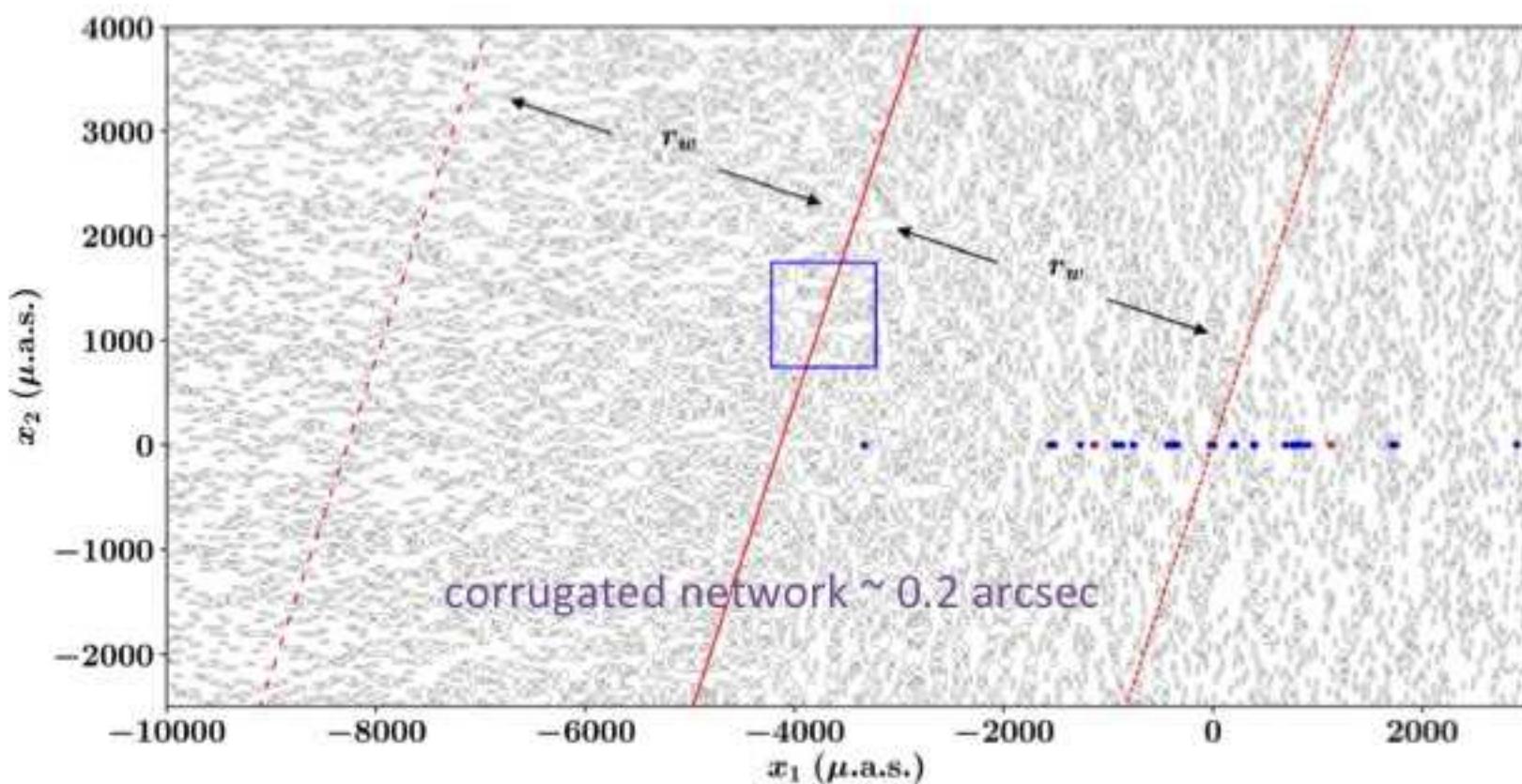
Venumadhav, Dai & Miralda-Escudé 17

Oguri+ 17

“Dumbbells” join and form a corrugated network of micro-critical curves even for small surface density of intracluster stars

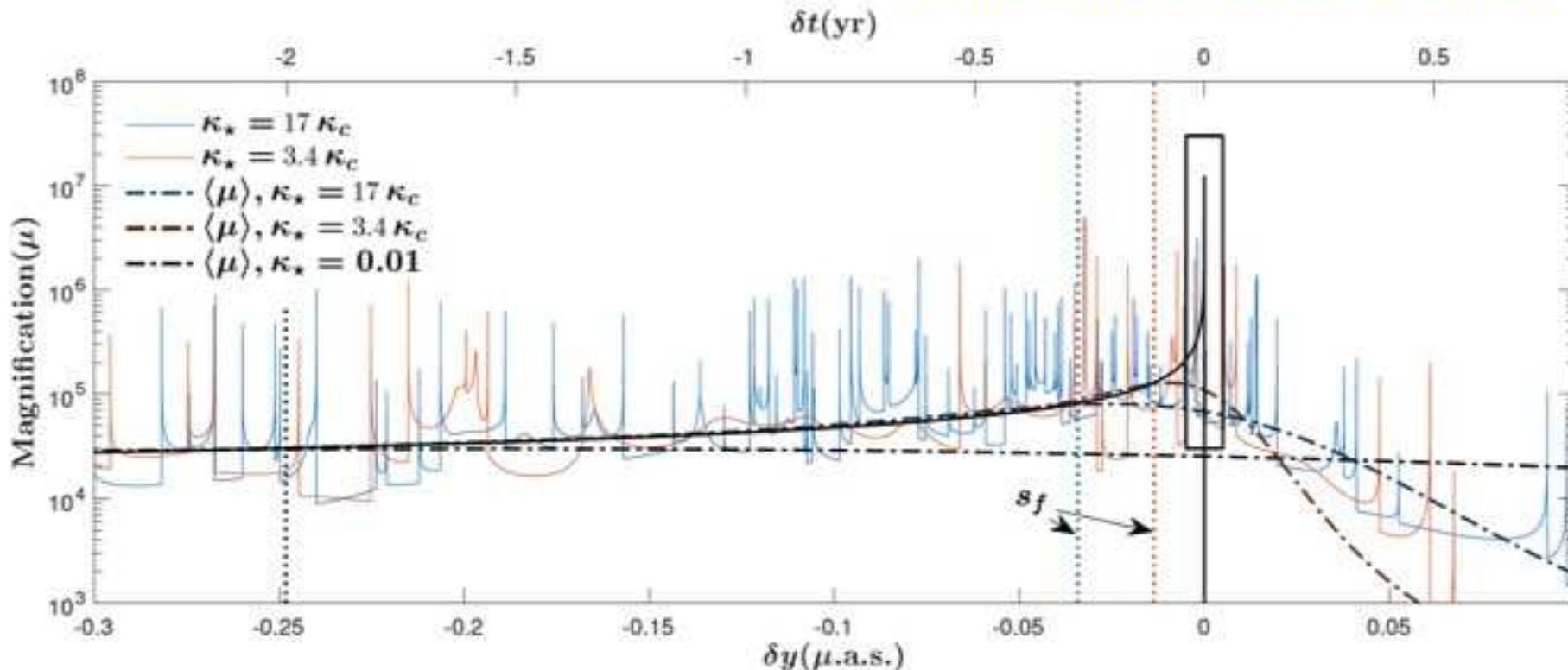
$$\kappa_{\text{crit}} \sim (\theta_{\star}/\theta_C)^{2/3} \sim \text{few} \times 10^{-5}$$

$$\kappa_{\star} \sim 10^{-3} - 10^{-2} \gg \kappa_{\text{crit}}$$



# Microlensing Light Curves

Venumadhav, Dai & Miralda-Escudé 17



- Maximum peak magnification reduced:  $\mu_{\text{peak}} \sim 10^4 (R/10 R_\odot)^{-1/2}$
- Spend **tens of thousands of years** crossing the network!!
- Frequent micro-caustic crossings: 1-100 per year ---- **better chance to see individual stars!**
- Culmination of each crossing lasts for  $\sim 5 \text{ hr} (R/10 R_\odot)$

# Re-appearance due to microlensing?

## Detection of a Microlensing Event of a Magnified Star at Redshift $z = 1.5$

ATel #11708; *Patrick Kelly (UMN), Jose Maria Diego (IFCA), Liliya Williams (UMN), Pablo Perez-Gonzalez (UCM), Alexei V. Filippenko (UC Berkeley), Masamune Oguri (Tokyo), Alberto Molino Benito (IAG), Jens Hjorth (DARK)*

on 12 Jun 2018; 03:36 UT

Credential Certification: *Patrick Kelly (pkelly@astro.berkeley.edu)*

Subjects: Optical, Request for Observations, Microlensing Event, Star, Transient, Gravitational Lensing

Referred to by ATel #: [11724](#)



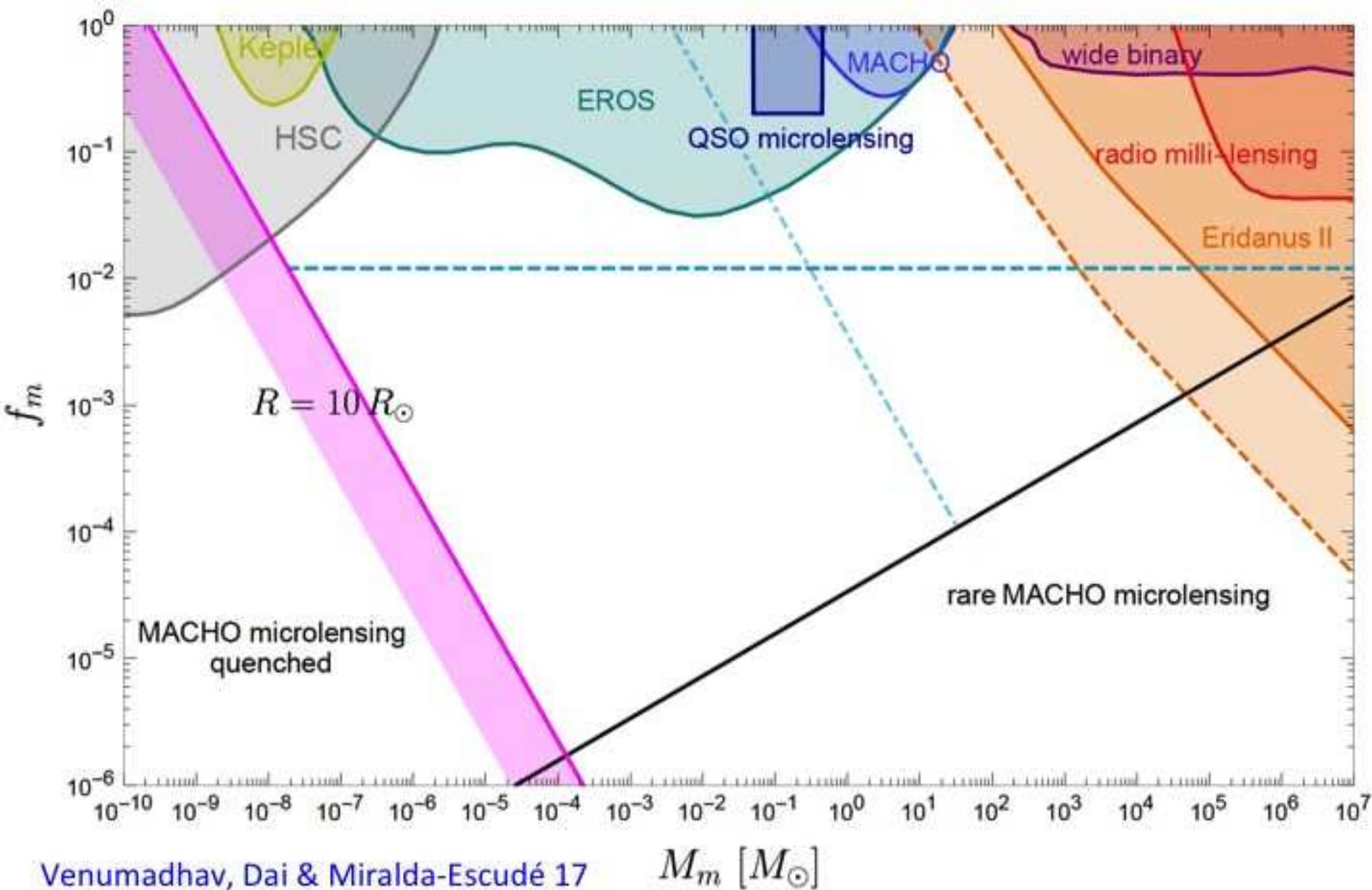
Tweet



Recommend 0

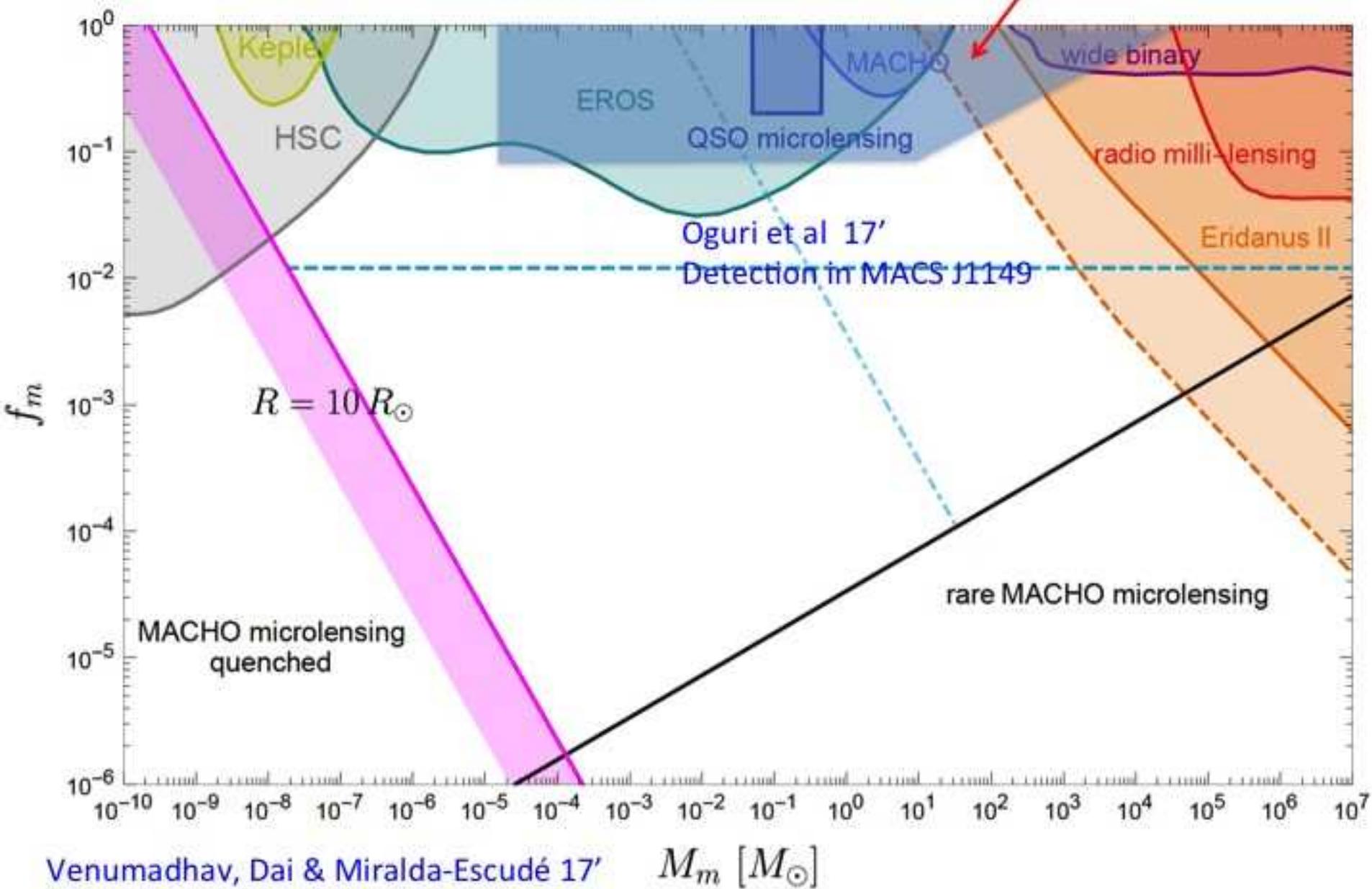
On June 5, 2018 UT, we acquired simultaneous g and r broadband imaging of the MACS J1149 galaxy cluster (redshift  $z = 0.54$ ) field with the Large Binocular Telescope (LBT). We report the detection of a bright unresolved source with a position consistent with the image (or likely counterimage) of Lensed Star 1 (LS1), a highly magnified blue supergiant star at  $z = 1.49$  (Kelly et al. 2018). We measure preliminary magnitudes of  $g = 26.9 +/- 0.2$  mag AB and  $r = 26.4 +/- 0.3$  mag AB, which imply a color statistically consistent with that of LS1. The measured optical flux densities are comparable to those observed during LS1's May 2016 microlensing peak, when the star's magnification likely reached several thousand. A Gemini Director's Discretionary Time program (GN-2018A-DD-107) for imaging follow up has been approved, and all observations of the microlensing event are encouraged.

# Constraints on Compact Dark Matter



# Constraints on Compact Dark Matter

LIGO black holes !!



Part II: Using highly magnified stars to probe dark matter subhalos

# “Halos inside halos”



Phoenix simulation, Gao et al 12'

# Why to look near critical curves?

Subhalos are puffy. Usually inefficient gravitational deflectors:

deflection angle       $\alpha \lesssim \frac{G M_{\text{sub}}}{b} \sim 1 \text{ mas} \left( \frac{M_{\text{sub}}}{10^7 M_{\odot}} \right) \left( \frac{b}{100 \text{ pc}} \right)^{-1}$

Argument due to [Katz 1986](#)

$$\mathbf{x} - \mathbf{y} = \boldsymbol{\alpha}_{\text{cl}}(\mathbf{x})$$

$$\mathbf{x}' - \mathbf{y} = \boldsymbol{\alpha}_{\text{cl}}(\mathbf{x}') + \boldsymbol{\alpha}_{\text{sub}}(\mathbf{x}') \quad \mathbf{x}' = \mathbf{x} + \Delta \mathbf{x}$$

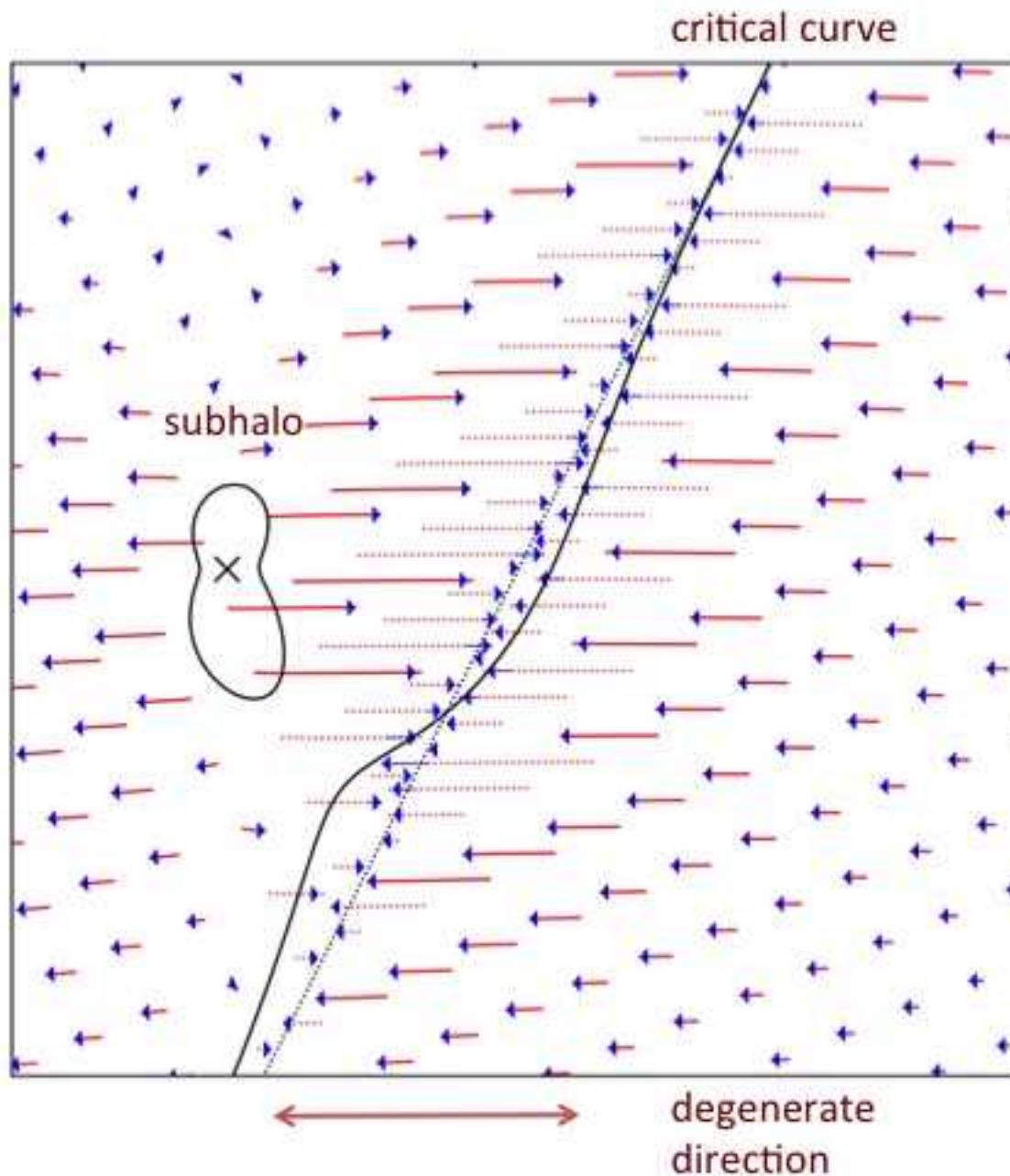
$$\Delta \mathbf{x} = \Delta \mathbf{x} \cdot \nabla \boldsymbol{\alpha}_{\text{cl}} + \boldsymbol{\alpha}_{\text{sub}}(\mathbf{x}')$$

$$\Delta \mathbf{x} = \boxed{(\mathbf{I} - \nabla \boldsymbol{\alpha}_{\text{cl}})^{-1}} \cdot \boldsymbol{\alpha}_{\text{sub}}(\mathbf{x}')$$

**lensing Jacobian matrix**

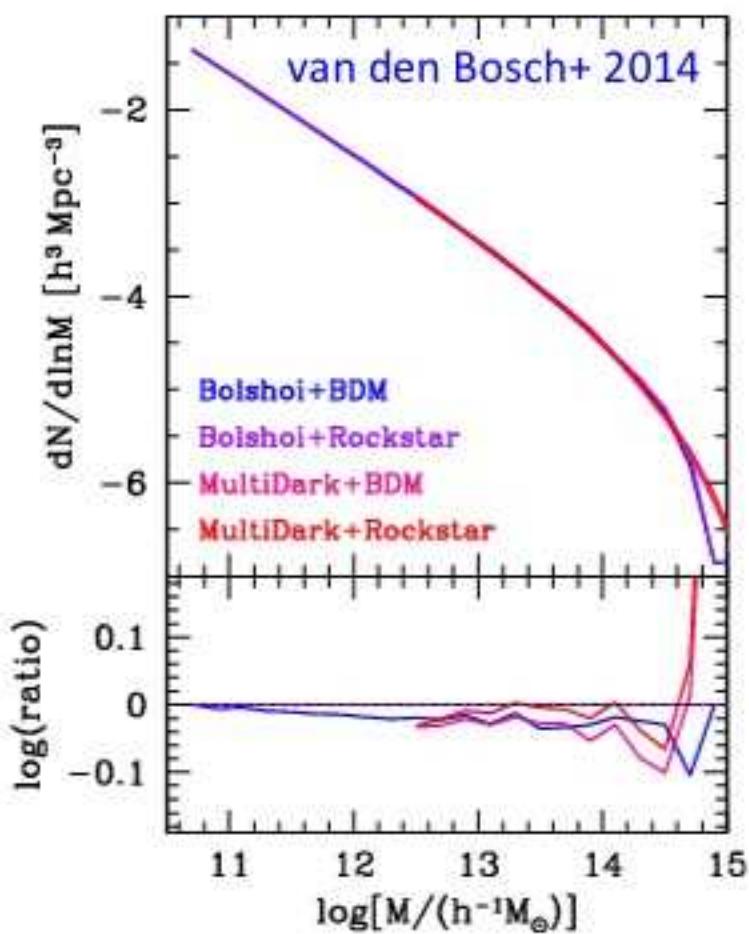
- Deflection angle “amplified” by a factor of the fold magnification  $\sim 10^2 - 10^3$
- Amplified deflection always along the **degenerate** direction.

# Astrometric perturbation near critical curves

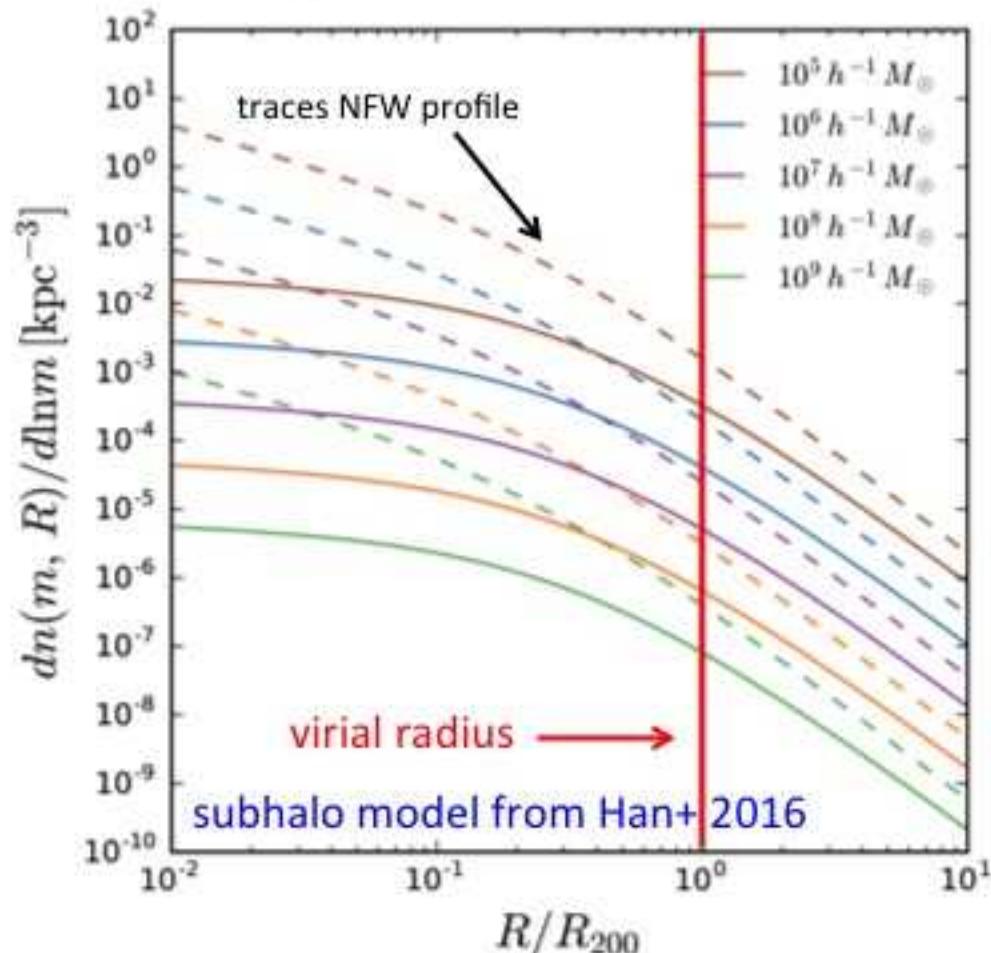


# Substructure in CDM halos

- Power-law mass function  $dN/d \log m \propto m^{-0.9}$  Mo+ 2010
- ~1-10% mass in subhalos across a wide mass range



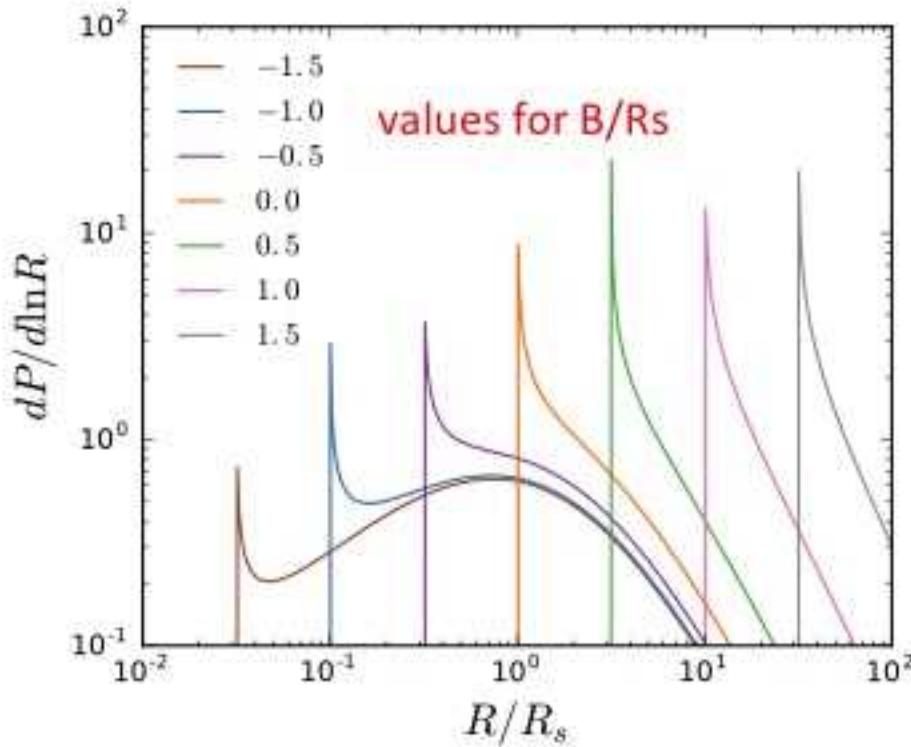
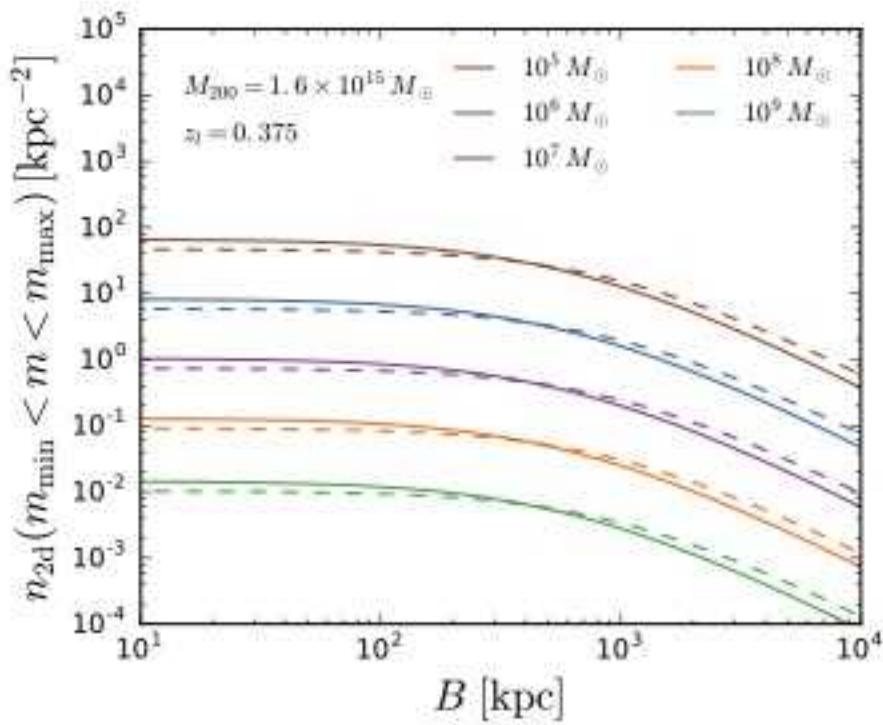
cluster size halo



Infall mass vs bound mass  
“unbiased accretion”

# Probability of intersecting subhalos

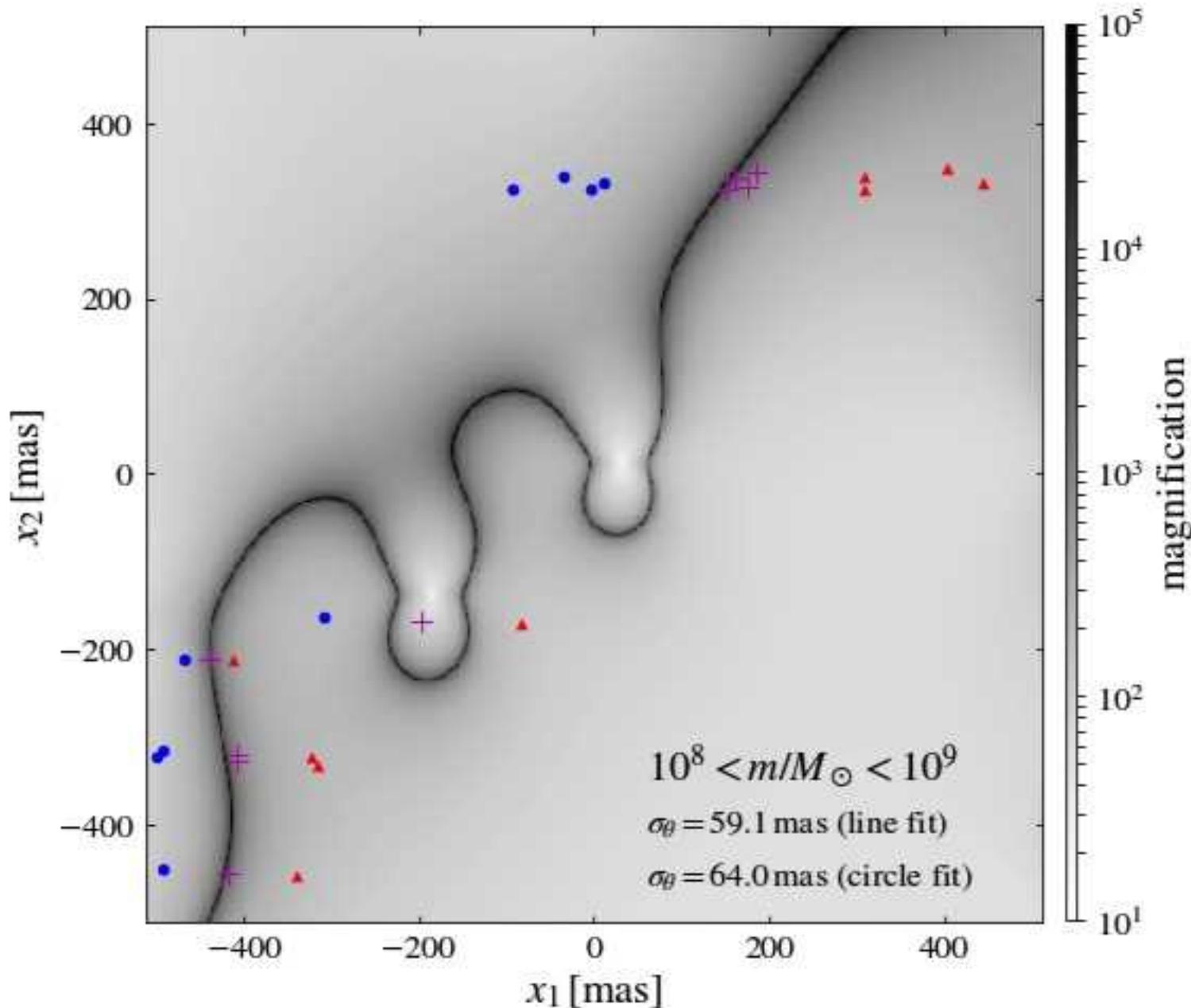
Massive galaxy cluster has size  $\sim 1$  Mpc;  
Huge column density of dark matter



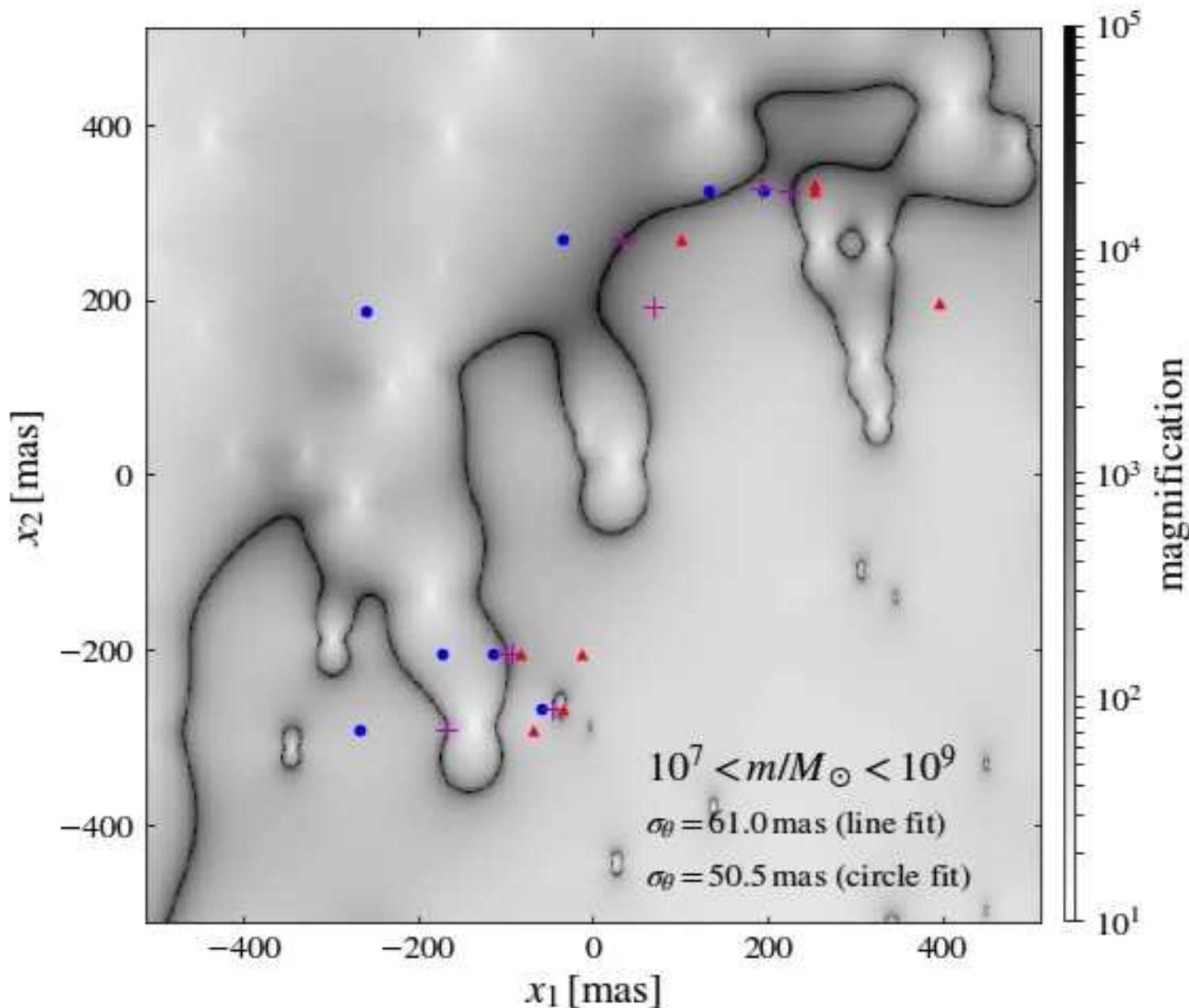
$\sim 1$  halo of  $10^6$  ---  $10^7$  Msun in each  $0.2'' \times 0.2''$  patch

Most likely to be at halocentric distance  $R_s \sim 300$  kpc

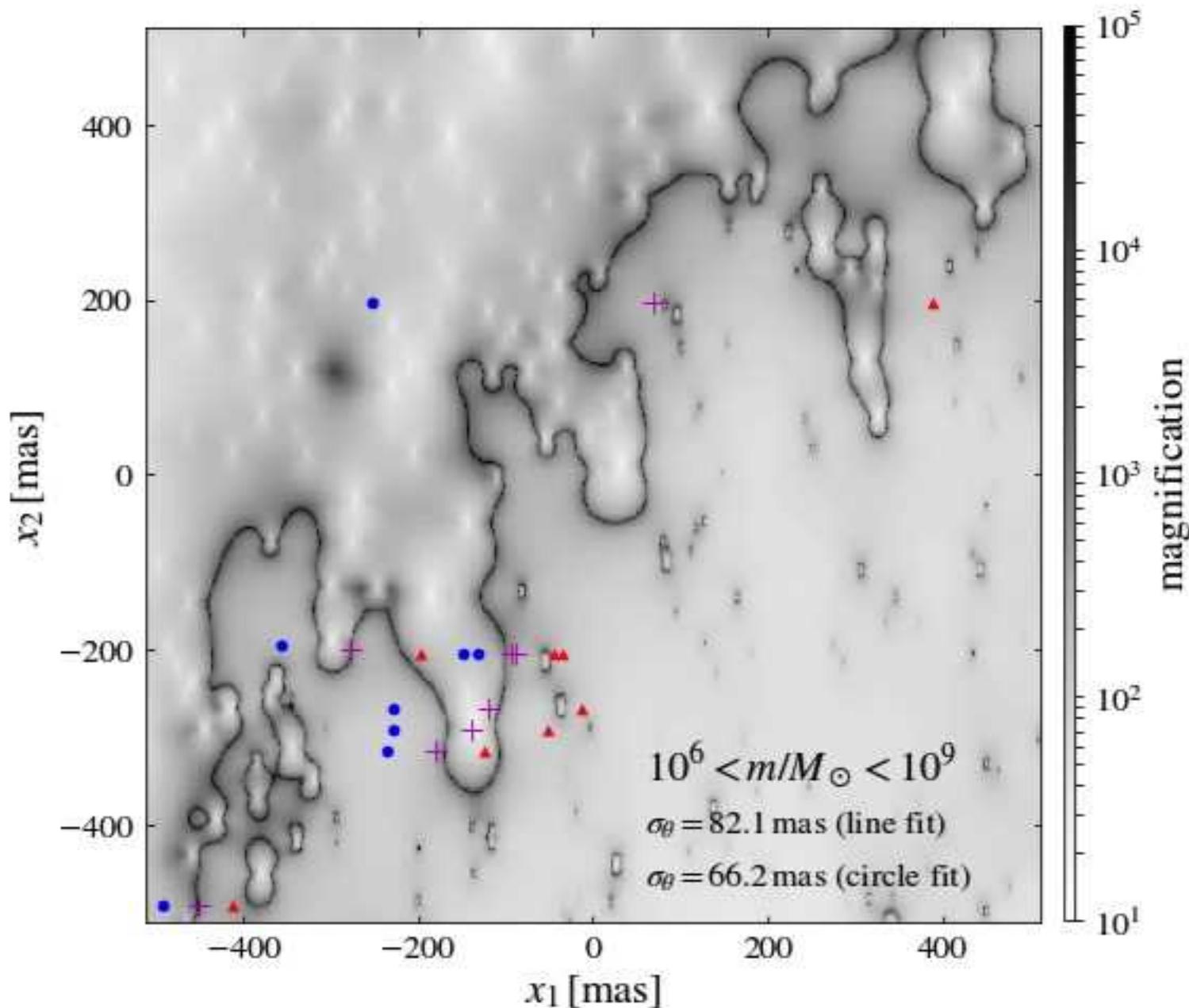
# Effect of Subhalos on Image Locations



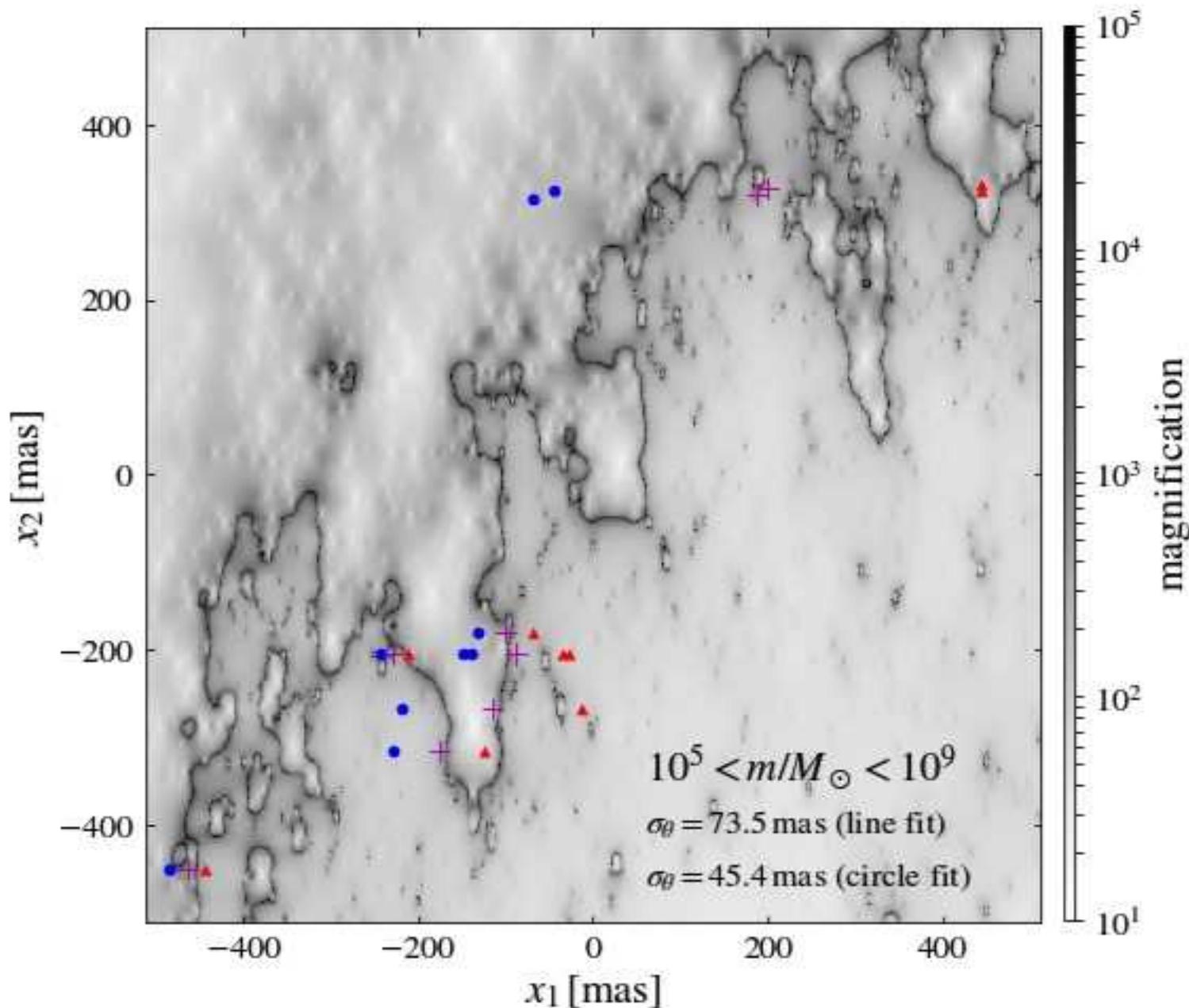
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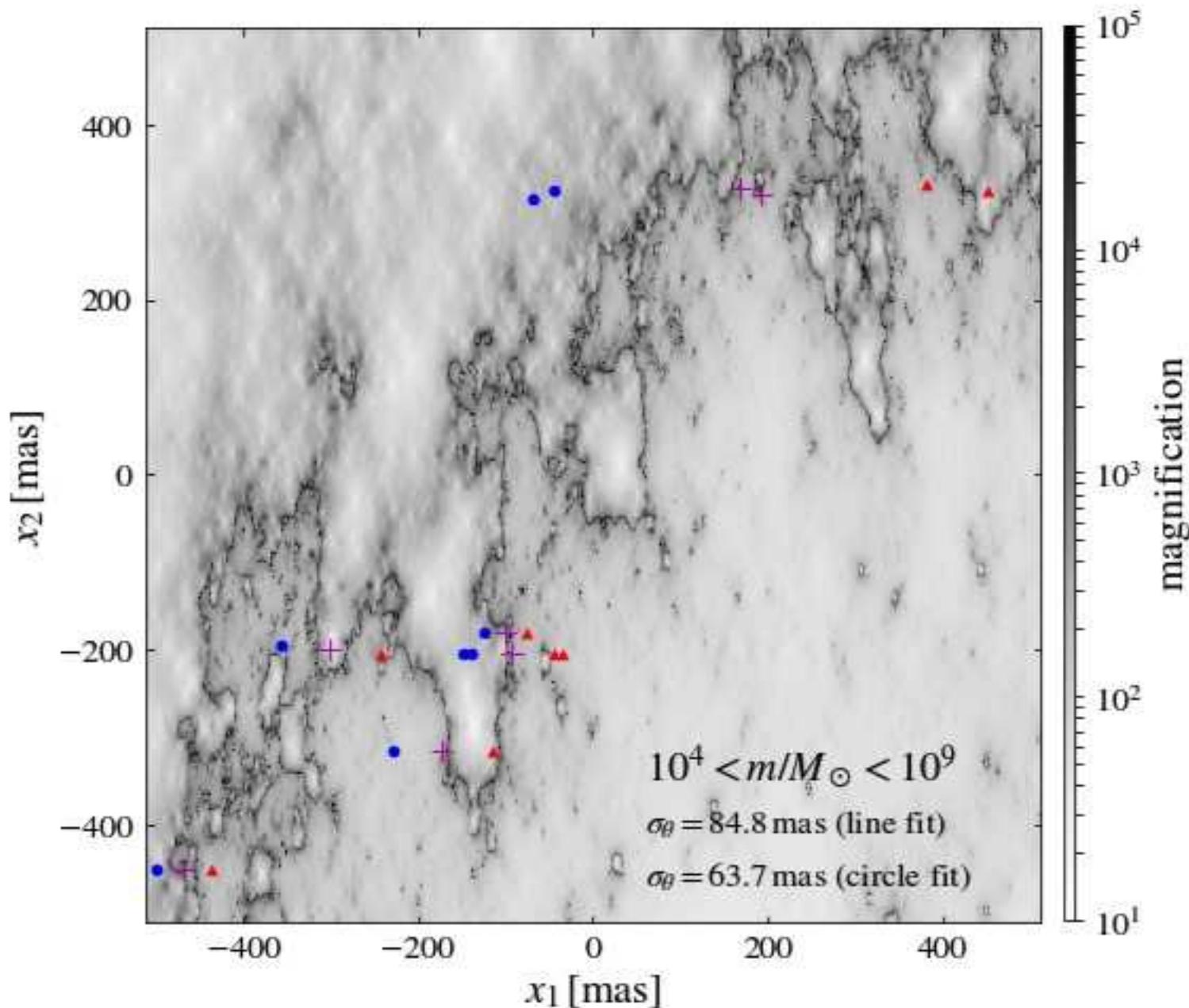
# Effect of Subhalos on Image Locations



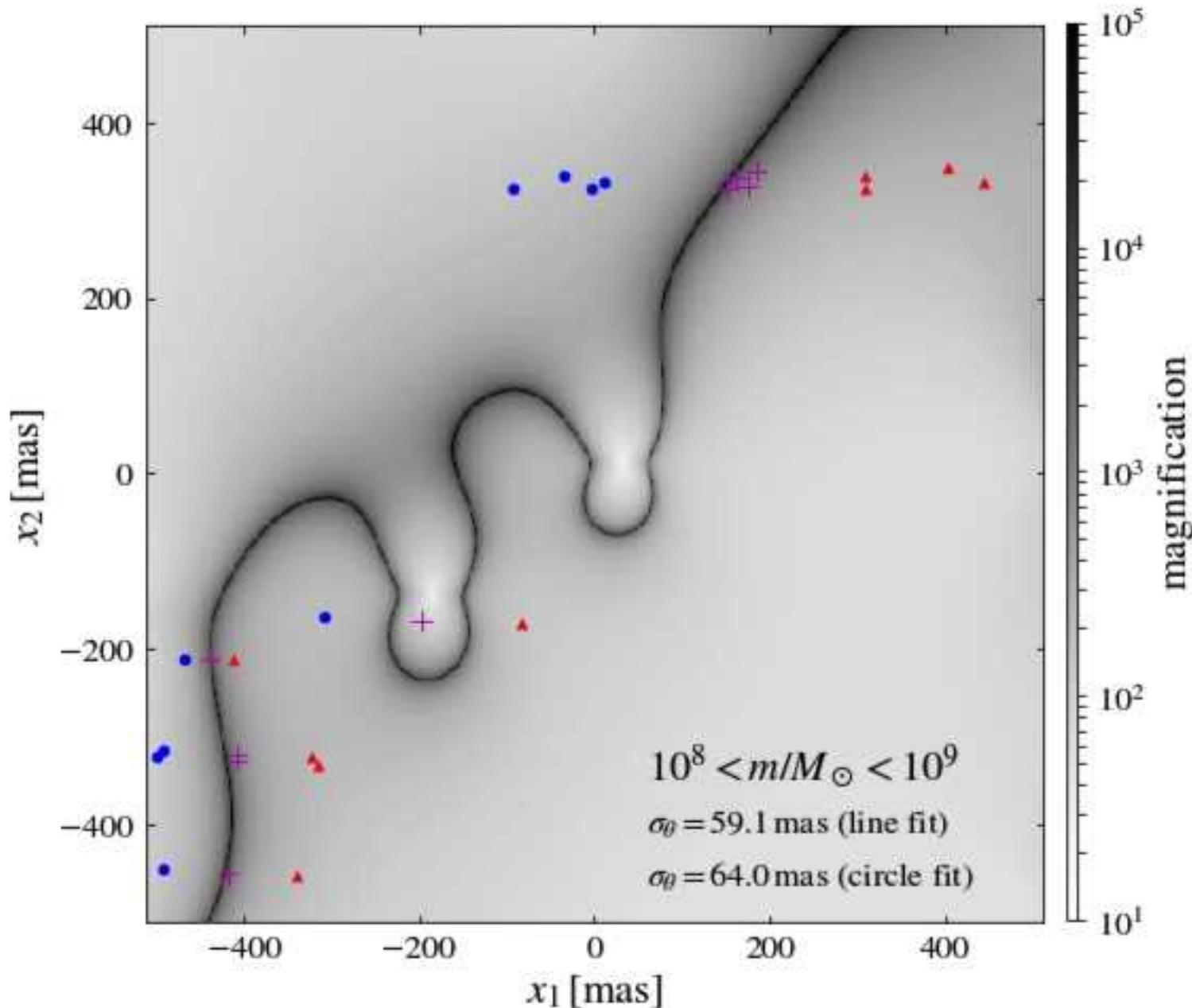
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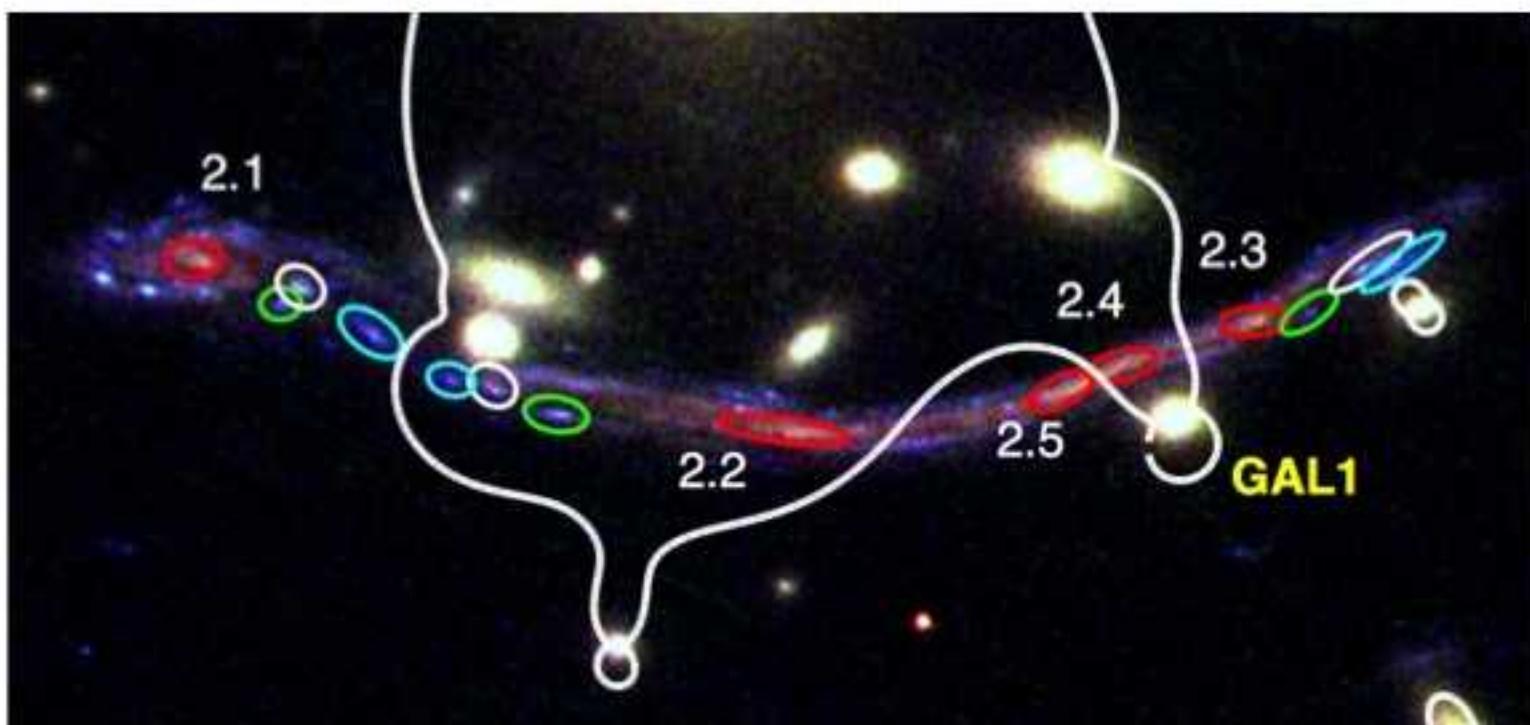
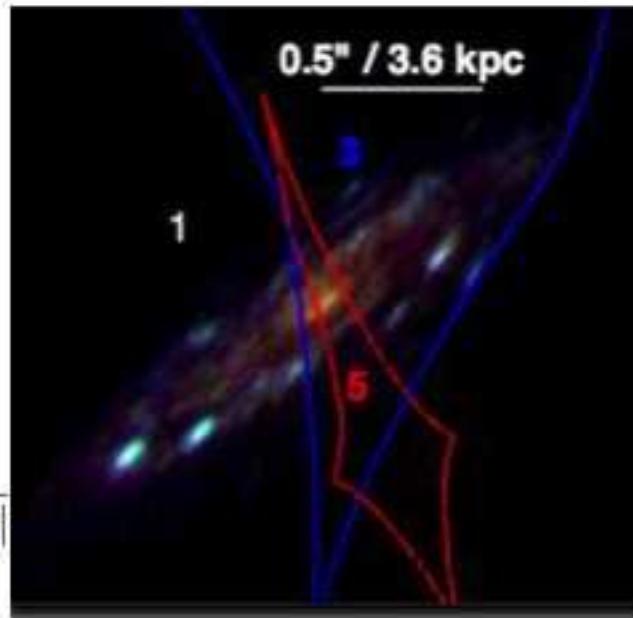


# What systems are the most promising?

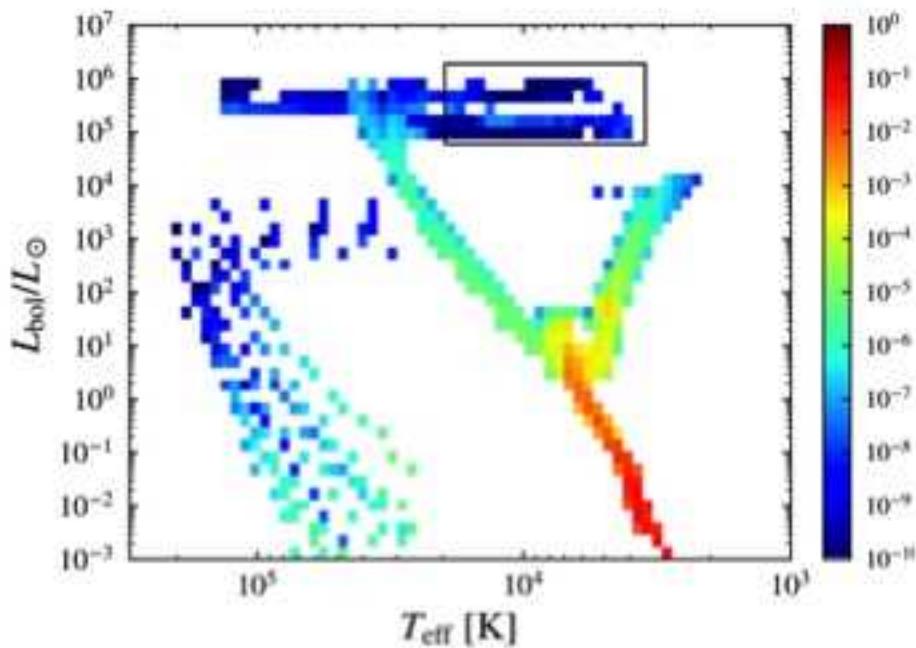
- Source galaxy at **low redshift**
- Active **star formation**
- **Wide giant arc favorable**
- Near **critical** convergence
- Shear has small derivative

$$\mu = \frac{1}{|2(1 - \kappa_0) d \cdot x|}$$

The “Dragon” of Abell 370      Richard+ 2009

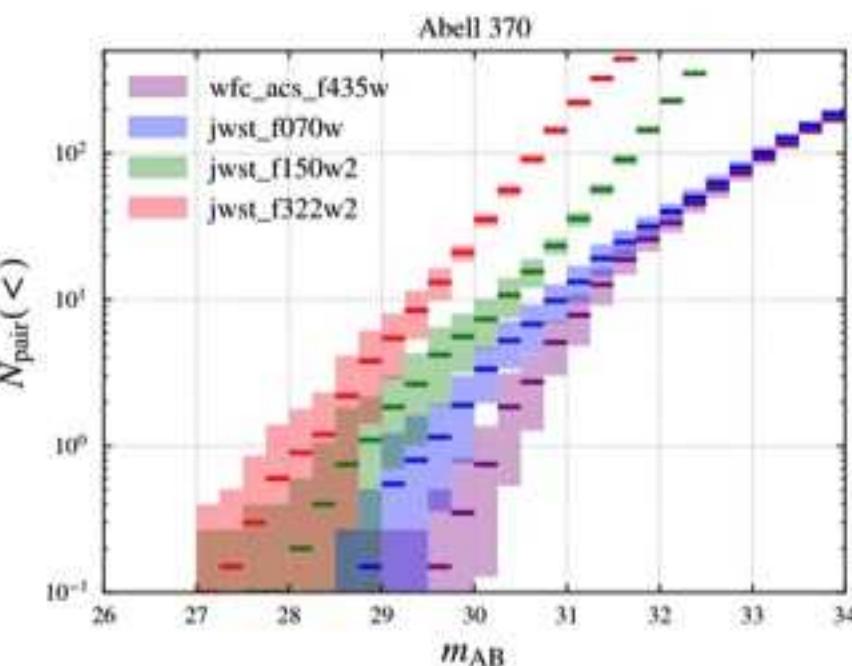


# Estimation of the abundance of bright source stars



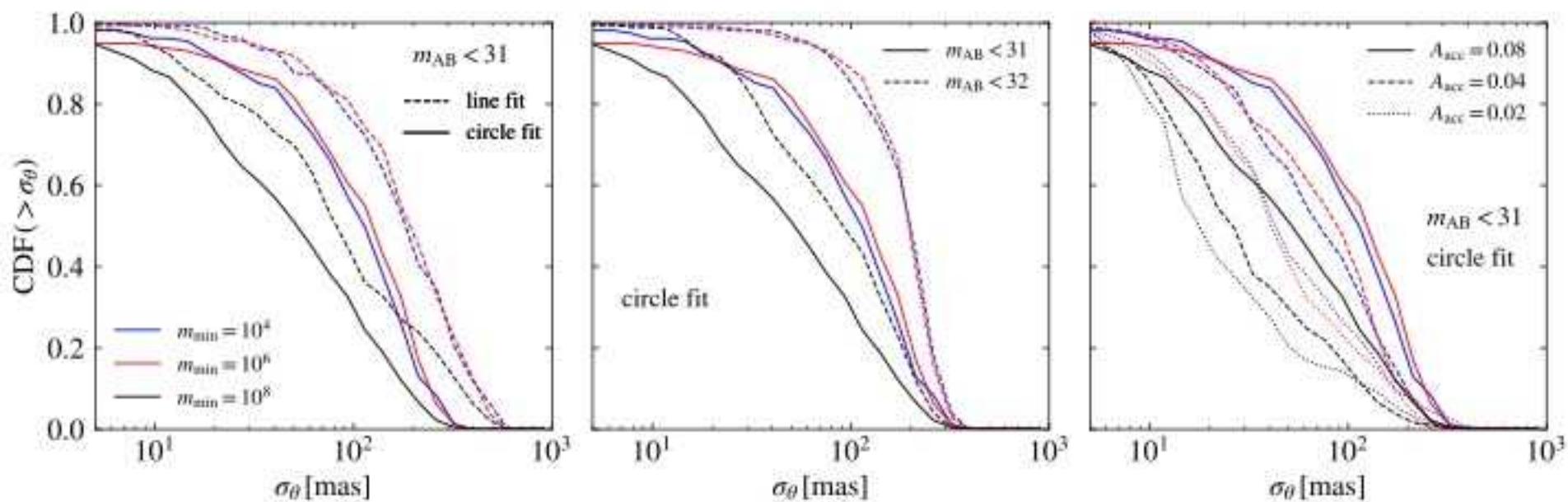
Red super-giants may be very promising (more than blue super-giants, or hot main-sequence stars)

Crude prediction for JWST  
Try go down to mag ~ 29-30 in  
the near-IR (but on top of a  
bright arc).



# Astrometric precision required to probe subhalos

Statistical forecast for the giant arc in Abell 370



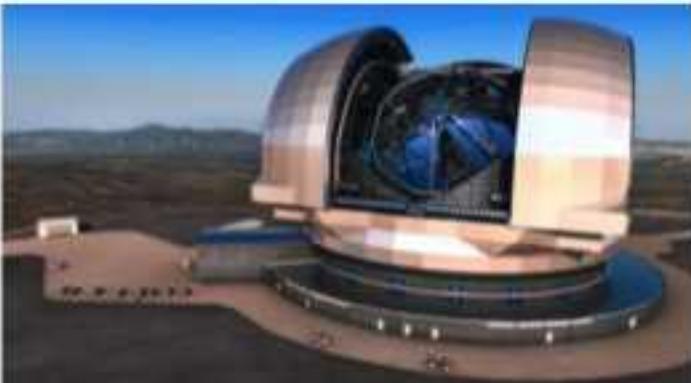
- Sensitive to  $10^6 - 10^8$  Msun subhalos, **7-8 orders of magnitude** down the cluster's mass scale; those expected to be non-luminous
- With clear detection of **5-10 pairs**, require mean astrometric precision **20-100 mas**

## Comments

- For individual stars, microlensing inevitably induces flux variability. However, image shift due to microlensing negligibly small  $< 1$  mas. Astrometric signal should be robust.
- How do we know which images belong to the same star? Make use of universal property of lensing near a fold. Color and variability information would be supplementary.
- Any **under-resolved, compact feature** in the surface brightness pattern equally useful for the astrometric test. Examples: star clusters, star-forming complexes, HII regions, dust lanes.
- Even without compact feature detected, a sharper image of the critical curve's neighborhood is highly desired. **Surface brightness reconstruction techniques** applicable.

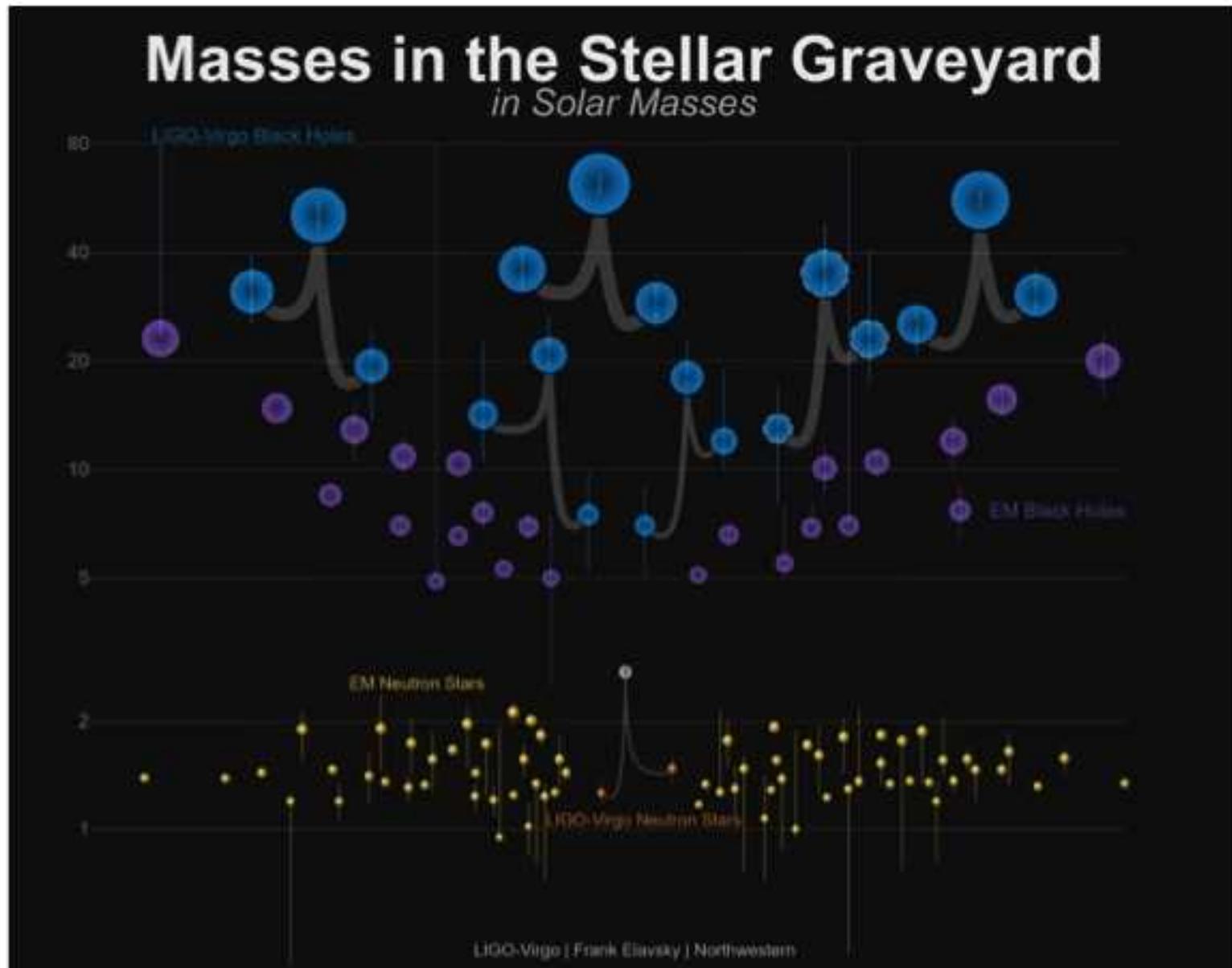
# Promise of future ELTs

- ❑ Caustic crossing sources are faint, even with huge lensing amplifications. Faster photon collecting rate is good.
- ❑ Smaller diffraction-limited PSF extremely beneficial for detecting point sources.
- ❑ Higher angular resolution crucial for detecting astrometric distortions from subhalos.
- ❑ Red/white super-giants very bright in J,H,K bands. These are very important and powerful bands for ELTs aided with AO.

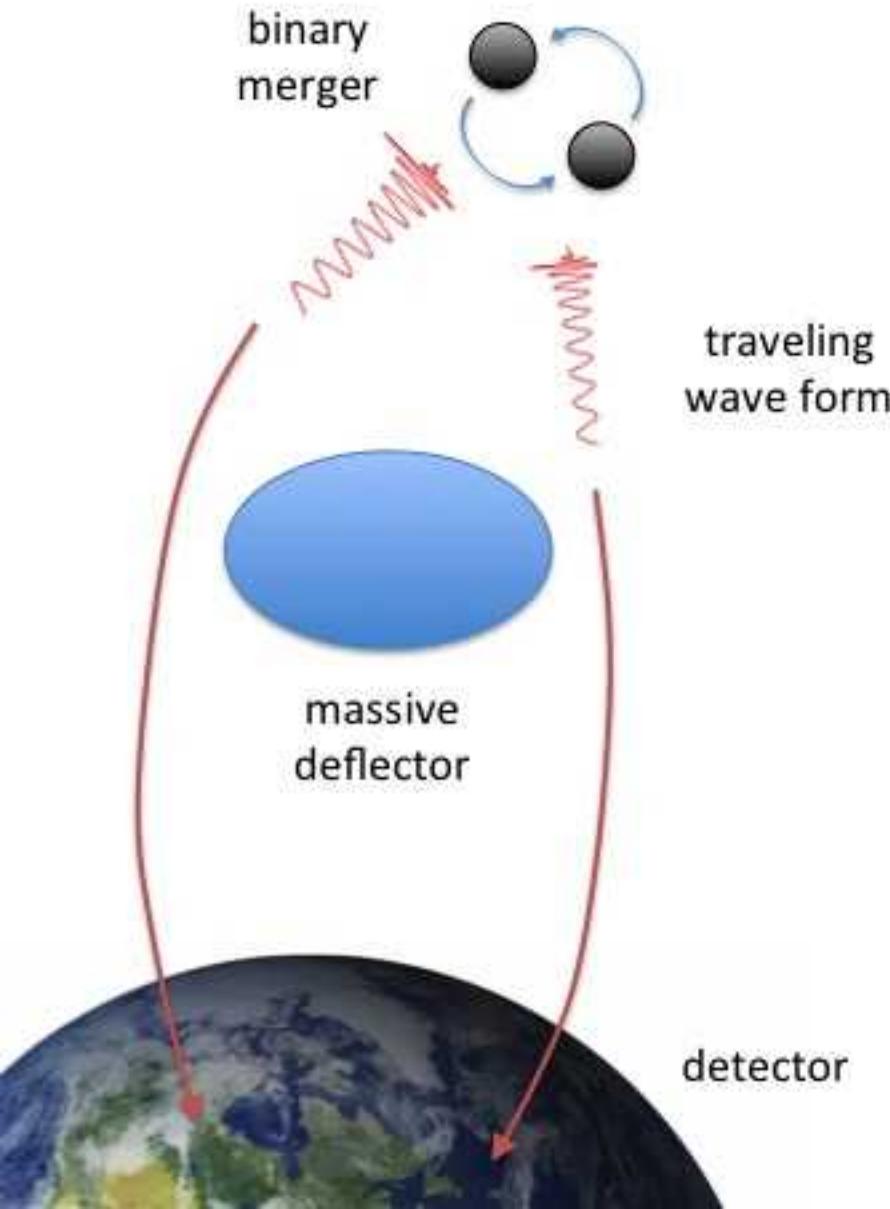


# Part III: Diffractive lensing of gravitational waves from binary mergers

# Stellar remnants uncovered by gravitational observatories



# Gravitational lensing of gravitational waves



- ◆ Geometrical limit

$$\lambda_{GW} \ll R_{de}$$

- ◆ Observable is strain **amplitude**

$$h' = \sqrt{\mu} h$$

- ◆ Frequency as received not changed

- ◆ Multiple bursts: without spatial localization, delayed events in time domain

# Biased black hole mass for strongly magnified events

Dai, Venumadhav & Sigurdson 17'

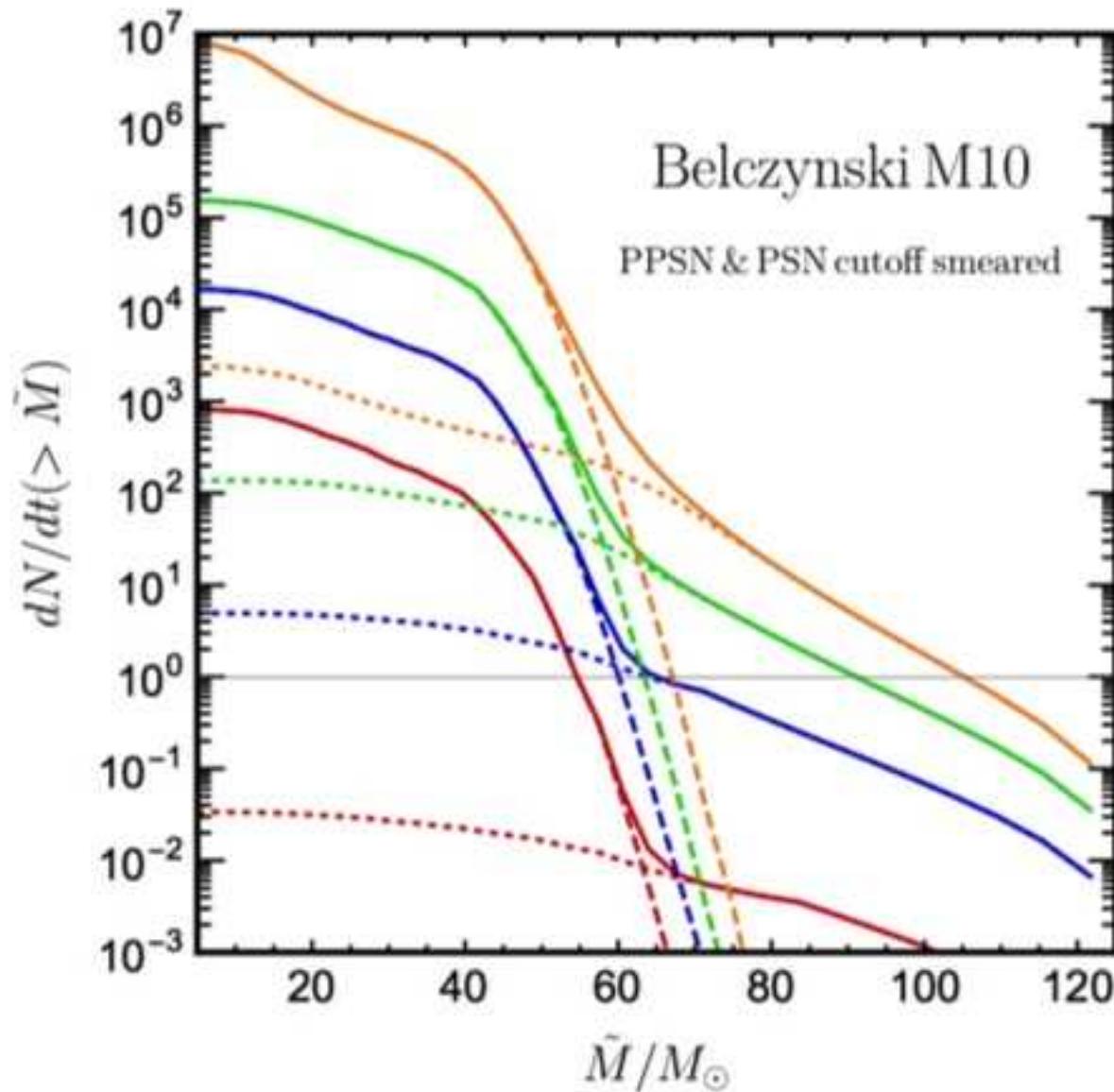
Broadhurst, Diego & Smoot 18'

Apparent mass scale  $M'$

Apparent source redshift  $z'$

$$M(1+z) = M'(1+z')$$

$$\frac{\sqrt{\mu}}{d_L(z)} = \frac{1}{d_L(z')}$$



# Wave diffraction regime of lensing

Takahashi & Nakamura 2003

Takahashi PhD thesis

$$w = 2\pi f (1 + z_L) G M_L / c^2 \sim \mathcal{O}(1)$$

In the band of ground-based detectors  
**f~10-1000 Hz**, sensitive to (interestingly)  
small lens masses **M ~ 100-1000 Msun**

## Geometric lensing

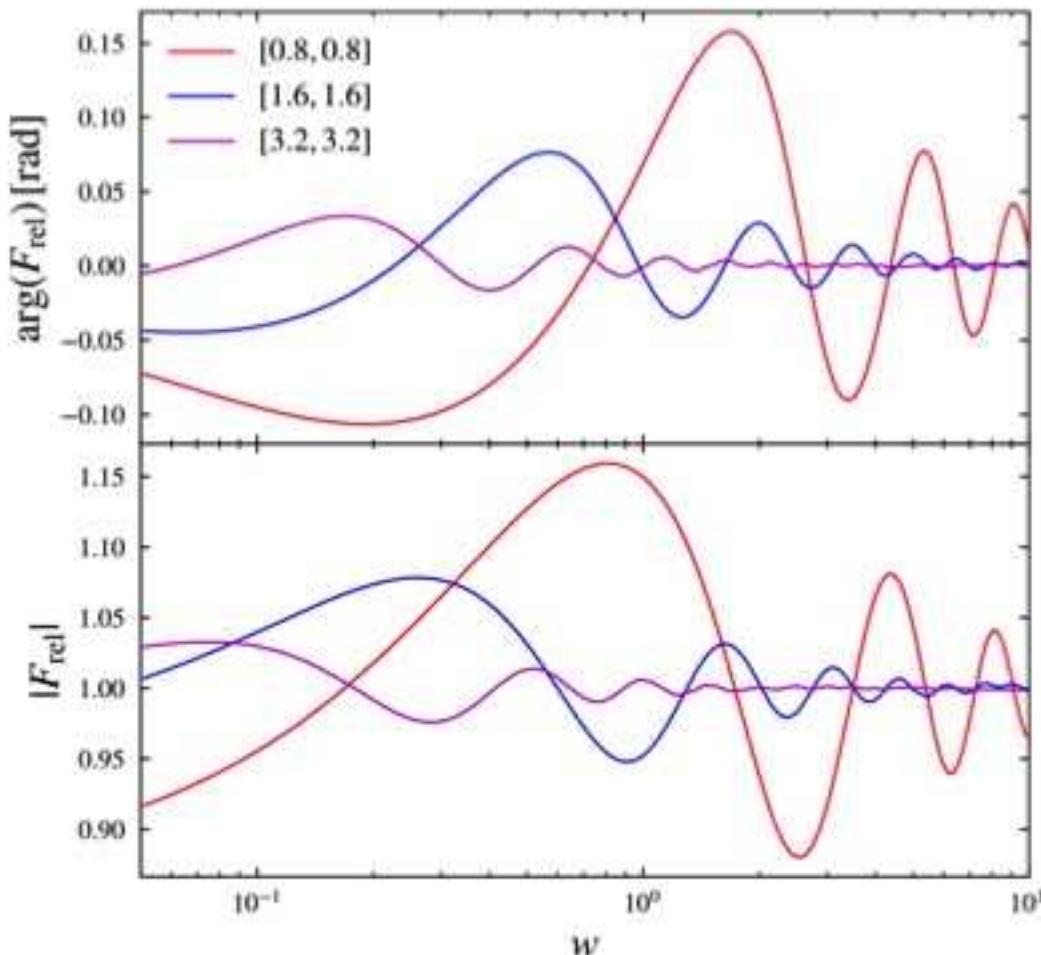
$$h_{\text{geo}}(f) = \sqrt{\mu} e^{i 2\pi f \tau} h_0(f)$$

## Wave-diffractive lensing

$$h(f) = F(f) h_0(f)$$

Define a ratio:

$$F_{\text{rel}}(f) = e^{-i 2\pi f \tau} F(f) / \sqrt{\mu}$$



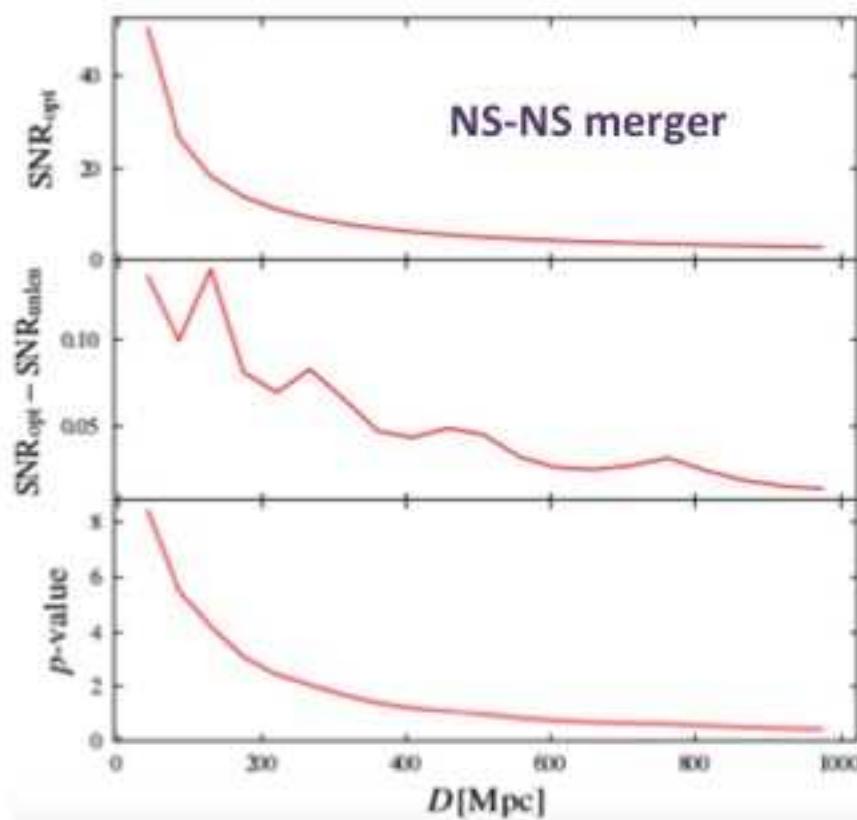
# Detection of diffraction signal can be challenging

- There may not be multiple **images** (lens may be puffy)
- Small modulus and phase perturbations  $\sim 10\text{-}20\%$

In fact, using **unlensed** templates can yield sufficiently good matches

Ideal (through **matched filtering**) detection significance for diffraction signature

$$\begin{aligned}\ln p &= -(\text{SNR}_{\text{opt}}^2 - \text{SNR}_{\text{unlen}}^2)/2 \\ &= -\frac{1}{2} \left( \langle h_L | h_L \rangle - \frac{\langle h_L | h_{\text{BF}} \rangle^2}{\langle h_{\text{BF}} | h_{\text{BF}} \rangle} \right)\end{aligned}$$

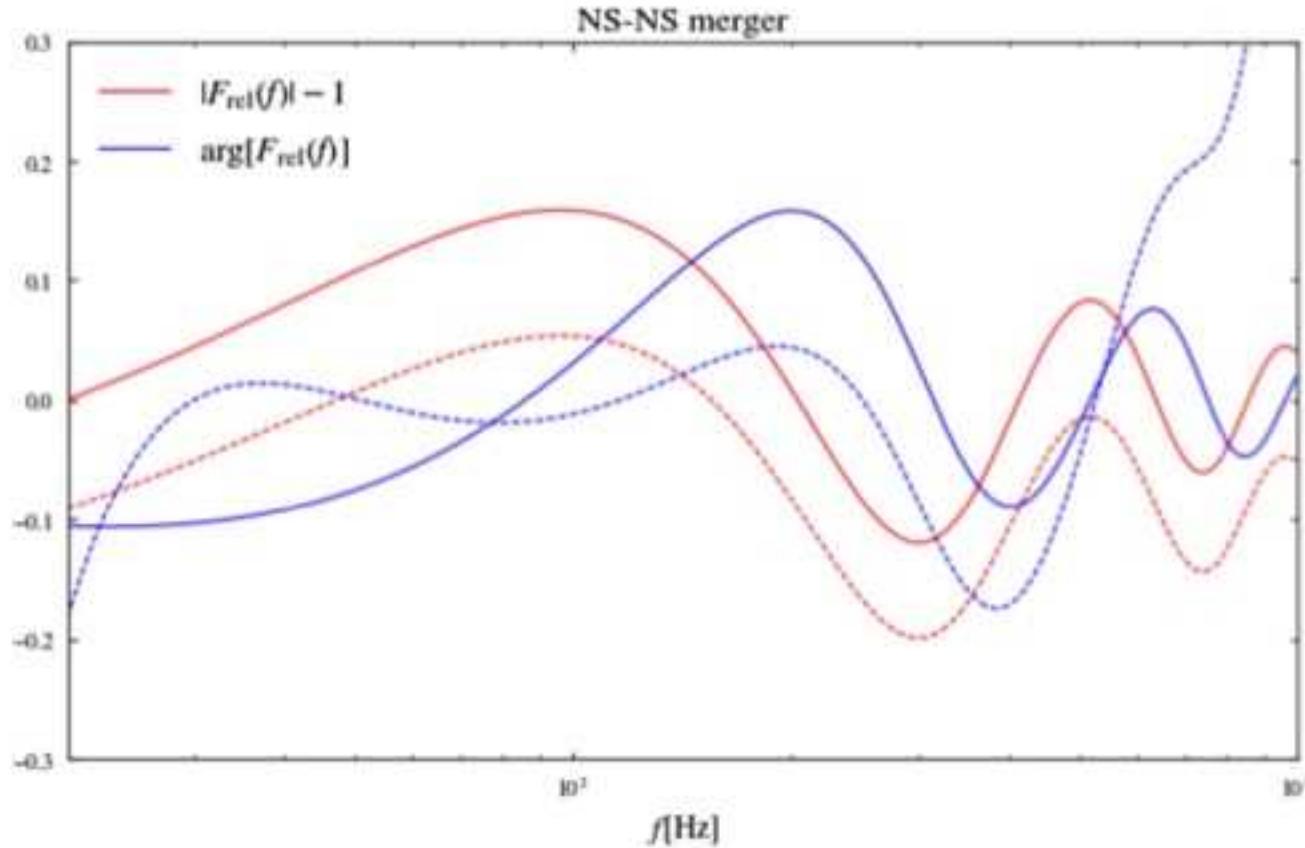


Difficulty with matched filtering: **lensed** template banks not known a priori

## Phase/amplitude distortions (partially) degenerate with binary parameters

- **Intrinsic parameters:** chirp mass, mass ratio, spins, tidal deformability
- **Extrinsic parameters:** arrival time

Before/after fitting  
for the best  
unlensed model



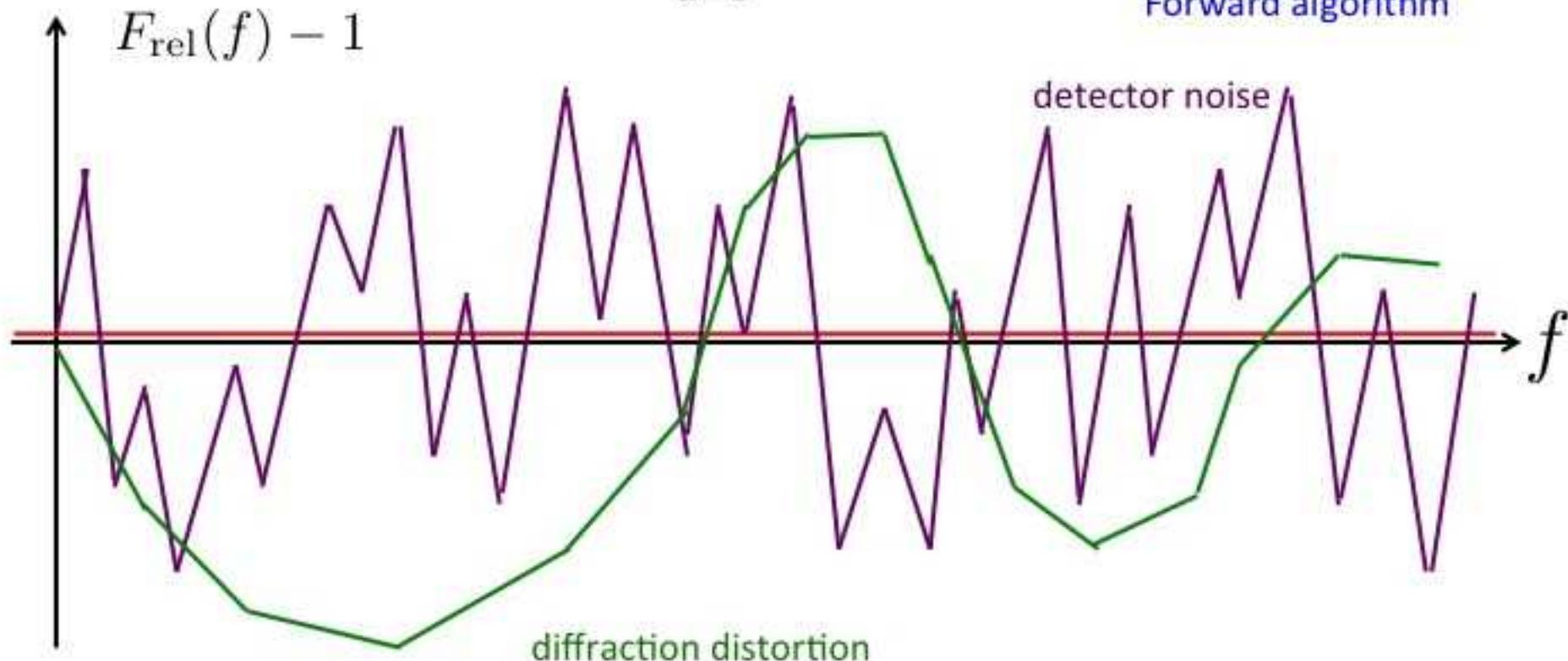
# Agnostic search for diffraction signature

Dai+ in preparation

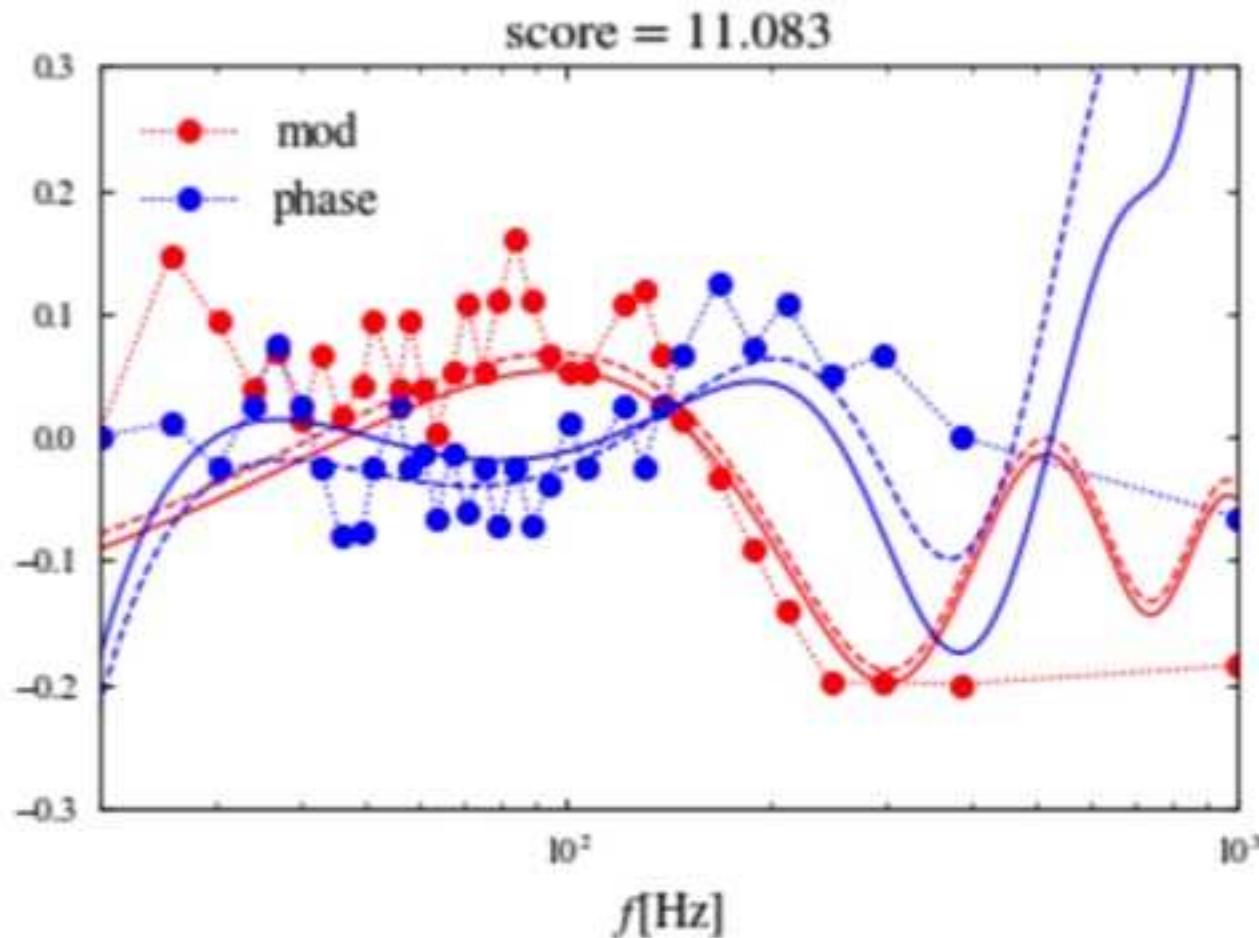
Marginalize over all distortions around the **best-fit** assuming a **Markovian** process

$$\mathcal{S} \equiv \int \mathcal{D}g(f) P(g(f)) \prod_{a=1}^{N_d} \frac{P(s_a | h_{L,a})}{P(s_a | h_{BF,a})}$$

Marginalization  
efficiently  
computable using  
**Forward algorithm**

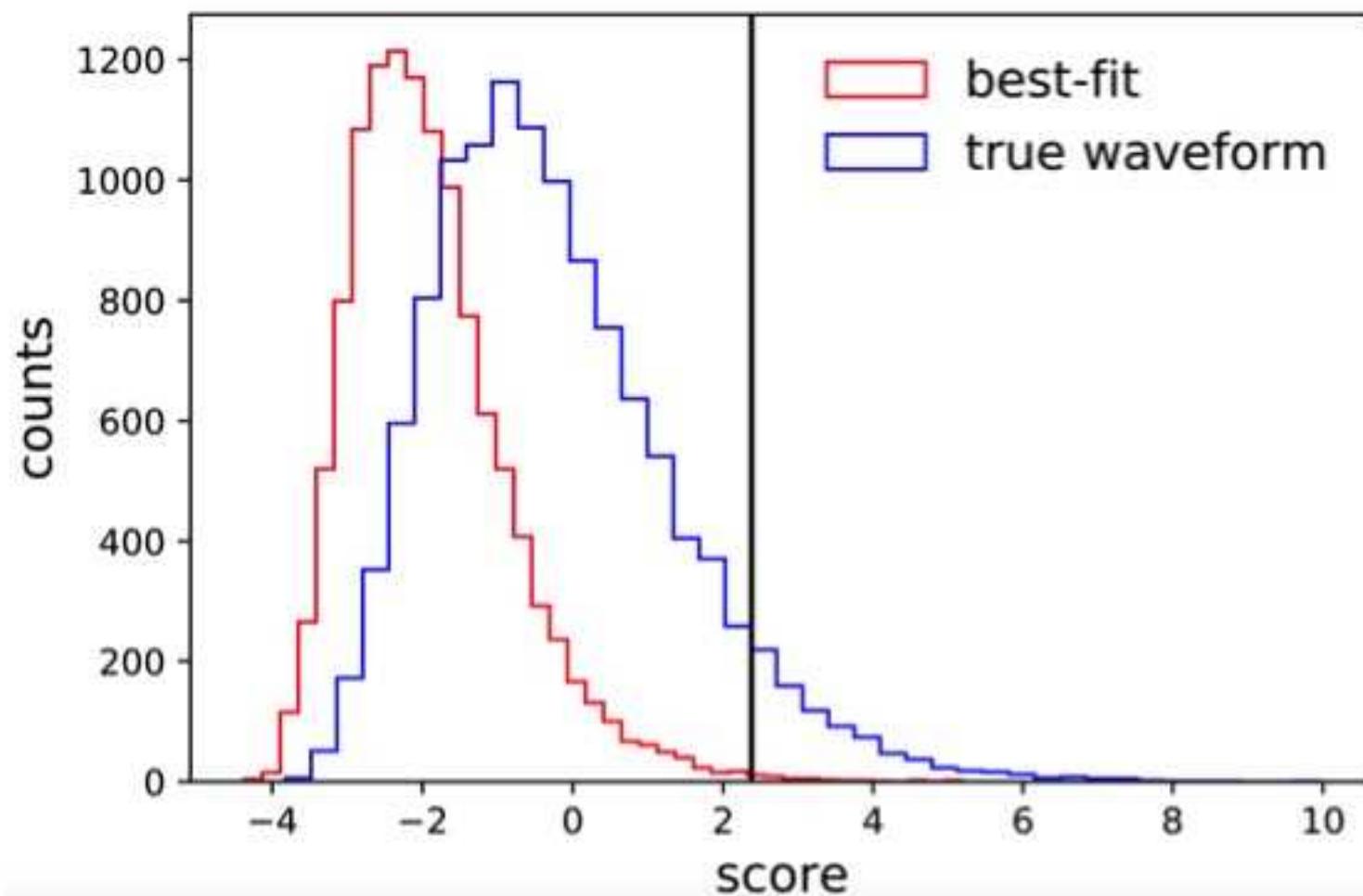


# Testing the Forward-Viterbi algorithm



- Suitably choose frequency bins and steps.
- Apply proper priors to impose frequency “smoothness”

## Determine statistical significance from mocks



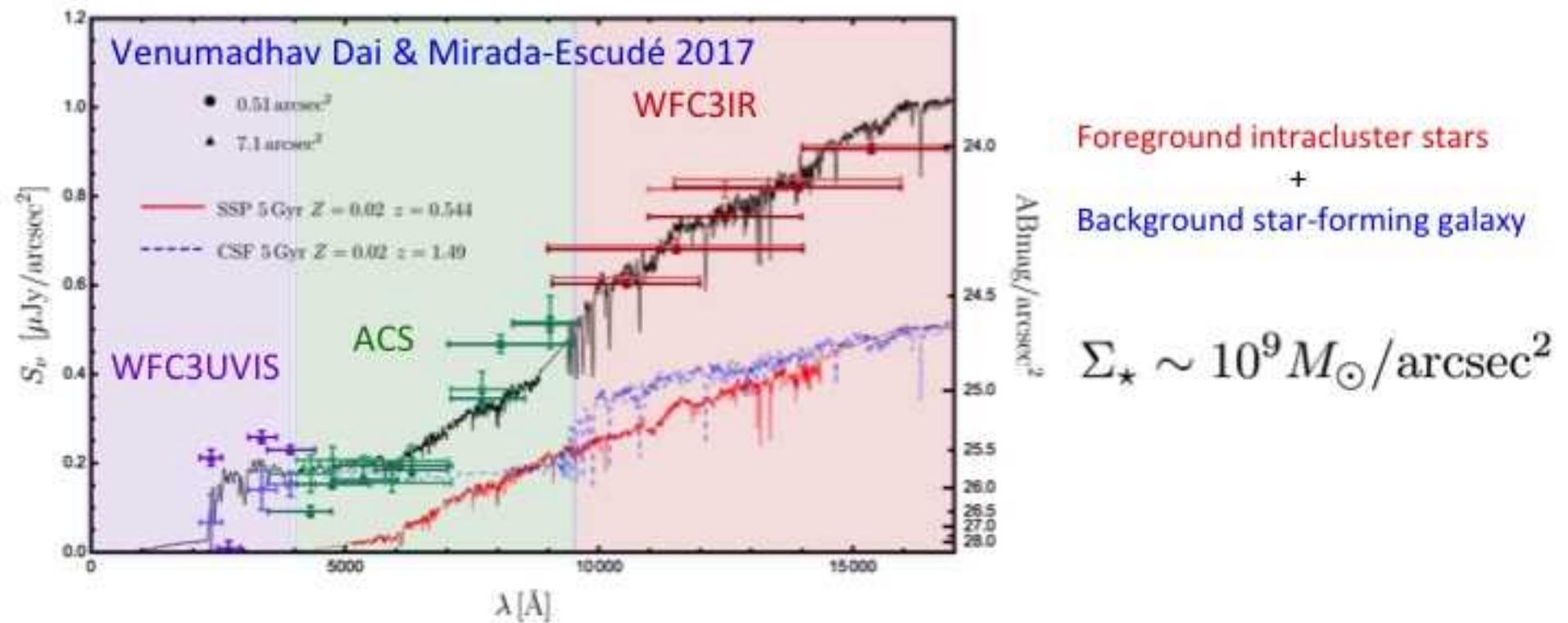
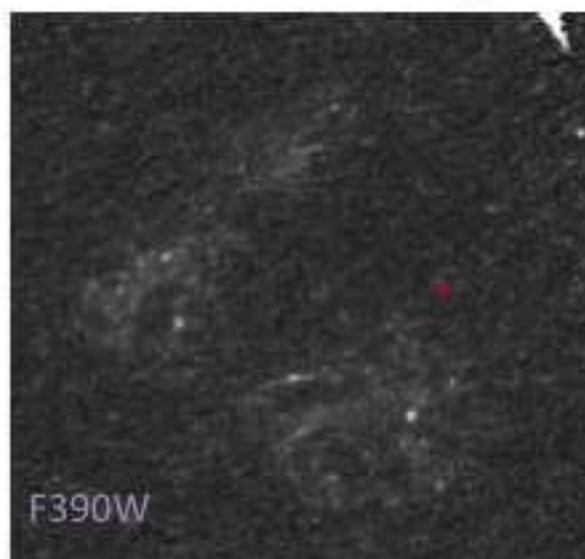
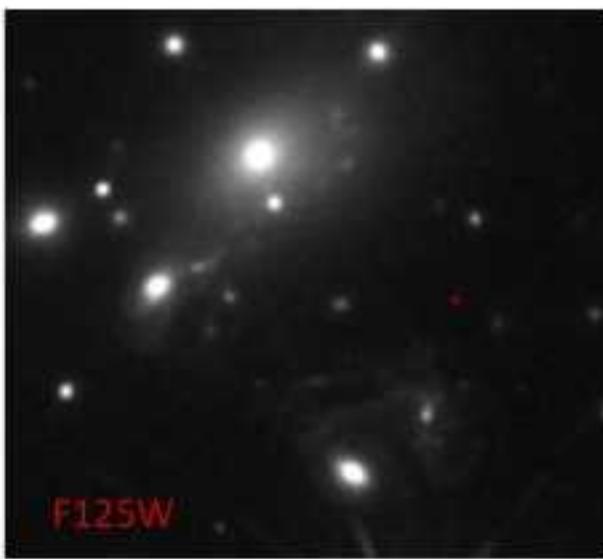
It's important that one perturbs the best-fit unlensed model

## Prospects

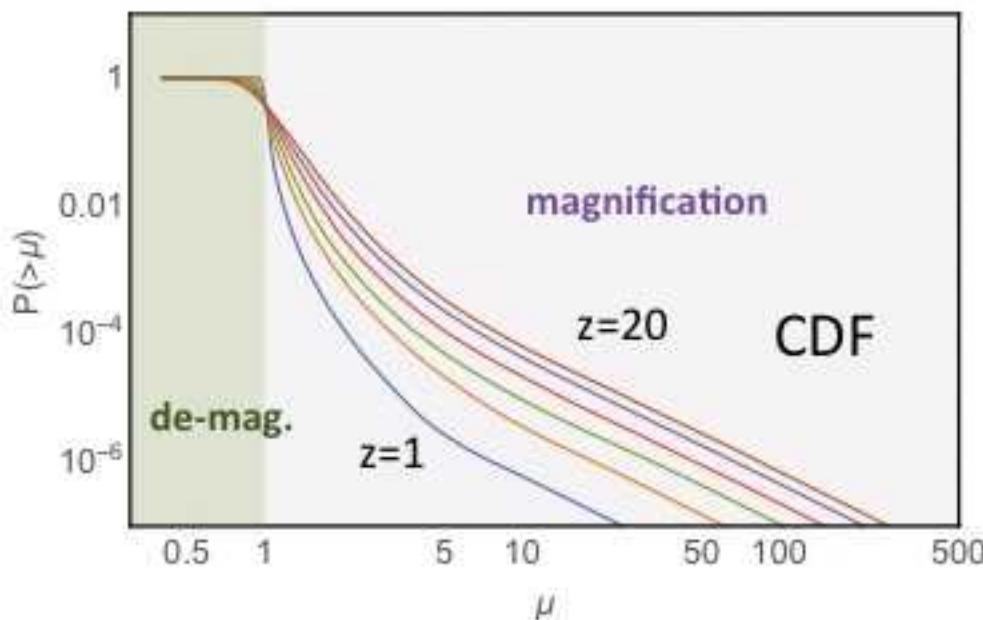
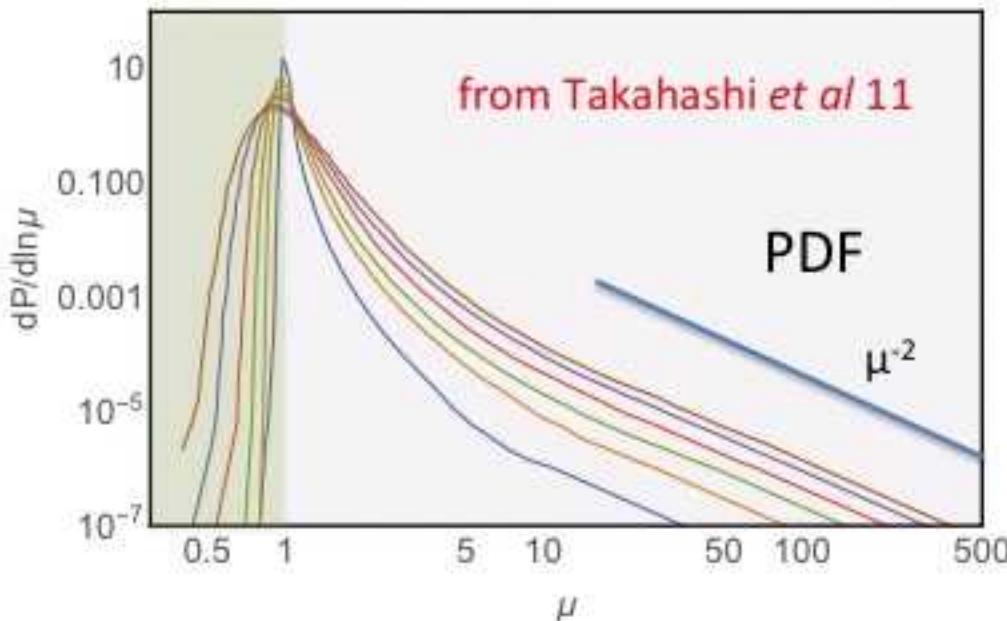
- ❑ Interesting mass range **100 – 1000 Msun** for DM halos.
- ❑ Applicable to both **NS-NS mergers** and **BH-BH mergers**.
- ❑ Computationally very cheap; readily applicable to every GW detection.
- ❑ Limitation is the required high SNR for performing this test.  
**(Lensing optical depth grows sharply with distance)**. For the two advanced LIGO detectors at design sensitivity, NS-NS out to  $\sim 100$  Mpc, and BH-BH out to  $\sim 700$  Mpc. Virgo/KAGRA/LIGOIndia joining the network is going to be very valuable. Also ET in the more distant future.

Thank you!

# Infer microlens abundance from surface brightness



# Lensing probability in $\Lambda$ CDM



normalization

$$\int d\ln\mu \frac{dP(\mu, z)}{d\ln\mu} = 1$$

conservation of solid angle

$$\langle \mu \rangle = \int d\ln\mu \frac{dP(\mu, z)}{d\ln\mu} \mu = 1$$

- Lensing by large-scale structure, DM haloes, galaxies
- Possible underestimation for point sources: lensing by compact stellar-mass objects?