Predicting Proton Decay from low energy SUSY spectrum

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in collaboration with

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26/12/2018 Seminar @ IPMU

◉ Motivation of SUSY

- Fine-tuning Problem ➡ tension with LHC; but better than non-SUSY

- Dark Matter ➡ well studied; consistent if its pure Higgsino (~1TeV) or pure Wino (~3TeV)

- Gauge Coupling Unification

⬇

not well studied compared to the other two

- How is the condition of GCU formulated?
- Is there an upper/lower limit on SUSY masses from GCU?
- Any relation between low energy SUSY and proton decay?

Contents

- Derivation of analytical formulae for GCU including the bulk 2-loop correction applicable for general SUSY and GUT spectra.

- Application:

  - Minimal SU(5)

    coloured Higgs mass as a function of general SUSY mass spectrum

    => prediction of D=5 proton decay from SUSY spectrum

  - Orbifold SUSY SU(5) [Hall-Nomula model]

    X,Y gauge boson mass as a function of general SUSY mass spectrum

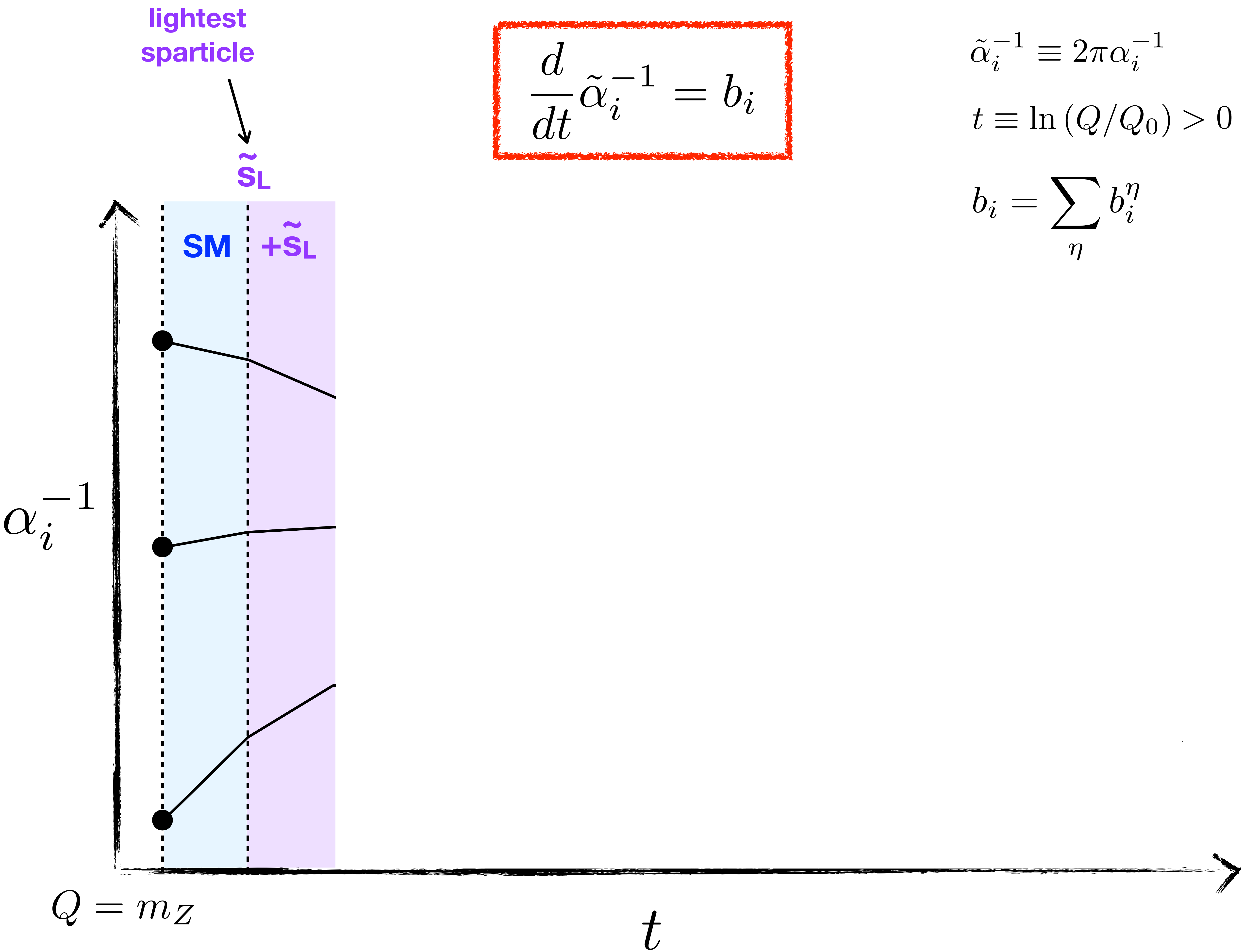
    => prediction of D=6 proton decay from SUSY spectrum

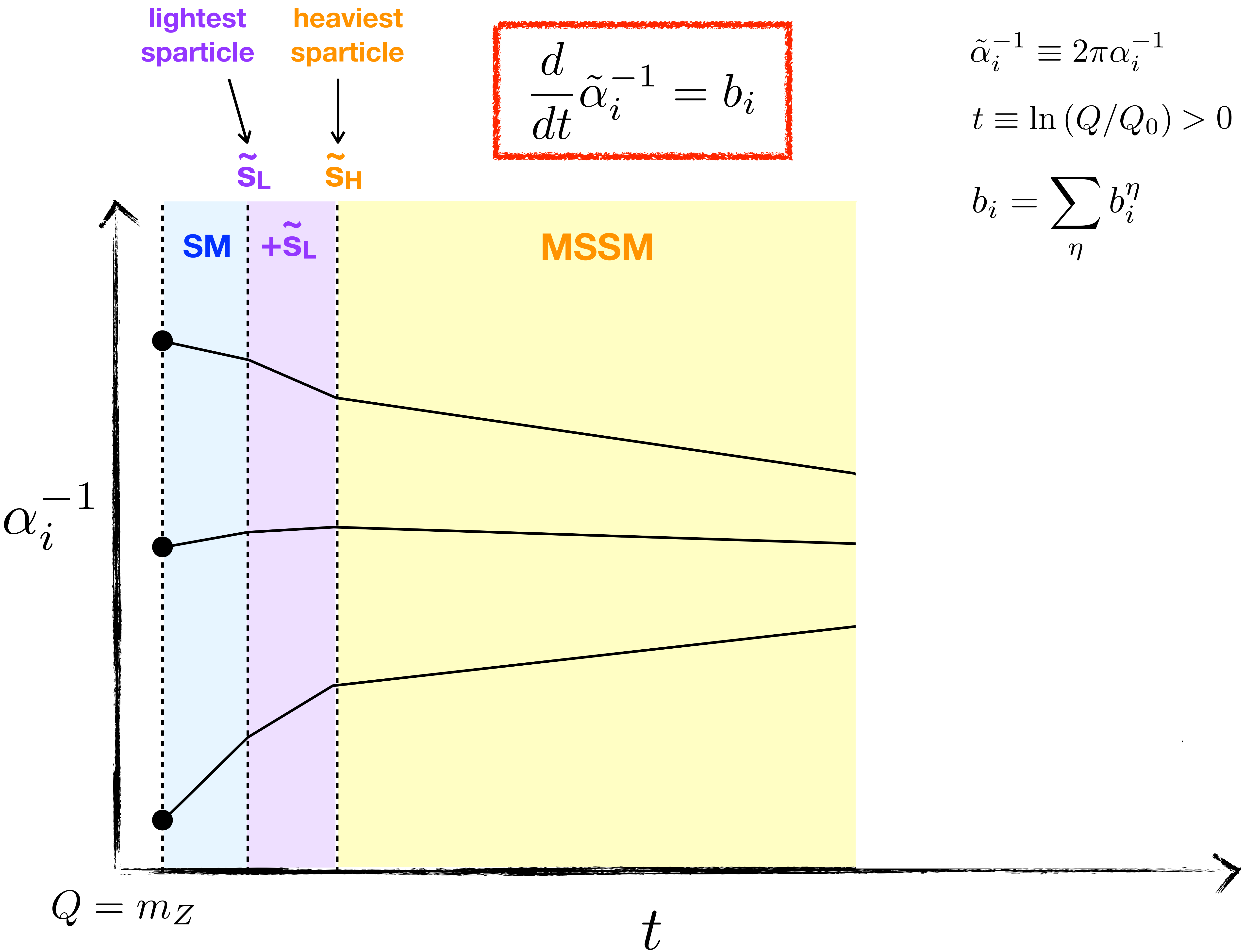
$$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$$

$$\tilde{\alpha}_i^{-1} \equiv 2\pi\alpha_i^{-1}$$

$$t \equiv \ln\left(Q/Q_0\right) > 0$$

$$b_i = \sum_\eta b_i^\eta$$





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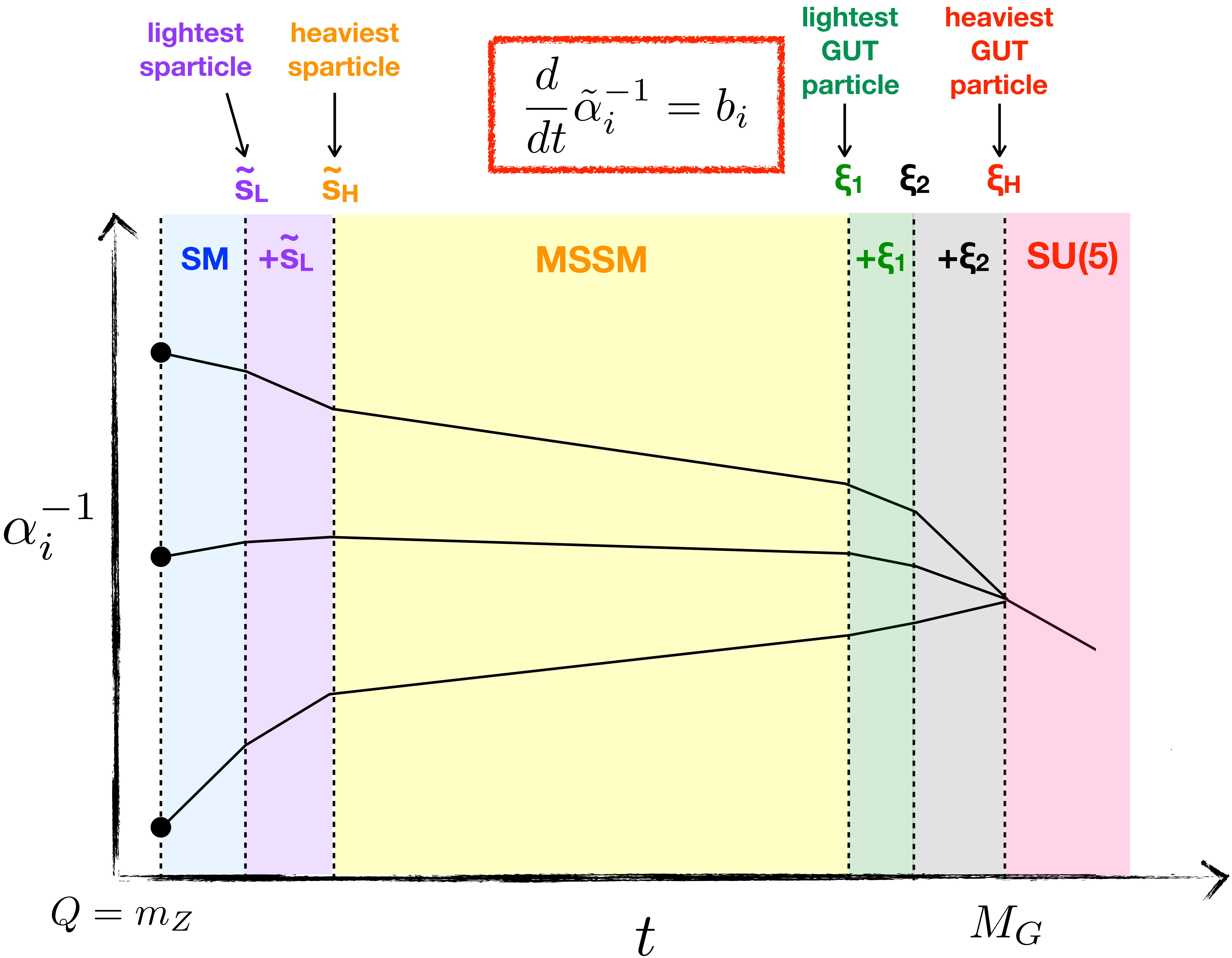
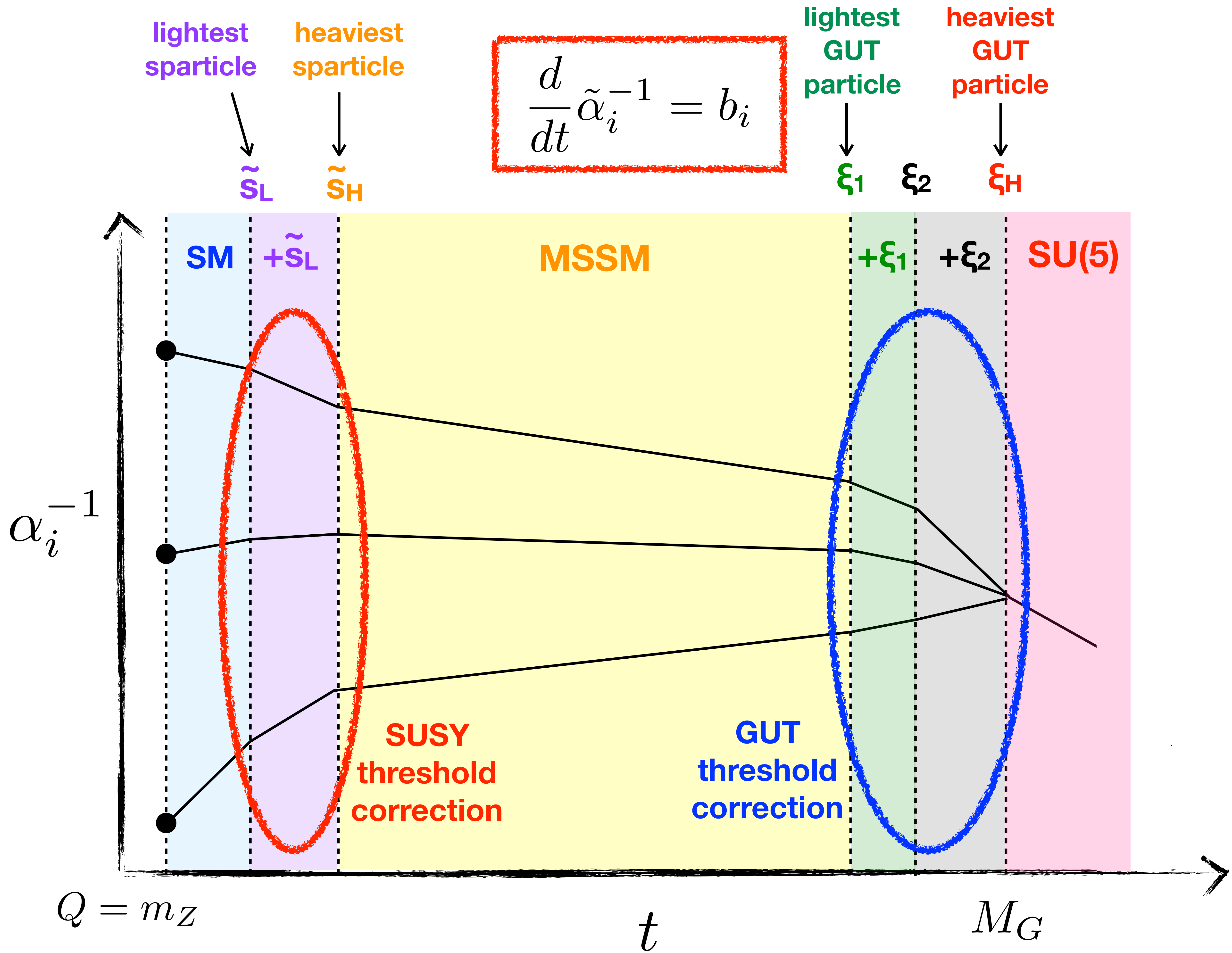


lightest sparticle
heaviest sparticle
$\frac{d}{dt}\tilde{\alpha}_i^{-1} = b_i$
lightest GUT particle
heaviest GUT particle
$\tilde{s}_L$
$\tilde{s}_H$
ξ_1
ξ_2
ξ_H
SM
$+\tilde{s}_L$
MSSM
$+\xi_1$
$+\xi_2$
SU(5)
α_i^{-1}
$Q = m_Z$
t
M_G



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SU(5)
α_i^{-1}
SUSY threshold correction
GUT threshold correction
$Q = m_Z$
t
M_G

General solution to RGE

$$\frac{2\pi}{\alpha(\Lambda)} = \frac{2\pi}{\alpha_i(m_Z)} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

- $\frac{2\pi}{\alpha(\Lambda)}$: unified coupling
- $\frac{2\pi}{\alpha_i(m_Z)}$: experimental input
- $- b_i \ln\left(\frac{\Lambda}{m_Z}\right)$: 1-loop MSSM
- s_i: SUSY threshold
- r_i: GUT threshold
- γ_i: bulk 2-loop contribution
- Δ_i: top-quark threshold, MSbar-DRber conversion

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

[bulk 2-loop contribution]

$$\gamma_i = -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(\frac{\alpha_j(\Lambda)}{\alpha_j(m_Z)}\right)$$

$$\simeq -\frac{1}{2}\sum_j \frac{b_{ij}}{b_j} \ln\left(1 + \frac{b_j \alpha(\Lambda)}{2\pi} \ln\frac{\Lambda}{m_Z}\right)$$

$$b_i = (\frac{33}{5}, 1, -3)$$

$$b_{ij} = \begin{pmatrix} 199/25 & 27/5 & 88/5 \\ 9/5 & 25 & 24 \\ 11/5 & 9 & 14 \end{pmatrix}$$

one can find γ_i by iteratively updating $\alpha(\Lambda)$ and Λ

General solution to RGE

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$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right)$$

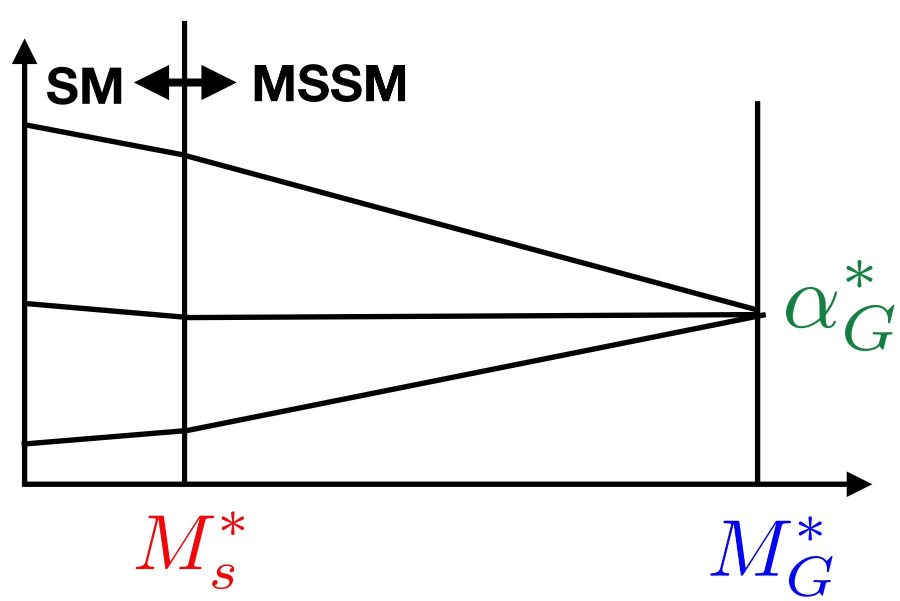
- We trade 3 exp inputs with the ***3 constants***: $[\alpha_1(m_Z), \alpha_2(m_Z), \alpha_3(m_Z)] \rightarrow [M_s^*, M_G^*, \alpha_G^*]$

$$\frac{2\pi}{\alpha_i(m_Z)} = \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{m_Z}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) - \gamma_i - \Delta_i$$

degenerate SUSY without GUT thres

$$b_i = \left(\frac{33}{5}, 1, -3\right)$$

$$\delta_i \equiv \sum_\eta b_i^\eta = b_i - b_i^{\rm SM} = \left(\frac{2}{5}, \frac{25}{6}, 4\right)$$



$$M_s^* = 2.13\,\text{TeV}$$

$$M_G^* = 1.26 \cdot 10^{16}\,\text{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

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degenerate SUSY without GUT thres

$$\frac{2\pi}{\alpha(\Lambda)} = \boxed{\frac{2\pi}{\alpha_i(m_Z)}} - b_i \ln\left(\frac{\Lambda}{m_Z}\right) + s_i + r_i + \gamma_i + \Delta_i$$

general solution

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + s_i + r_i$$

SUSY threshold (s_i) **GUT threshold** (r_i)

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SUSY threshold (s_i) **GUT threshold** (r_i)

Any 3D vector can be decomposed into a sum of 3 independent vectors: *1, b_i, δ_i*

$$\vec{1} = (1,1,1) \qquad \vec{b} = (\frac{33}{5}, 1, -3) \qquad \vec{\delta} \equiv \vec{b} - \vec{b}_{\rm SM} = (\frac{2}{5}, \frac{25}{6}, 4)$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

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general solution

i-independent

$$= \frac{2\pi}{\alpha_G^*} + b_i \ln\left(\frac{M_G^*}{\Lambda}\right) - \delta_i \ln\left(\frac{M_s^*}{m_Z}\right) + s_i + r_i$$

$$= \left[\frac{2\pi}{\alpha_G^*} + C_S + C_G\right] + b_i \ln\left(\frac{M_G^* \Omega_S}{T_G}\right) + \delta_i \ln\left(\frac{T_S}{M_s^* \Omega_G}\right)$$

Any 3D vector can be decomposed into a sum of 3 independent vectors:

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$$= \left[\frac{2\pi}{\alpha_G^*} + C_S + C_G\right] + b_i \ln\left(\frac{M_G^* \Omega_S}{T_G}\right) + \delta_i \ln\left(\frac{T_S}{M_s^* \Omega_G}\right)$$

must vanish

The condition of gauge coupling unification:

$$T_S = M_s^* \Omega_G \cap T_G = M_G^* \Omega_S$$

$$M_s^* = 2.13\,\text{TeV}$$

$$M_G^* = 1.26 \cdot 10^{16}\,\text{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}\left(C_S + C_G\right)$$

$$s_i = \sum_\eta b_i^\eta \ln\left(\frac{m_\eta}{m_Z}\right) = C_S + b_i \ln \Omega_S + \delta_i \ln\left(\frac{T_S}{m_Z}\right)$$

$$b_i = (\frac{33}{5}, 1, -3)$$

$$\delta_i = (\frac{2}{5}, \frac{25}{6}, 4)$$

$$\begin{pmatrix} \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \delta_1 \\ 1 & b_2 & \delta_2 \\ 1 & b_3 & \delta_3 \end{pmatrix} \begin{pmatrix} C_S \\ \ln \Omega_S \\ \ln\left(\frac{T_S}{m_Z}\right) \end{pmatrix}$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega\right]^{\frac{1}{288}}$$

$$X_T \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^3}{m_{\tilde{d}_{Ri}}^3}\right)\left(\frac{m_{\tilde{q}_i}^7}{m_{\tilde{e}_{Ri}}^2 m_{\tilde{u}_{Ri}}^5}\right)$$

$$X_\Omega \equiv \prod_{i=1\ldots3} \left(\frac{m_{\tilde{l}_i}^8}{m_{\tilde{d}_{Ri}}^8}\right)\left(\frac{m_{\tilde{q}_i}^6 m_{\tilde{e}_{Ri}}}{m_{\tilde{u}_{Ri}}^7}\right)$$

$$C_S = \frac{125}{19}\ln M_3 - \frac{113}{19}\ln M_2 - \frac{40}{19}\ln \mu - \frac{10}{19}\ln m_A + \sum_{i=1\ldots3}\left[\frac{79}{114}\ln m_{\tilde{d}_{Ri}} - \frac{10}{19}\ln m_{\tilde{l}_i} - \frac{121}{114}\ln m_{\tilde{q}_i} + \frac{257}{228}\ln m_{\tilde{u}_{Ri}} + \frac{33}{76}\ln m_{\tilde{e}_{Ri}}\right]$$

$$r_i = \sum_\xi b_i^\xi \ln\left(\frac{m_\xi}{\Lambda}\right) = C_G - b_i \ln\left(\frac{T_G}{\Lambda}\right) - \delta_i \ln \Omega_G$$

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln \Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$C_G = \sum_\xi \left(\frac{165}{76}b_1^\xi - \frac{339}{76}b_2^\xi + \frac{125}{38}b_3^\xi\right) \ln\left(\frac{m_\xi}{\Lambda}\right)$$

The condition of gauge coupling unification:

$$T_S = M_s^* \Omega_G \quad \cap \quad T_G = M_G^* \Omega_S$$

$$M_s^* = 2.08\ \text{TeV}$$

$$M_G^* = 1.27 \cdot 10^{16}\ \text{GeV}$$

$$\alpha_G^{*-1} = 25.5$$

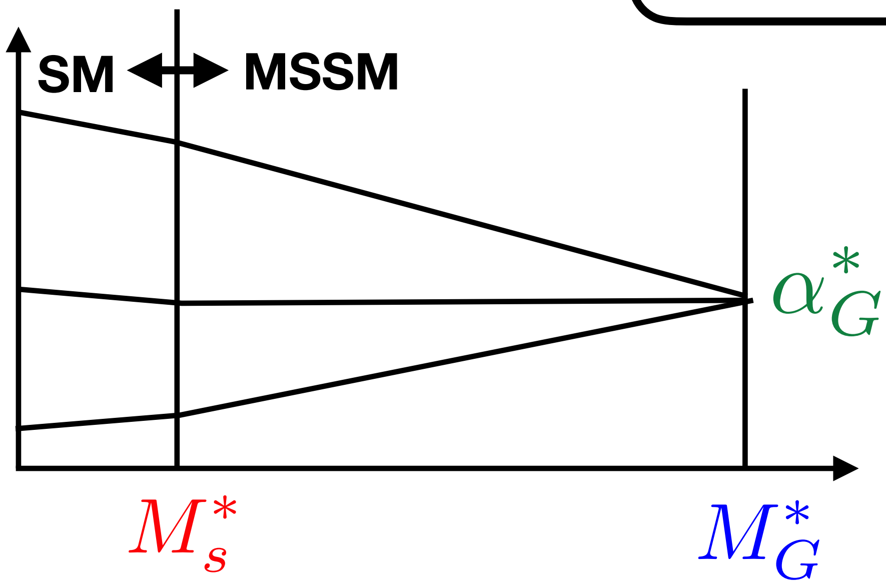
The unified coupling at Λ

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}\left(C_S + C_G\right)$$

Uncertainty of $\alpha_s(m_Z)$

$$\alpha_s(m_Z) = \underset{\alpha_s^0}{0.1183} \pm \underset{\Delta\alpha_s}{0.0008}$$

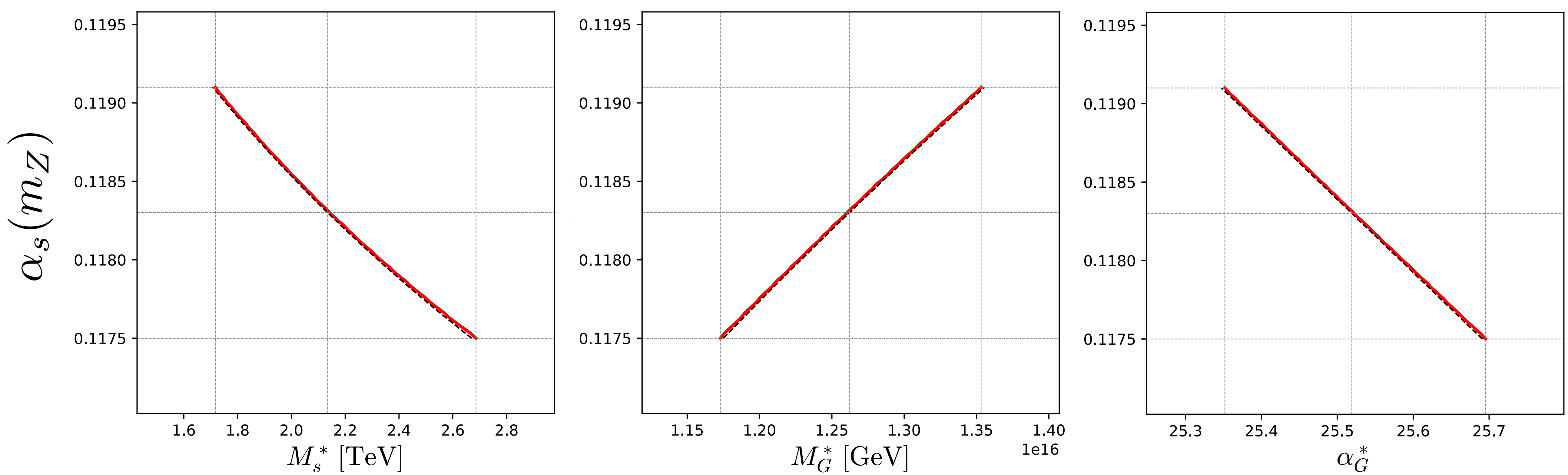
D. d'Enterria [1806.06156]



$$\frac{M_s^*}{\mathrm{TeV}} = \frac{2.13}{\mathrm{TeV}} \cdot \exp\left[-0.224\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right],$$

$$\frac{M_G^*}{\mathrm{GeV}} = \frac{1.26 \cdot 10^{16}}{\mathrm{GeV}} \cdot \exp\left[0.0715\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right)\right]$$

$$\alpha_G^{*-1} = 25.5 - 0.172\left(\frac{\alpha_s - \alpha_s^0}{\Delta\alpha_s}\right).$$



$M_s^* \in [2.69,\ 1.72]$ TeV $\quad M_G^* \in [1.17,\ 1.35] \cdot 10^{16}$ GeV $\quad \alpha_G^{*-1} \in [25.7,\ 25.4]$

Minimal SU(5)

Higgs and gage fields

$$H(\mathbf{5}) = (H_C, H_u) \qquad \Sigma(\mathbf{24}) = (\Sigma_8, \Sigma_3, \Sigma_1, \Sigma_{(2,3)}, \Sigma_{(2,3^*)})$$

$$\overline{H}(\overline{\mathbf{5}}) = (\overline{H}_C, H_d) \qquad \mathcal{V}(\mathbf{24}) = (G, W, B, (X,Y), (X,Y)^\dagger)$$

$$W_H = \frac{1}{2} M \mathrm{Tr}\Sigma^2 + \frac{1}{3}\lambda_\Sigma \mathrm{Tr}\Sigma^3 + \lambda_H \overline{H}(\Sigma + 3V)\Sigma H,$$

$$\langle \Sigma \rangle = V \cdot \mathrm{diag}(2,2,2,-3,-3)$$

doublet triplet splitting

$$M_{H_C} = 5\lambda_H V$$

$$M_{H_{u/d}} = 0$$

$$M_V = 5\sqrt{2} g_5 V$$

$$M_\Sigma = \frac{2}{5}\lambda_\Sigma V$$

mass	$(U(1)\times SU(2)\times SU(3))$	(b_1, b_2, b_3)
M_{H_C}	$(-\frac{1}{3}, \mathbf{1}, \mathbf{3})$, $(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$	$(\frac{2}{5}, 0, 1)$
M_V	$(-\frac{5}{6}, \mathbf{2}, \mathbf{3})$, $(\frac{5}{6}, \mathbf{2}, \overline{\mathbf{3}})$	$(-10, -6, -4)$
M_Σ	$(0, \mathbf{3}, \mathbf{1})$, $(0, \mathbf{1}, \mathbf{8})$	$(0, 2, 3)$

formulae for T_G and Ω_G

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln\Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$T_G = M_{H_C}^{\frac{4}{19}}\left(M_V^2 M_\Sigma\right)^{\frac{5}{19}}$$

$$\Omega_G = M_{H_C}^{\frac{18}{19}}\left(M_V^2 M_\Sigma\right)^{-\frac{6}{19}}$$

Condition for GCU

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{\frac{5}{6}}$$

$$(M_V^2 M_\Sigma)^{\frac{1}{3}} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*}\right)^{-\frac{2}{9}}$$

$$(3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1})(m_Z) = \frac{1}{2\pi}\left\{\frac{12}{5}\ln\frac{M_{H_C}}{m_Z} - 2\ln\frac{m_{SUSY}}{m_Z}\right\},$$

$$(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1})(m_Z) = \frac{1}{2\pi}\left\{12\ln\frac{M_V^2 M_\Sigma}{m_Z^3} + 8\ln\frac{m_{SUSY}}{m_Z}\right\}.$$

Hisano, Murayama, Yanagida '92

$$M_{H_C} = M_G^* \Omega_S \left(\frac{T_S}{M_s^*} \right)^{\frac{5}{6}}$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T \right]^{\frac{1}{19}}$$

$$\Omega_S = \left[M_3^{-100} M_2^{60} \mu^{32} m_A^8 X_\Omega \right]^{\frac{1}{288}}$$

$$M_s^* = 2.08\ \text{TeV}$$

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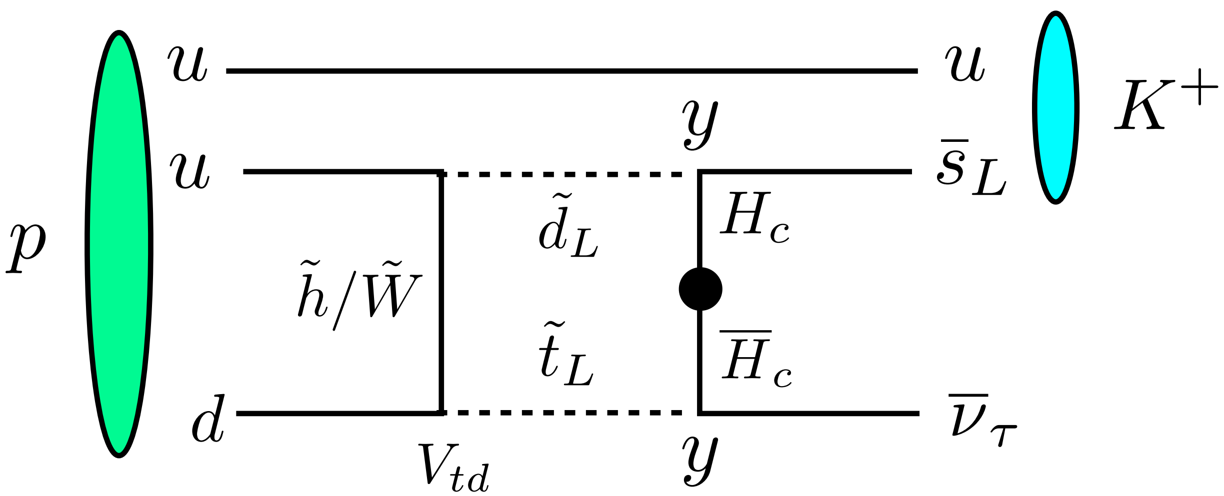
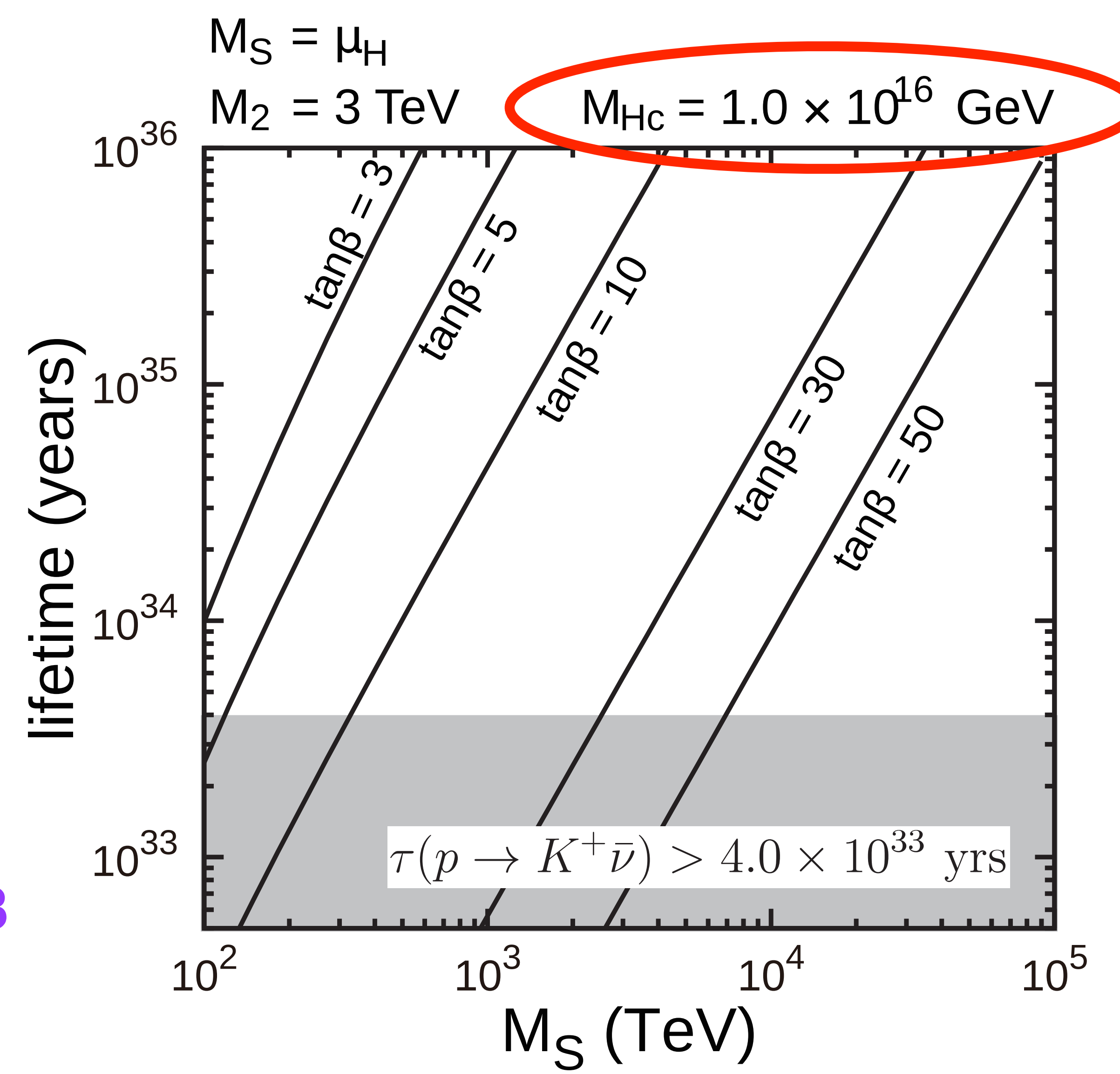
D=5 proton decay



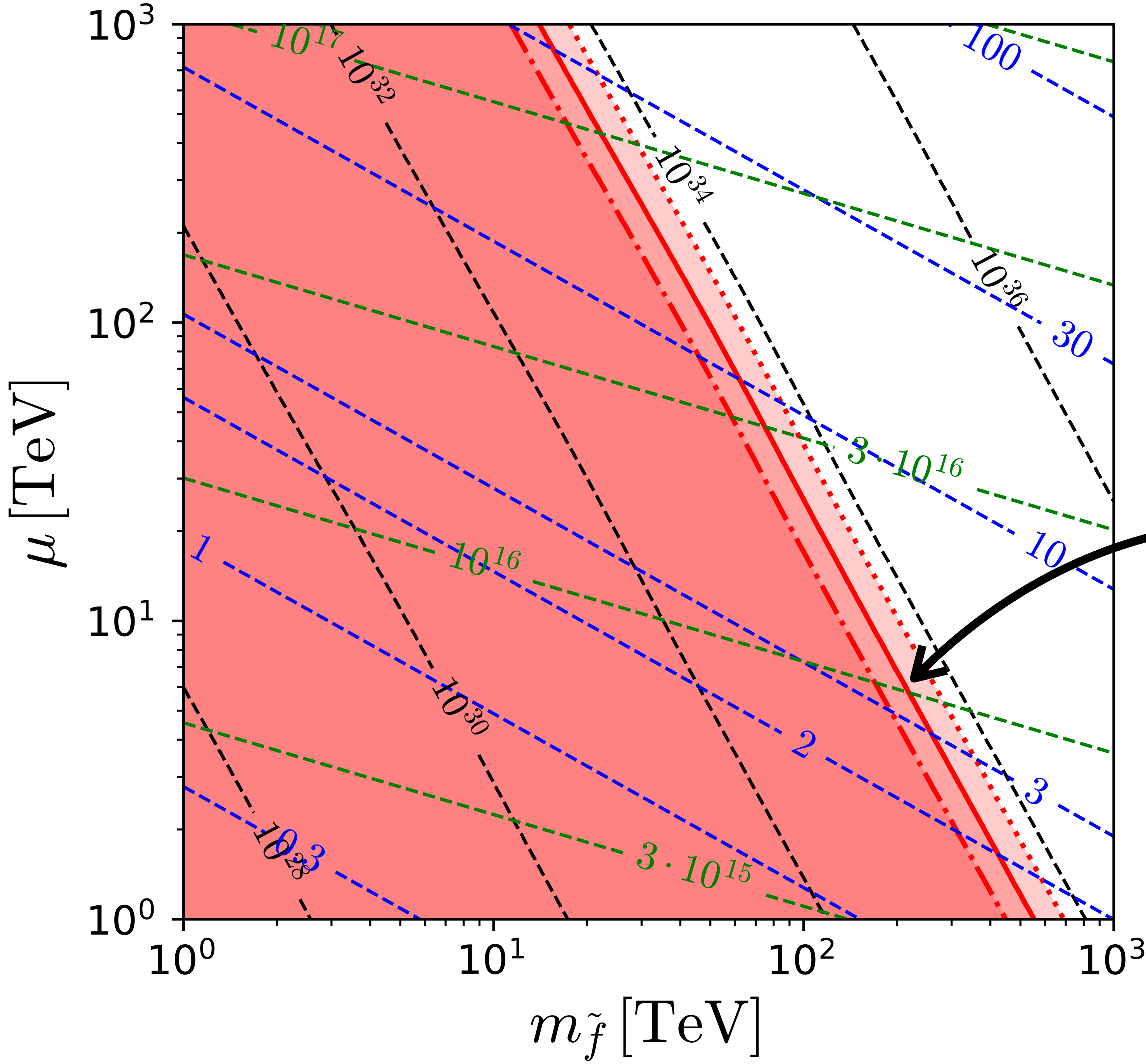


Hisano, Kobayashi, Kuwahara, Nagata '13

Vanilla SUSY

$(M_V = M_\Sigma)$

$M_3 = m_{\tilde f} = m_A = 3M_2,\ \tan\beta = 2$



$\tau(p \to K^+\bar\nu)/\mathrm{yrs}$

M_{H_C}/GeV

T_S/TeV

$\tau(p \to K^+\bar\nu) > 4.0 \times 10^{33}\,\mathrm{yrs}$

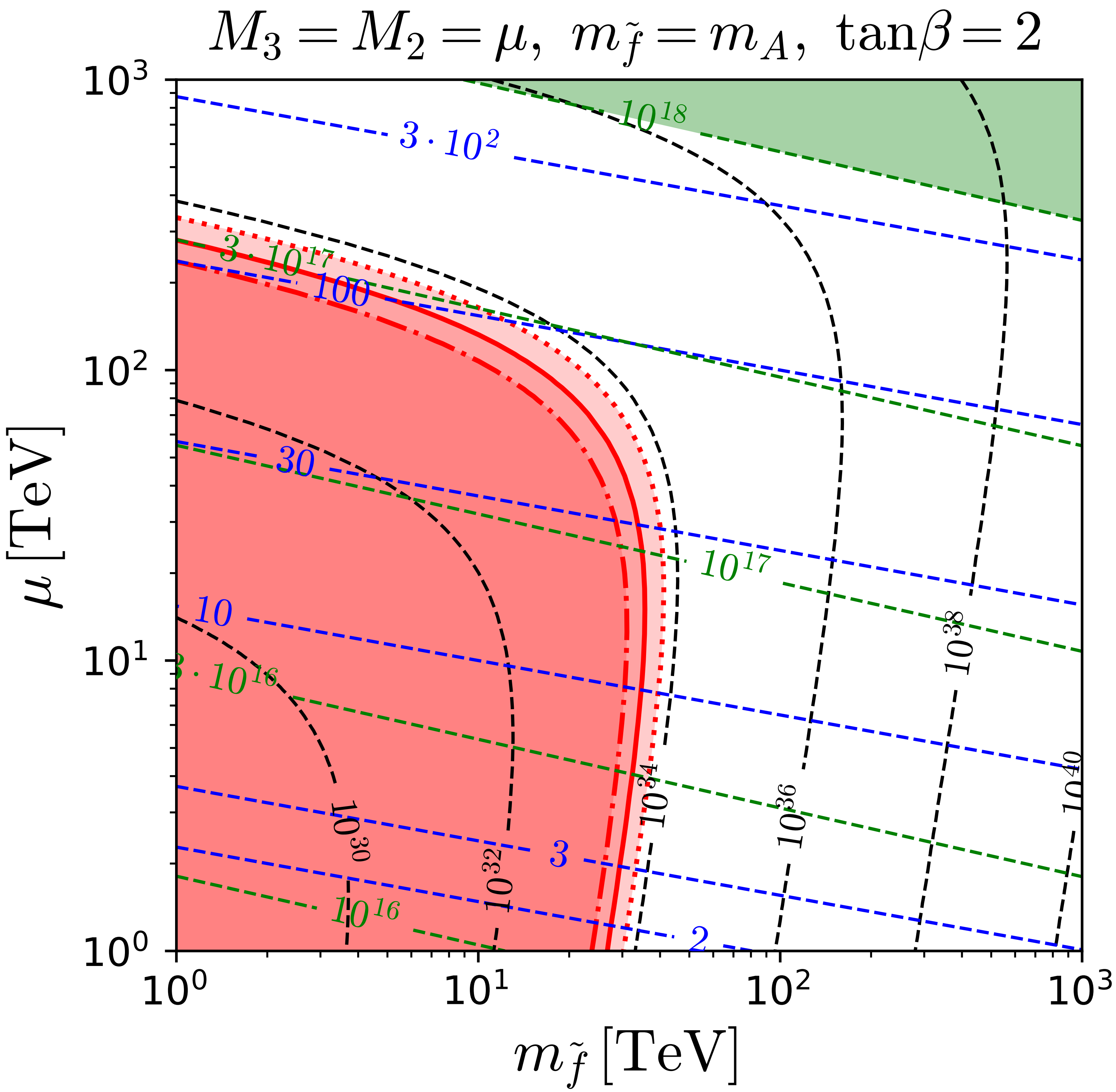
$\alpha_s(m_Z) = 0.1183 \pm 0.0008$

$\Lambda \equiv \max\{M_{H_C}, M_V\}$

$24.7 < \alpha^{-1}(\Lambda) < 31.8 \ \ (M_V = M_\Sigma)$

$3.77 \cdot 10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.83 \cdot 10^{16}\,\mathrm{GeV}$

Universal Gaugino Mass



$$\tau(p \to K^+\bar{\nu}) > 4.0 \times 10^{33}\,\mathrm{yrs}$$

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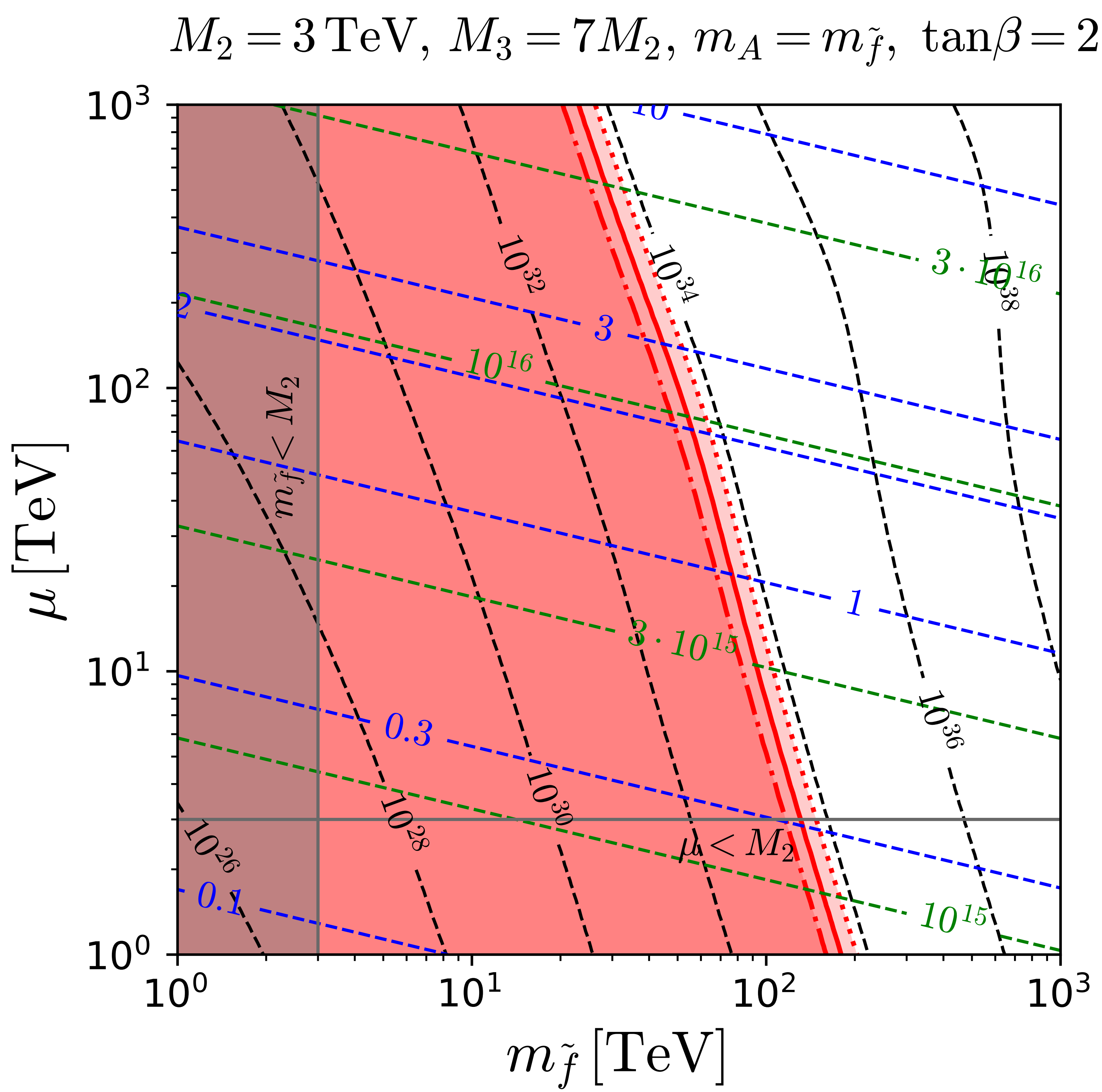
$$\Lambda \equiv \max\{M_{H_C}, M_V\}$$

$$25.0 < \alpha^{-1}(\Lambda) < 33.0 \quad (M_V = M_\Sigma)$$

$$3.14 \cdot 10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.52 \cdot 10^{16}\,\mathrm{GeV}$$

Pure Gravity Mediation



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$\tau(p \to K^+\bar{\nu}) > 4.0 \times 10^{33}\,\mathrm{yrs}$

$\alpha_s(m_Z) = 0.1183 \pm 0.0008$

$\Lambda \equiv \max\{M_{H_C}, M_V\}$

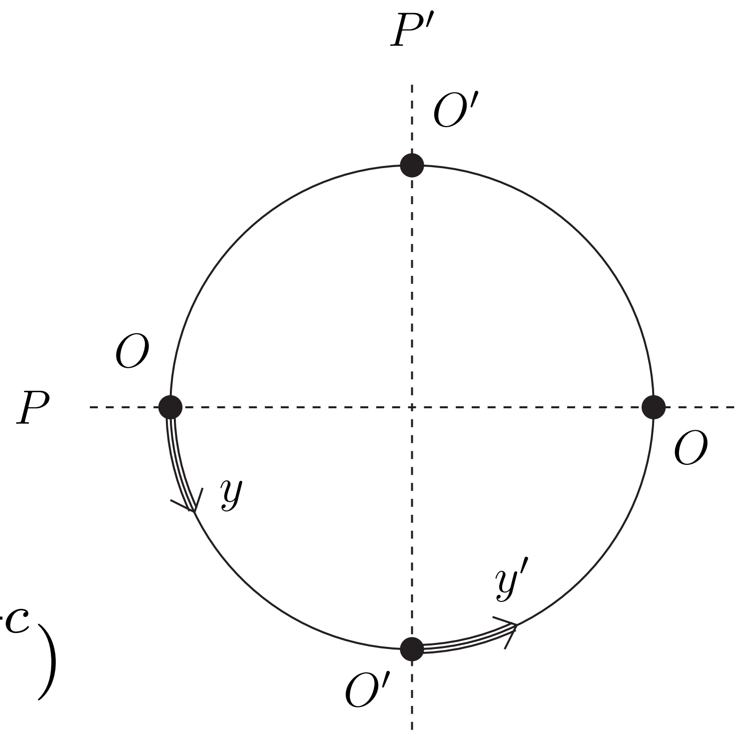
$25.3 < \alpha^{-1}(\Lambda) < 29.5\ \ (M_V = M_\Sigma)$

$8.90 \cdot 10^{15}\,\mathrm{GeV} < (M_V^2 M_\Sigma)^{\frac{1}{3}} < 1.19 \cdot 10^{16}\,\mathrm{GeV}$

Orbifold SU(5)

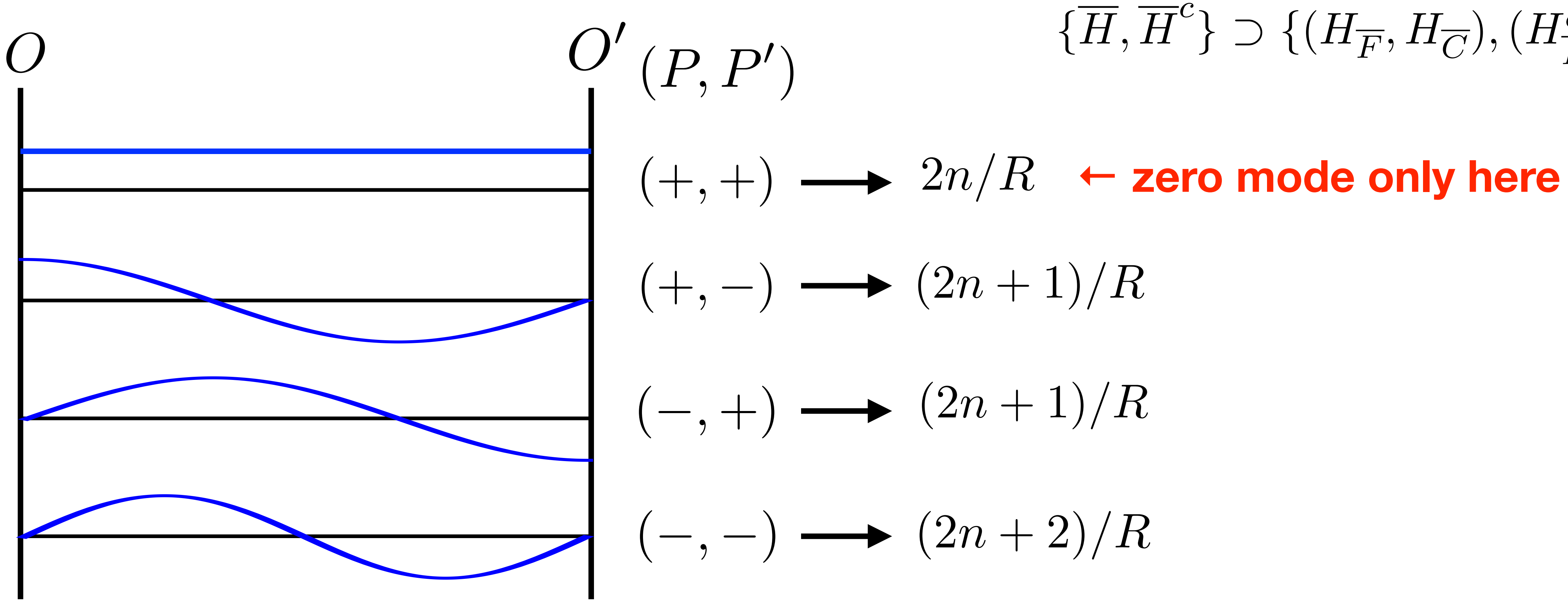
[Hall, Nomura '01]

- space-time = 4d Minkowski x $S^1/(Z_2 \times Z_2')$
- two fixed points O, O', & two orbifold parities P, P'
- N=2 4d SUSY in the bulk (8 spinor dof)
- 5d vector multiplets: $\mathcal{V}(\mathbf{24}) = (V, \Sigma)$
- 5d hipermultiplets: $\mathcal{H}(\mathbf{5}) = (H, H^c)$, $\overline{\mathcal{H}}(\overline{\mathbf{5}}) = (\overline{H}, \overline{H}^c)$



$$\{H,\ H^c\} \supset \{(H_F, H_C), (H^c_F, H^c_C)\}$$

$$\{\overline{H}, \overline{H}^c\} \supset \{(H_{\overline{F}}, H_{\overline{C}}), (H^c_{\overline{F}}, H^c_{\overline{C}})\}$$



a = unbroken generators $\hat{a}$ = broken generators $n = 0, 1, 2, \cdots$

KK mode	mass	(P, P')	4d fields	$\sum(b_1, b_2, b_3)$
zero	0	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	
even	$(2n+2)/R$	$(+,+)$	$V^a,\ H_F,\ H_{\overline{F}}$	$(\frac{6}{5}, -2, -6)$
		$(-,-)$	$\Sigma^a,\ H^c_F,\ H^c_{\overline{F}}$	
odd	$(2n+1)/R$	$(+,-)$	$V^{\hat{a}},\ H_C,\ H_{\overline{C}}$	$(-\frac{46}{5}, -6, -2)$
		$(-,+)$	$\Sigma^{\hat{a}},\ H^c_C,\ H^c_{\overline{C}}$	

- doublet-triplet splitting can be achieved by the parity assignment
- X,Y fields are absent at the 3-brane at *O'* => SU(5) is explicitly broken at *O'*
- non-universal contribution to the gauge couplings from the 3-brane at *O'*

$$f^a \mathcal{W}^a \mathcal{W}^a \Big|_{O'}$$

the effect is negligible due to the dominant bulk contribution if the extra dimension is large, RΛ > 4 [Hall, Nomura '01]

- The success of charge quantisation in 4d GUTs can be kept by placing the matter fields at the SU(5) symmetric brane at *O*.

- D=5 proton decay is absent due to an accidental $U(1)_R$ sym.

	Σ	$H_{\mathbf{5}}$	$H_{\bar{\mathbf{5}}}$	$H^c_{\mathbf{5}}$	$H^c_{\bar{\mathbf{5}}}$	$T_{\mathbf{10}}$	$F_{\bar{\mathbf{5}}}$	$N_{\mathbf{1}}$
$U(1)_R$	0	0	0	2	2	1	1	1

- Above the KK mass scale, $M_c = 1/R$, *KK* modes appear and the three gauge couplings approach each other. They finally unify at the cut-off scale, Λ, where the 5d theory is incorporated into a more fundamental theory.

$$\ln\left(\frac{T_G}{\Lambda}\right) = \sum_\xi \left(-\frac{5}{288}b_1^\xi - \frac{15}{76}b_2^\xi + \frac{25}{114}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

$$\ln\Omega_G = \sum_\xi \left(\frac{10}{19}b_1^\xi - \frac{24}{19}b_2^\xi + \frac{14}{19}b_3^\xi\right)\ln\left(\frac{m_\xi}{\Lambda}\right)$$

the total contributions from (2k+1) and (2k+2) modes

$$c_o \ln\frac{2k+1}{R\Lambda} \qquad c_e \ln\frac{2k+2}{R\Lambda}$$

$$c_o = -c_e = \frac{24}{19} \text{ for } \ln\Omega_G \qquad c_o = -c_e = \frac{18}{19} \text{ for } \ln(T_G/\Lambda) \qquad c_o = \frac{4}{19},\ c_e = -\frac{156}{19} \text{ for } C_G$$

The index k runs from 0 to $k_{\max}^{o/e}$, where
$$\begin{cases} 2k_{\max}^o + 1 < R\Lambda & \text{for odd modes} \\ 2k_{\max}^e + 2 < R\Lambda & \text{for even modes} \end{cases}$$

$$\Omega_G = \left[\frac{\prod_k^{k_{\max}^o}(2k+1)}{\prod_k^{k_{\max}^e}(2k+2)}\left(\frac{1}{r}\right)^{k_{\max}^o - k_{\max}^e}\right]^{\frac{24}{19}},$$

$$\frac{T_G}{\Lambda} = \left[\frac{\prod_k^{k_{\max}^o}(2k+1)}{\prod_k^{k_{\max}^e}(2k+2)}\left(\frac{1}{r}\right)^{k_{\max}^o - k_{\max}^e}\right]^{\frac{18}{19}}, \qquad (r \equiv R\Lambda)$$

$$C_G = \frac{4}{19}\ln\left[\prod_{k=0}^{k_{\max}^o}\frac{2k+1}{r}\right] - \frac{156}{19}\ln\left[\prod_{k=0}^{k_{\max}^e}\frac{2k+2}{r}\right]$$

$(r \equiv R\Lambda)$

$r \in$	$(1,2]$	$(2,3]$	$(3,4]$	$(4,5]$	$\cdots$
Ω_G	$\left[\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1}{2}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{24}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{24}{19}}$	$\cdots$
T_G/Λ	$\left[\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1}{2}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2}\frac{1}{r}\right]^{\frac{18}{19}}$	$\left[\frac{1\cdot 3}{2\cdot 4}\right]^{\frac{18}{19}}$	$\cdots$

$$T_G/\Lambda = \Omega_G^{\frac{3}{4}}$$

GCU condition

$$T_S = M_s^* \Omega_G$$

$$T_G = M_G^* \Omega_S$$

$$T_S = \left[M_3^{-28} M_2^{32} \mu^{12} m_A^3 X_T\right]^{\frac{1}{19}} = M_s^* \Omega_G$$

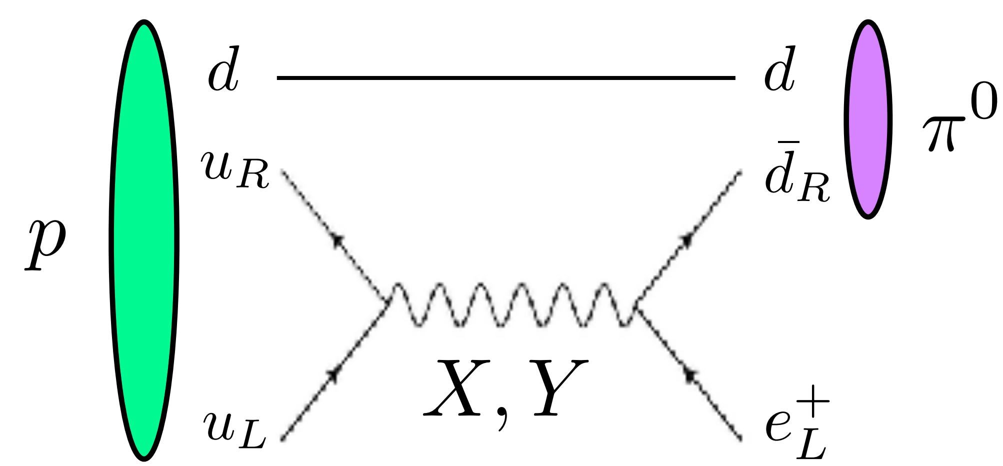
non-trivial constraint on SUSY masses

$$M_c = M_G^* \Omega_G^{-\frac{3}{4}} \Omega_S / r$$

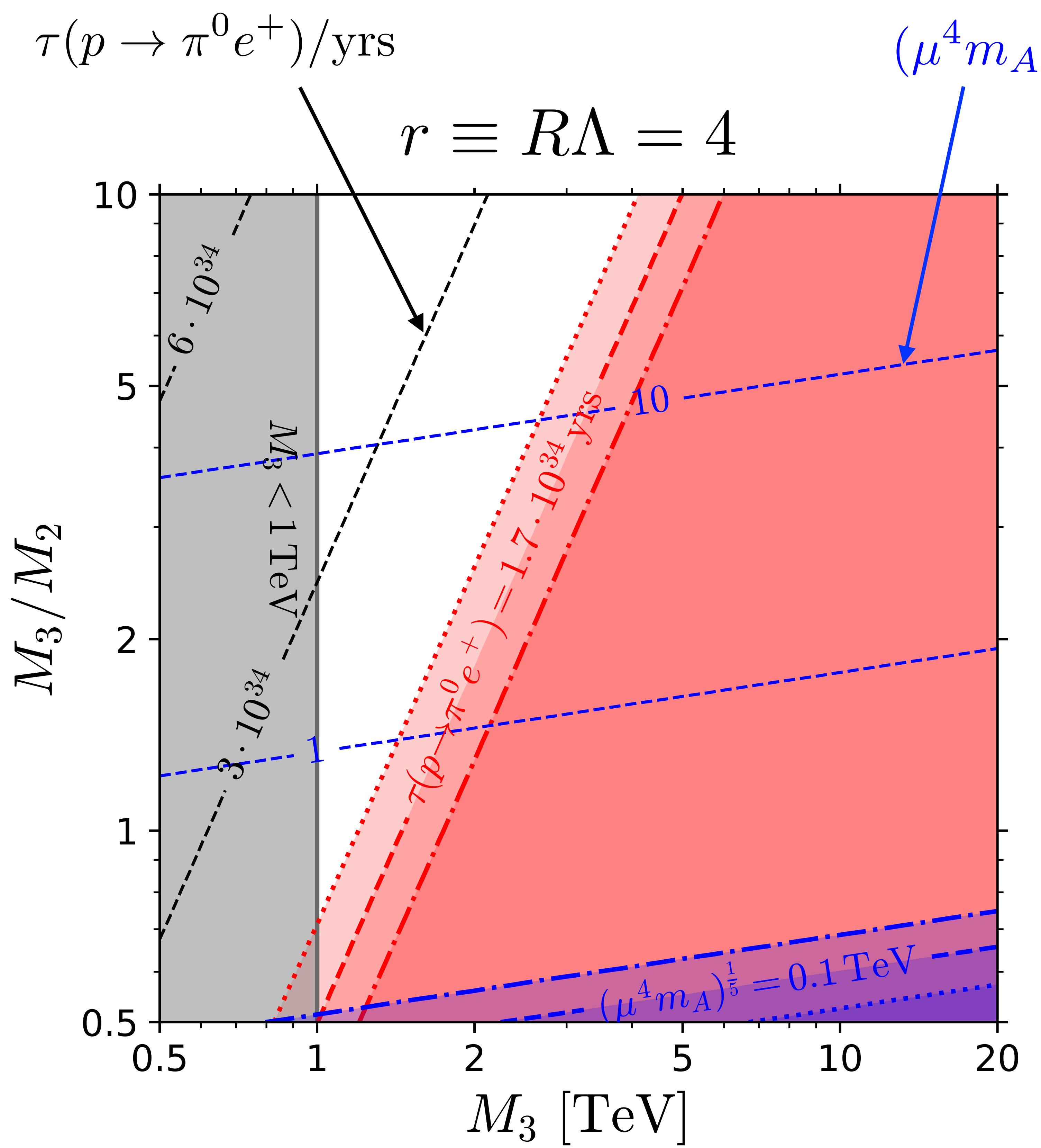
$$= M_G^* M_s^{*\,\frac{19}{108}} \Omega_G^{-\frac{31}{54}} M_3^{-\frac{19}{216}} M_2^{-\frac{19}{216}} X_T^{-\frac{1}{108}} X_\Omega^{-\frac{1}{288}} / r$$

compactification scale can be predicted from SUSY spectrum and r ⇒ allows to predict D=6 proton decay

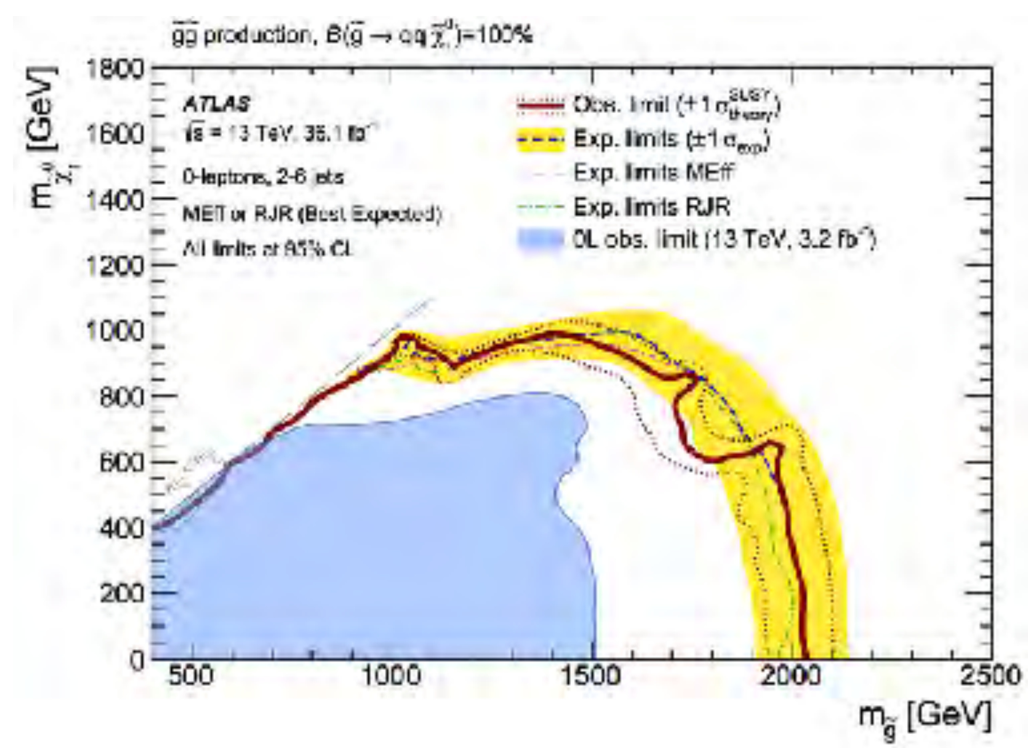
D=6 proton decay



A SUSY plane with GCU

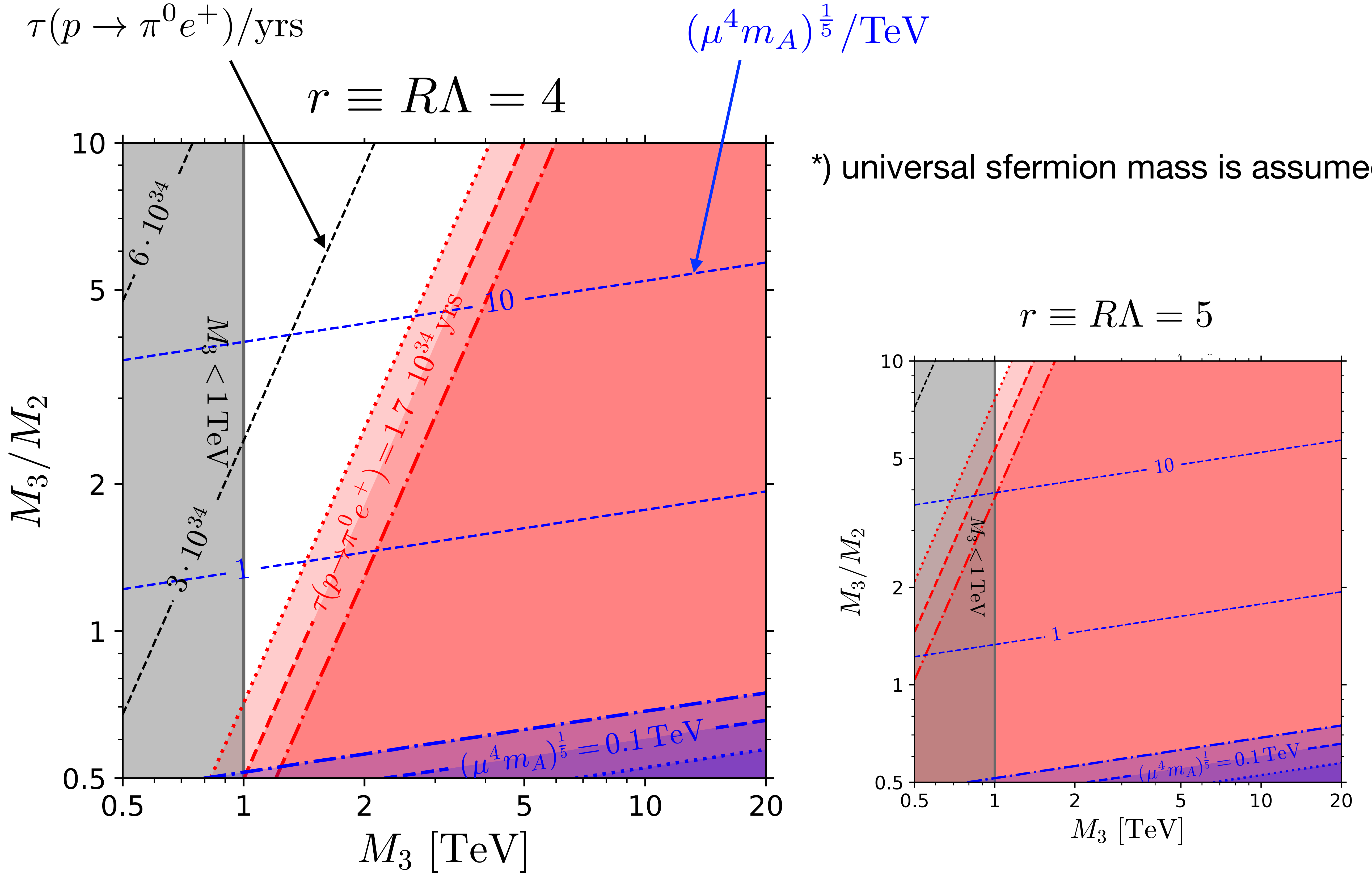


*) universal sfermion mass is assumed

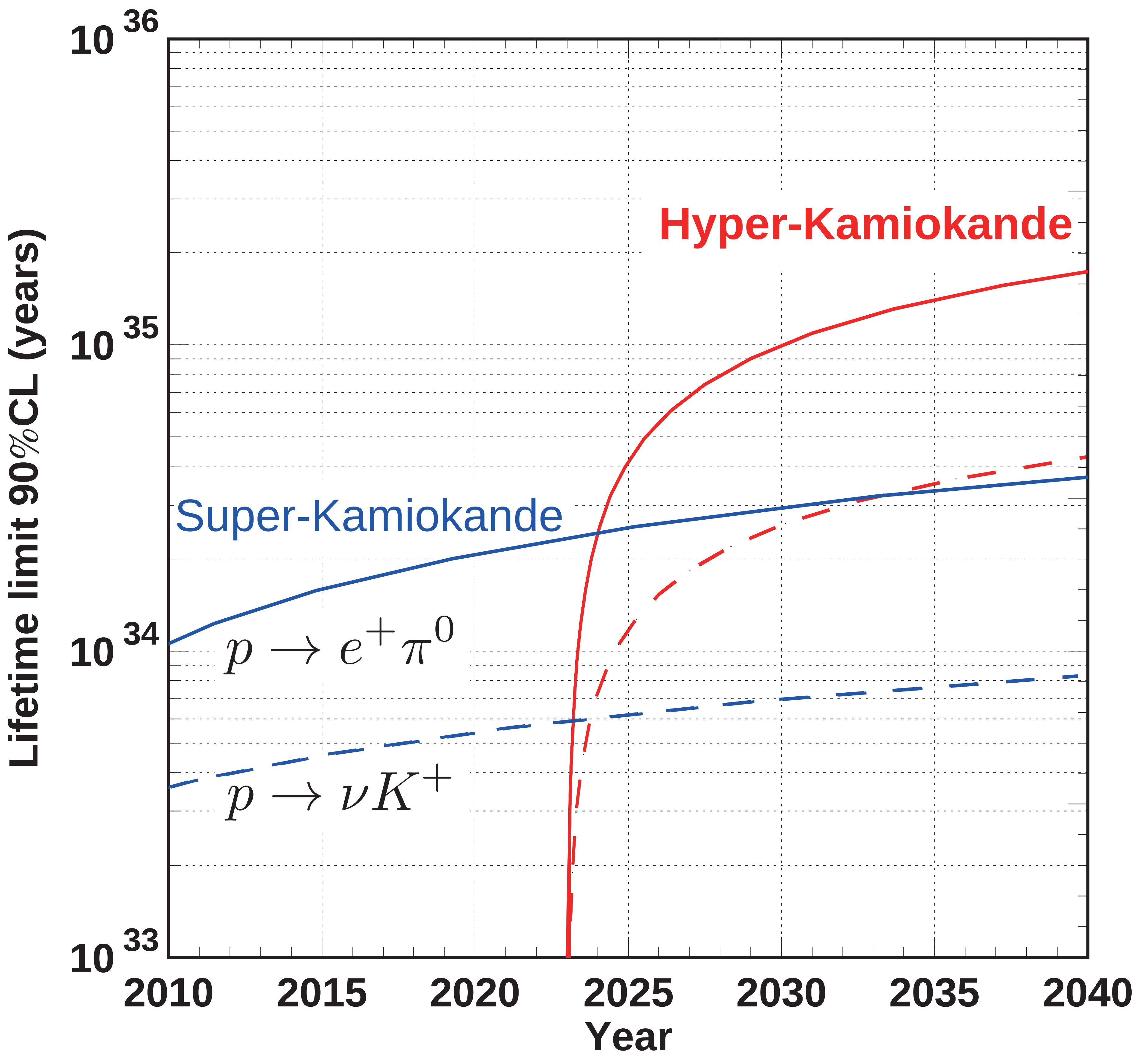


M_3 > 1 - 2 TeV from LHC

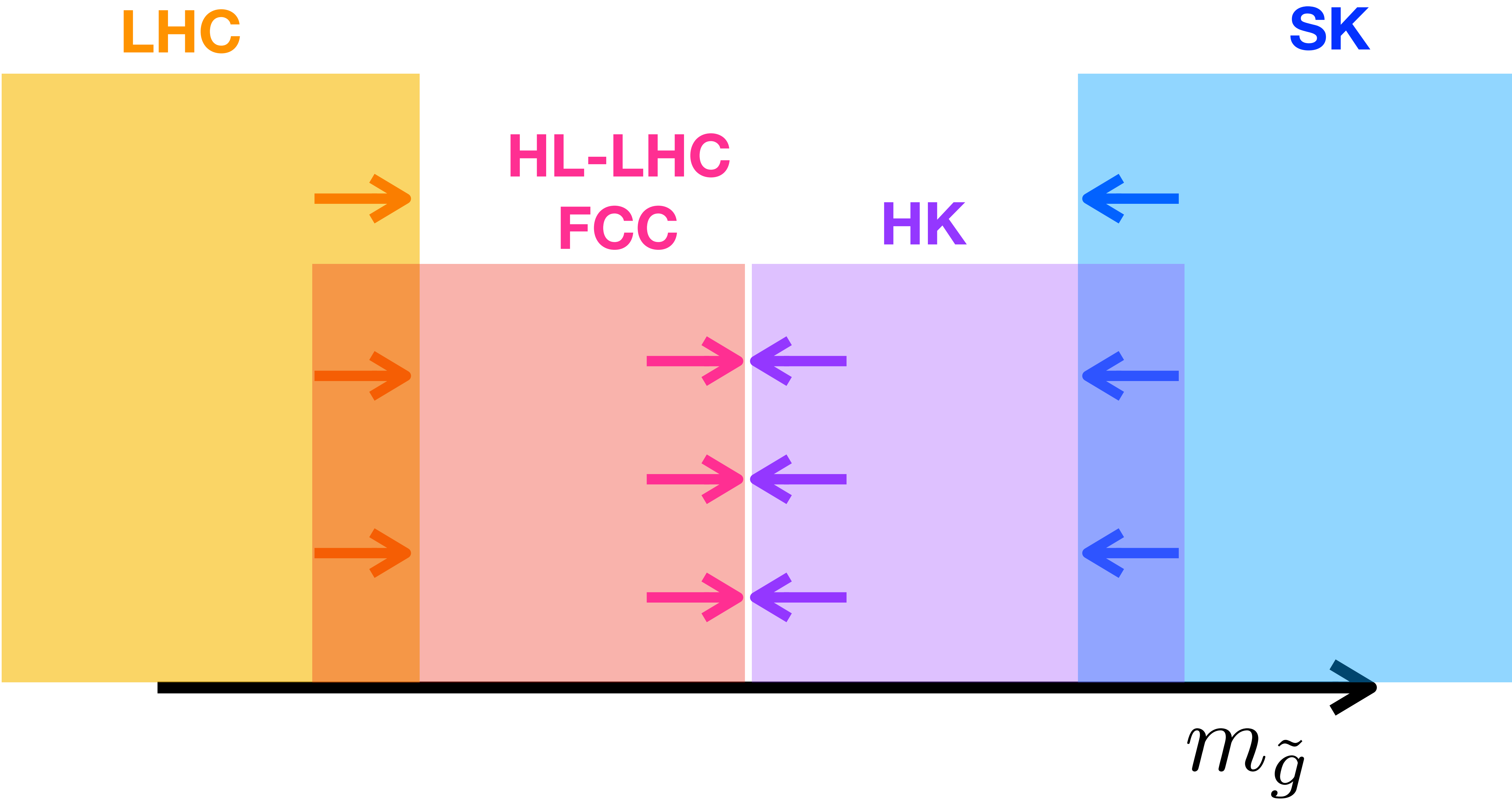
A SUSY plane with GCU



*) universal sfermion mass is assumed



Lifetime limit 90%CL (years)
10^{36}
10^{35}
10^{34}
10^{33}
Hyper-Kamiokande
Super-Kamiokande
$p \rightarrow e^+\pi^0$
$p \rightarrow \nu K^+$
2010
2015
2020
2025
2030
2035
2040
Year


LHC
SK
HL-LHC
FCC
HK
$m_{\tilde{g}}$

Discussion

- The gauge coupling unification does not constrain the Bino mass, since it is singlet. The LHC signature varies depending on the Bino mass.

- The formula can easily be extended to non-minimal SUSY models.

NMSSM => the same formula can be used.

Non-singlet extension => straightforward to update the formulae for C_S, Ω_S, T_S

$$\begin{pmatrix} \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_1^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_2^\eta}\right) \\ \ln\left(\prod_\eta \left[\frac{m_\eta}{m_Z}\right]^{b_3^\eta}\right) \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \delta_1 \\ 1 & b_2 & \delta_2 \\ 1 & b_3 & \delta_3 \end{pmatrix} \begin{pmatrix} C_S \\ \ln \Omega_S \\ \ln\left(\frac{T_S}{m_Z}\right) \end{pmatrix}$$

- The similar formula can be found for non-SUSY models.

Conclusions

- We have derived an analytic formula for the condition of GCU and the unified coupling at Λ in terms of the general SUSY and GUT spectra, including the dominant 2-loop effect and $\alpha_s(m_Z)$ uncertainty.

GCU condition

$$T_S = M_s^* \Omega_G \quad \cap \quad T_G = M_G^* \Omega_S$$

unified coupling

$$\alpha^{-1}(\Lambda) = \alpha_G^{*-1} + \frac{1}{2\pi}(C_S + C_G)$$

- Minimal SU(5):

The coloured Higgs mass is given as a function of low energy SUSY masses: D=5 proton decay can be predicted by the SUSY spectrum.

- Orbifold SUSY SU(5):

There is a non-trivial constraint on the SUSY spectrum. The X,Y boson mass is given as a function of low energy SUSY masses: D=6 proton decay can be predicted by the SUSY spectrum.

Backup

Is Minimal SU(5) excluded?

[Murayama, Piece '01]

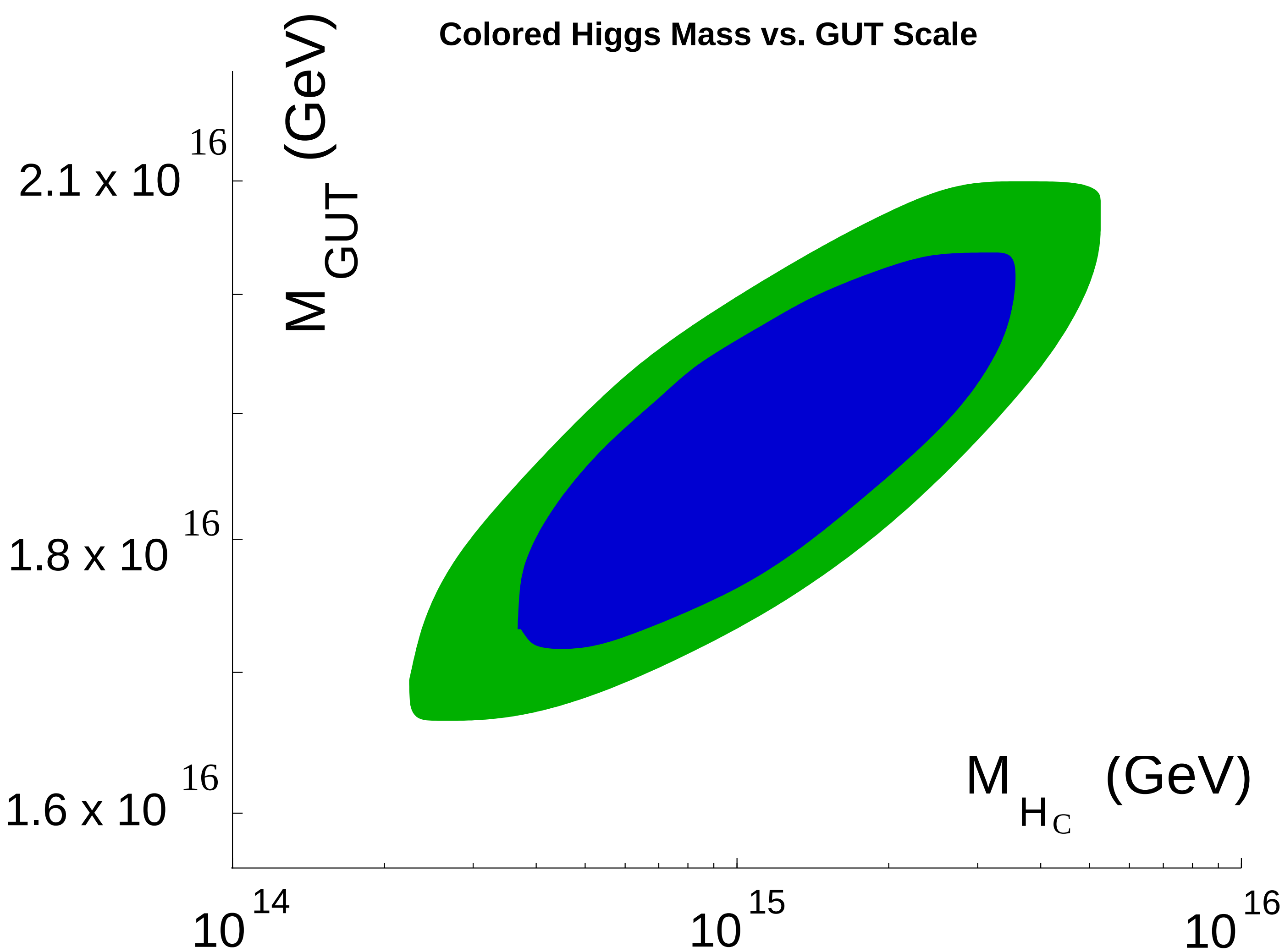


FIG. 2. Plot showing 68% and 90% contours allowed by the renormalization group analysis for the color Higgs triplet mass, M_{H_C}, and the GUT scale, $M_{GUT} \equiv (M_\Sigma M_V^2)^{1/3}$.

assumption:

$$M_3/M_2 = \alpha_3/\alpha_2$$

$$m_{\tilde{f}} = 1\,\mathrm{TeV}$$

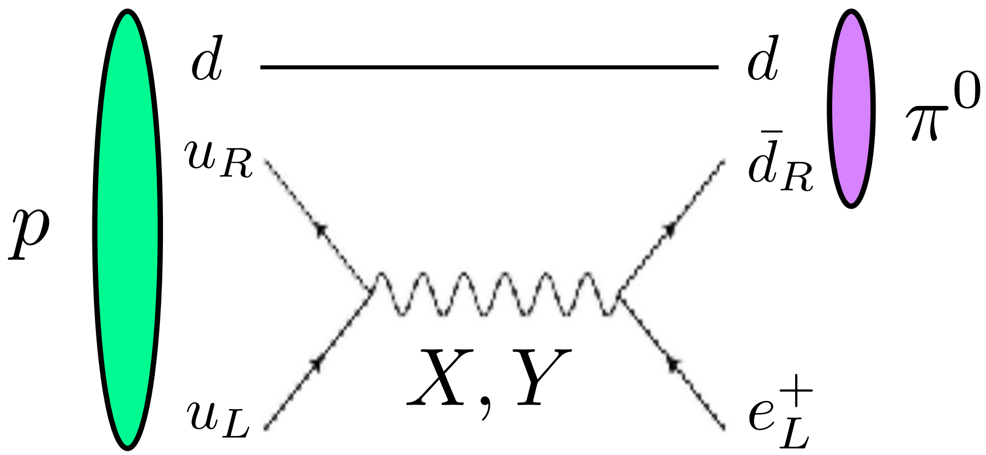
$$m_{\tilde{t}} \in (400, 800)\,\mathrm{GeV}$$

$$\tan\beta \in (1.8, 4)$$

$$M_2 \in (100, 400)\,\mathrm{GeV}$$

$$\mu \in (100, 1000)\,\mathrm{GeV}$$

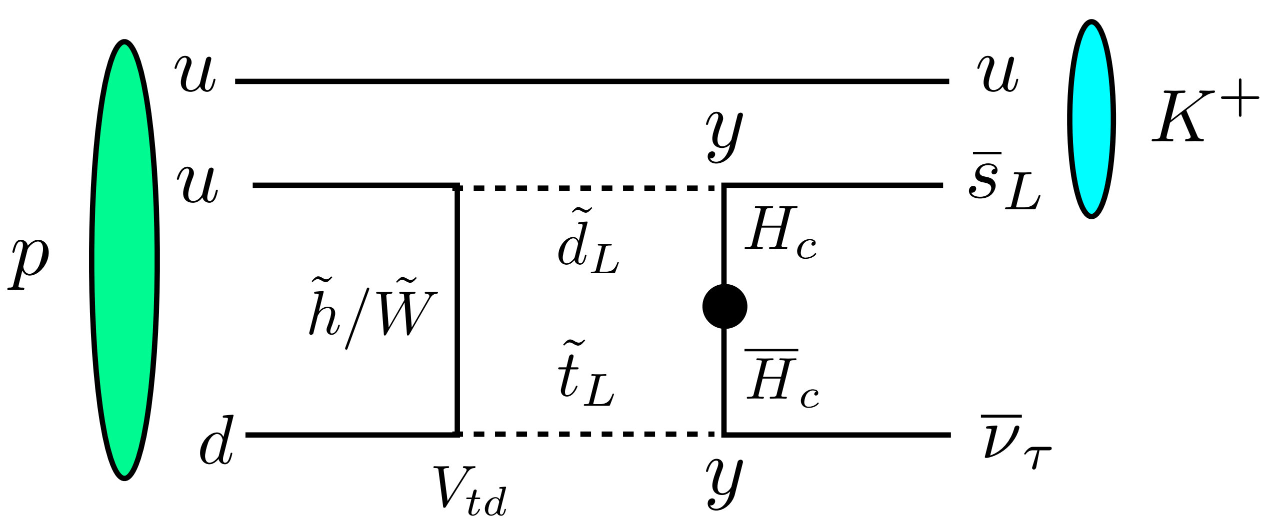
D=6 $\frac{g^2}{\Lambda^2}\,(\mathbf{10}_i^*\mathbf{10}_i)(\mathbf{10}_j^*\mathbf{10}_j)$ $\frac{g^2}{\Lambda^2}\,(\mathbf{10}_i^*\mathbf{10}_i)(\overline{\mathbf{5}}_j^*\overline{\mathbf{5}}_j)$



$$\tau_{p\to e^+\pi^0} \sim \frac{1}{\alpha_G^2}\frac{M_{X,Y}^4}{m_p^5} > 1.7\cdot 10^{34}\ \text{years}$$

$$M_{X,Y} \gtrsim 6\cdot 10^{15}\ \text{GeV}\cdot\left(\frac{\alpha_G}{25.}\right)^2$$

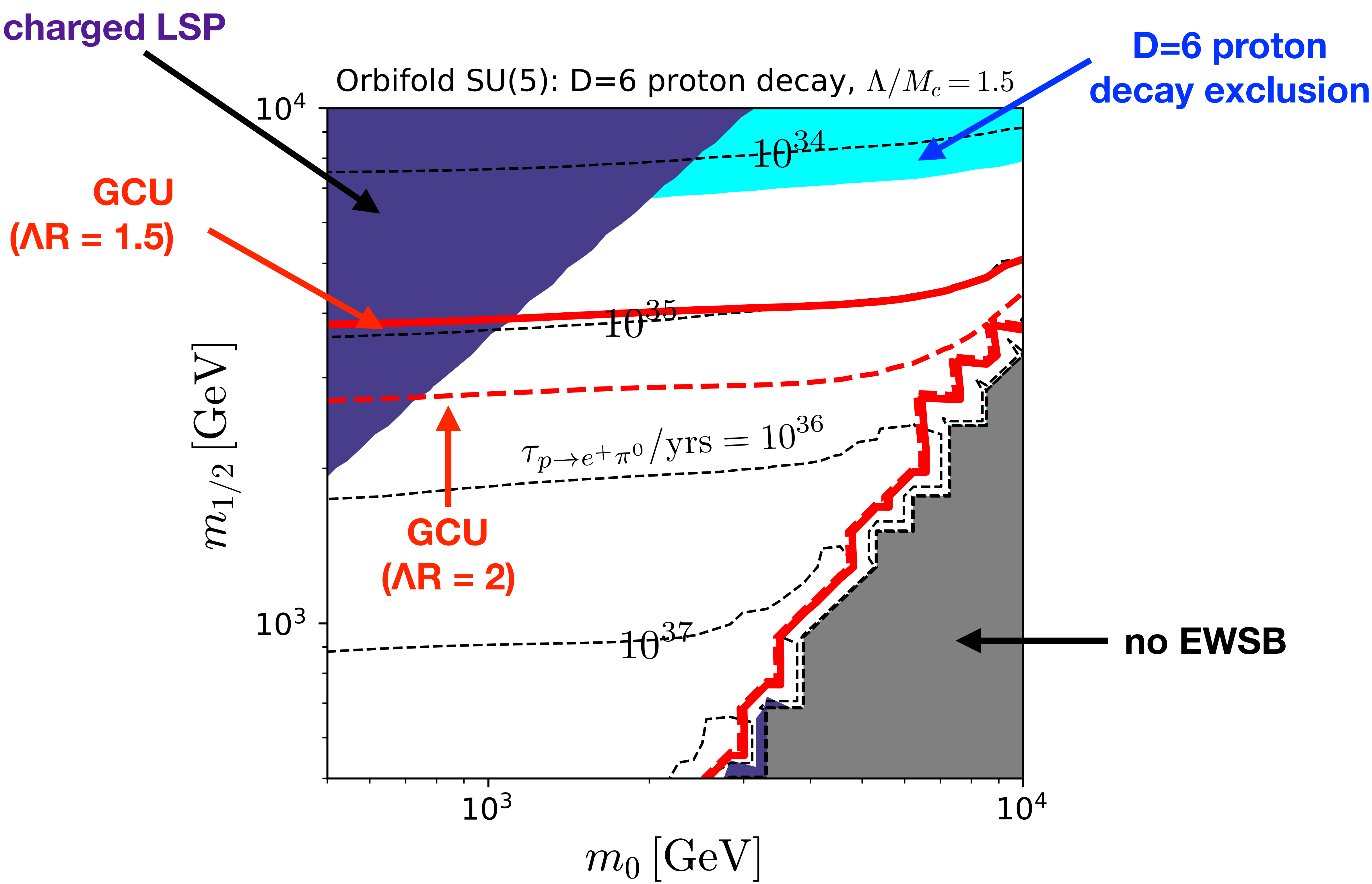
D=5 $\frac{y_i y_j}{\Lambda}\,\mathbf{10}\cdot\mathbf{10}\cdot\mathbf{10}\cdot\overline{\mathbf{5}}$



$$\tau(p\to K^+\bar{\nu}) > 5.9\times 10^{33}\ \text{yrs}$$

$$t_\beta^4\left(\frac{10^3\ \text{GeV}}{M_{\text{SUSY}}}\right)^2\left(\frac{10^{19}\ \text{GeV}}{M_{H_c}}\right)^2 \lesssim 1$$

CMSSM


charged LSP
D=6 proton
decay exclusion
GCU
(ΛR = 1.5)
GCU
(ΛR = 2)
no EWSB
Orbifold SU(5): D=6 proton decay, $\Lambda/M_c = 1.5$
10^{34}
10^{35}
$\tau_{p\to e^+\pi^0}/\mathrm{yrs} = 10^{36}$
10^{37}
10^4
10^3
$m_{1/2}\,[\mathrm{GeV}]$
$m_0\,[\mathrm{GeV}]$