Weak Gravity Conjecture from Unitarity and Causality

Toshifumi Noumi

(Kobe Univ, Wisconsin-Madison)

mainly based on 1810.03637 w/Y. Hamada, G. Shiu

6th Nov 2018 @ IPMU

Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\mathrm{Pl}}^2}$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']

Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\mathrm{Pl}}^2}$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06’]

Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\mathrm{Pl}}^2} \xrightarrow{M_{\mathrm{Pl}} \to \infty} 0$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06']

Weak Gravity Conjecture

in quantum gravity,

∃ a charged state satisfying

$$g^2 q^2 \geq \frac{m^2}{2M_{\mathrm{Pl}}^2}$$

for each U(1) gauge force

[ArkaniHamed-Motl-Nicolis-Vafa 06’]

in QED, the electron trivially satisfies it:

$$10^{-2} \sim g^2 q^2 \geq \frac{m^2}{2M_{\mathrm{Pl}}^2} \sim 10^{-44}$$

however, its generalization (ex. axion) constrains

models of inflation, dark matter, $\cdots$

in this talk, I will argue [Hamada-TN-Shiu '18]

existence of a charged state (BH) satisfying

$$g^2 q^2 > \frac{m^2}{2M_{\rm Pl}^2}$$

follows from unitarity & causality

in a wide class of theories

(ex. stingy setups w/dilaton or moduli stabilized below M_s)

plan

1. Introduction: Landscape & Swampland

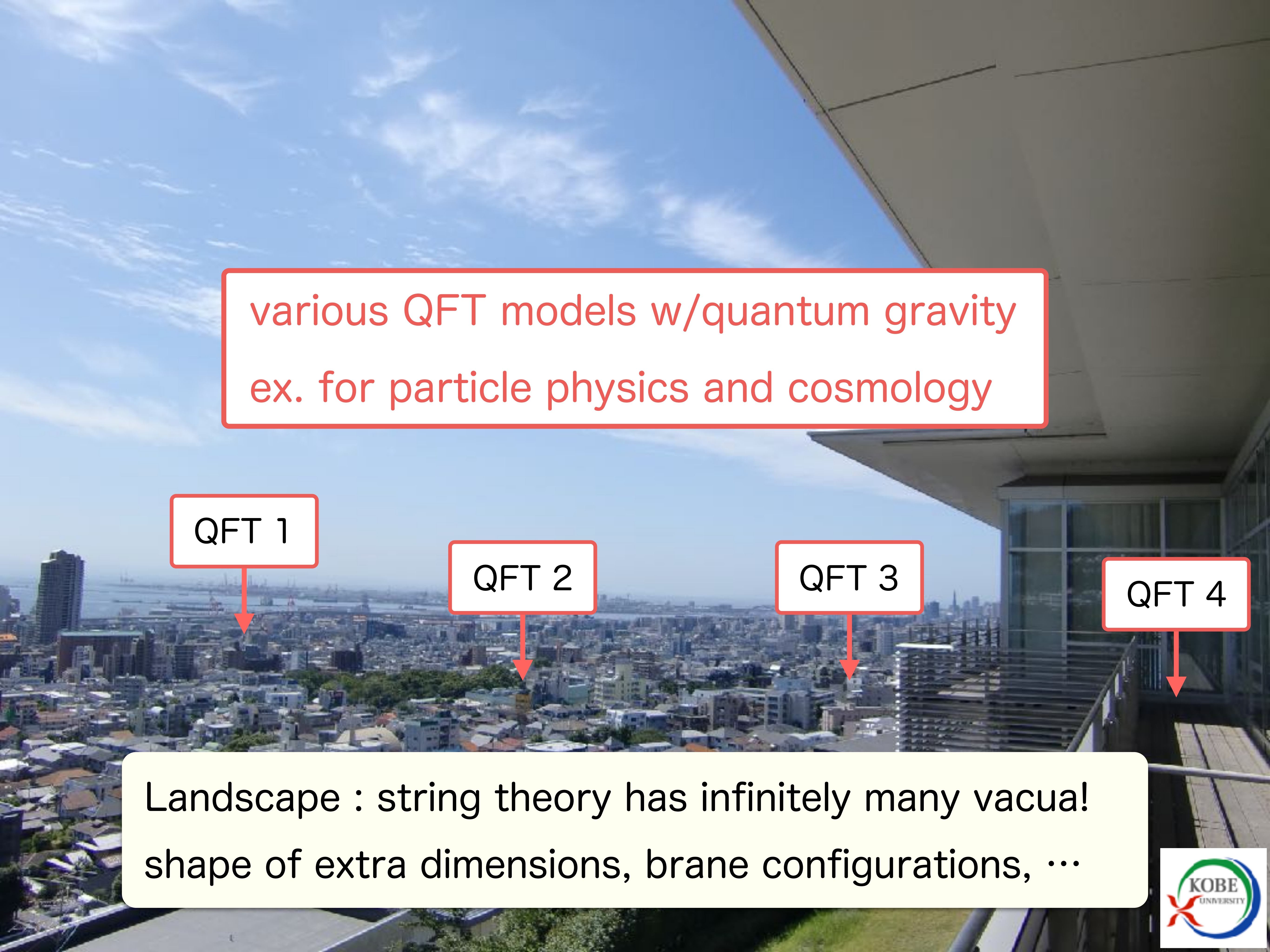
2. Weak Gravity Conjecture

3. WGC from unitarity and causality

4. Summary and prospects

1. Landscape & Swampland

1. Landscape & Swampland


various QFT models w/quantum gravity
ex. for particle physics and cosmology
QFT 1
QFT 2
QFT 3
QFT 4
Landscape : string theory has infinitely many vacua!
shape of extra dimensions, brane configurations, …
KOBE UNIVERSITY

string theory

= generator of QFT models w/quantum gravity

Q. every QFT model is realized in string theory?

A. NO!!!

no global symmetry in string theory

\# continuous symmetries in string theory are gauged!

- world sheet theory analysis [Banks-Dixon ’88, $\cdots$]

  conserved current $\to$ gauge boson vertex operator

- if we assume AdS/CFT $\cdots$

conserved current J^μ in CFT $\rightleftarrows$ gauge field A_M in AdS

\# holographic proof including discrete symmetries

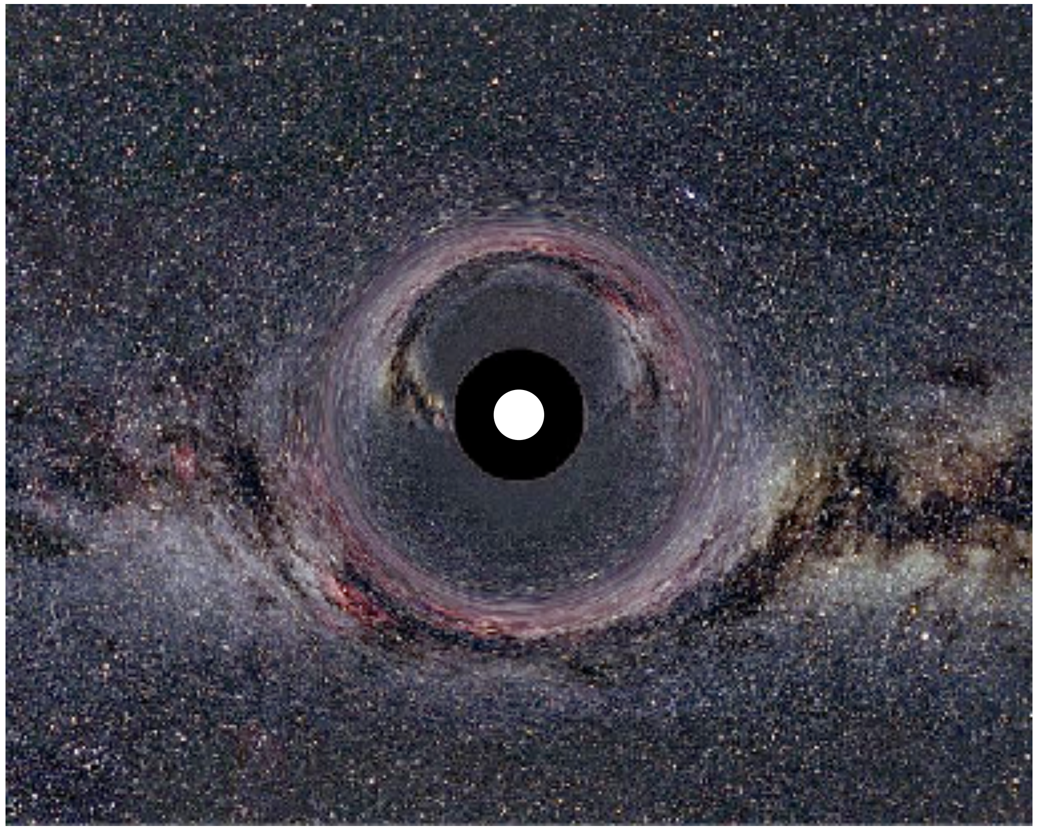
[Harlow-Ooguri 18’]

more generally,

black hole (BH) thought experiments motivate

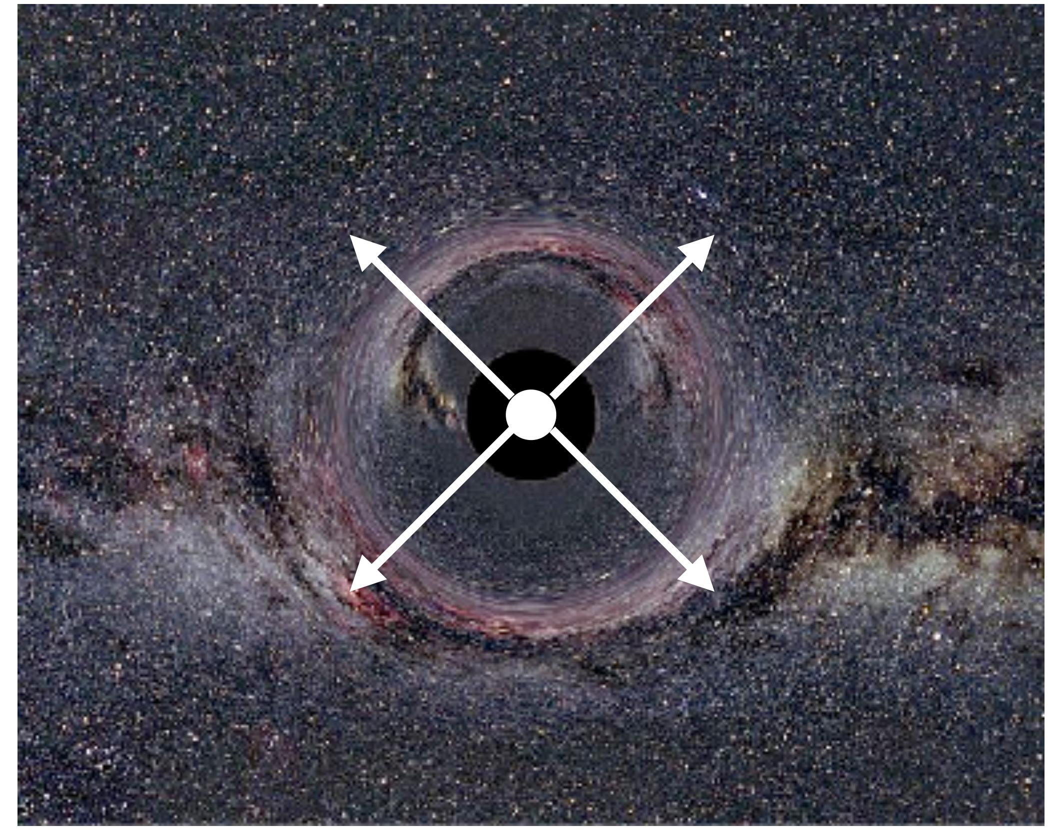
no global symmetry in quantum gravity!

global vs gauge in the BH context



global symmetry

ex. $B - L$



gauge symmetry

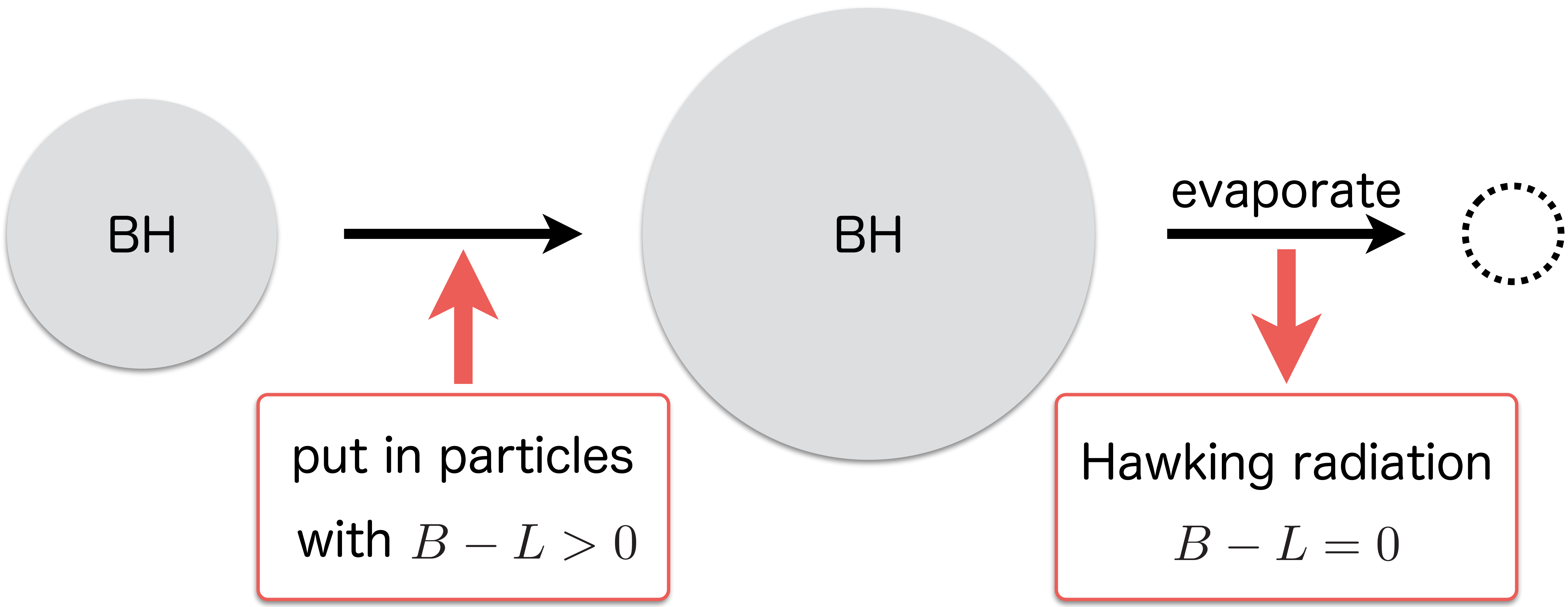
ex. $U(1)_{EM}$ Q

no-hair theorem:

event horizon $\rightarrow$ global symmetry charge is not observable

cf. EM fluxes outside the horizon tell us the EM charge

no global symmetry in quantum gravity



global charge is not conserved due to BH evaporation

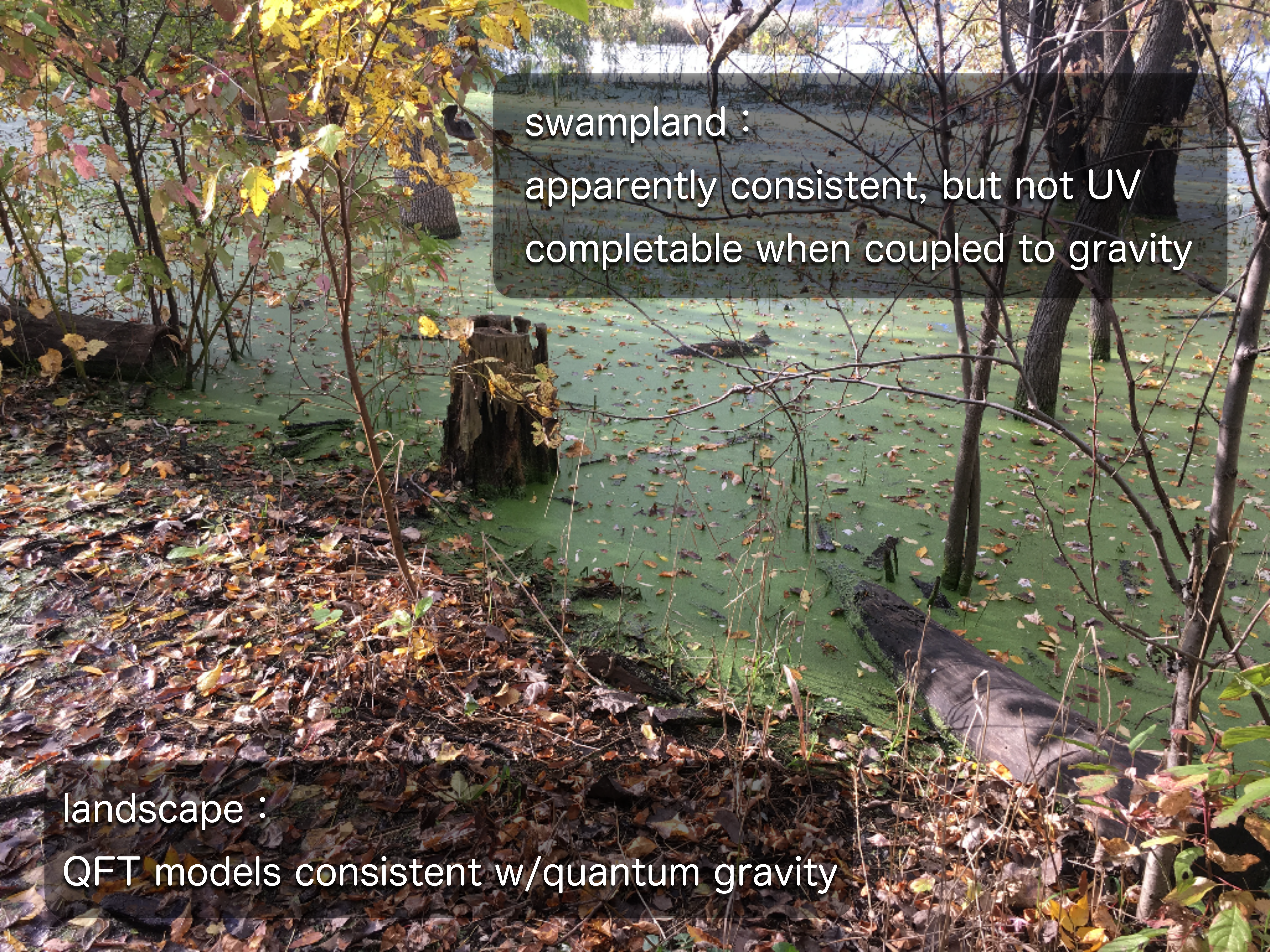
$\rightarrow$ global symmetry is approximate symmetry (if exists)

cf. for gauge symmetry, Hawking radiation is not neutral in this way,

nontrivial constraints on symmetry & matter contents

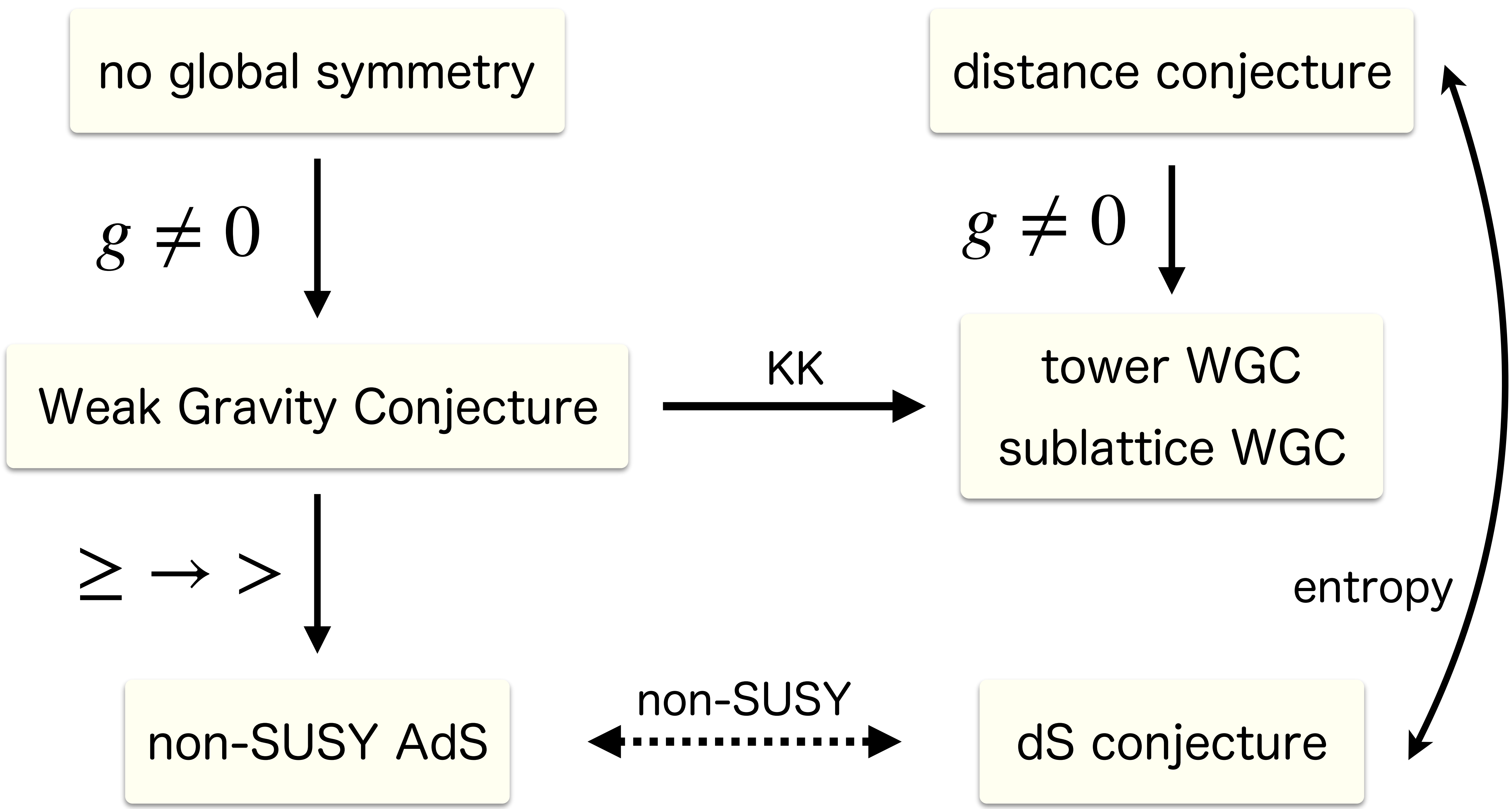
in string theory (quantum gravity in more general)

$\rightarrow$ Landscape & Swampland [Vafa '06]

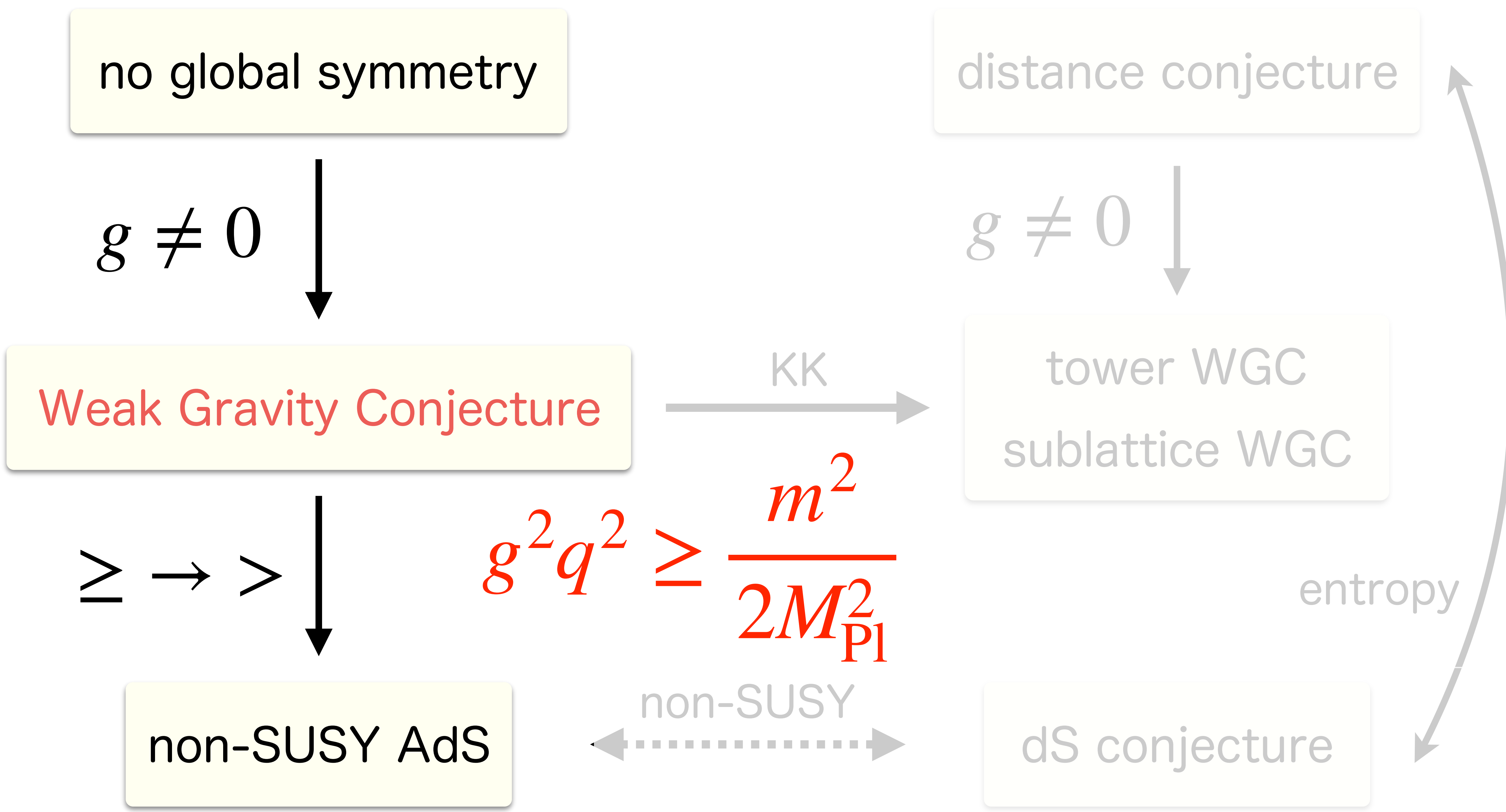

swampland :
apparently consistent, but not UV
completable when coupled to gravity
landscape :
QFT models consistent w/quantum gravity


- where is the boundary?
- phenomenological implications?
swampland
landscape

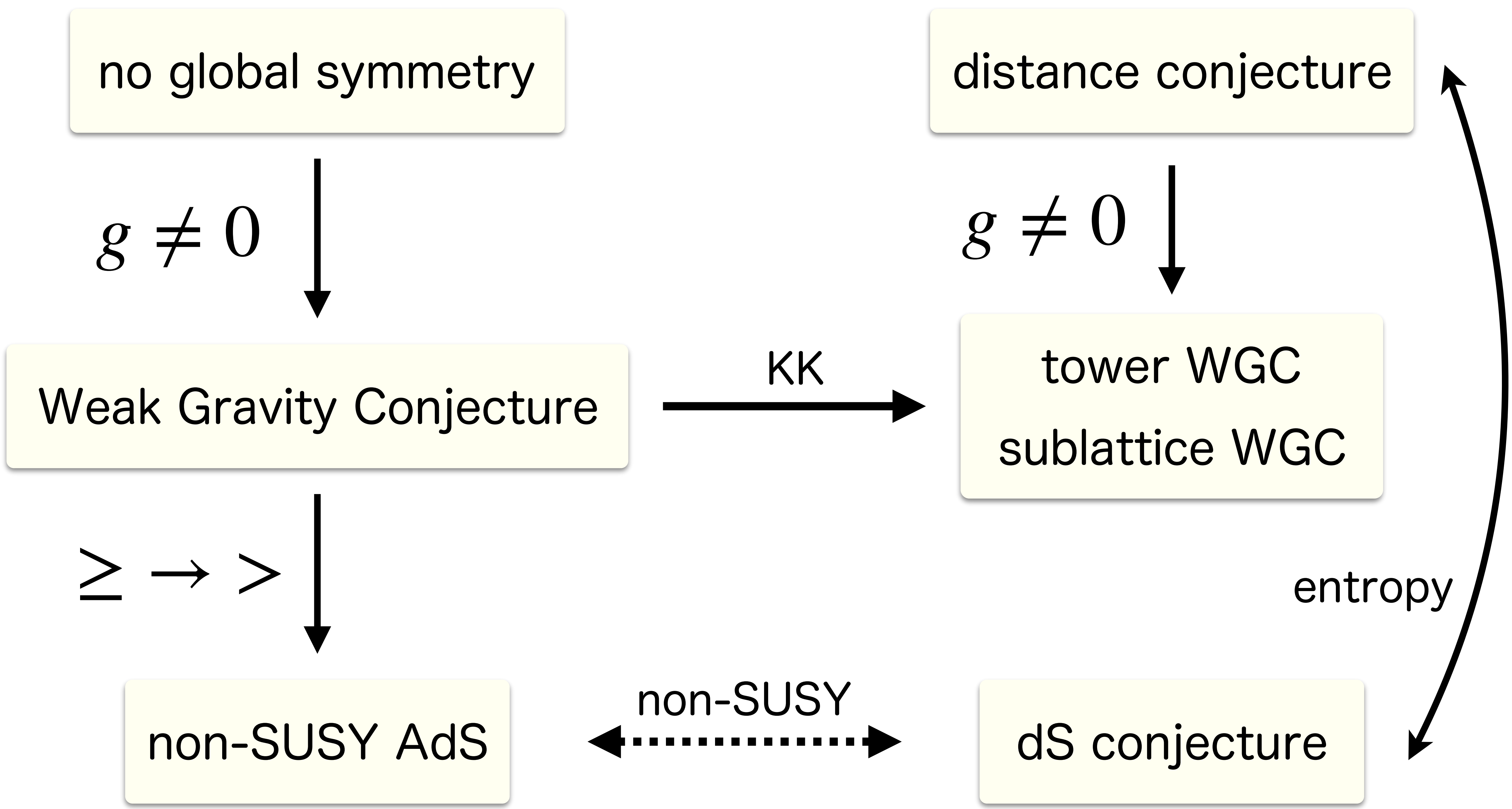
web of swampland conjectures



web of swampland conjectures



web of swampland conjectures



plan

1. Introduction: Landscape & Swampland ✔
2. Weak Gravity Conjecture
3. WGC from unitarity and causality
4. Summary and prospects

2. Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06’]

global symmetry = gauge symmetry @ $g = 0$

$\rightarrow$ any lower bound on gauge coupling g ??

a simplest possibility will be

$$g^2 q^2 \geq (\text{constant}) \times \frac{m^2}{M_{\text{Pl}}^2}$$

BHs in Einstein-Maxwell theory

1) sub-extremal BH: $g\,|\,Q\,| < M/\sqrt{2}M_{\rm Pl}$

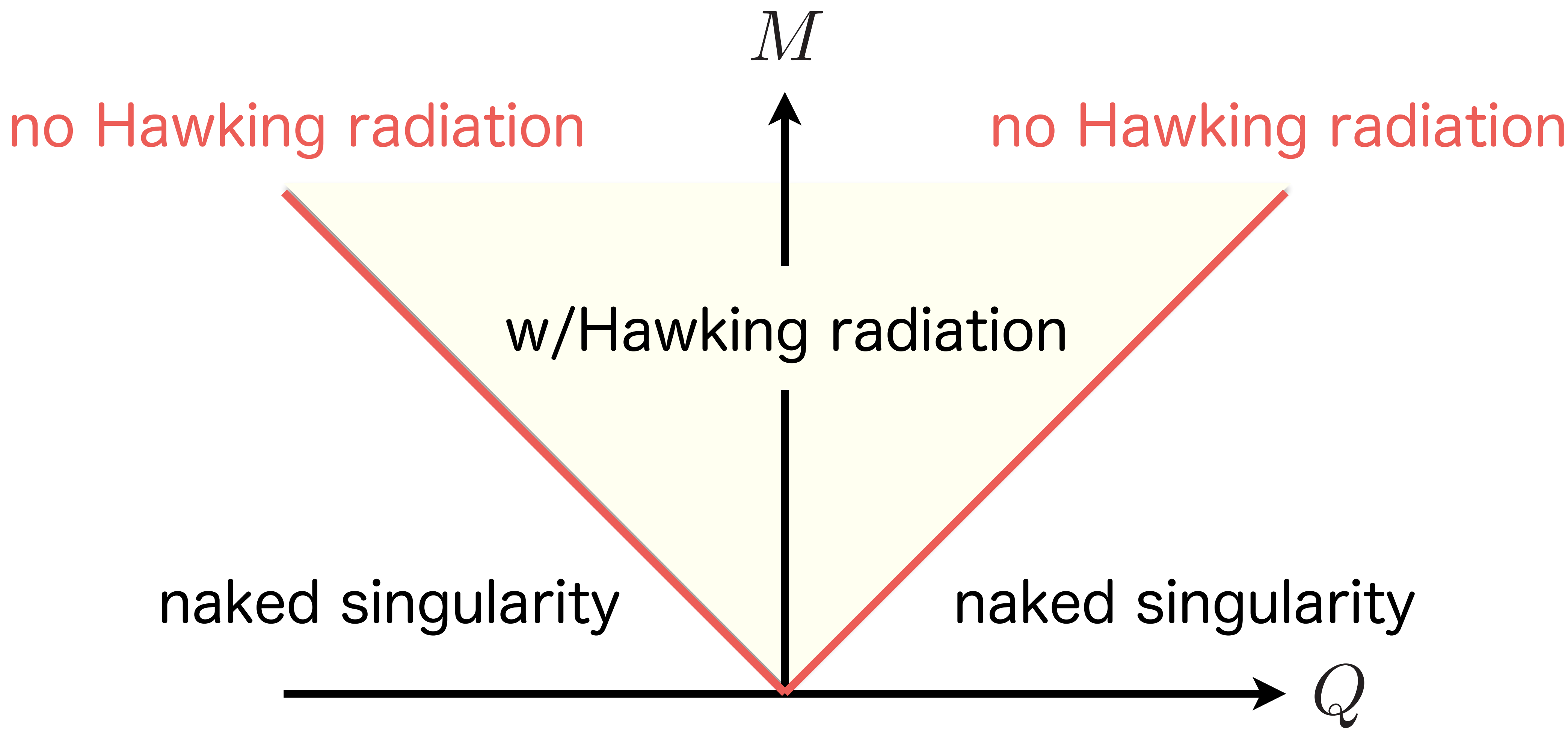
emit Hawking radiation ($T \neq 0$) to decay; unstable

2) extremal BH: $g\,|\,Q\,| = M/\sqrt{2}M_{\rm Pl}$

no Hawking radiation ($T = 0$)

$\rightarrow$ stable unless $\exists$ some other decay mechanism

※ $g\,|\,Q\,| > M/\sqrt{2}M_{\rm Pl}$: naked singularity (cf. cosmic censorship)



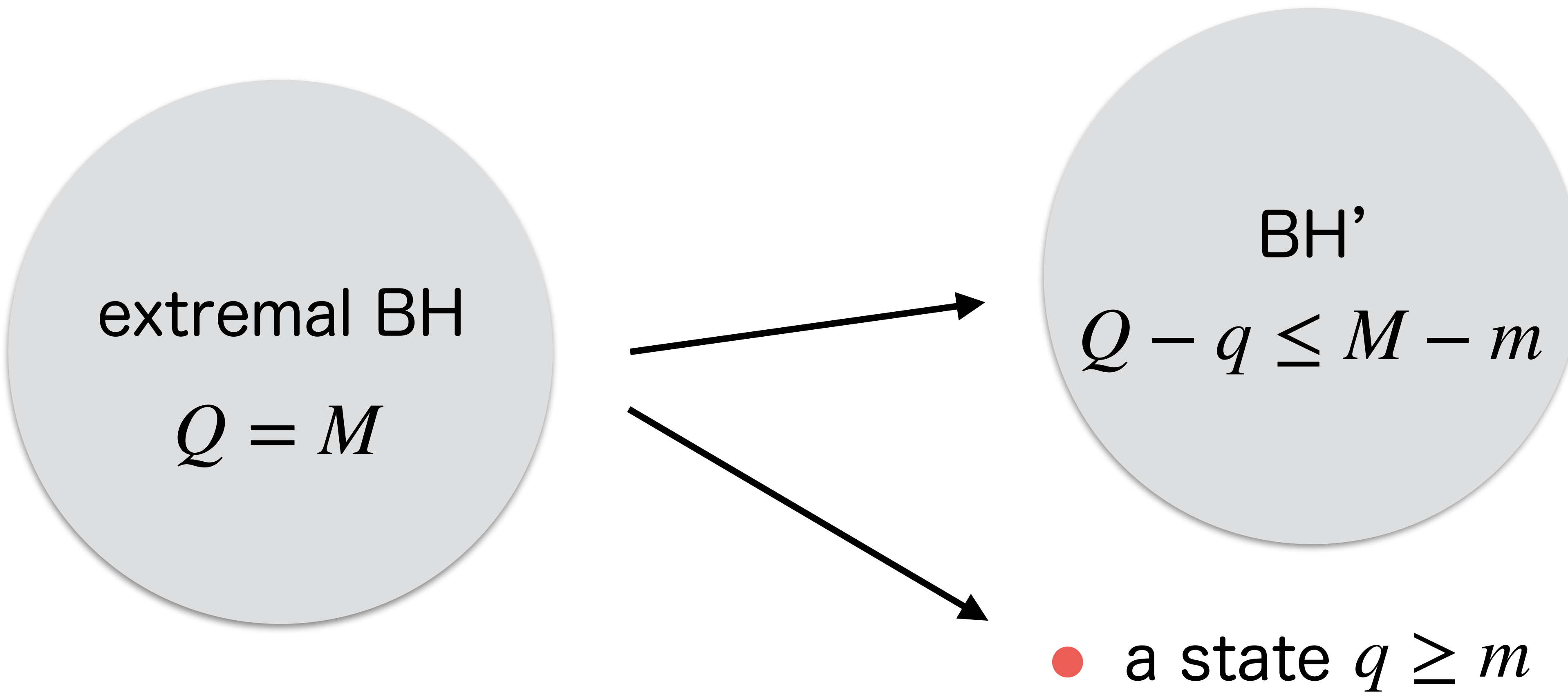
a proposal by [ArkaniHamed-Motl-Nicolis-Vafa 06']:

postulate that extremal BHs have to decay unless not protected by some symmetry (ex. SUSY)

- "∞ stable states w/o sym protection" seems strange
- revisit from unitarity & causality perspective later

Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06']



- extremal BH has a decay channel

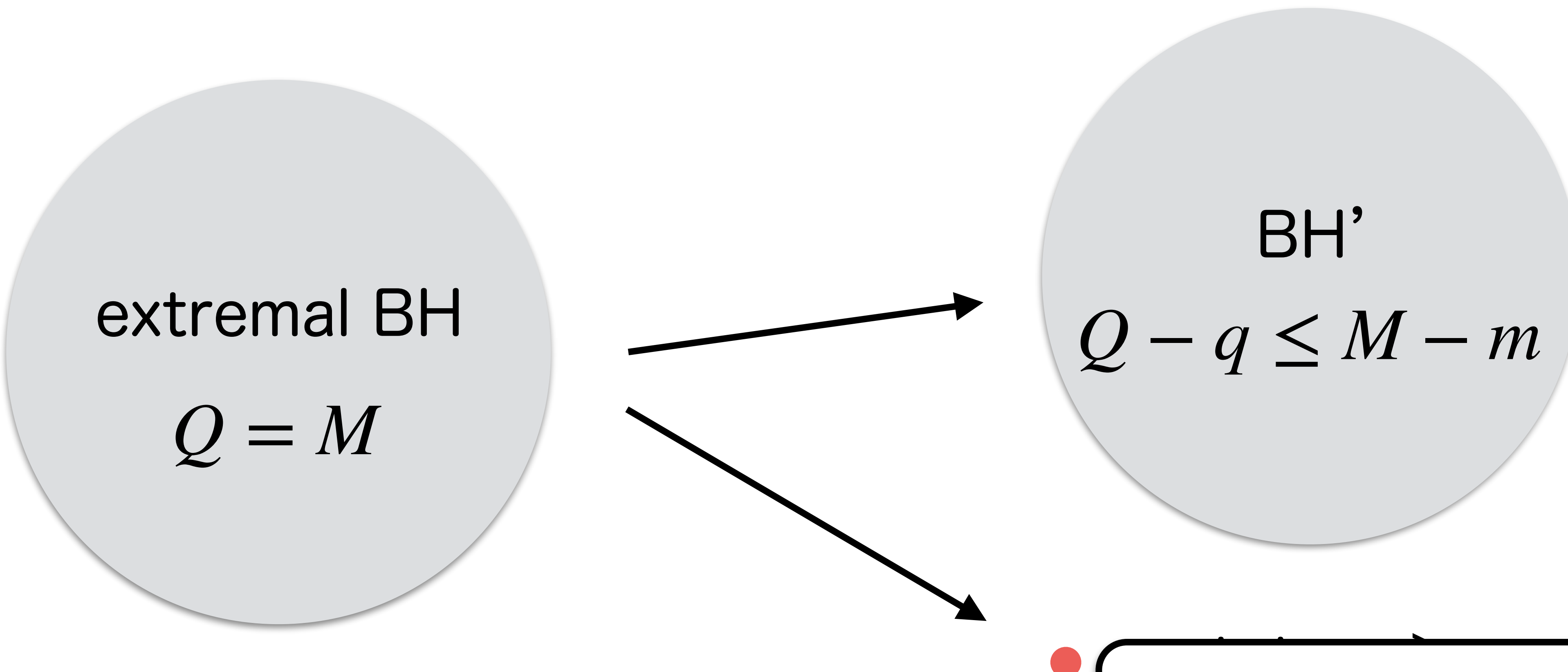
- the BH' after decay has no naked singularity

$\rightarrow \exists$ a charged state satisfying $q \geq m$

in the unit $Q_{\text{ext}} = M_{\text{ext}}$ for simplicity

Weak Gravity Conjecture

[ArkaniHamed-Motl-Nicolis-Vafa 06']



$$gq \geq \frac{m}{\sqrt{2}M_{\mathrm{Pl}}}$$

- extremal BH has a decay chann
- the BH' after decay has no naked singularity

$\rightarrow \exists$ a charged state satisfying $q \geq m$

in the unit $Q_{\mathrm{ext}} = M_{\mathrm{ext}}$ for simplicity

- no rigorous proof, so it is still a conjecture
- but consistent with all known examples in string theory
- if true, various phenomenological implications

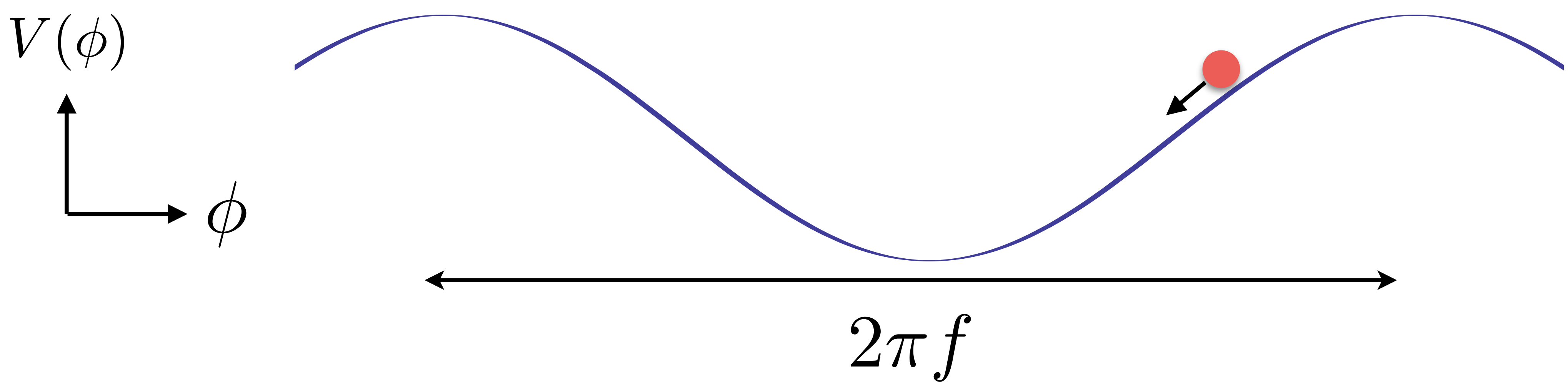
  ex. mili-charged dark matter, axion inflation, axion DM, …

- no rigorous proof, so it is still a conjecture
- but consistent with all known examples in string theory
- if true, various phenomenological implications

ex. mili-charged dark matter, axion inflation, axion DM, $\cdots$

"charge > mass" $\leftrightarrows$ $\dfrac{1}{f} > \dfrac{S_{\text{inst}}}{M_{\text{Pl}}}$ $\leftrightarrows$ $\dfrac{f}{M_{\text{Pl}}} \cdot S_{\text{inst}} < 1$

implications to axion inflation



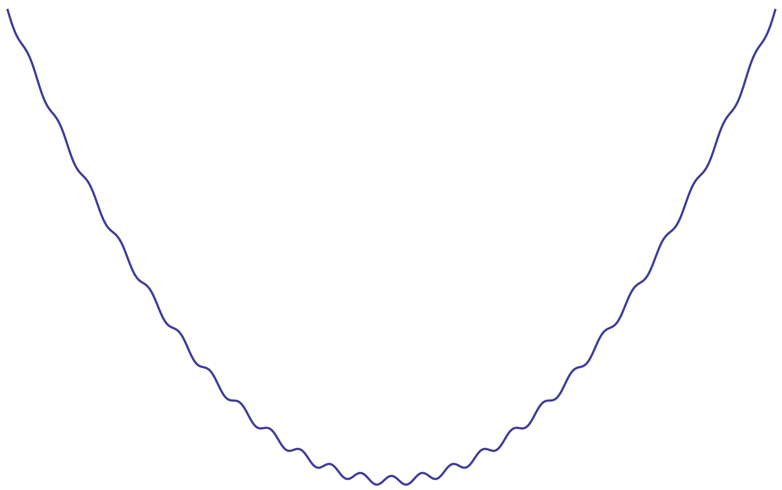
inflaton potential has to be flat enough (slow-roll condition)

$$V(\phi) \propto e^{-S_{\rm inst}}\left(1-\cos\frac{\phi}{f}\right) + \cancel{\sum_{n\geq 2} e^{-nS_{\rm inst}}\left(1-\cos\frac{n\phi}{f}\right)}$$

- negligible higher harmonics ($n \geq 2$) $\rightarrow$ $S_{\rm inst} > 1$
- long enough periodicity $\rightarrow$ $f > M_{\rm Pl}$

※ inconsistent with WGC $\dfrac{f}{M_{\rm Pl}} \cdot S_{\rm inst} < 1$

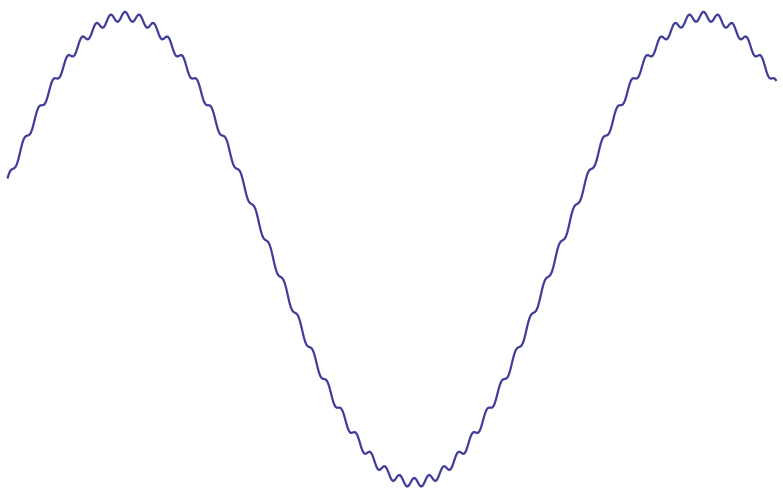
loophole and prediction



axion monodromy (ex. pure natural inflation)

multi-valued potential

$$V(\phi) = V_{\mathrm{s.r.}}(\phi) + e^{-S_{\mathrm{inst}}}\left(1 - \cos\frac{\phi}{f}\right)$$



spectator instanton

add an instanton satisfying the WGC bound

$$V(\phi) = e^{-S_{\mathrm{inst}}}\left(1 - \cos\frac{\phi}{f}\right) + e^{-S'_{\mathrm{inst}}}\left(1 - \cos\frac{\phi}{f'}\right)$$

- large field inflation is realized by

$$V_{\mathrm{s.r.}} \text{ or } e^{-S'_{\mathrm{inst}}}\left(1 - \cos\frac{\phi}{f'}\right) \ (f' > M_{\mathrm{Pl}})$$

- WGC is satisfied by the instanton with $S_{\mathrm{inst}} \cdot \dfrac{f}{M_{\mathrm{Pl}}} \lesssim 1$

$\rightarrow$ wiggy potential $\rightarrow$ oscillating feature in power spectrum

plan

1. Introduction: Landscape & Swampland ✔

2. Weak Gravity Conjecture ✔

3. WGC from unitarity and causality

4. Summary and prospects

3. WGC from unitarity and causality

[Hamada-TN-Shiu ’18]

constraints on signs and/or amplitudes

of effective interactions (ex. positivity bound)

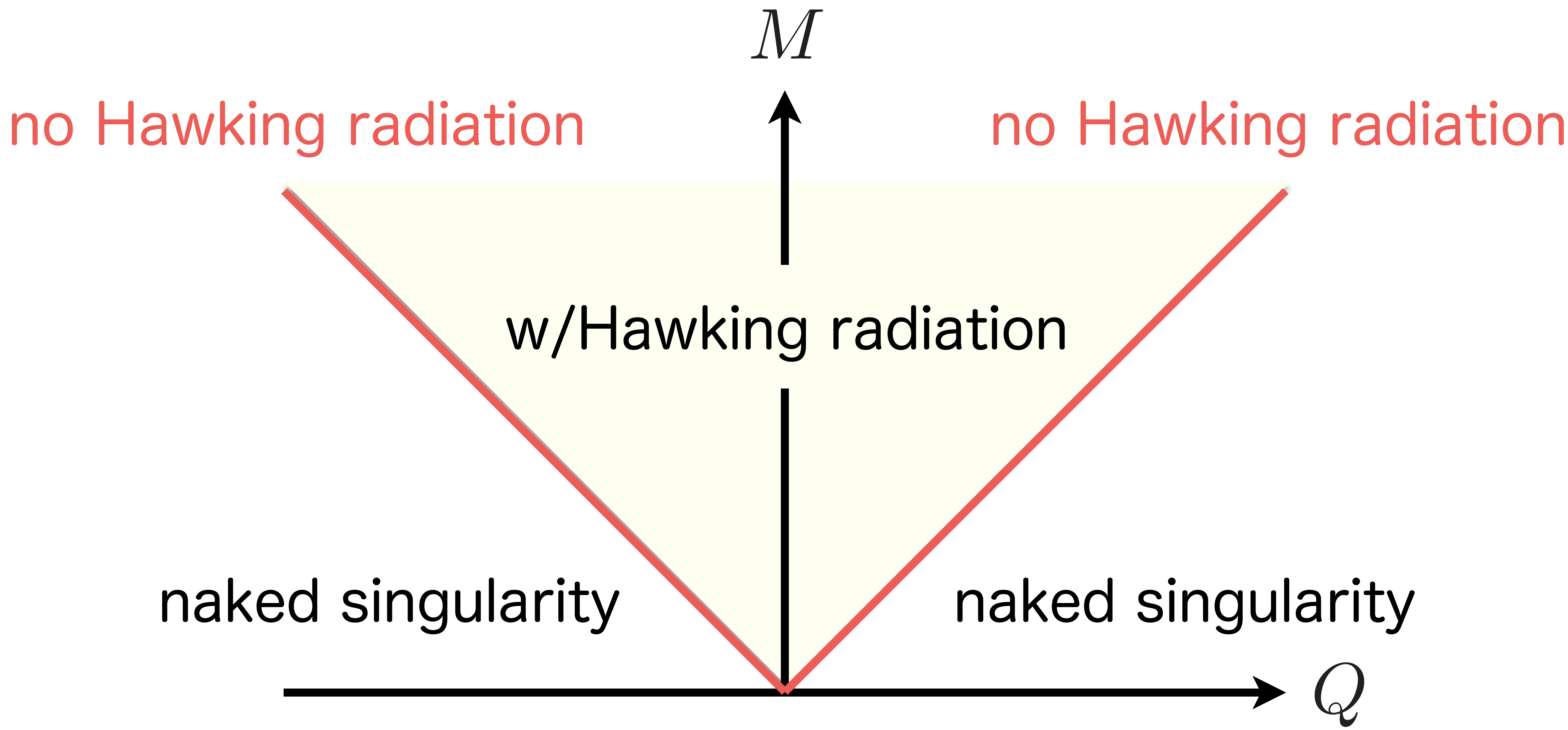
3. WGC from unitarity and causality

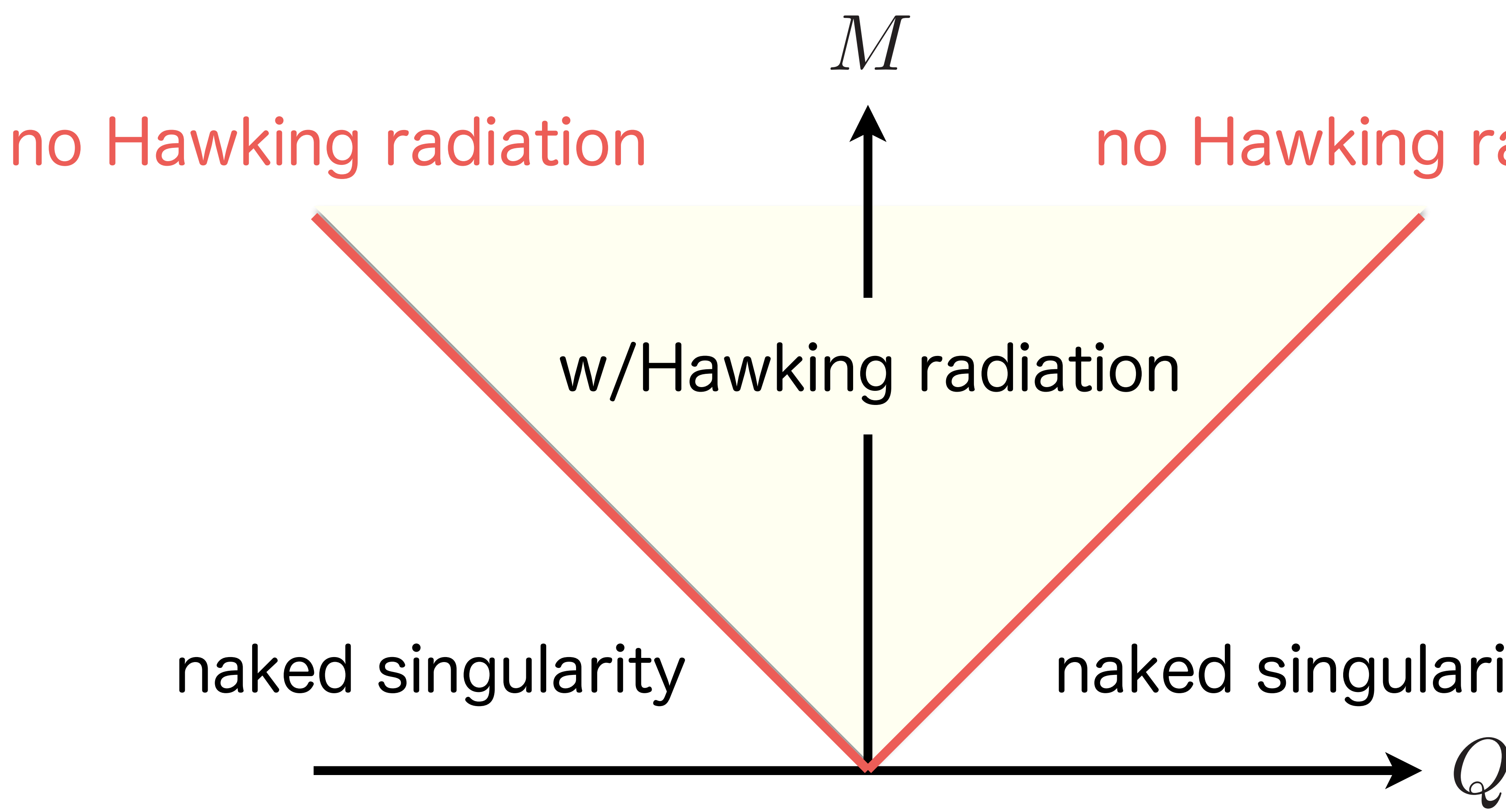
[Hamada-TN-Shiu '18]

∃ heavy BHs satisfying the WVC bound $g|Q| > \frac{M}{\sqrt{2}M_{\mathrm{Pl}}}$

if higher derivative interactions have a certain sign

[Kats-Motl-Padi '06]


M
no Hawking radiation
no Hawking radiation
w/Hawking radiation
naked singularity
naked singularity
Q



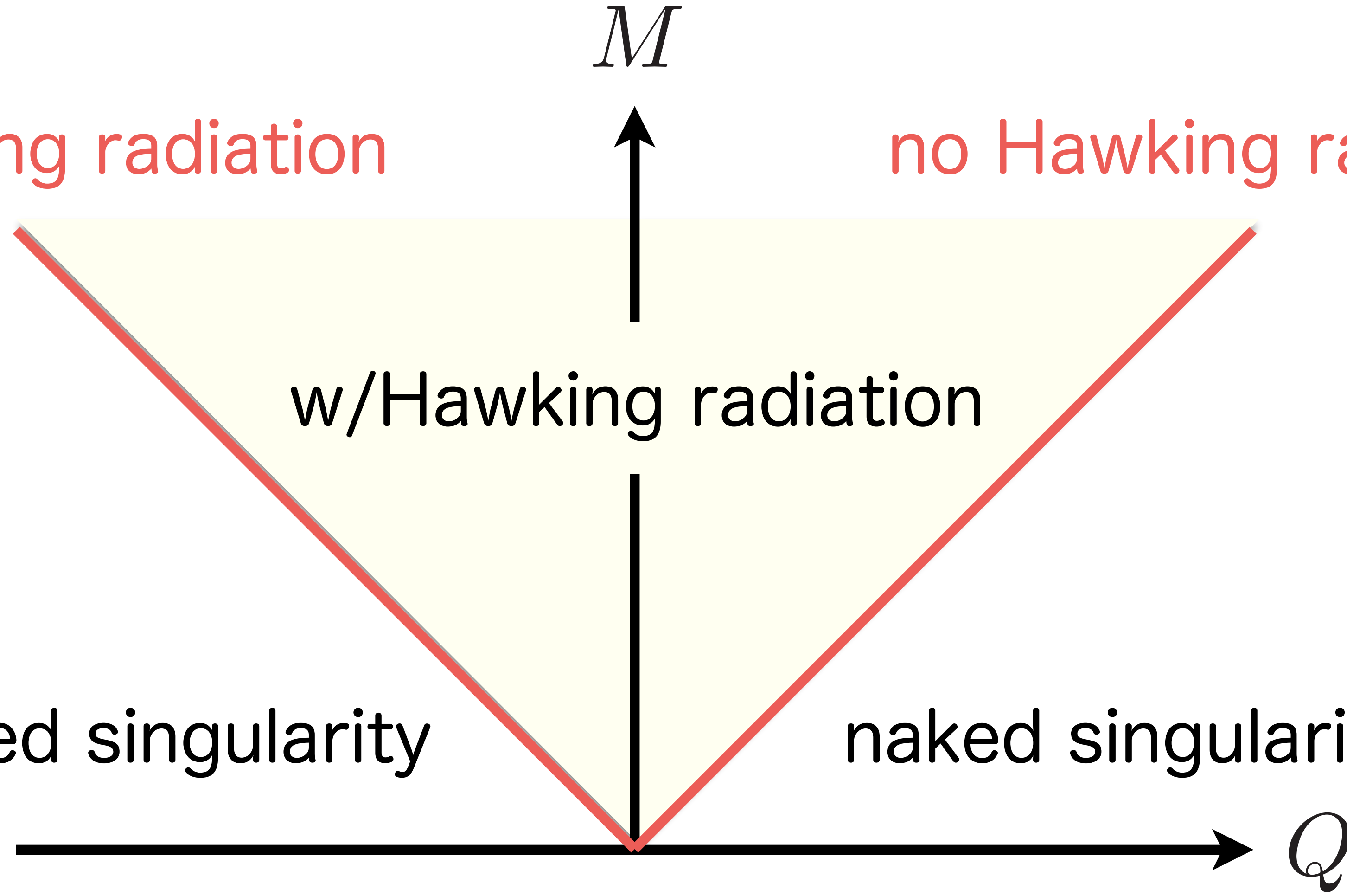
higher derivative corrections to Einstein-Maxwell theory

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma} + \cdots\Big]$$

※ work in the unit $2M_{\rm Pl}^2 = 1,\ g = 1$ in the following

※ higher order terms are negligible for heavy BHs

$$F^2 \sim R \sim 1/M^2$$

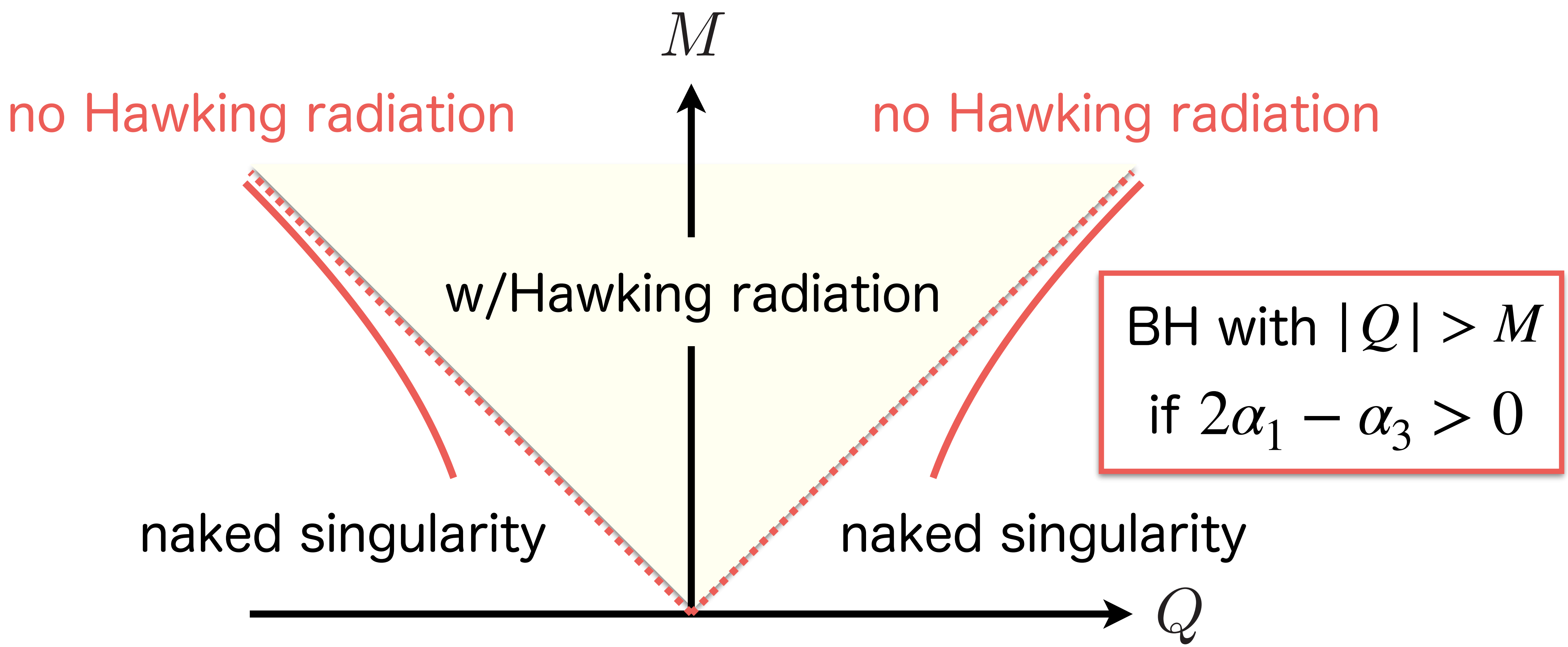


higher derivative corrections to Einstein-Maxwell theory

$$S=\int d^4x\sqrt{-g}\Big[\frac{1}{4}R-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\alpha_1(F_{\mu\nu}F^{\mu\nu})^2 +\alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2+\alpha_3F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma}+\cdots\Big]$$

$\rightarrow$ modify BH solutions and the horizon structure

no naked singularity if $\dfrac{|Q|}{M}\leq 1+\dfrac{2}{5}\dfrac{(4\pi)^2}{Q^2}(2\alpha_1-\alpha_3)+\mathcal{O}(1/Q^4)$



higher derivative corrections to Einstein-Maxwell theory

$$S = \int d^4x\sqrt{-g}\Big[\frac{1}{4}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \alpha_1(F_{\mu\nu}F^{\mu\nu})^2 + \alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2 + \alpha_3 F_{\mu\nu}F_{\rho\sigma}W^{\mu\nu\rho\sigma} + \cdots\Big]$$

$\rightarrow$ modify BH solutions and the horizon structure

no naked singularity if $\dfrac{|Q|}{M} \leq 1 + \dfrac{2}{5}\dfrac{(4\pi)^2}{Q^2}(2\alpha_1 - \alpha_3) + \mathcal{O}(1/Q^4)$

if we can derive $2\alpha_1 - \alpha_3 > 0$,

- an existence proof of WGC (more precisely, mild version)
- decay of heavy extremal BH is kinematically allowed

in the following, I demonstrate

that the inequality follows from unitarity & causality

$$2\alpha_1 - \alpha_3 \underset{\text{causality}}{\simeq} 2\alpha_1 \underset{\text{unitarity}}{>} 0$$

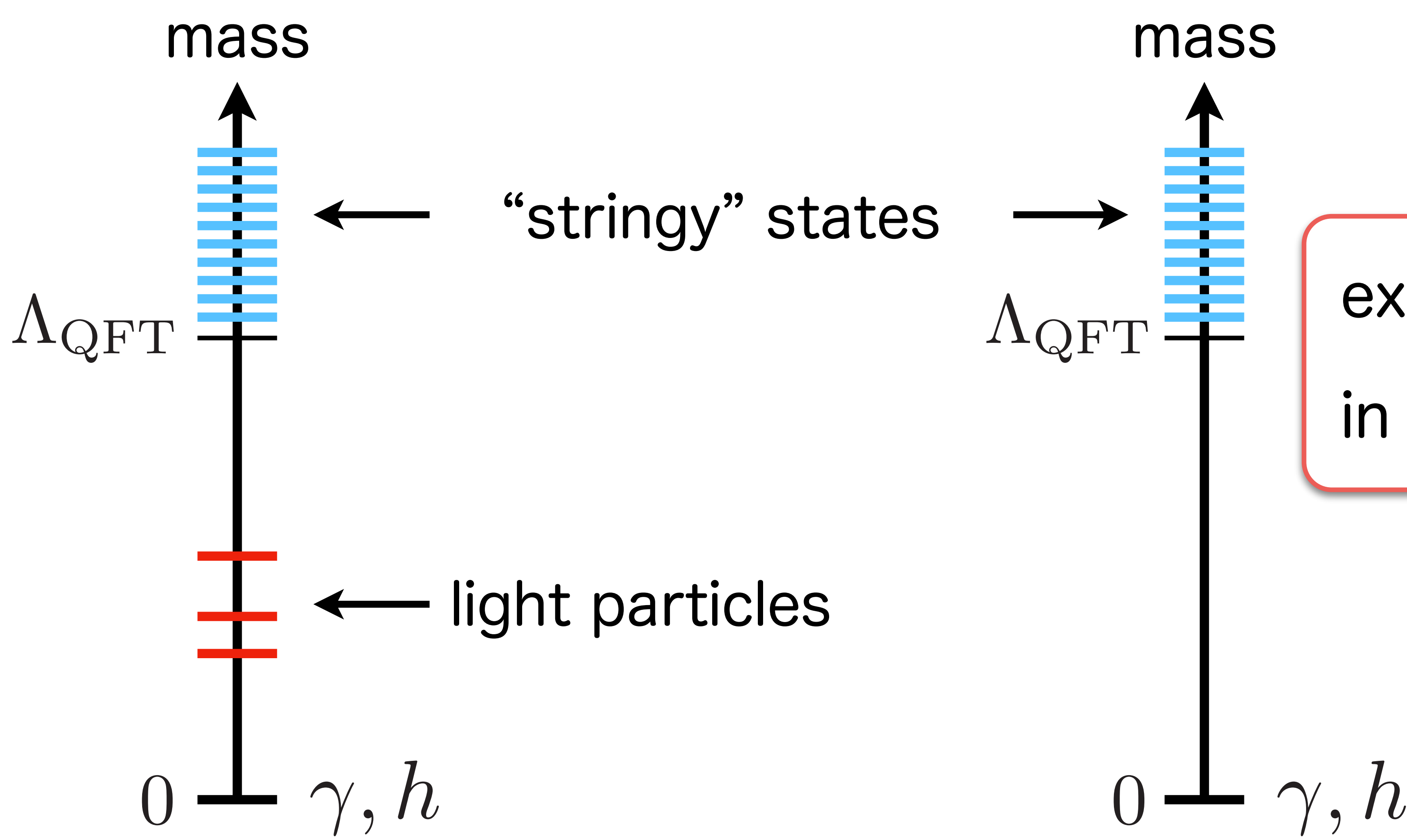
in a class of theories including

- theories with light neutral particles (ex. dilaton, moduli)

- open string theory type UV completion

setup and assumptions

particle spectrum



ex. $\Lambda_{\mathrm{QFT}} \sim M_s$

in string theory

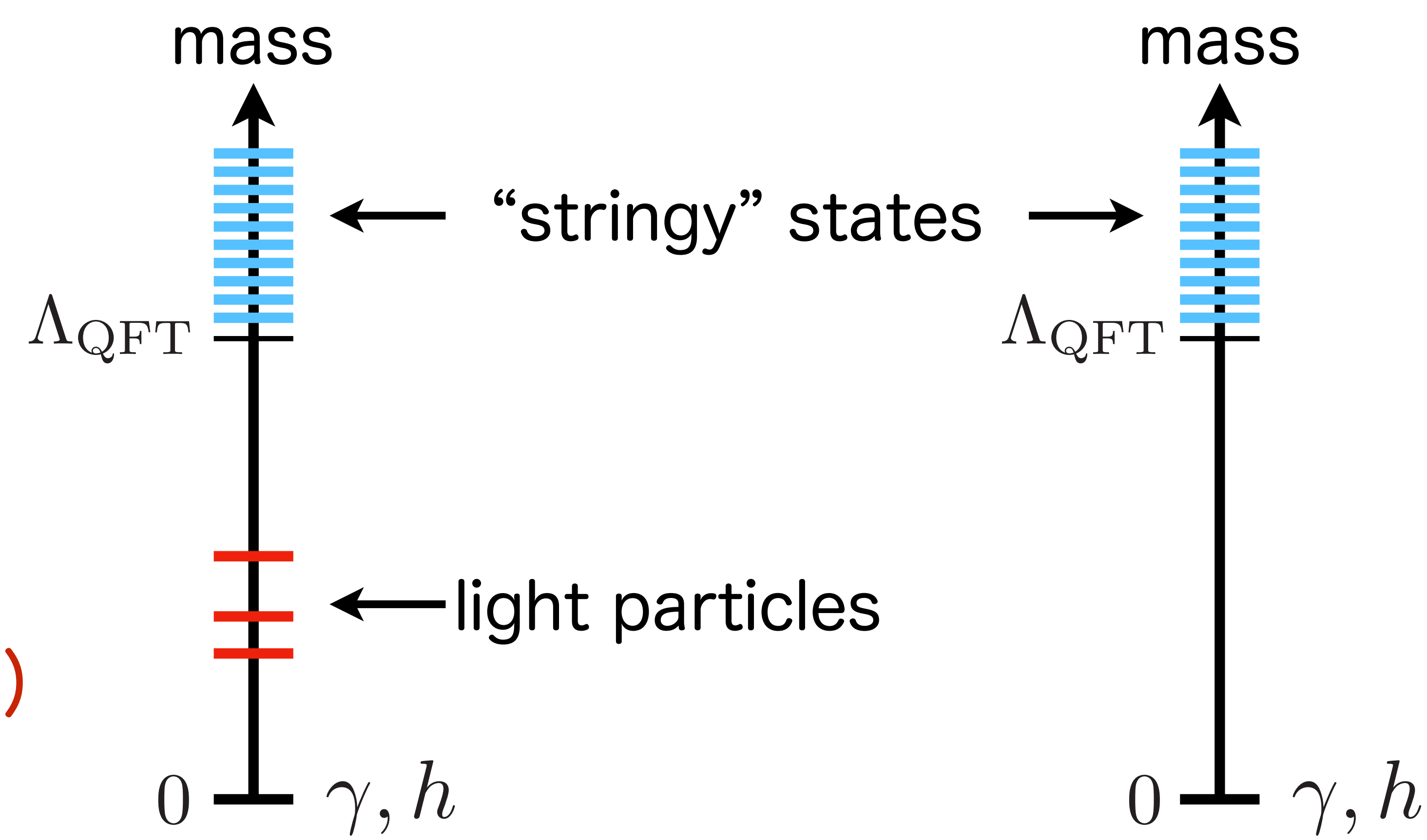
- BH dynamics is controlled by photon and graviton at IR
- Λ_{QFT} : scale beyond which QFT description breaks down
- assume a weakly coupled UV completion of gravity

Sources of higher dimensional operators

(a) neutral light bosons (dilaton, axion)

(b) loop effects (charged particles)

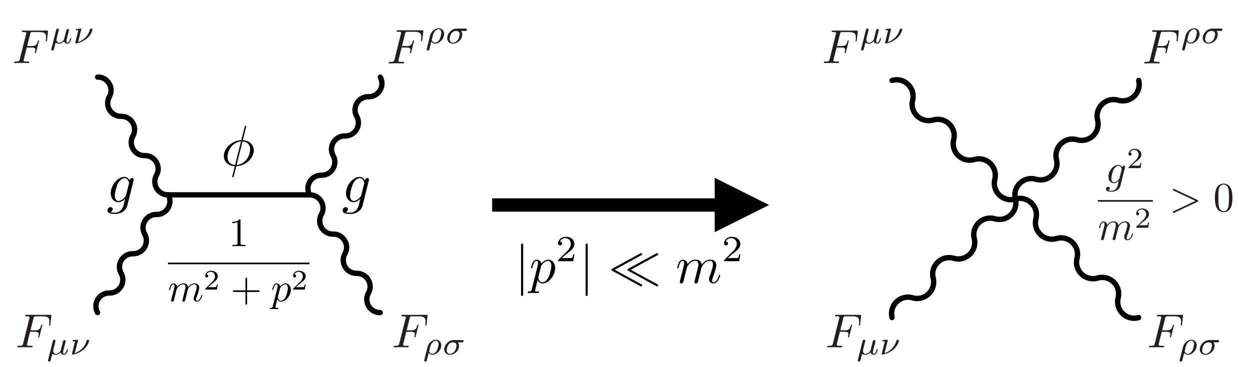
(c) UV effects (stringy states)



Sources of higher dimensional operators

(a) neutral light bosons (dilaton, axion)

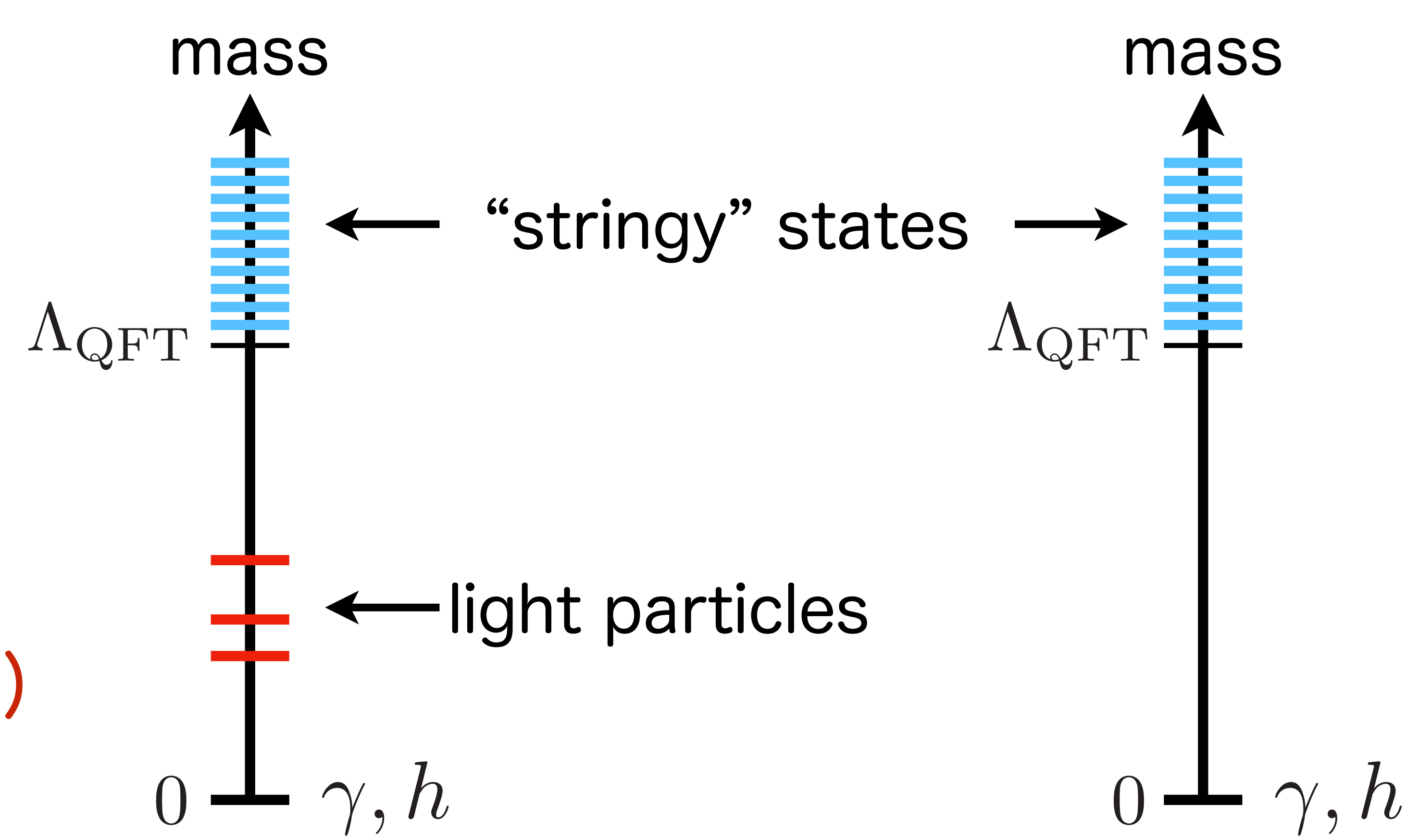
$$\alpha_{1,2} : F^4, \quad \alpha_3 : F^2 W$$



$$|\alpha_i| \gtrsim \frac{1}{m^2}$$

(b) loop effects (charged particles)

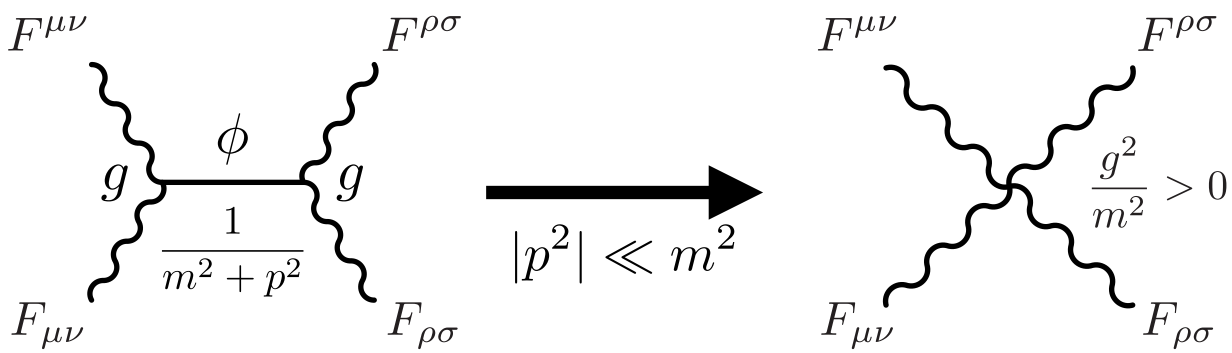
(c) UV effects (stringy states)



Sources of higher dimensional operators

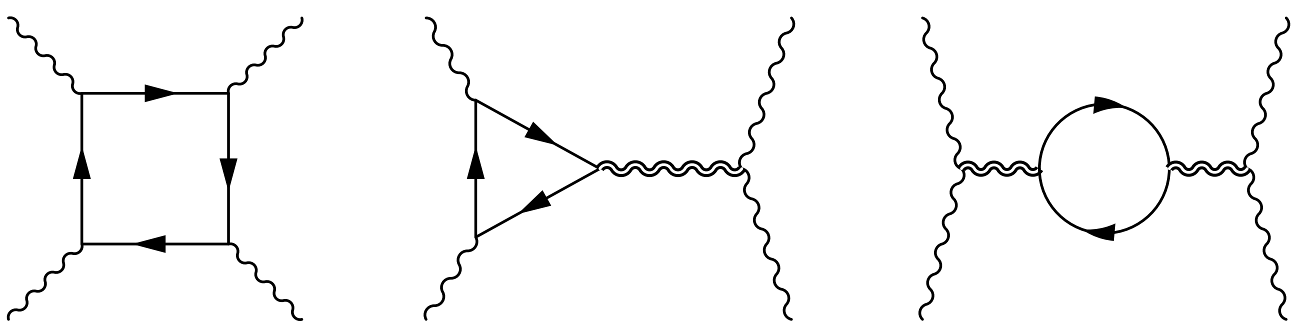
$$\alpha_{1,2} : F^4, \quad \alpha_3 : F^2 W$$

(a) neutral light bosons (dilaton, axion)



$$\boxed{|\alpha_i| \gtrsim \frac{1}{m^2}}$$

(b) loop effects (charged particles)



$\mathcal{O}(z^4)$ $\mathcal{O}(z^2)$ $\mathcal{O}(z^0)$

※ $z = \left| q/m \right|$: the charge-to-mass ratio

$|\alpha_i| \gg 1$ for $z \gg 1$

$\alpha_i = \mathcal{O}(1)$ for $z = \mathcal{O}(1)$

(c) UV effects (stringy states)

$$\alpha_{1,2} \sim \frac{1}{\Lambda_{\mathrm{QFT}}^4}, \quad \alpha_3 \sim \frac{1}{\Lambda_{\mathrm{QFT}}^2}$$

loop effects (b) dominates only when $z \gg 1$

- this particle satisfies the WGC bound $z > 1$ (✔ WGC satisfied)

→ let's focus on the case either (a) or (c) is dominant

- even in this case $2\alpha_1 - \alpha_3 \simeq 2\alpha_1 > 0$ follows from unitarity

$$2\alpha_1 - \alpha_3 \simeq 2\alpha_1 > 0$$

causality

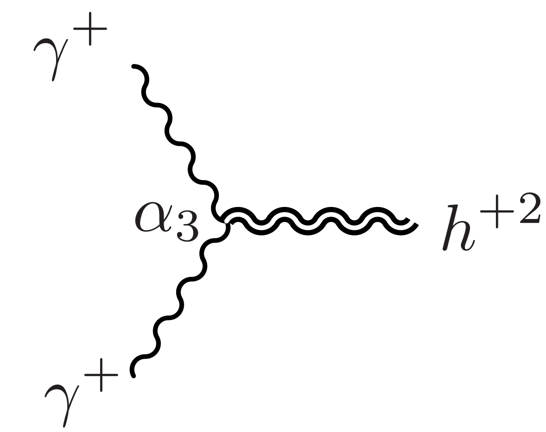
$\alpha_1 : (FF)^2, \;\; \alpha_2 : (F\tilde{F})^2, \;\; \alpha_3 : F^2W$

Causality constraints

the FFW coupling α_3 is significantly constrained by causality!

[Camanho-Edelstein-Maldacena-Zhiboedov '14]

- generates a new 3pt helicity amplitudes
- leads to causality violation (time-advancement)
  unless $\exists$ an infinite tower of higher spin particles
  with the mass $m \sim \alpha_3^{-1/2}$ just like string theory!



cf. this amplitude is incompatible with SUSY, so $\alpha_3 = 0$ in SUSY theories

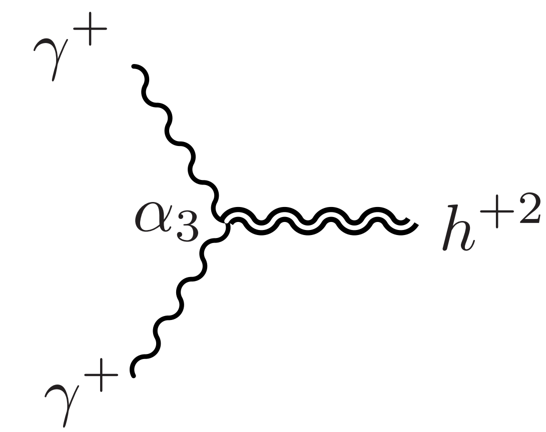
$$\alpha_1 : (FF)^2, \ \ \alpha_2 : (F\tilde{F})^2, \ \ \alpha_3 : F^2 W$$

Causality constraints

the FFW coupling α_3 is significantly constrained by causality!

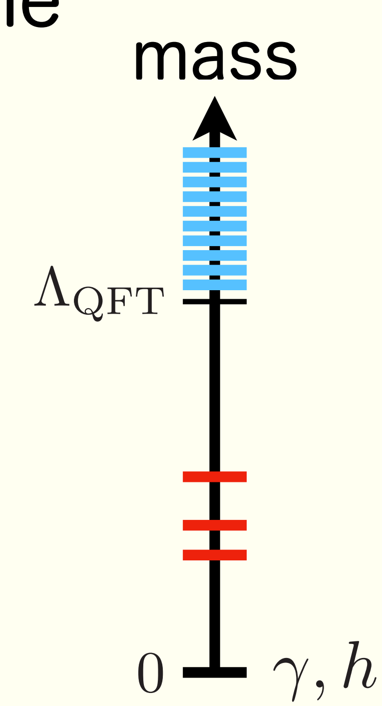
[Camanho-Edelstein-Maldacena-Zhiboedov '14]

- generates a new 3pt helicity amplitudes
- leads to causality violation (time-advancement)
 unless $\exists$ an infinite tower of higher spin particles
 with the mass $m \sim \alpha_3^{-1/2}$ just like string theory!



- α_3 is tightly bounded by the graviton Regge state scale
 (closed string states)

$$|\alpha_3| \lesssim \frac{1}{\Lambda_{\rm QFT}^2}$$ → classified into the UV effect (c)

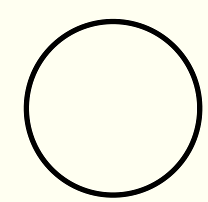


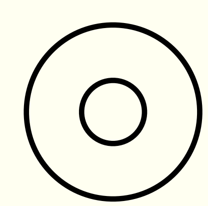
cf. the same bound is available from conformal bootstrap

[Meltzer-Poland '17, Afkhami-Jeddi et al '18] the Wilson coefficients enjoy the hierarchy $|\alpha_1|, |\alpha_2| \gg |\alpha_3|$

- if the effect (a) of light neutral particles is dominant
- if photon and graviton carry different sets of Regge states and the photon Regge state effect is dominant

ex. open string theory

open string Regge stats: $\alpha_{1,2} \sim \dfrac{1}{g_s M_s^2}$  disk

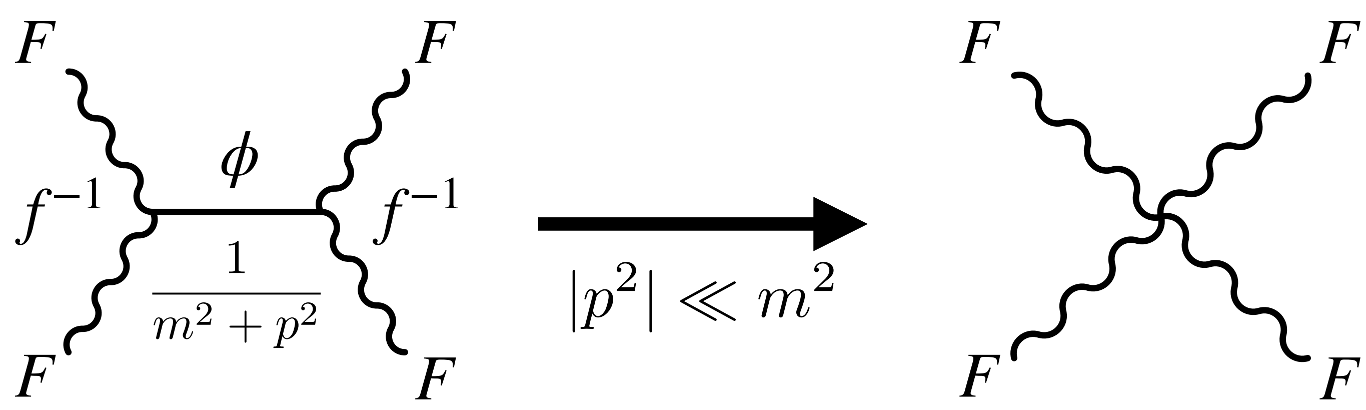
closed string Regge stats: $\alpha_{1,2,3} \sim \dfrac{1}{M_s^2}$  annulus

$$2\alpha_1 - \alpha_3 \underset{\substack{\uparrow \\ \text{causality}}}{\simeq} 2\alpha_1 \underset{\substack{\uparrow \\ \text{unitarity}}}{>} 0$$

$\alpha_1 : (FF)^2, \;\; \alpha_2 : (F\tilde{F})^2, \;\; \alpha_3 : F^2W$

Unitarity constraints

unitarity implies positivity of $\alpha_1(F_{\mu\nu}F^{\mu\nu})^2$ and $\alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2$!



$$\alpha_i = \frac{1}{2m^2f^2} > 0$$

※ sign of α depends on sign of propagator (norm positivity)

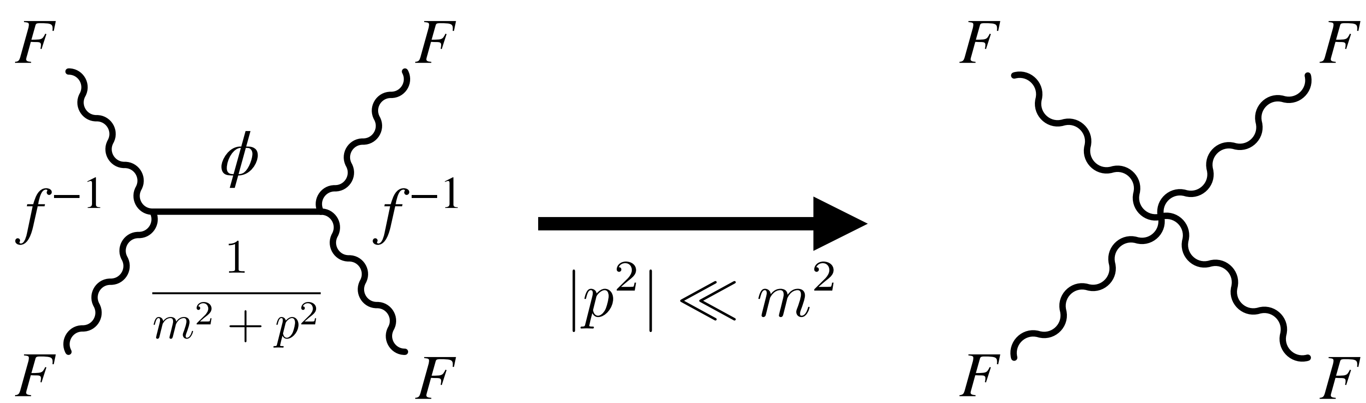
ex. exchange of dilaton and axion

$$\mathscr{L}_\phi = -\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m_\phi^2\phi^2 + \frac{\phi}{f_\phi}F_{\mu\nu}F^{\mu\nu} \xrightarrow[|p^2| \ll m^2]{} \frac{1}{2m_\phi^2 f_\phi^2}(F_{\mu\nu}F^{\mu\nu})^2$$

$$\mathscr{L}_a = -\frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 + \frac{a}{f_a}F_{\mu\nu}\widetilde{F}^{\mu\nu} \xrightarrow[|p^2| \ll m^2]{} \frac{1}{2m_a^2 f_a^2}(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2$$

Unitarity constraints

\# unitarity implies positivity of $\alpha_1(F_{\mu\nu}F^{\mu\nu})^2$ and $\alpha_2(F_{\mu\nu}\widetilde{F}^{\mu\nu})^2$!



$$\alpha_i = \frac{1}{2m^2 f^2} > 0$$

more generally, we can explicitly show [Hamada-TN-Shiu '18]

$\alpha_1 > 0$ $(\alpha_2 > 0)$ follows from unitarity

when photon is coupled to parity even (odd) neutral scalars

or spin $s \geq 2$ neutral particles

by using factorization and UV mildness of scattering amplitudes,

and assuming that graviton Regge states are subdominant effects

cf. spinning polynomials basis of [ArkaniHamed-Huang-Huang '17] to summarize,

unitarity and causality implies $2\alpha_1 - \alpha_3 \underset{\text{causality}}{\simeq} 2\alpha_1 \underset{\text{unitarity}}{>} 0$

※ WGC bound is satisfied by heavy extremal BHs!

※ decay of heavy extremal BH is kinematically allowed

in the following classes of theories:

- theories with light neutral particles

  (parity even scalar or spin $s \geq 2$ particles; dilaton, moduli, KK-graviton)

- open string theory type UV completion

  (photon Regge state effect dominates over graviton Regge state one)

4. Summary and prospects

Summary

as a consequence of unitarity & causality,

heavy charged BH satisfies the WGC bound

$$g^2q^2 > \frac{m^2}{2M_{\rm Pl}^2}$$

- in theories w/light neutral particles (ex. dilaton, moduli)
- in open string theory type UV completion

✔ extension to higher dimension, multiple U(1)'s

✔ entropy correction is positive in these theories

(cf. [Cheung-Liu-Remmen 18'])

Prospects

\# enlarge applicability of our argument

- heterotic string type setup was not covered

  (photon and graviton are from closed string)

- detailed study of Regge amplitudes will be necessary

\# connection to non-SUSY AdS conjecture [Ooguri-Vafa '16]

- AdS = near horizon limit of extremal BHs

- decay of extremal BHs is kinematically allowed in the aforementioned theories

\# extension to WGC for axion

- corrections to extremality condition of black instantons

Thank you!