

# Stellar rejuvenation and gravitational waves in AGN discs: Analog of planetary systems around massive black holes.

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in collaboration with

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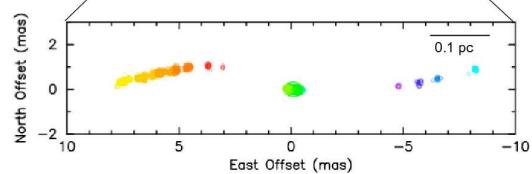
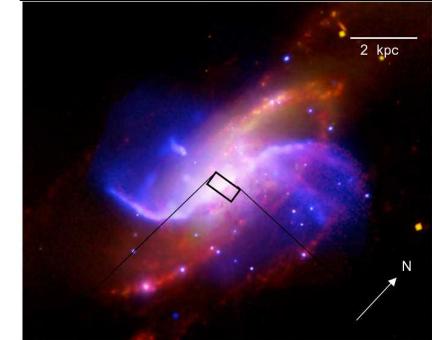
ITC, CfA, Harvard, Sep 2018



# Main motivations

- What caused the emergence of a disk of young stars around the Galactic Center?
- Why are high-redshift quasars so metal rich?
- Can stellar remnants power X-rays in AGNs?
- Can the dynamical evolution of nuclear clusters be affected by gaseous or stellar disks?
- How do AGN become inactive?
- Can a fraction of gravitational wave events be due to stellar black hole mergers around AGNs ?

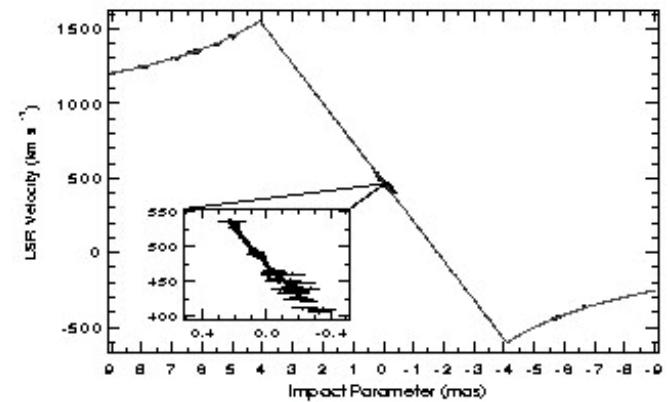
# Unified AGN model (Antonucci & Miller)



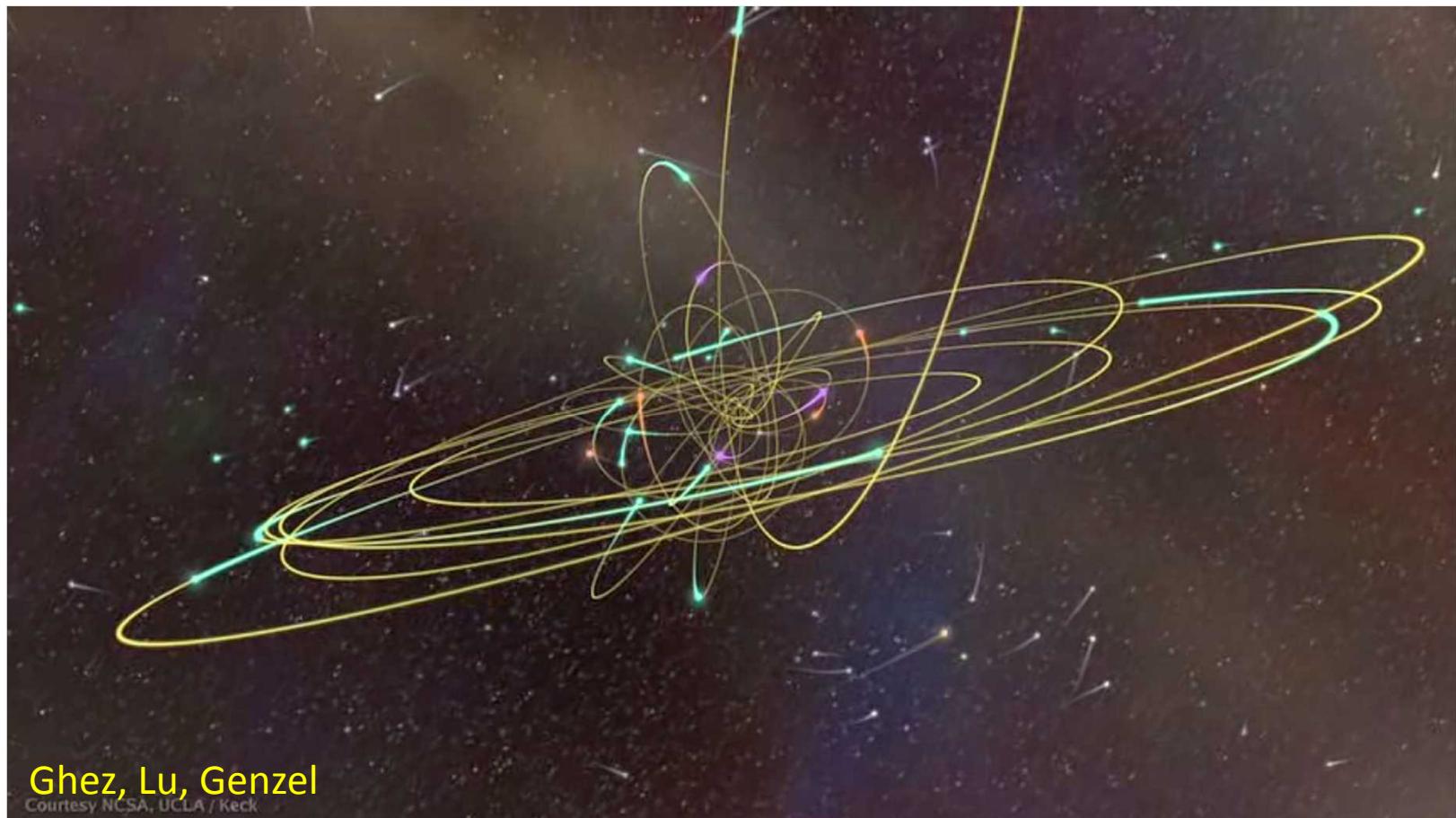
Massive Black Holes:  
Lynden-Bell 1969

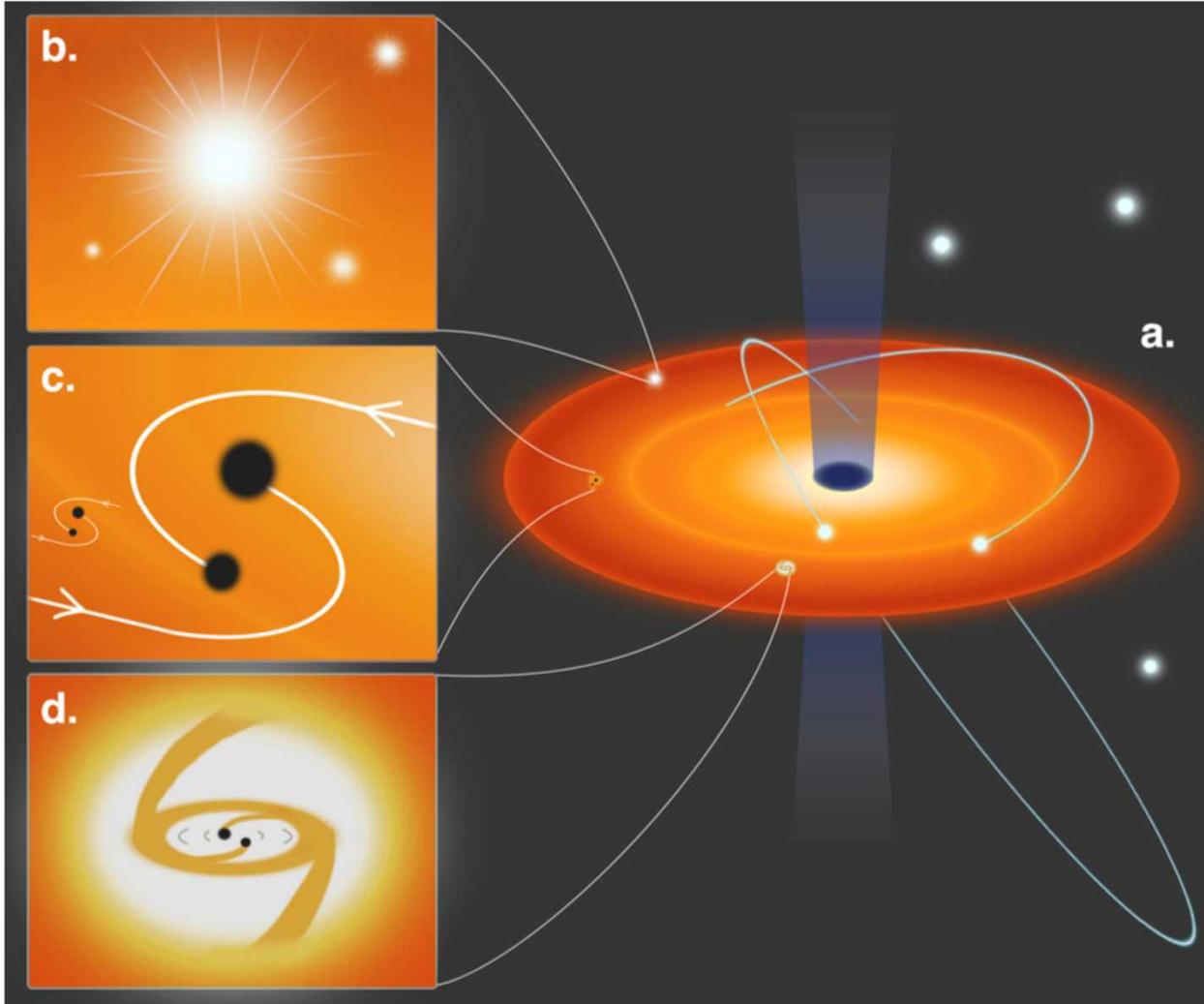
Accretion disk theory:  
Lynden-Bell & Pringle

Keplerian disk in NGC4258



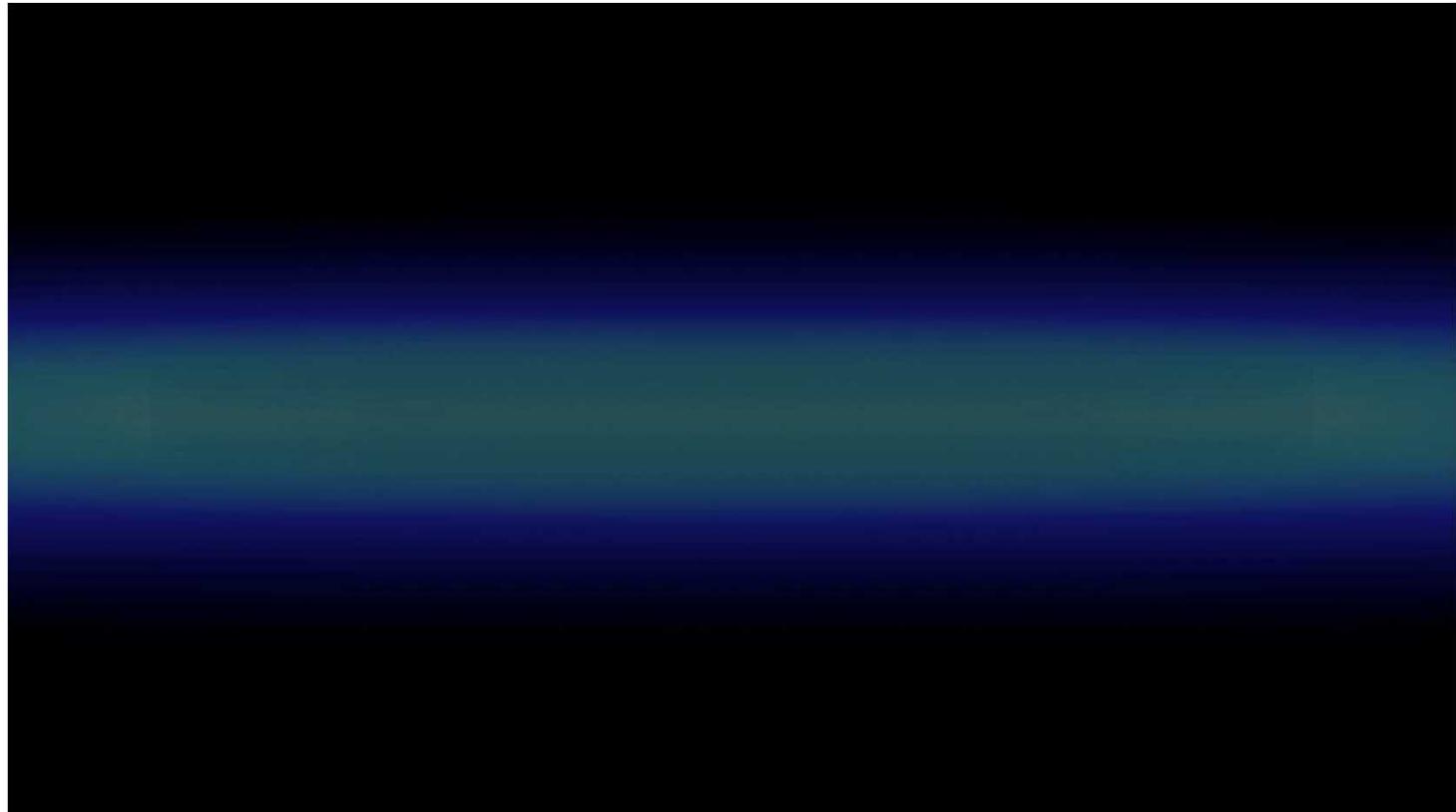
# Young stars in the Galactic Center



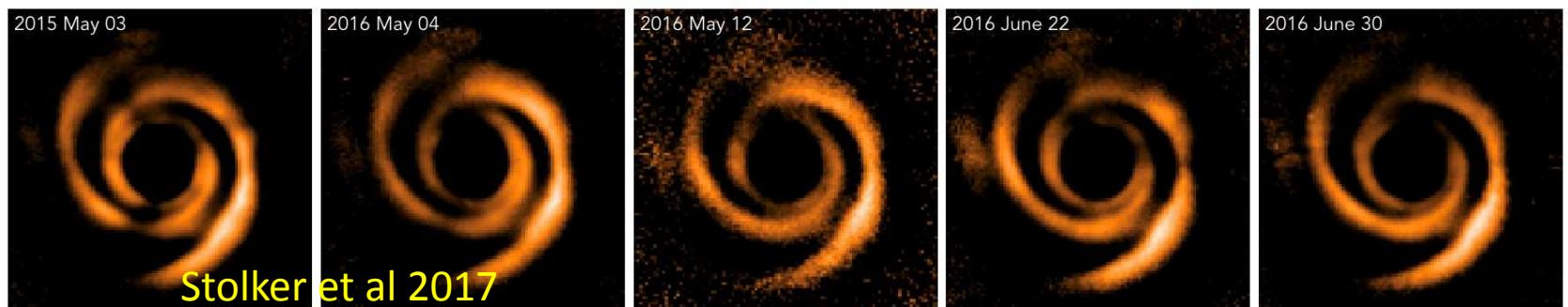


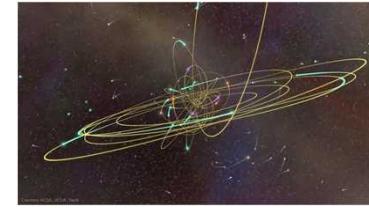
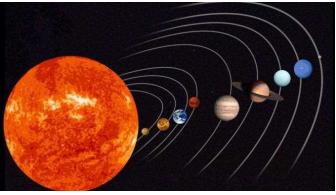
1. There is a MBH in every galaxy
2. Around each MBH there is a nuclear cluster
3. AGN occurs when a MBH is fed by a disk
4. What happens during stars' disk passage ?

# Companions inclined to disks



R Naylor





# Relevant physical parameters

Planetary systems:

1. Mass ratio:  $10^{-6}$ - $10^{-3}$
2. Period: days-centuries
3. Radius/semi major axis:  $10^{-4}$

Galactic Center system:

1. Mass ratio:  $10^{-6}$ - $10^{-3}$
2. Period: yrs- millenium
3. Radius/semi major axis:  $10^{-5}$

Protostellar disks

1. Disk mass/star mass: 0.01-0.1
2.  $H/r = 0.05-0.2$
3.  $Q > 10$
4. Persistent time scale: 3-10My

AGN and young stellar disk

1. Disk mass/star mass:  $\sim 0.01$
2.  $H/r \sim 0.01-0.1$
3.  $Q: \sim 1$
4. Persistent time scale: 1-100My

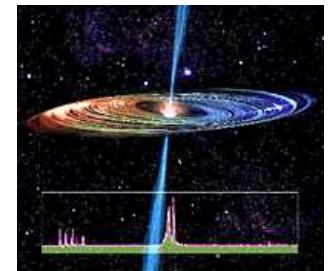
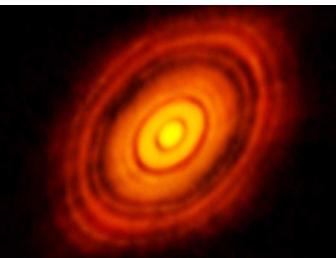
## Required model parameters

Nuclear star clusters:

1. Stellar density
2. Dynamical property
3. Connection to host galaxy

Accretion disks:

1. Capture rate
2. Accretion & stellar IMF
3. Contamination & BH formation



# Dissipation & accretion rate in AGN disks

$$\dot{M} = L_{\text{Bol}} / \epsilon c^2 \quad \epsilon \sim 0.065$$

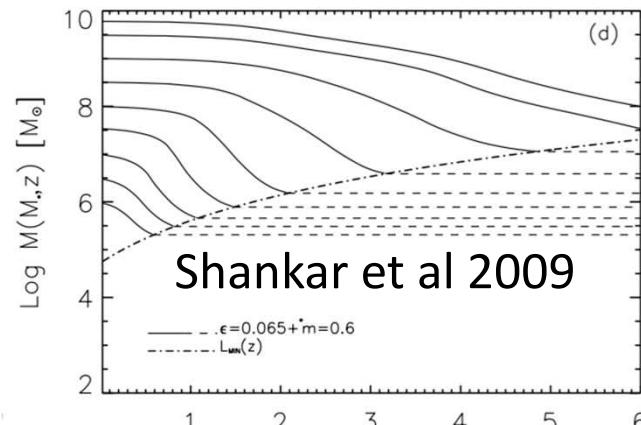
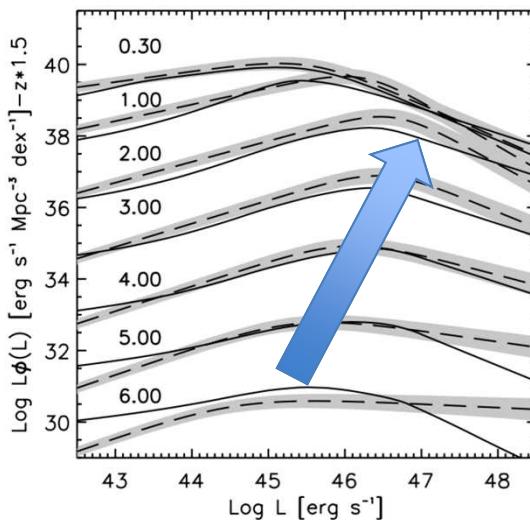
$$= \lambda \dot{m}_E \quad \lambda \sim 0.6$$

$$\dot{m}_E = 4\pi G m_p M_h / \epsilon \sigma_T c$$

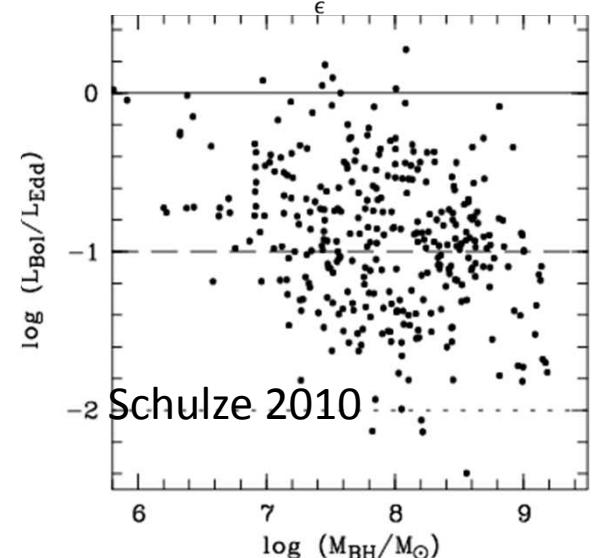
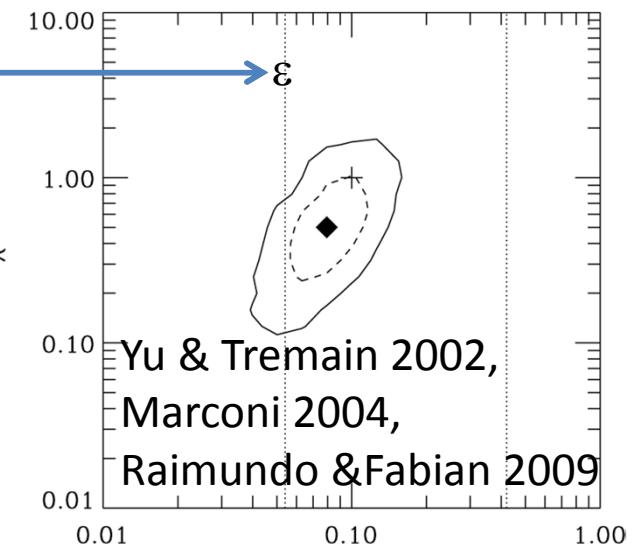
$$\text{Timescales } \tau_{\text{Sal}} = M_h / \dot{m}_E$$

$$P(R) = (R^3 / GM_h)^{1/2}$$

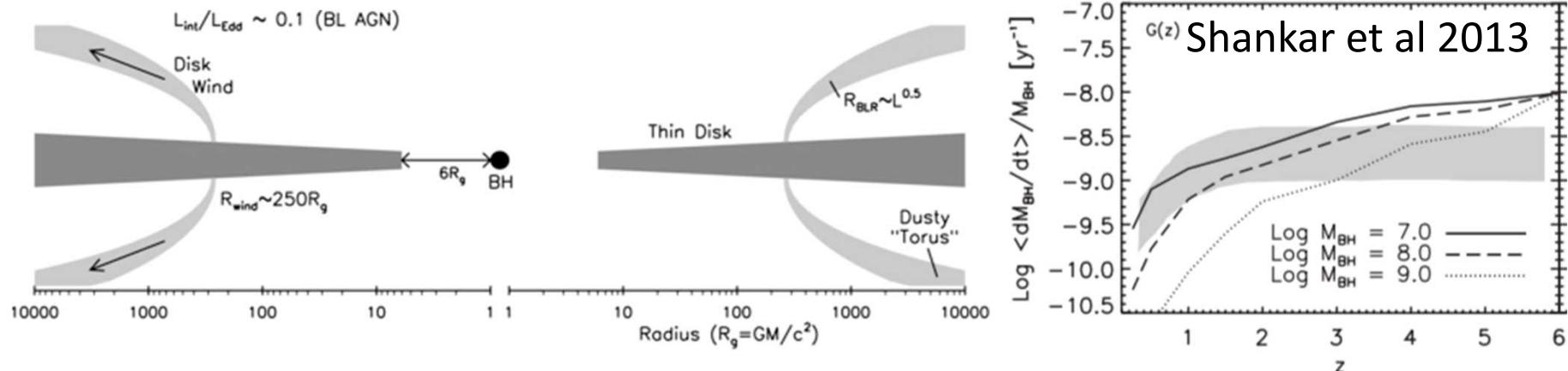
$$\text{Diffusion: } \tau_{\text{diff}}(R_{\text{out}}) = P(R_{\text{out}}) / 2\pi\alpha h^2$$



Black hole growth  
fed by disk accretion



# A generic quantitative AGN accretion disk model



$$R_o = GM_h/\sigma_o^2 = 10m_8\sigma_{200}^{-2}\text{pc} \quad m_8 = M_h/10^8M_\odot \quad \sigma_{200} = \sigma_o/200\text{km s}^{-1}$$

- Steady state alpha disk ( $h=H/R$ ,  $R_{\text{pc}}=R/1\text{pc}$ ) Shakura & Sunyaev

$$\dot{M} = 3\pi\Sigma\nu \quad \nu = \alpha H^2\Omega = \alpha h^2\Omega R^2$$

- Marginal gravitational stability (Safronov, Toomre)

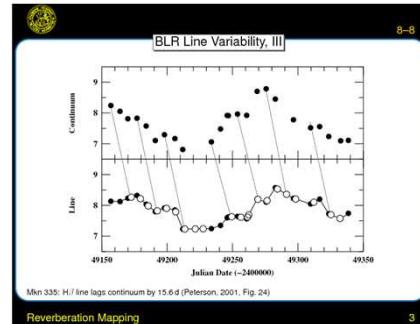
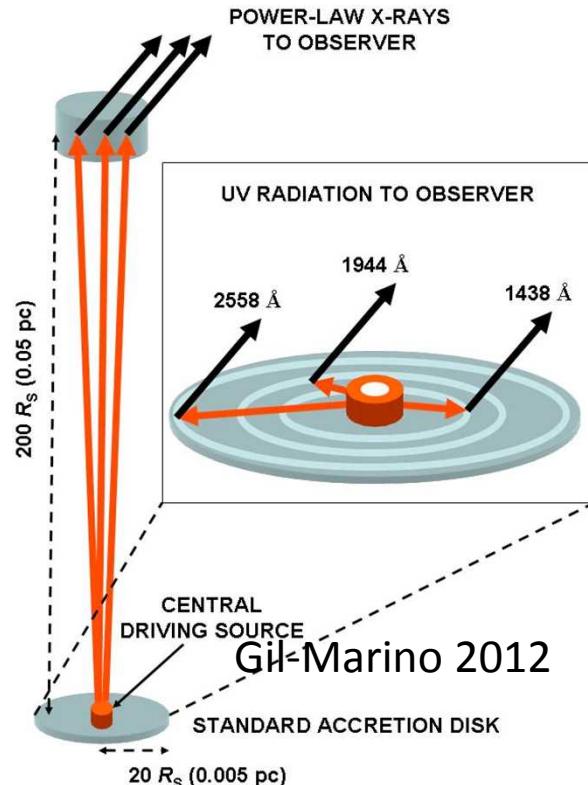
$$\Sigma = \Sigma_Q/Q$$

$$\Sigma_Q = h(M/\pi R^2),$$

$$\alpha h^3/Q \sim (\lambda/\varepsilon) (4\pi/3\sigma_{\text{es}})(Gm_p/c\Omega) \sim 10^{-5} m_8^{-1/2} R_{\text{pc}}^{3/2} \quad (1)$$

XJZhang

# Reverberation from accretion disk to dusty torus



DB: snapshot\_032.hdf5  
Time: 039.986

PowerLaw  
Vor PartType/Density

~60000778

~313e+05

~5.62e+06

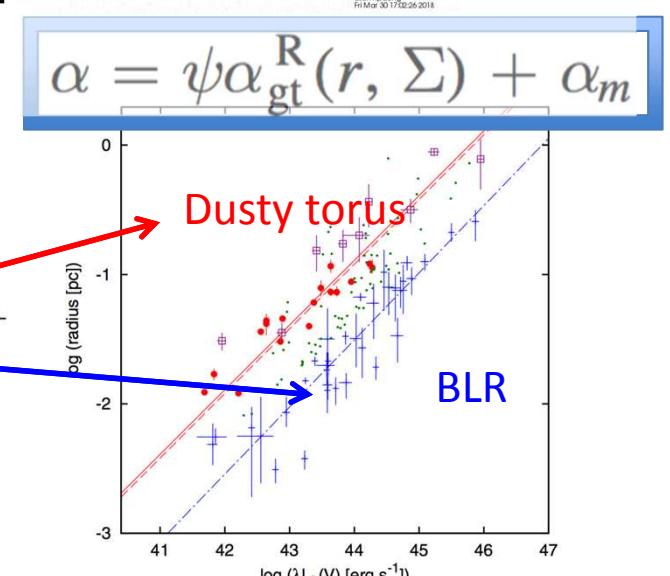
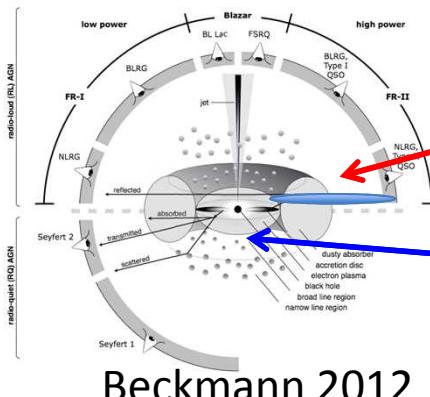
~1.23e+08

Max: 0.5425

Min: 0.403e-10



L-Pringle, Rees  
Rafikov, Rice,  
**HPDeng**  
Riols, Latter



3)

At  $R_{pc} > 1$ ,  $Q \sim 1-10$ ,  $\alpha \sim 0.1-1$ ,  $h \sim 0.02-0.1 m_8^{-1/6} R_{pc}^{1/2}$ ,  
 $\tau_{disk} > 40 m_8^{5/6} R_{pc}^{-3/2}$  (optical depth)

XJZhang, ZXWang

# Capture by the disk

Hydro drag: Artymowicz et al 1993

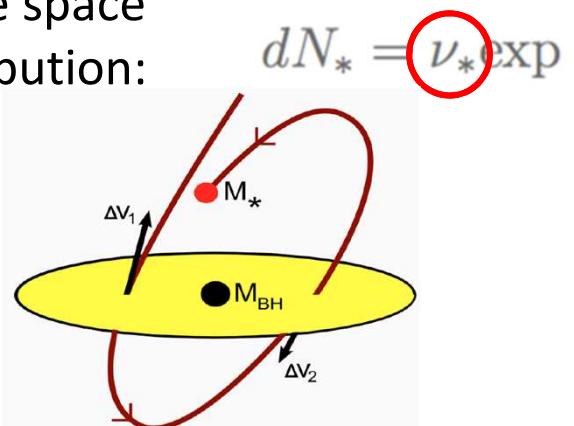
$$F_d = 4\pi G^2 m_*^2 \rho \frac{C_d}{V_c^2} \left[ \left( \frac{V_c}{V} \right)^2 + \left( \frac{V}{V_c} \right)^2 \right],$$

$$C_d = C_d^{\text{gas}} + C_d^{\text{wave}} \sim 6$$

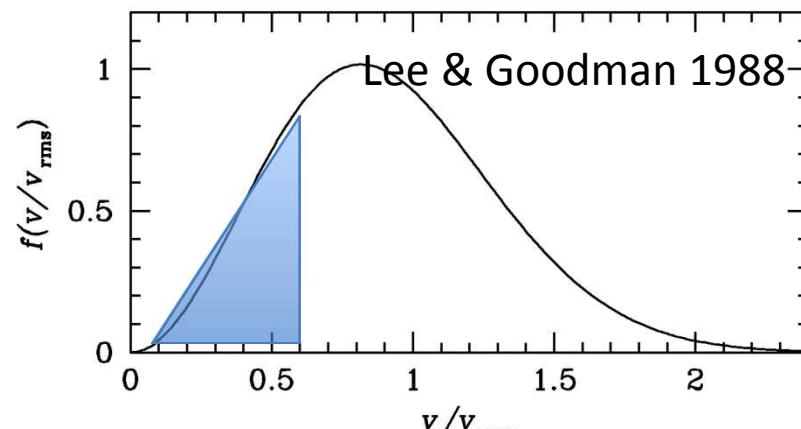
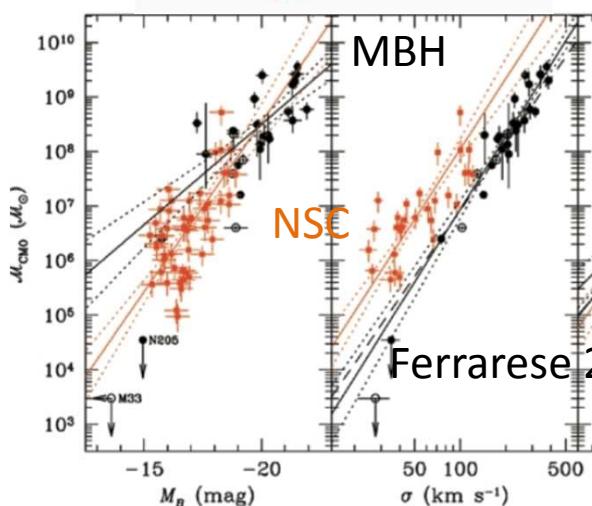
Condition for disk trapping within  $\Delta T$ :

$$\frac{V_z V^3}{V_K^4} \leq \xi = 32 C_d \frac{m_*}{M_h} \frac{\pi R^2 \Sigma}{M_h} \frac{\Delta T}{P}$$

Phase space distribution:



$$dN_* = \nu_* \exp \left[ -\frac{v_r^2 + (v_\phi - V_{\text{rot}})^2 + v_z^2}{2\sigma^2} \right] \frac{d^3 r d^3 v}{(2\pi\sigma^2)^{3/2}}$$



Phase space fraction of trapped stars:

$$dN = q dN_* = 4\pi q R^2 v_* dR$$

$$q(\zeta) = \left( \frac{\sigma}{v_K} \right)^3 \int_{u_z u^3 \leq \zeta^4} e^{-(E_x + E_z)} \frac{d^2 u d^2 \zeta}{(2\pi)^2}$$

# Nuclear clusters

$$\nu_o \sim 2.5 \times 10^6 M_{c,8} \text{ pc}^{-3}$$

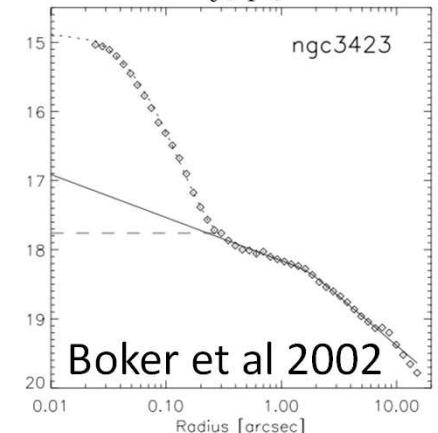
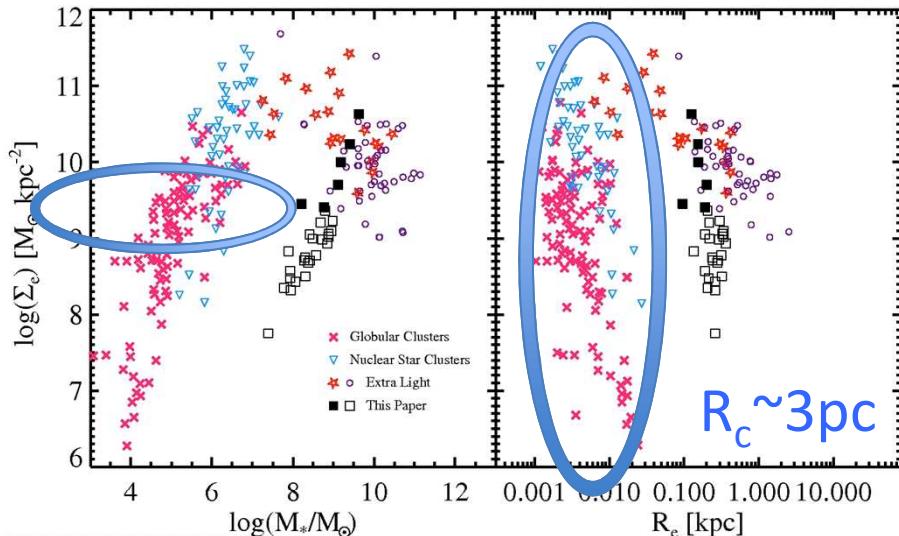
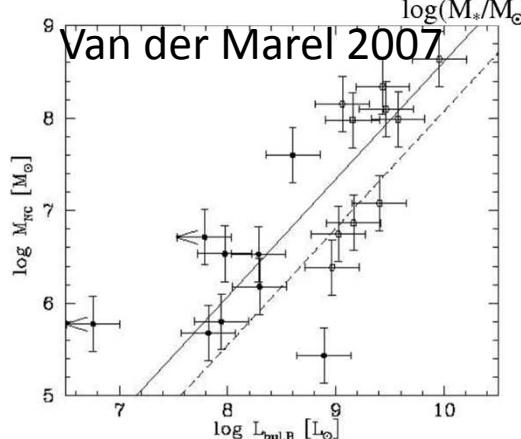
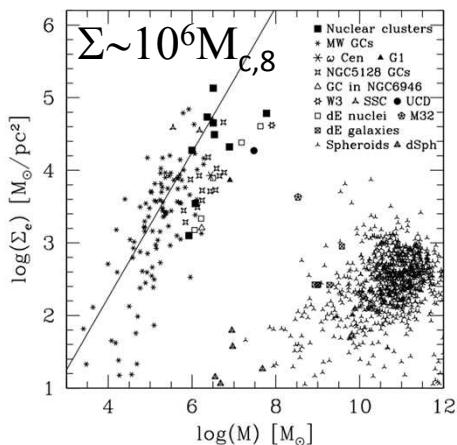
$$P_o \sim 5 \times 10^4 M_{c,8}^{1/2} \text{ yr}$$

$$\sigma_o \sim 350 M_{c,8}^{1/2} \text{ km s}^{-1}$$

$$M_{c,8} \sim 21 \sigma_{200}^{4.3}$$

$$m_8 \sim 1.8 \sigma_{200}^{4.4} \sim 0.08 M_{c,8}$$

Efficiency of NSC  $\leftrightarrow$  MBH



Capture rates:  
XJZhang

$$d\dot{N} \simeq \frac{\pi \xi^4 \nu_* R^2 dR}{2\Delta\tau} \simeq \frac{48C_d}{\sqrt{\pi} P_0} \frac{h}{Q} \frac{dR}{R} \simeq \frac{2.7 \times 10^{-4} \sigma_{200}^{1.4}}{\text{yr}} \frac{dR_{\text{pc}}}{R_{\text{pc}}^{1/2}}$$

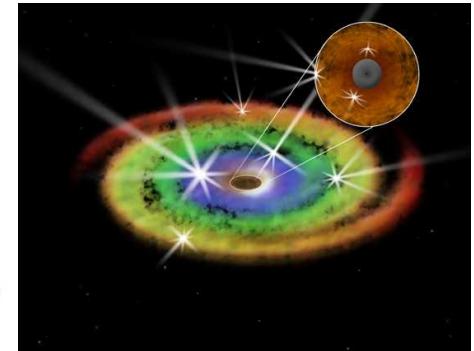
# Accretion rate and stellar rejuvenation

If  $R_R < H$ ,  $R_B < R_R$  (**hot**) Bondi accretion (**runaway growth**)

$$\dot{m}_* \simeq \frac{4\pi G^2 m_*^2 \rho}{(v^2 + c_s^2)^{3/2}} \simeq \frac{2\Omega}{Q} \frac{m_*}{h^3} \frac{m_*}{M_h}$$

**Bondi** accretion time scale:  
(independent of  $M_h$ )

$$\tau_B = m_*/\dot{m}_* \simeq 0.6(M_\odot/m_*)R_{pc}^3 \text{Myr}$$



Wind loss



$$\tau_w = m_*/\dot{m} \sim (60M_\odot/m)^3 \text{Myr}$$

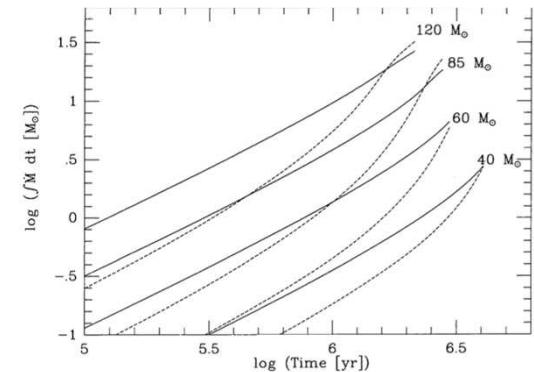
$$\log\left(\frac{\dot{m}}{M_\odot \text{yr}^{-1}}\right) \simeq 1.74\log\left(\frac{L_*}{L_\odot}\right) - 1.35\log T_{\text{eff}} - 9.55$$

$$m_{*w} \sim 120R_{pc}^{-3/2}M_\odot$$

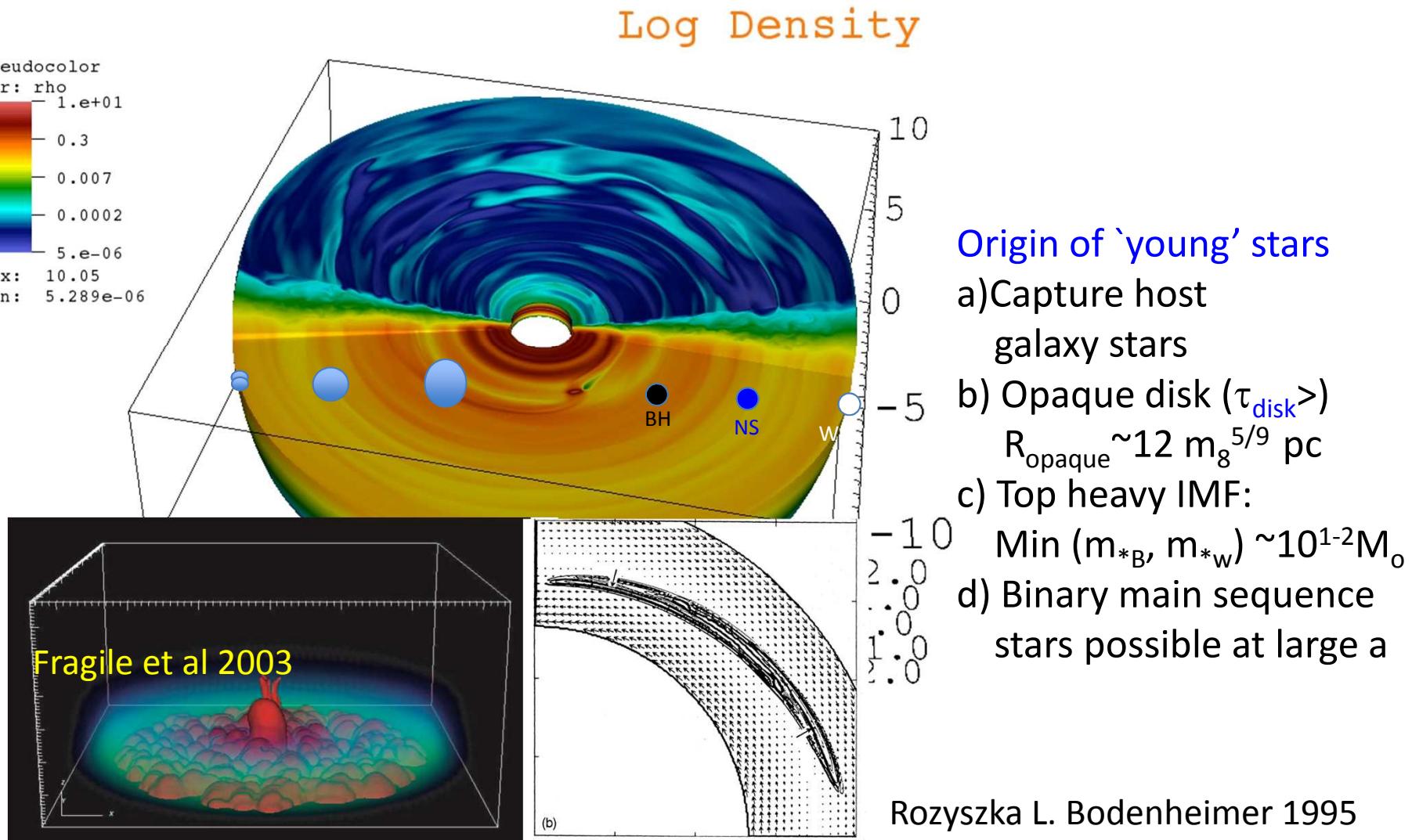
Main sequence evolution time:

$$\tau_* \sim 10(m/M_\odot)^{-2.5} \text{Gyr}$$

YTang, J Szulagyi

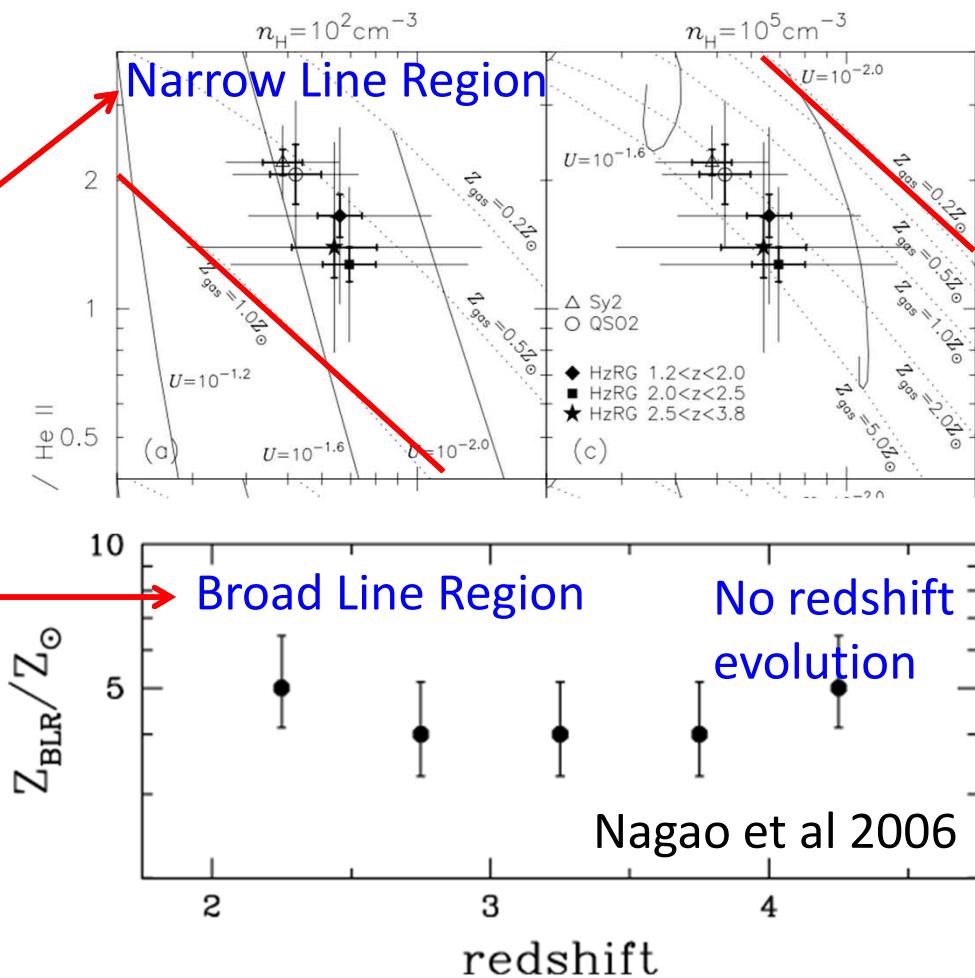
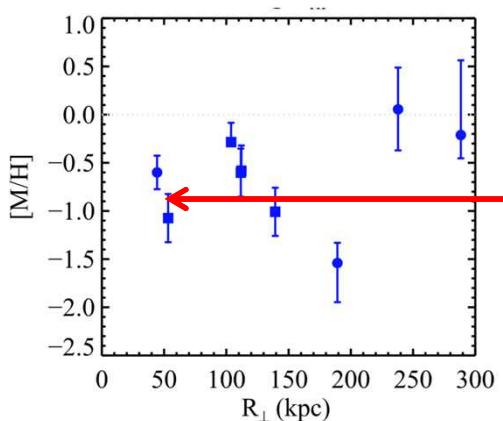
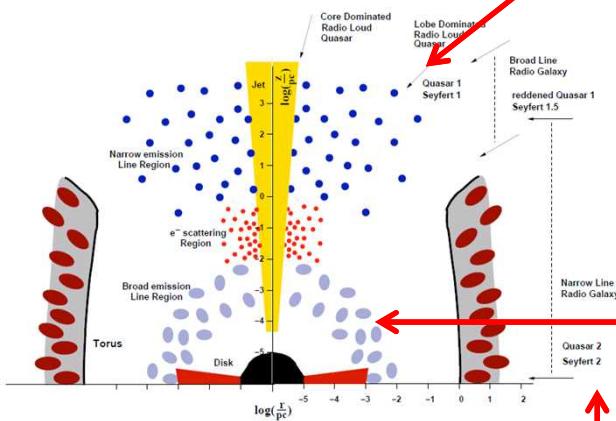


# Stellar rejuvenation



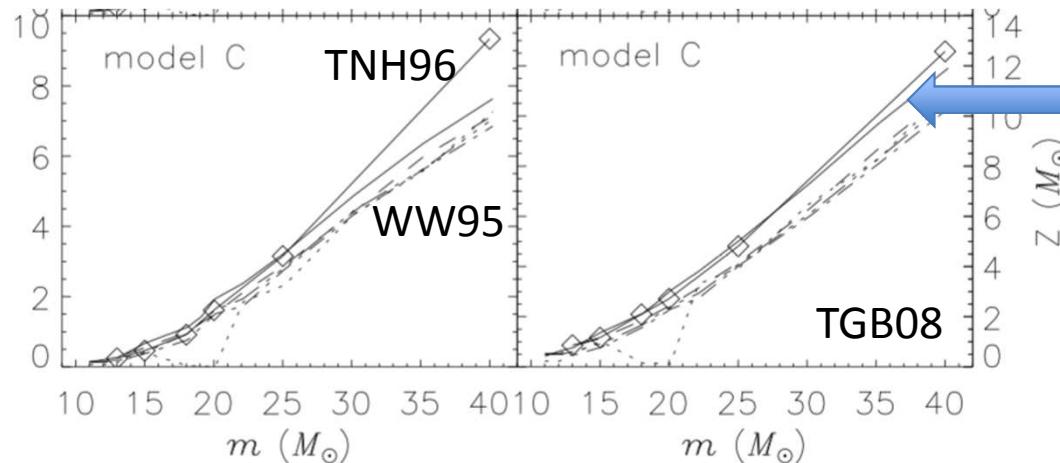
# Super-solar metallicity in high-redshift AGNs

$Z_{\text{BLR}} > Z_{\text{NLR}}$

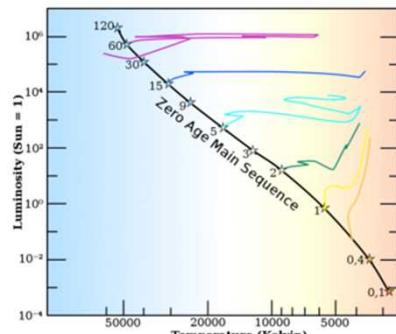


Quasars probing metal-poor outskirts of quasars  
Lau, Prochaska, Hennawi 2016

# In situ metallicity enrichment



a) Type II supernovae yield:  
 $m_{zSN} \simeq 0.3(m_* - 12M_\odot)$   
 $m_O \sim 2m_z/3$  up to  $\sim 10M_\odot$

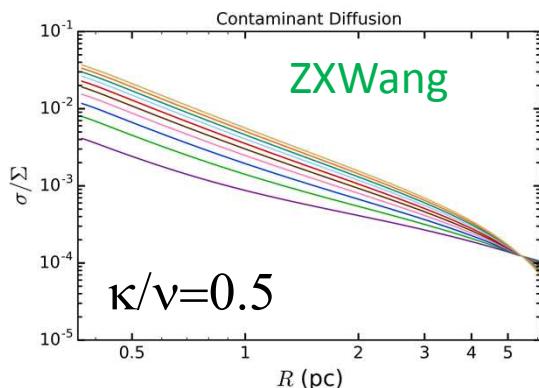
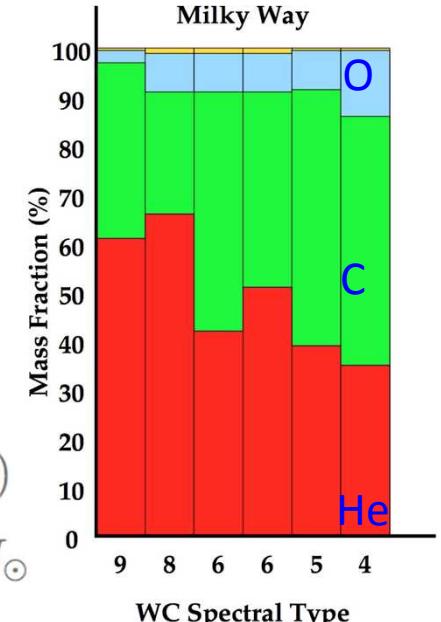


b) WR and AGB winds:

$$m_{zw}(m_{*w}) = \dot{m}\tau_* / 2 = 35R_{pc}^{-9/4} M_\odot$$

c) Local enrichment:  $\dot{\Sigma}_Z = m_z(d\dot{N}_Z/2\pi R dR)$

$$m_z = m_{zw} + m_{zSN} = 35(R_{pc}^{-3/2} + R_{pc}^{-9/4} - 0.1)M_\odot$$

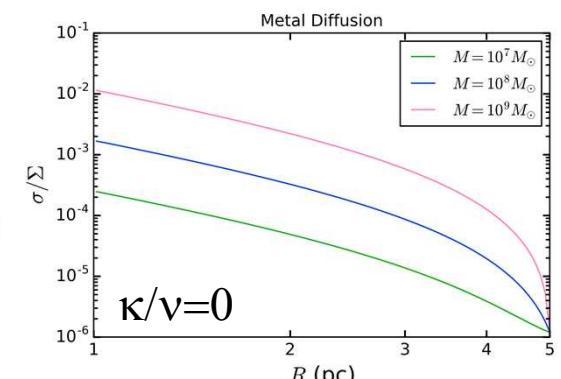


d) [Fe/H] diffusion (Clarke 1988)

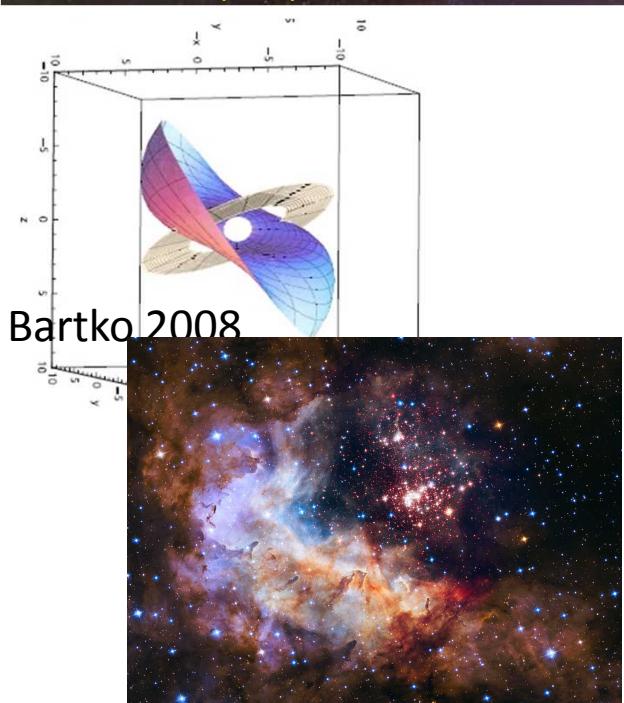
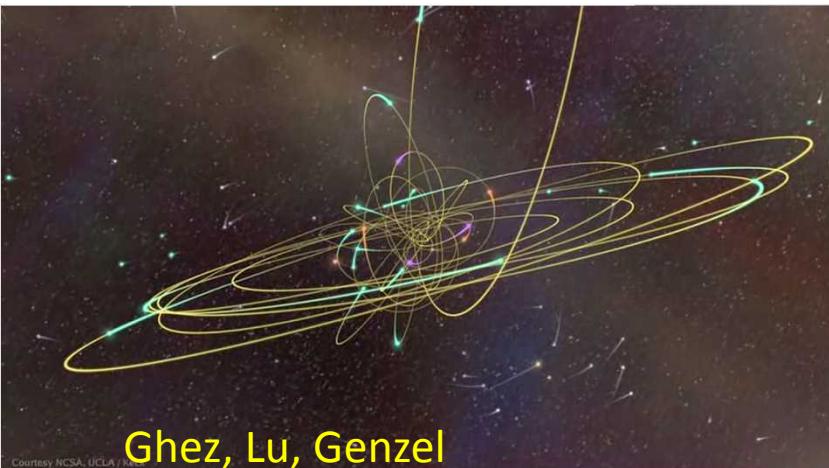
$$\Sigma \frac{\partial C}{\partial t} + \Sigma U_r \frac{\partial C}{\partial R} = \frac{1}{R} \frac{\partial}{\partial R} \left( R \kappa \Sigma \frac{\partial C}{\partial R} \right) + \dot{\Sigma}_Z$$

$$\kappa = 0 \Rightarrow \Delta Z(R) = \frac{m_z dN R}{U_r 2\pi \Sigma R dR} \simeq 3 \times 10^{-3} \sigma_{200}^3$$

$$\kappa/\nu = 0.5 \quad \Delta [\text{Fe}/\text{H}] \sim O(1 - 10)$$

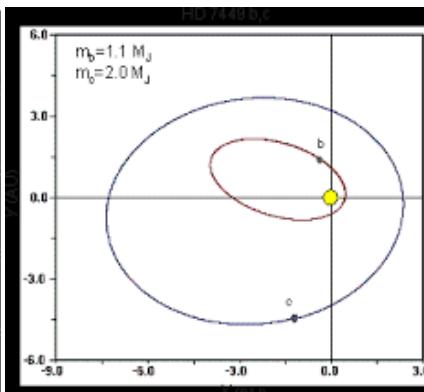
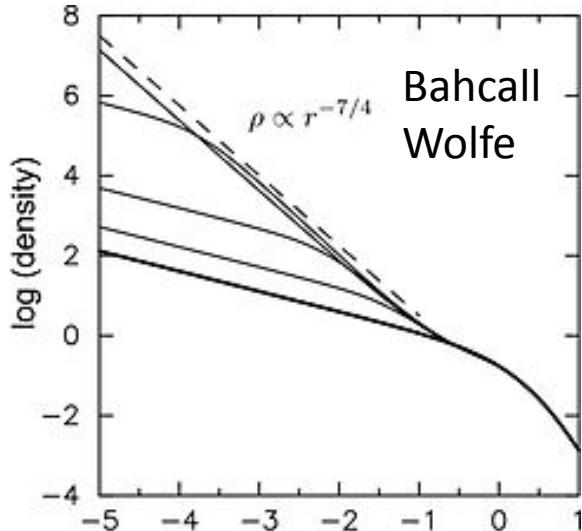


# Disk's reorientation due to infall of turbulent gas

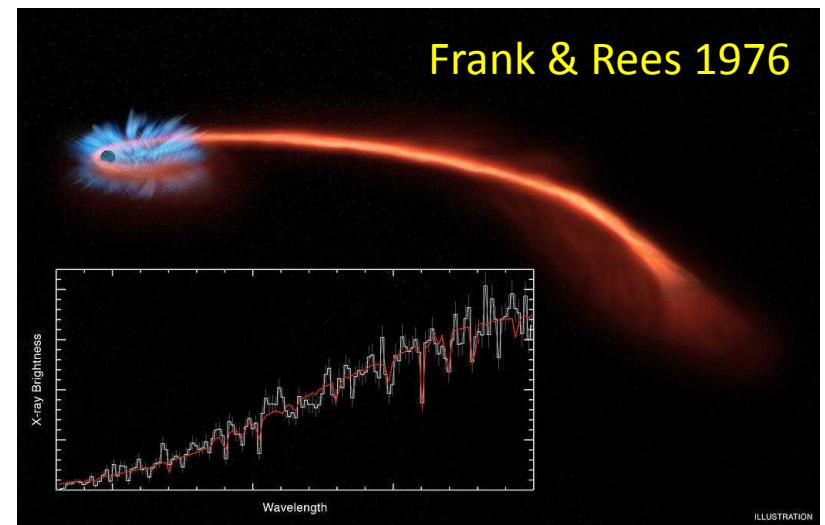
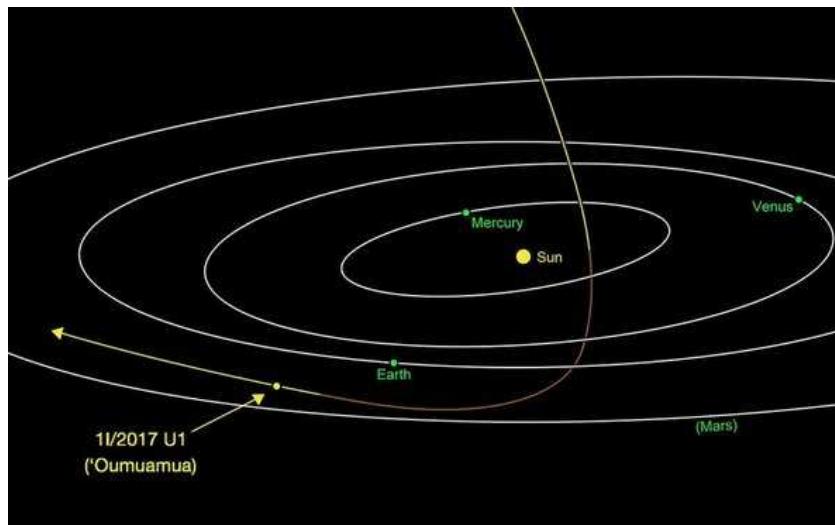
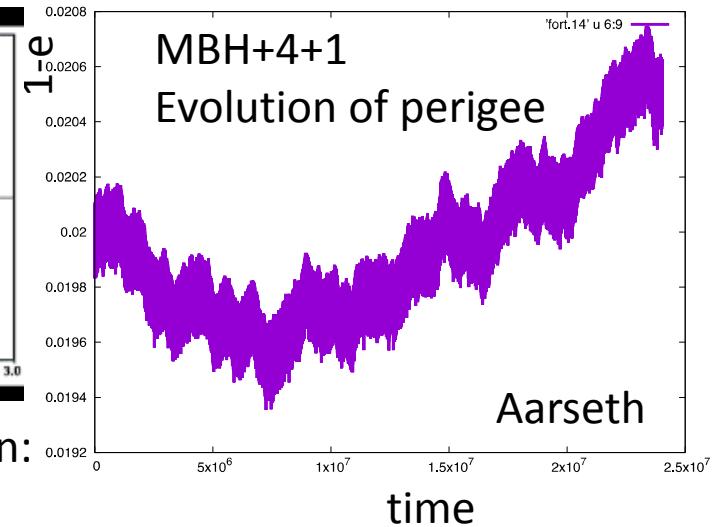


HPDeng, RNaylor, XJZhang

# *cusp* profile, relaxation, & TDEs

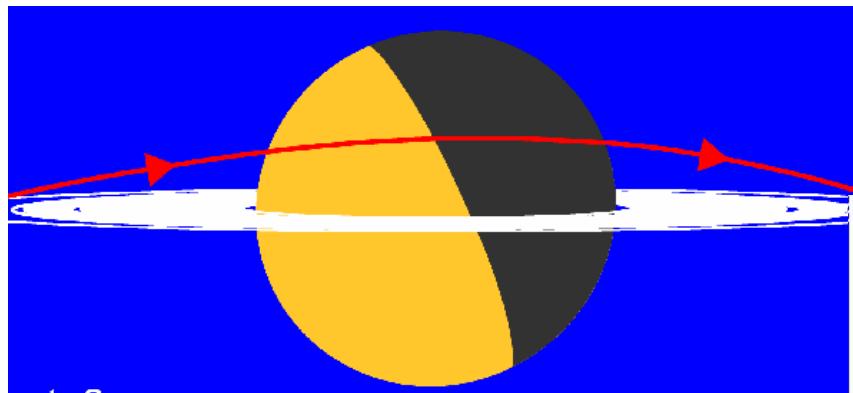


Resonant relaxation:  
Rauch & Tremaine



ILLUSTRATION

# Recapture of neutron stars and seed black holes

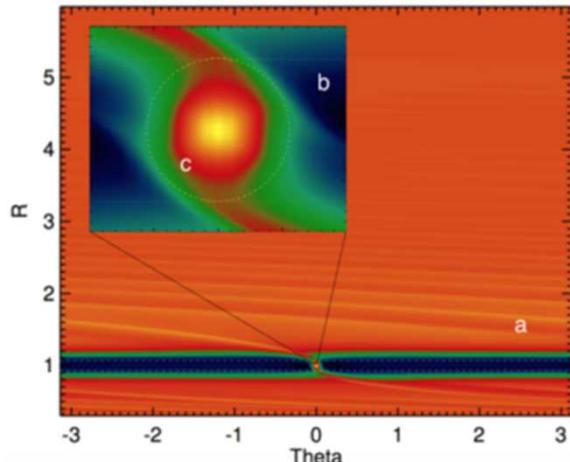


accretion radius = min [R<sub>B</sub>, R<sub>R</sub>]

$$\tau_{\text{sal}} = m_*/\dot{m}_E = 4.5 \times 10^8 \eta \text{ yr}$$

**Mass growth:** Eddington limited if

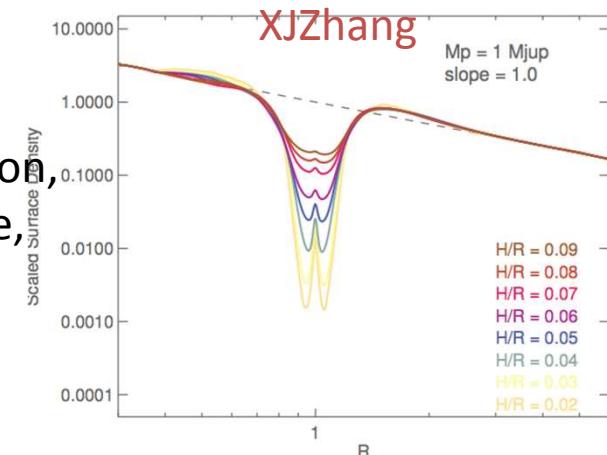
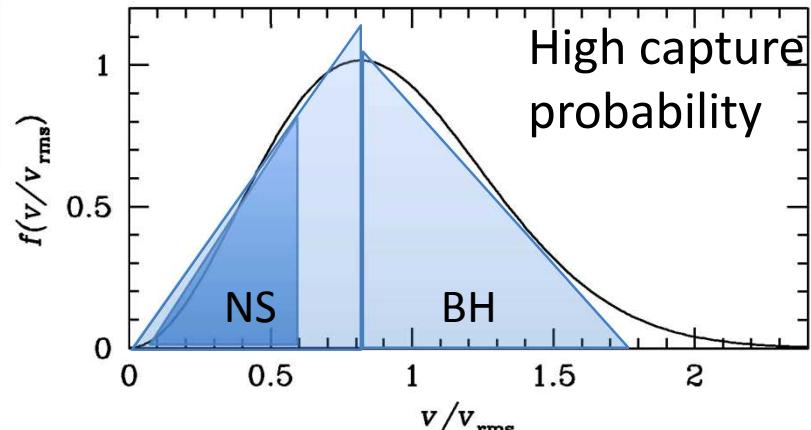
$$\tau_{\text{sal}} > \tau_B \text{ or } m_* > 10^{-3} \eta^{-1} R_{\text{pc}}^3 M_\odot$$



L Papaloizou 1986  
Artymowicz, Lubow, Nelson,  
Bryden, Masset, Armitage,  
Li, Dobbs-Dixon, Kley  
**Mass limited by gaps:**  
Thermal Condition for  
gap formation R<sub>R</sub>>H.

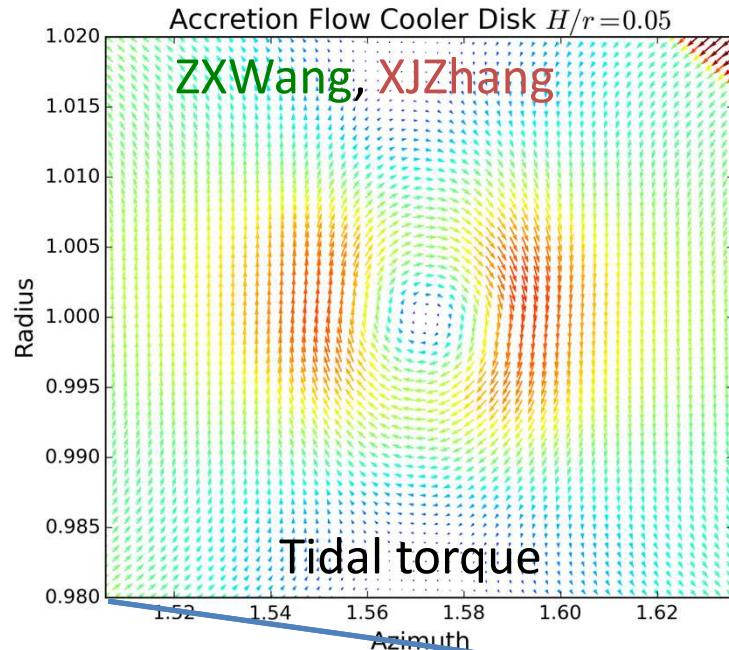
$$\dot{N}_{bh,t} = \int_{R_{in}}^{R_{bh}} d\dot{N} \simeq \frac{8 \times 10^{-4}}{\text{yr}} \sigma_{200}^{1.4}$$

modest kick speed: V<sub>rms</sub>(NS)>V<sub>rms</sub>(BH)

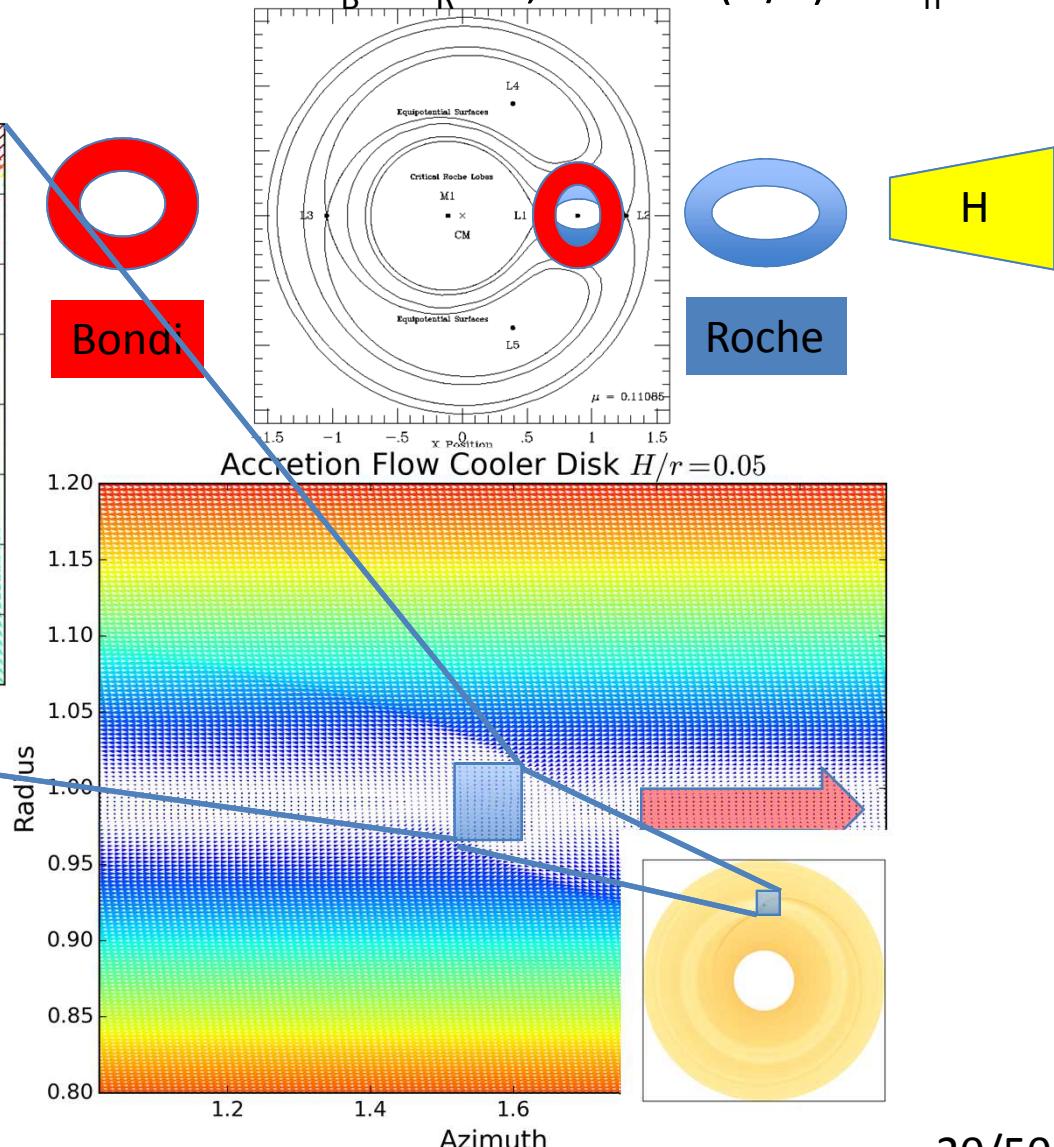


# Angular momentum evolution ( $h=0.05, q=10^{-4}$ )

Modest- $m_*$  seed black holes in **warm** disks with  $R_B > R_R \sim H$ , i.e.  $m_* \sim (H/a)^{1/3} M_h$

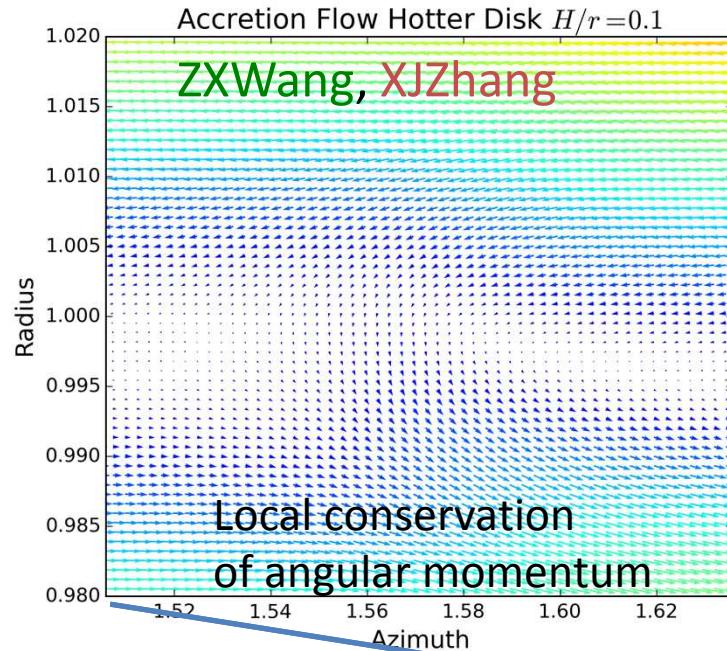


Prograde  
with  $j_a \sim R_R^2 \Omega$   
**large disk**  
With  $R_{cen} \sim R_R$

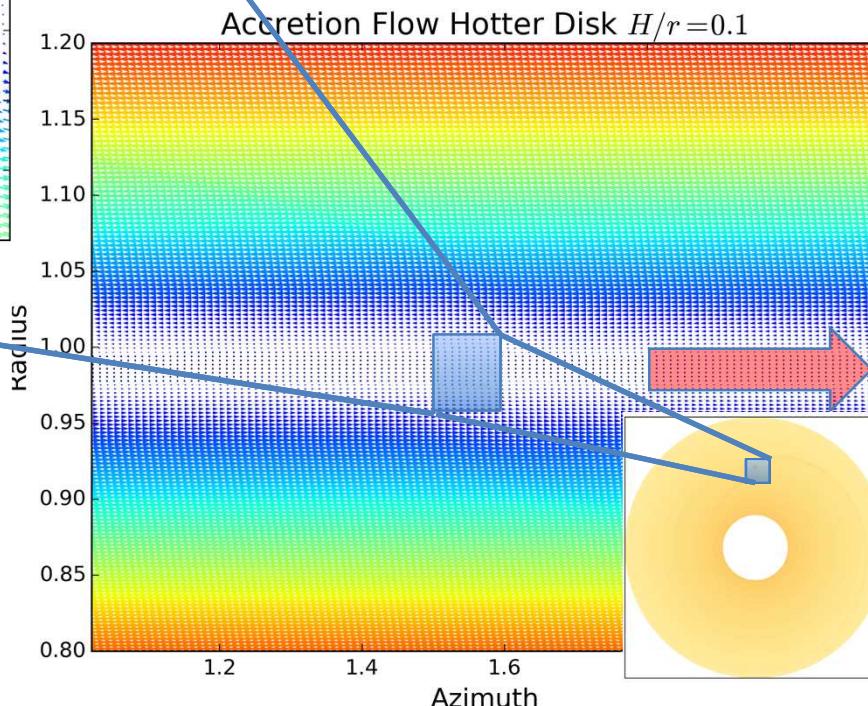
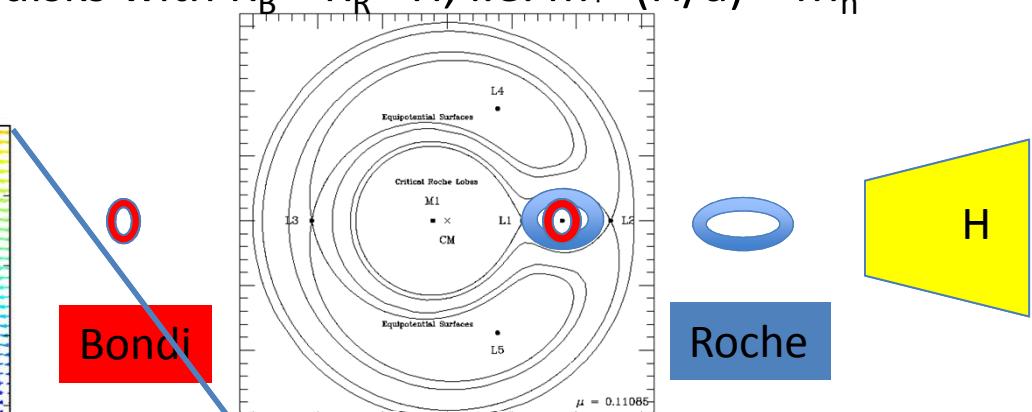


# Spin due to local shear ( $h=0.1, q=10^{-4}$ )

Low- $m_*$  seed black holes in **hot** disks with  $R_B < R_R < H$ , i.e.  $m_* < (H/a)^{1/3} M_h$

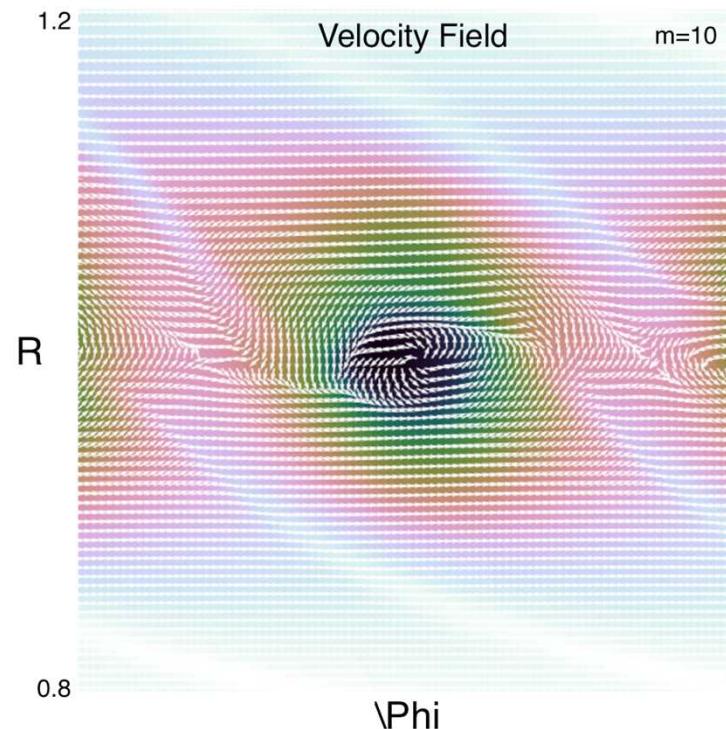
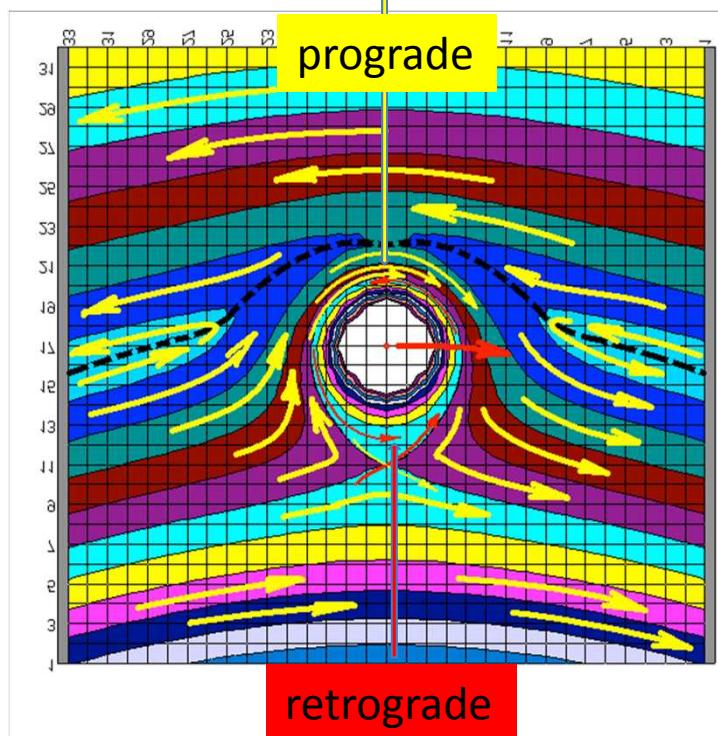


Retrograde with  $j_a = R_B^2 \Omega$ , small disk size  
 $R_{cen} \sim (R_R/H)^8 R_B$   
 $\sim (R_R/H)^{14} h^3 a R_{*}/R_H$



# Seed black holes in hot turbulent disks

Low- $m_*$  seed black holes in hot turbulent disks with  $R_B < R_R < H$  &  $v_{\text{tur}} < c_s$



Eddies with  $\lambda < H$ , can be  $> R_B$

$$v_{\text{tur}}(\lambda) \sim (\lambda/H)^{1/3} v_{\text{tur}}(H) < c_s, \text{ can be } > R_B \Omega$$

$$\tau_{\text{tur}} \sim (\lambda/H)^{2/3} [c_s/v_{\text{tur}}(H)] \Omega^{-1}, \text{ can be } > \Omega^{-1}$$

Spin determined by local vorticity  $j_a = \lambda v_{\text{tur}}$   $\dot{J}_{\text{turb}} = \dot{m} \cdot j_a$

$$R_{\text{cen}} = A(H/R_R)^4 R_R = A(H/R_R)^6 R_B \text{ with } A = (\lambda/H)^{8/3} (V_{\text{tur}}/c_s)^2$$

# Multiple black holes' eccentricity excitation/damping

Scattering:  $d\sigma_h^2/dt \simeq n_0 \Omega^3 R_{roc}^6 / \Delta^2 R^2$

$$\tau_{e+} = \frac{\sigma_h^2}{2(d\sigma_h^2/dt)} = \left( \frac{R}{R_{roc}} \right)^6 \frac{e^4 P}{4N_{trap}}$$

Type I damping:  $\tau_{e-} = h^2 \tau_I = \frac{h^3 QPM}{4f_\Gamma m_*}$ .

Dynamical equilibrium: low-e orbits

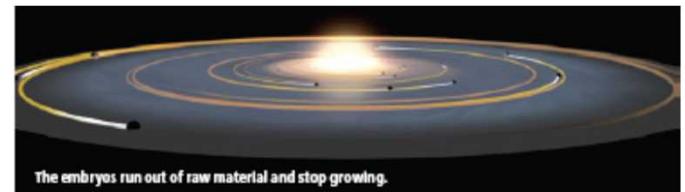
$$\tau_{e+} = \tau_{e-}$$

$$\frac{eR}{R_{roc}} = \left( \frac{N_{trap} h^3 Q}{3f_\Gamma} \frac{R}{R_{roc}} \right)^{1/4} < 1$$

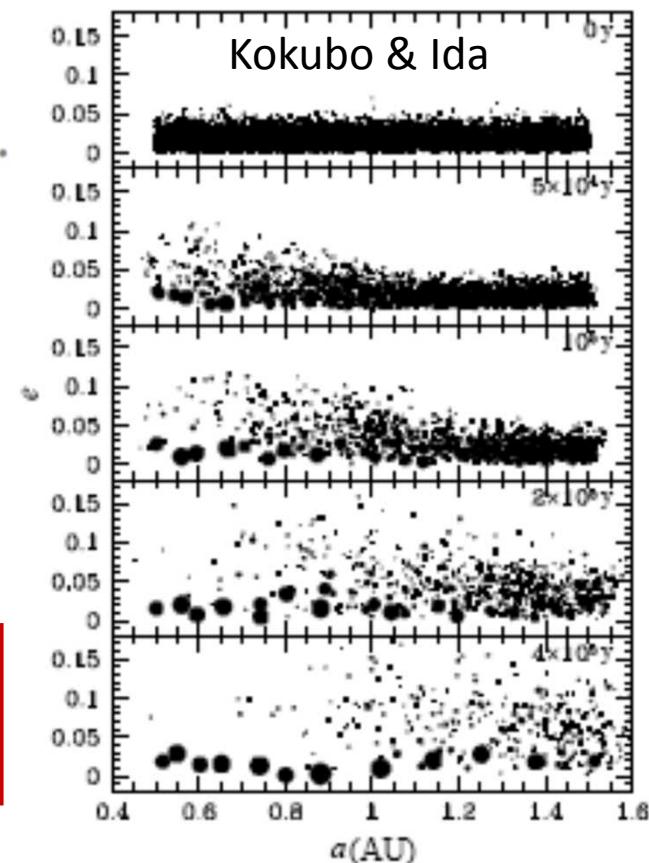
Feeding zones:

$$\Delta \sim 10 r_{\text{Hill}}$$

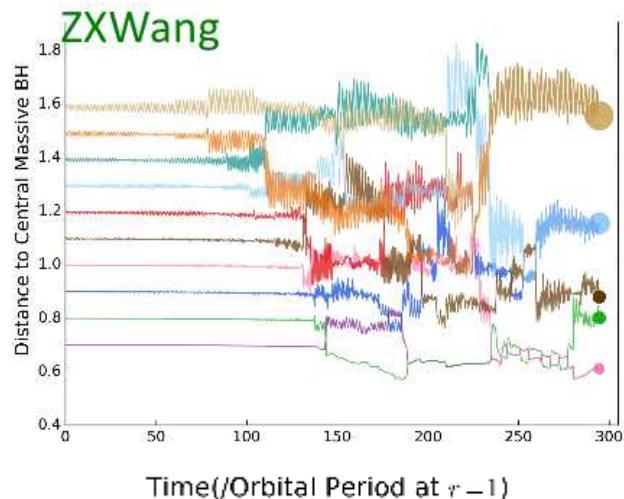
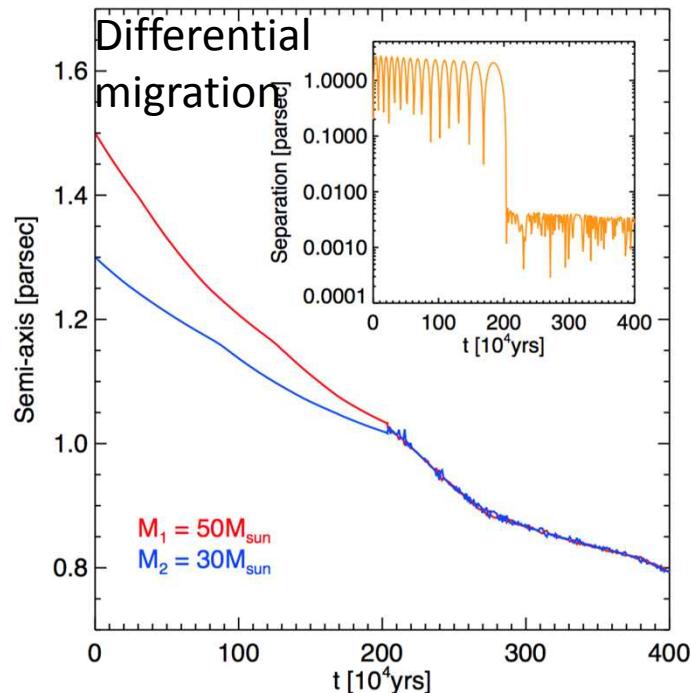
Isolation mass:  $M_{\text{isolation}} \sim \Sigma^{1.5} a^3 M_h^{-1/2}$



The embryos run out of raw material and stop growing.



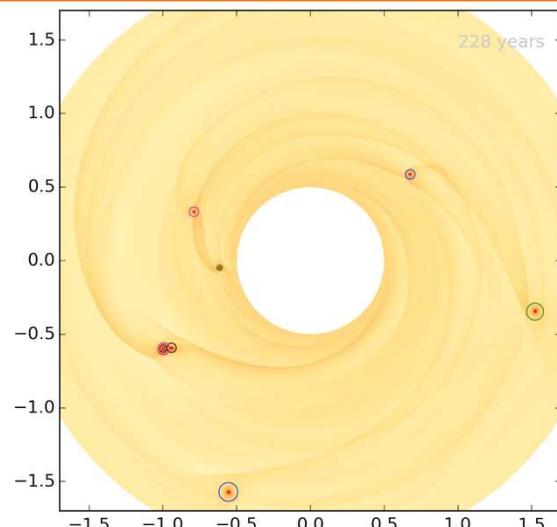
# Differential migration & groups with isolation masses



Goldreich & Tremaine 1982, Ward



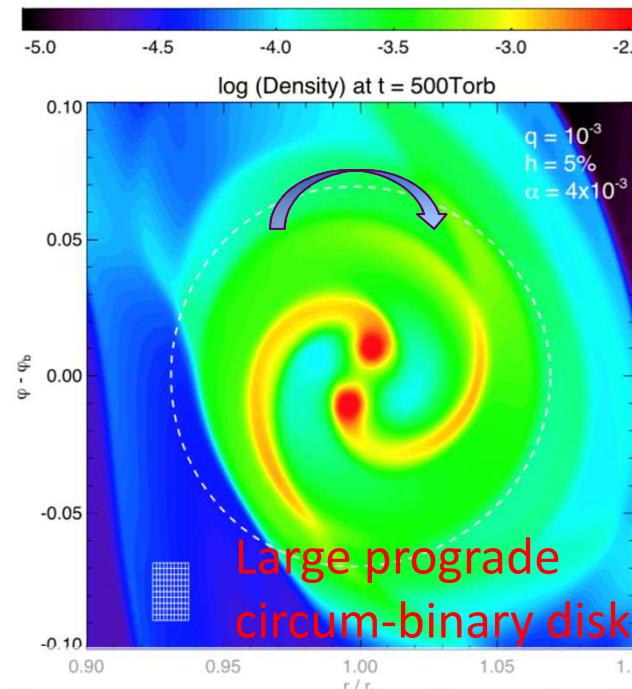
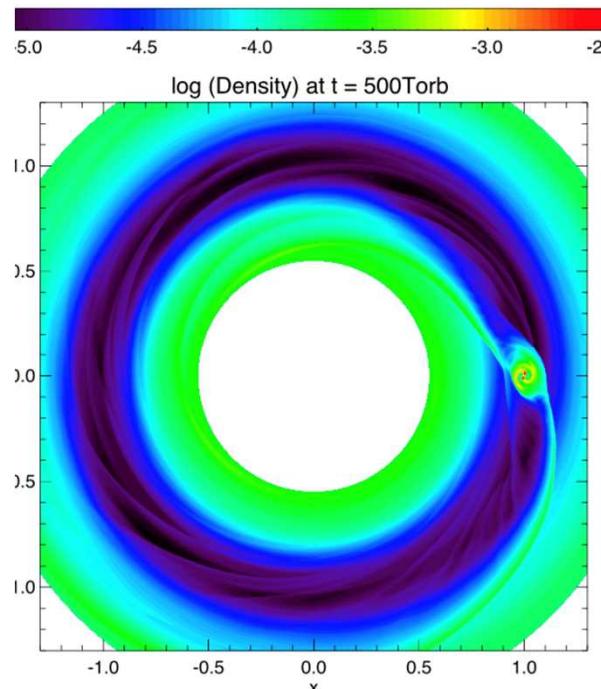
Nelson, Paardekooper, Baruteau



# Scales of binary seed black holes in disks

- 1) Bound binary:  $R_R > a_{12}$
- 2) Gap formation  $R_R > H$  (large  $m_*$ )
- 3) Common envelope  $a_{12} > R_b$  (wide)
- 4) Accretion-enhanced drag  $R_b > a_{12}$  (compact)
- 5) Prograde orbit  $R_b > R_R$  (medium  $m_*$ )
- 6) Retrograde orbit  $R_R > R_b$  (small  $m_*$ )

Gap formation by relatively massive binary with  $H = C_s / \Omega < R_R = (m_{12}/3M_h)^{1/3}a$   
 (thermal condition for gap formation) and  $R_R > a_{12}$  (bound)



$R_o$	20
$R_D$	18
$H$	16
$R_R$	14
$R_B$	12
$a_{12}$	10
$R_{cen}$	8
$R_{isco}$	6
$a_{gr}$	4
$a_{LT}$	2
$a_{mer}$	0
$R_{gbh}$	-2

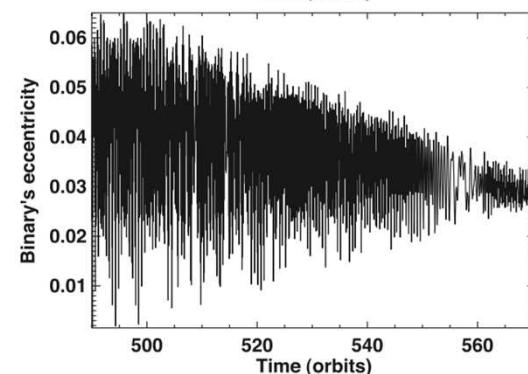
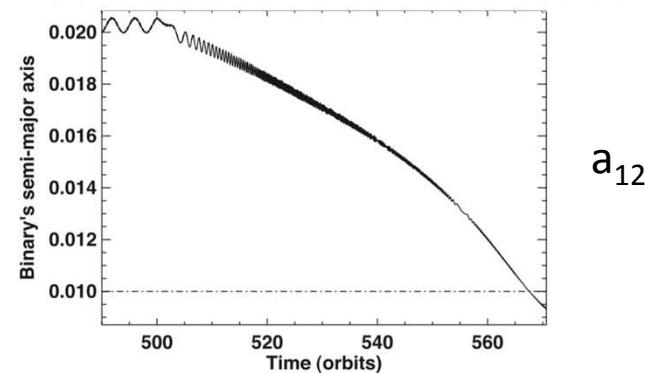
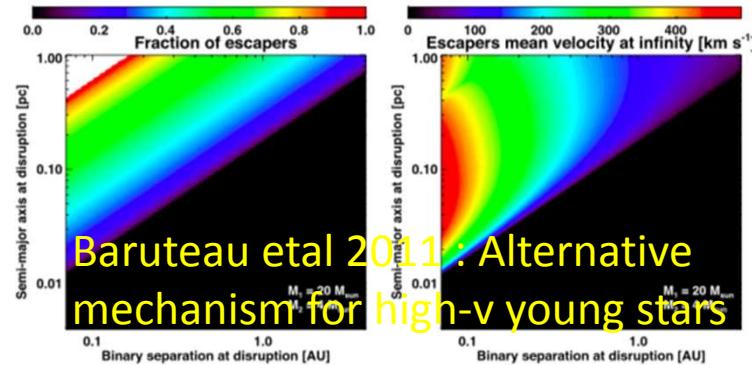
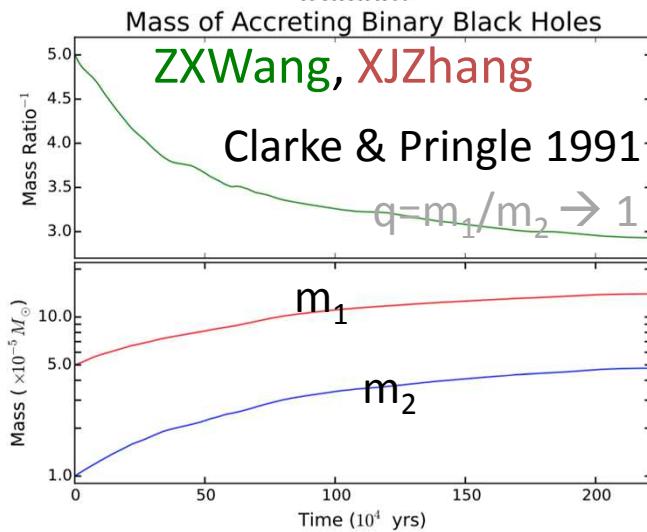
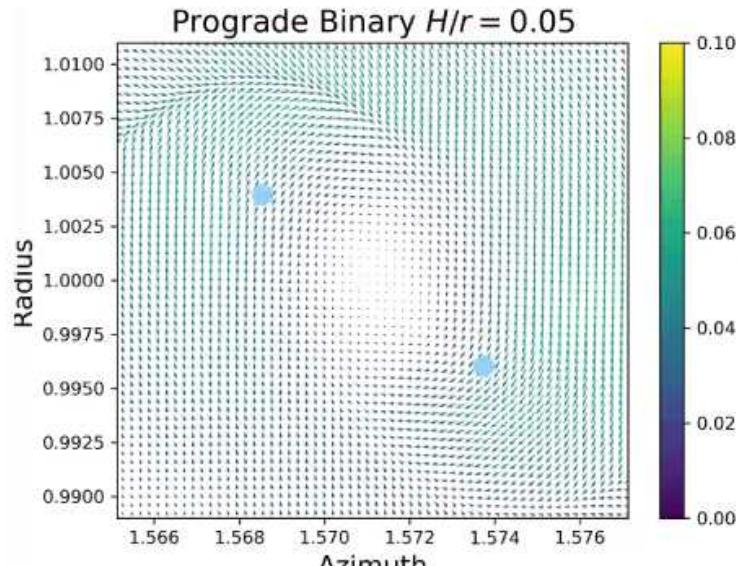
Initial binary tightening

Baruteau, Cuadra L 2011 Stone, Haiman 2017

$$j_t \simeq 0.23(m_2/m_1)^2 \Sigma_b a_{12}^4 \omega_{12}^2 h_b^{-3}$$

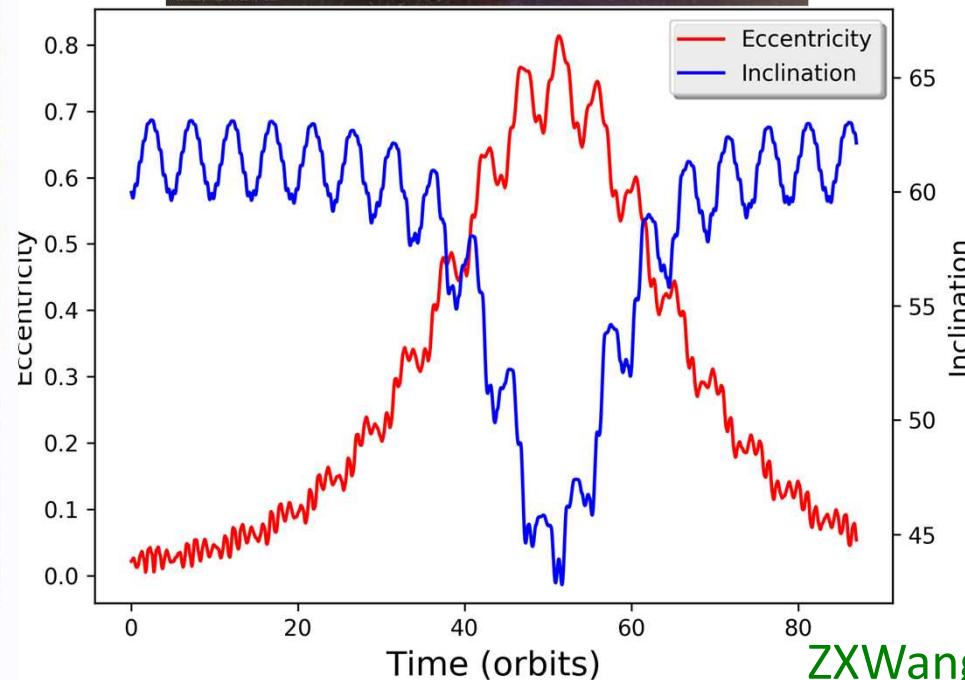
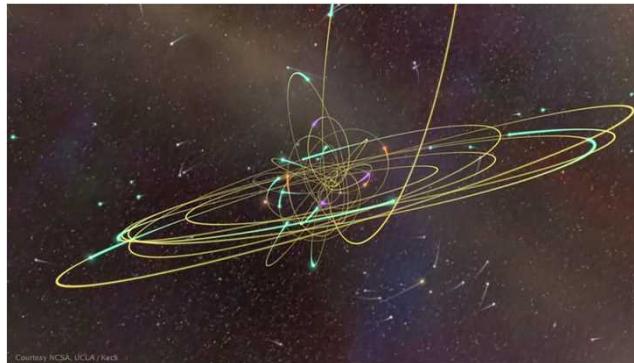
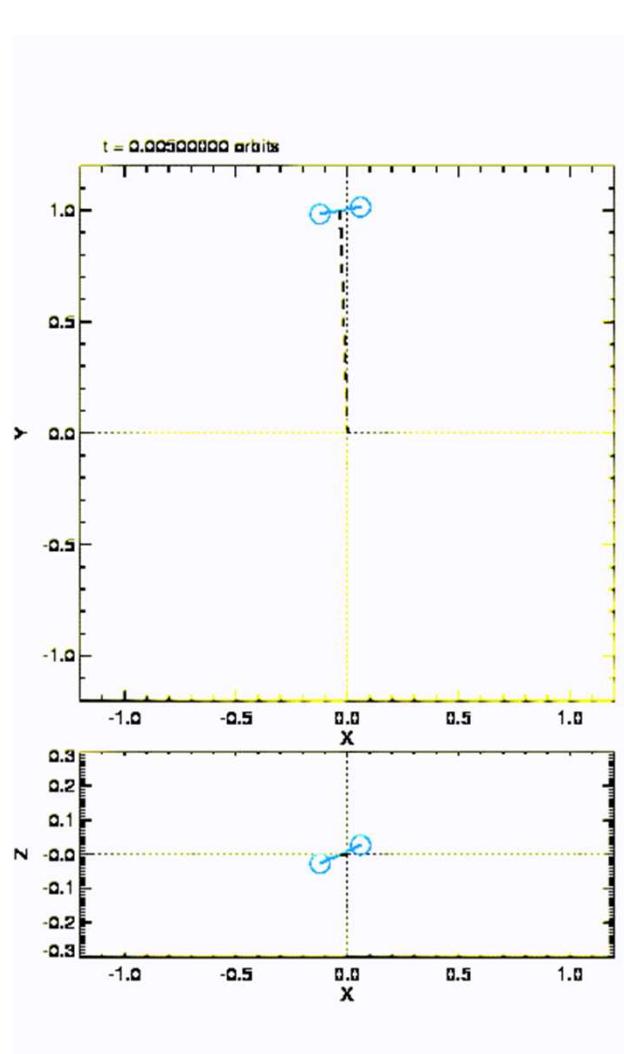
# Modest- $m_*$ binary with modified disk structure

$H=C_s/\Omega > R_R$  (no gap)  $\sim R_B$  (perturbed, prograde)  $\sim a_{12}$  (bound, no enhancement)



# Lidov-Kozai oscillation of binaries in AGN disks

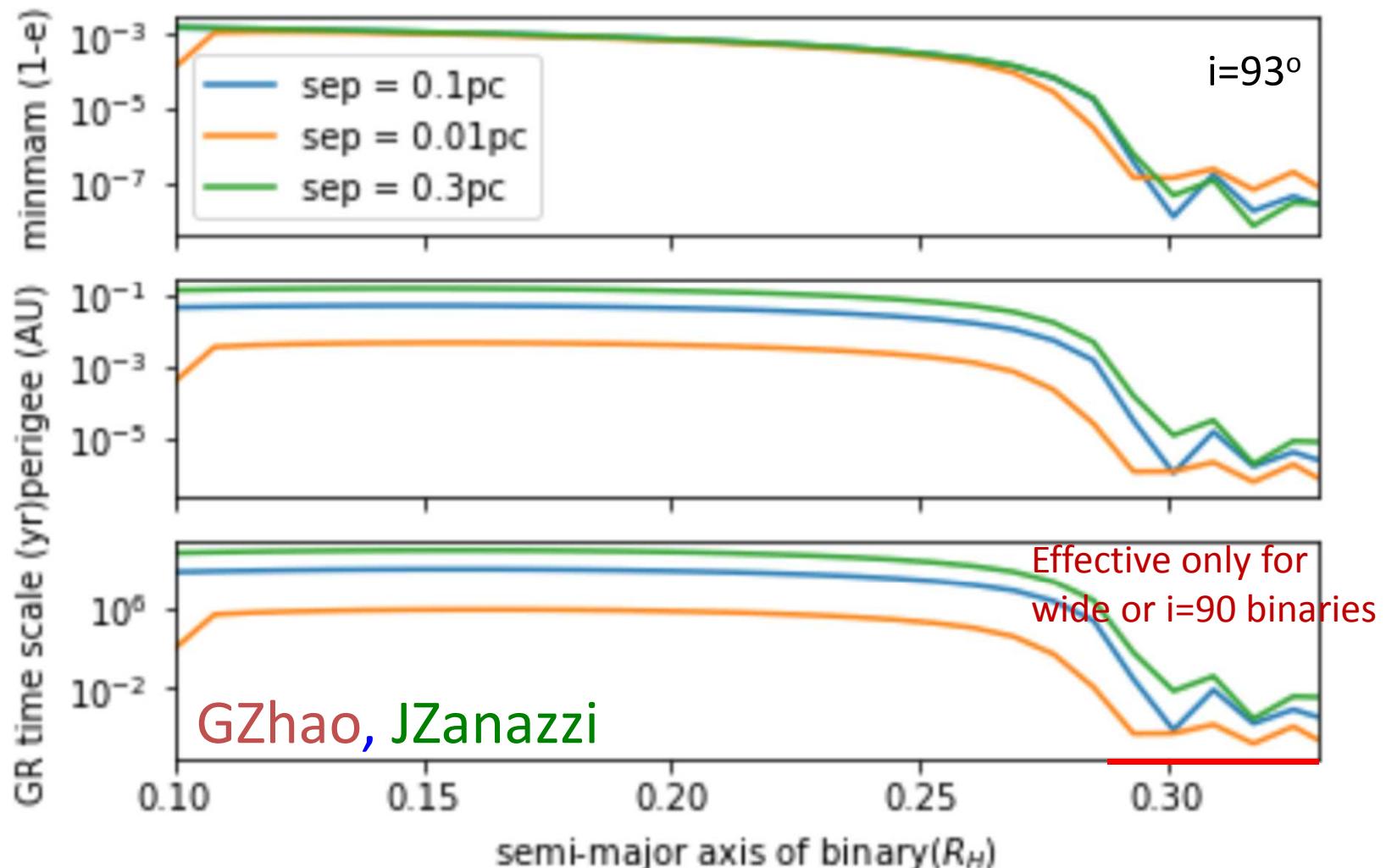
obliquity between the MBH-IMBH-SBH system + disk torque & gas drag



ZXWang, XJZhang

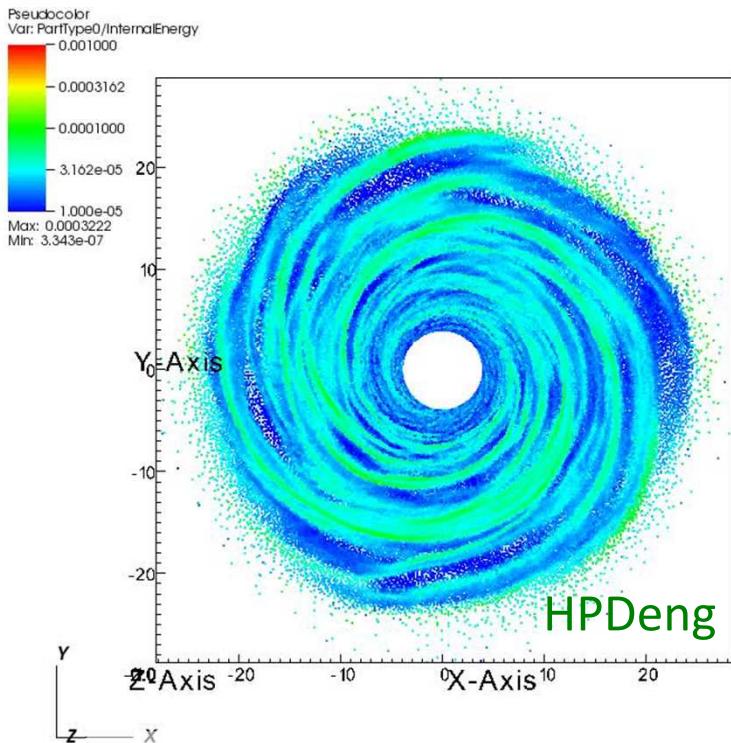
# Binary separation near supermassive black holes

Combining Lidov-Kozai effect and gravitational radiation



# Accretion & tidal torque due to circum-binary disk

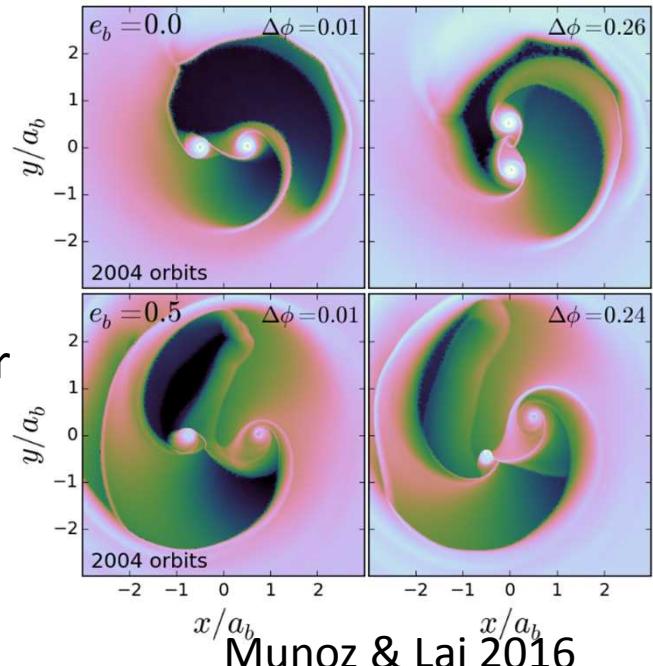
DB: snapshot\_000.hdf5  
Time: 0



Directly rotating, self-gravitating, circum-binary disk

Accretion onto  
seed binary  
black holes

Potential sites for  
Enhanced  
star formation



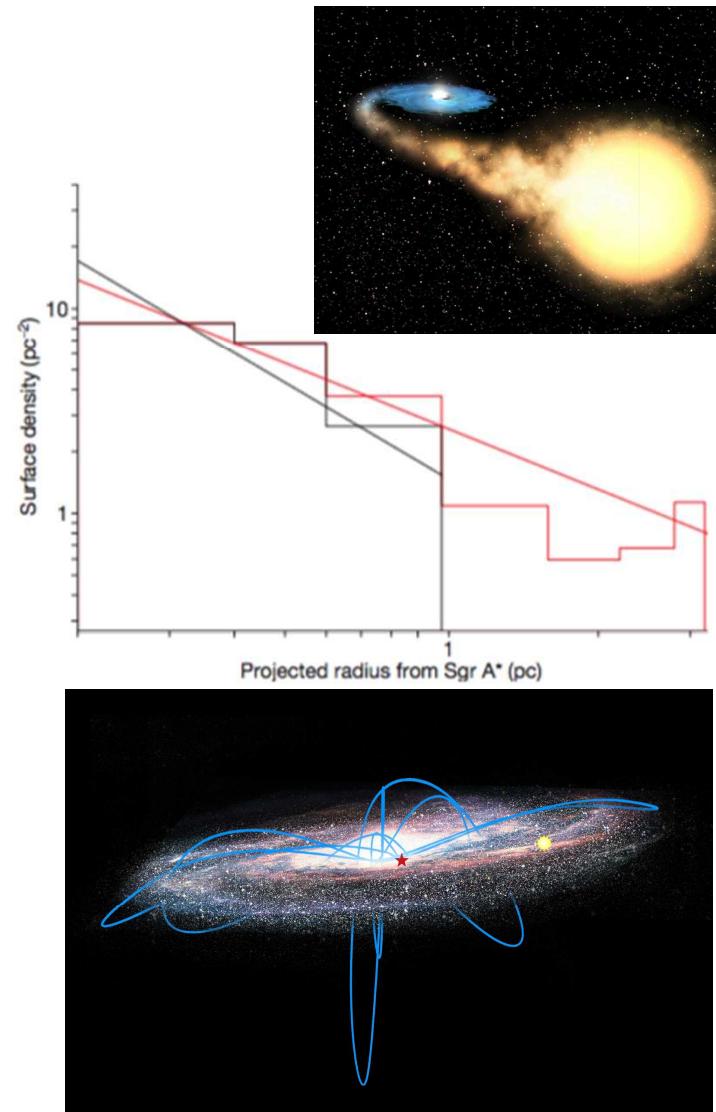
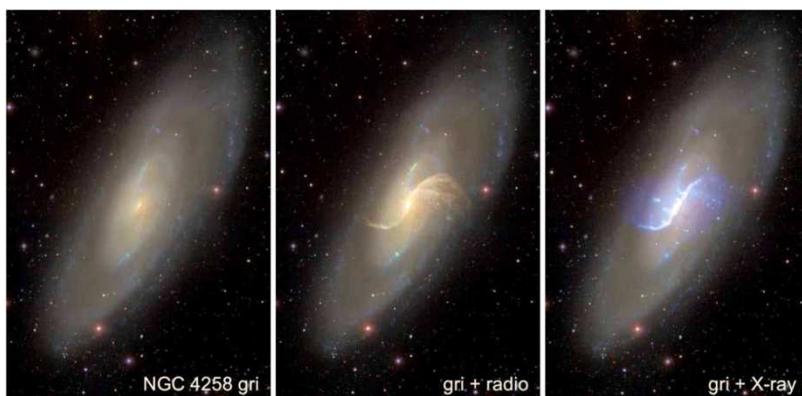
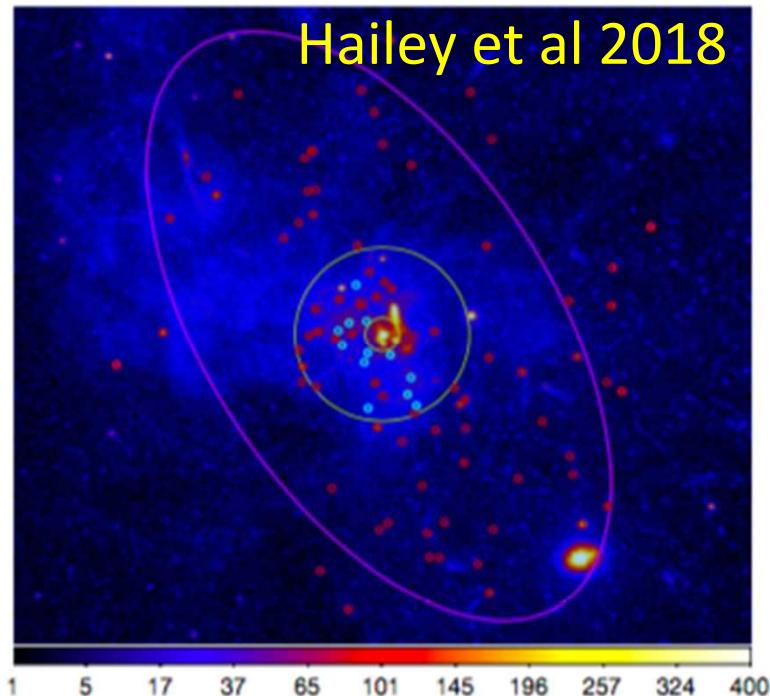
$$\Gamma_{\text{drag}} = 4\pi C_d \rho a_{12} V_{12}^2 \left( \frac{Gm_{12}}{V_{12}^2} \right)^2 \left( \frac{m_1}{m_2} + \frac{m_2}{m_1} - 1 \right) \quad \Gamma_{\text{drag}} \sim V_{12} a_{12} \dot{m}.$$

$$\tau_{\text{at}} \simeq \frac{m_1 m_2 \omega_{12} a_{12}^2}{m_{12} \dot{J}_t} \simeq \frac{4m_1^3}{m_2 m_{12} \Sigma_b a_{12}^2} \frac{h_b^3}{\omega_{12}}. \quad \text{XJZhang}$$

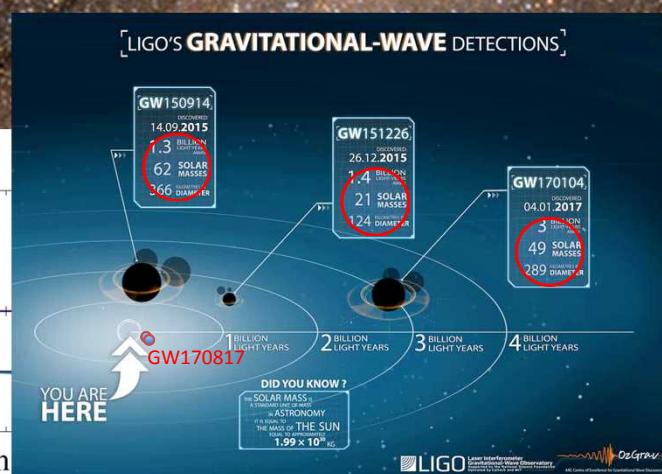
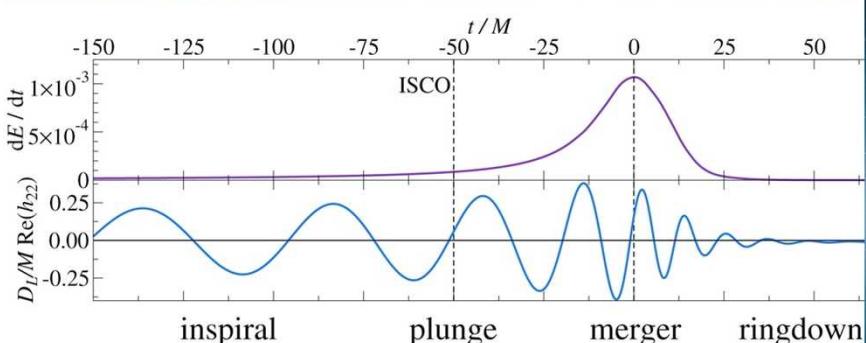
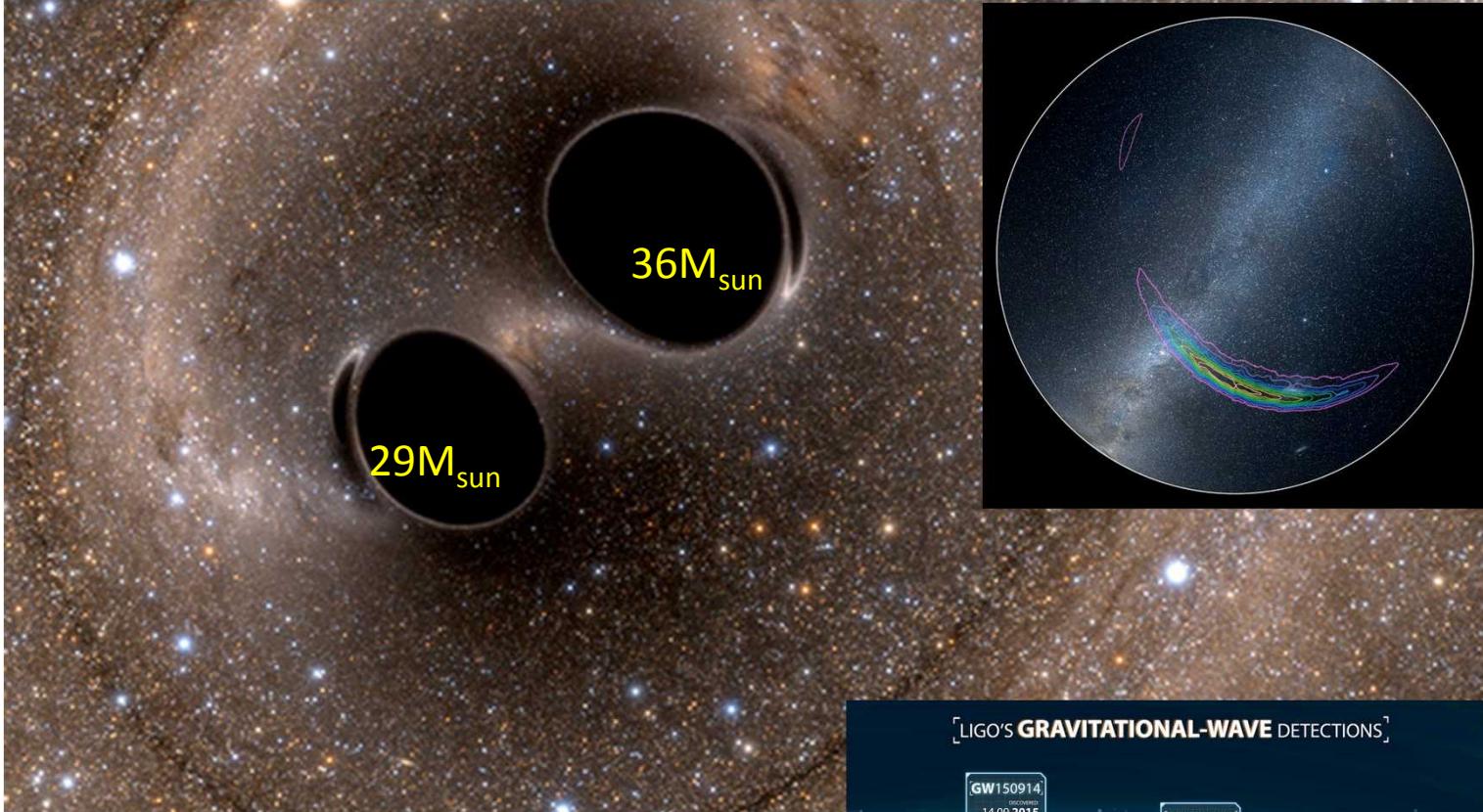
$$\tau_{\text{gr}} = \frac{5a_{12}^4 c^5}{256 G^3 m_1 m_2 m_{12}} \simeq \left( \frac{a_{12}}{1\text{pc}} \right)^4 \left( \frac{M_\odot^3 10^{39} \text{yr}}{m_1 m_2 m_{12}} \right). \quad \text{Peters 1964}$$



# LMXBs, X-ray Luminosity, high-V stars



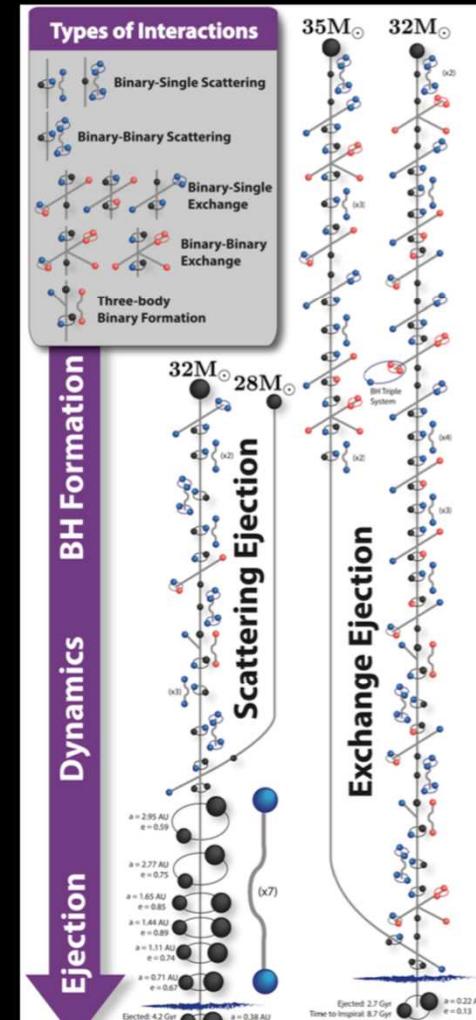
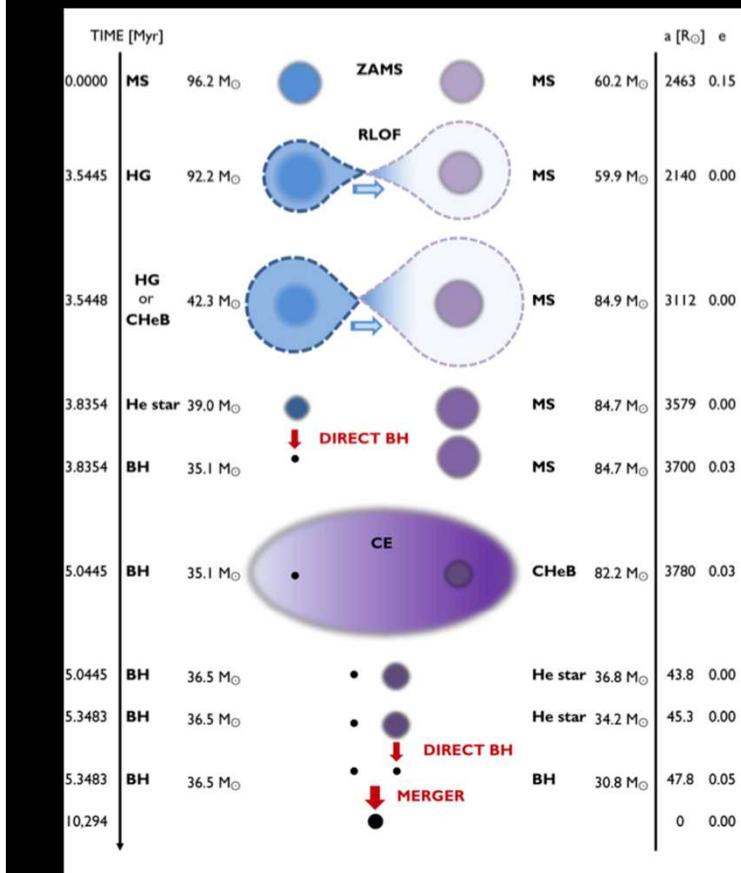
# Binary stellar black holes



# Common-Envelope vs stellar cluster scenarios

Cluster - Rodriguez et al., 2016

Isolated Binary - Belczynski et al., 2016



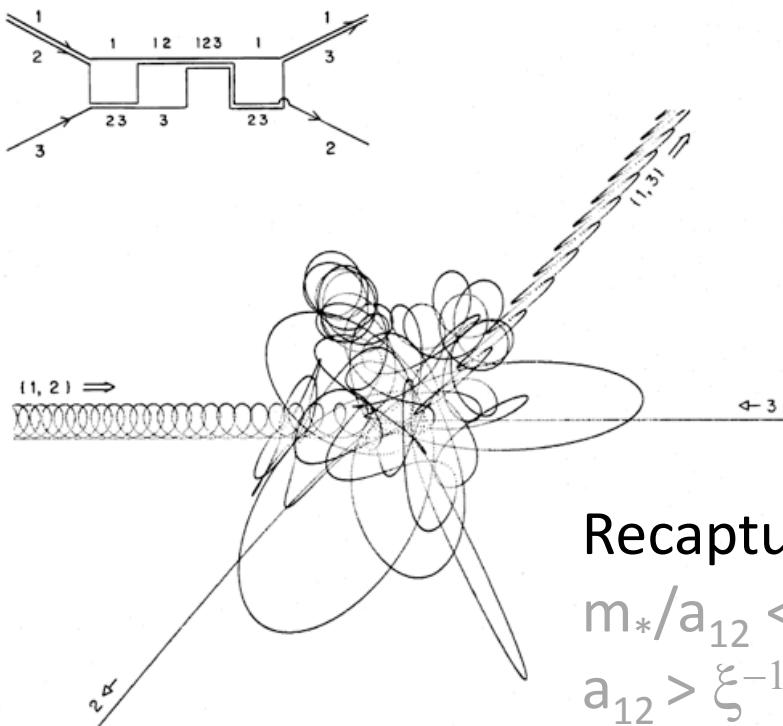
# AGN model: channels of binaries' dynamical evolution: Multiple capture, tightening of binaries, *slingshot*, recapture

Feeding zones:

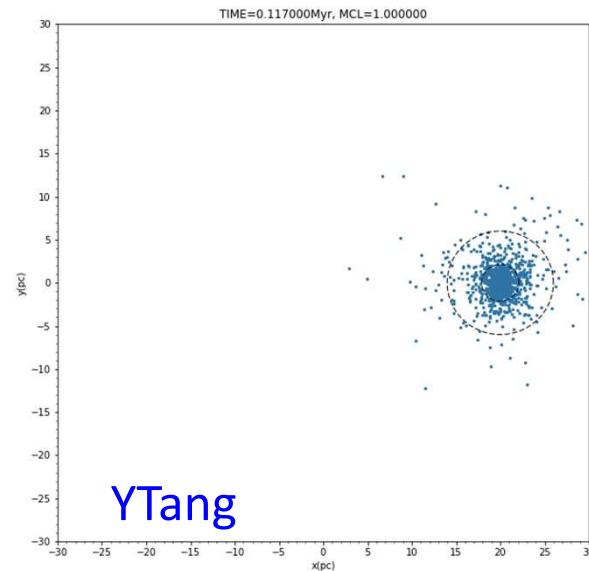
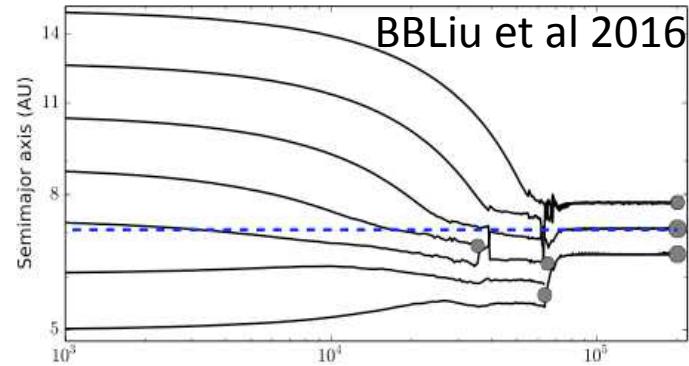
$$\Delta \sim 10 r_{\text{Hill}}$$

**Isolation mass:**  $M_{\text{isolation}} \sim \Sigma^{1.5} a^3 M_h^{-1/2}$

Heggie 1975, Saslaw et al *slingshot* 1975  
 Hut & Bahcall, 1983, Myllari Valtonen 2018  
 $(1-e_3)a_3 < (4-5)a_{12}$  groups of 10+ possible

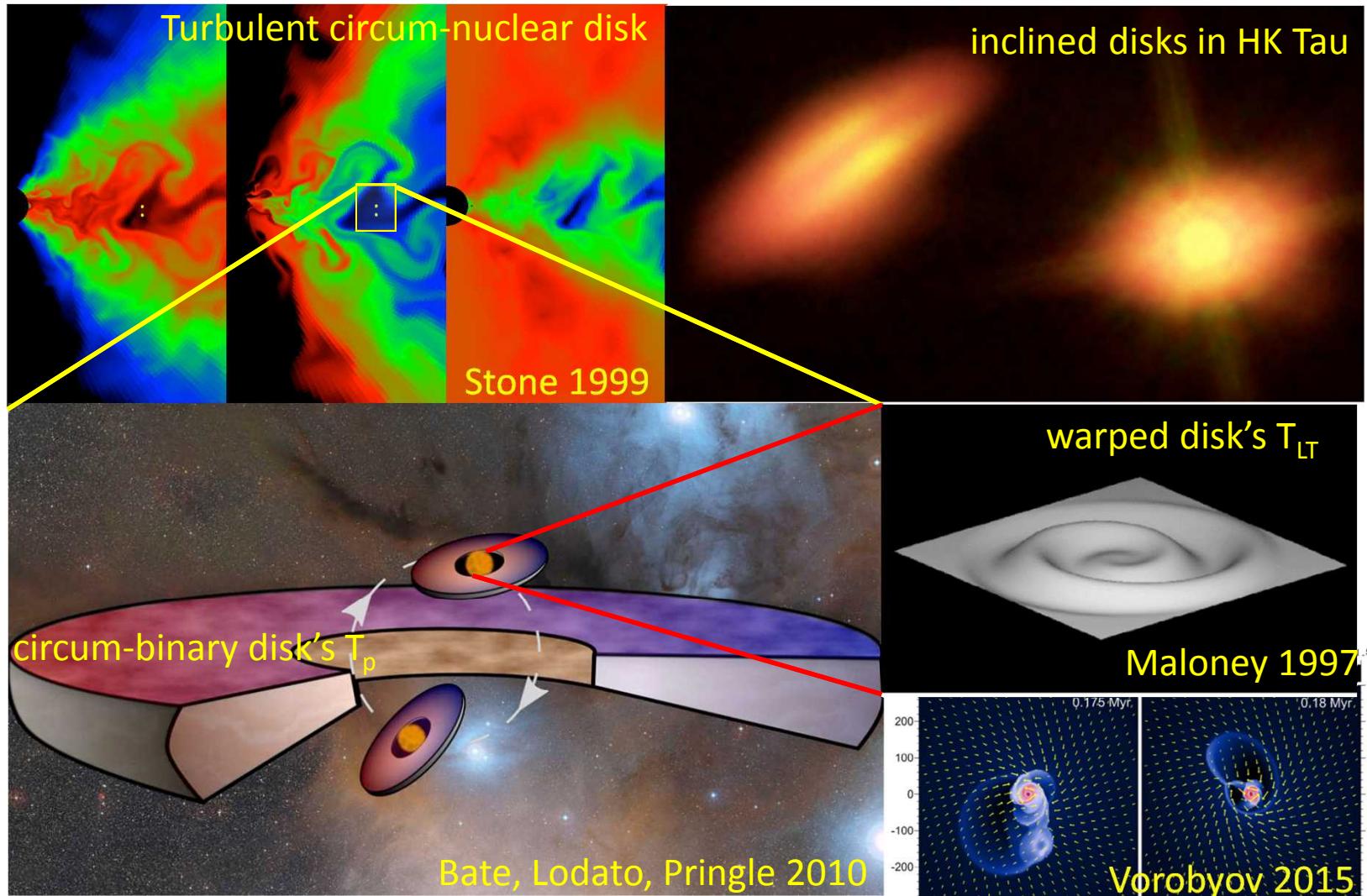


Recapture possible if  
 $m_*/a_{12} < \xi^{1/2} M_h/a$  or  
 $a_{12} > \xi^{-1/2} a m_{12}/M_h$



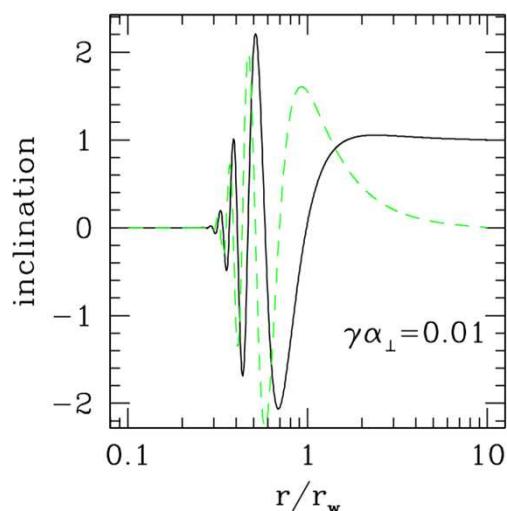
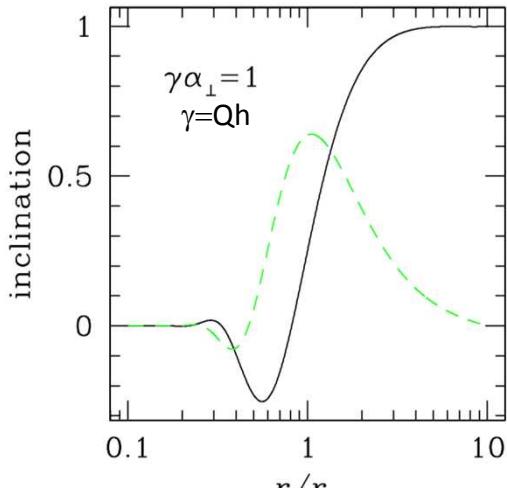
Shangfei Liu

# Accretion of turbulent gas onto seed bh's & spin angular momentum of circumbinary disks



# Torque through warped accretion disks

*S. Tremaine and S. W. Davis*



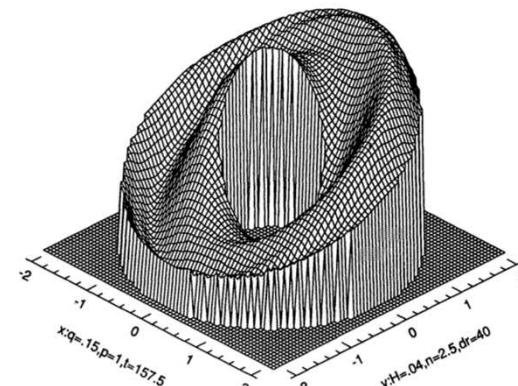
Evolution time scales of binary:

- a) turbulent accretion  $\tau_{\text{tur}}$  and drag by common envelope  $\tau_{\text{CE}} \Rightarrow \Delta J_b/J_b \sim \tau_{\text{tur}}/\tau_{\text{CE}}$  per turn over.  
For  $t > \tau_{\text{tur}}$ ,  $\Delta J_b/J_b \sim \tau_{\text{tur}}/\tau_{\text{CE}} (t/\tau_{\text{tur}})^{1/2}$
- b) circumbinary disks: for  $\tau_{\text{tur}} < \tau_{\text{Sal}}$ ,  $\Delta J_b/J_b \sim \tau_{\text{tur}}/\tau_{\text{Sal}}$   
for  $\tau_{\text{tur}} > \tau_{\text{Sal}}$ ,  $\Delta J_b/J_b \sim 1$ .
- c) Individual black holes' spin angular momentum  
For  $\tau_{\text{tur}} < \tau_{\text{Sal}}$ ,  $\Delta J_*/J_* \sim \tau_{\text{tur}}/\tau_{\text{Sal}}$  &  $\tau_{\text{tur}} > \tau_{\text{Sal}}$ ,  $\Delta J_*/J_* \sim 1$
- d) Individual black holes' spin alignment due to LT  
For  $\tau_{\text{tur}} < \tau_{\text{Sal}}$ ,  $\Delta J_*/J_* \sim \tau_{\text{tur}}/\tau_{\text{Sal}} (r_w/R_*)^{1/2}$

The Lense–Thirring torque  $T_{\text{LT}}$  and the companion torque  $T_*$  are equal at

$$r_w \simeq \left( a_* \frac{M}{M_*} R_g^{3/2} r_*^3 \right)^{2/9},$$

Papaloizou & L 1995  
Ogilvie & Dubus 2001  
Nixon & King 2016



# Gravitational radiation from binary black hole



$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64}{5} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^3 (1 - e^2)^{7/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304}{15} \frac{G^3 m_1 m_2 (m_1 + m_2)}{c^5 a^4 (1 - e^2)^{5/2}} \left( 1 + \frac{121}{304} e^2 \right)$$

$$\tau_{\text{gr}} = \frac{5a_{12}^4 c^5}{256 G^3 m_1 m_2 m_{12}} \simeq \left( \frac{a_{12}}{1 \text{pc}} \right)^4 \left( \frac{M_\odot^3 10^{39} \text{yr}}{m_1 m_2 m_{12}} \right).$$

Peters 1964

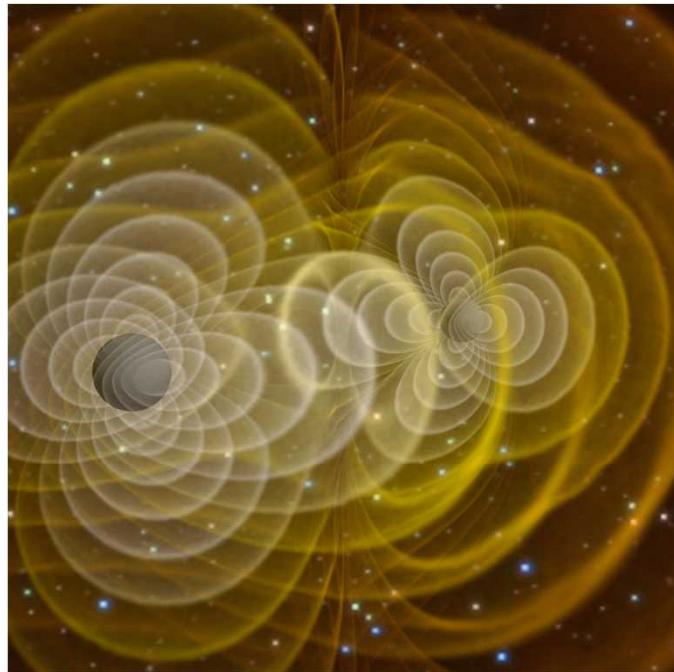
With  $m_1 \sim m_2 \sim 30 M_\odot$ , binary BHs  $\tau_{\text{gr}}$  would be less than 1Gyr in the limit  $a_{12} \sim 0.1 \text{ AU}$ .

Decoupled from disk's tides if  $a_{12} < a_{\text{gr}} = \left( (1+q)\alpha h_b^5 \frac{384}{10\pi} \frac{\eta \sigma_T}{R_{s1}} \frac{m_1}{m_p} \right)^{1/4} R_{s1} \sim 10^{10} \text{cm}$ .

For nearly parabolic orbit ( $e \sim < 1$ )

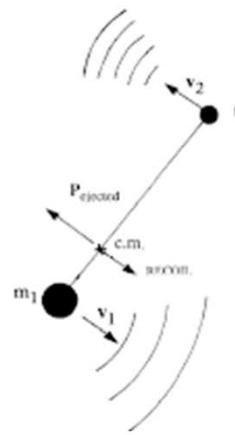
$$\tau_{\text{gr}}(e) = (768/425)(1-e^2)^{7/2} \tau_{\text{gr}}(0)$$

# Merger & recoil: binaries with spin-orbit obliquity



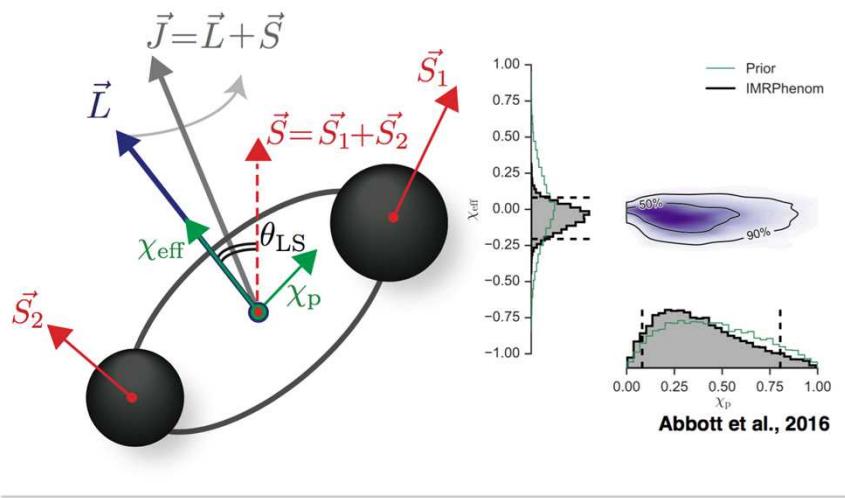
$$V_{\text{rec}} \sim \frac{q^2 V_H (a_1 - q a_2)}{(1+q)^5}$$

Resettle into the disk if  $V_{\text{rec}}/V_k < \xi$

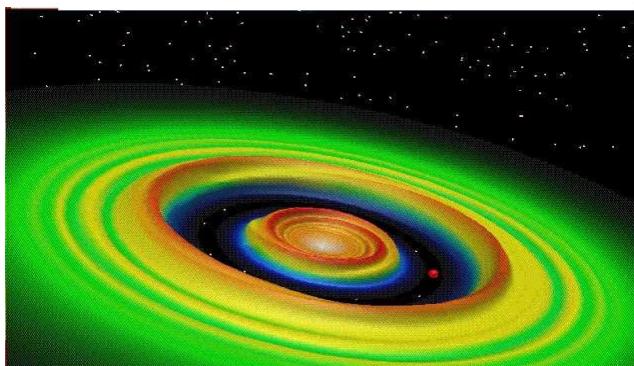
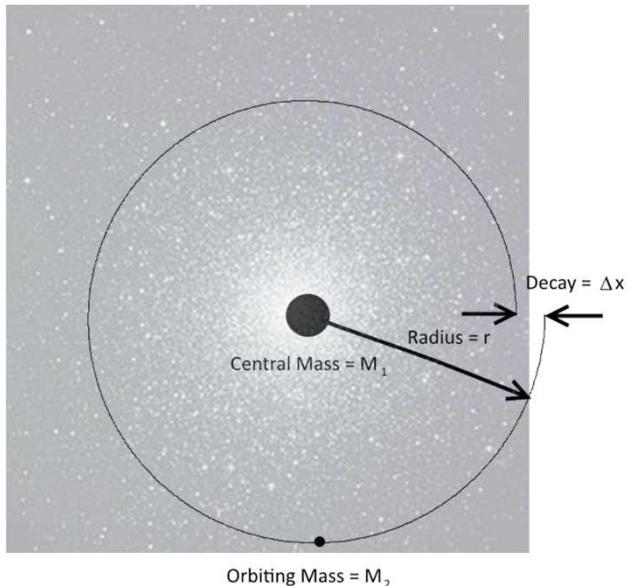


Recoil does not lead to significant disk perturbation

ZXWang, XJZhang



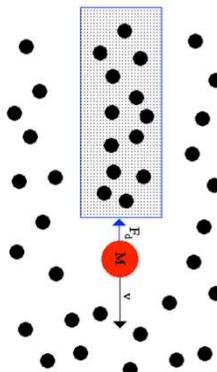
# IMBH: gap formation & dynamical friction



Inefficient type I migration

accretion for  $\tau_{\text{BH}} \sim 10 \tau_{\text{Sal}}$

IMBH with  $M > 10^3 M_{\text{sun}}$



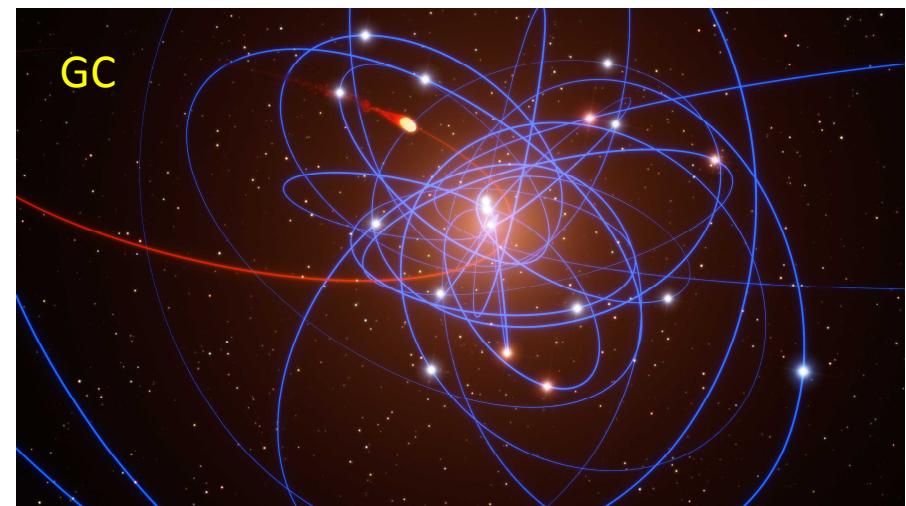
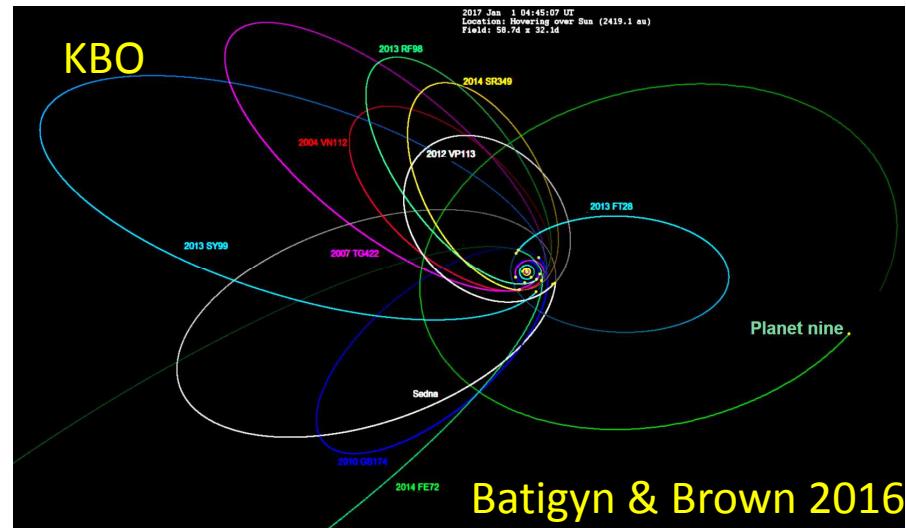
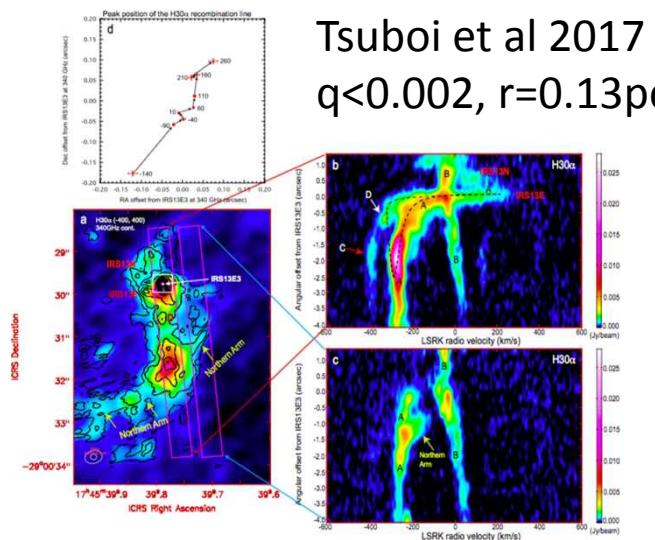
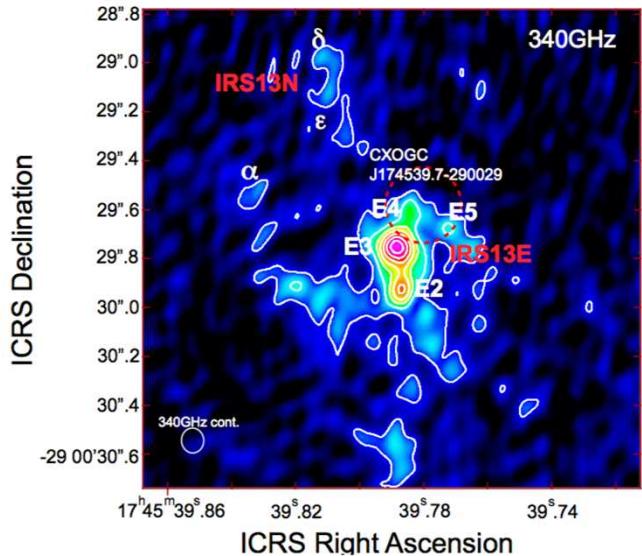
Dynamical friction, decay of black hole's orbit leads to efficient angular Momentum transport

Gap formation:  
Angular momentum transfer

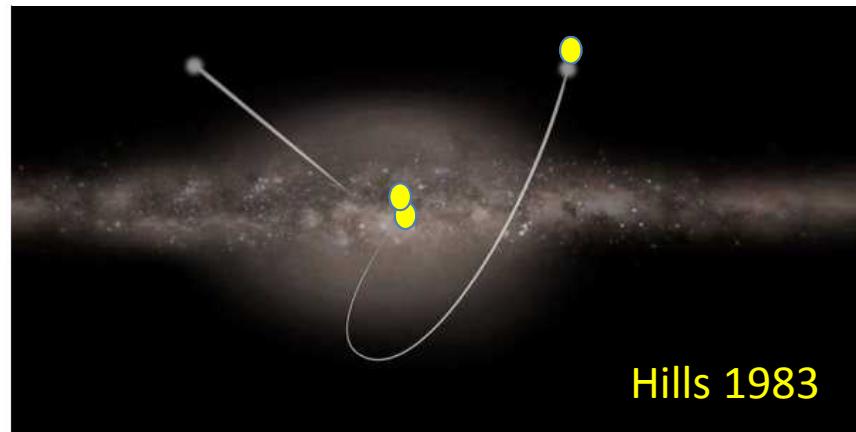
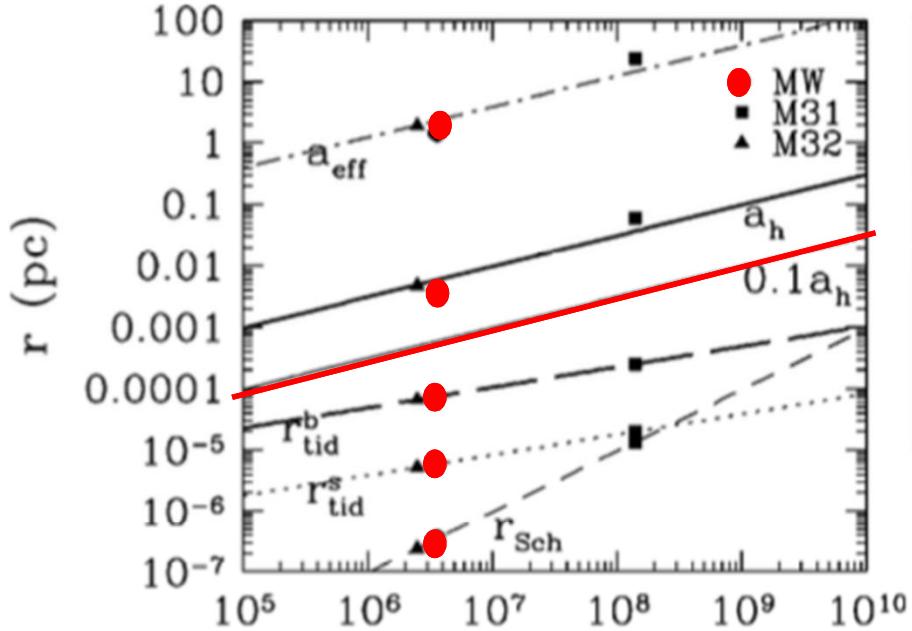
$$\frac{3hR}{4R_{\text{roc}}} + \frac{50\alpha h^2}{q} \leq 1.$$

Gap formation with  $M > 10^3 M_{\odot}$  L. Papaloizou 1986,  
Bryden 2000, Crida & Morbidelli 2007, many others

# IMBH's secular perturbation on nearby stars

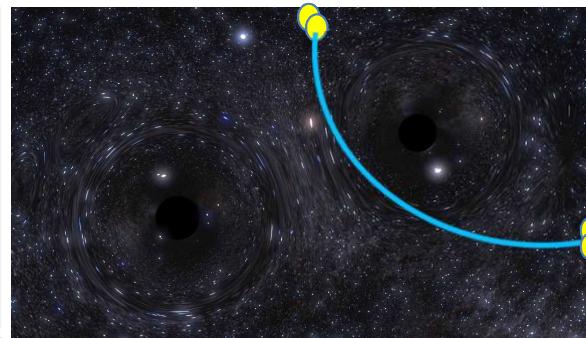
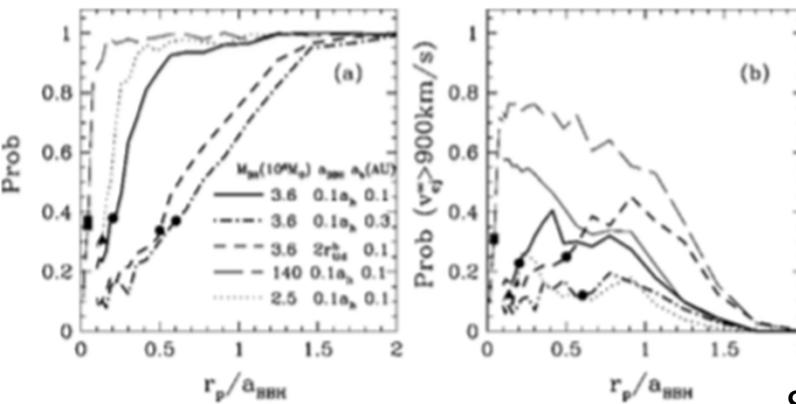


# Test: hypervelocity binary stars



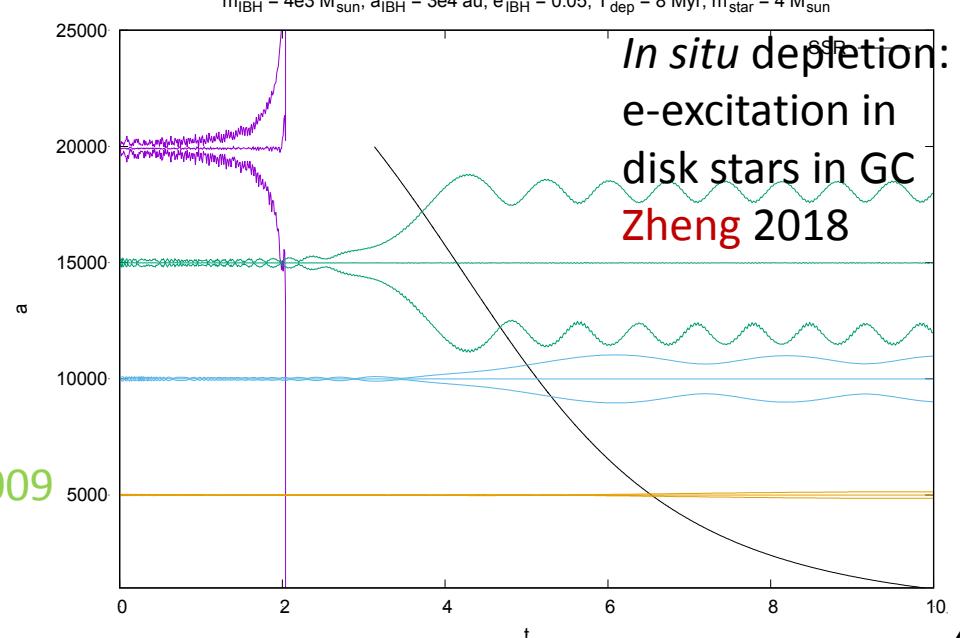
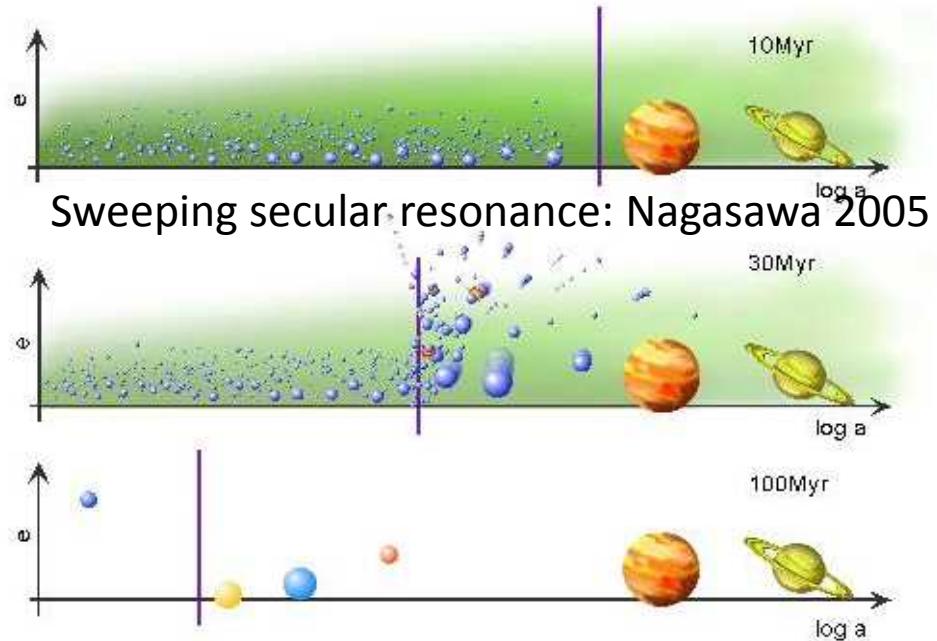
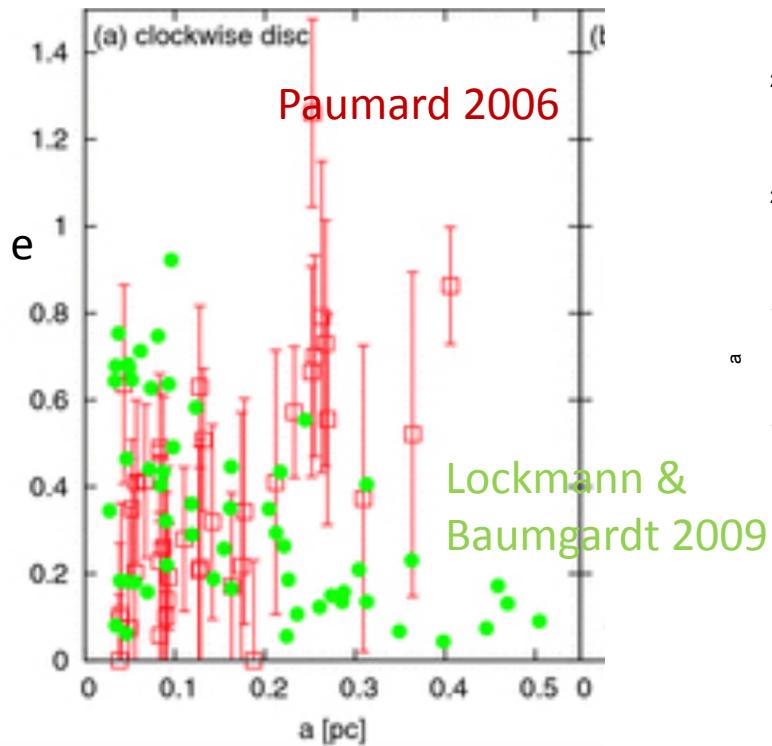
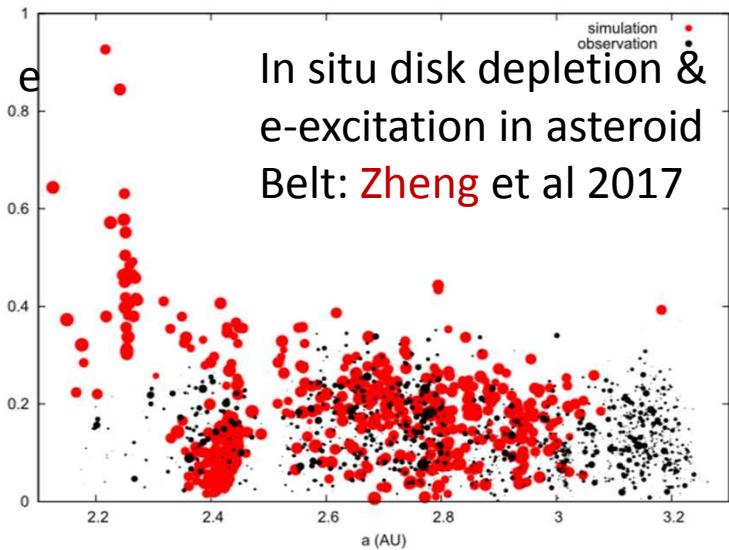
Hills 1983

Lu, Yu, L 2007

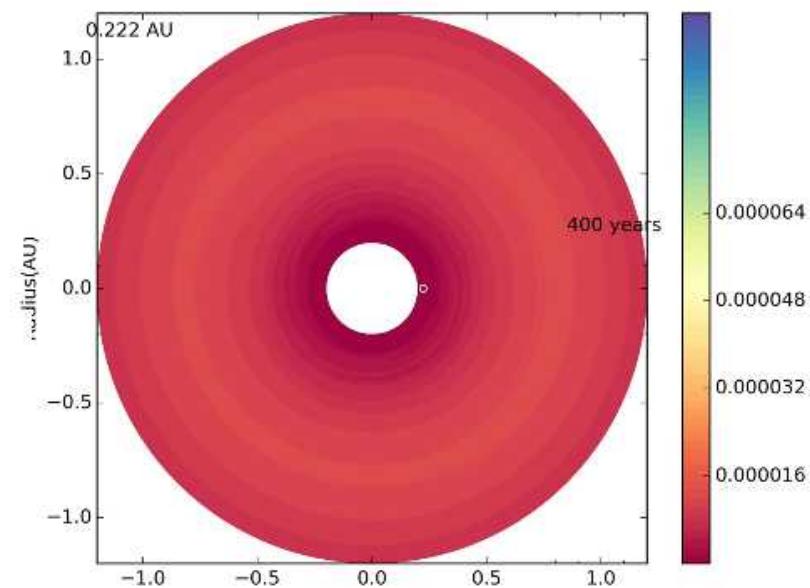
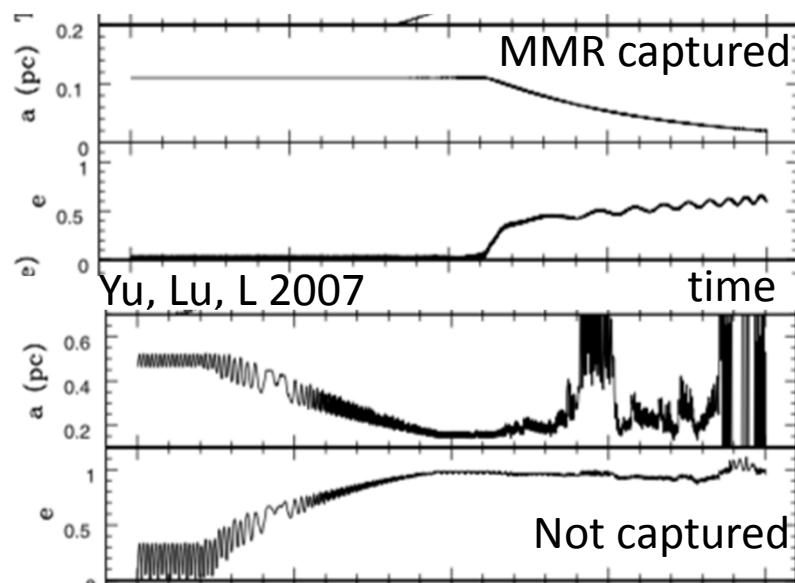
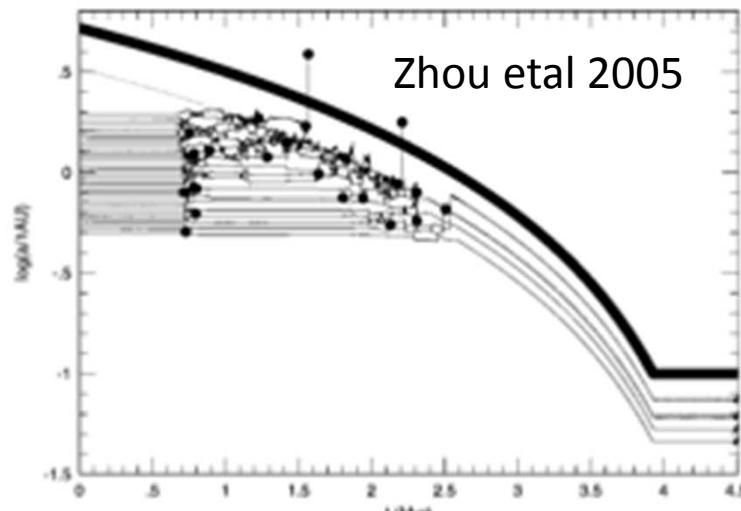
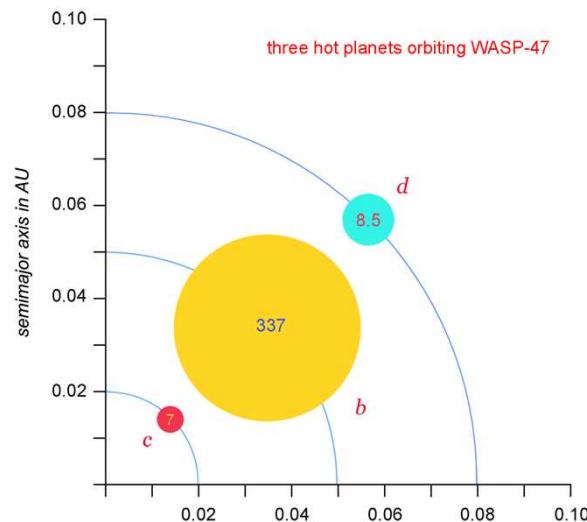


Outcome	Binary Ejection
$a_{\star}/a_{\bullet}$	
0.0005	92.8%
0.001	87.9%
0.0025	76.4%
0.005	60.9%
0.01	35.0%
0.05	3.09%
0.1	0.936%
Integrated	45.1%

Search for HV binaries with GIAI Coughlin et al 2018

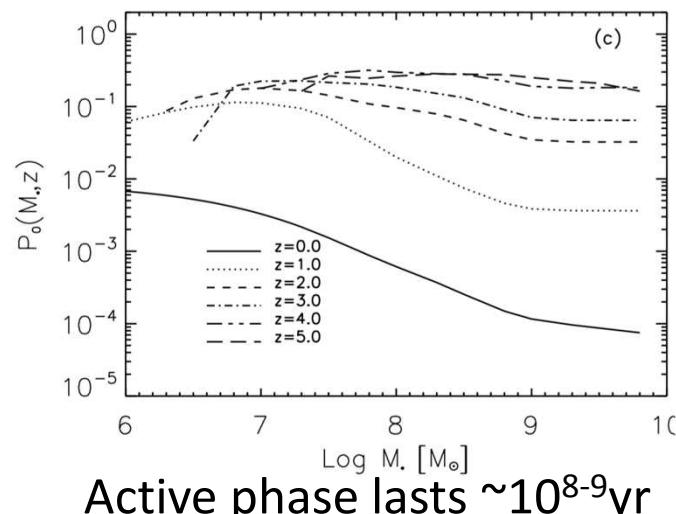


# Migrating IMBH's resonant capture & disk clearing



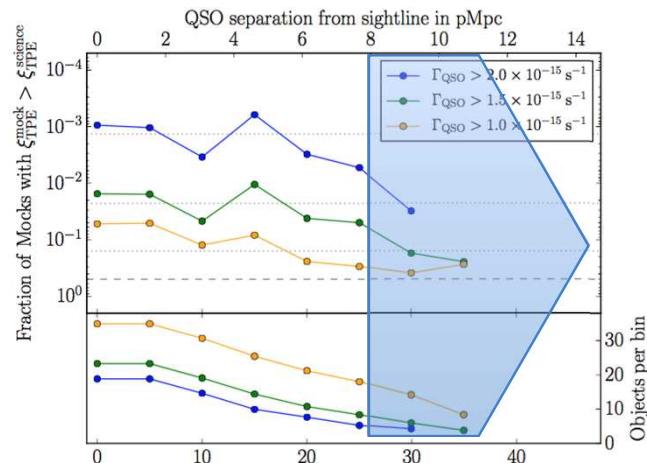
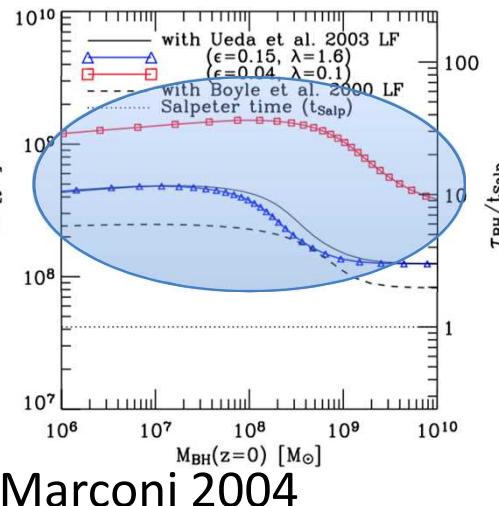
# AGNs' duty cycle and disk persistent time scale

duty cycle

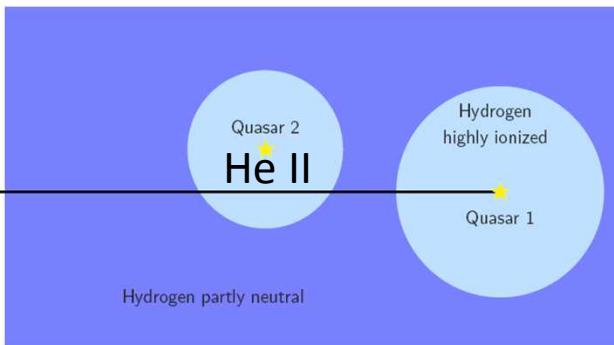


$$\frac{(z \cdot \mathcal{W})\Phi}{|\mathcal{W} \frac{\partial \Omega}{\partial p}|(z \cdot T)\tau\Phi} = (z \cdot \mathcal{W})\Phi$$

Duration of AGN



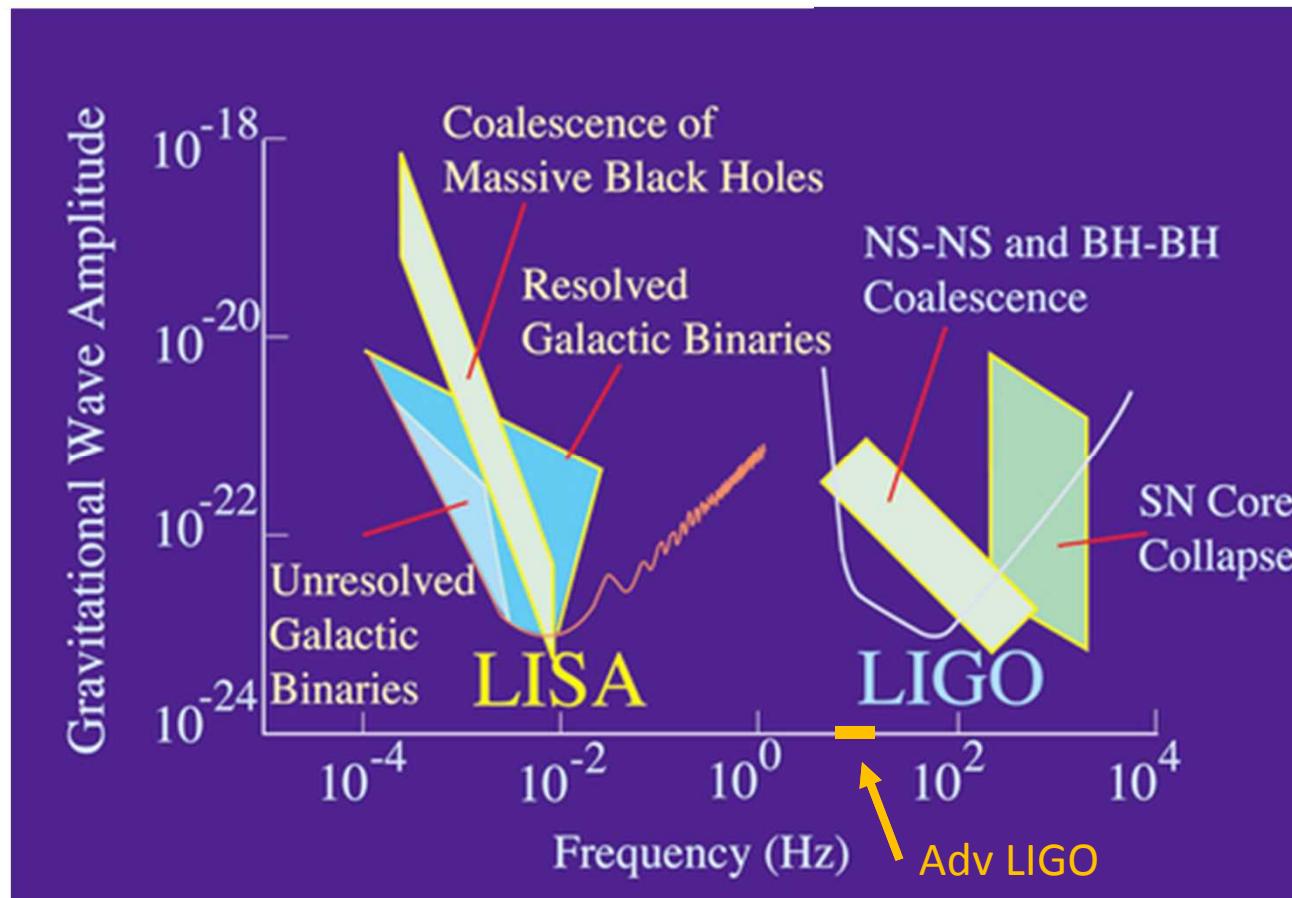
$\tau_{\text{diffusion}}(R_{\text{out}}) > 10^8 \text{ yr} \text{ & } R_{\text{out}} > 10 \text{ pc}$



Quasar-on-quasar transverse proximity effect, Schmidt 2017

# Intermediate- $m_*$ seed black holes' decay into MBH

$$\tau_{\text{df}} = \frac{R_{12}}{\dot{R}_{12}} = \frac{m_{12} R_{12}^2 \Omega_{12}}{\dot{J}_{\text{df}}} \simeq \frac{P_0}{24\sqrt{\pi} f_{\text{df}} \ln \Lambda} \frac{M}{m_{12}} \frac{M}{M_c} \sim 10^{8-9} \text{yr} \Rightarrow P_o(M, z < 1) \sim 10^{-2}$$



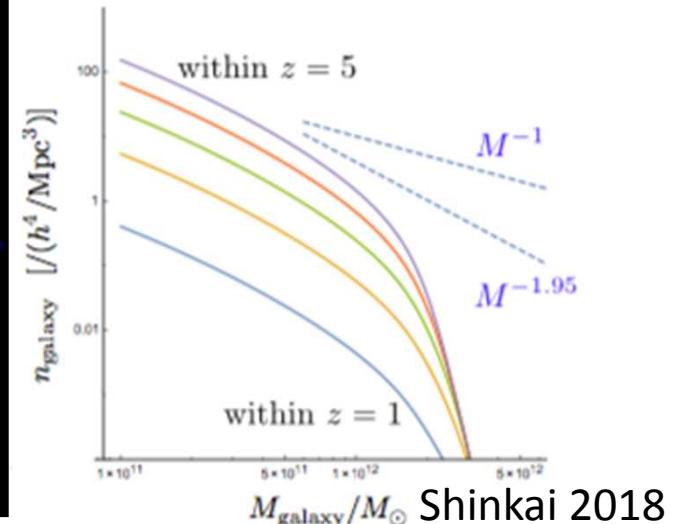
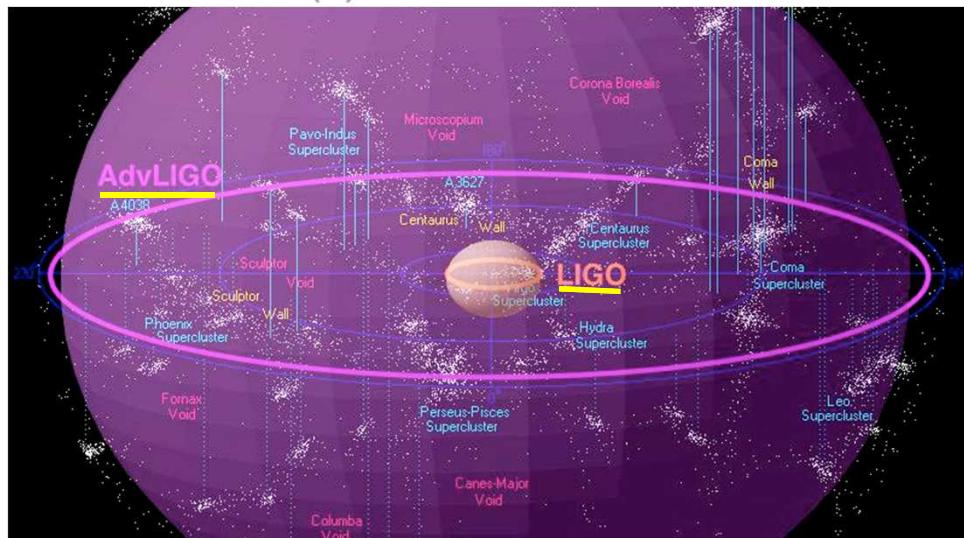
Occurrence rate of BH-MBH may be a fraction that for BH-BH merger events.  
Possible to detect intermediate mass BH with  $M_H \sim 10^3 M_o$

# Occurrence rate of binary black hole merger

$$\dot{N}_{\text{tot}} = \int \int \dot{N} \frac{dV_{\text{cm}}}{dZ} \frac{dn_A(Z)}{d\sigma_{200}} d\sigma_{200} dZ \quad D_c(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}$$

$$\frac{dV_{\text{cm}}}{dz} = \frac{4\pi c}{H_0} \frac{D_c^2(z)}{E(z)}$$

$$E(z) = \sqrt{\Omega_m(1+z)^3 + \Omega_\lambda}$$



Order of magnitude estimate in co-moving volume:

$$N(L_*) \sim 10^7 \text{ Gpc}^{-3}, \quad N_{\text{AGN}} \sim (\tau_{\text{active}} / \tau_{\text{Hubble}}) N(L_*) \sim 10^5 \text{ Gpc}^{-3}$$

$$\tau_{\text{trap}} \sim (M_{\text{disk}} / M_*) (R_{\text{bondi}} / a)^2 P \quad \text{with } R_{\text{bondi}} / a \sim (M_* / M_H) h^{-2}$$

$$\ln \tau_{\text{active}}, \quad R_{\text{bondi}} / a \sim (PM_* / \tau_{\text{active}} M_{\text{disk}})^{1/2} \Rightarrow h \sim 0.1$$

$$\text{Trapping rate} \sim (h M_H / M_*) / \tau_{\text{active}} \sim 10^{-4} \text{ yr}^{-1} \text{ per AGN}$$

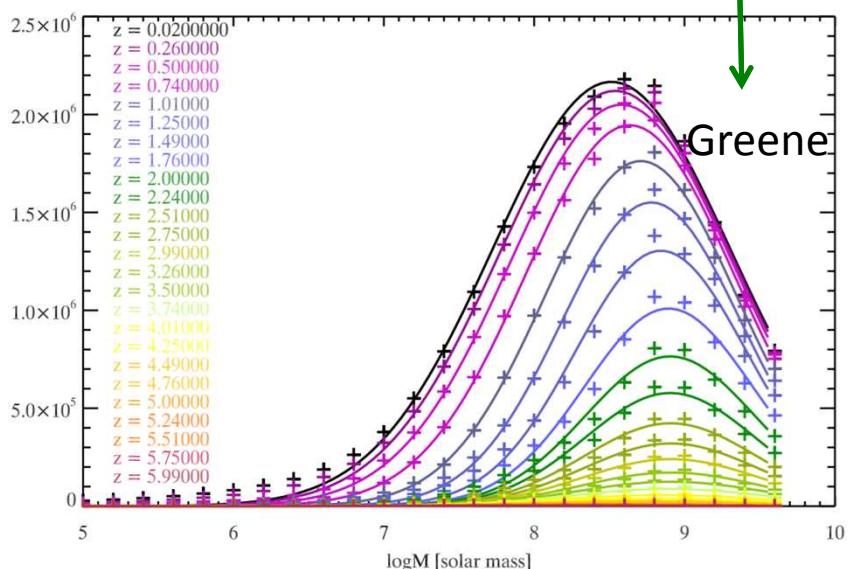
Total rate  $\sim O(1)$  wk<sup>-1</sup> Gpc<sup>-3</sup> at z=0. Statistical characterization!

# Mass density & M– $\sigma$ relation

$$\frac{dn_A(Z)}{d\sigma_{200}} = \left( \frac{dn_A(Z)}{dM} \right)$$

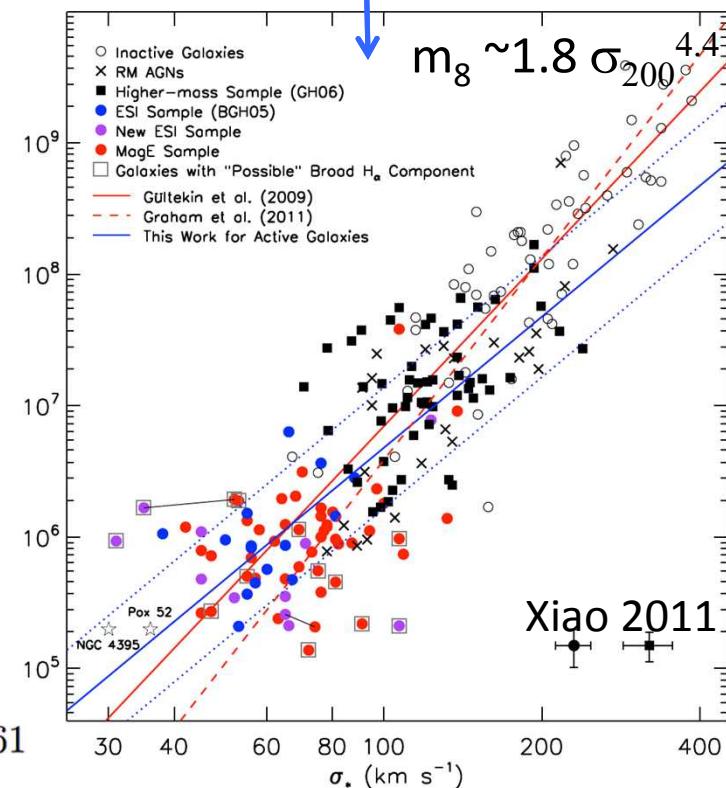
$$\left( \frac{dM}{d\sigma_{200}} \right)$$

Magorrian, Tremaine, Faber  
Gebhardt, Ferraresi, Merritt



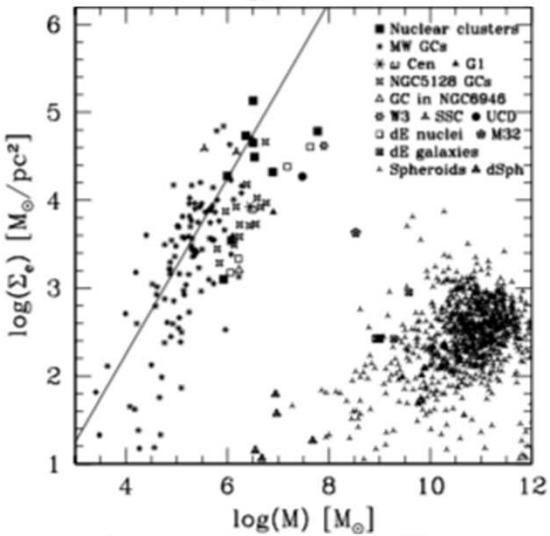
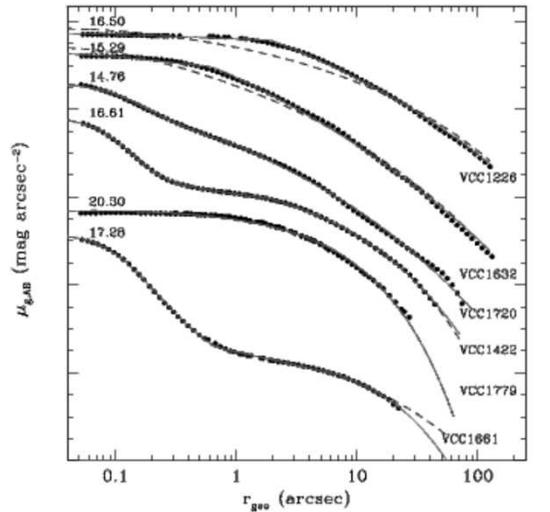
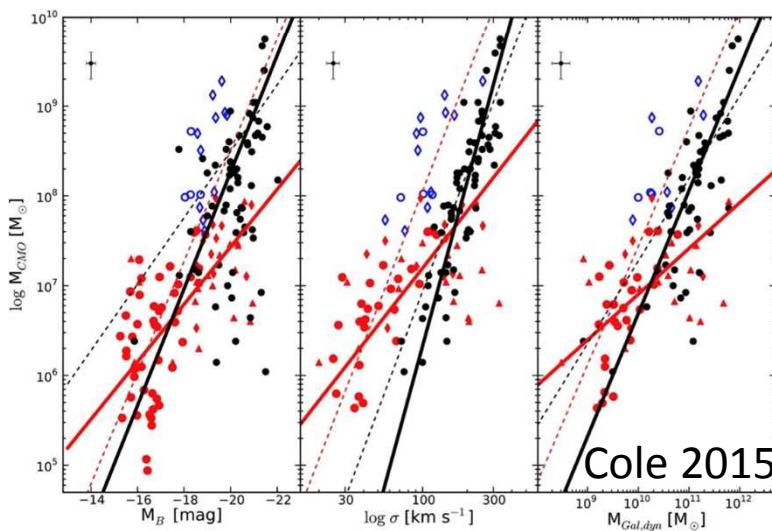
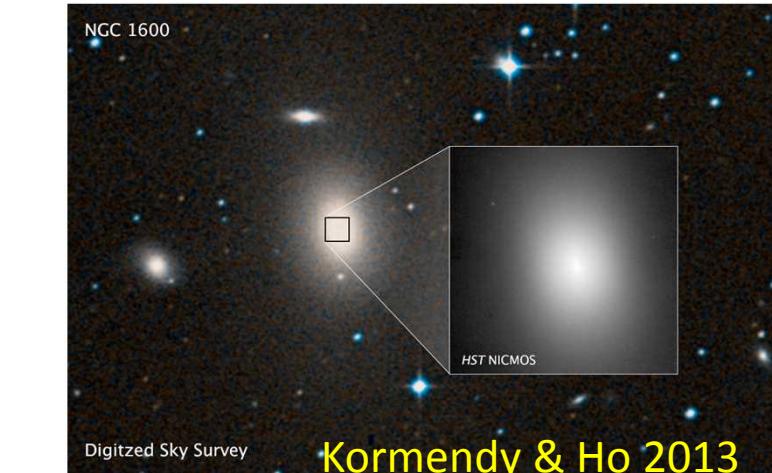
$$\frac{dn_A}{d\sigma_{200}} = \frac{4.4\phi_0}{\sqrt{2\pi}\sigma_{200}\sigma_M} \exp\left(-\frac{(\log m_8 - \nu_1)^2}{2\sigma_M^2}\right)$$

$$\phi_0 = 3.4 \times 10^{-5} \text{ Mpc}^{-3}, \quad \nu_1 = 6.7, \quad \text{and} \quad \sigma_M = 0.61$$



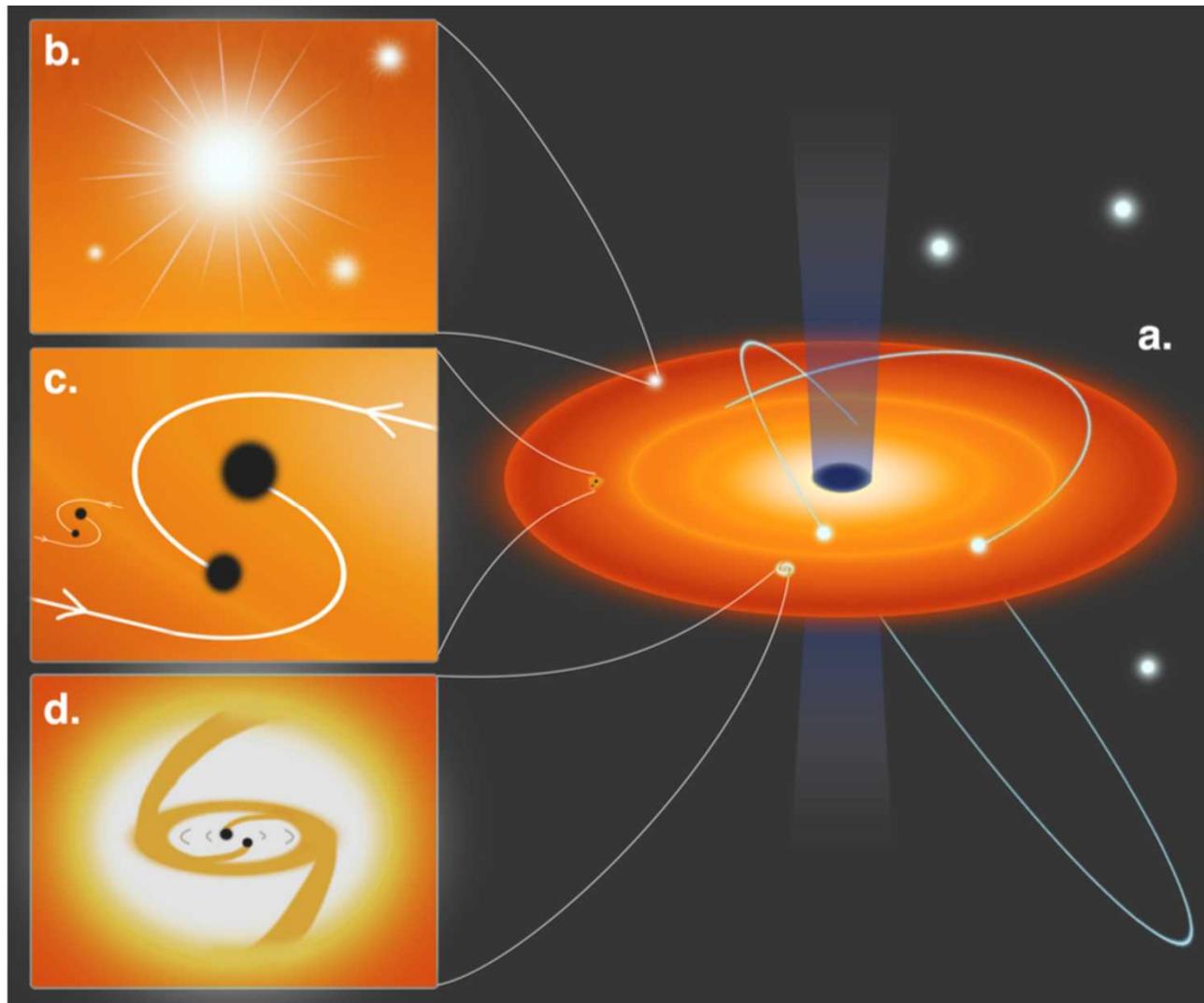
XJZhang, SDMao

# $\dot{N}(\sigma)$ of nuclear stellar clusters (NSC)



$\dot{N}_{\text{tot}} \sim 4A$  LIGO events per year with  $\int \sigma_{200}^{-3} \exp\left(-\frac{(4.4 \log \sigma_{200} - \nu_1)^2}{2\sigma_M^2}\right) d\sigma_{200} \approx \mathcal{O}(1)$

# Take-home cartoon



## Take-home messages

- AGN disks resemble protostellar disks & may trap nearby stars.
- Trapped stars are rejuvenated, gain mass, and evolve into SNs.
- Supernovae lead to formation of seed black holes with a few  $M_{\text{sun}}$  and the contamination of AGN disks.
- Seed black holes are retained, grow, migrate, capture partners closely analogous to planetary formation and dynamics.
- Single & multiple seed black holes' mass, spin and orbital angular momenta evolve as they mutually interact & accrete turbulent gas
- Binaries tighten by tides, drag by circum-binary disks, endure Lidov-Kozai effect, & merge through gravitational radiation.
- Events occur  $\sim 10^2 \text{ yr}^{-1} \text{ Gpc}^{-3}$  around metal-rich AGN with wide masses, spin angular momenta, and spin-orbit obliquity.
- Intermediate-mass ( $> 10^3 M_{\text{sun}}$ ) black-hole merger may be detectable by Advanced LIGO. They undergo orbital decay, clear disk gas, & regulate AGN duty cycle and are visible to LISA.