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FOR FUNDAMENTAL PHYSICS

Compensating strong coupling with large charge
Part I

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based on arXiv:1505.01537, 1610.04495, 1707.00710, 1809.06371
and work in progress with:
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O. Loukas, D. Orlando (INFN Torino), M. Watanabe (IPMU)

TODAY: Introductory level

- Basic idea of the large-charge expansion, simplest example and generalization to other examples with the same type of behavior

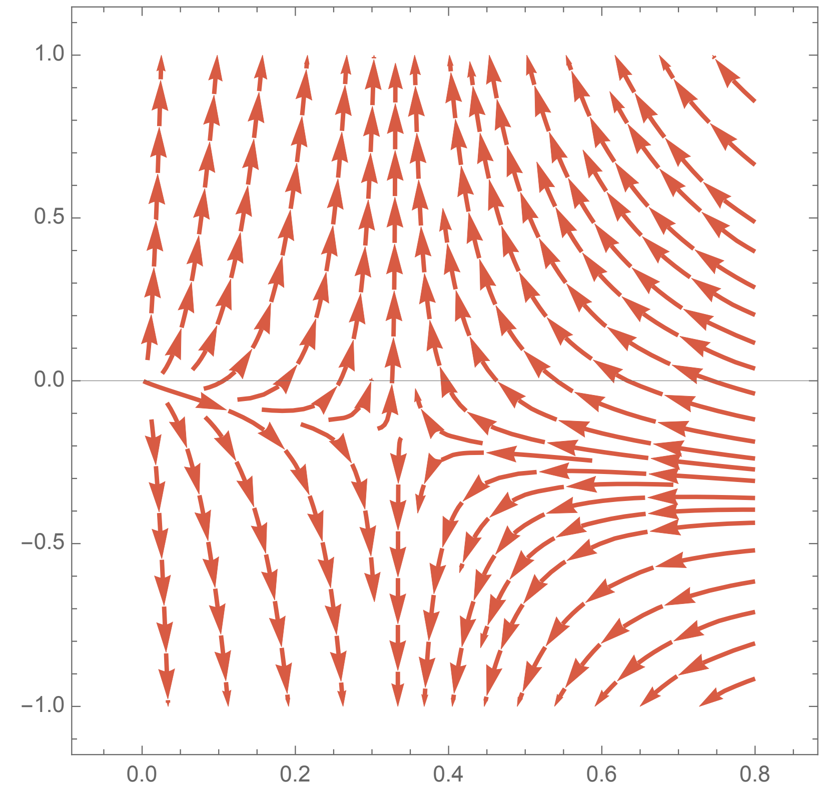
Domenico’s talk: new exciting (unpublished) results

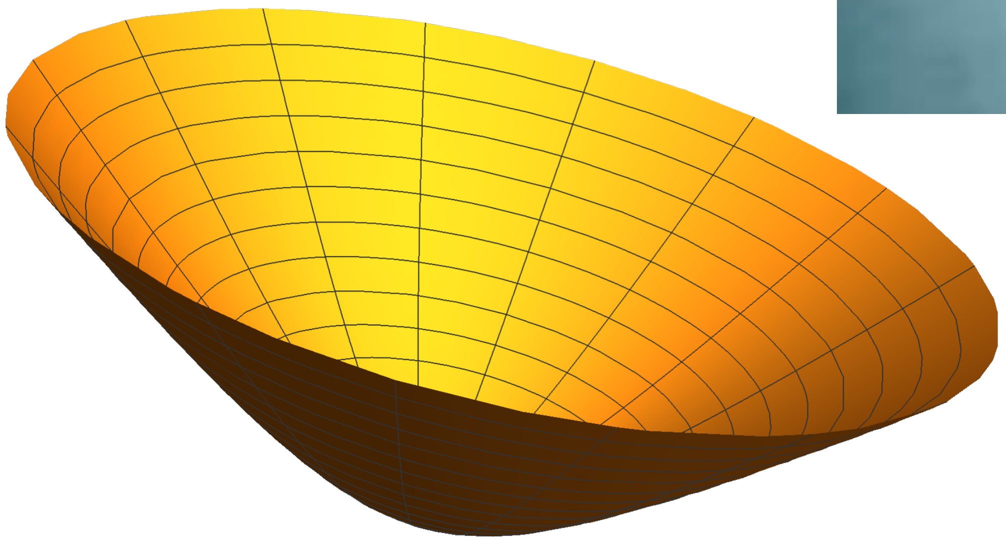
- qualitatively different behavior of large-charge expansion

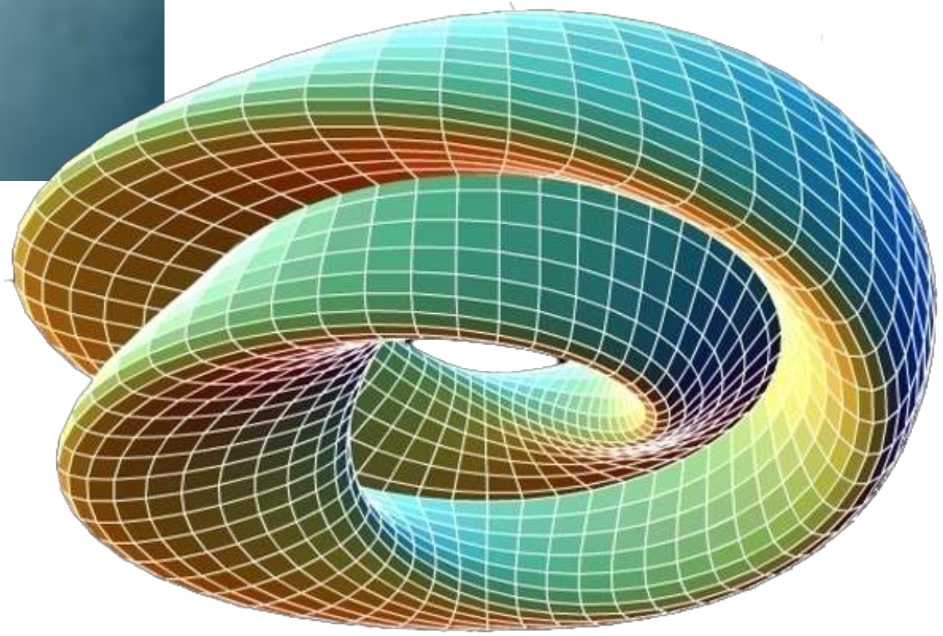
Introduction

Conformal field theories (CFTs) play an important role in theoretical physics:

- fixed points in RG flows
- critical phenomena
- quantum gravity
- string theory







Introduction

BUT: most CFTs do not have small parameters in which to do a perturbative expansion: couplings are O(1).

Difficult to access.
Possibilities: analytic (2d), conformal bootstrap (d>2), lattice calculations, non-perturbative methods…

Make use of symmetries, look at special subsectors where things simplify.

Here: study theories with a global symmetry group. Hilbert space of the theory can be decomposed into sectors of fixed charge Q under the action of the global symmetry group.
Study subsectors with large charge Q.

Large charge Q becomes controlling parameter in a perturbative expansion!

Introduction

The large-charge approach consists of 2 steps:

1. identify the possible fixed-charge symmetry breaking patterns for a given order parameter
2. write an effective action for the low-energy DOF and compute physical quantities

Step 1: start from the global symmetries of the system and how they act on the order parameter.

For example, in the superfluid transition of 4He, it is known that the system has an O(2) symmetry. Assume that, just like in the UV, the order parameter is a complex scalar that transforms the same way under O(2).

Introduction

Write down Wilsonian effective action. In general: infinitely many terms - not so useful.
Make self-consistent truncation at large charge:

- Set a cutoff Λ obeying

space dimension

typical scale of the system

$$\frac{1}{L} \ll \Lambda \ll \frac{1}{\ell_Q} = \frac{Q^{1/d}}{L}$$

- write a linear sigma model action for the order parameter. Want to describe a second-order phase transition: impose scale invariance of the action, assuming that the fields have vanishing anomalous dimension (at leading order in 1/Q)
- determine the fixed-charge ground state
- compute the quantum fluctuations to verify that they are parametrically small when Q >> 1.

Introduction

In a sector of fixed charge, the classical solution around which the quantum fluctuations are computed will generically break both spacetime (Lorentz) and global symmetries.

Step 2: write down EFT.
Similar techniques to chiral perturbation theory.
Important difference: the symmetry breaking comes from fixing the charge (NOT dynamical).

Use EFT to calculate the CFT data (anomalous dimensions, 3-pt functions).

Wilsonian action has only a handful of terms that are not suppressed by the large charge. Useful!

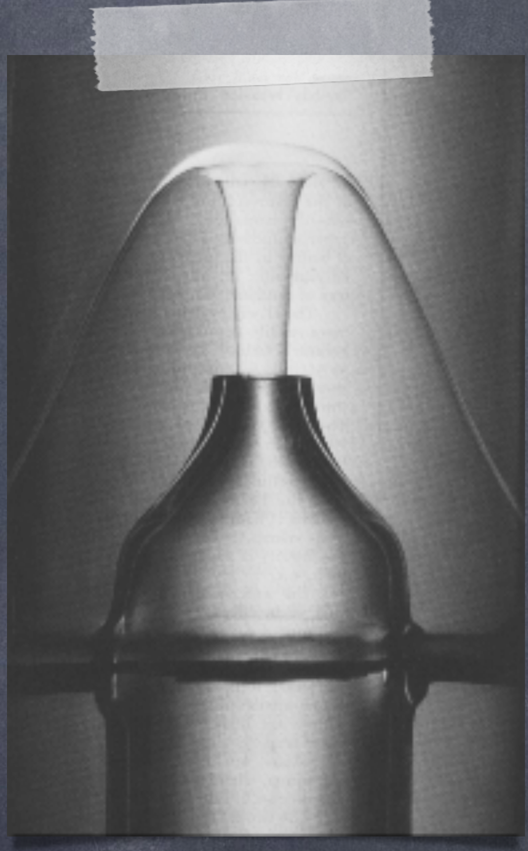
Introduction

Open questions:

- Does it work?
- For what kinds of theories does it work?
- In how many space-time dimensions?
- For what kinds of global symmetries does it work?
- What happens if we fix several charges independently?
- What can we learn via this approach?

Overview

- Introduction
- The O(2) model
  - semi-classical treatment
  - quantum treatment
  - results and lattice comparison
- The O(2N) vector model
  - counting Goldstone DoF
  - results and lattice comparison
- Beyond the vector models
- Nonrelativistic CFTs
- SCFTs at large R-charge
- Summary/Outlook



The O(2) model

The O(2) model

Consider simple model: O(2) model in (2+1)d.

$$\mathcal{L}_{\rm UV} = \partial_\mu \phi^* \, \partial^\mu \phi - g^2 (\phi^* \phi)^2$$

Flows to Wilson-Fisher fixed point in IR.

Assume that also the IR DOF are encoded by cplx scalar

Global U(1) symmetry: $\varphi_{IR} = a \, e^{ib\chi} \qquad \chi \to \chi + \text{const.}$

Look at scales: put system in box of scale R
Second scale given by U(1) charge Q: $\rho^{1/2} \sim Q^{1/2}/R$

Study the CFT at the fixed point in a sector with

$$\frac{1}{R} \ll \Lambda \ll \frac{Q^{1/2}}{R} \ll g^2$$

UV scale

cut-off of effective theory

Write Wilsonian action.

The O(2) model

Assume large vev for a: $\Lambda \ll a^2 \ll g^2$

scalar curvature infinitely many

$$\mathcal{L}_{\mathrm{IR}} = \tfrac{1}{2}\,\partial_\mu a\,\partial^\mu a + \tfrac{1}{2} b^2 a^2\,\partial_\mu \chi\,\partial^\mu \chi - \frac{R}{16} a^2 - \frac{\lambda}{6} a^6 + \text{higher derivative terms}$$

dimensionless constants suppressed by large Q

Lagrangian is approximately scale-invariant.

φ has approximately mass dimension 1/2 and the action has a potential term $\propto |\varphi|^6$

Do semi-classical analysis: solve classical e.o.m. at fixed Noether charge.

$$\rho = \frac{\delta \mathcal{L}_{\mathrm{IR}}}{\delta \dot{\chi}} = b^2 a^2 \dot{\chi} \qquad Q \sim 4\pi R^2 b \sqrt{\lambda} a^4$$

Classical solution at lowest energy and fixed global charge becomes the vacuum of the quantum theory.

The O(2) model

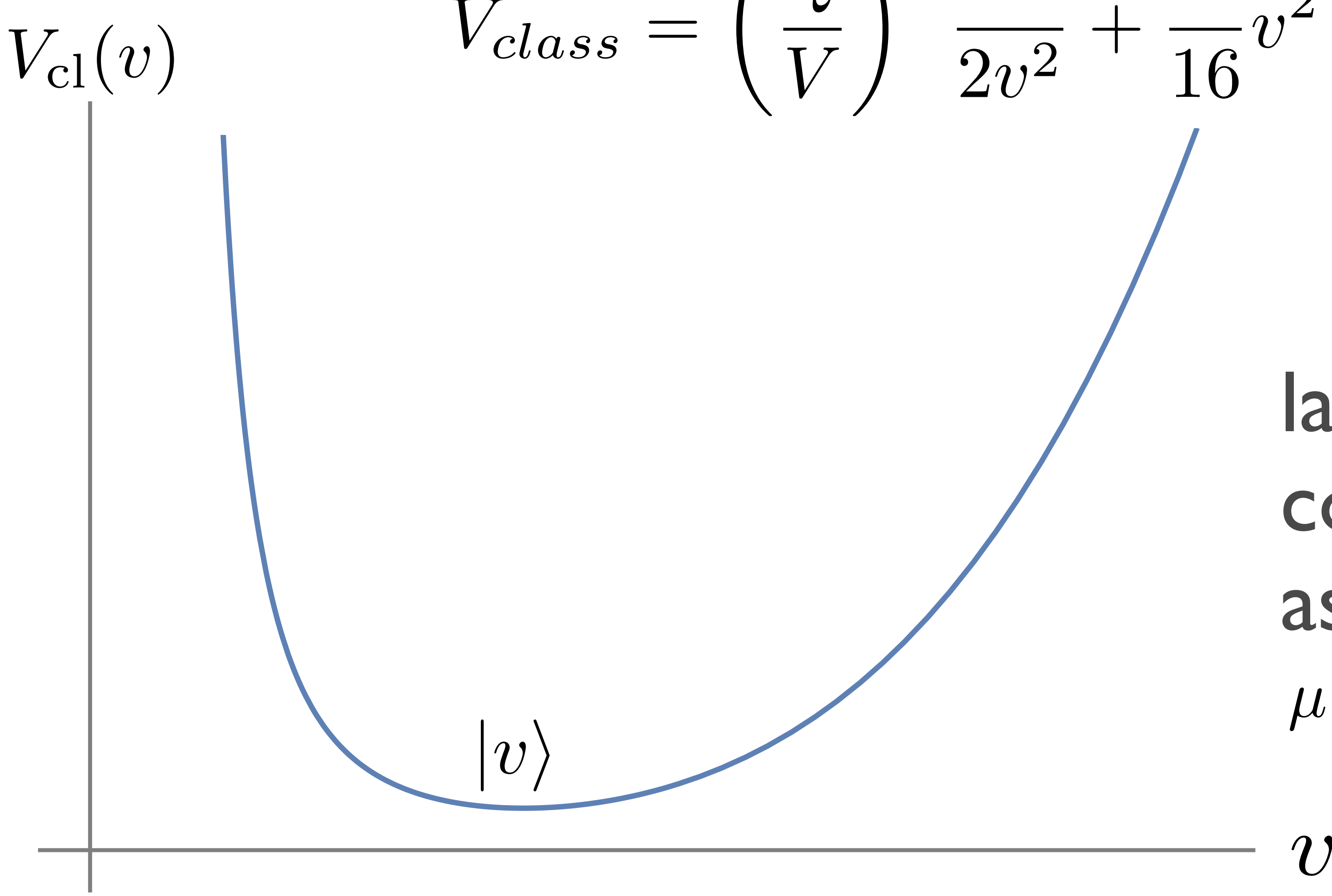
$\Rightarrow$ non-trivial condensate

non-const. vev

$$\langle a \rangle = v, \qquad \langle \dot{\chi} \rangle = \mu = \frac{Q}{V \cdot v^2}, \qquad \langle \chi \rangle = \mu\, t$$

Fixed charge ground state is homogeneous in space.
Determine radial vev by minimizing the classical potential:

$$V_{class} = \left(\frac{Q}{V}\right)^2 \frac{1}{2v^2} + \frac{R}{16} v^2 + \frac{\lambda}{6} v^6$$



$$v \sim Q^{1/4}$$

large condensate is compatible with our assumption $a \gg 1$

$$\mu \sim \rho^{1/2}$$

The O(2) model

Ground state at fixed charge breaks symmetries:

$$SO(1,3)_{\text{spacetime}} \times O(2)_{\text{global}} \to SO(3)_{\text{space}} \times D$$

linear comb. of time translation and the global O(2)

Quantum story: study the low-energy spectrum
Parametrize fluctuations on top of the classical vacuum

$$a = v + \hat{a} \qquad \chi = \mu\, t + \frac{\hat{\chi}}{v}$$

massive mode, not relevant for low-energy spectrum $m \sim \mathcal{O}(\sqrt{Q})$

Go to NLSM: Integrate out a (saddle point for LO).
Dynamics is described by a single Goldstone field χ:

$$\mathcal{L}_{LO} = k_{3/2}(\partial_\mu \chi\, \partial^\mu \chi)^{3/2}$$

can get this purely by dimensional analysis

The O(2) model

Use dimensional analysis and scale invariance to determine (tree-level) operators in effective action beyond LO (scalar operators of scaling dimension 3, including curvatures of the background metric)

Use ρ-scaling to determine which terms appear:

$$\partial\chi \sim \rho^{1/2}, \quad \partial\dots\partial\chi \sim \rho^{-1/4}$$

$\mathcal{O}(\rho^{3/2})$: $\quad \mathcal{O}_{3/2} = |\,\partial\chi|^3$ LO Lagrangian

$\mathcal{O}(\rho^{1/2})$: $\quad \mathcal{O}_{1/2} = R|\,\partial\chi| + 2\frac{(\partial|\,\partial\chi|)^2}{|\,\partial\chi|}$

conf. inv. combination, negative ρ-scaling

scale-inv. but NOT conformally inv.

For homogeneous solutions, there are no other terms contributing to the effective Lagrangian at non-negative ρ-scaling for d>1.

The O(2) model

Result:

$$\mathcal{L} = k_{3/2}(\partial_\mu\chi\partial^\mu\chi)^{3/2} + k_{1/2}R(\partial_\mu\chi\partial^\mu\chi)^{1/2} + \mathcal{O}(Q^{-1/2})$$

dimensionless parameters

suppressed by
inverse powers of Q

To be understood as an expansion around the classical ground state $\mu t + \hat{\chi}$

Expand action to second order in fields:

$$\mathcal{L} = k_{3/2}\mu^3 + k_{1/2}R\mu + (\partial_t\hat{\chi})^2 - \frac{1}{2}(\nabla_{S^2}\hat{\chi})^2 + \ldots$$

Compute zeros of inverse propagator and get dispersion relation.

$$\omega_{\vec{p}} = \frac{|\vec{p}|}{\sqrt{2}}$$

Spontaneous breaking of time-translation invariance
⇒ χ is relativistic Goldstone (type I)
⇒ superfluid phase of O(2) model

The O(2) model

Are also the quantum effects controlled?

All effects except Casimir energy are subleading (negative ρ-scaling)

Effective theory at large Q:

vacuum + Goldstone + 1/Q-suppressed corrections

Energy of classical ground state at fixed charge:

2 dimensionless parameters (b, λ)

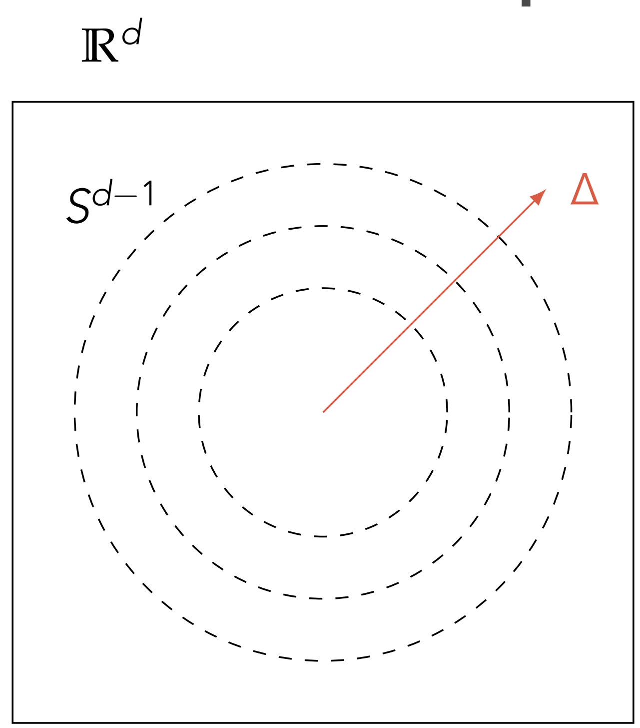
$$E_\Sigma(Q) = \frac{c_{3/2}}{\sqrt{V}} Q^{3/2} + \frac{c_{1/2}}{2} R\sqrt{V} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

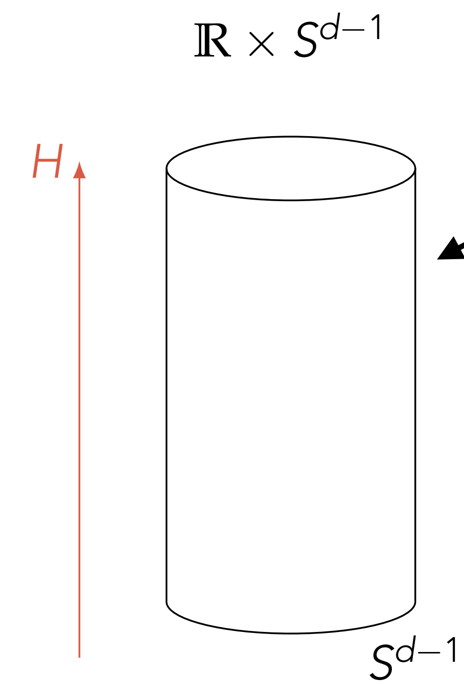
dependence on manifold

The O(2) model

Use state-operator correspondence of CFT:

anomalous dimension





energy

Anomalous dimension of lowest operator of charge Q:

one-loop vacuum energy of Goldstone

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

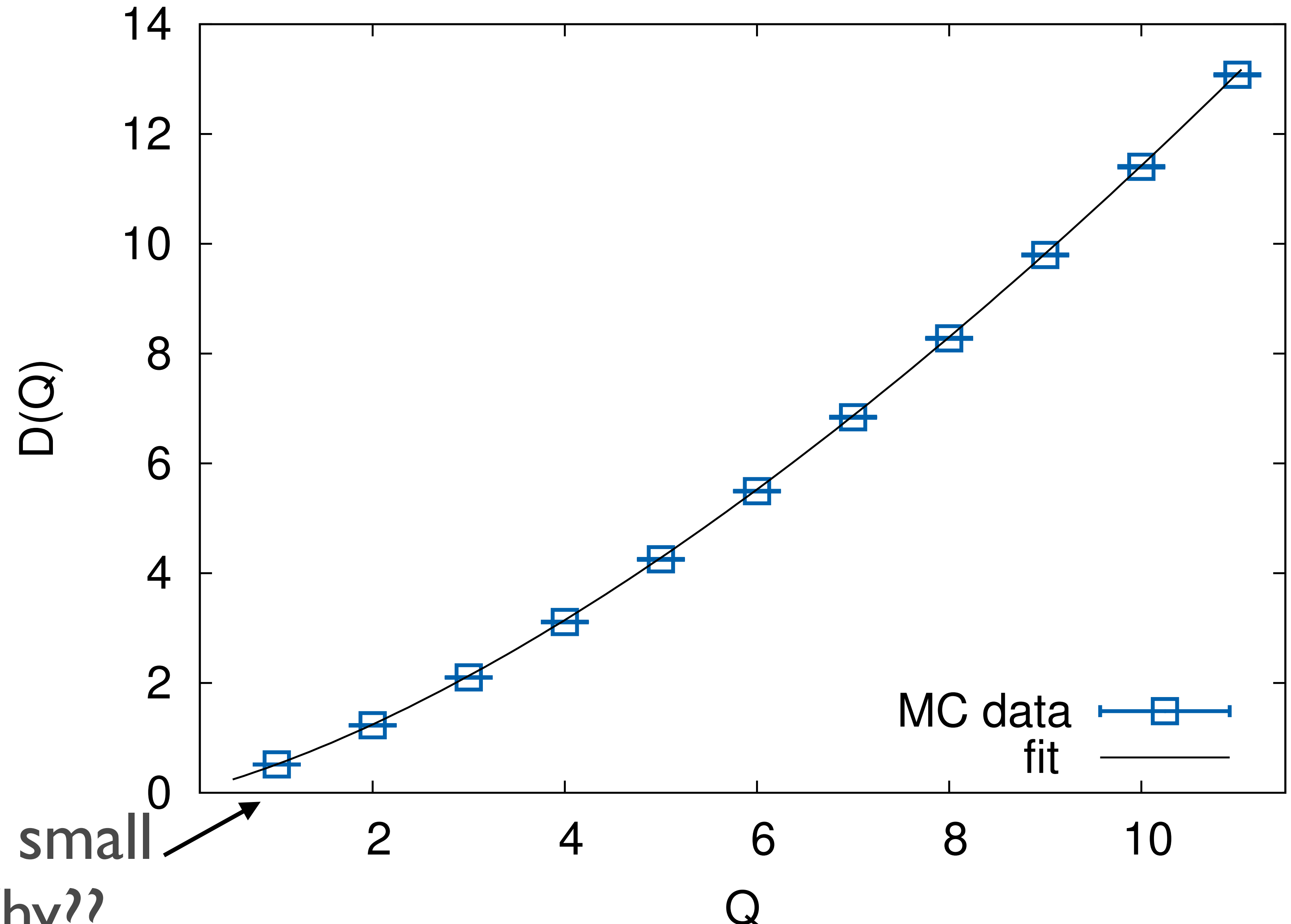
S. Hellerman, D. Orlando, S. R., M. Watanabe, arXiv:1505.01537 [hep-th]

$$E_{\text{VAC}} = \frac{1}{2\sqrt{2}r} \int \frac{\mathrm{d}\omega}{2\pi} \sum_{l=0}^{\infty} (2l+1) \log(\omega^2 + l(l+1)) = \frac{1}{2\sqrt{2}r} \zeta(-1/2|S^2) = -\frac{0.0937\ldots}{r}$$

The O(2) model

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Independent confirmation from the lattice:



works for small charge. Why??

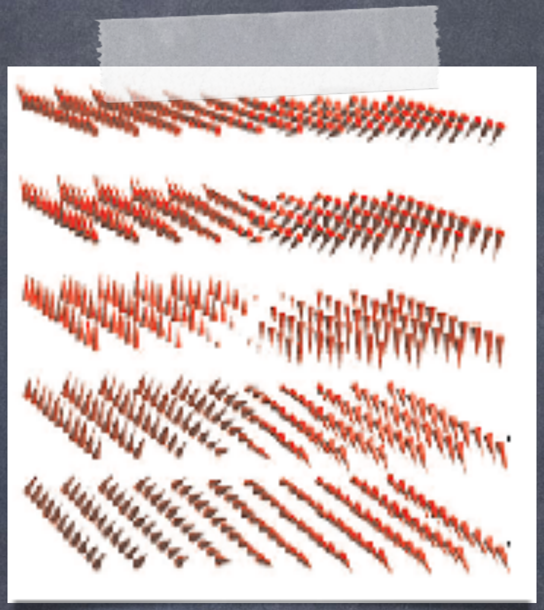
Excellent agreement!!

$c_{3/2} = 1.195(10)$

$c_{1/2} = 0.075(10)$

D. Banerjee, Sh. Chandrasekharan, D. Orlando [hep-th/1707.00711

Large-charge expansion works extremely well for O(2). Where else?



The O(2n) vector model

The O(2n) vector model

Generalize to O(2n).

$$\mathcal{L} = \tfrac{1}{2}\,\partial_\mu \phi^a\,\partial^\mu \phi^a - \tfrac{1}{2}\sum_{i=1}^{2n}\left(\frac{1}{8}R\phi_a^2 + \frac{\lambda}{12}\,\phi_a^6\right), \qquad a = 1, \dots, 2n \qquad \mathbb{R}_t \times \mathbb{R}^2$$

$U(n) \subset O(2n)$

$$\varphi_1 = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)\,, \qquad \varphi_2 = \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4)\,, \qquad \dots,$$

Fix $k \leq N$ U(1) charges:

$$\int \mathrm{d}^{d-1}x\, i\,(\dot{\varphi}_i\varphi_i^* - \dot{\varphi}_i^*\varphi_i) = \overline{Q}_i = \text{vol.} \times \bar{\rho}_i$$

Solution for homogeneous ground state:

$$\begin{cases} \varphi_i = \frac{1}{\sqrt{2}}A_i\, e^{i\mu t}, & i = 1, \dots, k\,, \\ \varphi_{k+j} = 0, & j = 1, \dots, n-k\,, \end{cases}$$

same for all fields!

$$A_i^2 = \frac{Q_i}{4\pi\mu}, \qquad \mu = \frac{1}{4}\sqrt{R + \sqrt{R^2 + \frac{2}{\pi^2}\lambda(\sum_i Q_i)^2}}$$

The O(2n) vector model

Fixing k charges explicitly breaks O(2n) to O(2n-2k) x U(k).

We can always rotate $\langle \vec{\varphi} \rangle = \frac{1}{\sqrt{2}}(A_1, \ldots, A_k, 0, \ldots)$ by a U(k) transformation into $(0, \ldots, 0, \sqrt{\frac{A_1^2 + \cdots + A_k^2}{2}}, 0, \ldots)$

Vacuum breaks symmetry spontaneously to O(2n-2k) x U(k-1).

We also see that all homogeneous states of minimal energy with fixed total charge $(Q_1 + Q_2 + \cdots + Q_k)$ are related by an U(k) transformation and have the same energies (and conformal dimensions).

What happens if instead, we choose a configuration with k different chemical potentials that cannot be rotated into the state $\big(\underbrace{0, \ldots, 0}_{k-1}, \frac{v}{\sqrt{2}}, \underbrace{0, \ldots, 0}_{n-k}\big)$?

Ground state must be inhomogeneous! Domenico's talk!

The O(2n) vector model

For quantum description, write effective theory for fluctuations around the ground state.

Expand Lagrangian around the ground state

$$\Big(\underbrace{0\,,\,\dots\,,0}_{k-1},\ \tfrac{v}{\sqrt{2}}\,,\underbrace{0\,,\,\dots\,,0}_{n-k}\Big)$$

U(1) sector: $\varphi_k = \frac{1}{\sqrt{2}}\, e^{i\mu t + i\hat{\phi}_{2k}/v}\left(v + \hat{\phi}_{2k-1}\right)$

$$\begin{cases} \hat{\phi}_{2k-1} \to \hat{\phi}_{2k-1} \\ \hat{\phi}_{2k} \to \hat{\phi}_{2k} + \theta\,, \end{cases}$$

U(k-1) sector: $\varphi_i = e^{i\mu t}\hat{\varphi}_i$

$$\hat{\varphi}_i \mapsto \tilde{U}_i{}^j\,\hat{\varphi}_j$$

Developing to second order in fields:

$$\mathcal{L}^{(2)} = \sum_{i=1}^{k}(\partial_t - i\mu)\varphi_i^*(\partial_t + i\mu)\varphi_i + \sum_{i=k+1}^{n}\dot{\varphi}_i^*\dot{\varphi}_i - \sum_{i=1}^{n}\nabla\varphi_i^*\nabla\varphi_i - \sum_{i=1}^{n}\mu^2\varphi_i^*\varphi_i - 2\mu^2\phi_{2k-1}^2$$

Find inverse propagators and dispersion relations.

The O(2n) vector model

We expect dim[U(k)/U(k-1)] = 2k-1 Goldstone d.o.f.

Massless modes:

$$\omega_{nr}^2 = \frac{p^4}{4\mu^2} - \frac{p^6}{8\mu^4} + \mathcal{O}(\mu^{-6}) \qquad k-1 \text{ times}$$

$$\omega_r^2 = \frac{1}{2}p^2 + \frac{p^4}{32\mu^2} + \mathcal{O}(\mu^{-4}) \qquad \text{one time}$$

There are

- 1 relativistic Goldstone $\omega \propto p$
- k-1 non-relativistic Goldstones (count double) $\omega \propto p^2$

Nielsen and Chadha; Murayama and Watanabe

$$1 + 2 \times (k-1) = 2k - 1 = \dim(G/H)$$

Non-relativistic Goldstones are suppressed by large Q. Low-energy physics again governed by single relativistic Goldstone.

The O(2n) vector model

Same formula for anomalous dimensions as for O(2):

n-dependent

universal for O(2n)

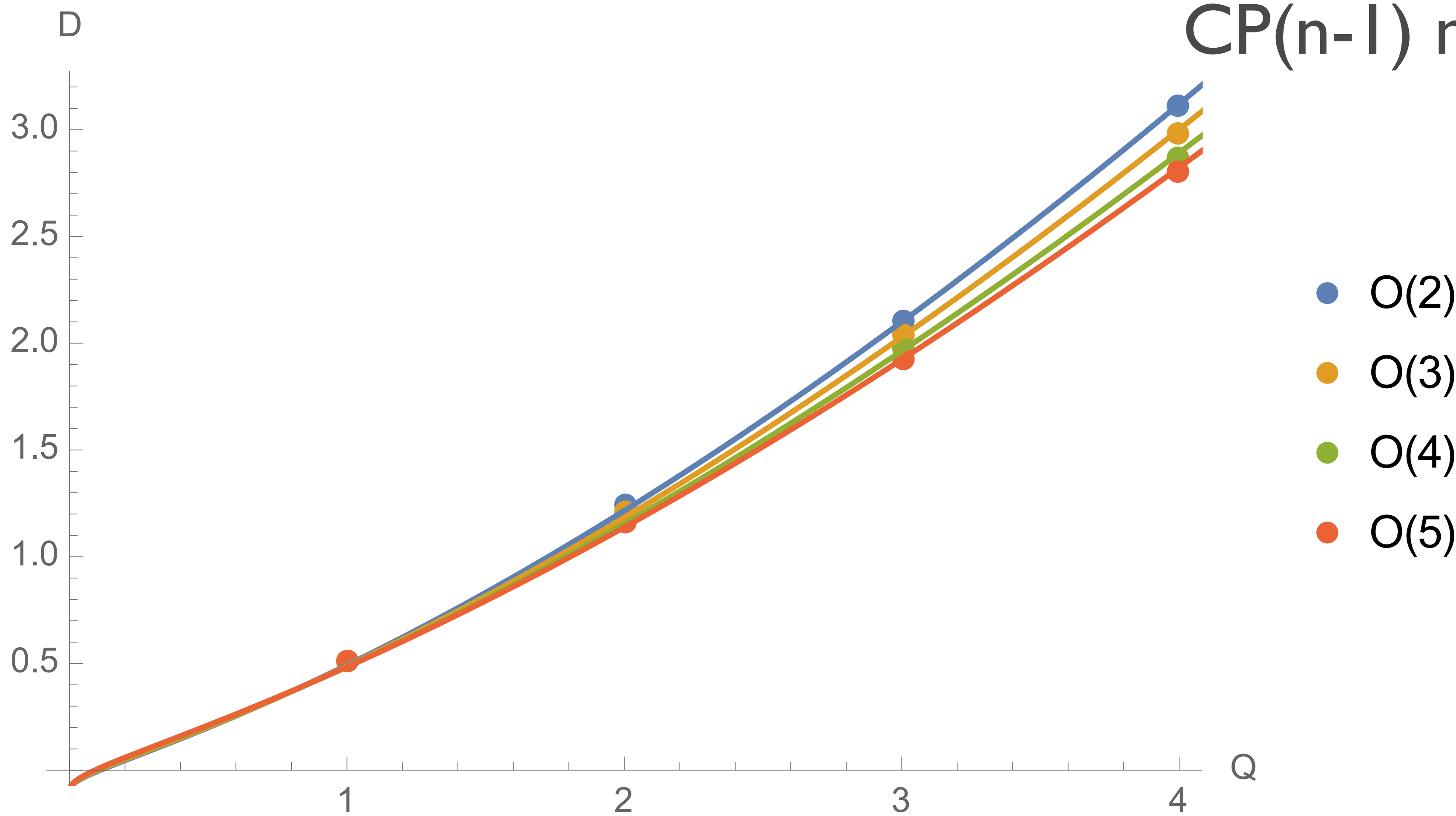
$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

L. Alvarez-Gaume, O. Loukas, D. Orlando and S. R., arXiv:1610.04495 [hep-th]

verified at large n for CP(n-1) model

de la Fuente

Confirmation with old lattice data:

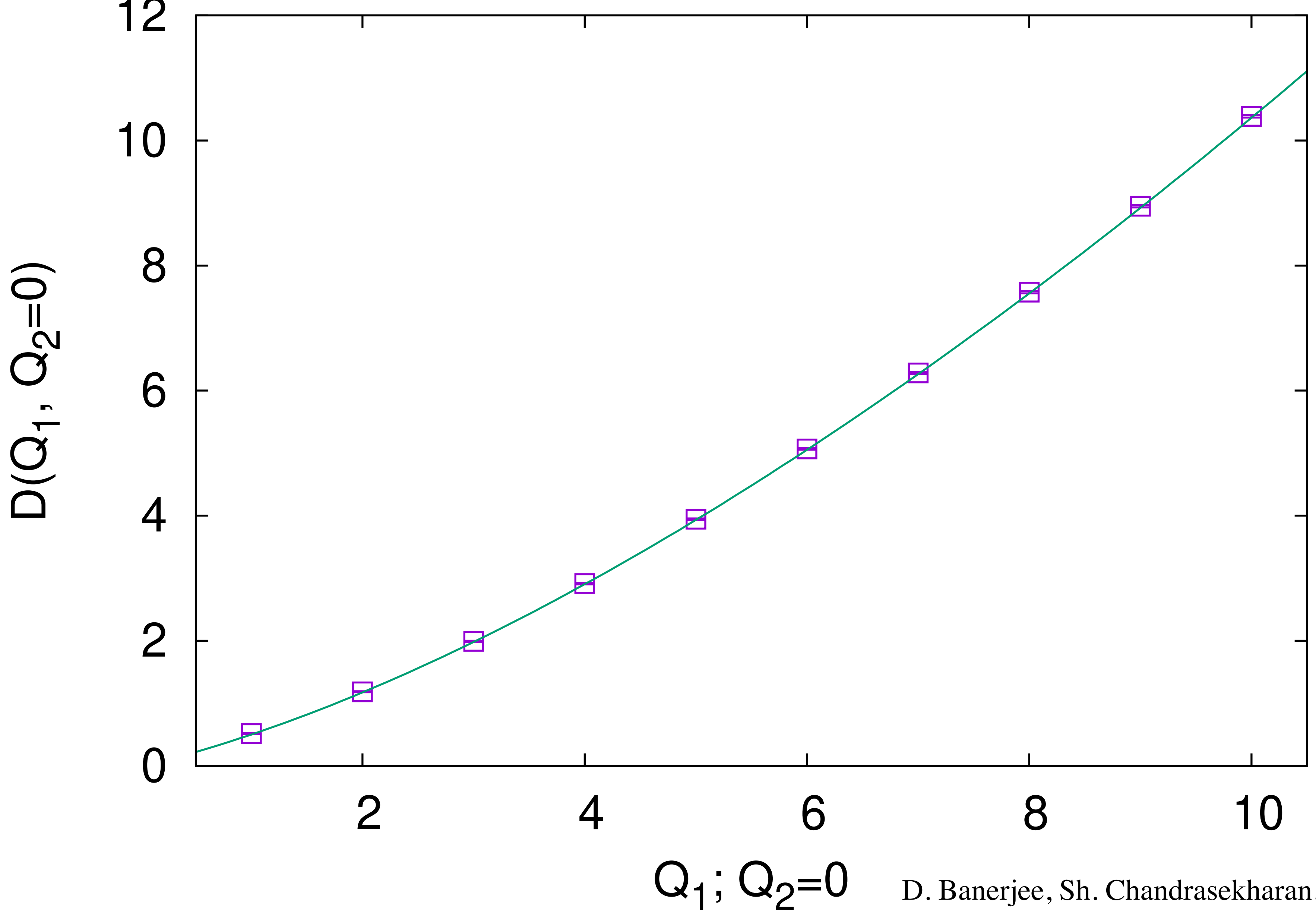


Hasenbusch, Vicari

Coefficients c become smaller for larger n.

The O(2n) vector model

New (preliminary) lattice results for O(4) model (homogeneous GS):



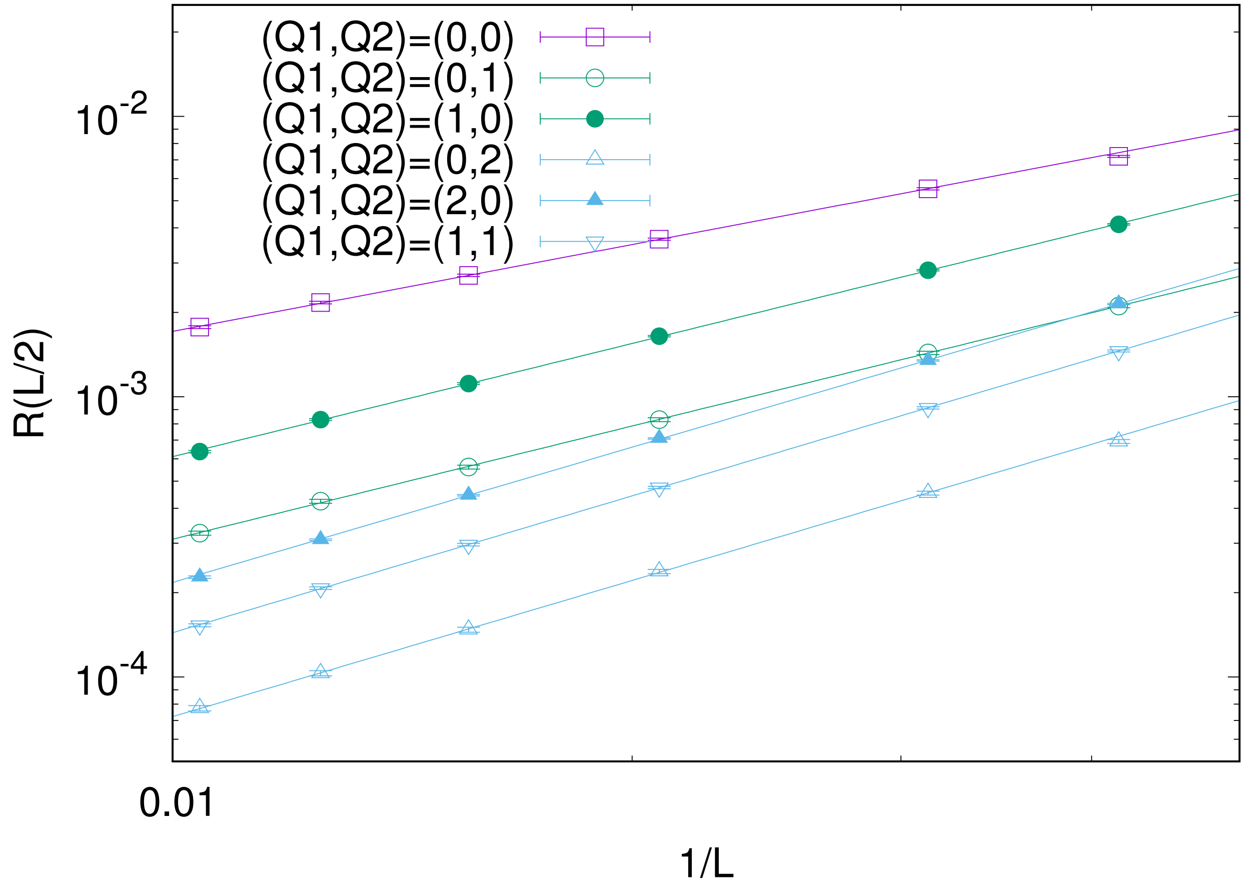
$c_{3/2} = 1.068(4)$
$c_{1/2} = 0.083(3)$

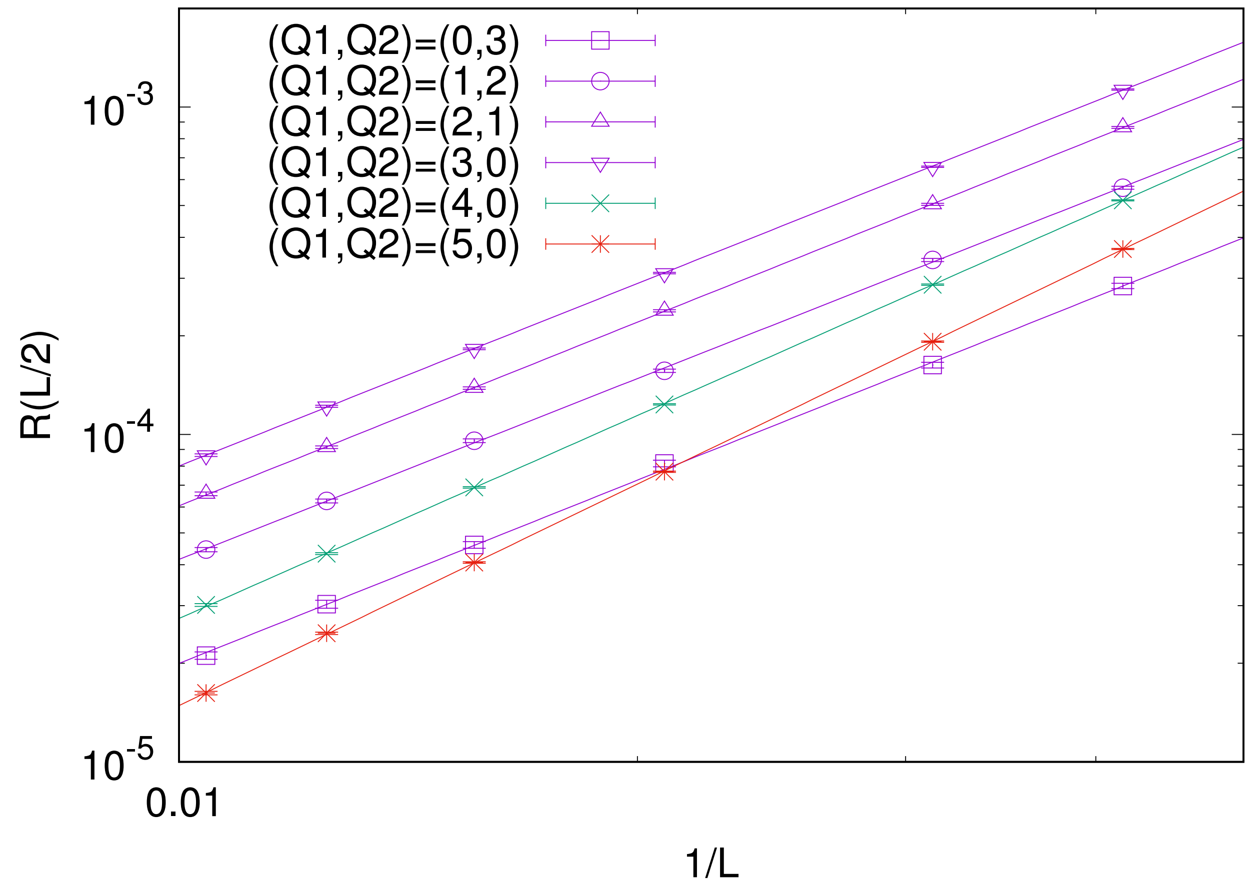
D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. to appear

Again very good agreement with large-Q prediction!

The O(2n) vector model

Only total charge matters for homogeneous case:



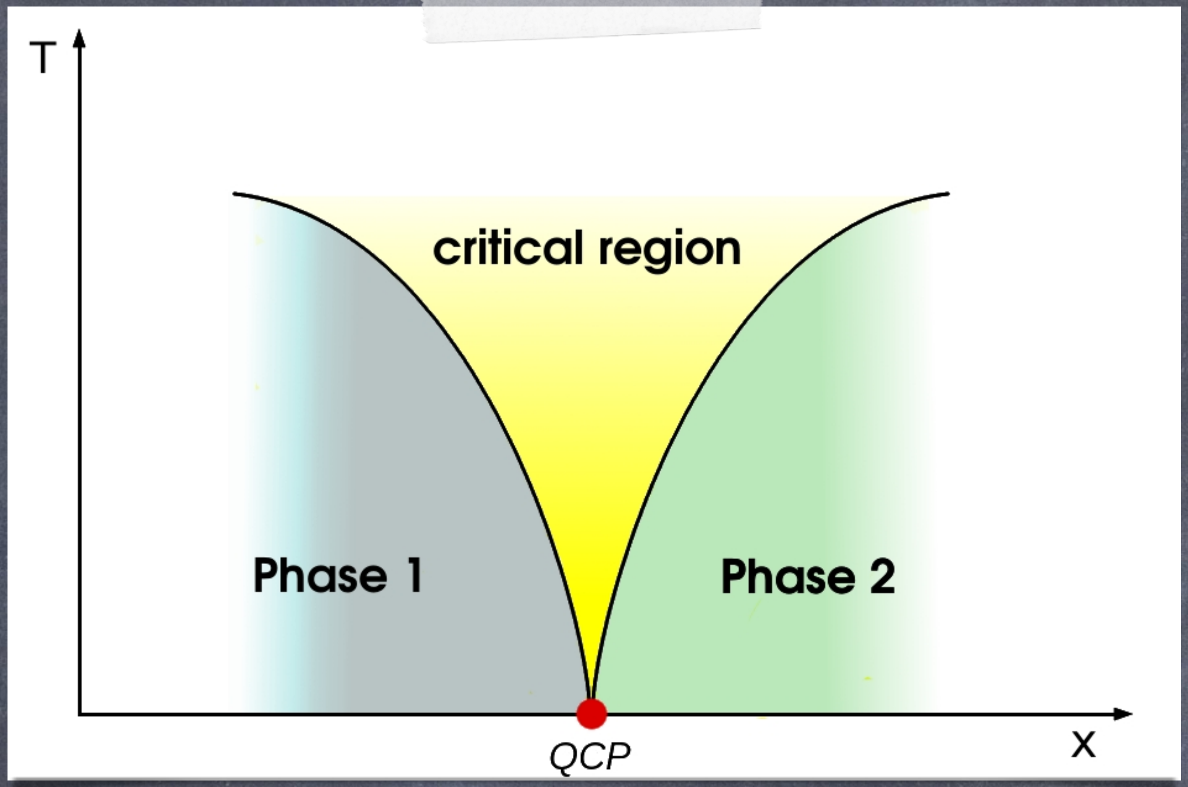


Correlation function:

D. Banerjee, Sh. Chandrasekharan, D. Orlando, S.R. to appear

$$C_Q(r) \sim \frac{a(Q)}{|\vec{r}|^{2D(Q)}} \qquad R(L/2) = \frac{C_Q(r = L/2)}{C_{Q-1}(r = L/2)} \qquad R(L) \sim 1/L^{2(D(Q)-D(Q-1))}$$

Parallel lines in log/log plot: anomalous dimensions are the same!



Beyond the vector models

Matrix models

Want to go beyond vector models.
Study models with matrix-valued order parameter, global SU(N) symmetry.

SU(3) matrix model in 3d: can fix only one U(1)-charge if you want a homogeneous ground state.

Low-energy physics is again governed by a single relativistic Goldstone boson.

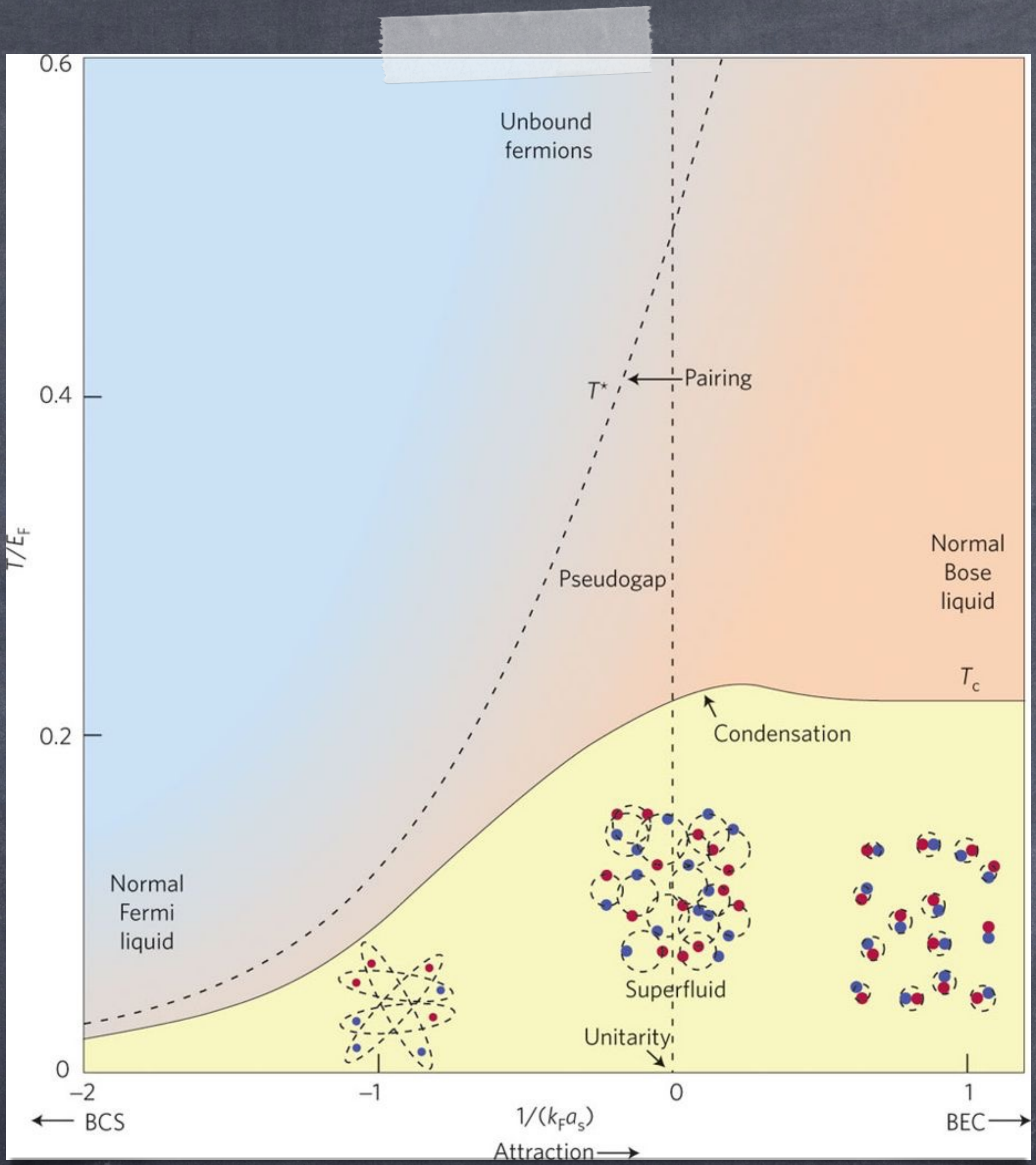
Anomalous dimension has the same form as for the vector model.

Calculated the 3-point functions as well.

O. Loukas, D. Orlando and S. R., [arXiv:1707.00710 [hep-th]]

SU(4) matrix model: new effects appear. Can fix more than one U(1) charge independently. Can distinguish more than one IR fixed point at large charge.

O. Loukas, [arXiv:1711.07990 [hep-th]]

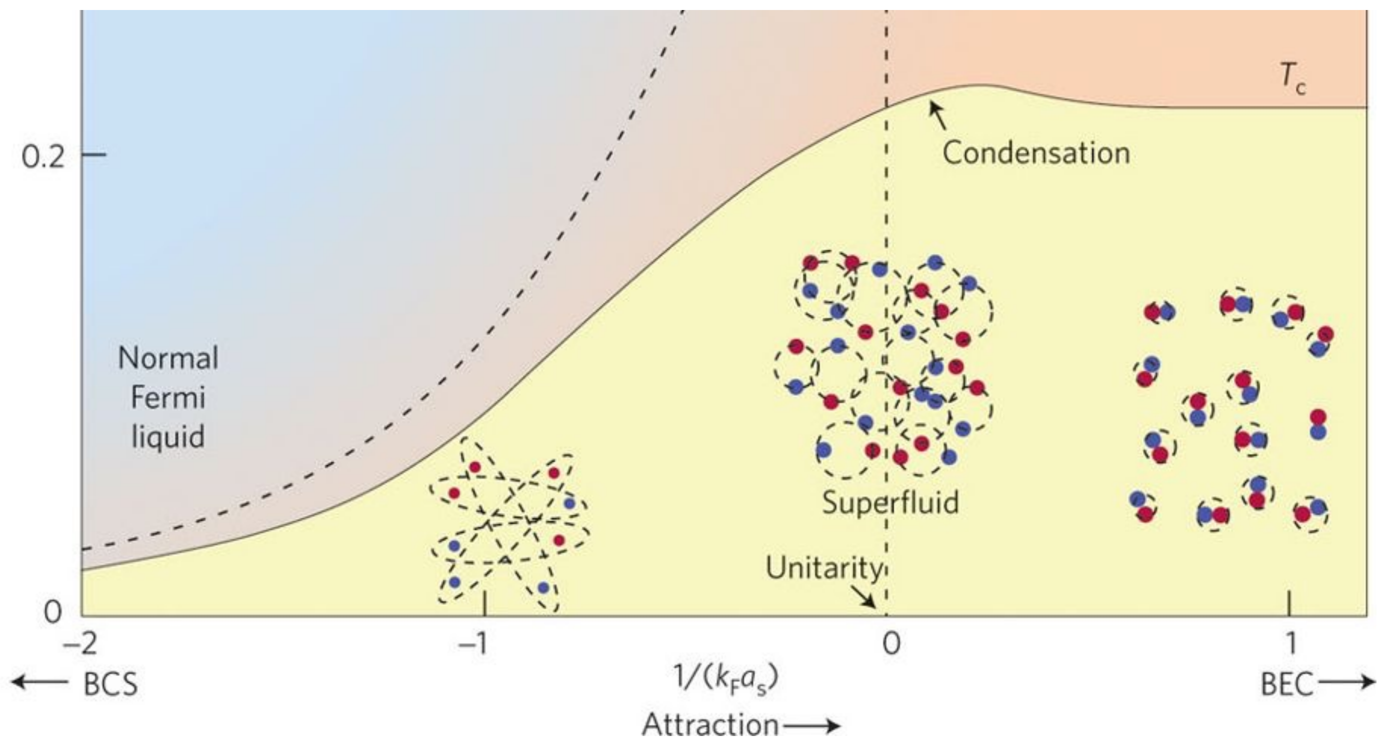


Non-relativistic CFTs

Nonrelativistic CFTs

Motivation: unitary Fermi gas (3+1)D

Can be realized in the lab via cold atoms in a trap.
Tuning via Feshbach resonances: unitary point,
correlation length = ∞, interaction length = 0



At unitary point: described by a non-relativistic superfluid
Effective action (small momentum expansion)

Son & Wingate

Nonrelativistic CFTs

Non-relativistic systems are not invariant under the full conformal group.

Schrödinger algebra: contains the Galilean algebra with central extension plus scale and special conformal transformations:

real parameters

$$(t, x_i) \to (t', x_i') = (e^{2\tau} t, e^{\tau} x_i)$$

$$(t, x_i) \to (t', x_i') = \left(\frac{t}{1 + \lambda t}, \frac{x_i}{1 + \lambda t} \right)$$

The Schrödinger Lagrangian (in d space-dim) is invariant under Schrödinger symmetry:

$$\mathcal{L}(\psi) = \frac{i}{2}(\psi^* \partial_t \psi - \psi \partial_t \psi^*) - \frac{\hbar}{2m} \partial_i \psi^* \partial_i \psi - \frac{k}{m} \hbar^{\frac{d-2}{d}} (\psi^* \psi)^{\frac{d+2}{d}}$$

scale

Nonrelativistic CFTs

System has again a global U(1) symmetry.

Follow the same recipe as for O(2): $\psi = a\, e^{i\theta}$

Homogeneous ground state:

$$\theta = \mu t + \chi \qquad\qquad \mu = k \frac{d+2}{d} \frac{\hbar}{m} \rho^{2/d}$$

The leading piece of the effective action for θ can be found by dimensional analysis:

$$\mathcal{L}^{(0)} = c_0\ \hbar^{(2-d)/2} m^{d/2} U^{(d+2)/2}$$

$$U = \partial_t \theta - \frac{\hbar}{2m}\, \partial_i \theta\, \partial_i \theta$$

The first quantum correction to this (semi-classical) result is the Casimir energy, it goes as $Q^{1/d}$

Nonrelativistic CFTs

Check for higher-derivative terms at tree level in the effective action (here w/o curvature terms = flat space).

Use Schrödinger symmetry to constrain the terms that can appear in the action (d=3):

Generic operator allowed by dimensional analysis and compatible with scale and SC transformations:

$$\mathcal{O}_\beta \propto \hbar^{\beta-1/2} m^{3/2-\beta}\, \partial_i^{2\beta} U^{5/2-\beta}$$

Invoke μ-scaling to exclude highly suppressed terms:

$$U \sim \mu, \qquad \partial_i \theta \sim \mu^{-1/4}, \qquad \partial_i U \sim \mu^{-1/4}$$

Term with highest μ-scaling:

$$\mathcal{O}_\beta^{\max} \propto \hbar^{\beta-1/2} m^{3/2-\beta} U^{5/2-\beta}\, \partial_i^n \theta\, \partial_i^m \theta \sim \mu^{2-\beta}, \quad n+m=2\beta$$

For positive μ-scaling, β<3.

Nonrelativistic CFTs

Check terms explicitly.

Result for d=2 and 3:

$$
\begin{aligned}
\mathcal{L}(\theta) = {} & c_0 \hbar^{1-d/2} m^{d/2} U^{(d+2)/2} && \leftarrow \beta = 0 \\
& + c_1 \hbar^{2-d/2} m^{-1+d/2} U^{(d-4)/2}\, \partial_i U\, \partial_i U && \leftarrow \beta = 1 \\
& + c_2 \hbar^{3-d/2} m^{-2+d/2} U^{(d-2)/2} (\partial_i \partial_i \theta)^2 + \mathcal{O}(\mu^{-2}) && \leftarrow \beta = 2
\end{aligned}
$$

Check loop corrections to the effective action.

Both quantum corrections and tree-level higher derivative terms are suppressed by inverse powers of μ for d>1.

Nonrelativistic CFTs

Speed of sound (leading order): $c_s^2 = \frac{2}{d}\frac{\hbar\mu}{m}$ ← different from relativistic case!

NLO-correction to the dispersion relation:

$$\omega = c_s p \left(1 - d_0^2 \frac{\hbar}{m} \left(2c_1 + d c_2\right) \frac{p^2}{\mu} + \mathcal{O}(\mu^{-2})\right)$$

again linear in p!

from NLO tree-level terms

Quantum corrections enter at higher order.

Energy of ground state (on the torus):

different from relativistic case!

$$E_{T^d} = \frac{\hbar^2}{m}\left[V b_1^2 \rho^{(d+2)/d} + \underbrace{\frac{b_1}{V^{1/d} d}\sqrt{\frac{d+2}{2}}\rho^{1/d}\zeta_{T^d}(-2) + \frac{b_2}{V^{2/d}}}\right] + \mathcal{O}\left(\frac{1}{\rho^{2/d}}\right)$$

class. ground state energy

Casimir energy

All other classical and quantum corrections are suppressed by inverse powers of ρ.

Nonrelativistic CFTs

Large-Q expansion also works for non-relativistic CFTs.

Reproduce results of Son, Wingate (different approach)

Include curvature

Work in harmonic potential to use non-relativistic state-operator correspondence

Kravec, Pal

Make connection to experimental results.

SCFT in 3D

SCFTs at large R-charge

Study supersymmetric CFTs.

Global symmetry: R-symmetry

SCFTs in sector of large R-charge!

Simplest example: N=2 theory in 3D with a single chiral superfield Φ:

$$W = \frac{1}{3}\Phi^3 \qquad\qquad K \propto \Phi^\dagger \Phi$$

Known to flow to interacting superconformal fixed pt

Barnes; Jafferis

No marginal deformations or small parameters.

No moduli space!

Concentrate on theory at IR fixed point:

$$W \propto \Phi^3 \qquad \Rightarrow \qquad \Phi \propto [\mathrm{mass}]^{2/3} \qquad \Rightarrow \qquad K \propto |\Phi|^{3/2}$$

SCFTs at large R-charge

Expand in component fields and parametrize the complex scalar like for the O(2) model in terms of radial and angular component:

$$\mathcal{L}_{IR} = a_K |\phi|^{3/2} (\partial\chi)^2 + a_K \frac{(\partial|\phi|)^2}{|\phi|^{1/2}} + \frac{1}{b_K} |\phi|^{9/2}$$
$$+ \text{higher derivatives} + \text{fermions}$$

what about them?

For $|\phi| = \text{const.}$ this is minimized for

$$(\partial\chi)^2 \propto |\phi|^3 \propto \rho$$

Same form of action as for the O(2) model!

Fermions decouple from dynamics as they receive large masses via the Yukawa couplings and have rest energy

$$E_0 \sim |\, \partial_0 \chi| \sim \sqrt{\rho}$$

SCFTs at large R-charge

SUSY is spontaneously broken by fixing the charge!

Same dynamics as in O(2) model - same universality class.

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

Formula for anomalous dimension remains the same!

Surprising, as due to SUSY, we'd expect for BPS states

$$D(Q) = Q + \mathcal{O}(Q^0)$$

Via a partition function calculation, it was instead found that for BPS states,

Eager

$$\begin{aligned} D(Q) =& Q + S + \dots \\ =& Q + Q^2 + \dots \end{aligned}$$

BPS states appear at a much higher energy than our ground state (no scalar BPS states!).

SCFTs at large R-charge

Unique ground state at large charge, same EFT as O(N) vector model.

Things are very different for SCFTs with a moduli space: Curvature of manifold always relevant: theory has a dimensionful parameter.

How can we write an EFT? Need extra ingredient. Make use of SUSY properties.

Simplest (class of) examples with moduli space: N=2 SCFT in 4D with 1D Coulomb branch.

More in Domenico's talk on Thursday!!

Summary

Summary

- Study CFTs in sectors of large global charge
- Concrete examples where a strongly-coupled CFT simplifies in a special sector.
- O(2N) model in 3d: in the limit of large U(1) charge Q, we computed the conformal dimensions in a controlled perturbative expansion:

$$D(Q) = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi}\, c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}(Q^{-1/2})$$

- Can be applied beyond vector model: SU(N) matrix models
- Theory "classicalizes" at large charge
- Excellent agreement with lattice results for O(2), O(4)

Summary

- Study non-relativistic CFTs with global U(1).
- Large-charge expansion exists, quantum corrections and higher-derivative terms are suppressed
- results in 3+1D match eff. theory for unitary Fermi gas
- qualitatively different behavior to relativistic case
- SCFTs without moduli space: works the same as for vector models
- Everything today: low-energy behavior encoded by one relativistic Goldstone boson, all relativistic cases look the same.
- Thursday: will see inhomogeneous ground states, different leading behavior for anomalous dimensions, new results and work in progress…

Summary

Open questions:

- Does it work?
  - For all the examples, we tried, yes! Confirmation from lattice data (O(2) and O(4))
- For what kinds of theories does it work?
  - (S)CFTs and non-relativistic CFTs
- In how many space-time dimensions?
  - d>1 space dimensions
- For what kinds of global symmetries does it work?
  - we checked U(1), O(2n) vector models, SU(N) matrix models

Summary

- What happens if we fix several charges?
  - k charges with same chemical potential: homogeneous solution with type I and type II Goldstones. Different chemical potentials - Domenico's talk
- What can we learn via this approach?
  - calculate CFT data at large charge!

Outlook

- Further study of non-homogeneous states

  Hellerman et al.

  - Domenico's talk!
- Further study of supersymmetric models at large R-charge (higher-dim. moduli spaces)

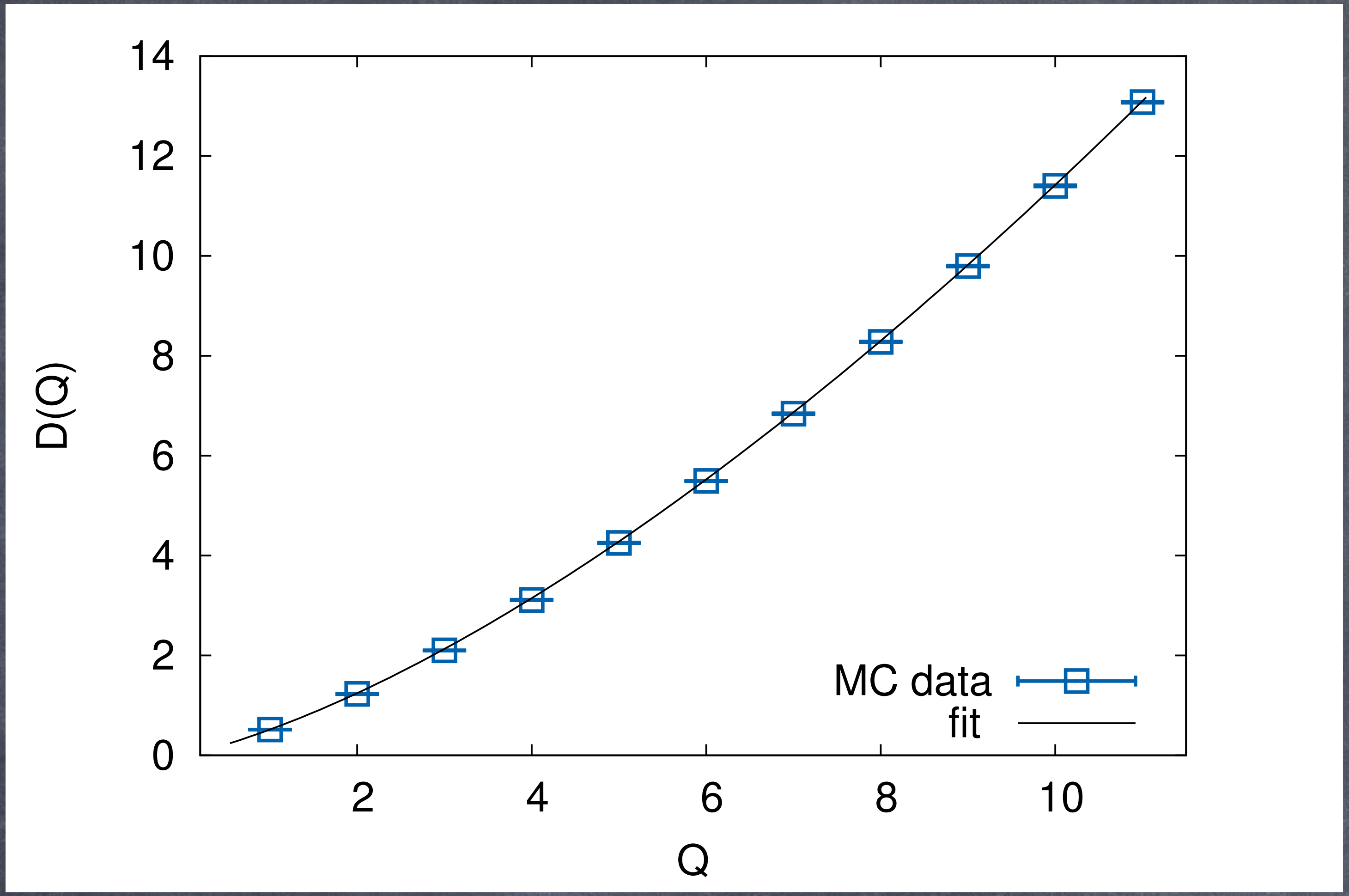
  Hellerman et al.

  - Domenico's talk!
- Connection to holography (gravity duals)

  Loukas, Orlando, Reffert, Sarkar
- Connection to large-spin results

  Rattazzi et al.
- Understanding dualities semi-classically at large charge
- Use/check large-charge results in conformal bootstrap

  Jafferis and Zhiboedov
- Comparison with large-N expansion
  - Domenico's talk!
- Can large charge approach be used for QCD (e.g. large baryon number)?



Thank you for your attention!