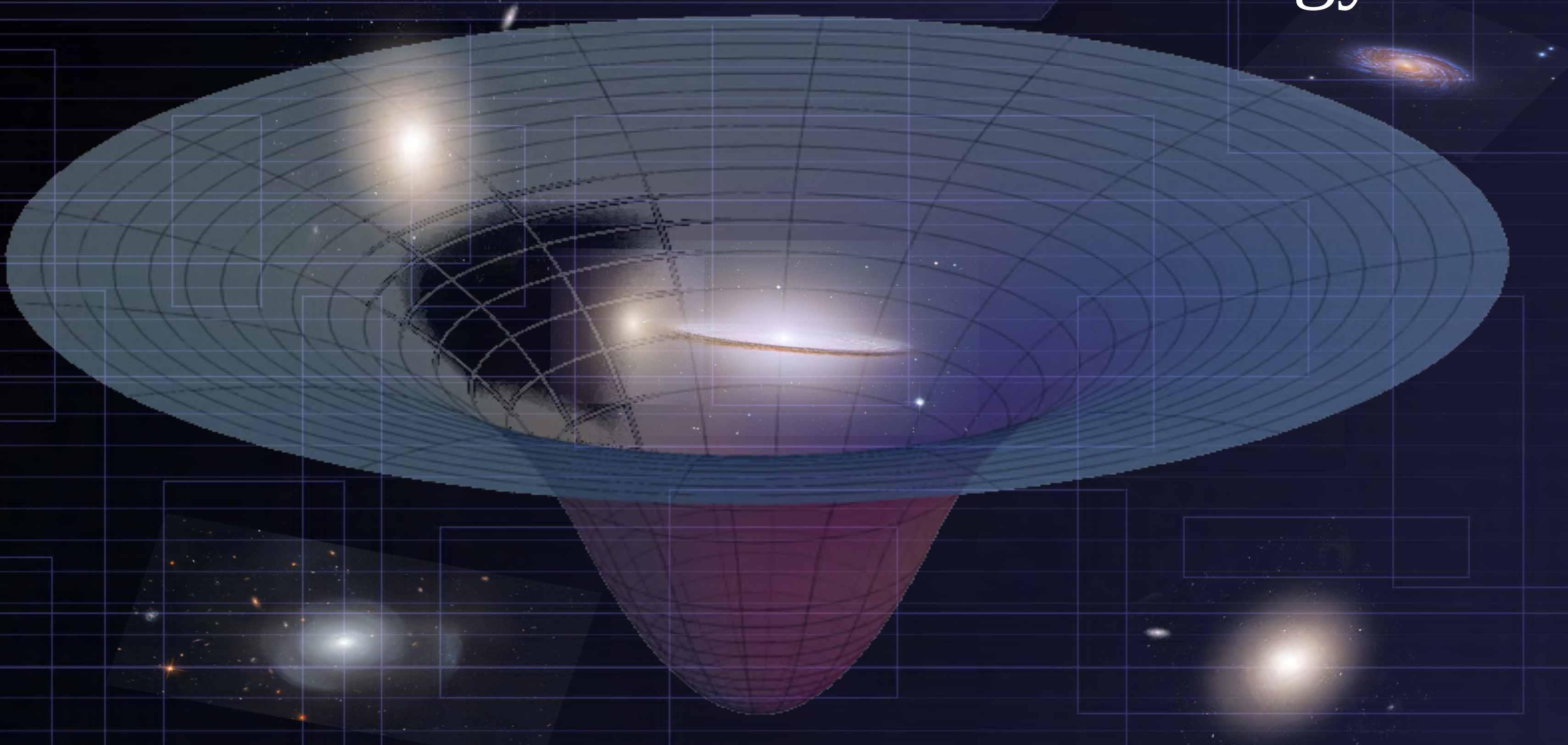
Galaxy Clusters:
A Standard Cannon for Cosmology



Christopher J. Miller

University of Michigan

Galaxy clusters provide us the opportunity to directly or indirectly measure their *gravitational potentials*

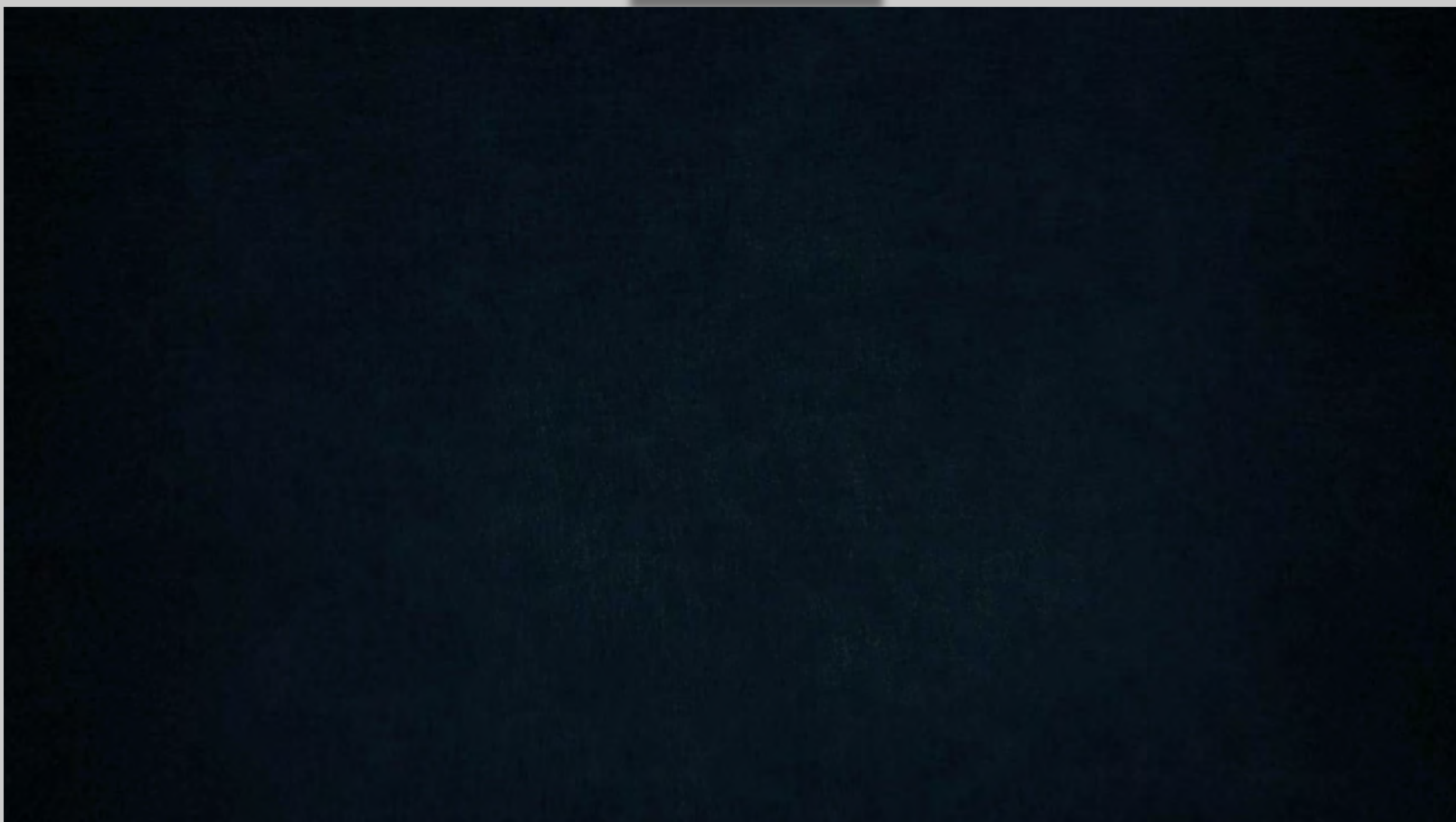
- Weak and Strong Lensing
- Galaxy Dynamics
- Emission of the intra-cluster medium
- Scattering of CMB photons by the ICM

Galaxy clusters provide us the opportunity to directly or indirectly measure their *gravitational potentials*

- Weak and Strong Lensing
- Galaxy Dynamics
- Emission of the intra-cluster medium
- Scattering of CMB photons by the ICM

From Escape Velocity to Mass: the radius-velocity phase space of clusters

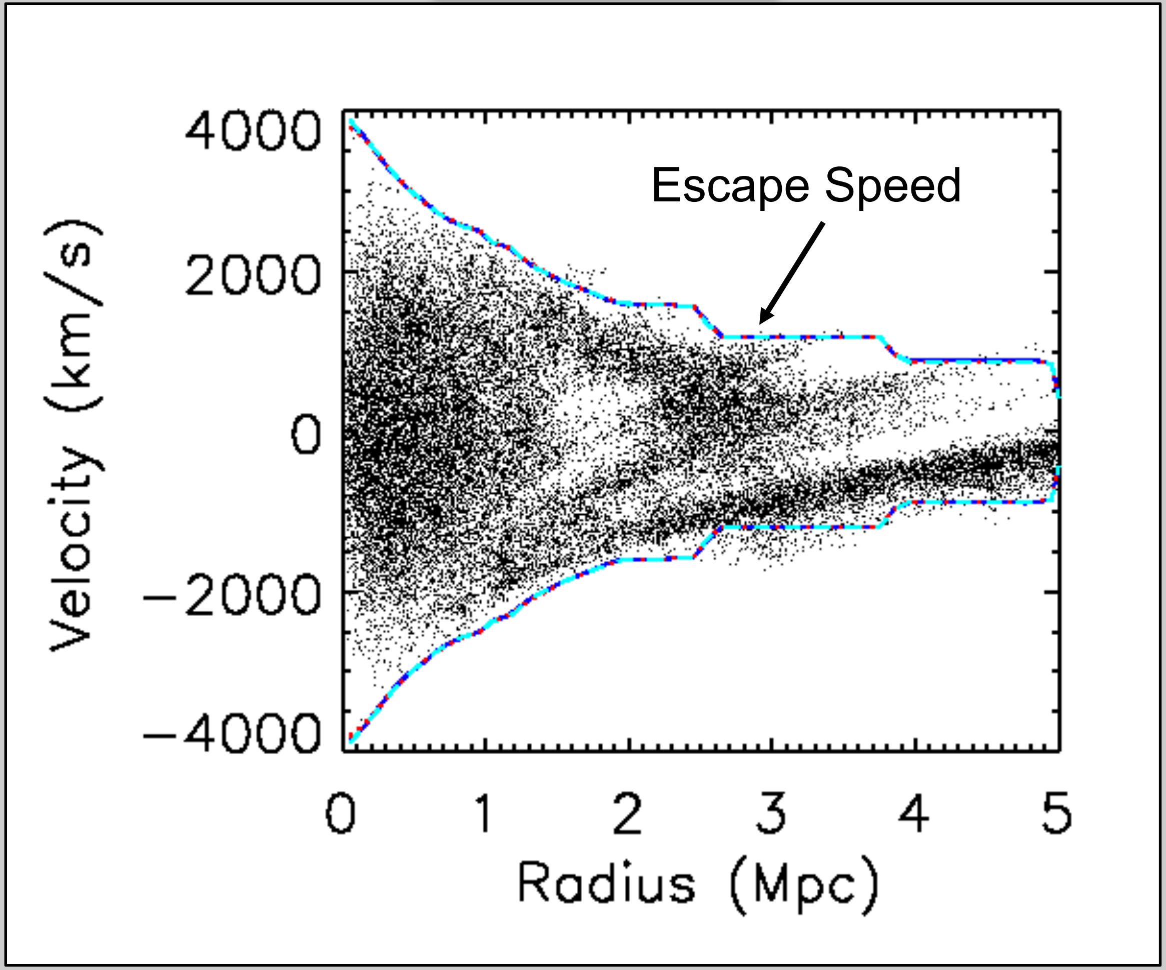
$$\nabla^2\Phi = 4\pi G\rho.$$



Simulation: Wu et al.

Radius-Velocity Phase Space

$$v_{\rm esc}^2(r) = -2\Phi(r).$$



Miller et al. 2016; Diaferio 1997

How do we relate the potential to the escape velocity?

Consider the acceleration on a particle near some mass M in an expanding Universe:

$$a = -\frac{GM}{r^2} - qH^2 r$$

e.g. Lahav et al. 1991, Nandra et al. 2012; Behroozi et al. 2013

Solve for the effective potential:

$$\frac{d\phi}{dr} = \frac{GM}{r^2} + qH^2 r$$

$$\int d\phi = \int \frac{GM}{r^2} dr + qH^2 \int r dr$$

Key 2: Cluster potential depths are governed not only by the mass present,
But also by the accelerated expansion of space. Both of these bend space
time and create curvature.

Aside: what is *q*?

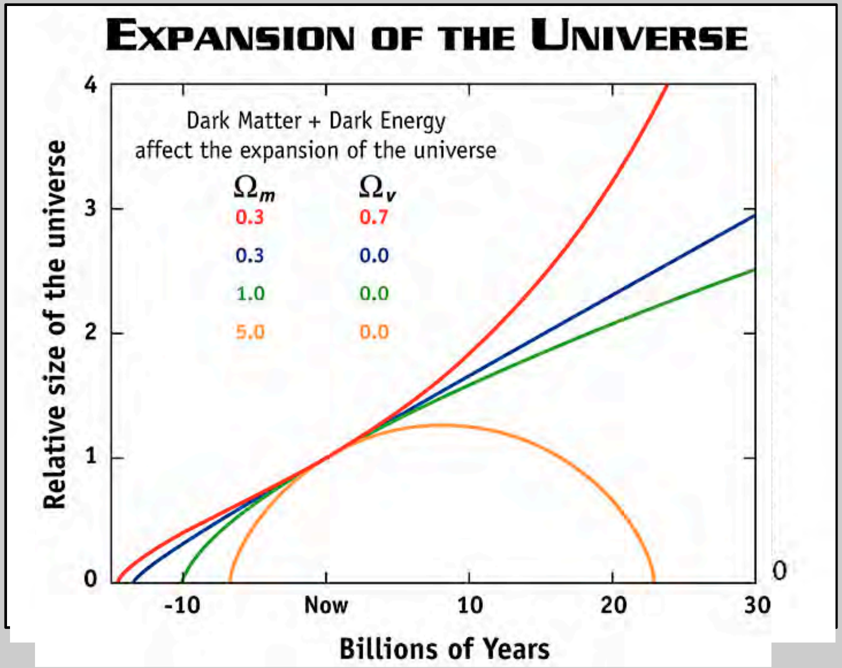
q is the "deceleration parameter", the deceleration of spacetime in the Universe:

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)}\frac{1}{H_0^2} = -\frac{a(t_0)\ddot{a}(t_0)}{\dot{a}^2(t_0)}$$

$$q_0 = \frac{4\pi G}{3}\left(\rho + 3p/c^2\right)\frac{3}{8\pi G\rho_c} = \frac{\Omega_0}{2}\left[1 + 3p/(\rho c^2)\right]$$

$$a = -\frac{GM}{r^2} - \boxed{qH^2 r}$$

Sandage (1961) suggests that modern cosmology will be about quantifying H_0 and q_0.



Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **422,** 2945–2959 (2012) doi:10.1111/j.1365-2966.2012.20617.x

The effect of an expanding universe on massive objects

Roshina Nandra,[1,2★] Anthony N. Lasenby[1,2★] and Michael P. Hobson[1★]

[1]*Astrophysics Group, Cavendish Laboratory, JJ Thomson Avenue, Cambridge CB3 0HE*
[2]*Kavli Institute for Cosmology, c/o Institute of Astronomy, Madingley Road, Cambridge CB3 0HA*

Consider the concept of escape in a Universe that is accelerating away from all locations. *Tracers no longer need to escape to infinity!* They only need to reach a radius where the pull of gravity is balanced by the ”push” of the accelerating expansion of the Universe:

$$a = -\frac{GM}{r^2} - qH^2 r$$

$$0 = -\frac{GM}{r^2} - qH^2 r$$

$$r_{eq}^3 \equiv -\frac{GM}{qH^2}$$

Behroozi et al. 2013

Instead of integrating from some distance *r* to infinity (typical case), go out to some radius, r_{eq}:

$$\phi(r_{eq}) - \phi(r) = GM \int_r^{r_{eq}} r'^{-2} dr' + qH^2 \int_r^{r_{eq}} r' dr'$$

How do we relate the potential to the escape velocity?

The result of the Poisson equation for a point mass in an expanding Universe:

$$\phi(r) = GM\left(\frac{1}{r_{eq}} - \frac{1}{r}\right) - \frac{qH^2}{2}(r_{eq}^2 - r^2) + \phi(r_{eq})$$

Where there is a boundary condition. Relative to the cluster, escape occurs at the equivalence radius:

$$v_{esc}^2 = -2\phi \qquad\qquad v_{esc}^2 = 0 \text{ as } r \rightarrow r_{eq}$$

$$v_{esc}^2 = \frac{2GM}{r} \boxed{- qH^2(r^2 - 3r_{eq}^2)}$$

The acceleration of the Universe at late times *lowers* the escape velocity.

Predicting the phase-space escape velocity edge from the potential

Example: NFW

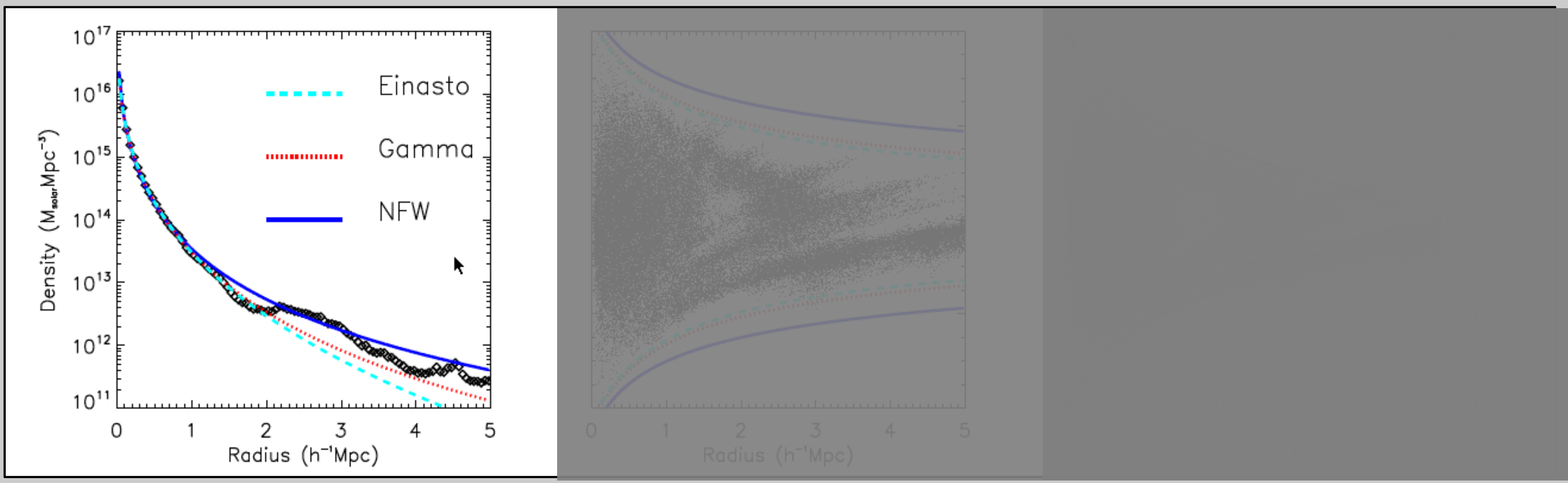
$$\rho(r) = \frac{\rho_0}{(r/r_0)(1+r/r_0)^2}$$

$$\phi(r) = -\frac{4\pi G\rho_0(r_0)^2 \ln(1+r/r_0)}{r/r_0}$$

$$\phi(r) = GM\left(\qquad \frac{1}{r}\right)$$

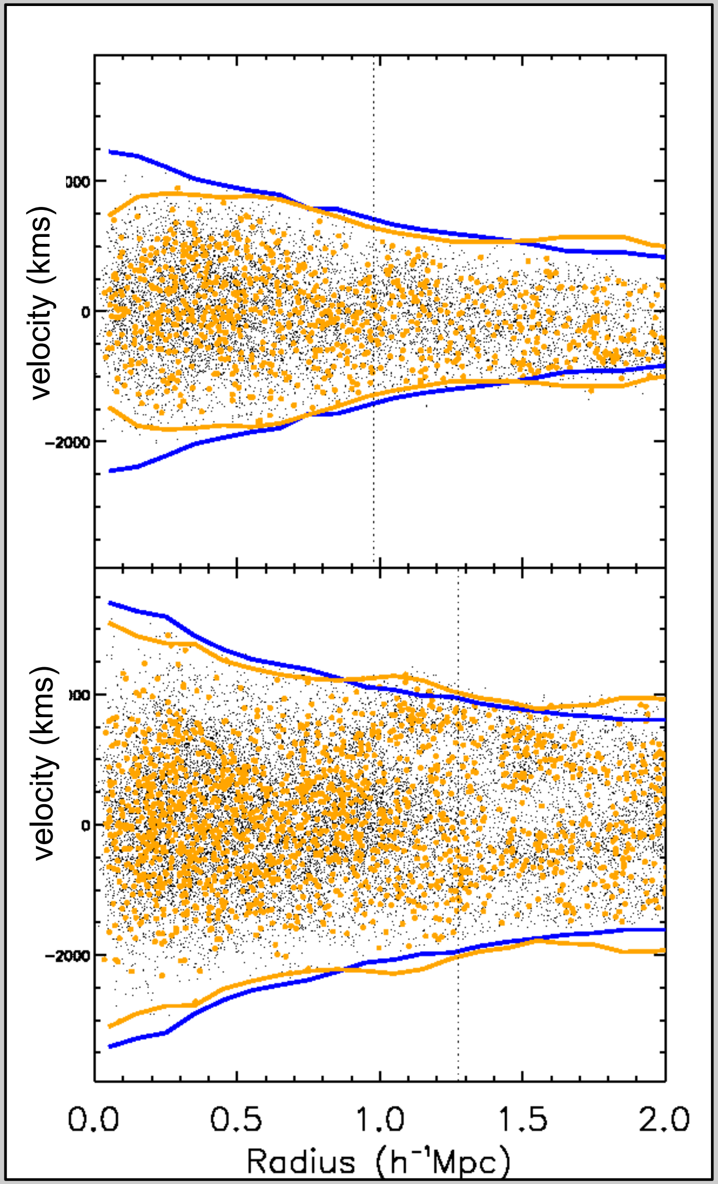
$$v_{esc}^2 = -2\phi$$

$$v_{esc}^2 = 0 \text{ as } r \to r_{eq}$$



Using the Millennium Simulation (Springel et al.)

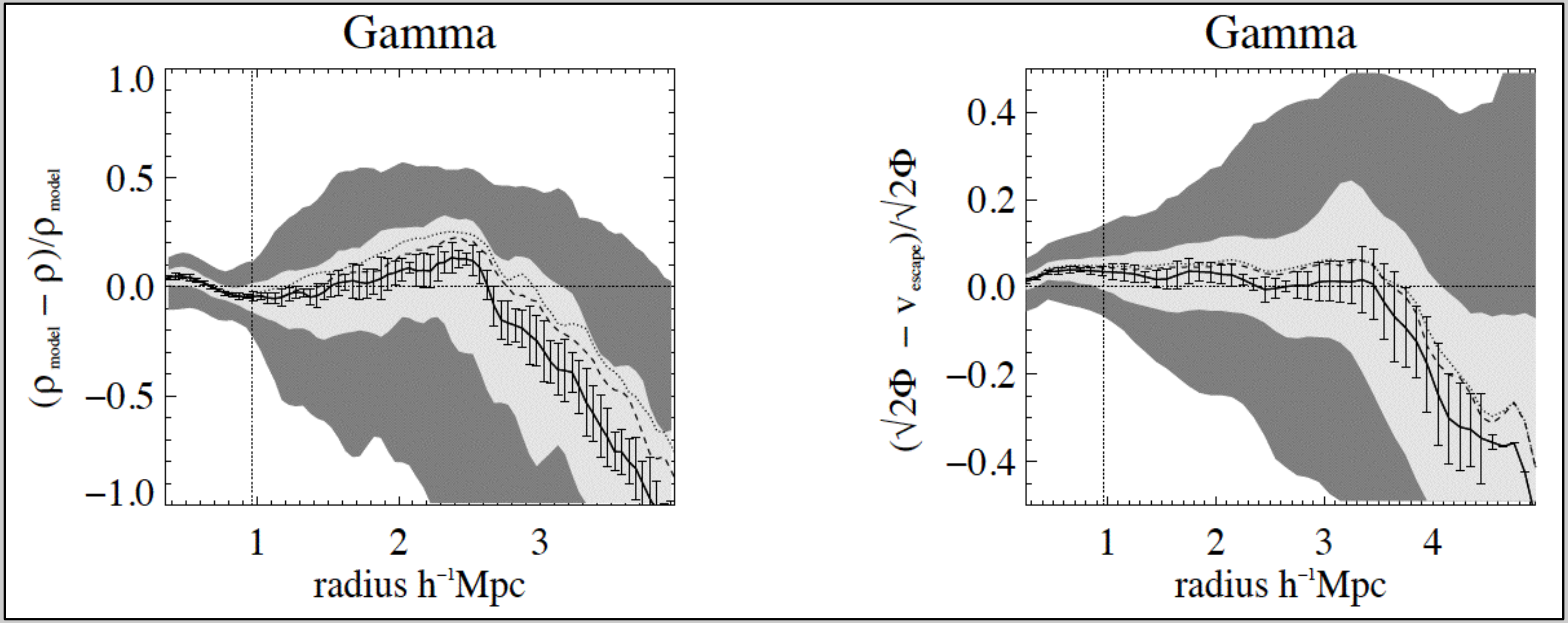
Measuring the edge



The edge is identified as the minimum of the two maximum surfaces defined by either particles or galaxies in the simulation.

Outside the core, we find no difference in the location of the edge whether we use particles, sub-halos, or "galaxies" (in simulations)

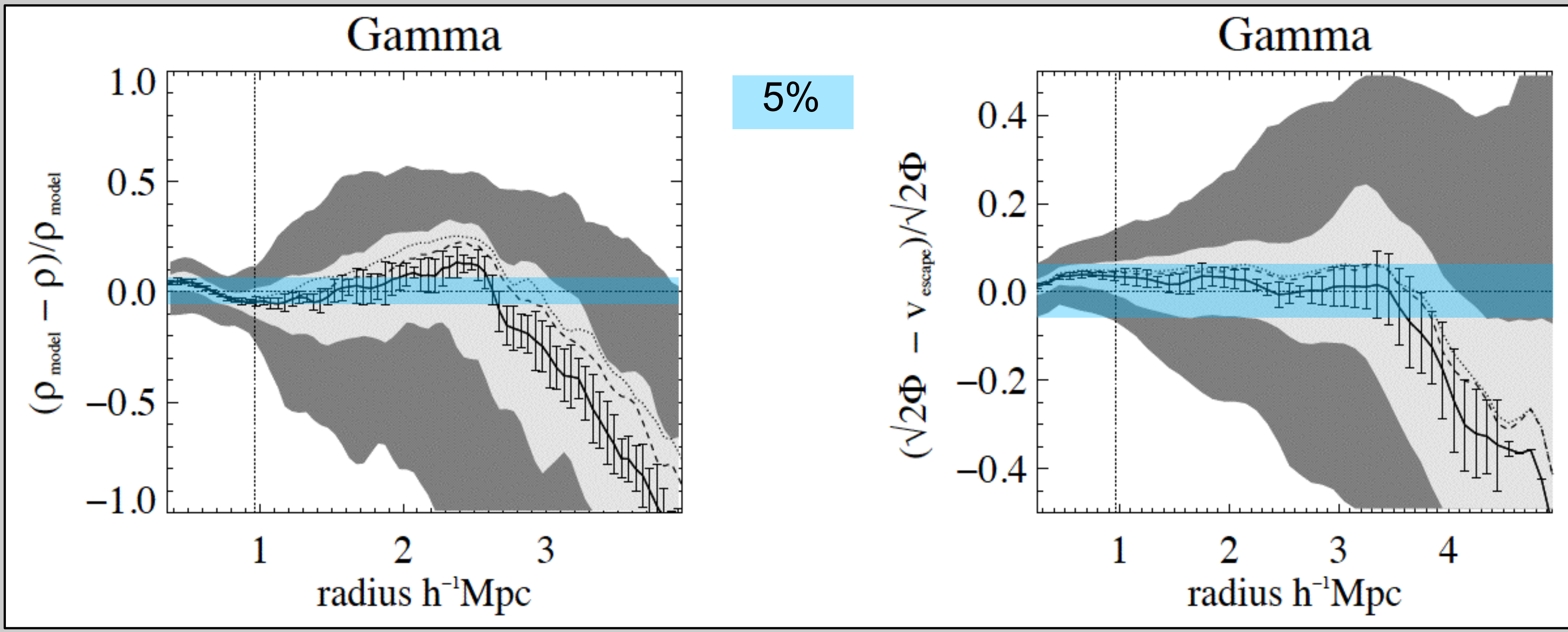
How well can we infer the *average* escape velocity given cosmology and an *average* cluster density profile?



Miller, et al. 2016

100 Clusters from the Millennium

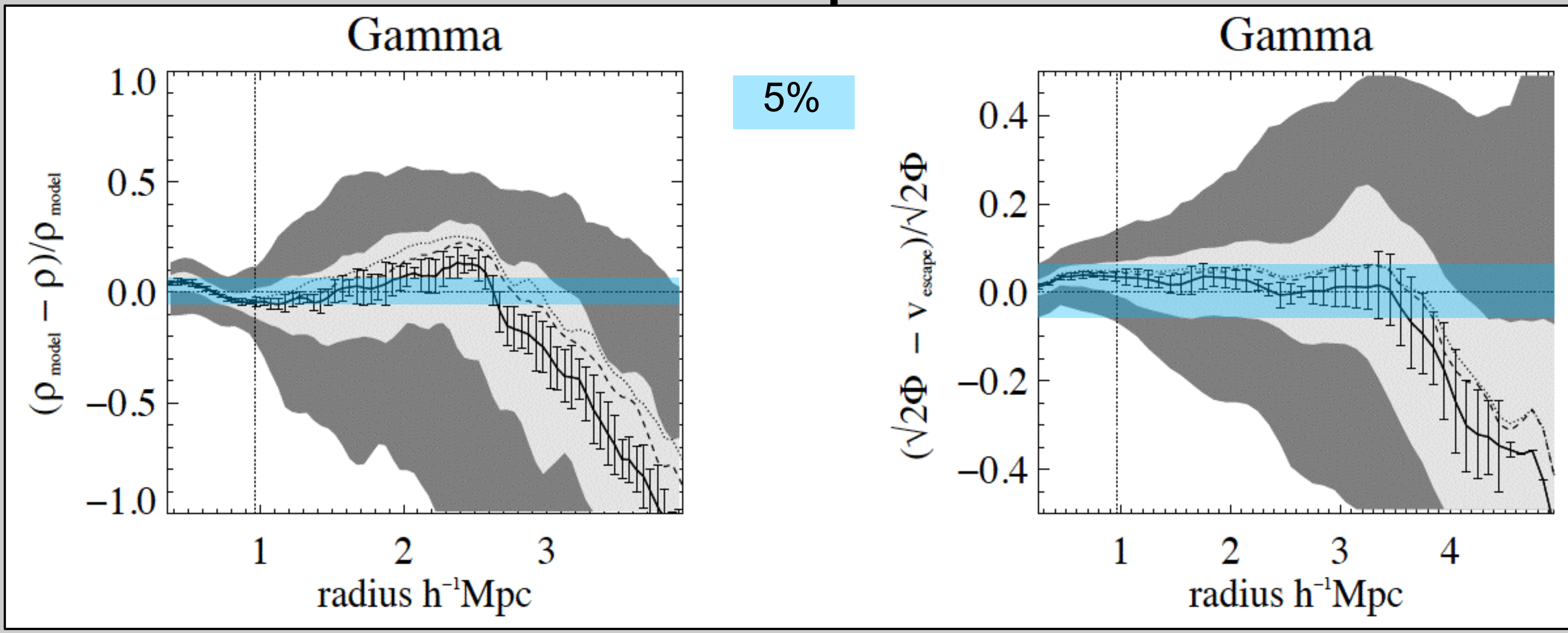
How well can we infer the *average* escape velocity given cosmology and an *average* cluster density profile?



Miller, et al. 2016

100 Clusters from the Millennium

Precision Cosmology with Cluster Phase-spaces



Einasto

$$\rho(r) = \rho_0 \exp\Big[-\Big(\frac{\mathrm{r}}{\mathrm{r}_0}\Big)^{1/\mathrm{n}}\Big] \tag{6a}$$

$$\phi(r) = \frac{-\mathrm{GM}}{r}\Big[1 - \frac{\Gamma(3n, \frac{r}{r_0}^{(1/n)})}{\Gamma(3n)} + \frac{r}{r_0}\frac{\Gamma(2n, \frac{r}{r_0}^{(1/n)})}{\Gamma(3n)}\Big] \tag{6b}$$

Gamma

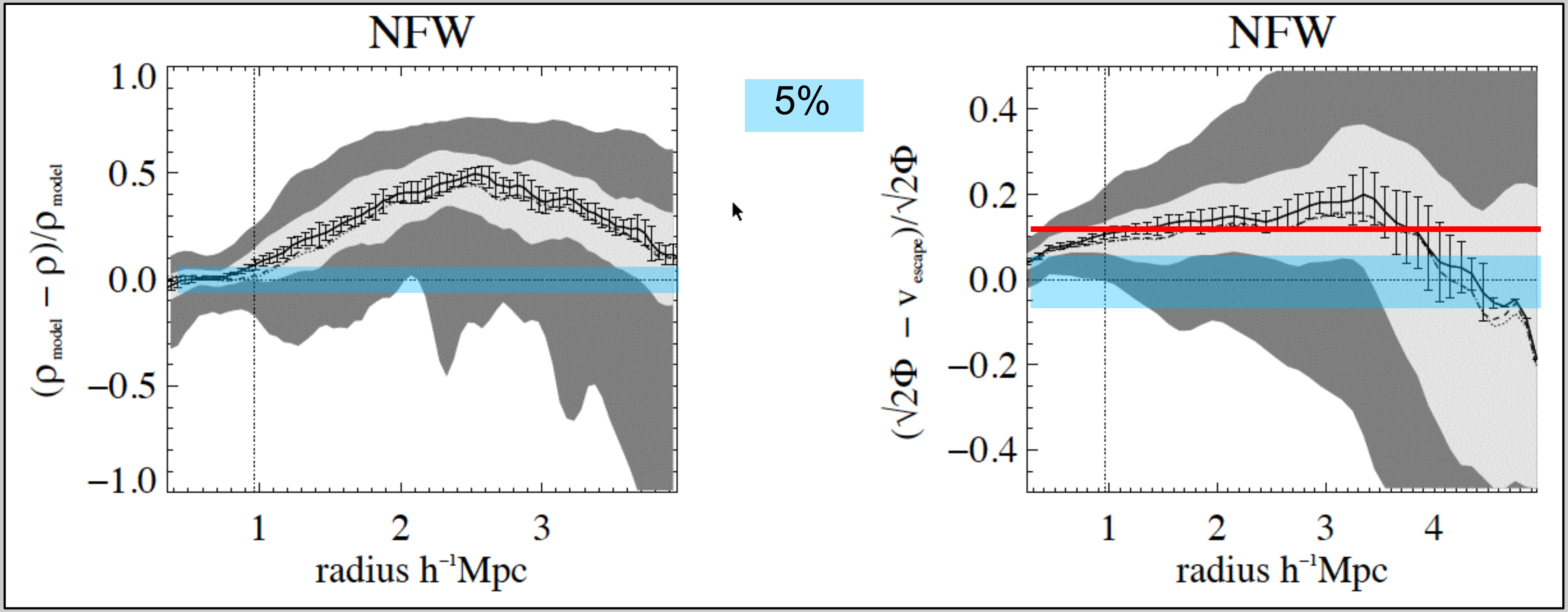
$$\rho(r) = \frac{(3-n)\mathrm{M}}{4\pi}\frac{r_0}{r^n}(r+r_0)^{4-n}$$

$$\phi(r) = \frac{\mathrm{GM}}{r_0}\frac{-1}{2-n}\Big[1-\Big(\frac{r}{r+r_0}\Big)^{2-n}\Big], n \neq 2$$

$$= \frac{\mathrm{GM}}{r_0}\ln\frac{r}{r+r_0}, n = 2$$

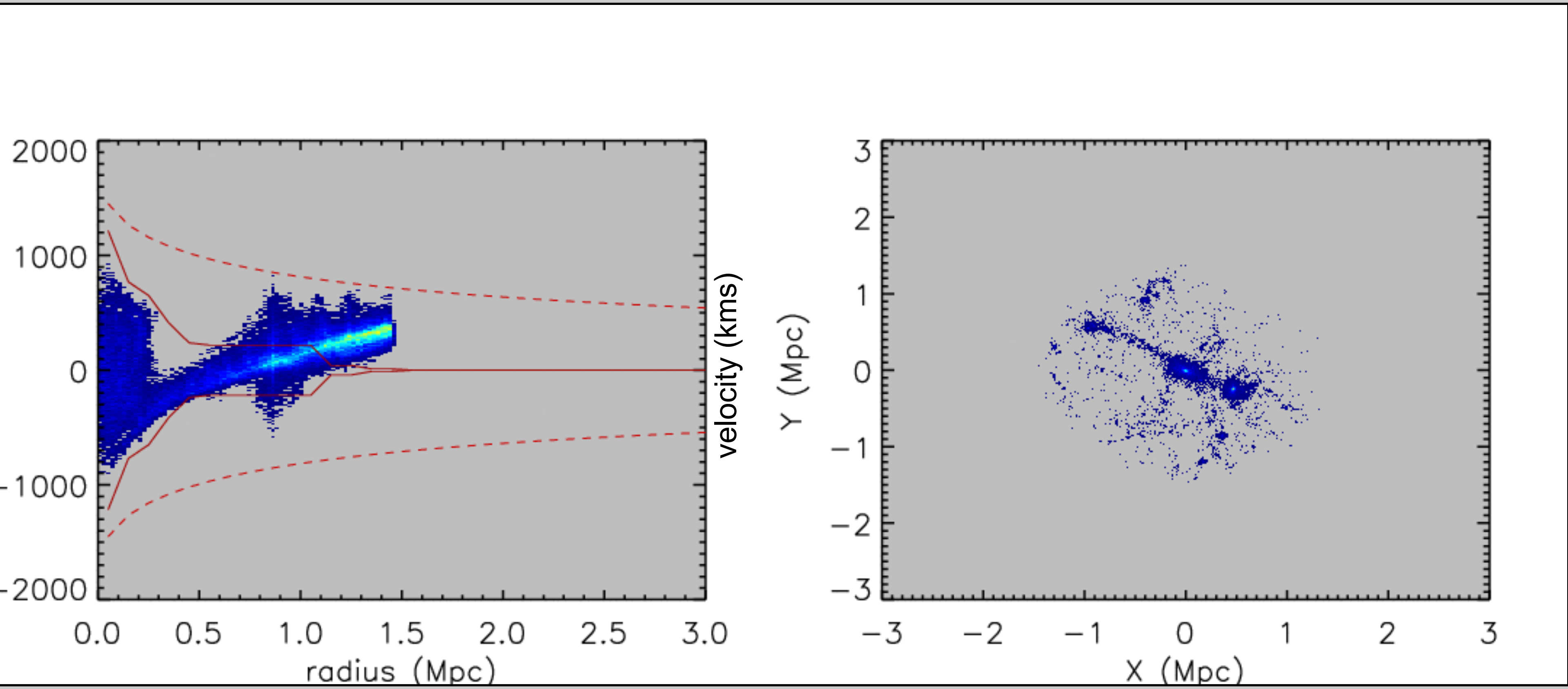
Key 3: A good density profile needs to work in conjunction with the dynamical potential. In 3D, theory predictions are accurate to within a few percent.

The NFW fails the test.



The NFW outer shape does not match the observed (simulated) density profiles. NFW suggests that there is more mass associated with the cluster than is actually allowed by the Poisson equation.

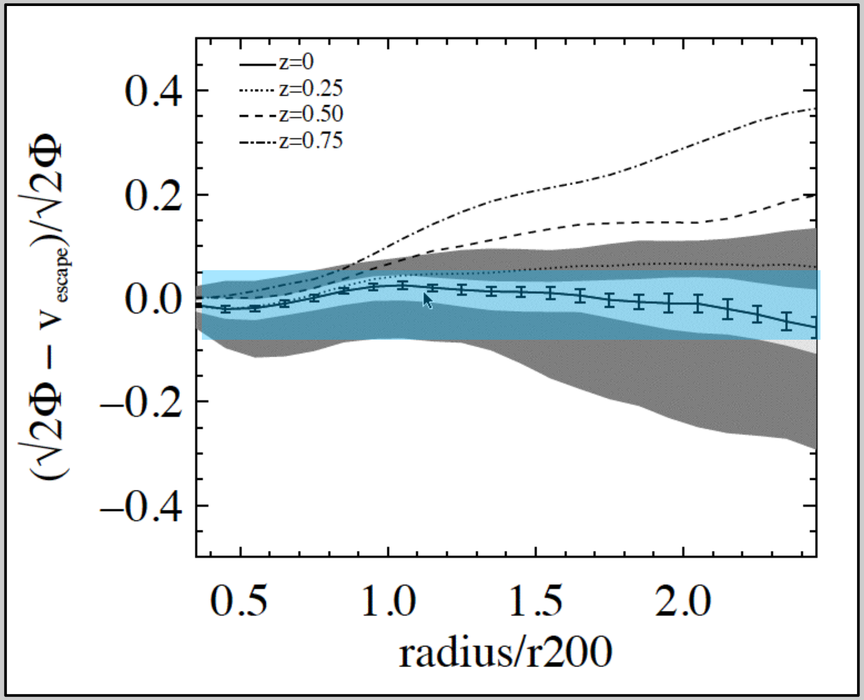
The Evolution of the Escape Edge



From z = 0.75 to z = 0

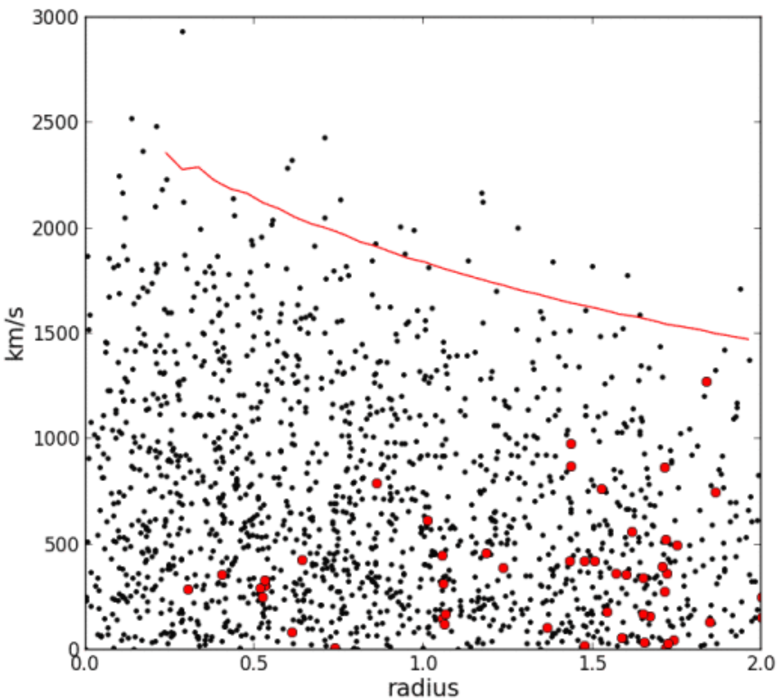
The Evolution of the Escape Edge

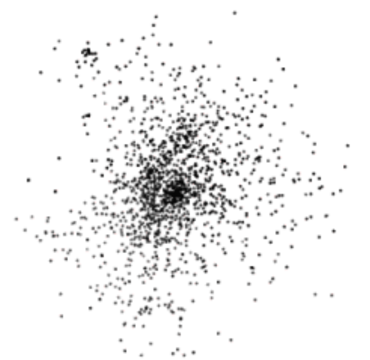
1. Outside the virial radius, the escape edge needs time to reach the expectation from the potential.
2. Tracers (particles or sub-halos) which have radial velocities above the potential escape. Everything else does not.
3. The predicted escape edge shrinks as acceleration of the Universe increases.

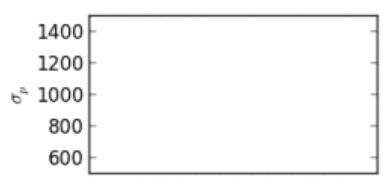


Miller, et al. 2016

Velocity Anisotropy and 2D reality



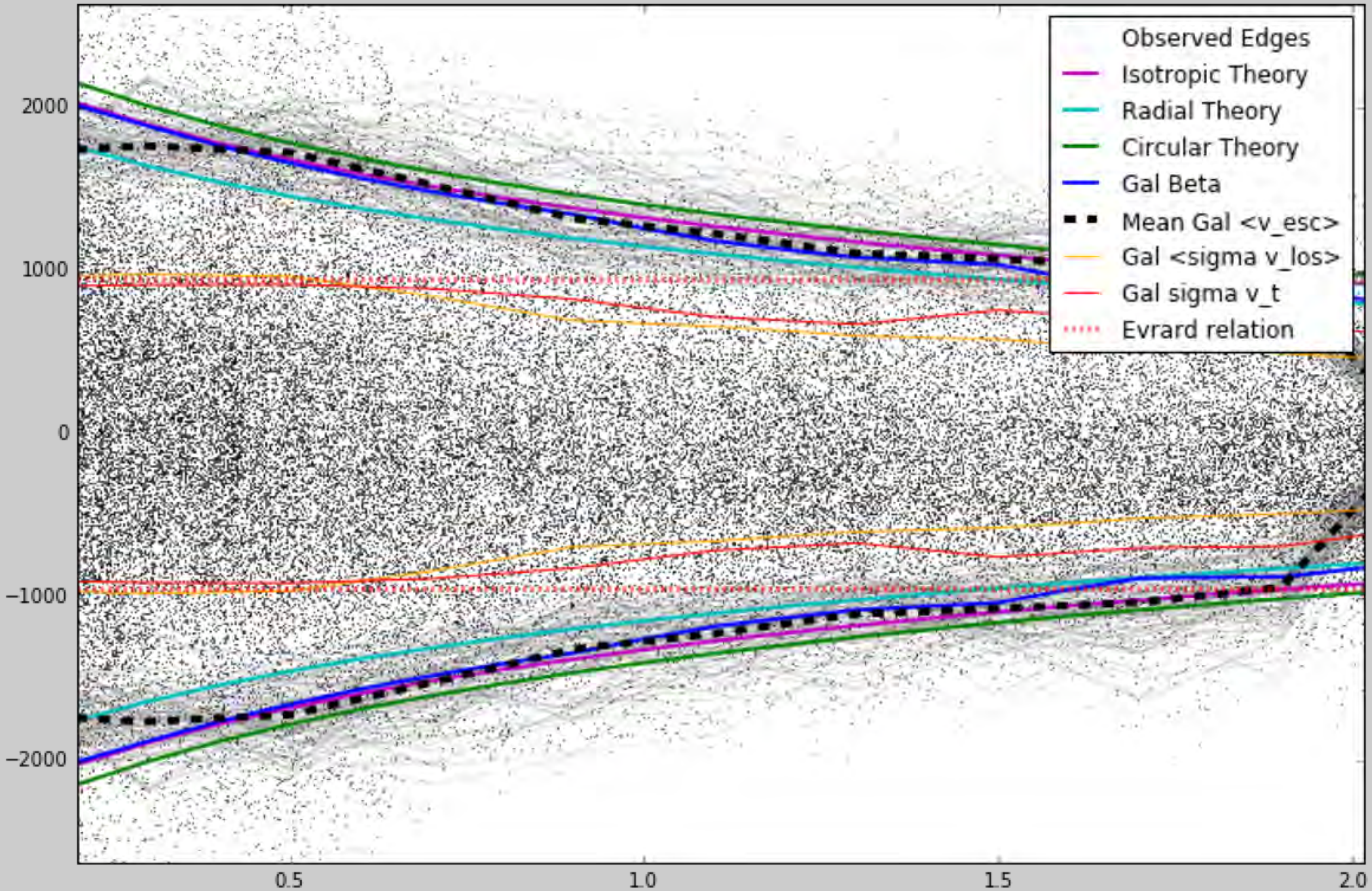




Gifford, Miller, Kern 2013

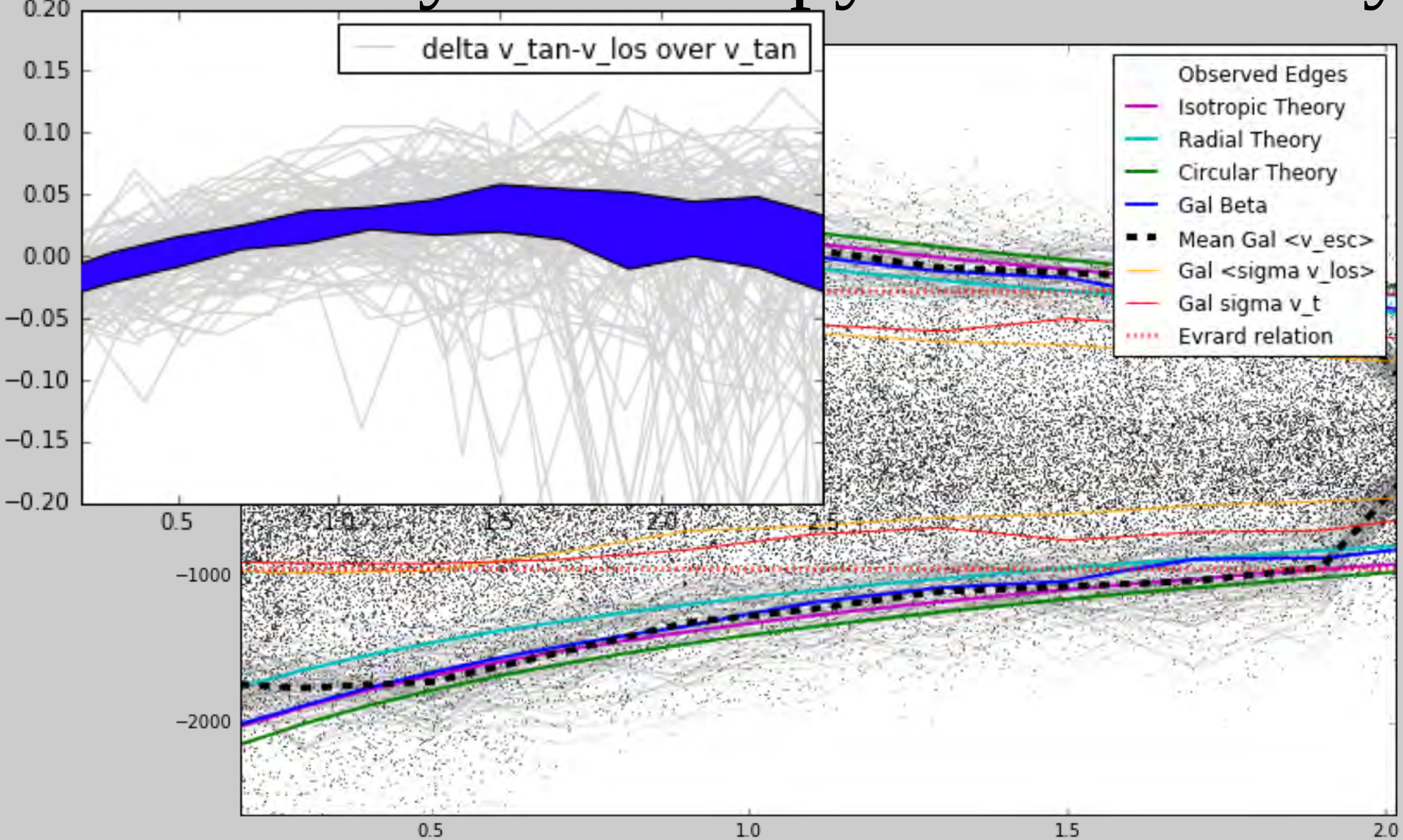
$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

Velocity Anisotropy and 2D reality



$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

Velocity Anisotropy and 2D reality



$$\beta \equiv 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

Escape Velocity Inferred Masses

We can invert the Poisson equation and solve for mass in terms of the escape velocity edge::

$$GM(< R) \quad = \quad \int_0^R \mathcal{F}_\beta(r)\langle v_{los,esc}^2\rangle(r)dr$$

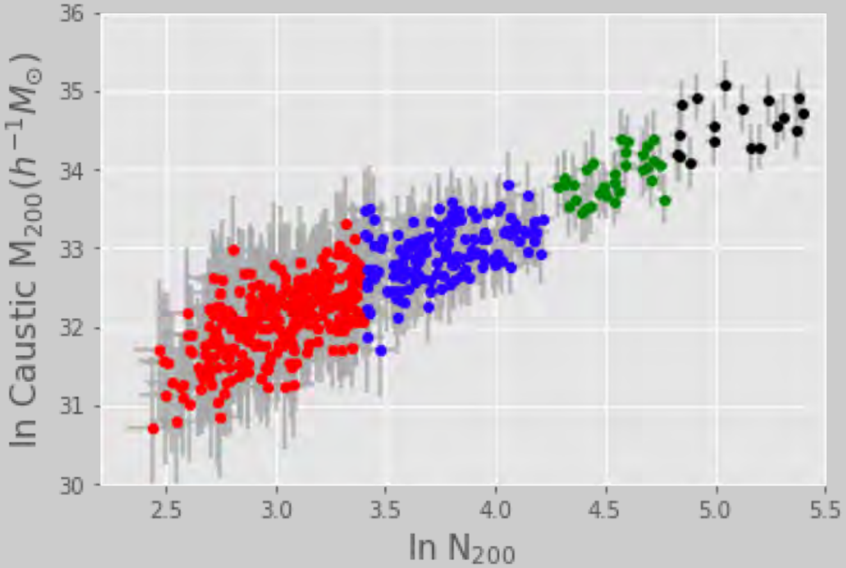
Now, all of the messiness of the profile and the velocity anisotropies are absorbed into F_β:

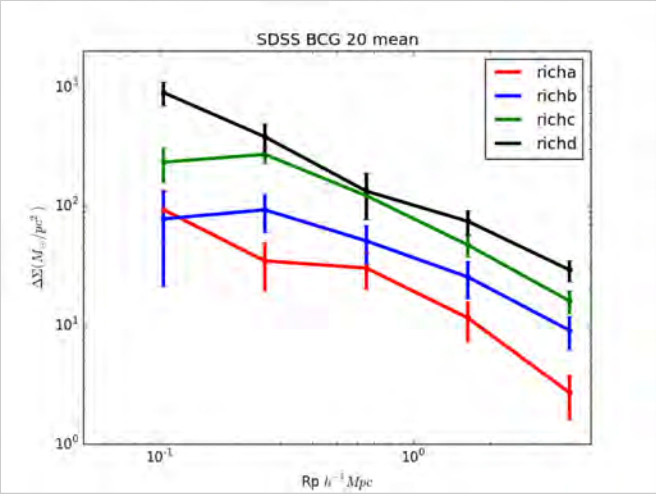
$$\mathcal{F}_\beta(r) = -2G\pi g(\beta(r))\frac{\rho(r)r^2}{\Phi(r)}$$

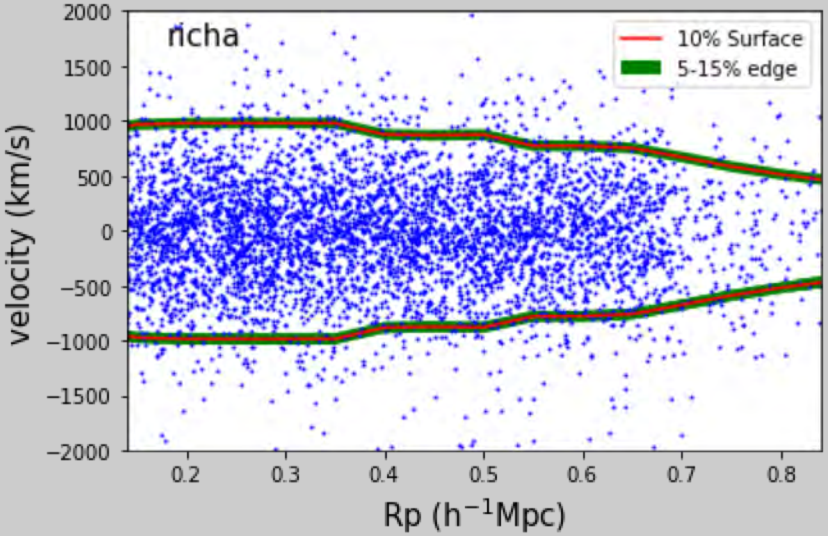
$$\mathcal{F}_\beta = 0.65$$

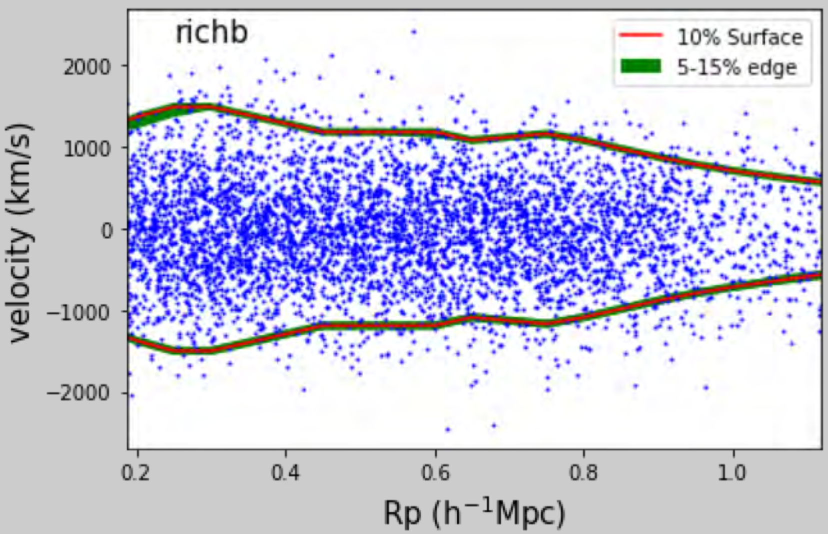
Key 4: The projected escape edge becomes a tool to measure galaxy cluster masses with systematics <5% if F_β can be calibrated.

Escape Velocity Inferred Masses: Precision and Accuracy

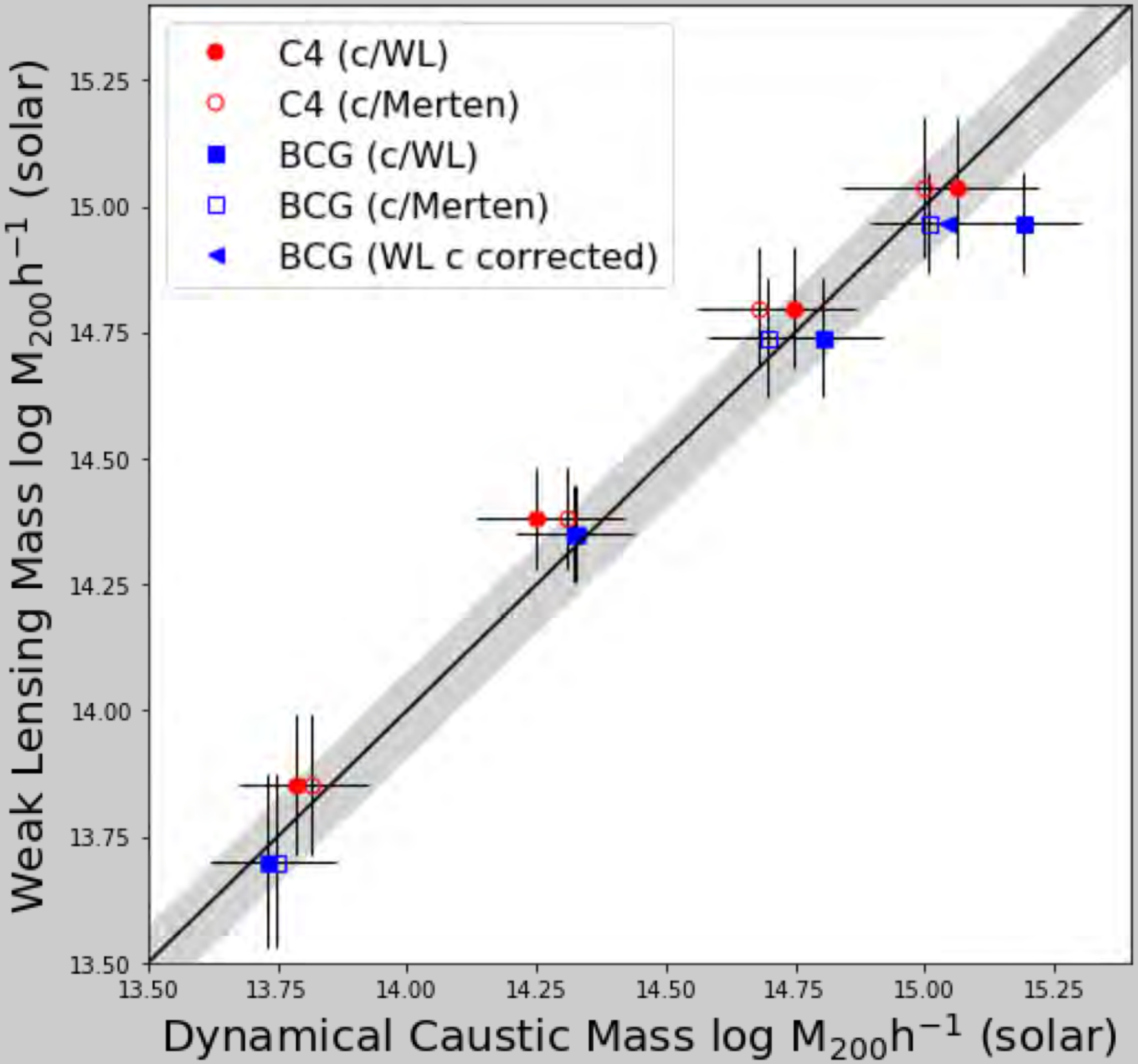








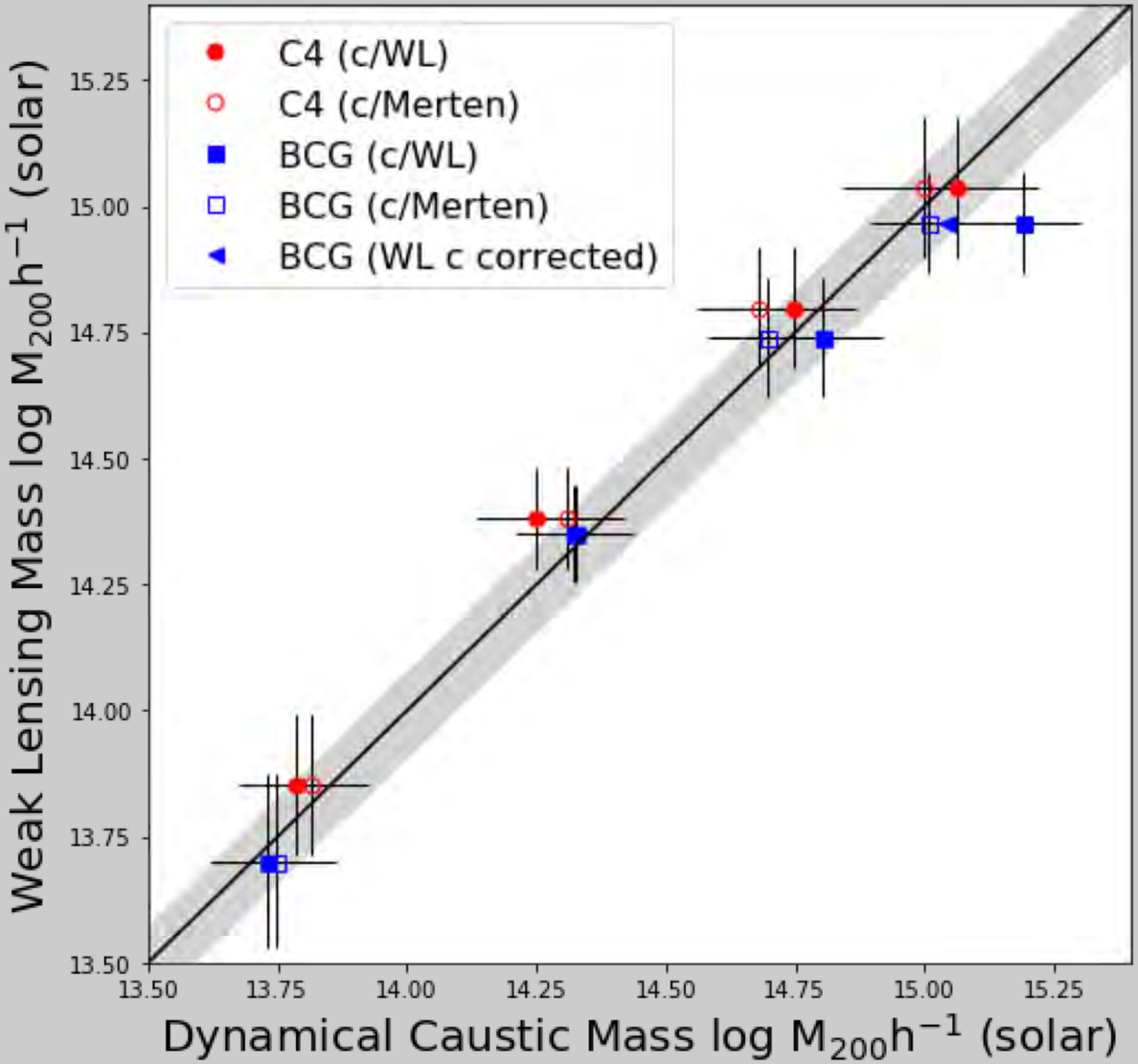
Escape Velocity Inferred Masses: Precision and Accuracy



If we assume GR and pick a (consistent) cosmology for ΛCDM, the bias between WL inferred cluster masses and escape-velocity masses is:

0.002 +/- 0.014 dex

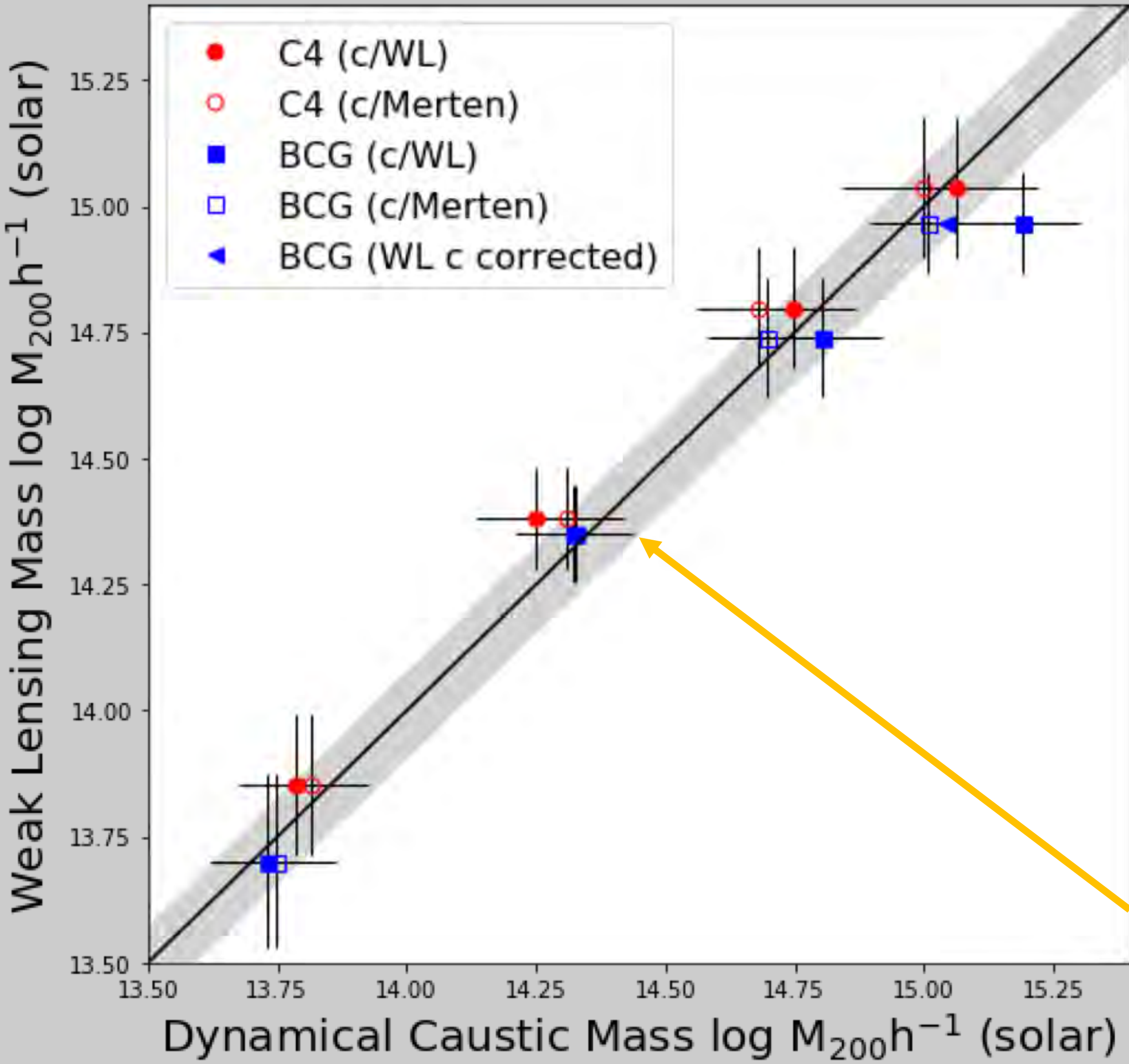
Escape Velocity Inferred Masses: Precision and Accuracy



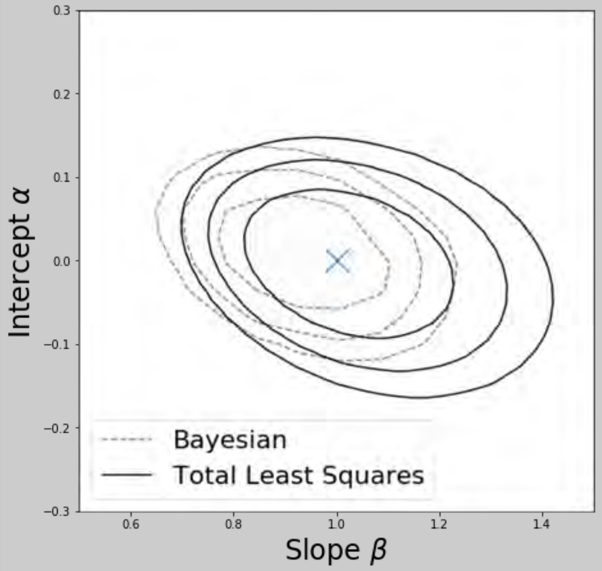
If we allow for centering and profile shape uncertainties when measuring the masses, the error on the bias goes up to:

+/- 0.07 dex

Escape Velocity Inferred Masses: Precision and Accuracy



If we allow for a mass dependent bias:



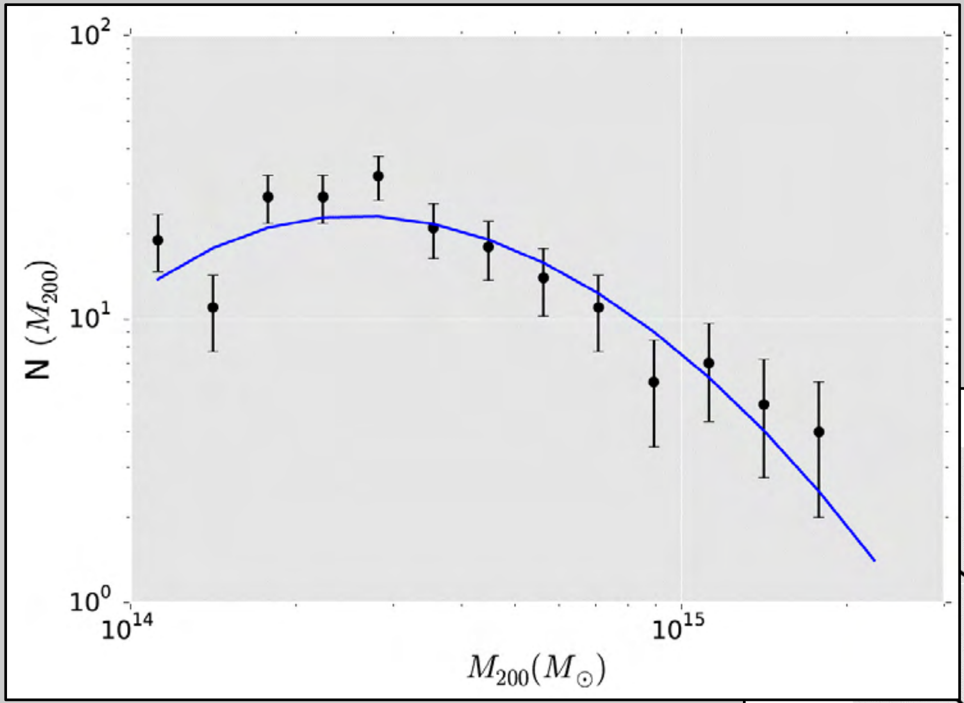
intercept: 0.01 +/- 0.07 dex

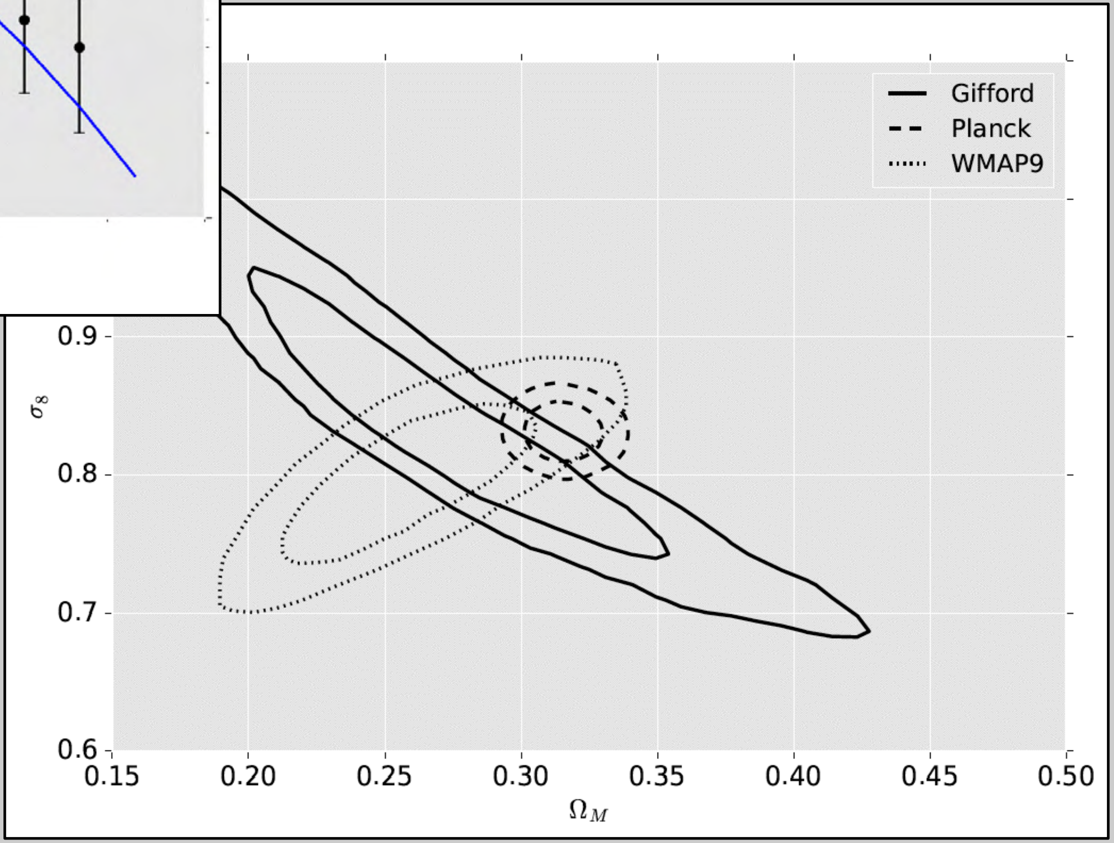
slope: 0.94 +/- 0.15 dex

Use Case #1: Constraining the Cluster Mass Function

- The normalization of the mass function on cluster scales is an important cosmological parameter. Planck and WMAP (and others) disagree on its value.
- For this we need:
  - **A cluster catalog**
    - We use the SDSS-C4 catalog
  - **Simulations to calibrate the catalog selection**
    - We calibrated against Millennium and MICE
  - **Cluster phase-space masses marginalized over F_β**
    - Using escape edges

The SDSS-C4 Cluster Mass Function



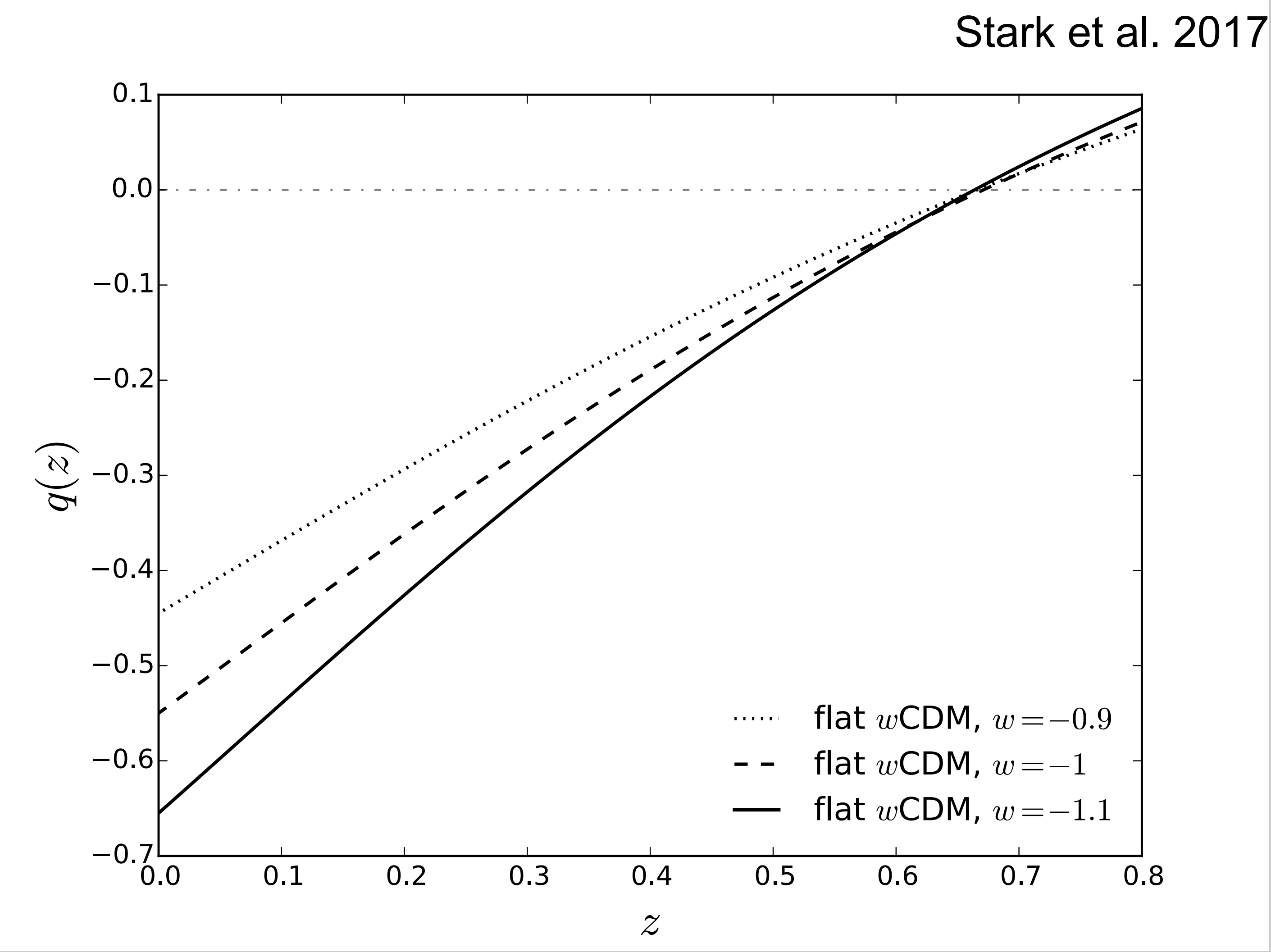


Miller et al. in prep
Gifford 2016, PhD Thesis

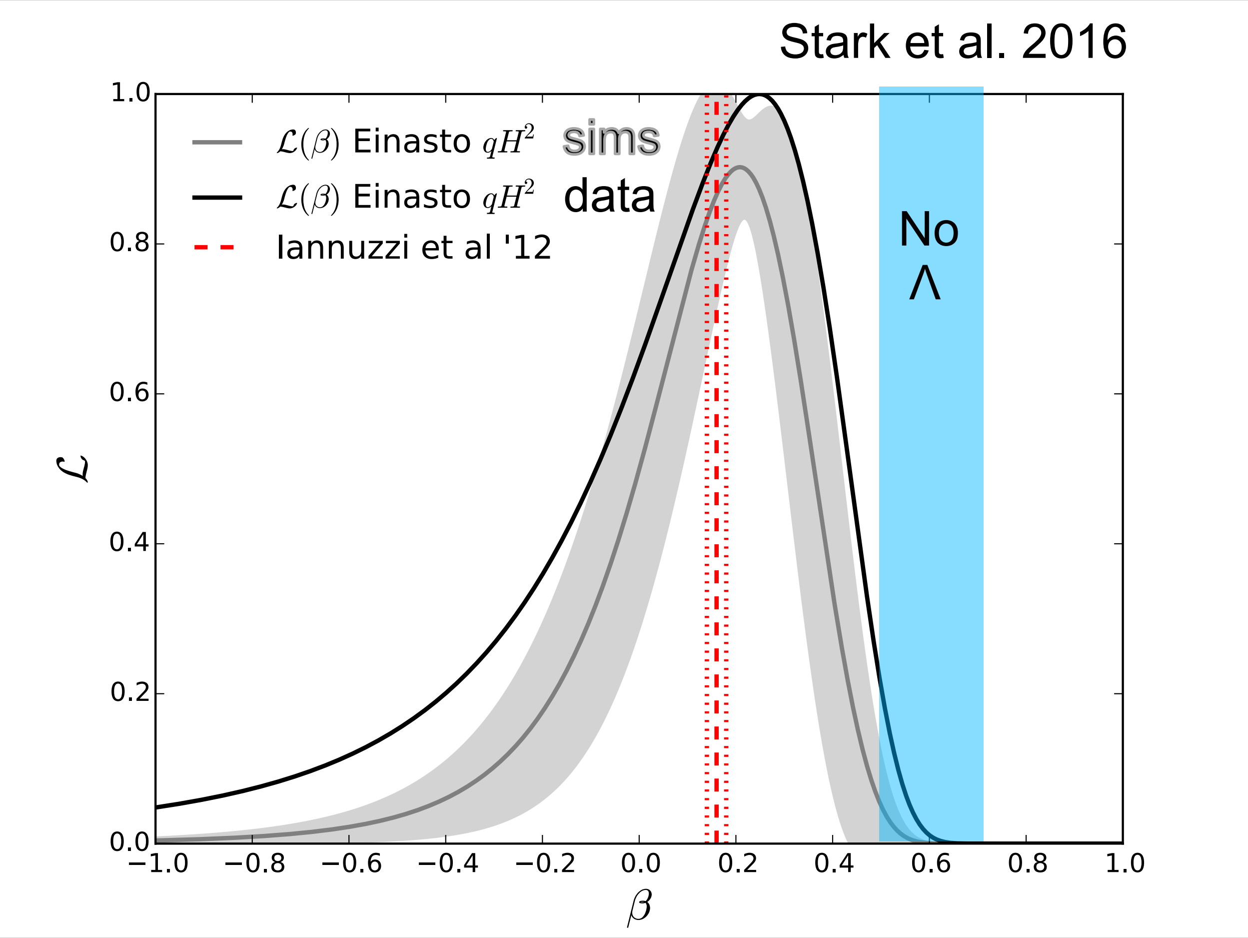
Use Case #2:
Detecting Dark Energy

- Compare theoretical predictions for the escape edge from weak lensing potentials to the edge. They should MATCH.
- For this we need:
  - **Cosmology** from Planck or of your choosing
  - **Mass profile** from weak lensing
    - Allows us to make a theoretical prediction for escape velocity profile as a function of cosmology
  - **Galaxy redshifts** from spectroscopy
    - Allows us to measure the observed escape velocity profile from phase-space
  - **Velocity Anisotropy**
    - Free parameter

Use Case #2: Detecting Dark Energy



Use Case #2:
Detecting Dark Energy

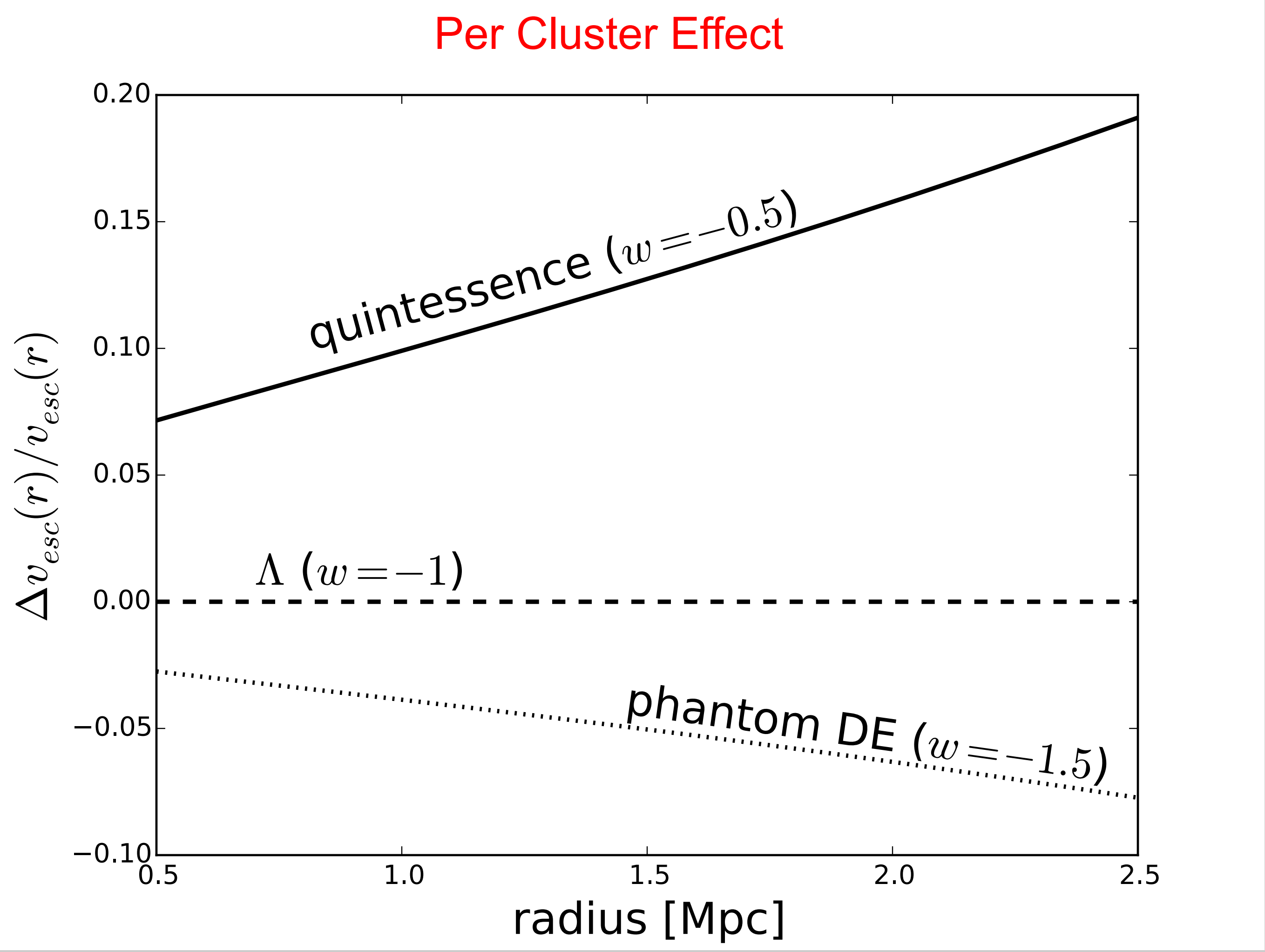


Key 5: Current data tell us that Λ is required to make sense of the observations and ΛCDM theory.

Use Case #3: Constraining Dark Energy

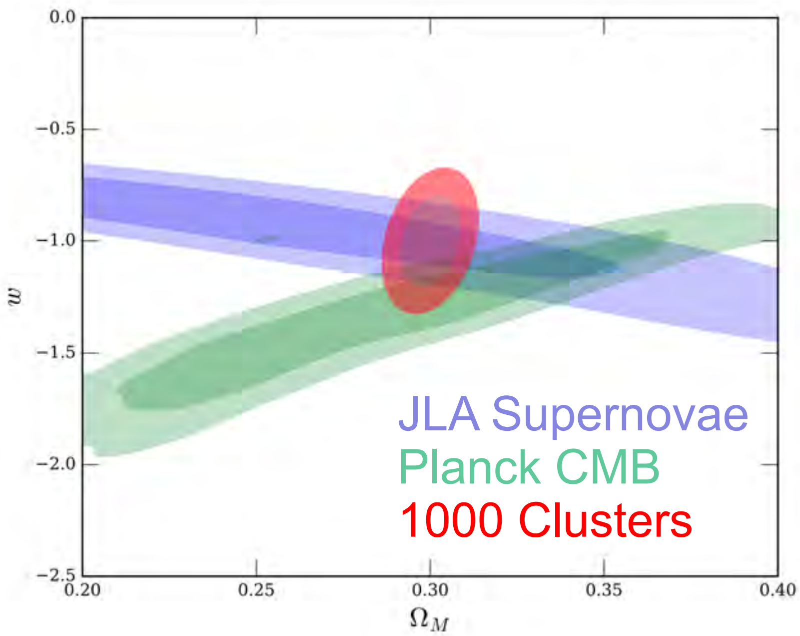
- Compare theoretical predictions for the escape edge from weak-lensing potentials to the edge. They should MATCH.
- For this we need:
  - **Mass profile** from weak lensing
    - Allows us to make a theoretical prediction for escape velocity profile as a function of cosmology
  - **Galaxy redshifts** from spectroscopy
    - Allows us to measure the observed escape velocity profile from phase-space
  - **Velocity Anisotropy** from other source (e.g., Jean's analysis)
    - Allows us to infer the underlying 3D potential

Use Case #3: Constraining Dark Energy

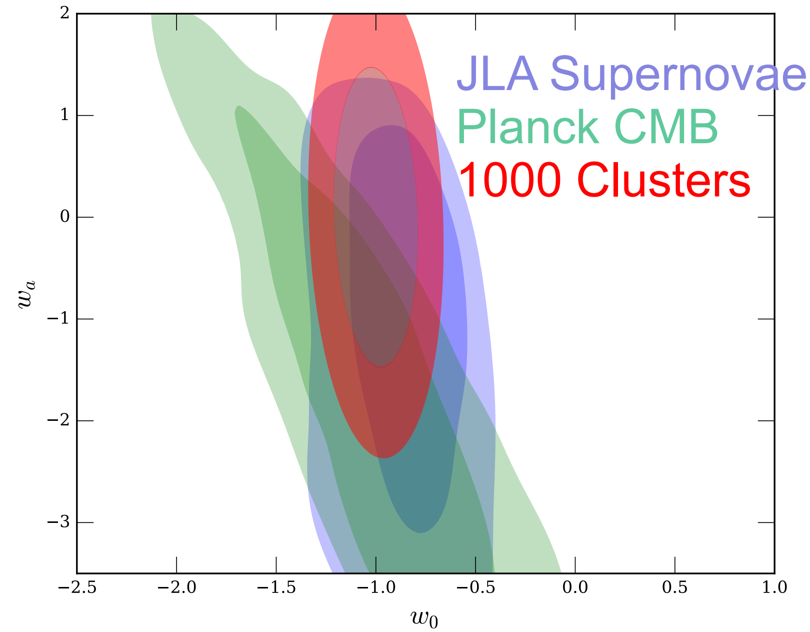


Stark et al. 2017

Use Case #3: Constraining Dark Energy



Stark et al. 2017



Key 6: Projected cluster phase-spaces are a powerful probe of cosmology and do not require a detailed understanding of the cluster selection process, centering accuracy, or mass function precision and accuracy. These are the three main challenges of traditional cluster cosmology.

Use Case 4:
Testing Chameleon gravity on Mpc Scales

modify the **left hand side** of Einstein Field Equations:

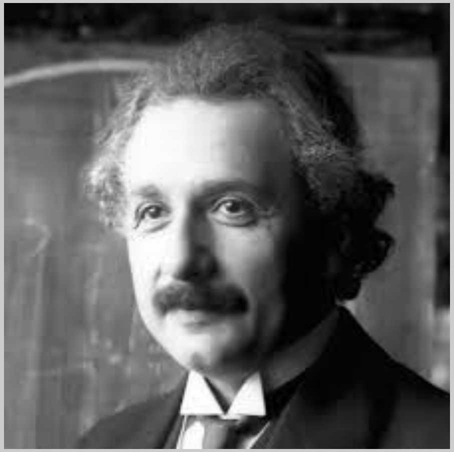
$\nabla^2 \Phi = 4\pi G \varrho$

geometry = mass, energy

$G_{\mu\nu} = 8\pi G \, (T^{M}_{\mu\nu} + T^{DE}_{\mu\nu})$

formally equivalent to...

$G_{\mu\nu} + \_\_\_\_\_ = 8\pi G \, T_{\mu\nu}{}^{M}$

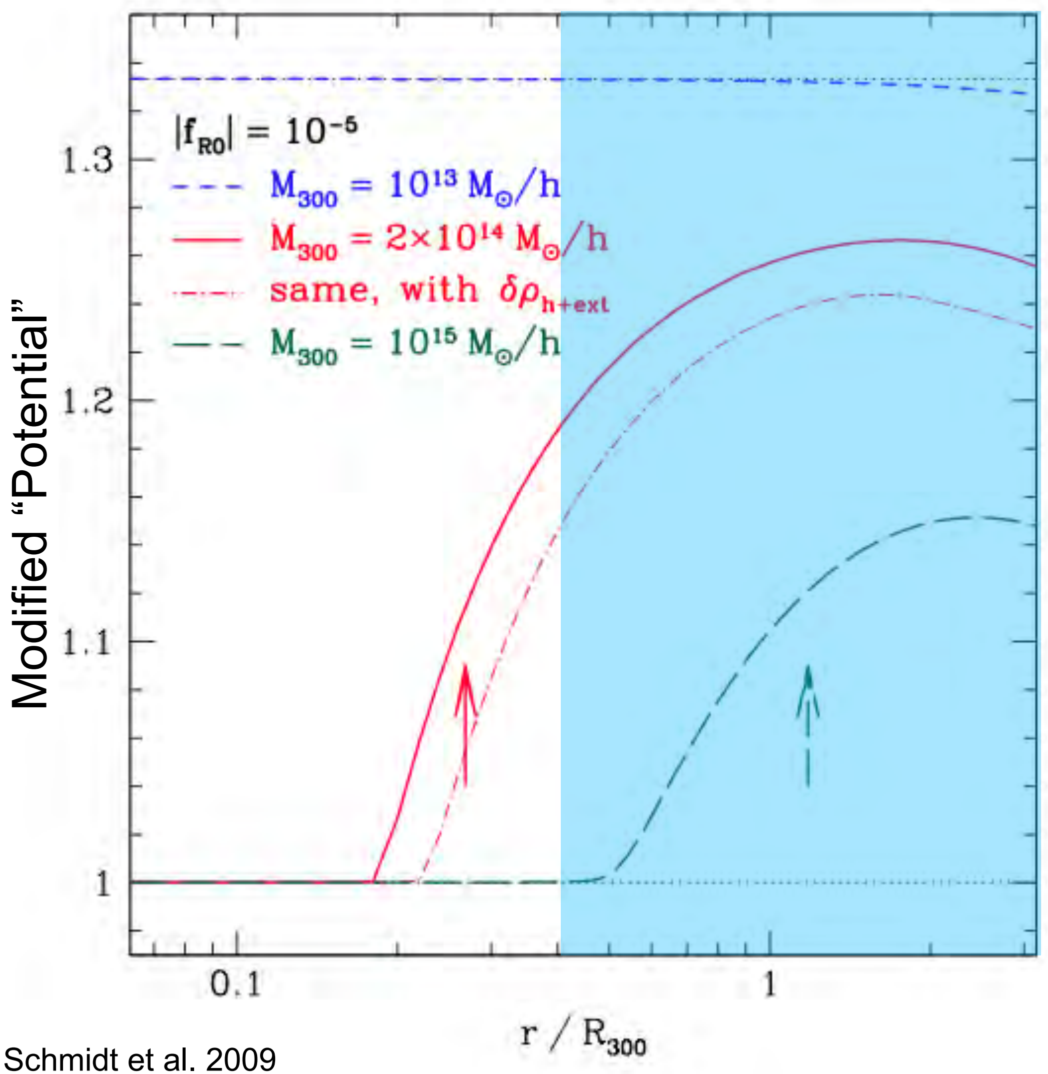


$G_{\mu\nu} + f_R R_{\mu\nu} - (½ f - \Box f_R) g_{\mu\nu} - \nabla_\mu \nabla_\nu f_R = 8\pi G \, T_{\mu\nu}$

modified gravitational potential:

$\Phi = \phi_N - ½ \; \delta f_R$

f(R) Gravity: Screening



Schmidt et al. 2009

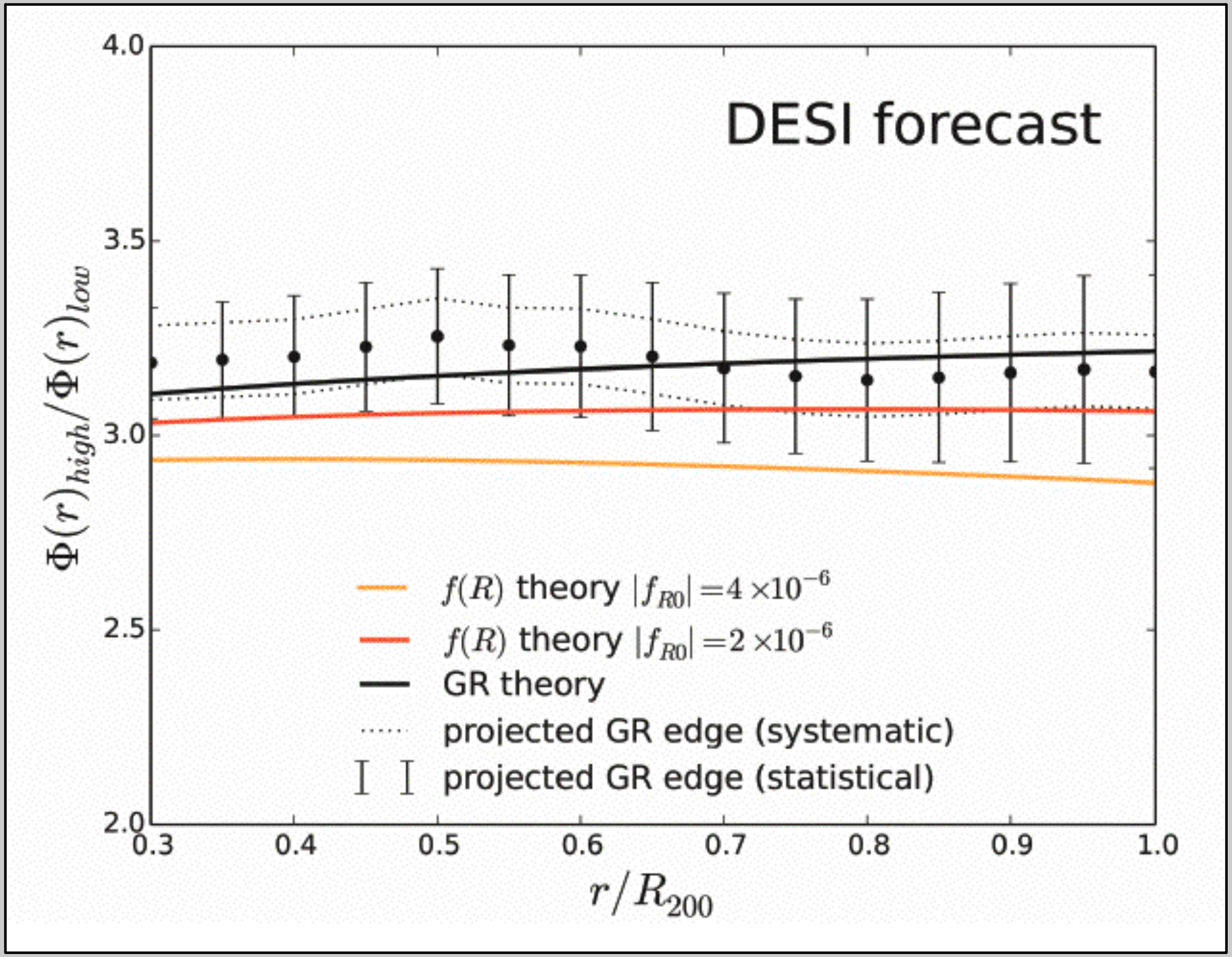
High mass clusters are mostly screened compared to low mass clusters.

Take the ratio of the potential in high mass and low mass cluster samples.

Divide out the systematics, including the velocity anisotropy.

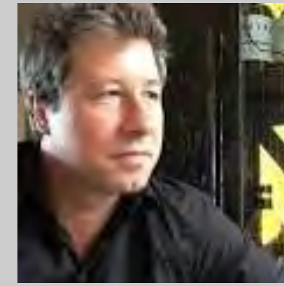
f(R) Gravity: Predictions

Stark et al. 2016



2D galaxies, 100 clusters per mass bin

Use Case 5:
Testing Emergent gravity on Mpc Scales



- An attempt to connect **String Theory to Gravity**
- Relies on space-time requiring “*quantum entanglement (QE).*”

QE means that the quantum states of two particles are coupled

Verlinde Showed that there is a ***surface contribution of Entropy*** which allowed him to derive Newton’s Laws and General Relativity from first principals.

Use Case 5:

Testing Emergent gravity on Mpc Scales

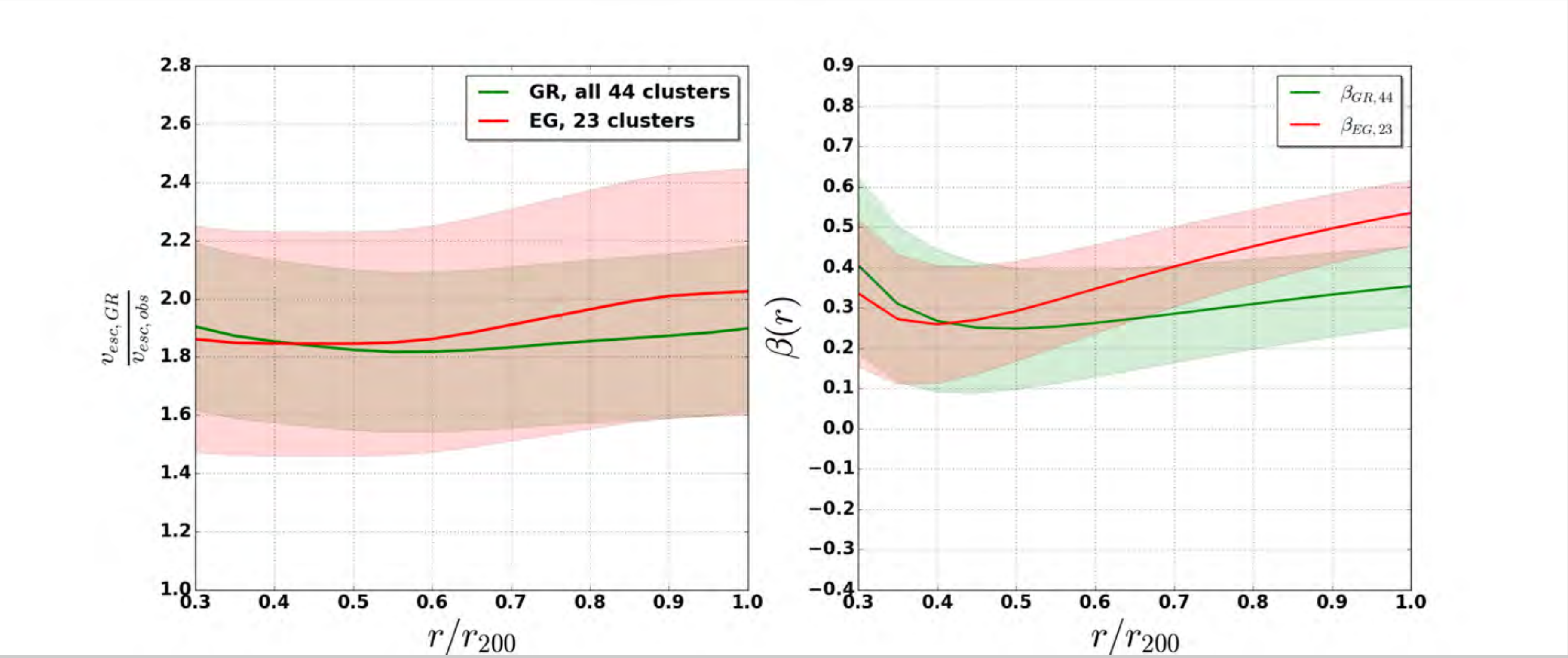
- In 2016, Verlinde extended his theory to de Sitter space-time (i.e., accelerated expansion of space).
    - Verlinde proposes that Λ and the accelerated expansion of the universe are due to the slow rate at which the emergent space-time thermalizes (e.g., like in thermodynamics and Black Holes).
    - This means that there is not only a surface contribution to the Entropy, but also a volume component, which is dominant at large-scales.
- Verlinde expresses the dark matter distribution as:

$$\int_0^r \frac{GM_D^2(r')}{r'^2} dr' = \frac{M_B(r) a_0 r}{6}.$$

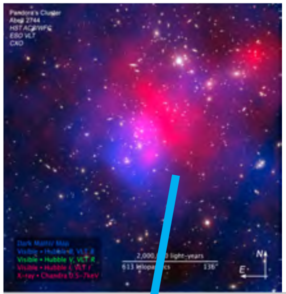
- On cosmological scales, Ω^2_{DM} – 4/3 $\Omega_{B,}$ which is consistent with Planck

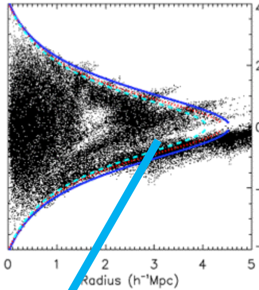
Use Case 5:

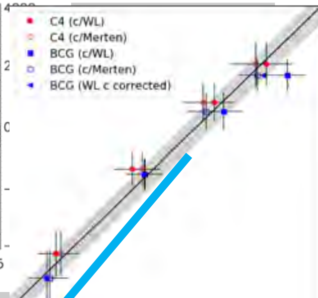
Testing Emergent gravity on Mpc Scales

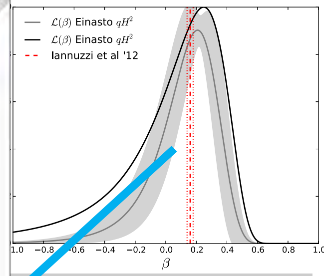


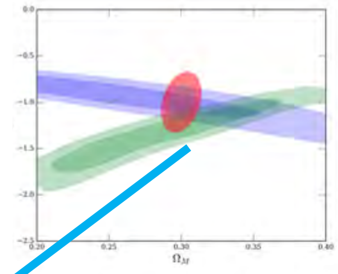
Key 7: The escape-edge tests GR and its extensions. \











Modern precision cosmology is (and will be) about quantifying H and q.

2. Cluster potential depths are governed not only by the mass present, but also by the accelerated expansion of space. Both of these bend space time and create curvature in the local space-time.

3. To understand and use the escape velocity of clusters, a “good” density profile needs to work well beyond the virial radius and must be a Poisson-pair to the potential. When doing so, theory is accurate to a few percent as tested in simulations.

4. The projected escape edge becomes a tool to measure galaxy cluster masses with systematics <5%.

5. Current data tell us that Λ is required to make sense of the observations and ΛCDM theory.

6. Projected cluster phase-spaces are a powerful probe of cosmology and do not require a detailed understanding of the cluster selection process, centering accuracy, or mass function precision and accuracy. These are the three main challenges of traditional cluster cosmology.

7: The escape-edge is a test of GR and extensions to it like fR Chameleon gravity and gravity as an emergent property of space-time. .

Vitali Helanka—Physics
--Testing Gravity with Phase-spaces

Alejo Stark- Astronomy
--Constraining Cosmology with phase-spaces

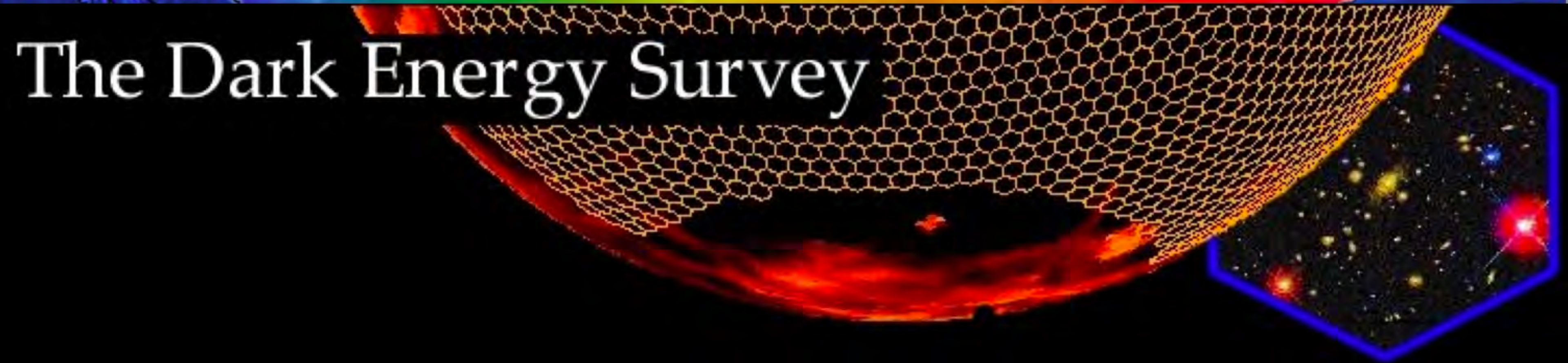
Jesse Golden-Marx-Astronomy
--Constraining Cluster Central galaxy and halo masses

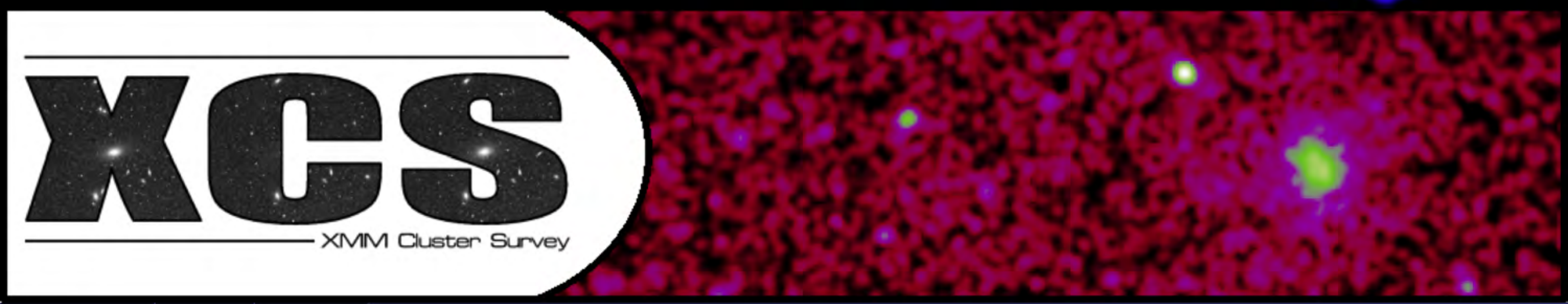
Rutuparna Das-Physics
--Measuring Weak Lensing cluster masses

Dan Gifford->
Getty Images

Anthony Kremin-Physics
--Measuring cluster phase-spaces and dynamical masses

The next generation of astronomical surveys





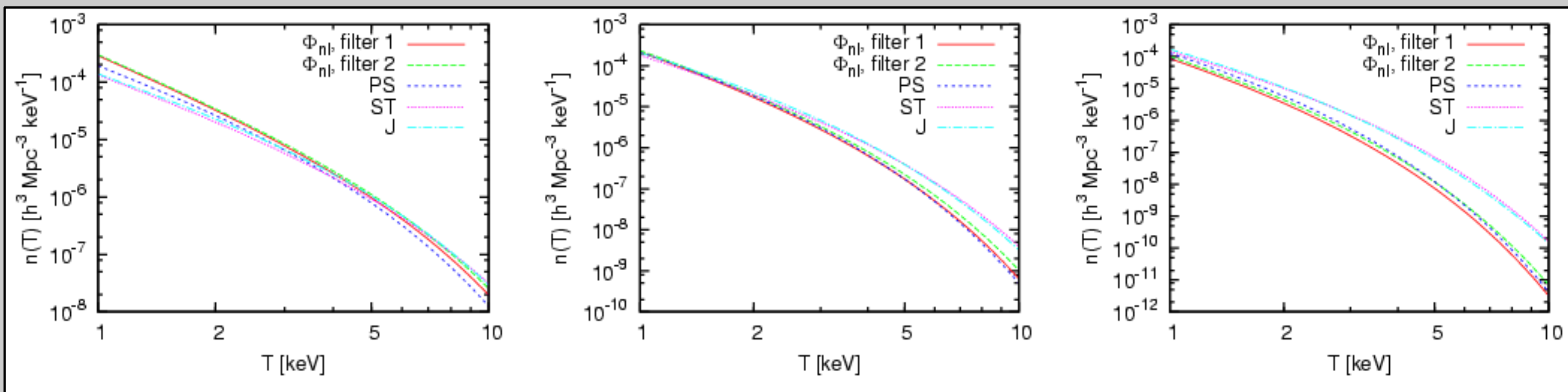
Cosmology directly via the potential instead of the mass density?

Angrick and Bartelmann 2009

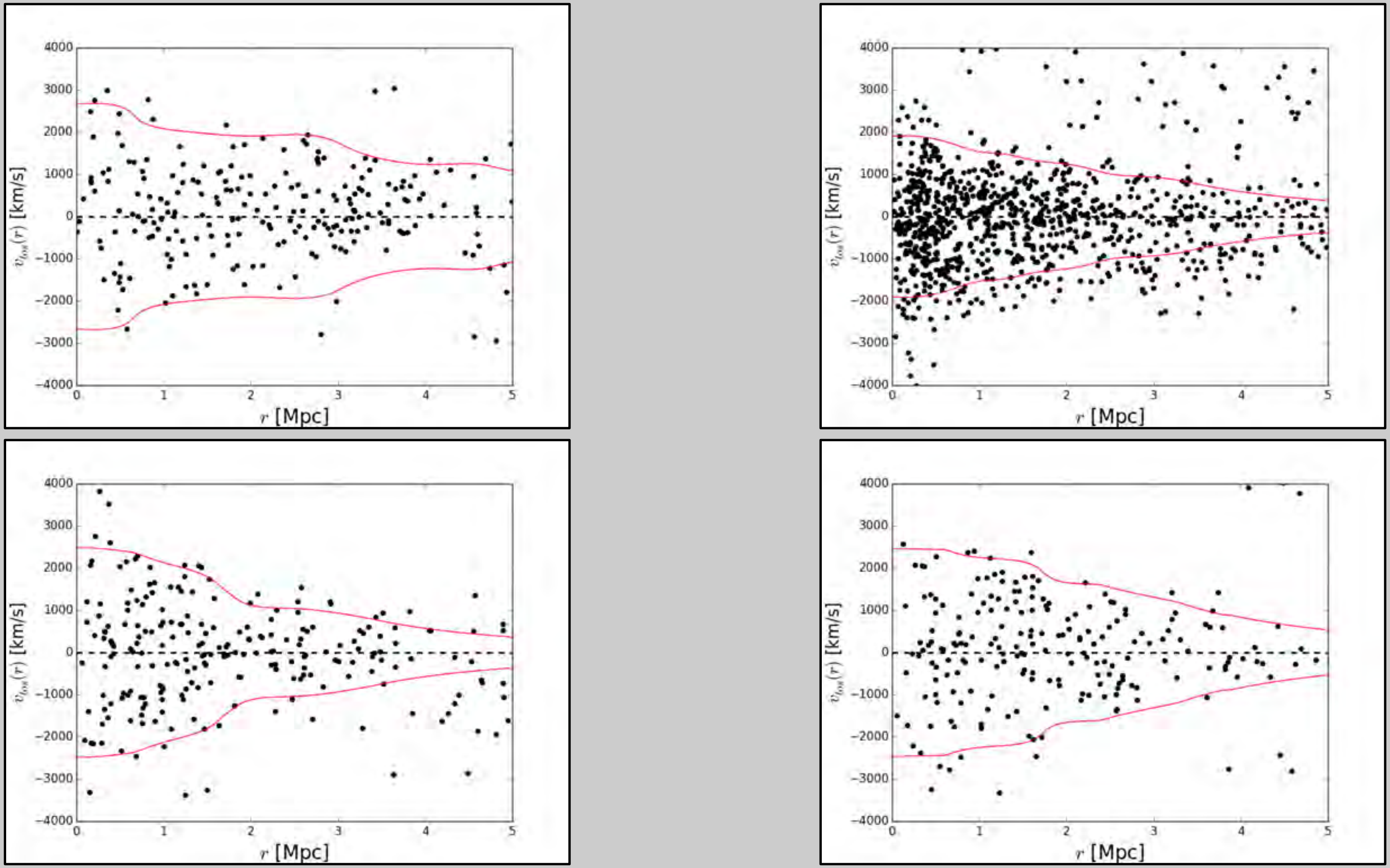
$$\Delta\Phi_c(a) = \frac{3}{2} H_0^2 \Omega_{m0} \frac{\delta_c(a)}{a}.$$

$$n(\Phi) = \int_{\Delta\Phi_c}^{\infty} d(\Delta\Phi)\,\tilde{n}(\Phi, \Delta\Phi),$$

$$P_\Phi(k) = \frac{9}{4} \frac{\Omega_{m0}^2}{a^2} \frac{H_0^4}{k^4} P_\delta(k).$$

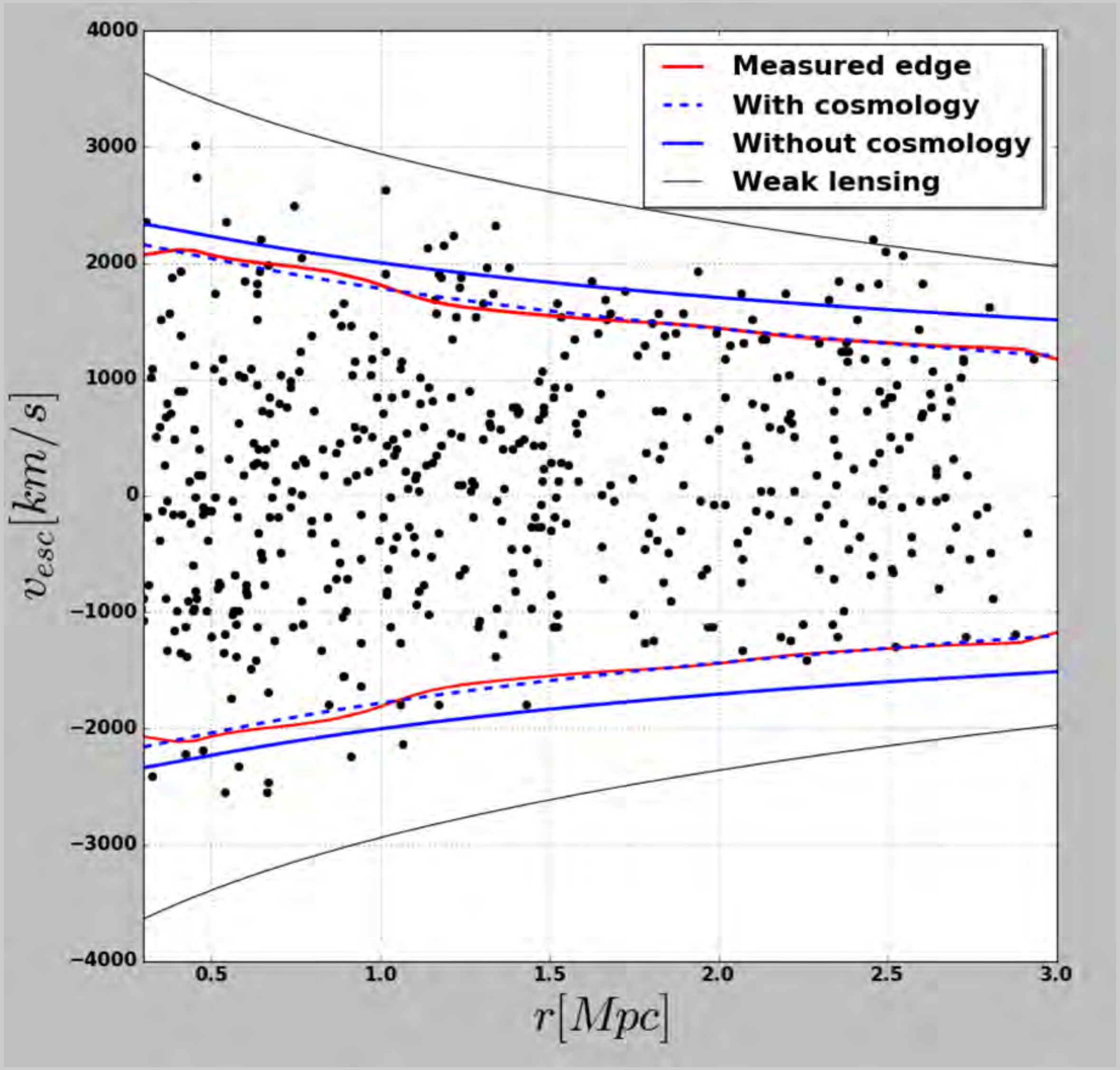


Using Hectospec+SDSS Velocities for Weak-lensing Clusters



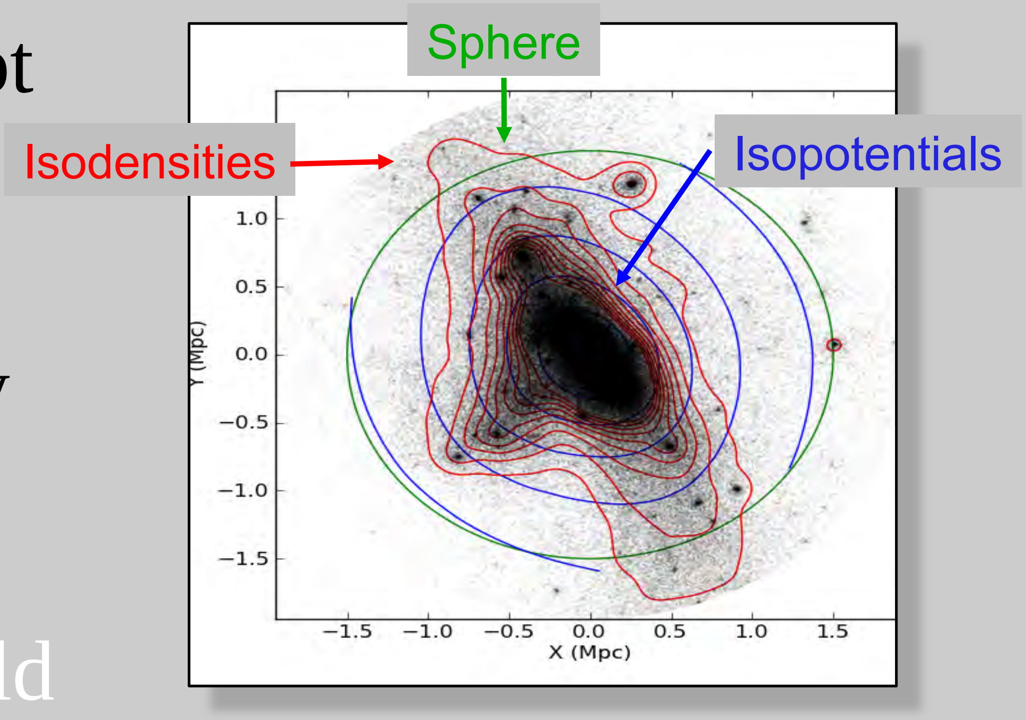
Data from SDSS & Geller et al. 2013

A real cluster

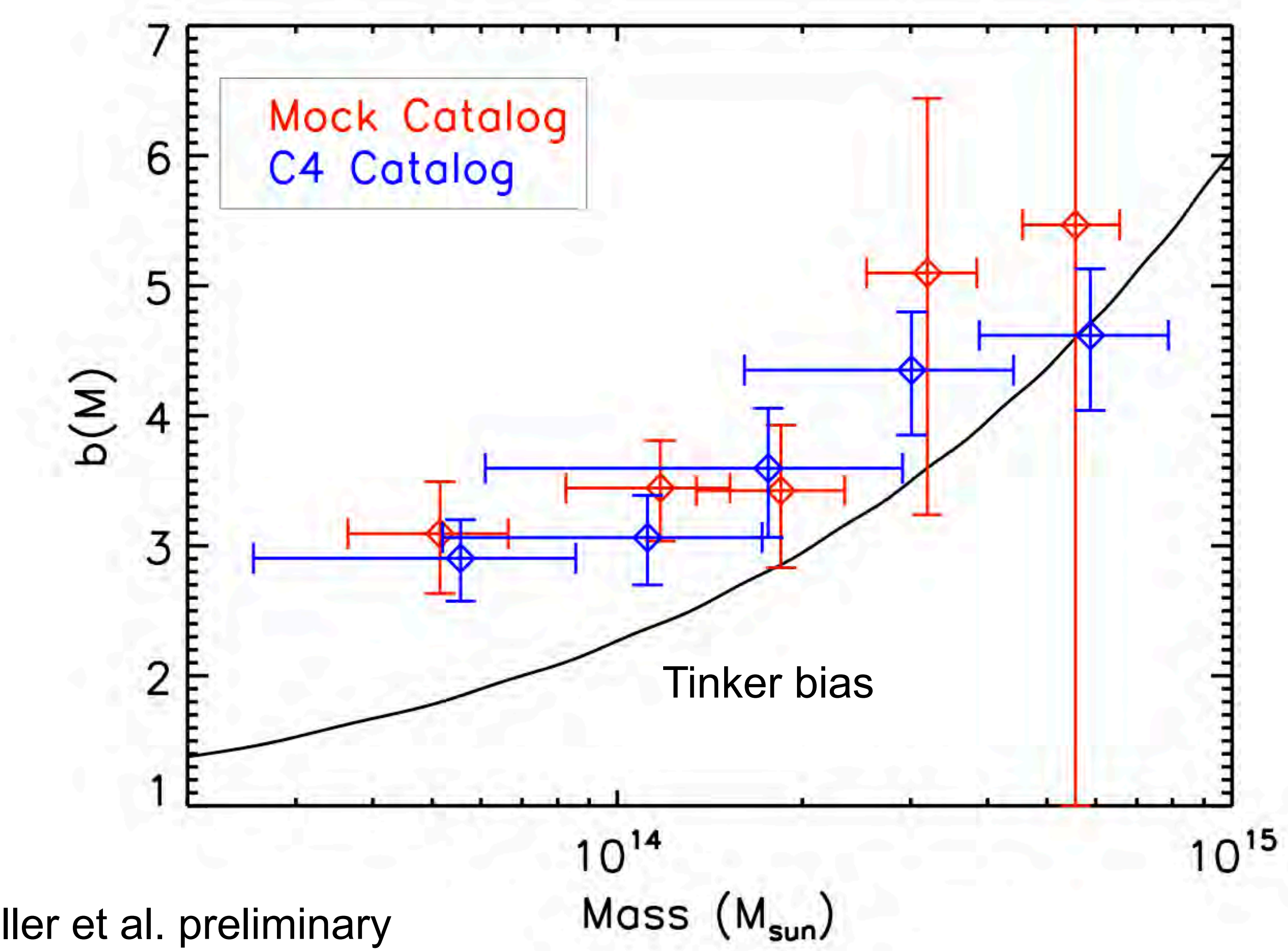


Density and Potential in N-body Halos

- Cluster-sized dark matter halos are not spherical in their density
- But they are nearly spherical in their local gravitational potential scalar field

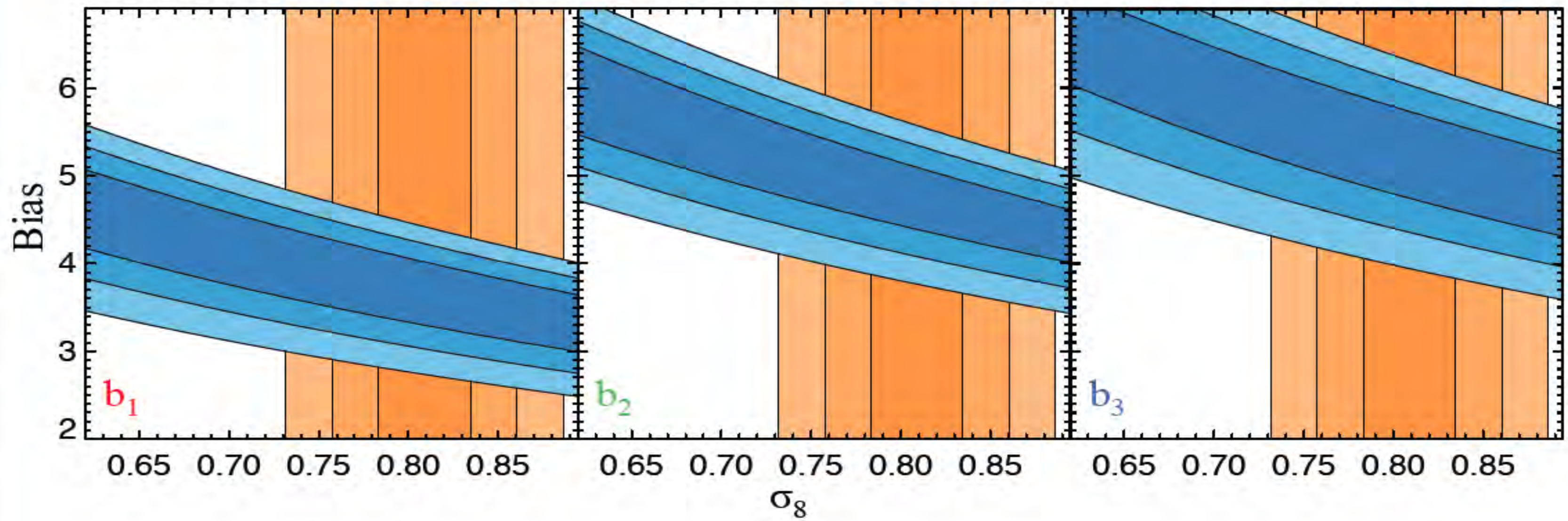


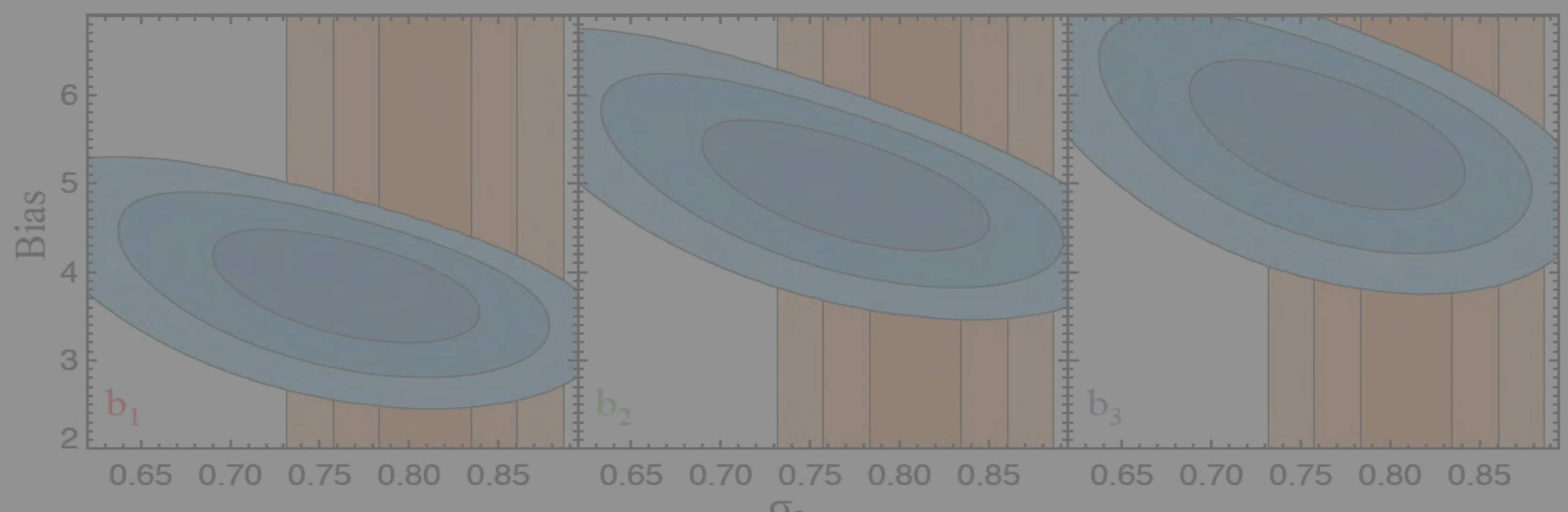
Directly Measuring the Bias



Miller et al. preliminary

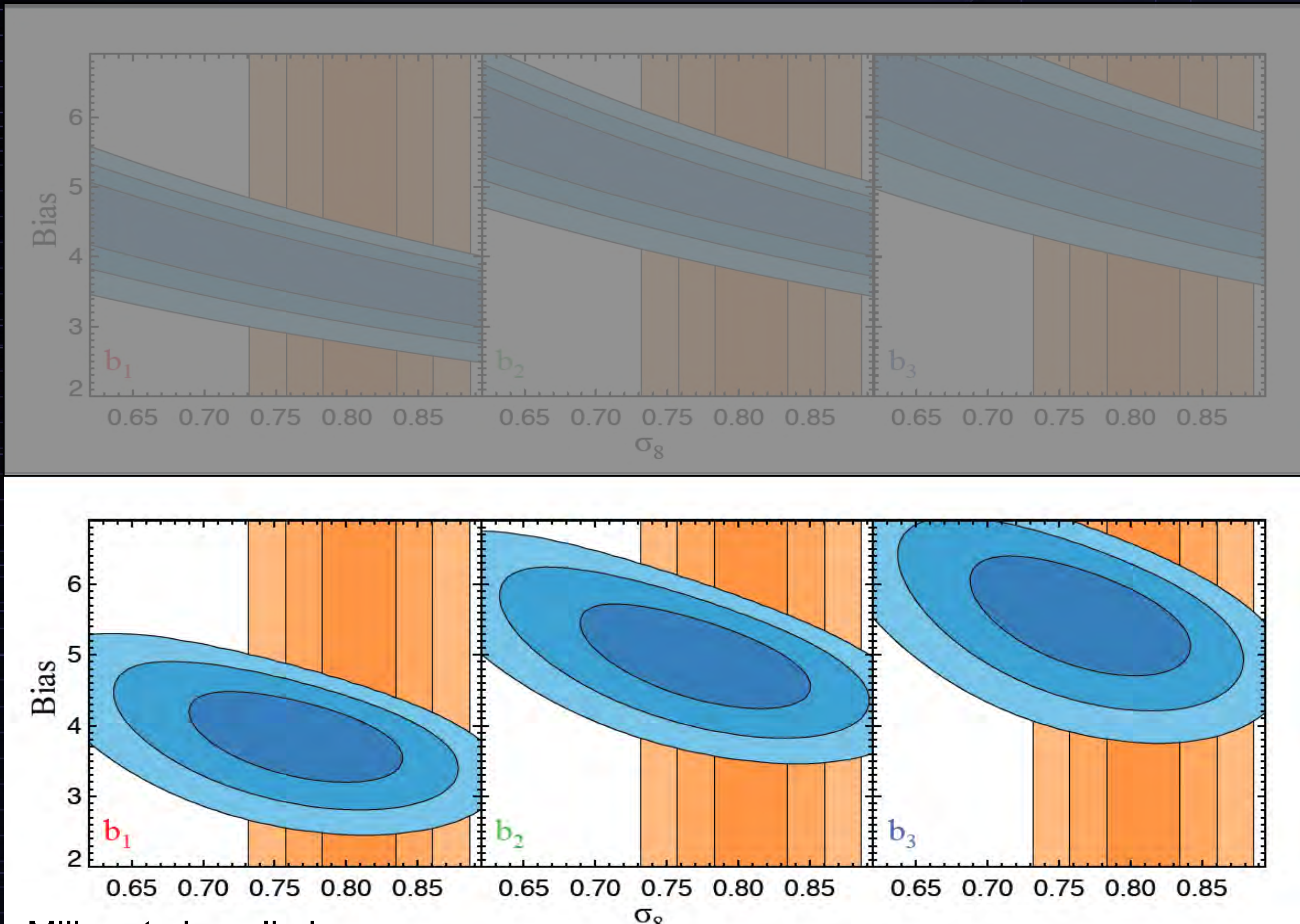
Without joint constraints on bias and σ_8





Miller et al. preliminary

With joint constraints on bias and σ_8



Miller et al. preliminary