Flux Compactifications & the Hierarchy Problem

Wilfried Buchmuller
DESY, Hamburg

in collaboration with
Markus Dierigl & Emilian Dudas
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UV Completion of the Standard Model

- Structure of Standard Model points towards "grand unification" of strong and electroweak interactions (quark and lepton content, gauge group, "unification" of gauge couplings, small neutrino masses ...)

- Strong theoretical arguments for supersymmetry at "high" energy scales (gravity, extra dimensions, string theory)

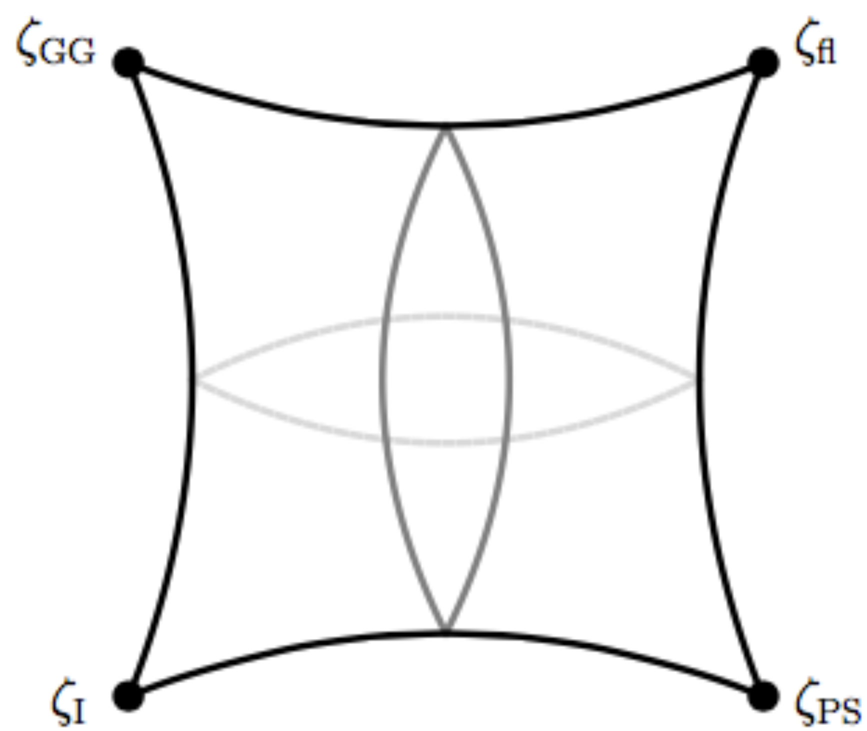
- Energy scale of grand unification: $\Lambda_{\mathrm{GUT}} \simeq 10^{15} \ldots 10^{16}$ GeV
  energy scale of supersymmetry breaking: $\Lambda_{\mathrm{SB}} \simeq$??

- 6d flux compactification: $\Lambda_{\mathrm{SB}} \sim R_{\mathrm{c}}^{-1} \sim \Lambda_{\mathrm{GUT}}$

Split symmetries

WB, Dierigl, Ruehle, Schweizer, ... '15, ...

Consider SO(10) GUT group in 6d, broken at orbifold fixed points to standard SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with SM group as intersection; bulk fields 45, 16, 16*, 10's [Asaka, WB, Covi '02; Hall, Nomura et al '02; ...]; full 6d gauge symmetry:



$$SO(10) \times U(1)_A$$

magnetic $U(1)_A$ flux

charged bulk 16-plet, with N flux quanta, yields N 16's of zero-modes:

$$\mathbf{16}\ [SO(10)] \sim \mathbf{5}^* + \mathbf{10} + \mathbf{1}\ [SU(5)] \sim \mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}}\ [G_{\rm SM}]$$

Higgs fields from two uncharged bulk 10-plets, form split multiplets:

$$H_1 \supset H_u\,, \quad H_2 \supset H_d$$

Flux **breaks supersymmetry** [Bachas '95], soft SUSY breaking only for quark-lepton families:

$$M^2 = m_{\tilde{q}}^2 = m_{\tilde{l}}^2 = \frac{4\pi N}{V_2} \sim (10^{15}\ \mathrm{GeV})^2$$

$$m_{3/2} \sim 10^{14}\ \mathrm{GeV}\,, \quad m_{\tilde{q}}^2 = m_{\tilde{l}}^2 > m_{3/2} \sim m_{1/2} \gg m_{\tilde{h}}$$

Emerging picture of **Split Symmetries** (cf. "split/spread SUSY" [Arkani-Hamed, Dimopoulos; Giudice, Romanino '04; Hall, Nomura '11]):

- *complete* GUT representations (quarks, leptons) come with a multiplicity, *incomplete* GUT representations (Higgs) only once

- masses of scalar quarks and leptons large, *because* they form *complete* GUT multiplets (magnetic flux)

- masses of Higgs/higgsinos small, *because* they form *incomplete* GUT multiplets (THDM); size of **quantum corrections**?

Can GUT-scale SUSY breaking be viable?

- Is a matching of THDMs to SUSY at GUT scale consistent with RG running and vacuum stability? (yes) Hope for the LHC: additional Higgs bosons, light higgsinos favoured

- Can all moduli be stabilized (D-term breaking, F-term breaking ...) with de Sitter (Minkowski) vacuum? (yes)

- Can the 6d SO(10)xU(1) SUGRA models be embedded into string theory? (yes, F-theory)

- How are **quantum corrections** affected by Kaluza-Klein tower? (observation 2016: cancellations at one loop)

Magnetic flux & quantum corrections

Consider 6d **gauge-Higgs unification** (Hosotani `83, Arkani-Hamed et al. `01, Antoniadis et al. `01, ...); Weyl fermion interacting with Abelian gauge field,

$$S_6 = \int d^6x \left(-\frac{1}{4} F^{MN} F_{MN} + i\overline{\Psi}\Gamma^M D_M \Psi \right) ,$$

$$D_M = \partial_M + iqA_M \, , \; F_{MN} = \partial_M A_N - \partial_N A_M \, , \; \Gamma_7 \Psi = -\Psi \, .$$

Compactification to 4d Minkowski space on square torus of area L^2 without magnetic flux, $\langle F_{56} \rangle = 0$; standard mode expansion of gauge and matter fields:

$$A_M(x_M, \theta, \bar{\theta}) = \sum_{n,m} A_{M;n,m}(x_\mu, \theta, \bar{\theta}) \, \lambda_{n,m}(x_m) \, ,$$

$$\Psi(x_M, \theta, \bar{\theta}) = \sum_{n,m} \Psi_{n,m}(x_\mu, \theta, \bar{\theta}) \, \lambda_{n,m}(x_m) \, ;$$

$$\lambda_{n,m}(x_m) = \frac{1}{L} \exp\left[\frac{2\pi i}{L} (n x_5 + m x_6) \right] , \quad M_{n,m} = \frac{2\pi}{L}(m + in)$$

Wilson-line scalar, $\phi = \frac{1}{2}(A_6 + iA_5)|_{n=m=0}$, is Higgs field in gauge-Higgs unification, massless at tree level,

$$m_\phi^2 = 0$$

one-loop correction, sum over Kaluza-Klein tower of states,

$$\delta m_\phi^2 = -4q^2 \sum_{n,m} \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 + |M_{n,m}|^2)^2}$$

after some manipulations finite result, due to discrete symmetry, remnant of gauge symmetry in extra dimensions [Antoniadis, Benakli, Quiros '01, Cheng, Matchev, Schmaltz '02, Ghilencea et al. '05; Dierigl '17] ($L = 2\pi R$),

$$m_\phi^2 \simeq 0.19 \; \frac{\alpha}{\pi} \frac{1}{R^2}$$

expected result: loop-factor times cutoff, i.e., small hierarchy; original application: electroweak symmetry breaking in models with large extra dimensions; can one also obtain a scalar much lighter than the cutoff ??

Compactification with magnetic flux

For convenience, rewrite 6d Lagrangian,

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} : \quad \gamma_5 \psi_L = -\psi_L \,, \quad \gamma_5 \psi_R = \psi_R \,,$$

$$\psi_L = \begin{pmatrix} \psi \\ 0 \end{pmatrix} \,, \quad \psi_R = \begin{pmatrix} 0 \\ \overline{\chi} \end{pmatrix} \,.$$

Complex coordinates and gauge fields,

$$z = \frac{1}{2}\left(x_5 + i x_6\right) \,, \quad \partial_z = \partial_5 - i\partial_6 \,, \quad \phi = \frac{1}{\sqrt{2}}\left(A_6 + iA_5\right)$$

constant magnetic flux background,

$$\langle A_5 \rangle = -\tfrac{1}{2} f x_6 \,, \ \langle A_6 \rangle = \tfrac{1}{2} f x_5 \,, \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} f \bar{z}$$

shift around quantized magnetic flux,

$$\frac{q}{2\pi} \int_{T^2} F = \frac{q}{2\pi} f = N \in \mathbb{Z} \,, \quad \phi = \frac{f}{\sqrt{2}} \bar{z} + \varphi$$

6d action with flux-dependent bilinear term of Weyl fermions,

$$
\begin{aligned}
S_6 = \int d^6x \Big(& -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \partial^\mu \overline{\varphi} \partial_\mu \varphi - \frac{1}{4} \left(\partial_z \overline{\varphi} + \partial_{\bar{z}} \varphi\right)^2 - \frac{1}{2} f^2 \\
& - \frac{1}{2} \partial_{\bar{z}} A^\mu \partial_z A_\mu - \frac{i}{\sqrt{2}} \partial_\mu A^\mu \left(\partial_z \overline{\varphi} - \partial_{\bar{z}} \varphi\right) \\
& - i\psi\sigma^\mu \overline{D}_\mu \overline{\psi} - i\chi\sigma^\mu D_\mu \overline{\chi} \\
& - \chi \left(\partial_z + qf\bar{z} + \sqrt{2} q\varphi\right) \psi - \overline{\chi} \left(\partial_{\bar{z}} + qfz + \sqrt{2} q\overline{\varphi}\right) \overline{\psi} \Big) \,,
\end{aligned}
$$

invariance under translations on torus; breaking of translational invariance by background gauge field compensated by **shift** of φ ,

$$
\delta_T \varphi = \left(\epsilon \partial_z + \overline{\epsilon} \partial_{\bar{z}}\right) \varphi + \frac{\bar{\epsilon}}{\sqrt{2}} f
$$

Lagrangian transforms into total divergence.

Lagrangian is invariant under local redefinition of fields, with change of boundary conditions (cf. Scherk-Schwarz '79),

$$\varphi_\Lambda = \varphi - \frac{1}{\sqrt{2}}\partial_z\Lambda\,,\ \psi_\Lambda = e^{q\Lambda}\psi\,,\ \chi_\Lambda = e^{-q\Lambda}\chi\,,\ \Lambda = f\left(\alpha\bar{z} - \bar{\alpha}z\right)\,,$$

for infinitesimal transformations φ shifts by a constant,

$$\delta_\Lambda\psi = q\Lambda\psi\,,\quad \delta_\Lambda\chi = -q\Lambda\chi\,,\quad \delta_\Lambda\varphi = -\frac{1}{\sqrt{2}}\partial_z\Lambda = \frac{\bar{\alpha}}{\sqrt{2}}f\,.$$

Mass spectrum of KK modes: Landau levels, harmonic oscillator algebra (Bachas '95, Alfaro et al. '07, Braun et al. '07; quantum hall effect):

$$a_+ = \frac{i}{\sqrt{2qf}}\left(\partial_z + qf\bar{z}\right)\,,\quad a_+^\dagger = \frac{i}{\sqrt{2qf}}\left(\partial_{\bar{z}} - qfz\right)\,,$$

$$a_- = \frac{i}{\sqrt{2qf}}\left(\partial_{\bar{z}} + qfz\right)\,,\quad a_-^\dagger = \frac{i}{\sqrt{2qf}}\left(\partial_z - qf\bar{z}\right)\,,$$

$$[a_\pm, a_\pm^\dagger] = 1\,,\quad [a_\pm, a_\mp] = 0\,,\quad [a_\pm, a_\mp^\dagger] = 0\,.$$

Mode expansion: ground state and higher levels:

$$\psi = \sum_{n,j} \psi_{n,j}\xi_{n,j}\,, \quad \chi = \sum_{n,j} \chi_{n,j}\bar{\xi}_{n,j}\,,$$

$$a_+\xi_{0,j} = 0\,, \quad a_-\bar{\xi}_{0,j} = 0\,,$$

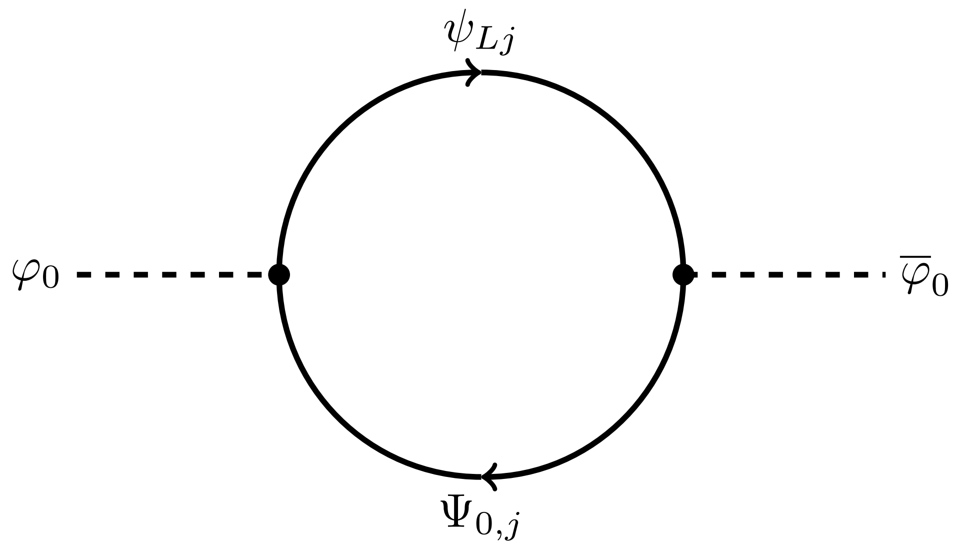
$$\xi_{n,j} = \frac{i^n}{\sqrt{n!}}\left(a_+^\dagger\right)^n \xi_{0,j}\,, \quad \bar{\xi}_{n,j} = \frac{i^n}{\sqrt{n!}}\left(a_-^\dagger\right)^n \bar{\xi}_{0,j}\,,$$

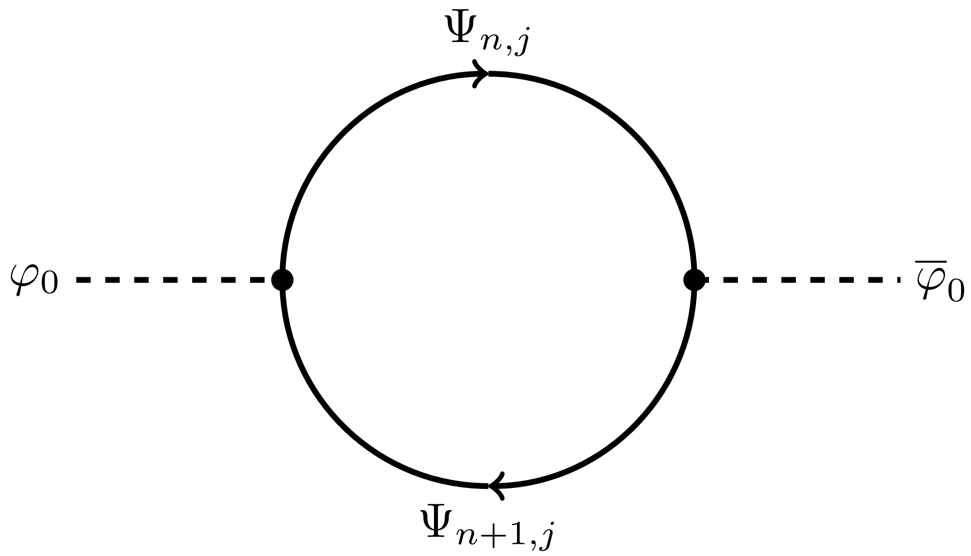
$$\mathcal{M}_+^2\xi_{n,j} = 2qfn\xi_{n,j}\,, \quad \mathcal{M}_-^2\bar{\xi}_{n,j} = 2qf\,(n+1)\,\bar{\xi}_{n,j}\,.$$

chiral spectrum (index theorem), zero mode contained in ψ. Action for fermions and gauge zero-modes (including Wilson-line scalar):

$$S_4 = \int d^4x\Big(-\partial^\mu\overline{\varphi}_0\partial_\mu\varphi_0 + \sum_{n,j}\Big(i\overline{\psi}_{Lj}\gamma^\mu D_\mu\psi_{Lj} + i\overline{\Psi}_{n,j}\gamma^\mu D_\mu\Psi_{n,j}$$
$$+ \sqrt{2qf(n+1)}\;\overline{\Psi}_{n,j}\Psi_{n,j} + \sqrt{2}q\varphi_0\left(\overline{\Psi}_{0,j}\tfrac{1-\gamma_5}{2}\psi_{Lj} + \overline{\Psi}_{n+1,j}\tfrac{1-\gamma_5}{2}\Psi_{n,j}\right)$$
$$+ \sqrt{2}q\overline{\varphi}_0\left(\overline{\psi}_{Lj}\tfrac{1+\gamma_5}{2}\Psi_{0,j} + \overline{\Psi}_{n,j}\tfrac{1+\gamma_5}{2}\Psi_{n+1,j}\right)\Big)\Big)\,.$$

Quantum corrections & shift symmetry





contribution of zero-mode and 1st Landau level yields usual quadratic divergence:

$$\delta m^2_{\varphi_0} = -2q^2|N| \int \frac{d^4k}{(2\pi)^4} \frac{2k^2}{k^2\,(k^2+2qf)}$$
$$= -\frac{q^2|N|}{4\pi^2}\left(\Lambda^2 - 2qf\ln\left(\frac{\Lambda^2}{2qf}\right) + \ldots\right)$$

Sum over all KK modes leads to cancellation (Schwinger representation of propagators, momentum integrations, perform sum first!):

$$\begin{aligned}
\delta m^2_{\varphi_0} &= -2q^2|N|\sum_n \int \frac{d^4k}{(2\pi)^4}\frac{2k^2}{(k^2+2qfn)\,(k^2+2qf(n+1))} \\
&= \frac{q^2}{4\pi^2}|N|\sum_n \int_0^\infty dt\,\frac{1}{t^2}\left(ne^{-2qfnt}-(n+1)e^{-2qf(n+1)t}\right) \\
&= \frac{q^2}{4\pi^2}|N|\int_0^\infty dt\,\frac{1}{t^2}\left(\frac{e^{2qft}}{(e^{2qft}-1)^2}-\frac{e^{2qft}}{(e^{2qft}-1)^2}\right) \\
&= \; 0\,.
\end{aligned}$$

WB, Dierigl, Dudas, Schweizer '16, Ghilencea, Lee '17

Cancellation can be traced back to shift symmetry of Wilson-line scalar, related to translation invariance of 6d action.

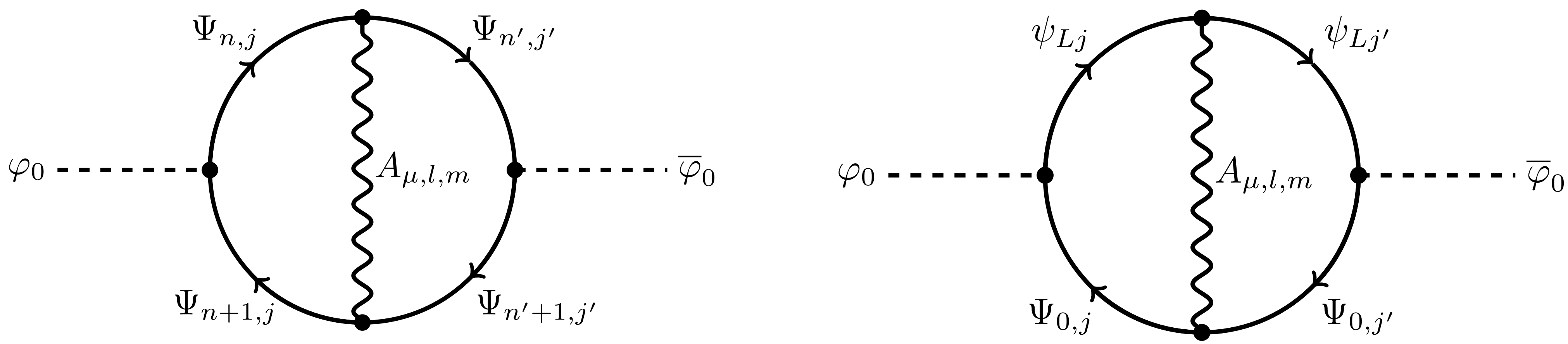
Symmetry of 4d effective action:

$$
\begin{aligned}
\delta\psi &= (\delta_T + \delta_{\Lambda,\alpha=\epsilon})\,\psi \\
&= -i\sqrt{2qf}(\epsilon a_+ + \bar{\epsilon} a_+^\dagger)\psi\,, \\
\delta\psi_{n,j} &= \sqrt{2qf}(\epsilon\sqrt{n+1}\;\psi_{n+1,j} - \bar{\epsilon}\sqrt{n}\;\psi_{n-1,j})\,, \\
\delta\chi &= (\delta_T + \delta_{\Lambda,\alpha=\epsilon})\,\chi \\
&= -i\sqrt{2qf}(\epsilon a_-^\dagger + \bar{\epsilon} a_-)\chi\,, \\
\delta\chi_{n,j} &= \sqrt{2qf}(-\epsilon\sqrt{n}\;\chi_{n-1,j} + \bar{\epsilon}\sqrt{n+1}\;\chi_{n+1,j})
\end{aligned}
$$

effective action invariant for shift of φ_0, analogous to translations on torus,

$$
\begin{aligned}
\delta S_4 = \int d^4x \Big(&- \partial^\mu \delta\overline{\varphi}_0 \partial_\mu \varphi_0 - \partial^\mu \overline{\varphi}_0 \partial_\mu \delta\varphi_0 \\
&+ (2qf\bar{\epsilon} - \sqrt{2}q\delta\varphi_0)\;\sum_{n,j} \chi_{n,j}\psi_{n,j} + \mathrm{h.c.}\Big)\,,
\end{aligned}
$$

$$
\delta\varphi_0 = \sqrt{2}\bar{\epsilon} f
$$



Shift symmetry for full 4d effective action; transformation of other fields:

$$\varphi = \varphi_0 + \varphi' \,, \quad A_\mu = A_{0\mu} + A'_\mu \,,$$
$$\varphi' = \sum_{l,m} \varphi_{l,m}\lambda_{l,m} \,, \quad A'_\mu = \sum_{l,m} A_{\mu,l,m}\lambda_{l,m} \,,$$
$$\lambda_{l,m} = e^{zM_{l,m} - \bar{z}\overline{M}_{l,m}} = \overline{\lambda}_{-l,-m} \,, \quad M_{l,m} = 2\pi(m+il) \,,$$
$$\delta\varphi_{l,m} = (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m})\varphi_{l,m} \,, \quad \delta A_{\mu,l,m} = (\epsilon M_{l,m} - \bar{\epsilon}\overline{M}_{l,m})A_{\mu,l,m}$$

Complete 4d effective action:

$$
\begin{aligned}
S_4 = \int d^6x \Big(&- \frac{1}{4} F_0^{\mu\nu} F_{0\mu\nu} - \partial^\mu \overline{\varphi}_0 \partial_\mu \varphi_0 - \frac{1}{2} f^2 \\
&+ \sum_{l,m} \Big(- \frac{1}{4} F^{\mu\nu}_{-l,-m} F_{\mu\nu,l,m} + \frac{1}{2} \overline{M}_{-l,-m} M_{l,m} A^\mu_{-l,-m} A_{\mu,l,m} \\
&\qquad - \partial^\mu \overline{\varphi}_{l,m} \partial_\mu \varphi_{l,m} - \frac{1}{4} \left| M_{-l,-m} \overline{\varphi}_{-l,-m} + \overline{M}_{l,m} \varphi_{l,m} \right|^2 \\
&\qquad - \frac{i}{\sqrt{2}} A^\mu_{-l,-m} \partial_\mu \left(M_{-l,-m} \overline{\varphi}_{-l,-m} - \overline{M}_{l,m} \varphi_{l,m} \right) \Big) \\
&+ \sum_{n,j} \Big(- i \psi_{n,j} \sigma^\mu \overline{D}_\mu \overline{\psi}_{n,j} - i \chi_{n,j} \sigma^\mu D_\mu \overline{\chi}_{n,j} \\
&\qquad - \sqrt{2qf(n+1)} \chi_{n,j} \psi_{n+1,j} - \sqrt{2} q \varphi_0 \chi_{n,j} \psi_{n,j} \\
&\qquad - \sqrt{2qf(n+1)} \overline{\chi}_{n,j} \overline{\psi}_{n+1,j} - \sqrt{2} q \overline{\varphi}_0 \overline{\chi}_{n,j} \overline{\psi}_{n,j} \Big) \\
+ \sum_{l,m;n,j;n',j'} & C^{l,m}_{n,j;n',j'} \Big(- q \psi_{n',j'} \sigma^\mu A_{\mu,l,m} \overline{\psi}_{n,j} + q \chi_{n,j} \sigma^\mu A_{\mu,l,m} \overline{\chi}_{n',j'} \\
&\qquad - \sqrt{2} q \varphi_{l,m} \chi_{n,j} \psi_{n',j'} - \sqrt{2} q \overline{\varphi}_{-l,-m} \overline{\chi}_{n',j'} \overline{\psi}_{n,j} \Big) \Big)
\end{aligned}
$$

The cubic couplings

$$C^{l,m}_{n,j;n',j'} = \int_{T_2} d^2x\, \lambda_{l,m}\bar{\xi}_{n,j}\xi_{n',j'}$$

satisfy recurrence relations,

$$\sqrt{2qf}\left(\sqrt{n}C^{l,m}_{n-1,j;n',j'} - \sqrt{n'+1}C^{l,m}_{n,j;n'+1,j'}\right) = \overline{M}_{l,m}C^{l,m}_{n,j;n',j'}\,,$$

$$\sqrt{2qf}\left(-\sqrt{n+1}C^{l,m}_{n+1,j;n',j'} + \sqrt{n'}C^{l,m}_{n,j;n'-1,j'}\right) = -M_{l,m}C^{l,m}_{n,j;n',j'}$$

which allow to prove invariance of effective action, $\delta S_4 = 0$, e.g.,

$$\delta \sum_{l,m;n,j;n',j'} C^{l,m}_{n,j;n',j'}\varphi_{l,m}\chi_{n,j}\psi_{n',j'} = 0$$

Conclusion: background flux leads to nonlinearly realized symmetry, which implies exact shift symmetry for Wilson-line scalar, despite the presence of gauge and Yukawa couplings. This forbids generation of scalar mass term to all orders in perturbation theory. Relevance for hierarchy problem of Higgs fields? Shortcoming in view of SM: vev of φ does not generate mass term for chiral fermions!

Current activities:

- Abelian gauge-higgs unification: neutral Higgs scalar; wanted: charged Higgs scalar, whose vev gives mass to chiral fermions, with SUSY broken by magnetic flux; requires to go from 6d to 10d

- Then, look for string theory model: intersecting D-brane models (type IIa strings), dual to magnetized D-brane models (D9-branes D5-branes); currently under investigation: type-I string model, i.e. SO(32), compactified on T^6, gauge group U(14)xU(1)xU(1), SUSY broken by magnetic flux, also Wilson-line background field; calculation of 1-loop quantum corrections in effective field theory and string theory; ground state without tachyons? (in the past always assumed, see Lust et al, Ibanez et al, Antoniadis et al)

Conclusions

- Higher-dimensional GUT models with flux lead to GUT scale for SUSY breaking; emerging low energy spectrum corresponds to "split symmetries" (THDM + higgsino)

- **Quantum corrections?** Example: gauge-higgs unification; without flux large mass for Wilson-line scalar

- Background magnetic flux drastically changes quantum corrections; nonlinearly realized symmetry leads to exact shift symmetry of Wilson-line scalar; forbids scalar mass term to all orders in perturbation theory. Protection mechanism independent of supersymmetry (Veneziano: IUVC?). Relevance for hierarchy problem of Higgs fields? Generalization to non-Abelian gauge interaction required