Supersymmetric Super-GUT Models

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Physics is About Correlations

- **Science explains correlation**
  - Naturalness: correlation between weak scale and $\Lambda$
  - Theory does not explain correlation, counter terms
  - Symmetries like (SUSY) explain correlation
  - Dynamics relaxion field explain correlation
Physics is About Correlations

- Science explains correlation
- Many correlations unknown
Physics is About Correlations

- Science explains correlation
- Many correlations unknown
- Good BSM explains more than one observable
Three Triumphs of SUSY: Naturalness

- SUSY enforces relationships among parameters

\[ H \, H = - y_f^2 \]

\[ m_f^2 = m_s^2 \]
Three Triumphs of SUSY: Naturalness

- SUSY enforces relationships among parameters
- Experiment tells us SUSY must be broken
  - If breaking too large → unnatural

\[ \Delta m_H^2 \approx \frac{|y_f|^2}{16\pi^2} m_f^2 \]

\[ m_f^2 \neq m_e^2 \]
Three Triumphs of SUSY: Naturalness

- SUSY enforces relationships among parameters
- Experiment tells us SUSY must be broken
- Not perfect but not so bad?
  \[ \Delta_{BG} \sim \frac{M^2_{SUSY}}{m_Z^2} \]
Three Triumphs of SUSY: Dark Matter

- SUSY has many dark matter candidates
  - Some are ruled out

Red: LUX(SI), Green: LUX(SD), Orange: (XENON1T), Yellow: (LZ)

Badziak, Olechowski, Szczerbiak
Three Triumphs of SUSY: Dark Matter

- SUSY has many dark matter candidates
  - Some are ruled out
- Stop/Gluino coannihilation still viable
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Three Triumphs of SUSY: Grand Unification

- Gauge couplings unify well in SUSY
- Quality of unification depends on $\mu$, $m_i$

$$g_i^{-2}(m_Z) = g_5^{-2}(M_{GUT}) + \frac{1}{8\pi^2} \left[ \beta_{SM_i} \ln \left( \frac{m_Z}{M_{GUT}} \right) + \frac{1}{3} \left( N_5 + 3N_{10} \right) \ln \left( \frac{M_{GUT}}{M_{SUSY}} \right) + \beta_\mu_i \ln \left( \frac{M_{GUT}}{\mu} \right) + \beta_m i \ln \left( \frac{M_{GUT}}{m_i} \right) \right]$$
Three Triumphs of SUSY: Grand Unification

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\[
g_i^{-2}(m_Z) = g_5^{-2}(M_{\text{GUT}}) + \frac{1}{8\pi^2} \left( \beta_{SM_i} \ln \left( \frac{m_Z}{M_{\text{GUT}}} \right) + \frac{1}{3} (N_5 + 3N_{10}) \ln \left( \frac{M_{\text{GUT}}}{M_{\text{SUSY}}} \right) + \beta_{\mu_i} \ln \left( \frac{M_{\text{GUT}}}{\mu} \right) + \beta_{m_i} \ln \left( \frac{M_{\text{GUT}}}{m_i} \right) \right)
\]
Three Triumphs of SUSY: Grand Unification

- Gauge couplings unify well in SUSY
- Quality of unification depends on $\mu$, $m_i$

CMSSM with $m_0 = 20$ TeV
Three Triumphs of SUSY: Grand Unification

- Gauge couplings unify well in SUSY
- Quality of unification depends on $\mu, m_i$
  - CMSSM with $m_0 = 20$ TeV vs $m_0 = 200$ TeV
Minimal Supersymmetric SU(5)

\[\Psi = 10 \supset Q_L, \bar{U}, \bar{E} \quad \Phi = 5 \supset L, \bar{D} \quad \text{(and} 1 \supset N)\]

\[W_5 = \mu \Sigma \text{Tr} \Sigma^2 + \frac{1}{6} \lambda' \text{Tr} \Sigma^3 + \mu H \bar{H} H + \lambda H \Sigma H + (h_{10})_{ij} \epsilon_{\alpha\beta\gamma\delta\zeta} \psi_i^{\alpha\beta} \psi_j^{\gamma\delta} H^\zeta + (h_5)_{ij} \psi_i^{\alpha\beta} \Phi_{j\alpha} \bar{H}_2, \]

\[\left[ + (h_1)_{ij} \bar{H}^\alpha \Phi_{i\alpha} N_j + \frac{1}{2} M_{ij} N_i N_j \right] \]
Unification, Thresholds, and Low-Scale Observables

- Minimal Supersymmetric SU(5)
- Threshold corrections → unification of couplings

\[
\frac{3}{g_2^2(M_{GUT})} - \frac{2}{g_3^2(M_{GUT})} - \frac{1}{g_1^2(M_{GUT})} = -\frac{3}{10\pi^2} \ln \left( \frac{M_{GUT}}{M_{H_C}} \right) \\
\frac{5}{g_1^2(M_{GUT})} - \frac{3}{g_2^2(M_{GUT})} - \frac{2}{g_3^2(M_{GUT})} = -\frac{9}{2\pi^2} \ln \left( \frac{M_{GUT}}{(M^2_{\chi} M_{\Sigma})^{\frac{1}{3}}} \right)
\]
Unification, Thresholds, and Low-Scale Observables

- Minimal Supersymmetric SU(5)
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  - SUSY breaking determines/constraints $M_{H_C}, M_X, M_\Sigma$

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Unification, Thresholds, and Low-Scale Observables

- Minimal Supersymmetric SU(5)
- Threshold corrections \(\rightarrow\) unification of couplings
  - SUSY breaking determines/constraints \(M_{H_C}, M_X, M_\Sigma\)

\[
\frac{3}{g_2^2(M_{GUT})} + \Delta_{SUSY2} - \frac{2}{g_3^2(M_{GUT})} - \frac{1}{g_1^2(M_{GUT})} = -\frac{3}{10\pi^2} \ln\left(\frac{M_{GUT}}{M_{H_C}}\right)
\]

\[
\frac{5}{g_1^2(M_{GUT})} - \frac{3}{g_2^2(M_{GUT})} - \frac{2}{g_3^2(M_{GUT})} = -\frac{9}{2\pi^2} \ln\left(\frac{M_{GUT}}{(M_X^2 M_\Sigma)^{\frac{1}{3}}}\right)
\]
Unification, Thresholds, and Low-Scale Observables

► Minimal Supersymmetric SU(5)
► Threshold corrections $\rightarrow$ unification of couplings
  - SUSY breaking determines/constraints $M_{H_C}, M_X, M_{\Sigma}$
► Color Higgs $\rightarrow B, L$ violating interactions
  - This leads to nucleon decay

$$ \mathcal{L}_{\Delta B = 1}^{\text{eff.}} = C_{5L}^{ijkl} \int d^2 \theta \frac{1}{2} (Q_i Q_j)(Q_k L_l) + C_{5R}^{ijkl} \int d^2 \theta \overline{U_i} \overline{E_j} \overline{U_k} \overline{D_l} + \text{h.c.} $$
Unification, Thresholds, and Low-Scale Observables

- Minimal Supersymmetric SU(5)
- Threshold corrections → unification of couplings
  - SUSY breaking determines/constraints $M_{H_C}, M_X, M_\Sigma$
- Color Higgs → $B, L$ violating interactions
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$$\mathcal{L}_{\text{eff.}}^{\Delta B=1} = C_{5L}^{ijkl} \int d^2 \theta \frac{1}{2} (Q_i Q_j)(Q_k L_l) + C_{5R}^{ijkl} \int d^2 \theta \bar{U}_i \bar{E}_j \bar{U}_k \bar{D}_l + \text{h.c.}$$

- Proton decay determined $M_{H_C}$ and so constrains SUSY

$$C_{5L}^{ijkl} = -\frac{1}{M_{H_C}} f^u_i e^{i\varphi_{u_i}} \delta^{ij} V^*_k l_l$$
$$C_{5R}^{ijkl} = -\frac{1}{M_{H_C}} f^u_i V_{ij} V^*_k f^d_l e^{-i\varphi_{u_k}}$$
Gauge Coupling Unification: Proton Decay

- Murayama and Pierce claim minimal SU(5) excluded
  - Non-minimal models needed (Pierce Murayama)
Gauge Coupling Unification: Proton Decay

Murayama and Pierce claim minimal SU(5) excluded
   - Non-minimal models needed

\[ m_{\tilde{f}_3} = 1 \text{ TeV} \quad m_{\tilde{f}_{2,3}} = 10 \text{ TeV} \quad \mu \in (100, 1000) \text{ GeV} \quad M_2 \in (100, 400) \text{ GeV} \]
Details of Proton Decay Amplitude

- Amplitudes for proton decay

\[ \mathcal{A}(p \rightarrow K^+ \bar{\nu}_i) = C_{RL}(usd\nu_i)\langle K^+|(us)_{R}d_{L}|p\rangle \]

\[ + C_{RL}(uds\nu_i)\langle K^+|(ud)_{R}s_{L}|p\rangle \]

\[ + C_{LL}(usd\nu_i)\langle K^+|(us)_{L}d_{L}|p\rangle \]

\[ + C_{LL}(uds\nu_i)\langle K^+|(ud)_{L}s_{L}|p\rangle \]

- Approximate low-scale WC of proton decay

\[ C_{LL} \approx \frac{2\alpha_2^2}{\sin 2\beta} \frac{m_t m_b M_2}{m_w^2 M_{HC} M_{SUSY}^2} V_{ub}^* V_{td} V_{ts} e^{i\phi_3} \left( 1 + e^{i(\phi_2 - \phi_3)} \frac{m_c V_{cd} V_{cs}}{m_t V_{td} V_{ts}} \right) \]

\[ C_{RL} \approx -\frac{\alpha_2^2}{\sin^2 2\beta} \frac{m_t m_d m_{\tau} \mu}{m_w^4 M_{HC} M_{SUSY}^2} V_{tb}^* V_{ud} V_{ts} e^{-i(\phi_2 + \phi_3)} \]
Details of Proton Decay Amplitude

- **Amplitudes for proton decay**

\[ A(p \to K^+ \bar{\nu}_i) = C_{RL}(usd\nu_i)\langle K^+|(us)_R d_L|p\rangle + C_{RL}(uds\nu_i)\langle K^+|(ud)_R s_L|p\rangle + C_{LL}(usd\nu_i)\langle K^+|(us)_L d_L|p\rangle + C_{LL}(uds\nu_i)\langle K^+|(ud)_L s_L|p\rangle \]

- **Respective \( \tau(p \to K\nu) \) for \( \phi_{2,3} = 0 \)
  
  \[ \tau(p \to K\nu) > 6.6 \times 10^{33} \text{ yr} \]

\[ \Gamma_{\tilde{W}} \simeq 3.3 \times 10^{31} \text{ yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{ GeV}} \right)^2 \left( \frac{M_{\text{SUSY}}}{11 \text{ TeV}} \right)^4 \left( \frac{3.5 \text{ TeV}}{M_2} \right)^2 \]

\[ \Gamma_{\tilde{h}} \simeq 2.8 \times 10^{31} \text{ yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{ GeV}} \right)^2 \left( \frac{M_{\text{SUSY}}}{11 \text{ TeV}} \right)^4 \left( \frac{12 \text{ TeV}}{M_{\tilde{h}}} \right)^2 \]
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - Strong Moduli Stabilization

\[ \frac{\langle Z \rangle}{M_P} \ll 1 \quad \frac{F_Z}{M_P} \sim m_{3/2} \]
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - Strong Moduli Stabilization
  - $Z$ not a singlet suppressed in gauge kinetic function

$$h_{\alpha\beta} = \frac{Z^n}{M^n_P} \quad \Rightarrow \quad m_{1/2} \sim \left(\frac{\langle Z \rangle}{M_P}\right)^{n-1} \quad m_{3/2} \ll m_{3/2}$$
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - Strong Moduli Stabilization
  - $Z$ not a singlet suppressed in gauge kinetic function
  - Gauginos are generated by anomalies

$$M_a = \frac{b_a g_a^2}{16\pi^2} m_{3/2} \quad b_a = (-33/5, -1, 3)$$
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - Strong Moduli Stabilization
  - $Z$ not a singlet suppressed in gauge kinetic function
  - Gauginos are generated by anomalies
  - $A$-terms are also suppressed

$$K = \frac{Z^\dagger Z}{M_P^2} H_u^\dagger H_u \quad \rightarrow \quad \frac{Z^\dagger}{M_P} \frac{F_Z}{M_P} H_u F_{H_u}^\dagger$$
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - Strong Moduli Stabilization
  - $Z$ not a singlet suppressed in gauge kinetic function
  - Gauginos are generated by anomalies
  - $A$-terms are also suppressed
  - Scalar masses the same as mSUGRA

$$m_0 = m_{3/2}$$
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression

\[ m_{\tilde{f}} = m_{3/2} \quad m_i = \beta_i m_{3/2} \quad A_0 = 0 \quad (\mu \sim m_{\tilde{t}}) \]
Proton Decay and Low-scale SUSY

- Large soft masses will help
- PGM/mini-split models give sufficient suppression
  - $m_{3/2} \lesssim 400$ TeV, DM not viable
Proton Decay and CMSSM

What about CMSSM like models?

- $M_{SUSY}$ is low

\[ \Gamma_{\tilde{W}} \sim 10^{31} \text{yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{GeV}} \right)^2 \left( \frac{M_{SUSY}}{11 \text{ TeV}} \right)^4 \left( \frac{3.5 \text{ TeV}}{M_2} \right)^2 \]

\[ \Gamma_{\tilde{h}} \sim 10^{31} \text{yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{GeV}} \right)^2 \left( \frac{M_{SUSY}}{11 \text{ TeV}} \right)^4 \left( \frac{12 \text{ TeV}}{\mu} \right)^2 \]

Contributions can add destructively

\[ A(p \to K^+ \bar{\nu}_i) = C_{RL}(usd\nu_i)\langle K^+|(us)_R d_L|p\rangle + C_{RL}(uds\nu_i)\langle K^+|(ud)_R s_L|p\rangle \]

\[ + C_{LL}(usd\nu_i)\langle K^+|(us)_L d_L|p\rangle + C_{LL}(uds\nu_i)\langle K^+|(ud)_L s_L|p\rangle \]

\[ C_{LL} \sim \frac{2\alpha_2^2}{\sin 2\beta} \frac{m_t m_b}{m_W^2 M_{HC}^2 M_{SUSY}^2} V_{ub}^* V_{td} V_{ts} e^{i\phi_3} \left( 1 + e^{i(\phi_2 - \phi_3)} \frac{m_c V_{cd} V_{cs}}{m_t V_{td} V_{ts}} \right) \]

\[ C_{RL} \sim -\frac{\alpha_2^2}{\sin^2 2\beta} \frac{m_t^2 m_d m_{\tau} \mu}{m_W^4 M_{HC}^2 M_{SUSY}^2} V_{tb}^* V_{ud} V_{ts} e^{-i(\phi_2 + \phi_3)} \]
Phase Suppression of Proton Decay

- Significant suppression afforded by phases
  - Misalignment of decay to different generations limits suppression
Phase Suppression of Proton Decay

- Significant suppression afforded by phases
  - Misalignment of decay to different generations limits suppression
- Suppression not so tuned in phase
- Other nucleon lifetimes also enhanced

![Phase Suppression Diagram]

\[ \phi_3 = 0 \]

 Lifetime [years]
CMSSM with Phase Suppressed Proton Decay

- Phase Suppression sufficient for viable dark matter

![Graphs showing parameter space](image)

- Parameter values:
  - $\tan \beta = 5$, $A_0 = 0$, $\mu < 0$
  - $\tan \beta = 6$, $A_0 = -4.2 m_0$, $\mu < 0$
Super-GUT Models

- SUSY breaking mediated at some scale $M_{in} > M_{GUT}$
- Parameters at $M_{in}$ for CMSSM
  
  $m_0$  $m_{1/2}$  $\tan \beta$  $A_0$  $\text{sgn}(\mu)$
Super-GUT Models

- SUSY breaking mediated at some scale $M_{in} > M_{GUT}$
  - Some matching conditions have consequences

\[ B = B_H + \frac{3\lambda V \Delta}{\mu} + \frac{6\lambda}{\lambda' \mu} \left[ (A_{\lambda'} - B_\Sigma)(2B_\Sigma - A_{\lambda'} + \Delta) - m_\Sigma^2 \right] \rightarrow A_{\lambda'} \gtrsim 8m_\Sigma^2 \]

\[ \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} - \frac{1}{g_1^2(Q)} = -\frac{3}{10\pi^2} \ln \left( \frac{Q}{M_{HC}} \right) \rightarrow M_{HC} \sim 10^{15}\text{GeV} \]

\[ \frac{5}{g_1^2(Q)} - \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} = -\frac{3}{2\pi^2} \ln \left( \frac{Q^3}{M_X^2 M_\Sigma} \right) \quad \& \quad M_{HC} = \lambda \left( \frac{2}{\lambda' g_5^2} \right)^{\frac{1}{3}} \left( \frac{M_X^2 M_\Sigma}{M_X^2 M_\Sigma} \right)^{\frac{1}{3}} \]

\[ \lambda \propto \lambda' \quad \& \quad \lambda' \sim 1 \rightarrow \text{Proton Decays too Quickly} \]
Super-GUT Models

▶ SUSY breaking mediated at some scale $M_{in} > M_{GUT}$
  – Some matching conditions have consequences
▶ Add higher dimensional operators

$$W_{\text{eff}}^{\Delta g} = \frac{c}{M_P} \text{Tr} [\Sigma \mathcal{W} \mathcal{W}]$$

– Alters matching condition $\rightarrow M_{H_C}$ Free

$$\frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} - \frac{1}{g_1^2(Q)} = - \frac{3}{10\pi^2} \ln \left( \frac{Q}{M_{H_C}} \right) - \frac{96cV}{M_P}$$
Super-GUT Models

- SUSY breaking mediated at some scale $M_{in} > M_{GUT}$
  - Some matching conditions have consequences
- Add higher dimensional operators
- Super-GUT CMSSM versus CMSSM ($\lambda = 0.6, \lambda' = 10^{-4}$)
Minimal SU(5) + Right-Handed Neutrinos

- Neutrino seemingly benign interactions
  - Leptogenesis is viable

\[ W_5 = (h_1)_{ij} \bar{H}^\alpha \Phi_{i\alpha} N_j + \frac{1}{2} M_{ij} N_i N_j \]
Minimal SU(5) + Right-Handed Neutrinos

- Neutrino seemingly benign interactions

\[ W_5 = (h_1)_{ij} \overline{H}^\alpha \Phi_{i\alpha} N_j + \frac{1}{2} M_{ij} N_i N_j \]

- Physical degrees of freedom in the Yukawa’s
  - We will assume \( M_R = \delta_{ij} M_{R_i} \)

\[ f_{ij}^u = f_i^u e^{i\varphi u_i} \delta_{ij} \quad f_{ij}^d = f_i^d V_i^* \quad f_{ij}^\nu = f_j^\nu e^{i\varphi d_i} U_i^* \quad (M_R)_{ij} = e^{i\varphi_{ij}} W_{ik} (M_R^D)_{jk} e^{2i\varphi_{kij}} W_{jk} e^{i\varphi_{ij}} \]
Minimal SU(5) + Right-Handed Neutrinos

- Neutrino seemingly benign interactions
  \[ W_5 = (h_1)_{ij} \bar H^\alpha \Phi_{i\alpha} N_j + \frac{1}{2} M_{ij} N_i N_j \]

- Physical degrees of freedom in the Yukawa’s
  - We will assume \( M_R = \delta_{ij} M_{Ri} \)

\[ 
\begin{align*}
  f^u_{ij} &= f^u_i e^{i\varphi_{ui}} \delta_{ij} \\
  f^d_{ij} &= f^d_i V^*_i \\
  f^\nu_{ij} &= f^\nu_j e^{i\varphi_{di}} U^*_i \\
  (M_R)_{ij} &= e^{i\varphi_{\nu_i}} W_{ik} (M^{D}_R)_{k} e^{2i\varphi_{\nu_k}} W^*_{jk} e^{i\varphi_{\nu_j}} 
\end{align*} \]

- PMNS has large CP violation and large flavor mixing
  \[ \sin^2 \theta_{12} = 0.297, \quad \sin^2 \theta_{23} = 0.425, \quad \sin^2 \theta_{31} = 0.0214, \quad \delta_{CP} = 1.38\pi \]
Kaon Mixing From Right-Handed Neutrinos

- Beta function of $\Phi_i, \overline{H}$ affected by $h_1$ ($N_i$ GUT Yukawa)
- Leading-log approximation for affected soft masses

\[
(m_d^2)_{ij} \simeq -\frac{1}{8\pi^2} [f^\nu f^{\nu\dagger}]_{ij} (3m_0^2 + A_0^2) \ln \frac{M_*}{M_{GUT}}, \quad (i \neq j)
\]
Kaon Mixing From Right-Handed Neutrinos

- Beta function of $\Phi_i, H$ affected by $h_1$
- Mixing in down-squark soft masses $\rightarrow$ kaon mixing

\[ H_{\text{eff.}} \simeq -\frac{\alpha_S^2}{36m_q^2} \left( \Delta_{12}^{(R)} \right)^2 F_1 \left( \frac{m_q^2}{m_q^2} \right) d_R \gamma_\mu s_R \gamma_\mu s_R - \frac{\alpha_S^2}{3m_q^2} \Delta_{12} \left( \Delta_{12}^{(R)} \right) F_2 \left( \frac{m_q^2}{m_q^2} \right) d_L s_R \gamma_\mu d_R s_R \]

\[
(m_{d_{12}}^2) \simeq -\frac{1}{8\pi^2} e^{i(\varphi_{d_1} - \varphi_{d_2})} \mathcal{U}_{1k} (f_k^\nu)^2 \mathcal{U}_{2k}^\ast (3m_q^2 + A_0^2) \ln \frac{M_{\text{in}}}{M_{\text{GUT}}}
\]

SUSY contributions

\[
\Delta m_K^{\text{SUSY}} = 2 \text{Re} \left( \frac{K}{H_{\text{eff.}}} \right) \Delta m_{K}^{S=2} \left| K^0 \right\rangle,
\]

\[
\epsilon_K^{\text{SUSY}} = \frac{1}{\sqrt{2} \Delta m_K} \text{Im} \left( \frac{K}{H_{\text{eff.}}} \right) \Delta m_{K}^{S=2} \left| K^0 \right\rangle
\]

Hadron matrix elements:

SM op.: [FLAG average '16]
BSM ops.: [SWME collaboration '15]

Kuwamura Planck Talk
Kaon Mixing From Right-Handed Neutrinos

- Beta function of $\Phi_i, \overline{H}$ affected by $h_1$
- Mixing in down-squark soft masses $\rightarrow$ kaon mixing
- Irreducible contributions to Kaon mixing ($\alpha_d = \phi_{d_1} - \phi_{d_2}$)
Right-handed neutrino Yukawa’s also affect leptons

- Super-GUT RG running enhances FV+CP violation
- Effect maximized for $Y_{\nu} \sim 1 \rightarrow M_{NR} \sim 10^{15}$

\[
(m^2_{\tilde{L}})_{ij} \simeq -\frac{1}{8\pi^2} \sum_k \hat{f}^{-1}_{ik}(\hat{f}^{\nu\dagger})_{kj}(3m_0^2 + A_0^2) \ln \frac{M_*}{(M_R^D)_k}, \quad (i \neq j)
\]
eEDM From Right-Handed Neutrinos

- Right-handed neutrino Yukawa’s also affect leptons
- Generated FV+CP violation lead to eEDM’s
Right-handed neutrino Yukawas's also affect leptons

Generated FV+CP violation lead to eEDM's

\[
\frac{d_e}{e} \sim \frac{g_Y^2}{32\pi^2} \frac{m_\tau}{m^2_l} \frac{\mu M_1}{m^2_l} \text{Im}[(\Delta^{(L)}_{l})_{13}(\Delta^{(R)}_{l})_{31}] f(x)
\]

\[
(\Delta^{(L)}_{l})_{ij} \equiv \frac{(m^2_L)_{ij}}{m^2_l}, \quad (\Delta^{(R)}_{l})_{ij} \equiv \frac{(m^2_e)_{ij}}{m^2_l}
\]
Supersymmetric Super-GUT Models

- Right-handed neutrino Yukawa’s also affect leptons
- Generated FV+CP violation lead to eEDM’s
- Irreducible contribution to eEDM \( \beta_d = \phi_d^1 - \phi_d^3 \)

\[
|d_e| \times 10^{-28} \text{ e.cm} \\
\beta_d/\pi
\]
Consequences for Super-GUT Models

- Super-GUT CMSSM with right-handed neutrinos

\[ m_0 \ m_{1/2} \ A_0 \ \tan \beta \ \text{sgn}(\mu) \ \ M_{\text{in}} \ M_R \ \lambda \ \lambda' \ \phi_{ui} \ \phi_{di} \]
Consequences for Super-GUT Models

- Super-GUT CMSSM with right-handed neutrinos
  - yellow: $\mu \rightarrow e\gamma$
  - Cyan: $e$EDM($9.3 \times 10^{-29}$)
  - Red: $\varepsilon_K$

![Graph showing constraints on $M_{1/2}$ vs $m_0$](image)
Consequences for Super-GUT Models

- Super-GUT CMSSM with right-handed neutrinos
  - yellow: $\mu \rightarrow e\gamma$
  - Cyan: $\text{eEDM}(1.1 \times 10^{-29})$
  - Red: $\varepsilon_K$
Consequences for Super-GUT Models

- Super-GUT CMSSM with right-handed neutrinos
- Constraints depend on $\beta_d$
  - Cyan: $\text{eEDM}(9.3 \times 10^{-29})$
  - Red: $\varepsilon_K$

![Graphs showing parameter space](image-url)
Consequences for Super-GUT Models

- Super-GUT CMSSM with right-handed neutrinos
- Constraints depend on $\beta_d$
  - Cyan: eEDM($1.1 \times 10^{-29}$)
  - Red: $\varepsilon_K$
CMSSM required Planck suppressed operators

\[ \frac{3}{g_2^2(Q)} - \frac{2}{g_3^2(Q)} - \frac{1}{g_1^2(Q)} = \frac{3}{10\pi^2} \ln \left( \frac{Q}{M_{H_C}} \right) - \frac{96cV}{M_P} \]
Super-GUT PGM

- CMSSM required Planck suppressed operators
- PGM proton decay is suppressed by soft masses
  - $M_{HC}$ is relatively unconstrained
  - No Planck Suppressed operators needed

$$\Gamma_{\tilde{W}} \sim 10^{31} \text{ yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{ GeV}} \right)^2 \left( \frac{M_{SUSY}}{11 \text{ TeV}} \right)^4 \left( \frac{3.5 \text{ TeV}}{M_2} \right)^2$$

$$\Gamma_{\tilde{h}} \sim 10^{31} \text{ yr} \left( \frac{M_{HC}}{7 \times 10^{16} \text{ GeV}} \right)^2 \left( \frac{M_{SUSY}}{11 \text{ TeV}} \right)^4 \left( \frac{12 \text{ TeV}}{\mu} \right)^2$$
Super-GUT PGM

- CMSSM required Planck suppressed operators
- PGM proton decay is suppressed by soft masses
- PGM restricts $\tan \beta$
  - soft masses larger guagino small
  - Stop masses driven lighter
  - More difficult to drive Higgs mass negative
Super-GUT PGM

- CMSSM required Planck suppressed operators
- PGM proton decay is suppressed by soft masses
- PGM restricts $\tan \beta$
- Super-GUT RG running allows for larger $\tan \beta$

$$W_5 \supset \lambda \bar{H} \Sigma H \quad \rightarrow \quad \Delta \beta m_{H_u}^2 = \frac{|\lambda|^2}{8\pi^2} \left( m_{\Sigma}^2 + m_{H}^2 + m_{\bar{H}}^2 + |A\lambda|^2 \right)$$
Super-GUT PGM

- CMSSM required Planck suppressed operators
- PGM proton decay is suppressed by soft masses
- PGM restricts $\tan \beta$
- Super-GUT RG running allows for larger $\tan \beta$
  - Wino dark matter with acceptable density
Super-GUT PGM

- CMSSM required Planck suppressed operators
- PGM proton decay is suppressed by soft masses
- PGM restricts $\tan \beta$
- Super-GUT RG running allows for larger $\tan \beta$
  - Wino dark matter with acceptable density
Non-Renormalizeable Operators in Super-GUT PGM

- Supersymmetric Non-Renormalizable operators

\[ W_{\text{eff}}^{\Delta g} = \frac{c}{M_P} \text{Tr} [\Sigma \mathcal{W}^2] \rightarrow \lambda' \text{ Free Parameter} \]

- Heavy gauge bosons can be suppressed

\[
\frac{5}{g_1^2(M_{\text{GUT}})} - \frac{3}{g_2^2(M_{\text{GUT}})} - \frac{2}{g_3^2(M_{\text{GUT}})} = -\frac{9}{2\pi^2} \ln \left( \frac{M_{\text{GUT}}}{M_{\chi^\prime}^2 M_{\Sigma}} \right) \quad M_{\Sigma} = \frac{5}{2} \lambda' V
\]

- Large sfermion masses allow for large \( \lambda' \)
  - Dim-6 proton decay enhanced
Heavy gauge bosons violate B,L

\[
\mathcal{L}_{\text{int}} = \frac{g_5}{\sqrt{2}} \left[ -\overline{d}_{Ri}^c X L_i + e^{-i\phi_i} \overline{Q}_i X u_{Ri}^c + \epsilon_{Ri}^c X (V^\dagger)_{ij} Q_j + \text{h.c.} \right]
\]

Amplitudes for \( P \rightarrow \pi e \)

\[
A_L(p \rightarrow \pi^0 e^+) = -\frac{g_5^2}{M_X^2} A_1 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,
\]

\[
A_R(p \rightarrow \pi^0 e^+) = -\frac{g_5^2}{M_X^2} (1 + |V_{ud}|^2) A_2 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,
\]
$P \to \pi e$ in Minimal Supersymmetric SU(5)

- Heavy gauge bosons violate B,L

$$\mathcal{L}_{\text{int}} = \frac{g_5}{\sqrt{2}} \left[ -\overline{d}^c_{Ri} XL_i + e^{-i\varphi_i} \overline{Q}_i Xu^c_{Ri} + \overline{e}^c_{Ri} X (V^i)_{ij} Q_j + \text{h.c.} \right]$$

- Amplitudes for $P \to \pi e$

$$A_L(p \to \pi^0 e^+) = -\frac{g_5^2}{M_X^2} \cdot A_1 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,$$

$$A_R(p \to \pi^0 e^+) = -\frac{g_5^2}{M_X^2} (1 + |V_{ud}|^2) \cdot A_2 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,$$
$P \rightarrow \pi e$ in Minimal Supersymmetric SU(5)

- Heavy gauge bosons violate B,L

$$\mathcal{L}_{\text{int}} = \frac{g_5}{\sqrt{2}} \left[ -d_{Ri}^c X L_i + e^{-i\varphi_i} \bar{Q}_i X u_{Ri}^c + e_{Ri}^c X (V^i)_{ij} Q_j + \text{h.c.} \right]$$

- Amplitudes for $P \rightarrow \pi e$

$$\mathcal{A}_L(p \rightarrow \pi^0 e^+) = -\frac{g_5^2}{M_X^2} \cdot A_1 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,$$

$$\mathcal{A}_R(p \rightarrow \pi^0 e^+) = -\frac{g_5^2}{M_X^2} (1 + |V_{ud}|^2) \cdot A_2 \cdot \langle \pi^0 | (ud)_R u_L | p \rangle,$$
Planck Corrections to Gauge Kinetic Terms

- For larger $\lambda'$ Dim-6 Proton decay most important
  - Dim-5 suppressed by larger sfermion masses.

\[
A_0 = 0, \ sgn(\mu) > 0, \ x_G = 0.3, \ M_{in} = 10^{18} \text{ GeV}, \ \tan\beta = 2.1, \ m_{3/2} = 1 \text{ PeV}
\]
SUSY Breaking Planck Suppressed operators

Planck Suppressed operators and SUSY breaking field

\[ \Delta W = \frac{Z}{\sqrt{3}M_P} \mu \Phi \psi + \frac{Z}{\sqrt{3}M_P} \Phi \psi X \]

\[ \Delta K = \kappa_i \frac{Z}{\sqrt{3}M_P} |\Phi_i|^2 + \frac{Z}{\sqrt{3}M_P} \Phi \psi \]
SUSY Breaking Planck Suppressed operators

- Planck Suppressed operators and SUSY breaking field

\[ \Delta W = \frac{Z}{\sqrt{3}M_P} \mu \Phi \psi + \frac{Z}{\sqrt{3}M_P} \Phi \psi X \]

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- Shift symmetry on Z restricts allowed operators
Planck Suppressed operators and SUSY breaking field
Shift symmetry on $Z$ restricts allowed operators
A-terms and B-terms get corrections of order $m_{3/2}$

$$\Delta A = (\kappa_1 + \kappa_2 + \kappa_3) m_{3/2} \quad \Delta B = (\kappa_1 + \kappa_2) m_{3/2}$$

$$W = \mu \phi_1 \phi_2 + y \phi_1 \phi_2 \phi_3$$
SUSY Breaking Planck Suppressed operators

- Planck Suppressed operators and SUSY breaking field
- Shift symmetry on $Z$ restricts allowed operators
- A-terms and B-terms get corrections of order $m_{3/2}$

\[ \Delta A = (\kappa_1 + \kappa_2 + \kappa_3) m_{3/2} \quad \Delta B = (\kappa_1 + \kappa_2) m_{3/2} \]

- A and B-terms give large corrections to gaugino masses

\[
M_1 = \frac{g_1^2}{g_5^2} M_5 - \frac{g_1^2}{16\pi^2} \left[ 10M_5 - 10(A_{\lambda'} - B_\Sigma) - \frac{2}{5} B_H \right] - \frac{4cg_1^2 V(A_{\lambda'} - B_\Sigma)}{M_P}
\]
\[
M_2 = \frac{g_2^2}{g_5^2} M_5 - \frac{g_2^2}{16\pi^2} \left[ 6M_5 - 6A_{\lambda'} + 4B_\Sigma \right] - \frac{12cg_2^2 V(A_{\lambda'} - B_\Sigma)}{M_P}
\]
\[
M_3 = \frac{g_3^2}{g_5^2} M_5 - \frac{g_3^2}{16\pi^2} \left[ 4M_5 - 4A_{\lambda'} + B_\Sigma - B_H \right] + \frac{8cg_3^2 V(A_{\lambda'} - B_\Sigma)}{M_P}
\]
Guaginos change so much that LSP changes
Dark Matter in PGM

- Guaginos change so much that LSP changes
- Gluino and Wino Coannihilation with Bino arises
Conclusions

- Science is about interrelating parameters
- Supersymmetric models are still true to goals of science
- Gauge couplings unify in SUSY
  - Important guided for BSM
- Min-SUSY SU(5) models have low-scale consequences
  - Proton decay requires large soft masses
  - Matching conditions prefer large $A$-terms
- Right-handed neutrinos lead to additional constraints
  - Generated mixing in $m_d^2$ can lead to large $\epsilon_K$
- Right-handed neutrinos also leads to additional signals
  - Irreducible FV+CPV lead to future detectable eEDM’s
- Higher Dim Operators can have important Consequences