# The one-loop matter bispectrum as a probe of gravity and dark energy


Ben Bose

@ IPMU


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in collaboration with Atsushi Taruya [1808.01120]

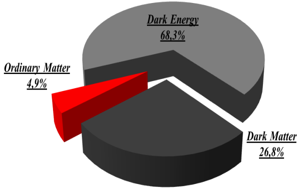


# Talk Outline:

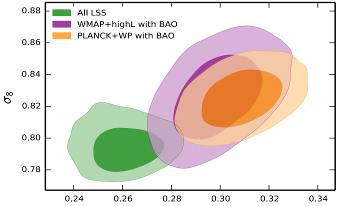
1. Motivation.
2. Description of **generalised perturbative** approach to Large Scale Structure (LSS).
3. The Importance of theoretical **consistency** for upcoming surveys.
4. The **1-loop matter bispectrum** in non-standard models of gravity and dark energy.
5. Applications and Extensions.

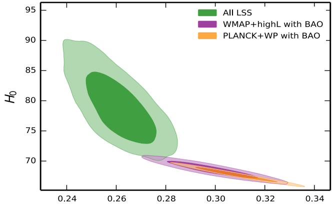
# Why Modify ΛCDM?

- Extrapolation of general relativity to cosmological scales.
- Dominant dark sector in standard picture.
- Tentative tensions in data sets.




[1409.2769]



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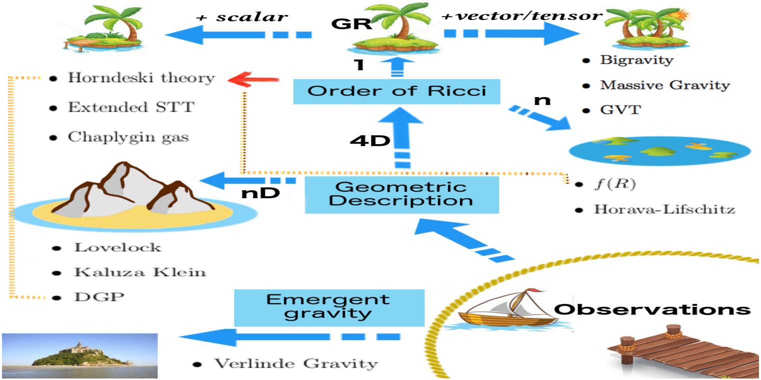

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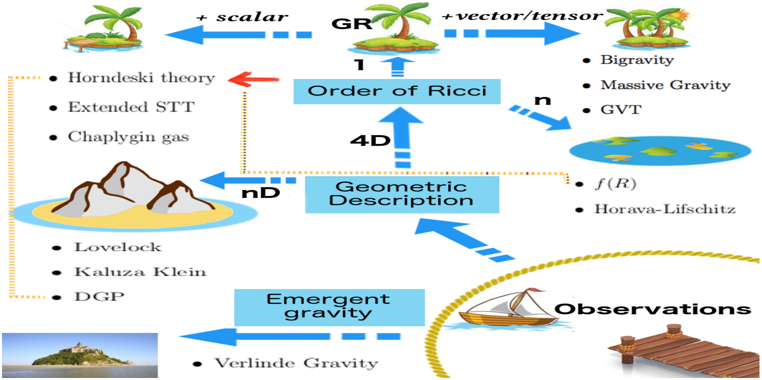
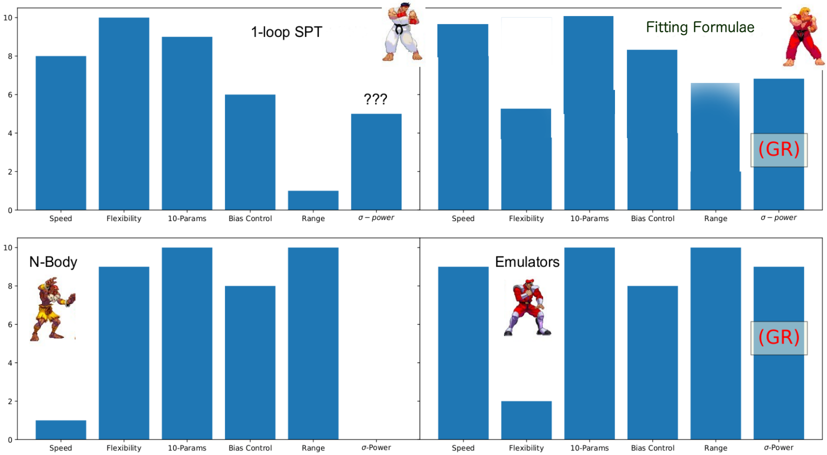



... but must respect solar system tests → **screening**.

# Comparisons of Approaches to LSS Modelling




[1607.03150,1704.05309,1804.05867]

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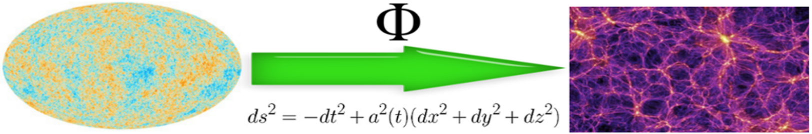
3. **Loop corrections** added to improve non-linear regime modelling.
4. **Additional modelling** required to connect with observables.



# Eulerian Perturbation Theory

Conservation of energy and momentum lead to the **Continuity** and **Euler** equations:

$$\boxed{a\frac{\partial \delta(\mathbf{k},a)}{\partial a} + \theta(\mathbf{k},a) = -\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3}\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\alpha(\mathbf{k}_1,\mathbf{k}_2)\,\theta(\mathbf{k}_1,a)\delta(\mathbf{k}_2,a),}$$

$$\boxed{\begin{aligned} a\frac{\partial \theta(\mathbf{k},a)}{\partial a} + \left(2+\left[???\right]+\frac{aH'}{H^2}\right)\theta(\mathbf{k},a) - \left(\frac{k}{a\,H}\right)^2 \Phi(\mathbf{k},a) = \\ -\frac{1}{2}\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3}\delta_{\rm D}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\beta(\mathbf{k}_1,\mathbf{k}_2)\,\theta(\mathbf{k}_1,a)\theta(\mathbf{k}_2,a),\end{aligned}}$$

$\cdots + \delta_q, \theta_q$ equations, etc.

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$\cdots+\delta_q,\theta_q$ equations, etc.

* $\delta_{NL}(\mathbf{k},a)=\sum_{n=1}^{\infty}\delta_n(\mathbf{k},a)$, $\qquad\theta_{NL}(\mathbf{k},a)=\sum_{n=1}^{\infty}\theta_n(\mathbf{k},a)$.
* $\delta_n(\mathbf{k};a)=\frac{1}{(2\pi)^{3(n-1)}}\int d^3\mathbf{k}_1...d^3\mathbf{k}_n\delta_D(\mathbf{k}-\mathbf{k}_{1...n})F_n(\mathbf{k}_1,...,\mathbf{k}_n;a)\delta_0(\mathbf{k}_1)...\delta_0(\mathbf{k}_n)$.

Evolution equations solved **numerically** using `MGCopter` up to 4th order in the perturbations.

## Gravitational Modelling

We can keep gravity general through Poisson equation in the evolution equations

$$-\left(\frac{k}{aH}\right)^2 \Phi = \frac{3\Omega_m(a)}{2}\mu(k;a)\,\delta(\mathbf{k}) + S(\mathbf{k};a),$$

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where the non-linear interaction term is given by

$$\begin{aligned} S(\mathbf{k},a) = &\int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3}\,\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{12})\gamma_2(\mathbf{k}_1,\mathbf{k}_2;a)\delta(\mathbf{k}_1)\,\delta(\mathbf{k}_2) \\ &+ \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3}{(2\pi)^6}\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{123})\gamma_3(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3;a)\delta(\mathbf{k}_1)\,\delta(\mathbf{k}_2)\,\delta(\mathbf{k}_3) \\ &+ \int \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3 d^3\mathbf{k}_4}{(2\pi)^9}\delta_{\rm D}(\mathbf{k}-\mathbf{k}_{1234})\gamma_4(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4;a)\delta(\mathbf{k}_1)\,\delta(\mathbf{k}_2)\,\delta(\mathbf{k}_3)\delta(\mathbf{k}_4). \end{aligned}$$

This term is responsible for **screening**.

[1606.02520]

# Dark Sector Modelling

We can incorporate **general interactions** within the dark sector *easily*. Some examples:

- Momentum exchange models:

$$\left(2 + A(a) + \frac{aH'}{H}\right)\theta(k;a),$$

[1412.1080]

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- Momentum exchange models:

$$\left(2 + A(a) + \frac{aH'}{H}\right)\theta(k;a),$$

[1412.1080]

or *NOT* so easily ...

- Clustering quintessence systems :

$$a\theta_q' + \left[2 - 3w + \frac{aH'}{H}\right]\theta_q - \frac{k^2}{a^2H^2}\Phi - \frac{c_s^2 k^2 \delta_q}{(1+w)a^2H^2} =$$
$$\int d^3k_1 d^3k_2 \delta_D(k - k_1 - k_2)\left[-\frac{\beta(k_1,k_2)}{2}\theta_q(k_1)\theta_q(k_2) + \frac{3(c_s^2 - w)}{(1+w)}\delta_q(k_1)\theta_q(k_2)\alpha(k_1\right.$$
$$\left. - \frac{ac_s^2}{(1+w)}\delta_q'(k_1)\theta_q(k_2)\alpha(k_1,k_2) + \frac{c_s^2(1+c_s^2)}{a^2H^2(1+w)^2}\delta_q(k_1)\delta_q(k_2)(k_1k_2\mu - k_2^2)\right].$$

[1106.0834]

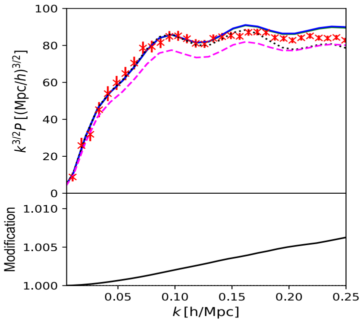
# Information Extraction: 2-point Statistics:

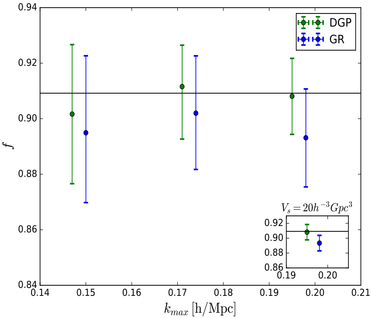
The **power spectrum** or **correlation function** are Fourier doubles of a 2-point correlation of the perturbations

$$\langle \delta(\mathbf{k})\delta(\mathbf{k}') \rangle = (2\pi)^3 \delta_{\rm D}(\mathbf{k}+\mathbf{k}')\, P(k).$$

- Has been well studied in perturbative regime. [ex. 1607.03150,1607.03148]
- High precision measurements demand **accurate theory**. There are many **difficulties** in this respect.
- Information rich scales difficult to model in a **useful** way.

# **Example:** Relevance of Consistent Modelling for Future Spectroscopic Surveys ($z = 1$)



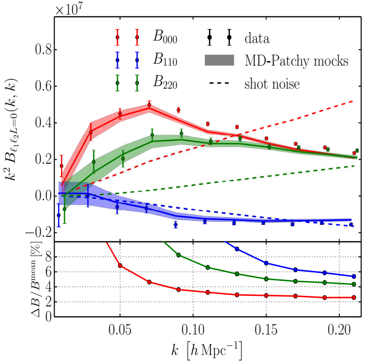




* Screening effects on **matter power spectrum** in above analysis $\leq 0.5\%$.

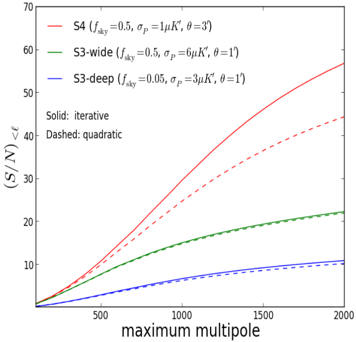
[1702.02348]

# Information Extraction: 3-point Statistics

## Galaxy Survey




## CMB Lensing




[1803.02132,1604.08578]

# The Matter Bispectrum

The **bispectrum** is a Fourier space 3-point correlation measurement of the perturbations:

$$\langle \delta(\mathbf{k}_1)\delta(\mathbf{k}_2)\delta(\mathbf{k}_3)\rangle = (2\pi)^3 \delta_{\rm D}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\, B(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3).$$

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To include non-linear information we can include loop corrections. The 1-loop bispectrum is then given by

$$B_{1-loop}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = B_{112} \; + \; [\; B_{222} + B_{321} + B_{411} \;],$$

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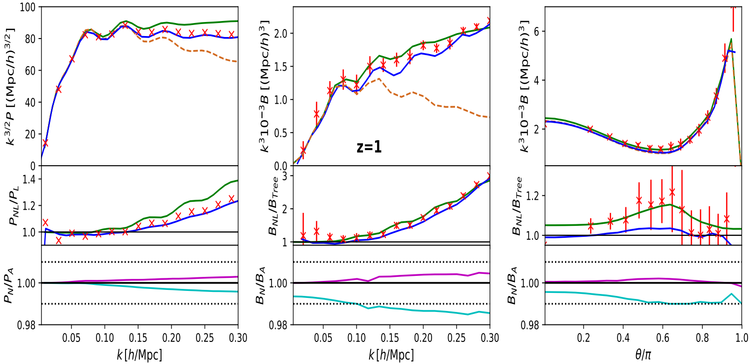
where

$$\langle \delta_2(\mathbf{k}_1)\delta_2(\mathbf{k}_2)\delta_2(\mathbf{k}_3)\rangle = (2\pi)^3\delta_{\rm D}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_{222},$$

$$\langle \delta_1(\mathbf{k}_1)\delta_2(\mathbf{k}_2)\delta_3(\mathbf{k}_3)\rangle \sim (2\pi)^3\delta_{\rm D}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_{321},$$

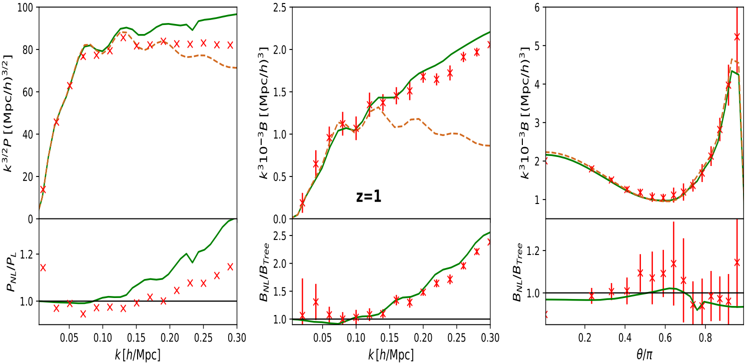
$$\langle \delta_1(\mathbf{k}_1)\delta_1(\mathbf{k}_2)\delta_4(\mathbf{k}_4)\rangle \sim (2\pi)^3\delta_{\rm D}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\, B_{411}.$$

# 1-loop Bispectrum: DGP

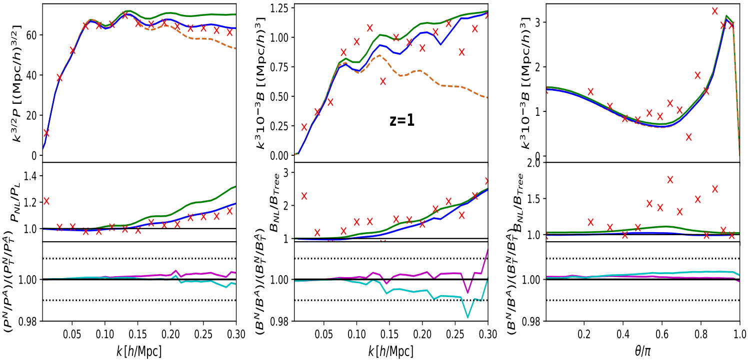



Fitting formula for (Beyond) Horndeski: [Namikawa et al. 1805.10567]

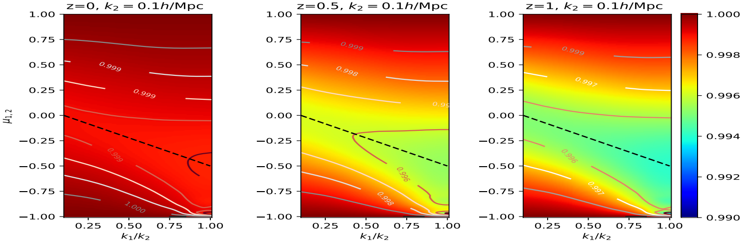
# 1-loop Bispectrum: f(R)



# 1-loop Bispectrum: Momentum Exchange



# Power of 3-point Statistics: **DGP** , $\Omega_{rc} = 0.438$.



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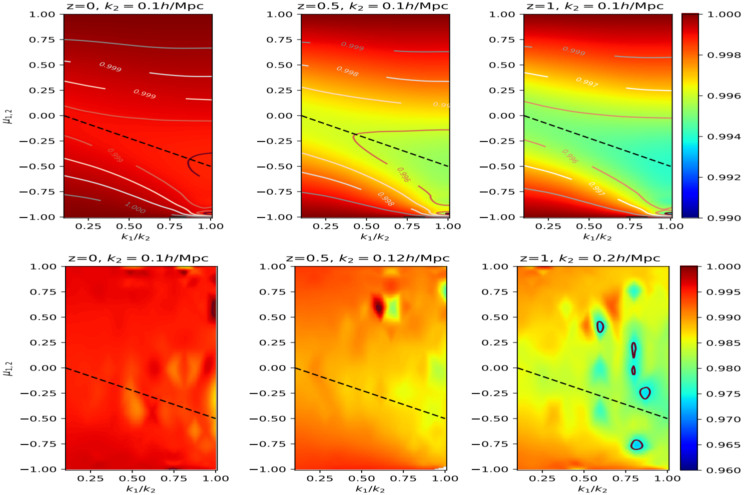

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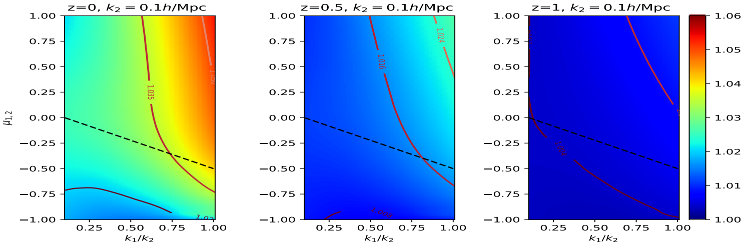

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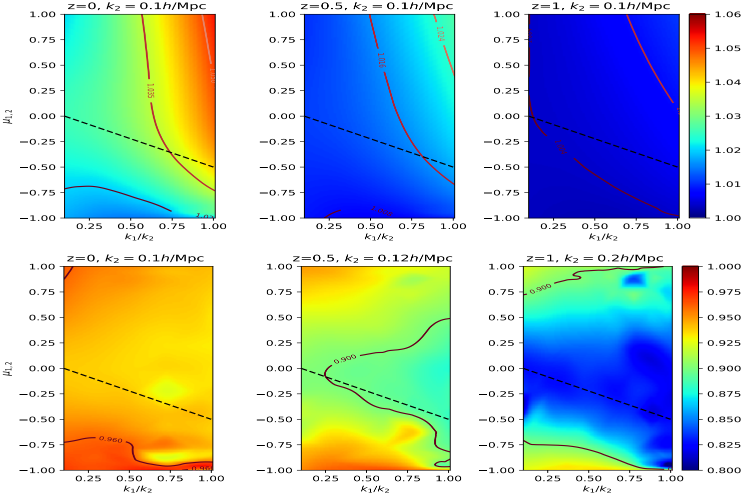
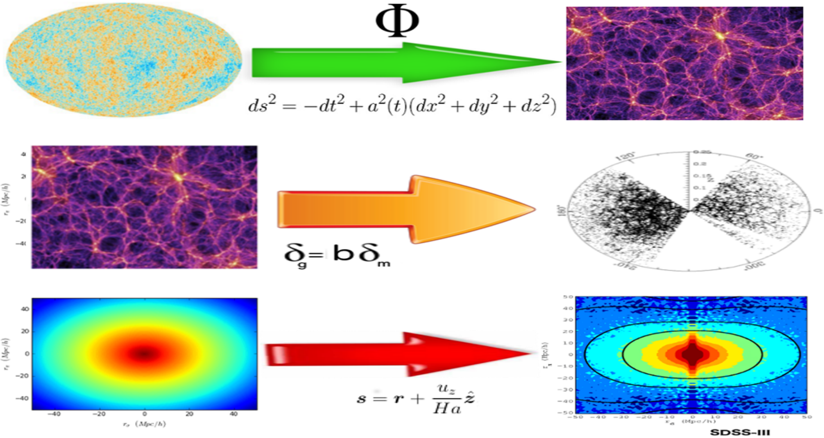


# Applications and Extensions: Galaxy Bispectrum



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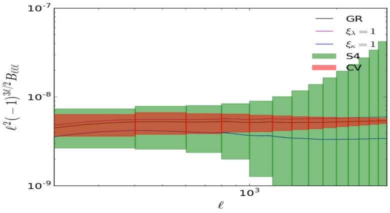
  [1705.02574,1806.04015]
* **Convergence spectra** has great scope to constrain gravitational theories but non-linear modelling must be accurate + intrinsic alignments, z-distribution.
  [1801.01741,1805.10567]

Direct application to CMB experiments:

* CMB lensing very clean but currently offers weaker constraints than convergence spectra.
* Next generation CMB experiments may be able to offer competitive constraints on gravity and dark energy. 1805.10567

**Equilateral**

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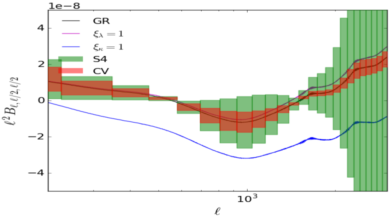

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- CMB lensing offers **clean measurement** but may not have strong enough signal even at stage 4 plus added complication of **post-born modelling**.

# Thanks for listening!