Stringy nonlocality and horizon physics

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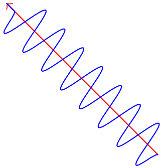


Based on: MD and Eva Silverstein, 1504.05536,1504.05537, 1703.10147
MD, Eva Silverstein, Gonzalo Torroba, 1704.02625

Summary

The goal of this talk is to revisit two old questions:
I. How big are strings?
II. Does this say anything about black holes?

Review of Hawking's information paradox

- A black hole is the unique spherically symmetric solution to Einstein's equation in vacuum with no charge or angular momentum (the no-hair theorem)

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2\, d\Omega^2, \qquad r_s = 2G_N M.$$

- Hawking showed in 1975 that black holes evaporate quantum mechanically, via production of Hawking radiation. To good approximation the radiation is thermal at temperature

$$T_{\text{Hawking}} \sim \frac{1}{r_s}.$$

- We can then think of black holes as thermal systems. Using $dS = T\, dE$, we get the Bekenstein-Hawking area law

$$S \sim \frac{\text{Area}}{G_N}.$$

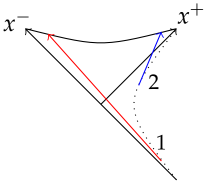
  This is formalized in AdS/CFT, where black holes are dual to a thermal ensemble in a conformal field theory.

- Once the black hole evaporates completely, we are left with radiation in a thermal density matrix. So we lose all information about what formed the black hole, which violates unitarity. This is Hawking's information paradox.
- If unitarity is not violated then the radiation must not be exactly thermal, so there are corrections to Hawking's calculation that must be understood. This is one motivation for understanding the role of string theory in black hole physics.
- Almheiri, Marolf, Polchinski and Sully intensified the argument: if unitarity is restored then modes near the horizon of an old black hole are entangled with each other and also with the early radiation (assuming EFT is valid). This violates "monogamy of entanglement."
- This is the firewall argument, which has led to many interesting proposals/solutions (Maldacena-Susskind's ER=EPR, Papadodimas-Raju's mirror operators, etc.). Here our goal is to investigate the assumption of the validity of EFT in string theory.

Conditions for validity of effective field theory

- AMPS[1] Postulate 2: the equivalence principle is valid near BH horizons (no drama). When is this true?
- Throw two objects with energies $E = m \ll M_{\mathrm{BH}}$ into a Schwarzschild black hole from a radius $R \gg r$,

$$\begin{aligned} ds^2 &= -\left(1 - \frac{r_s}{r}\right) dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 \\ &= -\frac{2r_s}{r} e^{1-r/r_s} dx^+ \, dx^- . \end{aligned}$$



- If the time separation between 1 and 2 is $\Delta t \gg r_s$, then in EFT 2 sees a black hole with mass $M_{\mathrm{BH}} + m \sim M_{\mathrm{BH}}$.
- Since 1 and 2 have small Schwarzschild energy, we naively conclude that EFT is valid, so there's no drama for $\Delta t \gg r_s$.

[1] Almheiri, Marolf, Polchinski, Sully

- However, black holes are powerful particle accelerators. For $|r - r_s| \ll r_s$, we can approximate the near horizon region by flat space. A time translation acts as a boost in the Rindler region with rapidity $\Delta t/(2r_s)$, since

$$x^{\pm} = \pm\sqrt{r_s(r - r_s)} \exp\left(\frac{r - r_s \pm t}{2r_s}\right).$$

- For a geodesic with Schwarzschild energy $E = m$,

$$p^+(r = r_s) = \frac{m}{r_s} x^+(r = r_s) \quad \Rightarrow \quad -\frac{p_1 \cdot p_2}{m^2} = \frac{x_2^+}{x_1^+} = \exp\left(\frac{\Delta t}{2r_s}\right)$$

- There are no large local invariants at $r = r_s$, but there is a large *nonlocal* invariant for $\Delta t \gg r_s$. If our UV theory has E-dependent nonlocalities, these could be important.
- In (weakly coupled) QFT we don't get to use this accelerator because the particles never hit.

- Let ΔX_{12}^{+} be the null size of 1 according to 2 in the flat space UV theory. In a frame where $p_1^+ \approx 0$, Lorentz invariance and dimensional analysis require

$$\Delta X_{12}^{+} = f\left(-\frac{p_1 \cdot p_2}{m^2}\right)\alpha' p_2^+ = f\left(\exp\left(\frac{\Delta t}{2r_{\mathrm{s}}}\right)\right)\alpha' p_2^+, \qquad [\alpha'] = L^2$$

- We are safe from drama if

$$\Delta X_{12}^{+} < x_2^+(r = r_{\mathrm{s}}) - x_1^+(r = r_{\mathrm{s}}) \quad \Rightarrow \quad \frac{\alpha' m}{r_{\mathrm{s}}} f\left(\exp\left(\frac{\Delta t}{2r_{\mathrm{s}}}\right)\right) < 1$$

- If $f' > 0$, EFT is violated drastically at late enough times. If $f' < 0$ the nonlocalities decouple at late times. $f' = 0$ is marginal: violations occur for $m \sim r_{\mathrm{s}}/\alpha'$ if α' satisfies

$$\frac{m}{M_{\mathrm{BH}}} \sim \frac{G_{\mathrm{N}}}{\alpha'} \ll 1 \quad \Rightarrow \quad \text{weak coupling.}$$

  In fact one finds violations for arbitrary m_1, as long as $m_2 \sim r_{\mathrm{s}}/\alpha'$.
- In the rest of the talk we will argue that $f = 1$ in tree level string theory.

THE SIZE OF STRINGS IN LIGHT-CONE GAUGE

- Choice of gauge can't matter, but light-cone gauge $X^- = x^- + p^- \tau$ is ideal for computing the size of strings[2]: local Hamiltonian system with only physical DOF.

$$S = -T \int d^2\sigma \, (\partial X^i)^2 \quad \Rightarrow \quad X^i = x^i + p^i \tau + \sqrt{\alpha'} \sum_n \frac{1}{n} \alpha_n^i e^{-in\tau} \cos(n\sigma).$$

- The transverse distribution of the endpoints is Gaussian, with width

$$\langle (\Delta X^i)^2 \rangle = \alpha' \sum_n \frac{1}{n} = \alpha' \log(n_{\max}) \to \infty$$

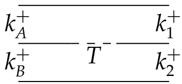
Strings are infinitely big, but in practice we can only measure up to a frequency $n_{\max}$.

- The longitudinal distribution is highly nonlinear, but has RMS size

$$X^+ = x^+ + p^+ \tau + \sqrt{\alpha'} \sum_n \frac{1}{n} \alpha_n^+ e^{-in\tau} \cos(n\sigma)$$

$$[\alpha_m^+, \alpha_n^+] = \frac{(m-n)\alpha_{m+n}^+}{\sqrt{\alpha'} p^-} + \frac{m^3 \delta_{m+n}}{\alpha' (p^-)^2} \quad \Rightarrow \quad \langle (\Delta X^+)^2 \rangle = \frac{1}{(p^-)^2} \sum_n n = \frac{n_{\max}^2}{(p^-)^2}.$$

[2] Susskind '69,'92; Karliner, Klebanov, Susskind '89.



- What is $n_{\max}$ for string scattering? The four-point function in light-cone gauge is an integral over light-cone time T. We are interested in the Regge limit $k_\perp^2 \ll E^2$, since this is dominated by $T \to 0$, [3]

$$\int \frac{dT}{T^2} e^{ET} \exp\left(-\sum_n \frac{\alpha' k_\perp^2}{n + n^2 T/(\alpha' E)} \right).$$

- For $E^2 \gg k_\perp^2 \gg 1/\alpha'$, this has a saddle point at

$$T \sim \frac{\alpha' k_\perp^2}{E} \to 0 \qquad \Rightarrow \qquad n_{\max} = \frac{E^2}{k_\perp^2}.$$

  The short time resolution means that the interaction probes large n.

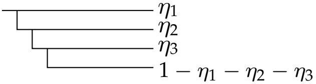
- The size of String A according to String B is

$$\Delta X^i = \sqrt{\alpha' \log(s/k_\perp^2)}, \qquad \Delta X^+ = \frac{p_B^+}{k_\perp^2}.$$

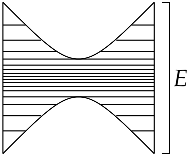
  Note that ΔX^+ doesn't depend on p_A^+: String A fails to Lorentz contract. In the CM frame $\Delta X^+ \sim E$.

[3] Brower, Polchinski, Strassler, Tan

- Strings are hadrons[4], so we should cross-check against the size of hadrons.
- The multiperipheral model generates the cloud of partons by splitting a bare hadron (with a strongly damped cascade $\eta_{i+1} \ll \eta_i$). The partons form a random walk with transverse RMS size $\log E$.



- Near-neighbor hypothesis: partons can only interact if they're close in phase space. For hadron-hadron scattering this means that only wee (low energy) partons can interact. So hadrons don't Lorentz contract - a boost introduces new wee partons into the spectrum.[5]
- The time it takes for two hadrons to cascade, interact, and recombine in the CM frame scales like E.



[4] Nambu; Olesen; Polyakov; Susskind; Maldacena, etc.

[5] Feynman; Kogut, Susskind; Gribov

The Gross-Mende saddle and the interaction scale

- For $s, |t| \gg 1/\alpha'$, the string path integral is dominated by a complex saddle point

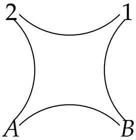
$$X^\mu = i\alpha' \sum_i k_i^\mu \log |z - z_i|^2$$

- Use $SL(2, \mathbf{C})$ to choose $z_A = 0, z_B = 1, z_2 = \infty$. Then

$$z_1 \sim \frac{p_\perp^2}{E^2} \to 0.$$

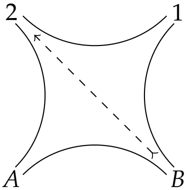
- The exchanged state is defined by $0 \ll |z| \ll 1$. In this limit one finds large spikes in the saddle,

$$X^+ X^- \leq \alpha'^2 p_\perp^2, \qquad |X^+|, |X^-| \leq \alpha' E.$$



- A simple test of this saddle: we should get corrections to string scattering in AdS when the hyperbola is larger than an AdS radius. And indeed there are corrections controlled by $\alpha'^2 p_\perp^2 / L_{\text{AdS}}^2$.

- To connect the Gross-Mende saddle to the longitudinal spreading scale, we slice the saddle in light cone time X^-,



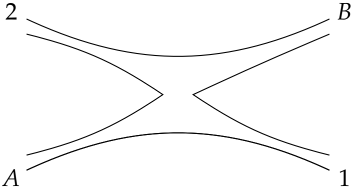
- This slicing begins at $X^- \sim -\alpha' E$, where String B is very small. String B grows to a size $\Delta X^+ \sim \alpha' E$ and joins with String A at time $X^- = -p_\perp^2/E^2$. Then they split and String B shrinks again.
- Note that String B grows bigger than the longitudinal spreading scale $X^+ \sim E/p_\perp^2$. Assume a linear distribution since $X^+ \sim (X_\perp)^2$. Then we pay a suppression factor

$$e^{-k_\perp^2 |\Delta X^+|/E} = e^{-\alpha' k_\perp^2}.$$

- This factor is present in the amplitude at large $k_\perp^2$,

$$A(s, k_\perp^2) = e^{-\alpha' k_\perp^2} \left(\frac{s}{k_\perp^2} \right)^{-\alpha' k_\perp^2}.$$

- We can also draw a picture of the saddle in the spatial dimensions,



- Focus on the endpoint connecting A and 1. This satisfies the equations

$$\ddot{X}_\perp = \frac{p_\perp \mathrm{sech}^2(T/(\alpha' E))}{E^2}$$
$$\ddot{X}_{||} = \frac{p_\perp^2 \mathrm{sech}^2(T/(\alpha' E))\tanh(T/(\alpha' E))}{E^3}.$$

This is equivalent to the motion of a particle in a potential,

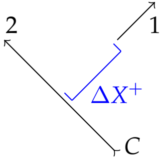
$$U = \frac{p_\perp^2 \mathrm{sech}^2(X_{||}/(\alpha' E))}{E^2}.$$

- This potential is constant for $X_{||} \ll \alpha' E$, and approaches a linear distribution at large $X_{||}$,

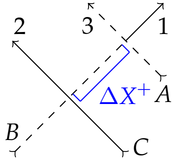
$$U \sim \frac{p_\perp^2}{E^2} e^{-|X_{||}|/(\alpha' E)}.$$

SIMULATING HORIZON PHYSICS IN FLAT SPACE

- We're interested in extracting the large longitudinal scale $\Delta X^+ \sim \alpha' E$ from some gauge-invariant quantity. The strategy is to set up a situation like in the black hole and compute $A(\Delta X^+)$,



- This is not an S-matrix element. But we can add a few auxiliary particles (dashed lines) to set up the above picture in an on-shell way:



- This is a six-point function. Longitudinal spreading predicts a long range over which $A(\Delta X^+)$ is supported, $\Delta X^+ \sim \alpha' E_1$. Of course we need to make sure that there is no early collision between the dashed and solid lines.

- Use the rules for perturbative string theory (in the ordering B2C1A3):

$$A = \int_1^\infty dy_A \int_0^1 dy_C \int_0^{y_C} dy_2\, y_2^{K_{B2}} (1-y_2)^{K_{12}} y_C^{K_{BC}} (1-y_C)^{K_{C1}} (y_C - y_2)^{K_{C2}}$$
$$y_A^{K_{AB}} (y_A - y_2)^{K_{A2}} (y_A - y_C)^{K_{AC}} (y_A - 1)^{K_{A1}}.$$

- This integral is dominated by saddle points, assuming all invariants $\gg 1/\alpha'$. We found several interesting saddles in Regge kinematics, I'll focus on one of them here. Introduce momenta

$$k_{1'} = k_1 + k_C + k_2, \qquad k_{B'} = k_B + k_C + k_2.$$

The saddle of interest is then

$$\left|\frac{K_{A1}}{K_{A3}}\right|^{-K_{A3}} \times \left|\frac{K_{12}}{K_{C2}}\right|^{-K_{C2}} \times \frac{\Gamma(k_{1'}^2)\Gamma(k_{B'}^2)}{\Gamma(k_{B'}^2 + k_{1'}^2)}.$$

- The first two factors are Regge four-point interactions with momentum transfer K_{A3} and K_{C2}. The third factor will turn out to contain all the longitudinal dynamics.
- Our goal is to compute the dependence of the amplitude on ΔX^+,

$$\int d(E_C - E_A)\, e^{i(E_C - E_A)\Delta X^+} A(K_{IJ}).$$

- Again we look for a saddle point, this time in the integral over energies. At $\Delta X^+ = 0$, the integrand is peaked at $k_{1'}^2 = k_{B'}^2 = -k_\perp^2$, where $k_\perp^2$ is a combination of transverse momenta. Varying away from this peak, the integrand is

$$e^{i\Delta X^+ \delta E_C} \left(1 - \frac{E_B \delta E_C}{k_\perp^2}\right)^{-\alpha'(k_\perp^2 - E_B \delta E_C)} \left(1 + \frac{E_B \delta E_C}{k_\perp^2}\right)^{-\alpha'(k_\perp^2 + E_B \delta E_C)}$$

- This has a saddle point at

$$\delta E_C = \frac{ik_\perp^2}{E_B} \tan\left(\frac{\Delta X^+}{2\alpha' E_B}\right),$$

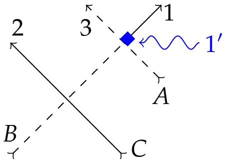
and the corresponding amplitude is

$$\cos\left(\frac{\Delta X^+}{2\alpha' E_B}\right)^{\alpha' k_\perp^2} \theta(\Delta X^+)$$

The longitudinal distribution is purely delayed, but is constant for $\Delta X^+ \ll \alpha' E_B$, as expected for an object with longitudinal size $\alpha' E_B$.

- There are also other saddles that contribute, corresponding to oscillations of on-shell strings in various channels. These have the same cosine shape, but are delayed by a multiple of $\alpha' E_B$.

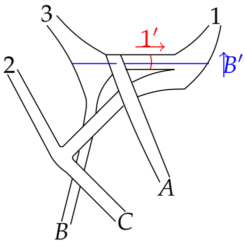
- For the black hole application, we are particularly interested in the case where $1'$ is produced by the AB collision,



- This is Reggeon exchange in the $1'$ channel, $|k_{1'}^2| \ll |k_{B'}^2|$. The amplitude factorizes,

$$A = \left|\frac{K_{A1'}}{K_{A3}}\right|^{-K_{A3}} \times \left|\frac{K_{1'2}}{K_{C2}}\right|^{-K_{C2}} \times e^{i\pi k_{1'}^2} \left|\frac{k_{B'}^2}{k_{1'}^2}\right|^{-k_{1'}^2} .$$

- The first factor is the four-point amplitude for preparing $1'$, and the second factor is a Regge interaction between C and $1'$. So far so good – the amplitude can be interpreted as production of a late-time particle $1'$ via an auxiliary process, and then interaction between C and $1'$.
- Finally there is the third factor, which is Pomeron exchange in the $k_{1'}$ channel. This has a phase $e^{i\pi k_{1'}^2}$, arising from production of on-shell states in the $k_{B'}$ channel.



- We set up localized wavepackets for the ingoing strings in this regime,

$$|i\rangle = \int d\delta E_A \, d\delta E_C \, e^{i(\delta E_C - \delta E_A)\Delta X^+} \exp\left(-\frac{\delta E_C^2 + \delta E_A^2}{2\sigma^2}\right) |k_A, k_B, k_C\rangle.$$

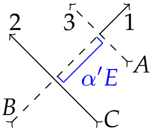
- Solve for δE_A in terms of δE_C. Then the amplitude is just

$$\langle k_1, k_2, k_3 | i \rangle = \int d\delta E_C \, e^{i\delta E_C \Delta X^+} \exp\left(-\frac{\delta E_C^2}{2\sigma^2}\right) A(K_{IJ}).$$

- Let's start by asking where this function is peaked. One finds

$$\partial_{E_C} k_{1'}^2 = E_B.$$

Recall that the phase of the amplitude is $e^{i\pi k_{1'}^2}$. This shifts the peak to $\Delta X^+ \sim -\alpha' E_B$!



- What about the spread of the amplitude? Expanding in δE_C,

$$A(K_{IJ}) \propto \exp\left(-\frac{\alpha' E_B}{2}\log\left(\frac{k_{1'}^2}{k_{B'}^2}\right)\delta E_C + \frac{\alpha' E_B^2}{4k_{1'}^2}(\delta E_C)^2 + \dots\right).$$

- If the width of the wavepacket dominates in the amplitude, then we only need to keep the linear term and we get

$$A(X^+) = \exp\left(-\frac{(\sigma^-)^2}{8}\left(\pi\alpha' E_B + \Delta X^+ - i\alpha' E_B \log\left(\frac{k_{B'}^2}{k_{1'}^2}\right)\right)^2\right).$$

This gets contributions up to $\Delta X^+ \sim \alpha' E_B \log(k_{B'}^2/k_{1'}^2)$, a huge scale.

- The conclusion persists if we use wavepackets with compact support in the longitudinal direction - it's not just an interaction on the tail of the wavepackets.

Probing the nonlocality with a background field

- Another simple test is to take the string coupling to depend on X^+, the simplest being a linear dependence,

$$g_s(X^+) = g_{s,0}e^{V^- X^+}.$$

Then we can track where the interaction is happening by looking for factors of $e^{V^- X^+}$ in the amplitude.

- The linear dilaton theory is exactly solvable (albeit strongly coupled in the far future). The dilaton basically just shifts the conserved momentum,

$$p^\mu \to p^\mu + iV^\mu.$$

- Repeating the worldsheet spreading calculation gives

$$\langle (\Delta X^+)^2 \rangle \sim \sum_{n>0} \frac{n}{(p^-)^2 + n^2(V^-)^2}.$$

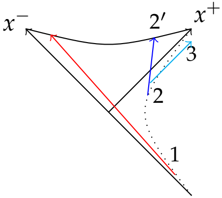
So for a weak enough dilaton the spreading prediction is the same.

- We found the expected factors of the dilaton in the scattering amplitude: for a term where a string oscillates n times we get $e^{\alpha' n E_B V^-}$, and for the above early six point interaction we get $e^{-\alpha' E_B V^-}$.

DRAMA FROM SECONDARY PROBES

- This was supposed to be a talk about black holes. From now on assume

$$\Delta X_{12}^{+} = \frac{p_2^+}{m^2} \text{ for } m^2 > 1/\alpha'.$$



- Can the late string 2 detect the early infaller 1 at times $\Delta t \gg r_s$? The best experiment it can do is to shoot a light ray 3 outward, decaying into $2'$.

- Momentum conservation gives

$$p_2^+ = p_{2'}^+ + p_3^+, \quad p_2^- = p_{2'}^- \quad \Rightarrow \quad p_3^+ = p_2^+ \left(1 - \frac{m_{2'}^2}{m_2^2}\right).$$

- Taking $E_2 = m_2$, the condition for drama is

$$\Delta X_{13}^{+} > x_2^+(r = r_s) - x_1^+(r = r_s) \quad \Rightarrow \quad \alpha' p_3^+ > \frac{r_s}{m_2} p_2^+$$

- So we get violations of EFT for $m_2 > r_s/\alpha'$, $m_2 - m_{2'} > r_s/\alpha'$.

- Easily extended to $m_1 \neq m_2$, and gives the same condition $m_2 > r_\mathrm{s}/\alpha'$.
- The size of a typical string at mass r_s/α' is

$$m_2^{1/2}\alpha'^{3/4} \sim r_\mathrm{s}^{1/2}\alpha'^{1/4} \ll r_\mathrm{s}.$$

- r_s/α' is parametrically smaller than the black hole mass at weak coupling,

$$\frac{r_\mathrm{s}}{\alpha' M_\mathrm{BH}} \sim g_\mathrm{s}^2 \ll 1.$$

- The condition for perturbative control is satisfied in a wide range of Δt,

$$-p_1 \cdot p_2 = m_1 m_2 \exp\left(\frac{\Delta t}{2r_\mathrm{s}}\right) \quad \Rightarrow \quad \Delta t \ll r_\mathrm{s} \log\left(\frac{M_\mathrm{p}^2}{m_1 m_2}\right).$$

- Giveon, Kutasov, Itzhaki computed the reflection amplitude off the Euclidean cigar with an additional time coordinate,

$$ds^2 = -dt^2 + k(dr^2 + \tanh^2 r\, d\theta^2), \qquad \Phi - \Phi_0 = -\log\cosh r.$$

  Found a new phase shift in the amplitude at $E \sim r_\mathrm{s}/\alpha'$, possibly related?

- Above we needed a secondary probe. Better idea: let's consider a situation where an outgoing particle is produced naturally by the black hole, i.e. Hawking quanta.
- Defining $\chi = \sqrt{1 - r_s/r}$,

$$ds^2 = -\chi^2\, dt^2 + r_s^2\, d\chi^2 + \text{sphere}.$$

- χ is the redshift factor so a typical Hawking quantum at radius r_0 has local energy

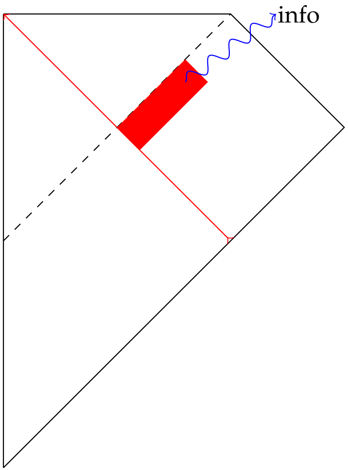
$$E \sim \frac{1}{\chi_0 r_s} \qquad \Rightarrow \qquad p^+(r = r_0) = \frac{1}{(\chi_0 r_s)^2} x^+(r = r_0)$$

  These can be mined so must be real.
- The condition for the Hawking quanta to detect the formation matter is

$$\alpha' p_2^+(r = r_0) \geq x_2^+(r = r_0) \qquad \Rightarrow \qquad \alpha' \geq \chi_0^2 r_s^2.$$

- So Hawking quanta emitted from the stretched horizon are sensitive to the shell that formed the BH.

- Could this be the solution to the information paradox[6]?
- Nice slice argument would fail because there's no Hamiltonian formulation of string theory on nice slices (need to go to light cone gauge).



[6]See also Susskind '92.

COSMOLOGICAL HORIZONS

- Need to check consistency of this effect with cosmological data. Take dS space in global coordinates,

$$ds^2 = \frac{L^2}{\cos^2\gamma}(-d\gamma^2 + d\theta^2 + \sin^2\theta\, d\Omega_2^2), \quad -\pi/2 < \gamma < \pi/2,$$

  where $L = H^{-1}$. Strings 1 and 2 are at θ_1 and θ_2.
- Transform to Kruskal-like coordinates in the static patch of the $\theta = 0$ observer,

$$ds^2 = \frac{1}{(L^2 - x^+x^-)^2}\left(-dx^+\, dx^- + (L^2 + x^+x^-)^2\, d\Omega_2^2\right).$$

  At the observer horizon $x^- = 0$ or $r = L$, we get

$$p^+(r = L) = \frac{m}{L}x^+(r = L) \quad \text{(Compare } p^+(r = r_\mathrm{s}) = \frac{m}{r_\mathrm{s}}x^+(r = r_\mathrm{s})\text{)}$$

- Using secondary probes as above, the analog of $m_2 > r_\mathrm{s}/\alpha'$ is

$$m_2 > \frac{1}{\alpha' H} \sim 10^{20} g_\mathrm{eff}^2 M_\mathrm{sun}.$$

  For $g_\mathrm{eff} \sim 1/10$ this is a bit bigger than a supercluster mass. Extra dimensions would lower g_eff. Cosmic strings could be the detector but haven't been found yet.

LONGITUDINAL SPREADING IN QCD?

- So far we've focused on fundamental string theory. Strings in AdS are dual to gauge theories, so we should ask whether gauge theories exhibit longitudinal spreading as well.
- This is not in direct conflict with locality. Local operators commute at spacelike separation but hadrons might have some intrinsic size. After all we already know they have transverse size $\log s$.
- The strategy is to set up the same six point amplitude from before, but now on the boundary of AdS. We take all kinematic invariants to not scale with L_{AdS}. Then the amplitude is[7]

$$\int_0^\infty dz \prod_i \psi_i(z) A_{6,\text{flat space}}\left(\alpha' k_i \cdot k_j z^2 / L_{AdS}^2\right).$$

- The wavefunctions go like z^Δ near the boundary. Folding in the flat space amplitude, the integrand is

$$z^{4\Delta}\left|\frac{K_{A1}}{K_{A3}}\right|^{-z^2 K_{A3}/L_{AdS}^2} \times \left|\frac{K_{12}}{K_{C2}}\right|^{-z^2 K_{C2}/L_{AdS}^2} \times \cos\left(\frac{\Delta X^+ L_{\text{AdS}}^2}{2\alpha' E_B z^2}\right)^{\alpha' k_\perp^2 z^2 / L_{\text{AdS}}^2}.$$

[7] See Polchinski-Strassler 2001

- At large Δ and small ΔX^+, this integral is dominated by a saddle point near the boundary, so that the power-law form of the wavefunctions is justified,

$$z_*^2 = \frac{\Delta^2 L_{\mathrm{AdS}}^2}{\alpha' \sum_i t_i \log s_i}.$$

- The leading time-dependence of the amplitude is then

$$1 - \frac{p_\perp^2 L_{\mathrm{AdS}}^2}{\alpha' E_B^2 z_*^2} (\Delta X^+)^2 + \ldots,$$

which is constant for

$$\Delta X^+ < \frac{\alpha' E_B \Delta}{p_\perp \sum_i t_i \log s_i}.$$

- Note the enhancement by Δ; more wee partons for higher operator dimensions? In hard-wall QCD the masses are given by the zeroes of the bessel function $J_\Delta(M/\Lambda)$. At large Δ the first zero is at $M \sim \Delta\Lambda$. Since length is mass over tension, the length should scale linearly with Δ.

Conclusion

- Off-shell calculations of longitudinal spreading in light-cone gauge imply that a detector with mass r_s/α' is not in its vacuum near the horizon of a black hole if it falls in a time $\Delta t \gg r_s$ after a string. In addition, Hawking quanta emitted from the stretched horizon are sensitive to the formation matter.
- Longitudinal spreading seems to be confirmed in on-shell scattering calculations; in particular, a situation mimicking the black hole can be set up in flat space. The amplitude exhibits long-range nonlocalities consistent with the longitudinal spreading scale.
- This is a testable effect, both in AdS/CFT and with real experiments (in principle), since it involves physics outside the horizon.