On dimensional reduction of N=1 Lagrangians for Argyres-Douglas theories

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Plan of the talk

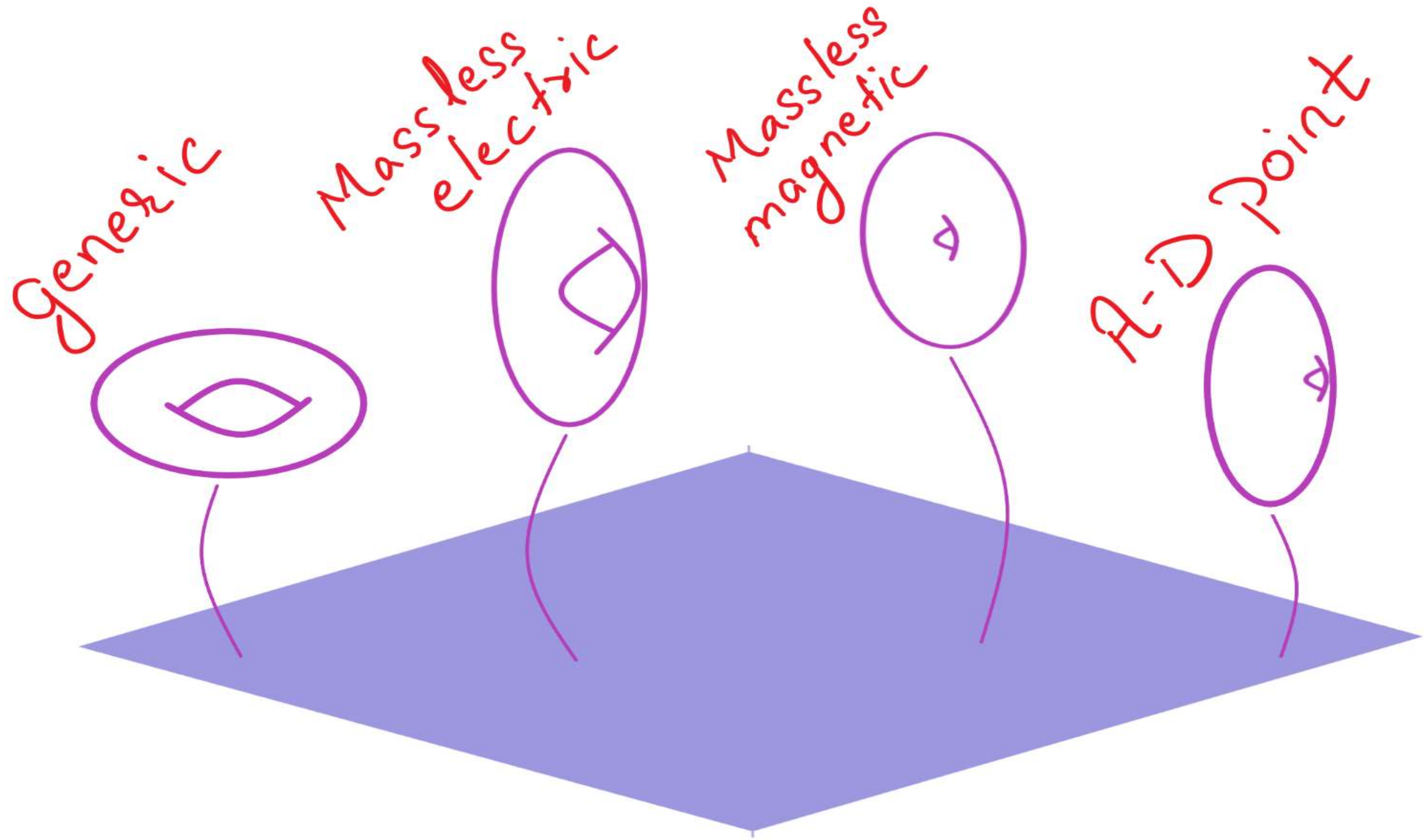
- ❑ Review of Argyres-Douglas (AD) theories
- ❑ Review of N=1 preserving nilpotent deformations
- ❑ 3d reduction of the N=1 Lagrangians
- ❑ Mirroring the N=1 preserving nilpotent deformations
- ❑ Summary and conclusion

Argyres-Douglas (AD) theories

- 4d N=2 Superconformal theories (SCFTs)

- Describe the low energy physics at special loci on the Coulomb branch of generic 4d N=2 theories [Argyres-Douglas `95] [Argyres-Plesser-Seiberg-Witten ‘95]

- At these special loci, magnetic monopoles and electrically charged particles simultaneously become massless


Generic
Massless electric
Massless magnetic
A-D Point

Simplest AD theory

- Supersymmetric U(1) gauge theory + electron + monopole/dyon
- AD point on the Coulomb branch of N=2 SU(2) gauge theory with 1 doublet hyper [Argyres-Douglas `95] [Argyres-Plesser-Seiberg-Witten '95]
- Often called as the H_0 theory

- H_0 has a single Coulomb branch operator with scaling dimension

$$\Delta_{\mathcal{O}} = \frac{6}{5}$$

- central charges are given by [Aharony-Tachikawa `08] [Shapere-Tachikawa `08]

$$a = \frac{43}{120}, c = \frac{11}{30}$$

Minimal 4d theory with N=2 SUSY

- H_0 is believed to be the minimal 4d superconformal theory with 8 supercharges

- 4d N=2 SCFTs obey an analytic lower bound on their central charge

[Liendo-Ramirez-Seo `15]

$$c \geq \frac{11}{30}$$

- H_0 theory saturates this bound

AD theories from type IIB

- AD theories can be obtained by compactification of type IIB on with an isolated singularity

$$CY_3 \subset \mathbf{C}^4 : W(x_i) = 0$$

$$dW = 0 \ \ iff \ \ x_i = 0$$

- Gives a (G, G') classification of AD theories [Cecotti-Neitzke-Vafa`10]

$$W(x,y,z,w) = W_G(x,y) + W_{G'}(z,w) = 0$$

- $W_G(x, y)$ is the superpotential defining ADE singularities

$$W_{A_n}(x,y) = x^{n+1} + y^2$$
$$W_{D_n}(x,y) = x^{n-1} + xy^2$$
$$W_{E_6}(x,y) = x^3 + y^4$$
$$W_{E_7}(x,y) = x^3 + xy^3$$
$$W_{E_8}(x,y) = x^3 + y^5$$

AD theories are Non-Lagrangian

- Impossible to write a manifestly Lorentz invariant Lagrangian with electrons as well as monopoles as elementary degrees of freedom
- Therefore AD theories are inherently non-perturbative
- Their Coulomb phase is well understood; much less is known about their conformal phase
- How to compute their partitions function on $S^4, \ S^3 \times S^1$ etc ?

N=1 preserving nilpotent deformations

- Start with any given 4d N=2 SCFT T_{UV} , with a flavor symmetry F
- The superconformal current multiplet contains a scalar : μ
- Deformation: Introduce chiral superfields M, in adjoint irrep. of F
- Superpotential: $\delta W = M\mu$
- Give M a nilpotent vev

$$\langle M \rangle = \rho(\sigma^+), \ \rho : SU(2) \hookrightarrow SU(2N)$$

- The low energy superpotential becomes [Gadde-Maruyoshi-Tachikawa-Yan `13]

$$\delta W = \mathrm{Tr}\rho(\sigma^{+})\mu_{j=1,m=-1} + \sum_{j,k} M_{j,-j,k}\mu_{j,j,k}$$

- This explicitly breaks supersymmetry down to N=1

- $I_3 \subset SU(2)_R$ and $U(1)_r$ remain unbroken

- N=1 R-symmetry $U(1)_R$: some linear combination of I_3 and $U(1)_r$

- Fix the linear combination via a-maximization [Intriligator-Wecht `03]

- IR central charges are given by [Anselmi-Freedman-Grisaru-Johansen `97]

$$a = \frac{3}{32}(3\mathrm{Tr}R^3 - \mathrm{Tr}R)$$

$$c = \frac{1}{32}(9\mathrm{Tr}R^3 - 5\mathrm{Tr}R)$$

N=1 deformations of N=2 Lagrangian SCFTs

- $T_{UV} = SU(n)_c + 2n$ hypers

  $F = SU(2n)$

  $\rho =$ principal

  $T_{IR} = (\mathrm{A}_1, \mathrm{A}_{2\mathrm{n}-})$

- $T_{UV} = USp(2n)_c + (4n + 4)$ half-hypers

  $F = SO(2n + 4)$

  $\rho =$ principal

  $T_{IR} = (\mathrm{A}_1, \mathrm{A}_{2n})$

[Maruyoshi-Song`16]

N=1 deformations of N=2 Lagrangian SCFTs

- $T_{UV} = SU(n)_c + 2n$ hypers

  $F = SU(2n)$

  $\rho: 2n \to (2n-1) \oplus 1$

  $T_{IR} = (A_1, D_{2n})$

- $T_{UV} = USp(2n)_c + (4n+4)$ half-hypers

  $F = SO(2n+4)$

  $\rho = (2n+1) \oplus 1 \oplus 1 \oplus 1$

  $T_{IR} = (\mathrm{A}_1, \mathrm{D}_{2n+1})$

[PA-Maruyoshi-Song`16]

- N=1 Lagrangians for (A_1, A_n) and (A_1, D_n) AD theories
- use these to compute RG protected quantities such as the superconformal index
- In all 4 cases, the Coulomb branch operators of T_{UV} decouple in the IR
- Some of the gauge singlets $M_{j,-j\ ,k}$ also decouple
- The remaining gauge singlets map to the Coulomb branch operators of the AD theory obtained in the IR

3d reduction of N=1 Lagrangians

- Decoupling of an operator $\mathcal{O}$ can be automatically accounted for by including a flipping field $\beta_{\mathcal{O}}$

$$\delta W = \beta_{\mathcal{O}}\, \mathcal{O}$$

- $\mathcal{O}$ may or may not decouple upon dimensional reduction

- In 3d, R-charges are fixed via Z- extremization

- Generically, extremization point is different if the flipping field β is included or not

- Proposal: the flipping fields are necessary for correct dimensional reduction [Benvenuti-Giacomelli `17]

- (A_1, A_{2n-1}) Lagrangians : no SUSY enhancement in 3d without flipping fields [Benvenuti-Giacomelli `17]

- RG flow to the mirror of (A_1, A_{2n-1}) AD theory upon including the flipping fields [Benvenuti-Giacomelli `17]

- Let's study the expected necessity of including flipping fields further

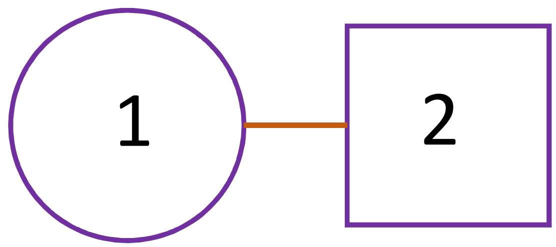
Lagrangian for the (A_1, D_3) AD theory

fields	$SU(2)_{\text{color}}$	$SO(3)_b$	$U(1)_T$	$U(1)_q$	$U(1)_R$	$U(1)_T - \frac{3}{2}U(1)_q$
q_1	$\mathbf{2}$	$\mathbf{3}$	$\frac{1}{4}$	$\frac{1}{2}$	$1-\frac{r_\phi}{2}$	$-\frac{1}{2}$
q_2	$\mathbf{2}$	$\mathbf{1}$	$-\frac{1}{4}$	$\frac{3}{2}$	$1-\frac{r_{M_3}+r_\phi}{2}$	$-\frac{5}{2}$
ϕ	adj	$\mathbf{1}$	$-\frac{1}{2}$	-1	r_ϕ	1
M_3	$\mathbf{1}$	$\mathbf{1}$	1	-2	r_{M_3}	4
β	$\mathbf{1}$	$\mathbf{1}$	1	2	$2-2r_\phi$	-2

$$W = Tr\, q_1\phi q_1 + M_3\, Tr\, q_2\phi q_2 + \beta\, Tr\phi^2$$

In 4d, non-anomalous R-charge : $r_{M_3} - 4\, r_\phi = 0$

- The mirror of the (A_1, D_3) theory : $T[SU(2)]$ theory (self-mirror)



- Thus we expect the above Lagrangian to flow to the $T[SU(2)]$ theory upon 3d reduction

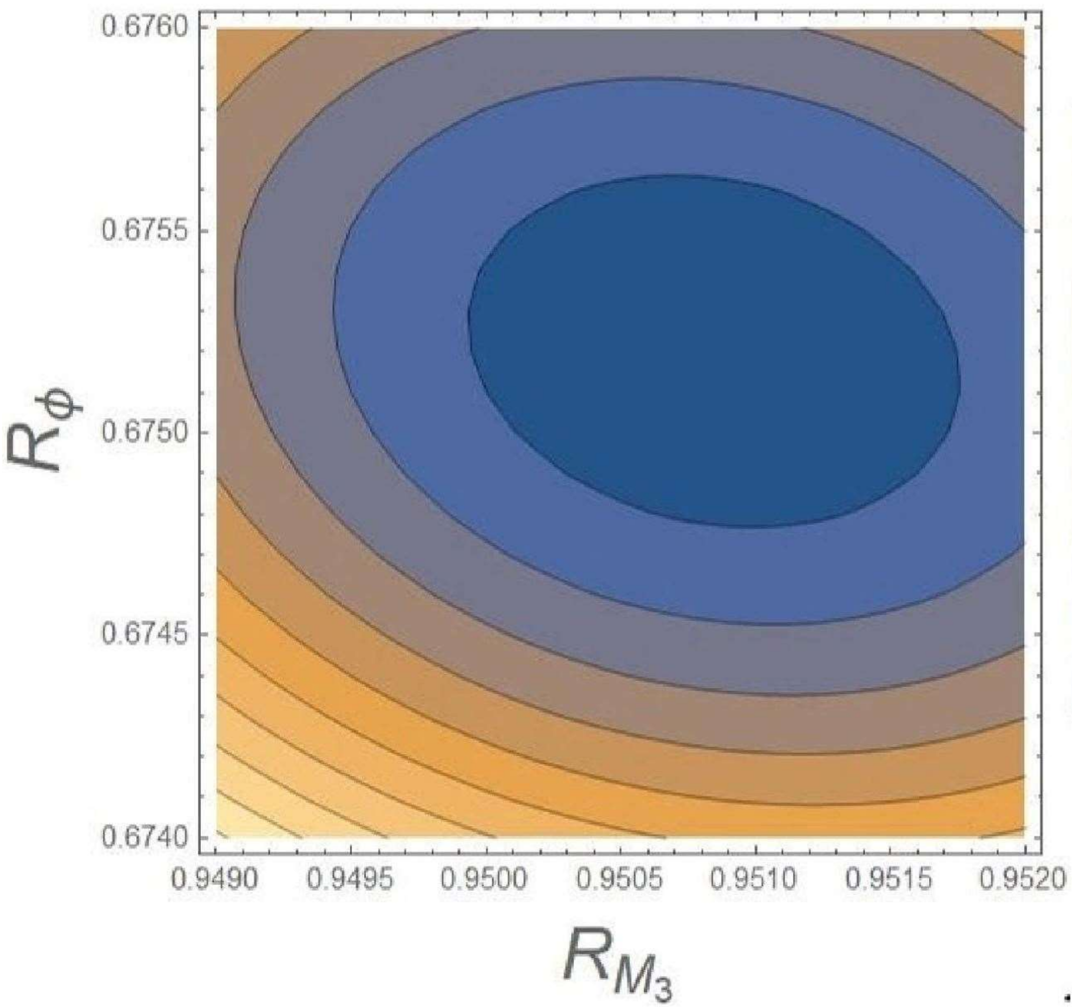
- Upon Z – extremization:

$$R_{M_3} \sim 0.95\,,\, R_\phi \sim 0.67$$

- The $SU(2)$ monopole operator decouples

- Remove the monopole operator contribution and re-extremize

$$R_{M_3} \sim 0.92\,,\, R_\phi \sim 0.68$$



- No SUSY enhancement to N=4 !

- How can we fix this?

- Let us try to find how the chiral operators in our Lagrangian are expected to match with those of the $T[SU(2)]$ theory

- T[SU(2)] theory has an $SU(2)_b$ flavor symmetry acting on its Higgs branch

- Expect this to map to the $SO(3)_b$ flavor symmetry of the (A_1, D_3) Lagrangian

- Constraint on $SO(3)_b$ moment map R-charge:

$$r_\mu = r_{q_1} + r_{q_2} = 1$$

- The Coulomb branch of $T[SU(2)]$ has an $SU(2)_T$ global symmetry
- In 4d : $M_3 \leftrightarrow \mathcal{O}_{Coulomb}$
- Thus, expect M_3 to be one of the 3 components of the $SU(2)_T$ moment map

$$r_{M_3} = 1$$

- Solving the two constraints

$$r_{q_1} + r_{q_2} = 1$$
$$r_{M_3} = 1$$

- Solution : $r_\phi = \frac{1}{2}$, $r_{M_3} = 1$

- This implies :

$$r_{q_1} = \frac{3}{4}\ , \quad r_{q_2} = \frac{1}{4}\ , \quad r_\beta = 1$$

- If SUSY indeed enhances, then $SO(3)_b$ moment map can only be charged with respect to Cartan subgroup of $SO(4)_R$

- Therefore, $U(1)_q$ and $U(1)_r$ should become the Cartan subgroup of $SO(4)_R$ of the enhanced superalgebra

- $U(1)_T$ should correspond to the Cartan of $SU(2)_T$

- Had normalized the $U(1)_q$ and $U(1)_T$ charges with this hindsight

- What about the other two components of the $SU(2)_T$ moment map?
- Upon reduction to 3d, the Coulomb branch chiral will also contain $SU(2)_{color}$ monopole operators : $\mathfrak{m}$, $\{\mathfrak{m}\phi\}$
- The various charges of these are [Benini-Closset-Cremonesi `11]

$$r_{\mathfrak{m}} = \frac{1}{2}\,, \quad r_{\mathfrak{m}\phi} = 1$$

$$T_{\mathfrak{m}} = \frac{1}{2}\,, \quad T_{\mathfrak{m}\phi} = 0$$

$$Q_{\mathfrak{m}} = -1\,, \quad Q_{\mathfrak{m}\phi} = -2$$

- The monopole operator ɱ decouples as a free field

- The operators M_3 and $\{ɱ\phi\}$ has the right charges to become the spin = 1 and spin = 0 components of the $SU(2)_T$ moment map

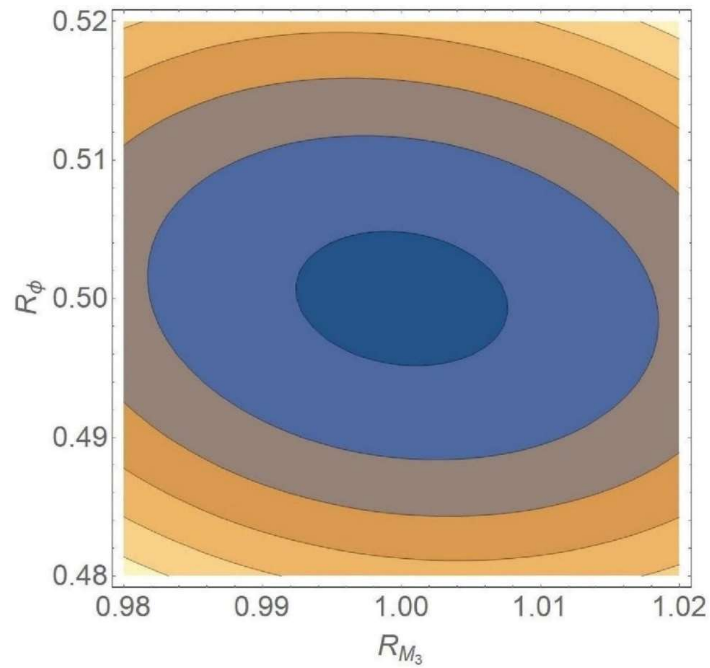
- Nothing has the right charges to give the spin = -1 component of the $SU(2)_T$ moment map

- Charges of β : $r_\beta = 1\,,\ T_\beta = 1,\ Q = 2$

An Observation

- Delete the superpotential term $\beta \; Tr\phi^2$
- β gets decoupled from the Lagrangian
- Now, $Tr\phi^2$ is part of the chiral operator spectrum
- $Tr\phi^2$ has just the right charges to become the spin = -1 component of $SU(2)_T$ moment map

Z – extremization without the flipping field



- The superconformal Index also matches with $T[SU(2)]$

3d Mirror of Nilpotent deformation

- (A_1, D_3) Lagrangian : from N=1 deformation of N=2 SCFT
- N=2 SCFT : $SU(2)_{gauge}$ + 4 fundamental hypers $\Rightarrow$ $SO(8)_{flavor}$
- Deformation:

$$\delta W = \mu_{e_2-e_3} + \mu_{e_3-e_4} + \mu_{e_3+e_4} + \mathrm{M}_3\, \mu_{-e_2-e_3}$$

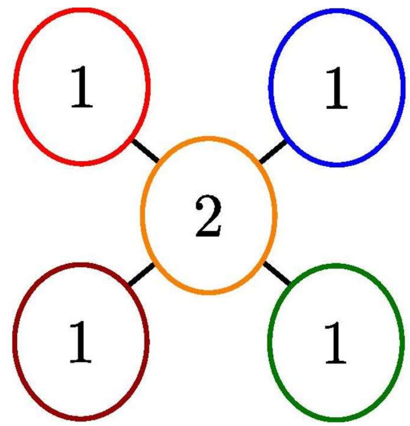
- $\mu_{e_i \pm e_j}$: component of moment map corresponding to the root $e_i \pm e_j$

- In 3d,

$SU(2)_{gauge}$ +
4 fundamental hypers

Mirror Symmetry

⟷



- $SO(8)$ enhancement in the D_4 quiver is due to monopole operators

Mirror Symmetry

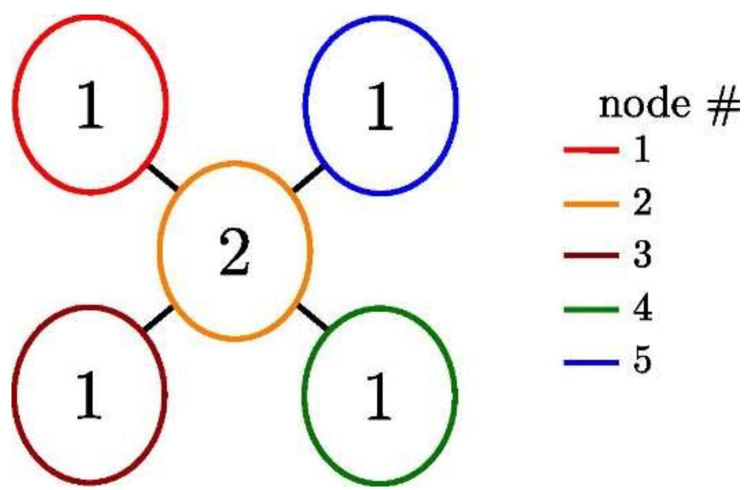
$\mu_{e_i \pm e_j}$ ⟷ $\mathfrak{m}_{e_i \pm e_j}$

- Mirror of the deformation

$$\delta W = \mathfrak{y}_{e_2-e_3} + \mathfrak{y}_{e_3-e_4} + \mathfrak{y}_{e_3+e_4} + \mathrm{M}_3\, \mathfrak{y}_{-e_2-e_3}$$

- The topological charges of a monopole operator are given by its root vectors in the α – basis [Cremonesi – Hanany – Zaffaroni `14]

roots	$U(1)_1$	$U(1)_2$	$U(1)_3$	$U(1)_4$
$\mathfrak{m}_{e_2-e_3}$	0	1	0	0
$\mathfrak{m}_{e_3-e_4}$	0	0	1	0
$\mathfrak{m}_{e_3+e_4}$	0	0	0	1
$\mathfrak{m}_{-e_2-e_3}$	0	-1	-1	-1

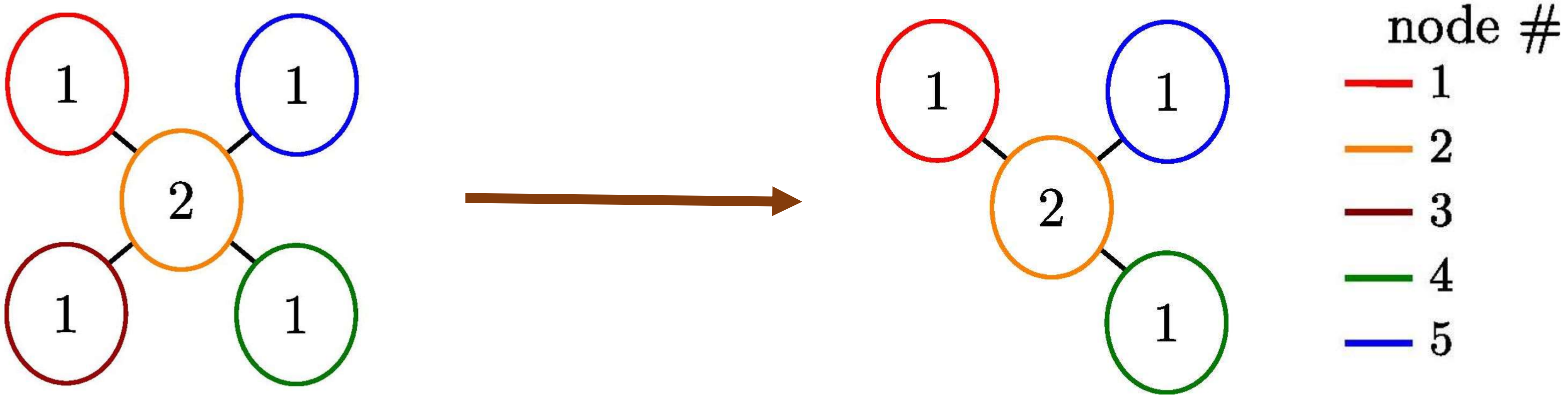


- 3d $\mathrm{N}=2$ $U(N_c)$ gauge theory with N_c+1 fundamental flavors and a monopole superpotential $\mathrm{W}=\mathfrak{m}^+$, undergoes confinement [Benini – Benvenuti – Pasquetti `17]

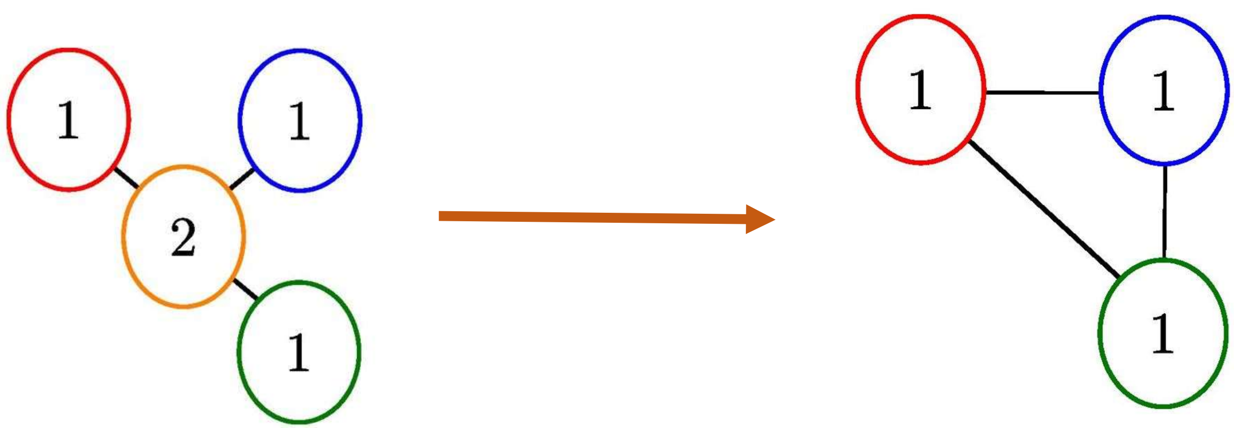
- In our deformed quiver, we can therefore consider the following sequence of nodes to confine

Node # 3 → Node # 2 → Node # 4

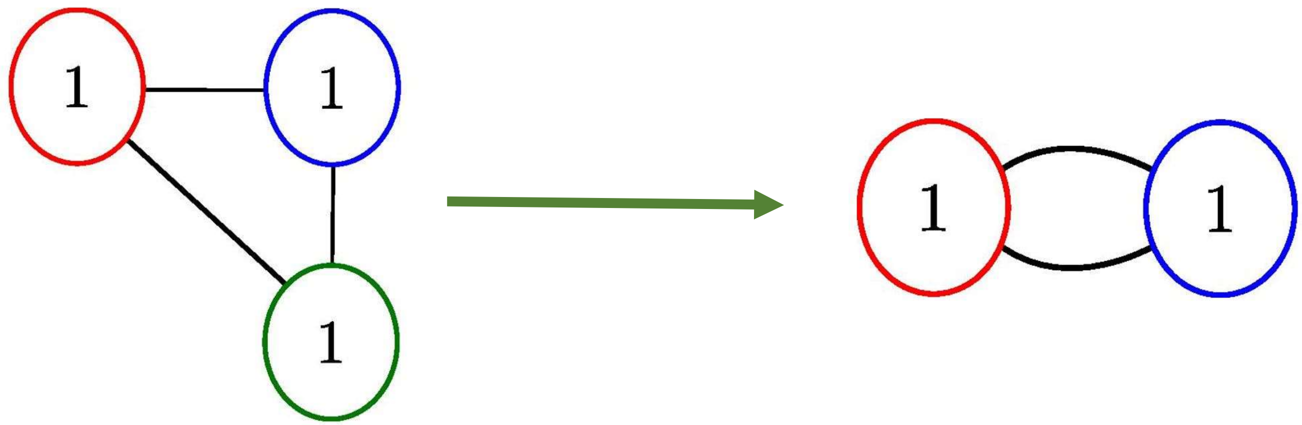
- Confinement of Node # 3



- Confinement of Node # 2



- Confinement of node # 4



- We recover $T[SU(2)]$ from mirror side

- More detailed analysis gives the correct superpotential

Summary and Conclusion

- Argyres – Douglas theories are simplest N=2 SCFTs
- Their non-Lagrangianity poses a major hurdle in understanding their conformal phase
- We have been successful in constructing N=1 Lagrangians whose IR fixed points describe AD theories
- Can use these to compute RG protected quantities such as the superconformal index

- It is also interesting to study dimensional reduction of these Lagrangians

- For (A_1, A_{2n-1}) type cases, correct dimensional reduction requires flipping fields

- However, including the flipping field does not always work

- (A_1, D_3) Lagrangian is a counter example to this expected necessity

- Is there a uniform way to understand when to include the flipping fields ?

- Need to understand the caveats which arise due non-commutation of the RG flow and dimensional reduction

THANK YOU!