New life in quadratic theories of gravity

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Motivation

General Relativity successful low energy theory

\[ S_{EH} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \ R \]
\[ \kappa = \sqrt{8\pi G} \]
Motivation

General Relativity successful low energy theory

\[ S_{EH} = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \ R \quad \kappa = \sqrt{8\pi G} \]

When we try to make a quantum formulation of the theory several problems appear
Motivation

In usual QFT, causality is defined as

\[ [\phi(x), \phi(y)] = 0 , \quad (x - y)^2 < 0 \]  

Spacelike separations
Motivation

In usual QFT, causality is defined as

\[ [\phi(x), \phi(y)] = 0 \ , \quad (x - y)^2 < 0 \]
Motivation

In usual QFT, causality is defined as

\[ [\phi(x), \phi(y)] = 0 , \quad (x - y)^2 < 0 \]
Motivation

In usual QFT, causality is ensured if

\[ [\phi(x), \phi(y)] = 0, \quad (x - y)^2 < 0 \]

Does a fundamental QFT exist for the gravitational interaction?

Möller (1952)  Rosenfeld (1957)
Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity
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At low energies, **effective description** in terms of spacetime variables.
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At low energies, effective description in terms of spacetime variables.

Even if we do not know anything about the non-perturbative part, we can do a perturbative analysis.
Motivation

It could be that at high energies the metric is not the fundamental quantity in a quantum theory of gravity.

At low energies, **effective description** in terms of spacetime variables.

Even if we do not know anything about the **non-perturbative** part, we can do a **perturbative analysis**.

**Background field method**

Metric perturbations around fixed background

Motivation

Around a fixed background spacetime we can use the usual QFT formalism and study the theory in the perturbation limit.
Motivation

Perturbatively, GR → Non renormalizable
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Gravitational coupling is dimensionful

\[[G] = -2\]
Motivation

Perturbatively, GR $\rightarrow$ Non renormalizable

Gravitational coupling is dimensionful $[G] = -2$

$R + R^2 + R^3 + \ldots$

One-loop Two-loop
Motivation

Perturbatively, GR \[ G = -2 R + R^2 + R^3 + \ldots \] is non renormalizable.

Gravitational coupling is dimensionful \([G] = -2\).

Pure gravity non renormalizable at two loops \cite{GoroffSagnotti}.
Motivation

What about quadratic theories of gravity? \[ S_Q \sim \int d^4x \sqrt{-g} \, R^2 \]
Motivation

What about **quadratic** theories of gravity? \[ S_Q \sim \int d^4x \sqrt{-g} \, R^2 \]

**PROS**

- Dimensionless couplings
- Renormalizable

K. Stelle (1977)
Motivation

What about quadratic theories of gravity? \( S_Q \sim \int d^4 x \sqrt{-g} \ R^2 \)

**PROS**

Dimensionless couplings \hspace{2cm} Renormalizable

Closest analogy to a **YM theory of gravity**

\[
R_{\mu\nu\rho\sigma} = \partial_\rho \Gamma^\mu_{\nu\sigma} - \partial_\sigma \Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\lambda\rho} \Gamma^\lambda_{\nu\sigma} - \Gamma^\mu_{\lambda\sigma} \Gamma^\lambda_{\nu\rho}
\]

Field strength

K. Stelle (1977)
Motivation

What about **quadratic** theories of gravity? \[ S_Q \sim \int d^4 x \sqrt{-g} \ R^2 \]

**CONS**

Propagators falling as \( \sim \frac{1}{p^4} \)
Motivation

What about **quadratic** theories of gravity?

\[ S_Q \sim \int d^4x \sqrt{-g} \ R^2 \]

**CONS**

**Propagators** falling as

\[ \sim \frac{1}{p^4} \]

Källen-Lehmann spectral representation

\[ \Delta(p) = \int_0^\infty d\mu^2 \rho(\mu^2) \frac{1}{p^2 - \mu^2 + i\epsilon} \quad \rho(\mu^2) \geq 0 \]
Motivation

What about quadratic theories of gravity?

$$S_Q \sim \int d^4x \sqrt{-g} \ R^2$$

**CON**

Propagators falling as

$$\sim \frac{1}{p^4}$$

Källen-Lehmann spectral representation

$$\frac{1}{p^4 - m^4} = \frac{1}{p^2 - m^2} - \frac{1}{p^2 + m^2}$$

Ghost and/or tachyons
Motivation

What about quadratic theories of gravity? $S_Q \sim \int d^4 x \sqrt{-g} \ R^2$

CONS

Propagators falling as $\sim \frac{1}{p^4}$

Källen-Lehmann spectral representation

$$\frac{1}{p^4 - m^4} = \frac{1}{p^2 - m^2} - \frac{1}{p^2 + m^2}$$

Unitarity is lost
Motivation

What about quadratic theories treated in first order formalism?
Motivation

What about **quadratic theories** treated in **first order** formalism?

First order → metric and connection independents
Motivation

What about quadratic theories treated in first order formalism?

→ Renormalizability
Motivation

What about **quadratic theories** treated in **first order** formalism?

- Renormalizability
- Still room for unitarity
Motivation

What about quadratic theories treated in first order formalism?

→ Renormalizability

→ Still room for unitarity

\[ R_{\nu\rho\sigma}^\mu \sim \nabla_{\nu} \Gamma_{\rho\sigma}^\mu \]

A single derivative in the curvature
Motivation

What about **quadratic theories** treated in **first order** formalism?

- Renormalizability
- Still room for unitarity

Possible **UV completion** of GR?
Motivation

What about **quadratic theories** treated in **first order** formalism?

- Renormalizability

- Still room for unitarity

Possible **UV completion** of GR?

A. Salvio and A. Strumia (2014)
M. B. Einhorn and T. Jones (2017)
Motivation

Lee-Wick type of mechanisms

Able to fix unitarity diagram by diagram

Nevertheless, this mechanism cannot be implemented into the path integral formalism
Outline

- First order formalism vs. Second order formalism
- First order quadratic gravity
- Physical content of the connection
- The coupling to matter in first order formalism
- Summary and Outlook
Outline

First order formalism vs. Second order formalism

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First order vs Second order

Let us take the simple case of the EH action
First order vs Second order

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\[ g_{\mu\nu} \quad \Gamma^\mu_{\nu\rho} \]
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\[ \Gamma^\mu_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left( \partial_\nu g_{\lambda\rho} + \partial_\rho g_{\lambda\nu} - \partial_\lambda g_{\nu\rho} \right) \]
First order vs Second order

Let us take the simple case of the EH action

\[
\delta S_{SO} = \int d^4x \sqrt{-g} \left( -R^\mu_\mu + \frac{1}{2} R g^\mu_\mu \right) \delta g_{\mu\nu}
\]

Einstein's field equation

\[
\Gamma^\mu_{\nu\rho} = \frac{1}{2} g^{\mu\lambda} \left( \partial_\nu g_{\lambda\rho} + \partial_\rho g_{\lambda\nu} - \partial_\lambda g_{\nu\rho} \right)
\]

Fixed relation
First order vs Second order

Let us take the simple case of the EH action

$g_{\mu\nu}$, $\Gamma^\mu_{\nu\rho}$

Independent fields
Let us take the simple case of the EH action

\[ \Gamma_{\nu\rho}^{\mu} = \left\{ \begin{array}{c} \mu \\ \nu \\ \rho \end{array} \right\} + K_{\nu\rho}^{\mu} + L_{\nu\rho}^{\mu} \]

First order vs Second order

**FO**

\[ g_{\mu\nu} \]

\[ \Gamma_{\nu\rho}^{\mu} \]

Independent fields

Levi-Civita connection
First order vs Second order

Let us take the simple case of the EH action

\[ \Gamma_{\nu\rho}^{\mu} = \left\{ \begin{array}{c} \mu \\ \nu \\rho \end{array} \right\} + K_{\nu\rho}^{\mu} + L_{\nu\rho}^{\mu} \]

Levi-Civita connection

Contorsion tensor

\[ \frac{1}{2} g^{\rho\sigma} \left( T_{\mu\nu\sigma} + T_{\nu\sigma\mu} - T_{\mu\nu\sigma} \right) \]
First order vs Second order

Let us take the simple case of the EH action

\[ \Gamma^\mu_{\nu\rho} = \left\{ \begin{array}{c} \mu \\ \nu \\ \rho \end{array} \right\} + K^\mu_{\nu\rho} + L^\mu_{\nu\rho} \]

- **Levi-Civita connection**
- **Contorsion tensor**
- **Related to non-metricity**

\[ Q_{\mu\nu\rho} = \nabla_\mu g_{\nu\rho} \]
First order vs Second order

Let us take the simple case of the EH action

\[ \Gamma^\mu_{\nu\rho} = \begin{cases} \mu \\ \nu \\ \rho \end{cases} + K^\mu_{\nu\rho} + L^\mu_{\nu\rho} \]

Levi-Civita connection

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First order vs Second order

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\[ \delta S_{FO} = \int d^4x \sqrt{-g} \left\{ \left( -R^{\mu\nu} + \frac{1}{2} R g^{\mu\nu} \right) \delta g_{\mu\nu} + g^{\mu\nu} \delta_\mu^\alpha \left( \delta_\nu^\beta \nabla_\lambda - \delta_\lambda^\beta \nabla_\nu \right) \delta \Gamma^\lambda_{\alpha\beta} \right\} \]
First order vs Second order

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Einstein’s field equation
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\[ \nabla_\lambda g^{\alpha \beta} = 0 \]
First order vs Second order

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First order vs Second order

Let us take the simple case of the EH action

\[\Gamma_{\nu\rho}^{\mu} = \left\{ \begin{array}{c} \mu \\ \nu \\ \rho \end{array} \right\} \quad + \quad K_{\nu\rho}^{\mu} \quad + \quad L_{\nu\rho}^{\mu}\]

Levi-Civita connection

Contorsion tensor

Related to non-metricity

\[Q_{\mu\nu\rho} = \nabla_\mu g_{\nu\rho}\]
First order vs Second order

Let us take the simple case of the EH action

$g_{\mu\nu}$  $\Gamma^\mu_{\nu\rho}$

For the EH action SO and FO classically equivalent

Palatini
First order vs Second order

Let us take the simple case of the EH action

\[ g_{\mu \nu} \quad \Gamma^\mu_{\nu \rho} \]

For the EH action SO and FO \textit{classically equivalent}

The equivalence also holds at one loop order \cite{AneroRS2017}
Outline

- First order formalism vs. Second order formalism

First order **quadratic gravity**

- Physical content of the **connection**

- The **coupling to matter** in first order formalism

- Summary and Outlook
First order quadratic gravity

Let us focus on the features of FO quadratic theories
First order quadratic gravity

Let us focus on the features of FO quadratic theories

The Riemann tensor does not enjoy the usual symmetries

\[
\begin{align*}
\text{LC} & \quad R_{[\mu\nu]\rho\sigma} & \quad R_{\mu\nu[\rho\sigma]} & \quad R_{\mu\nu\rho\sigma} = R_{\rho\sigma\mu\nu}
\end{align*}
\]
First order quadratic gravity

Let us focus on the features of FO quadratic theories

The Riemann tensor does not enjoy the usual symmetries

Two different traces of the Riemann tensor

\[ R^+[\Gamma]_{\nu\sigma} = g^{\mu\rho} R[\Gamma]_{\mu\nu\rho\sigma} \]
\[ R^-[\Gamma]_{\mu\sigma} = g^{\nu\rho} R[\Gamma]_{\mu\nu\rho\sigma} \]
\[ R[\Gamma]_{\rho\sigma} = g^{\mu\nu} R[\Gamma]_{\mu\nu\rho\sigma} \]

\[ R_{\mu\nu} = R^+_{\mu\nu} - R^-_{\nu\mu} \]
\[ R^+ = g^{\mu\nu} R^+_{\mu\nu} = - g^{\mu\nu} R^-_{\mu\nu} = - R^- \]
First order quadratic gravity

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Two different traces of the Riemann tensor

\[ R^+[\Gamma]_{\nu\sigma} = g^{\mu\rho}R[\Gamma]_{\mu\nu\rho\sigma} \]
\[ R^-[\Gamma]_{\mu\sigma} = g^{\nu\rho}R[\Gamma]_{\mu\nu\rho\sigma} \]
\[ \mathcal{R}[\Gamma]_{\rho\sigma} = g^{\mu\nu}R[\Gamma]_{\mu\nu\rho\sigma} \]

\[ R_{\mu\nu} = R^+_{\mu\nu} - R^-_{\nu\mu} \]
\[ R^+ = g^{\mu\nu}R^+_{\mu\nu} = - g^{\mu\nu}R^-_{\mu\nu} = - R^- \]

\[ R_{\mu\nu\rho\sigma}, \quad R^+_{\mu\nu}, \quad R^-_{\mu\nu}, \quad R \]
First order quadratic gravity

The most general first order quadratic action reads

\[ S_{FOQ} = \int d^4 x \sqrt{-g} \sum_{I=1}^{I=12} g_I O^I \]

\[ O_I = R^\mu_{\nu\rho\sigma}(D_I)^{\nu\rho\sigma\nu'\rho'\sigma'} R^{\mu'}_{\nu'\rho'\sigma'} \]

\( D_I \): Function of metrics and deltas
First order quadratic gravity

The most general first order quadratic action reads

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$$O_I = R_{\nu\rho\sigma}(D_I)_{\mu\mu'}^{\nu\rho\sigma\rho'\sigma'} R_{\nu'\rho'\sigma'}^{\mu'}$$

$D_I$: Function of metrics and deltas

The theory is Weyl invariant

$$g_{\mu\nu} \rightarrow \Omega^2(x) \ g_{\mu\nu}$$
$$\Gamma_{\mu\nu}^\lambda \rightarrow \Gamma_{\mu\nu}^\lambda$$
$$\sqrt{-g} \rightarrow \Omega^4 \sqrt{-g}$$
$$O_I \rightarrow \Omega^{-4} O_I$$
First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken.
First order quadratic gravity

The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken

\[ L_s = \sqrt{-g} \left( \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right) \]
First order quadratic gravity

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Renormalizing this sector we get \[ \Delta L_s = C_e R \phi^2 \]
First order quadratic gravity

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The spontaneous breaking of the symmetry generates an EH term

\[ L_{EH} = M^2 \sqrt{-g} \ R \]

\[ < \phi > = v \]
The theory is up to now in the conformal phase so the symmetry has to be spontaneously broken.

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The spontaneous breaking of the symmetry generates an EH term

\[ L_{EH} = M^2 \sqrt{-g} \, R \]

Dominates in the IR

S. L. Adler (1988)
First order quadratic gravity

More general connections are allowed
First order quadratic gravity

More general connections are allowed

\[ \Delta H_{\mu\nu} = H_{\mu\nu}^{SO} - H_{\mu\nu}^{FO} = -\frac{1}{2} \nabla_{\lambda} K_{(\mu\nu)}^{\lambda} + \frac{1}{4} g_{\lambda\mu} \nabla^{\rho} K_{(\rho\nu)}^{\lambda} + \frac{1}{4} g_{\lambda\nu} \nabla^{\rho} K_{(\rho\mu)}^{\lambda} \]

\[ H_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \quad K_{\mu\nu}^{\lambda} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \Gamma_{\mu\nu}^{\lambda}} \]

M. Borunda, B. Jansen and M. Bastero-Gil (2008)
First order quadratic gravity

More general connections are allowed

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\[ H_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta g^{\mu\nu}} \]

\[ K^\lambda_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S}{\delta \Gamma^\mu_{\lambda\nu}} \]

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First order quadratic gravity

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M. Borunda, B. Jansen and M. Bastero-Gil (2008)

Bigger solution space, where does gravitation live?
Outline

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  Physical content of the connection

- The coupling to matter in first order formalism

- Summary and Outlook
Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.
Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.

Our aim is to find a complete basis of spin projectors so that we can decompose the three index tensor $A_{\mu\nu\lambda}$ in its propagating spin pieces.
Physical content of $A_{\mu\nu\lambda}$

The connection being an independent field, can introduce new degrees of freedom.

Our aim is to find a complete basis of spin projectors so that we can decompose the three index tensor $A_{\mu\nu\lambda}$ in its propagating spin pieces.

As we will see, a proliferation of spins occurs. It is crucial to check that we do not have ghosts encoded in those propagating spin components.
Physical content of $A_{\mu \nu \lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu \nu} = \eta_{\mu \nu} + \kappa h_{\mu \nu}$$
Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

To quadratic order in the perturbation we get

$$S = \frac{1}{2} \int d^4 x \ h^{\mu\nu} \ K^{EH}_{\mu\nu\rho\sigma} \ h^{\rho\sigma}$$
Physical content of $A_{\mu \nu \lambda}$

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$$S = \frac{1}{2} \int d^4x \ h^{\mu \nu} K_{\mu \nu \rho \sigma}^{EH} h^{\rho \sigma}$$

Interaction between two index tensors
Physical content of $A_{\mu\nu\lambda}$

We take the EH action and expand the metric around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

To quadratic order in the perturbation we get

$$S = \frac{1}{2} \int d^4x \ h^{\mu\nu} \ K_{\mu\nu\rho\sigma}^{EH} h^{\rho\sigma}$$

We want to decompose a two index symmetric tensor in its spin components

Spin projectors $\rightarrow$ Four index operators that project onto a certain spin
Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta_0^\mu$$

In the rest frame

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$
Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$ k^\mu = \delta^\mu_0 $$

$$ \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \quad \text{Projects onto spatial indices} $$

$$ \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2} \quad \text{Projects onto time indices} $$
**Physical content of $A_{\mu\nu\lambda}$**

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**Barnes-Rivers Projectors**

- $s = 2$ : \( h_{ij}^T \equiv h_{ij} - \frac{1}{3} h_{ij} \delta_{ij} \)
- $s = 1$ : \( h_{0i} \)
- $s = 0$ : \( h_{00} \)
- $s = 0$ : \( h \equiv \delta^{ij} h_{ij} \)

Barnes (1963)  
Rivers (1964)  
P. Van Nieuwheinzen (1973)

Different spin representations $SO(3)$
Physical content of $A_{\mu\nu\lambda}$

To project into the different components we have

$$k^\mu = \delta^\mu_0$$

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$$

Projects onto spatial indices

$$\omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$$

Projects onto time indices

**Barnes-Rivers Projectors**

$s = 2 : \quad h^T_{ij} \equiv h_{ij} - \frac{1}{3} h \delta_{ij} \quad \rightarrow \quad (P_2)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2} \left( \theta^\rho_{\mu} \theta^{\sigma}_{\nu} + \theta^\sigma_{\mu} \theta^{\rho}_{\nu} \right) - \frac{1}{3} \theta_{\mu\nu} \theta^{\rho\sigma}$

$s = 1 : \quad h_{0i} \quad \rightarrow \quad (P_1)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{2} \left( \theta^\rho_{\mu} \omega^{\sigma}_{\nu} + \theta^\sigma_{\mu} \omega^{\rho}_{\nu} + \theta^\rho_{\nu} \omega^{\sigma}_{\mu} + \theta^\sigma_{\nu} \omega^{\rho}_{\mu} \right)$

$s = 0 : \quad h_{00} \quad \rightarrow \quad (P_0^w)^{\rho\sigma}_{\mu\nu} \equiv \omega_{\mu\nu} \omega^{\rho\sigma}$

$s = 0 : \quad h \equiv \delta^{ij} h_{ij} \quad \rightarrow \quad (P_0^s)^{\rho\sigma}_{\mu\nu} \equiv \frac{1}{3} \theta_{\mu\nu} \theta^{\rho\sigma}$
Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$
(P_2)^{\rho\sigma}_{\mu\nu} + (P_1)^{\rho\sigma}_{\mu\nu} + (P_0^w)^{\rho\sigma}_{\mu\nu} + (P_0^s)^{\rho\sigma}_{\mu\nu} = I^{\rho\sigma}_{\mu\nu}
$$
Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$\left(P_2\right)^{\rho\sigma}_{\mu\nu} + \left(P_1\right)^{\rho\sigma}_{\mu\nu} + \left(P_0^w\right)^{\rho\sigma}_{\mu\nu} + \left(P_0^s\right)^{\rho\sigma}_{\mu\nu} = I^{\rho\sigma}_{\mu\nu}$$

We are interested in forming a basis of four index projectors

5 independent monomials with this symmetry

$$M_1 \equiv k_\mu k_\nu k_\rho k_\sigma$$
$$M_2 \equiv k_\mu k_\nu \eta_{\rho\sigma}$$
$$M_3 \equiv k_\mu k_\sigma \eta_{\nu\rho}$$
$$M_4 \equiv \eta_{\mu\nu} \eta_{\rho\sigma}$$
$$M_5 \equiv \eta_{\mu\rho} \eta_{\nu\sigma}$$
Physical content of $A_{\mu \nu \lambda}$

These projectors add up to the identity

$$\left( P_2 \right)^{\rho \sigma}_{\mu \nu} + \left( P_1 \right)^{\rho \sigma}_{\mu \nu} + \left( P_0^w \right)^{\rho \sigma}_{\mu \nu} + \left( P_0^s \right)^{\rho \sigma}_{\mu \nu} = I^{\rho \sigma}_{\mu \nu}$$

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$$M_1 \equiv k^\mu k^\nu k^\rho k^\sigma$$
$$M_2 \equiv k^\mu k^\nu \eta^\rho \sigma$$
$$M_3 \equiv k^\mu k^\sigma \eta^\nu \rho$$
$$M_4 \equiv \eta^\mu \nu \eta^\rho \sigma$$
$$M_5 \equiv \eta^\mu \rho \eta^\nu \sigma$$

One extra operator

in the basis

$$\left( P_0^x \right)^{\rho \sigma}_{\mu \nu} = \frac{1}{\sqrt{3}} \left( \omega^\mu \nu \theta^\rho \sigma + \theta^\mu \nu \omega^\rho \sigma \right)$$
Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$(P_2)^{\rho\sigma}_{\mu\nu} + (P_1)^{\rho\sigma}_{\mu\nu} + (P_0^w)^{\rho\sigma}_{\mu\nu} + (P_0^s)^{\rho\sigma}_{\mu\nu} = I^{\rho\sigma}_{\mu\nu}$$

Strategy

Take $K_{\mu\nu\rho\sigma} = \sum_i c_i P_{i\mu\nu\rho\sigma}$
Physical content of $A_{\mu\nu\lambda}$

These projectors add up to the identity

$$(P_2)^{\rho\sigma}_{\mu\nu} + (P_1)^{\rho\sigma}_{\mu\nu} + (P_0^w)^{\rho\sigma}_{\mu\nu} + (P_0^s)^{\rho\sigma}_{\mu\nu} = I^{\rho\sigma}_{\mu\nu}$$

Strategy

Take

$$K_{\mu\nu\rho\sigma} = \sum_i c_i P_{i\mu\nu\rho\sigma}$$

Divide the quadratic piece in the different spin components

$$h^{\mu\nu} \left( \sum_i c_i P_{i\mu\nu\rho\sigma} \right) h^{\rho\sigma} = \sum_i h^{\mu\nu}_{i} \Box h^{i}_{\mu\nu}$$
Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2} \int d^4x - \frac{1}{4} h^{\mu\nu} \left( P_2 + P_1 - \frac{1}{2} P^s_0 + \frac{1}{2} P^w_0 - \frac{\sqrt{3}}{2} P^x \right)_{\mu\nu\rho\sigma} \Box h^{\rho\sigma}$$
Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2} \int d^4x \ - \frac{1}{4} h^{\mu\nu} \left( P_2 + P_1 - \frac{1}{2} P_0^s + \frac{1}{2} P_0^w - \frac{\sqrt{3}}{2} P^x \right)_{\mu\nu\rho\sigma} \Box h^{\rho\sigma}$$

Inverting the operator we get the propagator and the free energy

$$W \left[ T_{(1)}, T_{(2)} \right] = \int d^4x \Delta^{\mu\nu}_{(1)} \Delta^{\rho\sigma}_{(2)} = \int d^4x \left( T^{\mu\nu}_{(1)} (P_2 - \frac{1}{2} P_0^s)_{\mu\nu\rho\sigma} T^{\rho\sigma}_{(2)} \right)$$

Interaction between external sources
Physical content of $A_{\mu \nu \lambda}$

So now we can decompose any four index operator into the spin projectors

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Inverting the operator we get the propagator and the free energy

$$W\left[T_{(1)}, T_{(2)}\right] = \int d^4x T_{(1)}^{\mu \nu} \Delta_{\mu \nu \rho \sigma} T_{(2)}^{\rho \sigma} = \int d^4x \left( T_{(1)}^{\mu \nu} \left( P_2 - \frac{1}{2} P_0^s \right)_{\mu \nu \rho \sigma} T_{(2)}^{\rho \sigma} \right)$$

D. Dicus and S. Willenbrock (2004)
Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2} \int d^4x - \frac{1}{4} h^{\mu\nu} \left( P_2 + P_1 - \frac{1}{2} P_0^s + \frac{1}{2} P_0^w - \frac{\sqrt{3}}{2} P^x \right)_{\mu\nu\rho\sigma} \Box h^{\rho\sigma}$$

Inverting the operator we get the propagator and the free energy

$$W\left[ T_{(1)}, T_{(2)} \right] = \int d^4x T^{\mu\nu}_{(1)} \Delta_{\mu\nu\rho\sigma} T^{\rho\sigma}_{(2)} = \int d^4x \left( T^{\mu\nu}_{(1)} (P_2 - \frac{1}{2} P_0^s)_{\mu\nu\rho\sigma} T^{\rho\sigma}_{(2)} \right)$$

On-shell asymptotic states

A single spin 2 field propagates

Graviton
Physical content of $A_{\mu\nu\lambda}$

So now we can decompose any four index operator into the spin projectors

$$S_{EH+gf} = \frac{1}{2} \int d^4x \ - \frac{1}{4} h^{\mu\nu} \left( P_2 + P_1 - \frac{1}{2} P_0^s + \frac{1}{2} P_0^w - \frac{\sqrt{3}}{2} P^x \right)_{\mu\nu\rho\sigma} \Box h^{\rho\sigma}$$

Summary

Spin-2 and Spin-0 components mediating the interaction between external sources (off-shell)

Spin-2 massless unique asymptotic state Graviton
Physical content of $A_{\mu\nu\lambda}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathcal{A} \equiv T_x \otimes \text{Sym} \left( T_x \otimes T_x \right)$$
Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathcal{A} \equiv T_x \otimes \text{Sym} \left( T_x \otimes T_x \right)$$

40 independent components

$\begin{bmatrix} \bullet & \otimes & \bullet \end{bmatrix} = \begin{bmatrix} \boxdot & \oplus & \bullet \end{bmatrix}$

Hook part

Totally Symmetric part

$$\{2,0\} \otimes \{1\} = \{3,0\} \oplus \{2,1\}$$
Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathcal{A} \equiv T_x \otimes \text{Sym} \left(T_x \otimes T_x\right)$$

40 independent components

Hook part

Totally Symmetric part

$$20_H = 2\ (2) \oplus 3\ (1) \oplus (0)$$

$$20_S = (3) \oplus (2) \oplus 2\ (1) \oplus 2\ (0)$$

$$P_H + P_S = I$$
We are working with symmetric connections

\[ A_{\mu\nu\lambda} \in \mathcal{A} \equiv T_x \otimes \text{Sym} \left( T_x \otimes T_x \right) \]

\[ \begin{array}{cc} \square \otimes \square & = & \boxed{} \oplus \boxed{} \end{array} \]

We have 22 independent monomials with this symmetry

22 projectors in the basis \((P_i)^{\mu\nu\lambda}_{\alpha\beta\gamma}\)
Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

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22 projectors in the basis $\left( P_i \right)^{\mu\nu\lambda}_{\alpha\beta\gamma}$

We need 10 extra spin operators with mixed symmetry
Physical content of $A_{\mu(\nu\lambda)}$

We are working with symmetric connections

$$A_{\mu\nu\lambda} \in \mathcal{A} \equiv T_x \otimes \text{Sym} \left( T_x \otimes T_x \right)$$

$$\begin{array}{ccc}
\oplus & \otimes & \oplus \\
\oplus & \oplus & \oplus \\
\end{array} = \begin{array}{ccc}
\oplus & \oplus & \oplus \\
\oplus & \oplus & \oplus \\
\end{array}$$

We have 22 independent monomials with this symmetry

22 projectors in the basis $(P_i)^{\mu\nu\lambda}_{\alpha\beta\gamma}$

We need 10 extra spin operators with mixed symmetry

One spin-3, four spin-2, eleven spin-1 and six spin-0
Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$$

$$\Gamma^\lambda_{\mu\nu} = \bar{\Gamma}^\lambda_{\mu\nu} + A^\lambda_{\mu\nu}$$

$$A_{\alpha\beta\gamma} \ K^{\alpha\beta\gamma}_{\mu\nu\lambda} A^{\mu\nu\lambda} + h^{\mu\nu} M_{\mu\nu\rho\sigma} h^{\rho\sigma} + h^{\mu\nu} N^{\rho\sigma}_{\mu\nu\lambda} A^\lambda_{\rho\sigma}$$
Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space

$A_{\alpha\beta\gamma} \ K_{\mu\nu\lambda} \ A^{\mu\nu\lambda}$

All the dynamics encoded in the gauge field
Physical content of $A_{\mu(\nu\lambda)}$

In FOQG around flat space

$$A_{\alpha\beta\gamma} \; K_{\mu\nu\lambda}^\alpha \; A^{\mu\nu\lambda}$$

All the dynamics encoded in the gauge field

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left( \alpha \; R[\Gamma]^2 + \beta \; R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma \; R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$
Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left( \alpha R[\Gamma]^2 + \beta R[\Gamma]_{\mu\nu} R[\Gamma]^\mu\nu + \gamma R[\Gamma]_{\mu\nu}\rho\sigma R[\Gamma]^\mu\nu\rho\sigma \right)$$

$\mathcal{P}$ Hook \hspace{1cm} $\mathcal{P}$ Symmetric \hspace{1cm} $\mathcal{P}$ Mixed

$$(K_{FOQ})^\mu\nu_{\tau}^{\rho\sigma} = \left( -2(2\gamma + \beta) P_0^s - (4\gamma + 9\alpha + 2\beta) P_0^s + (2\gamma - \beta) P_0^x - \frac{4}{3}(3\gamma + 5\beta) P_1^s 
- 2\gamma P_1^s - \frac{4}{3}(3\gamma + \beta) P_1^t - (2\gamma + \beta) P_1^{wx} + 4\beta P_1^{ss} - 2(2\gamma + \beta) (P_2 + P_2) 
- 4\gamma P_2^s + 2(\beta + \gamma) P_2^x - 4\gamma P_3 \right)_{\tau}^{\mu\nu}_{\lambda}^{\rho\sigma}$$
Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left( \alpha R[\Gamma]^2 + \beta R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

<table>
<thead>
<tr>
<th>$\mathcal{P}$ Hook</th>
<th>$\mathcal{P}$ Symmetric</th>
<th>$\mathcal{P}$ Mixed</th>
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</thead>
</table>

$$(K_{FOQ})_{\tau \lambda}^{\mu\nu \rho\sigma} = \left( -2(2\gamma + \beta) \mathcal{P}_{0}^{s} - (4\gamma + 9\alpha + 2\beta) \mathcal{P}_{0}^{s} + (2\gamma - \beta) \mathcal{P}_{0}^{s} - \frac{4}{3}(3\gamma + 5\beta) \mathcal{P}_{1}^{s} 
- 2\gamma \mathcal{P}_{1}^{s} - \frac{4}{3}(3\gamma + \beta) \mathcal{P}_{1}^{t} - (2\gamma + \beta) \mathcal{P}_{1}^{w} + 4\beta \mathcal{P}_{1}^{ss} - 2(2\gamma + \beta) (\mathcal{P}_{2} + \mathcal{P}_{2}) 
- 4\alpha \mathcal{P}_{2}^{s} + 2(\beta + \gamma) \mathcal{P}_{2}^{s} - 4\gamma \mathcal{P}_{3} \right)_{\tau \lambda}^{\mu\nu \rho\sigma}$$

Spin-3 is present and comes from the Riemann squared terms
Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{\text{FOQ}} \equiv \int d^nx \sqrt{-g} \left( \alpha R[\Gamma]^2 + \beta R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook \hspace{1cm} $\mathcal{P}$ Symmetric \hspace{1cm} $\mathcal{P}$ Mixed

$$(K_{\text{FOQ}})^{\mu\nu}_{\tau\lambda \rho\sigma} = \left( -2(2\gamma + \beta) \mathcal{P}_0^s - (4\gamma + 9\alpha + 2\beta) \mathcal{P}_0^s + (2\gamma - \beta) \mathcal{P}_0^x - \frac{4}{3}(3\gamma + 5\beta) \mathcal{P}_1^s 
- 2\gamma \mathcal{P}_1^s - \frac{4}{3}(3\gamma + \beta) \mathcal{P}_1^t - (2\gamma + \beta) \mathcal{P}_1^{wx} + 4\beta \mathcal{P}_1^{ss} - 2(2\gamma + \beta)(\mathcal{P}_2 + \mathcal{P}_2)
- 4\gamma \mathcal{P}_2^s + 2(\beta + \gamma) \mathcal{P}_2^x - 4\gamma \mathcal{P}_3 \right)^{\mu\nu}_{\tau\lambda \rho\sigma}$$

We have in principle three Spin-2 components
Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left( \alpha R[\Gamma]^2 + \beta R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook \quad $\mathcal{P}$ Symmetric \quad $\mathcal{P}$ Mixed

$$(K_{FOQ})_{\tau\lambda}^{\mu\nu \rho\sigma} = \left( -2(2\gamma + \beta) \ P_0^s - (4\gamma + 9\alpha + 2\beta) \ P_0^s + (2\gamma - \beta) \ P_0^x - \frac{4}{3}(3\gamma + 5\beta) \ P_1^s ight.$$

$$- 2\gamma \ P_1^s - \frac{4}{3}(3\gamma + \beta) \ P_1^t - (2\gamma + \beta) \ P_1^{wx} + 4\beta \ P_1^{ss} - 2(2\gamma + \beta) \ (P_2 + P_2)$$

$$- 4\gamma \ P_2^s + 2(\beta + \gamma) \ P_2^x - 4\gamma P_3 \right)_{\tau\lambda}^{\mu\nu \rho\sigma} \square$$

We have in principle three Spin-0 and four Spin-1 components.
Physical content of $A_{\mu(\nu\lambda)}$

Taking for instance the simpler action

$$S_{FOQ} \equiv \int d^n x \sqrt{-g} \left( \alpha R[\Gamma]^2 + \beta R[\Gamma]_{\mu\nu} R[\Gamma]^{\mu\nu} + \gamma R[\Gamma]_{\mu\nu\rho\sigma} R[\Gamma]^{\mu\nu\rho\sigma} \right)$$

$\mathcal{P}$ Hook \hspace{1cm} $\mathcal{P}$ Symmetric \hspace{1cm} $\mathcal{P}$ Mixed

$$(K_{FOQ})_{\tau\lambda}^{\mu\nu\rho\sigma} = \left( -2(2\gamma + \beta) \mathcal{P}_0^s - (4\gamma + 9\alpha + 2\beta) \mathcal{P}_0^s + (2\gamma - \beta) \mathcal{P}_0^{x} - \frac{4}{3}(3\gamma + 5\beta) \mathcal{P}_1^s - 2\gamma \mathcal{P}_1^s - \frac{4}{3}(3\gamma + \beta) \mathcal{P}_1^t - (2\gamma + \beta) \mathcal{P}_1^{w} + 4\beta \mathcal{P}_1^{ss} - 2(2\gamma + \beta) (\mathcal{P}_2 + \mathcal{P}_2) \right.$$  

$$\left. - 4\gamma \mathcal{P}_2^s + 2(\beta + \gamma) \mathcal{P}_2^{x} - 4\gamma \mathcal{P}_3^s \right)^{\mu\nu\rho\sigma}_{\tau\lambda} \square$$

Which ones appear in the propagator? Which ones survive on-shell?
Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi} \int d^4 x \, \eta^{\mu\nu} \, \eta^{\rho\sigma} \, \eta_{\tau\lambda} \, A^\tau_{\mu\nu} \, \Box \, A^\lambda_{\rho\sigma}$$
Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi} \int d^4x \, \eta^{\mu\nu} \eta^{\rho\sigma} \eta_{\tau\lambda} \, A_{\mu\nu}^{\tau} \Box A_{\rho\sigma}^{\lambda}$$

$\mathcal{P}$ Hook \hspace{1cm} $\mathcal{P}$ Symmetric \hspace{1cm} $\mathcal{P}$ Mixed

$$(K_{gf})^{\mu\nu}_{\tau\lambda} = \frac{1}{\chi} \left( P_0^w + 3 \, P_0^s + 3 \, \mathcal{P}_0^s - 3 \, \mathcal{P}_0^x + \mathcal{P}_0^{sw} + \mathcal{P}_0^{ws} + P_1 - \frac{5}{3} \, P_1^s + \mathcal{P}_1^w + \frac{2}{3} \, \mathcal{P}_1^t - \mathcal{P}_1^{wx} + \mathcal{P}_1^{ws} + \mathcal{P}_1^{sw} + \mathcal{P}_1^{sx} + 4 \, \mathcal{P}_1^{ss} \right)^{\mu\nu}_{\tau\lambda} \Box$$
Physical content of $A_{\mu(\nu\lambda)}$

We need a gauge fixing to invert the operator

$$S_{gf} = \frac{1}{\chi} \int d^4 x \; \eta^{\mu\nu} \eta^{\rho\sigma} \eta_{\tau\lambda} \; A_{\mu\nu}^\tau \, \square \, A_{\rho\sigma}^\lambda$$

$P$ Hook    $P$ Symmetric    $P$ Mixed

$$(K_{gf})_{\tau \lambda}^{\mu\nu} \rho\sigma = \frac{1}{\chi} \left( P_0^w + 3 \, P_0^s + 3 \, P_0^s - 3 \, P_0^x + P_0^{sw} + P_0^{ws} + P_1 - \frac{5}{3} \, P_1^s + P_1^w + \frac{2}{3} \, P_1^t - P_1^{wx} + P_1^{ws} + P_1^{sw} + P_1^{sx} + 4 \, P_1^{ss} \right)_{\tau \lambda}^{\mu\nu} \rho\sigma \, \square$$

Important to note: no spin-2 or spin-3 in the gauge fixing!
Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(KR^2 + gf)^{\mu\nu}_{\tau} \rho^\sigma = \frac{1}{\chi} \left( P^w_0 + 3 P^s_0 + (3 - 9\chi) P^x_0 - 3 P^w_0 + P^{sw}_0 + P^{ws}_0 + P^w_1 - \frac{5}{3} P^s_1 + P^w_1 + \frac{2}{3} P^t_1 - P^{wx}_1 + P^{ws}_1 + P^{sw}_1 + P^{sx}_1 + 4 P^{ss}_1 \right)^{\mu\nu}_{\tau} \rho^\sigma$$
Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+gf})_{\tau \lambda}^{\mu \nu \rho \sigma} = \frac{1}{\chi} \left( P_0^w + 3 P_0^{s \lambda} - 3 \mathcal{P}_0^x + \mathcal{P}_0^{sw} + \mathcal{P}_0^{ws} + P_1^w - \frac{5}{3} P_1^s \right)$$

No spin-2 or spin-3
Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

$$(K_{R^2+gf})^{\mu\nu}_{\tau\lambda} = \frac{1}{\chi} \left( P_0^w + 3 P_0^s + (3 - 9\chi) P_0^s - 3 P_0^x + P_0^{sw} + P_0^{ws} + P_1^w - \frac{5}{3} P_1^s \right)$$

No spin-2 or spin-3

The projection onto spin-2 and spin-3 are zero modes

We would need to gauge fix them
Physical content of $A_{\mu(\nu\lambda)}$

If we make $\beta = \gamma = 0$

\[
(K_{R^2} + g_f)^{\mu\nu \rho \sigma}_{\tau \chi} = \frac{1}{\chi} \left( P^w_0 + 3 P^s_0 + (3 - 9\chi) P^s_0 - 3 P^x_0 + P^{sw}_0 + P^{ws}_0 + P^w_1 - \frac{5}{3} P^s_1 \right)
\]

No spin-2 or spin-3

\[+ P^w_1 + \frac{2}{3} P^t_1 - P^{wx}_1 + P^{ws}_1 + P^{sw}_1 + P^{sx}_1 + 4 P^{ss}_1 \]$

The projection onto spin-2 and spin-3 are zero modes

We would need to gauge fix them

In fact, for $R^2$ there are 13 zero modes
Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.
Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

The next step is to couple external sources for the connection and see the spin components that survive.
Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

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To see the asymptotic states, we need to compute the equations of motion for the different spin pieces of the connection.
Physical content of $A_{\mu(\nu\lambda)}$

For $\alpha \neq \beta \neq \gamma \neq 0$ we can invert the operator in order to get the propagator. In doing so, all the projectors of the basis appear.

The next step is to couple external sources for the connection and see the spin components that survive.

To see the asymptotic states, we need to compute the equations of motion for the different spin pieces of the connection.

We have not found any obvious problem with the spin-3 piece so far. A full analysis of this piece is needed in order to see if inconsistencies appear.
Outline

- First order formalism vs. Second order formalism
- First order quadratic gravity
- Physical content of the connection

The coupling to matter in first order formalism

- Summary and Outlook
The coupling to matter in FO

The coupling of bosons in first order formalism does not give any new feature as they do not couple to the connection.
The coupling to matter in FO

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We are interested in the coupling of fermions to gravity in first order formalism, as they turn out to be a source of torsion.
The coupling to matter in FO

The coupling of bosons in first order formalism does not give any new feature as they do not couple to the connection.

We are interested in the coupling of fermions to gravity in first order formalism, as they turn out to be a source of torsion.

This constitutes a difference between first order linear gravity and second order linear gravity, not present in the case of pure gravity.
Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

\[
\begin{align*}
  e_\mu^a & \quad \omega^{ab}_\mu \\
  \text{Independent}
\end{align*}
\]
Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

\[
S_{DEH} \equiv -\frac{1}{\kappa^2} \int d^4x \ e \ \frac{1}{2} e_a^\mu e_b^\nu R^{ab}_{\mu\nu}[\omega] + \frac{i}{2} \int d^4x \ e \left( \bar{\psi} e^\mu_a \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e^\mu_a \gamma^a \psi \right)
\]
We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

\[ S_{DEH} \equiv - \frac{1}{\kappa^2} \int d^4x \, e^{\frac{1}{2}} e_a^\mu e_b^\nu R^{ab}_{\mu\nu}[\omega] + \frac{i}{2} \int d^4x \, e \left( \bar{\psi} e_a^\mu \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e_a^\mu \gamma^a \psi \right) \]

Variations with respect to the vierbein

\[ G_a^\mu + \kappa T_a^\mu = 0 \]

Not symmetric!

\[ T_a^\mu = \frac{i}{2} \kappa \left( \bar{\psi} \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^a \psi \right) - \frac{i}{2} \kappa e_a^\mu \left( \bar{\psi} \gamma^\nu \nabla_\nu \psi - \nabla_\nu \bar{\psi} \gamma^\nu \psi \right) \]
Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

\[ S_{DEH} \equiv -\frac{1}{\kappa^2} \int d^4x \ e^{\mu a} e^{\nu b} R^{ab}_{\mu\nu}[\omega] + \frac{i}{2} \int d^4x \ e^{\mu a} \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e^{\mu a} \gamma^a \psi \]

Variations with respect to the vierbein

\[ G^a_\mu + \kappa T^a_\mu = 0 \]

\text{Not symmetric!}

Fermions act like a source of torsion
Coupling of fermions in FO

We take the minimal coupling of fermions to gravity. To do that, we need to change to the vielbein formalism

\[ S_{DEH} \equiv -\frac{1}{\kappa^2} \int d^4x \, e \, \frac{1}{2} e_a^\mu e_b^\nu R_{\mu\nu}^{ab}[\omega] + \frac{i}{2} \int d^4x \, e \left( \bar{\psi} e_\mu^a \gamma^a \nabla_\mu \psi - \nabla_\mu \bar{\psi} e_\mu^a \gamma^a \psi \right) \]

The antisymmetric part of the variations with respect to the spin connection

\[ T_{\nu\lambda\rho} = \frac{\kappa^2}{2} \epsilon_{\nu\lambda\rho\sigma} j_5^\sigma \]

Totally antisymmetric torsion proportional to the axial current
Coupling of fermions in FO

We can reintroduce it in the action
Coupling of fermions in FO

We can reintroduce it in the action

\[ \tilde{S}_{DEH} = S_{DEH} + \frac{3\kappa^2}{16} \int d^4x \, e \, \bar{\psi} \gamma^\sigma \gamma_5 \psi \, \bar{\psi} \gamma_\sigma \gamma^5 \psi \]

H. Weyl (1950)
Coupling of fermions in FO

We can reintroduce it in the action

\[ \tilde{S}_{DEH} = S_{DEH} + \frac{3\kappa^2}{16} \int d^4x \ e \bar{\psi}\gamma^\sigma\gamma_5\psi \bar{\psi}\gamma_\sigma\gamma^5\psi \]

Dirac-EH action with torsion

H. Weyl (1950)
We can reintroduce it in the action

\[ \tilde{S}_{DEH} = S_{DEH} + \frac{3\kappa^2}{16} \int d^4x \, e \, \bar{\psi} \gamma^\sigma \gamma_5 \psi \, \bar{\psi} \gamma_5 \psi \]

\text{Dirac-EH action with torsion} \quad \text{Dirac-EH action with no torsion}

Extra quartic contact interaction between fermions

H. Weyl (1950)
Coupling of fermions in FO

We can reintroduce it in the action

\[
\tilde{S}_{DEH} = S_{DEH} + \frac{3\kappa^2}{16} \int d^4x \, e \, \bar{\psi} \gamma^\sigma \gamma_5 \psi \, \bar{\psi} \gamma^\sigma \gamma^5 \psi
\]

Dirac-EH action with torsion  Dirac-EH action with no torsion

Extra quartic contact interaction between fermions

This is precisely the difference between FO and SO formalisms

H. Weyl (1950)
Coupling of fermions in FO

We can reintroduce it in the action

\[ \tilde{S}_{DEH} = S_{DEH} + \frac{3\kappa^2}{16} \int d^4x \, e \, \bar{\psi} \gamma^\sigma \gamma_5 \psi \, \bar{\psi} \gamma_\sigma \gamma^5 \psi \]

Dirac-EH action with torsion
Dirac-EH action with no torsion

Extra quartic contact interaction between fermions

This is precisely the difference between FO and SO formalisms

Nevertheless, suppressed by the Planck mass
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Dirac-EH action with **torsion**  
Dirac-EH action with **no torsion**

Extra quartic contact interaction between fermions

This is precisely the **difference** between FO and SO formalisms

What form does the **torsion** have when coupling fermions in FOQG?
Summary and Outlook

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- FOQG is a renormalizable gauge theory with room for unitarity
- In principle, this theory is richer in its field content: we have spin-3, spin-2, spin-1 and spin-0 components
- The solution space is bigger than that of second order quadratic theories
- The theory must undergo a spontaneous symmetry breaking so that EH dominates in the IR
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What are the sources of the connection?

Is the free energy positive definite?
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Are there ghostly degrees of freedom?
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  Can we reintroduce it in the action?

  What kind of new interactions do we get?
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- The coupling of fermions and the resulting torsion needs more study
- The spin-3 part is worth of deeper study

C. Aragone and S. Deser (1979)
M. A. Vasiliev (1988)
Summary and Outlook

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- The coupling to external sources will shed light into the unitarity of the theory.

- The equations of motion of each component are needed to analyse the asymptotic states.

- The coupling of fermions and the resulting torsion needs more study.

- The spin-3 part is worth of deeper study.

Are there any inconsistencies in the interactions?
Thank you for your attention
Backup

Possible sources

Zero modes

\[ J_{\alpha\beta\gamma} \equiv Ak_{\alpha} \eta_{\beta\gamma} + B(k_{\beta}\eta_{\alpha\gamma} + k_{\gamma}\eta_{\alpha\beta}) \]

\[ J_{\alpha\beta\gamma} = A j_{\alpha} T_{\beta\gamma} + B(j_{\beta} T_{\alpha\gamma} + j_{\gamma} T_{\alpha\beta}) \]

\[ Z_1 \equiv (P_{1}^{w} + P_{0}^{s} - \mathcal{P}_{0}^{ws})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_2 \equiv (-P_{1}^{w} + P_{1}^{s} + 3P_{1}^{w} - \frac{3}{8}P_{1}^{sw} - \frac{3}{2}P_{1}^{wst})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_3 \equiv (2P_{1}^{w} + P_{1}^{l} - \frac{3}{2}P_{1}^{sw})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_4 \equiv (-2P_{1}^{w} + P_{1}^{w} + P_{1}^{ws} - \frac{3}{8}P_{1}^{sw} - \frac{3}{2}P_{1}^{wst})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_5 \equiv (-2P_{1}^{w} + P_{1}^{w} - \frac{3}{4}P_{1}^{sw} + P_{1}^{sx} - P_{1}^{wst})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_6 \equiv (-\frac{7}{6}P_{1}^{w} + \frac{14}{3}P_{1}^{w} - \frac{21}{16}P_{1}^{ws} + P_{1}^{ss} - \frac{7}{4}P_{1}^{wst})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_7 \equiv (P_{1}^{s})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_8 \equiv (P_{1}^{sw})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_9 \equiv (P_{2})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_{10} \equiv (P_{2})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_{11} \equiv (P_{3})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_{12} \equiv (P_{3})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]

\[ Z_{13} \equiv (P_{3})_{\lambda\mu\nu}^{\alpha\beta\gamma} \Omega_{\alpha\beta\gamma} \]