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Electron Hydrodynamics & Black Holes in Anti de Sitter Space-time

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WIP w. E. Hankiewicz & C. Tutschku
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Gauge/Gravity Duality: Holography

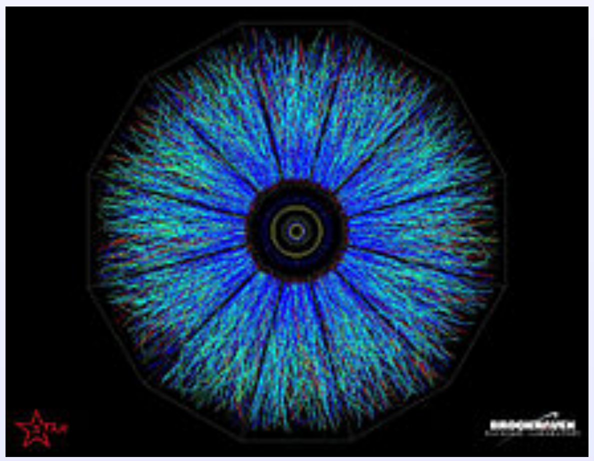
Map between certain
strongly coupled quantum field theories and
gravity in AdS space-times

Usual Question:
What does gauge/gravity duality
tell us about universal properties
of strongly coupled systems?

Gauge/Gravity Duality: Holography

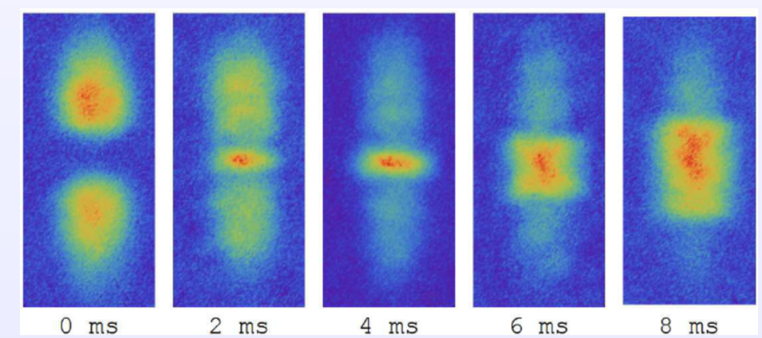
Unusual Question:

How to test AdS/CFT predictions in experiment?

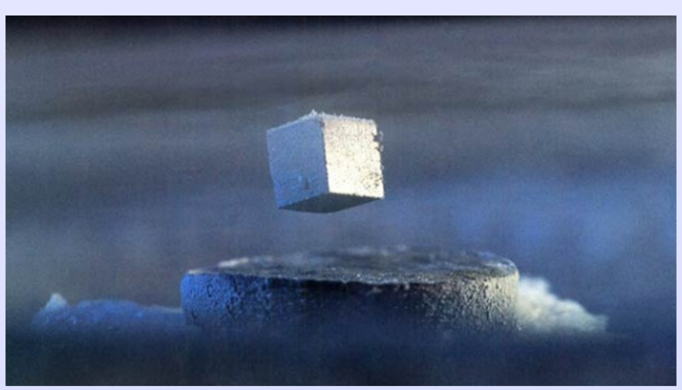


Quark-Gluon Plasma

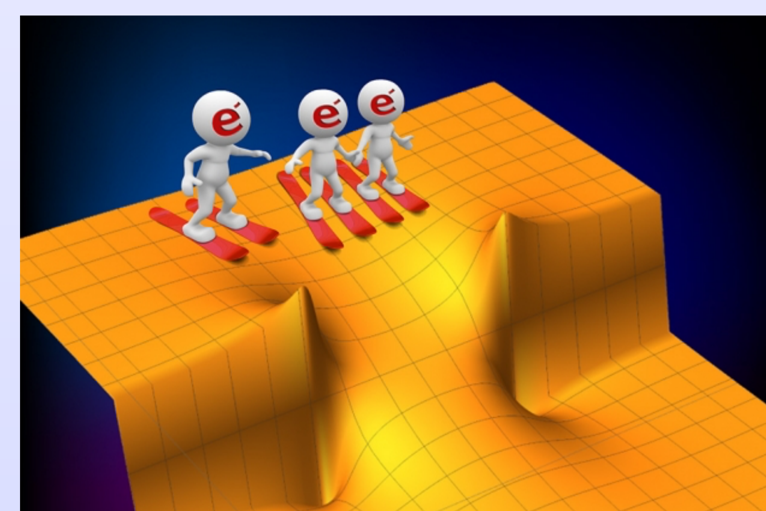
Hydrodynamics



Unitary Fermions



High Tc Superconductors
Quantum Critical Phases



Electronic Fluids

Hydrodynamics

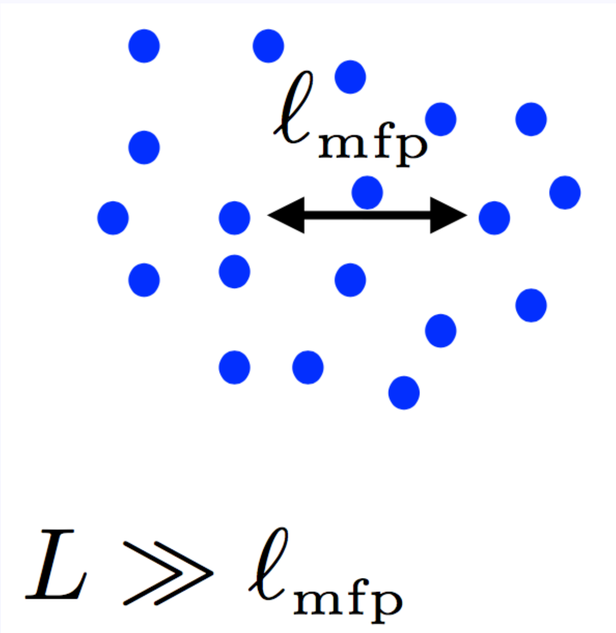
Hydrodynamics: Long wavelength, low frequency perturbations of a fluid away from global equilibrium

Theory of transport of (approx.) conserved quantities (energy, momentum, charges)

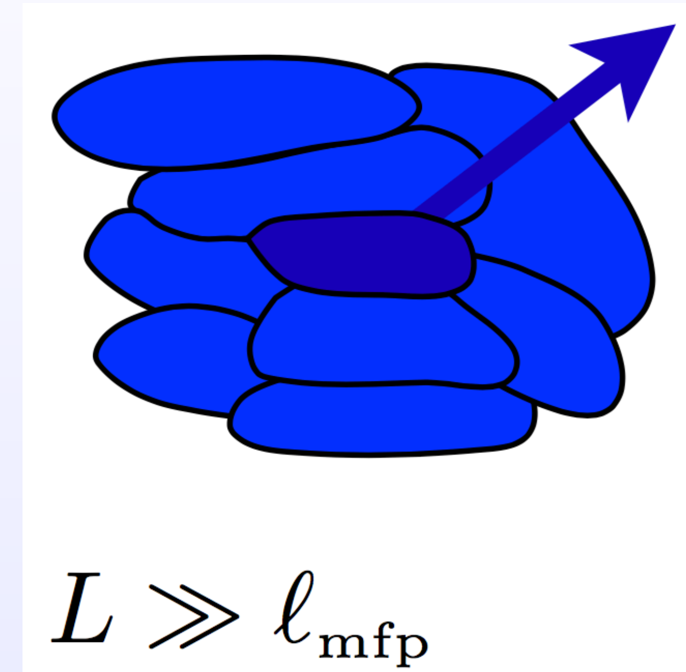
Black hole horizons show hydrodynamic response

[T. Damour 1970s, AdS/CFT]

The Hydrodynamic Regime



$\epsilon(x^\mu)$	Energy density	
$\rho(x^\mu)$	Charge density	
$u^\nu(x^\mu)$	Velocity field	$(u_\mu u^\mu = -1)$



Other scattering (Impurities, Walls): $\ell_{mfp} \ll \ell_{imp,wall,\ldots}$

$$\tau_{therm} \ll \tau_{imp,wall,\ldots}$$

Hydrodynamics applies with or without quasiparticles $\ell_{Compton} \ll \ell_{mfp}\,,\ \ \Gamma \ll \tau^{-1}_{longest}$

Theory of Transport of (approx.) conserved quantities:

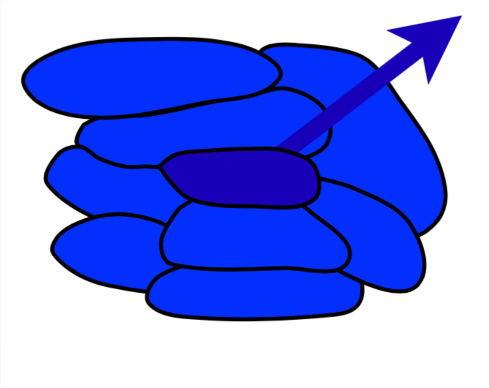
$$\nabla_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho - \frac{T^{0i}\delta^\nu_i}{\tau_{\rm imp}}$$

$$\nabla_\rho J^\rho = 0$$

Hydrodynamics as EFT

Expand $T_{\mu\nu}$ and J_μ in

$$\ell_{mfp}\frac{\partial}{\partial x^\mu} = \frac{\ell_{mfp}}{L}\frac{\partial}{\partial \xi^\mu}$$

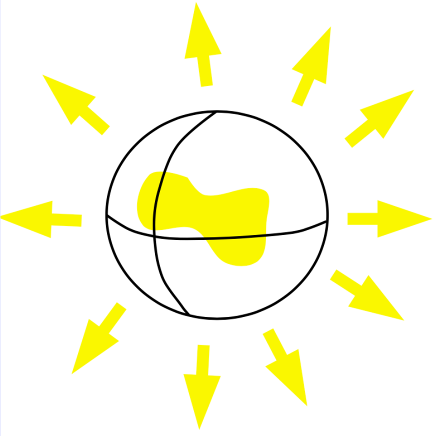


$$T^{\mu\nu} = T^{\mu\nu}_{ideal} + T^{\mu\nu}_{(1)} + \ldots \qquad J^\mu = \rho(T,\mu)u^\mu + J^\mu_{(1)} + \ldots$$

$$T^{\mu\nu}_{ideal} = \epsilon(T,\mu)u^\mu u^\nu + p(T,\mu)\underbrace{\left(u^\mu u^\nu + g^{\mu\nu}\right)}_{\equiv \Delta^{\mu\nu}}$$

$$T^{\mu\nu}_{(1)} = \eta\,\underbrace{\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho - g_{\rho\sigma}(\nabla u)\right)}_{shear} - \zeta\Delta^{\mu\nu}(\nabla u)$$

$$J^\mu_{(1)} = \sigma\left(E^\mu - T\Delta^{\mu\nu}\nabla_\nu\left(\frac{\mu}{T}\right)\right)$$



Local version of the 2nd law: $\partial_\mu J^\mu_s \geq 0$

Hydrodynamics as EFT

$$T^{\mu\nu}_{(1)} = \eta \underbrace{\Delta^{\mu\rho}\Delta^{\nu\sigma}\left(\nabla_\rho u_\sigma + \nabla_\sigma u_\rho - g_{\rho\sigma}(\nabla u)\right)}_{shear} - \zeta \Delta^{\mu\nu}(\nabla u)$$

$$J^{\mu}_{(1)} = \sigma\left(E^\mu - T\Delta^{\mu\nu}\nabla_\nu(\tfrac{\mu}{T})\right)$$

Linear Response:
(e.g. d=2+1)

$$G_R^{ij,kl}(\omega,0) = \langle T^{ij}(-\omega)T^{kl}(\omega)\rangle_R$$

$$G_R^{i,j}(\omega,0) = \langle J^{i}(-\omega)J^{j}(\omega)\rangle_R$$

$$\eta = \lim_{\omega\to 0}\frac{1}{8\omega}(\delta_{ik}\delta_{jl} - \epsilon_{ik}\epsilon_{jl})\,\mathrm{Im}\,G_R^{ij,kl}(\omega,0)\,,$$

$$\sigma = \lim_{\omega\to 0}\frac{1}{2\omega}\delta_{ij}\,\mathrm{Im}\,G_R^{i,j}(\omega,0)\,,$$

$$\zeta = \lim_{\omega\to 0}\frac{1}{4\omega}\delta_{ij}\delta_{kl}\,\mathrm{Im}\,G_R^{ij,kl}(\omega,0)\,,$$

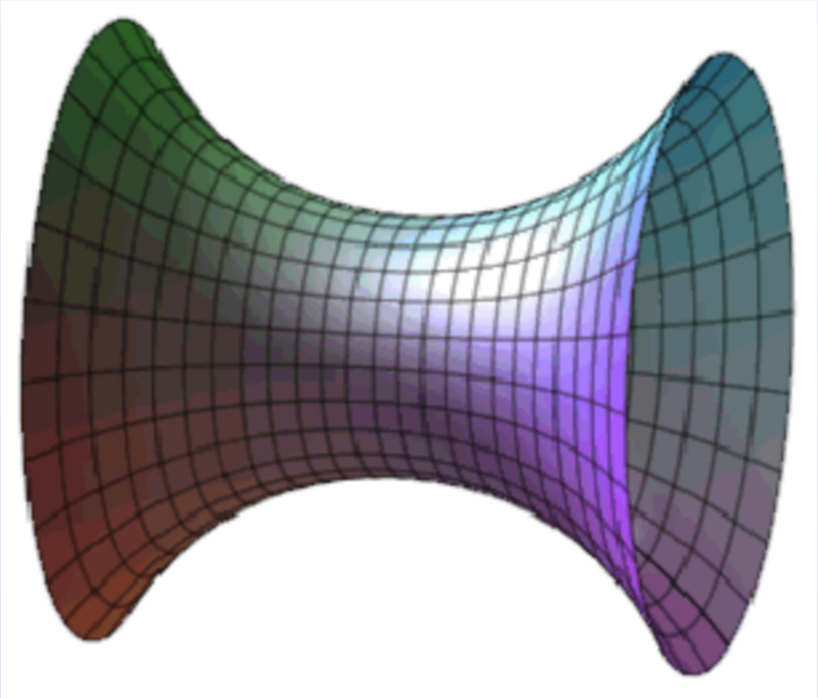
η, σ, ζ calculated from microscopic or other effective models (kinetic theory, AdS/CFT)

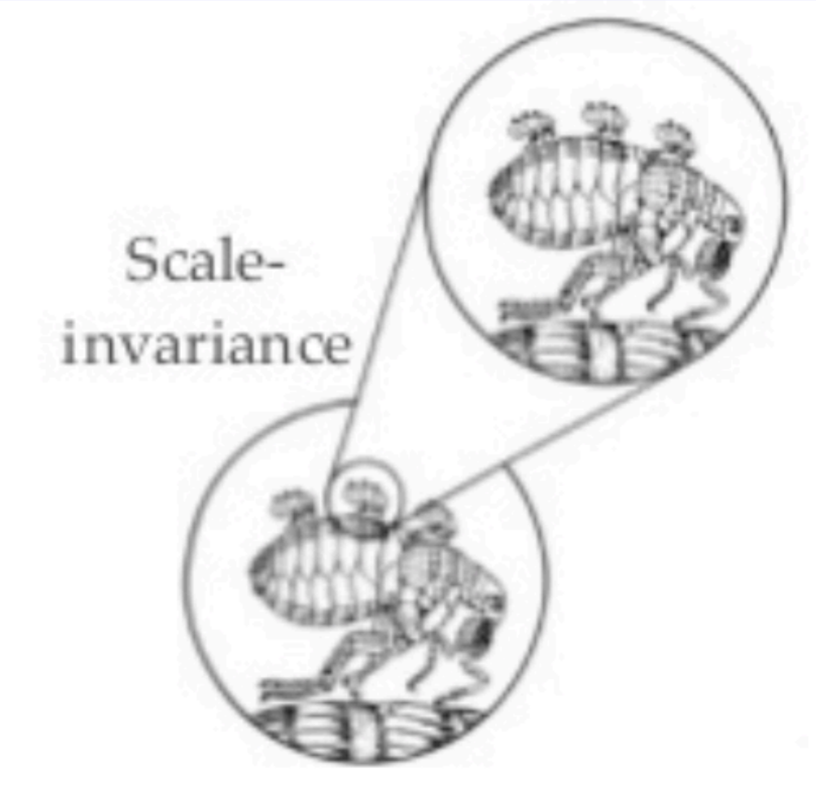
The AdS/CFT Correspondence

Maldacena 1997

Gravity in
Anti de Sitter space
in d+1 dimensions

Strongly Coupled
Conformal Field Theory
in d dimensions

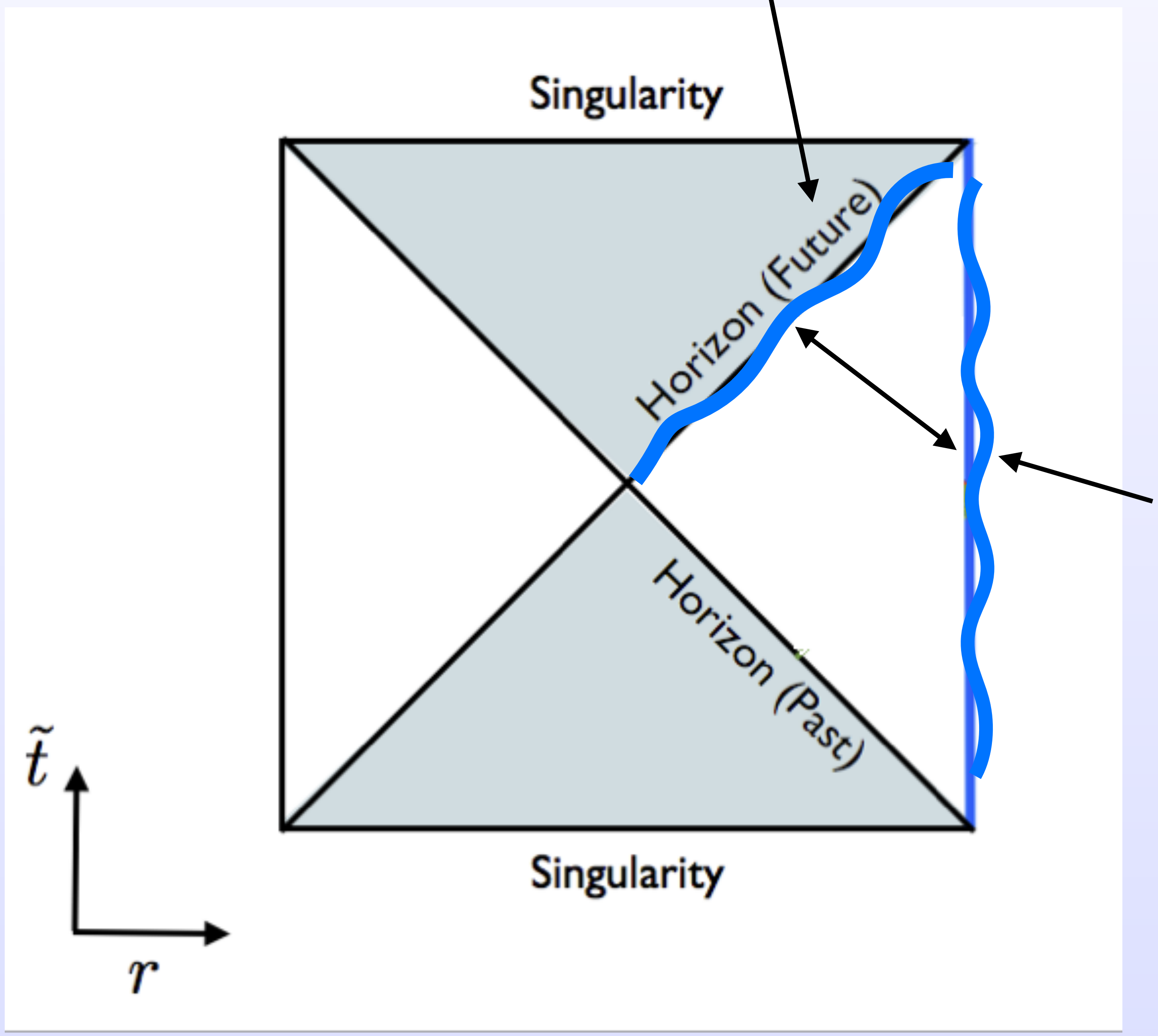




Deformations: Temperature, Density, Relevant Operators

Hydrodynamics in AdS/CFT

Perturbations of black hole horizon



Dual quantum field theory
Hydrodynamics

$$\nabla_\mu \langle T^{\mu\nu} \rangle = F^{\nu\rho} \langle J_\rho \rangle - \frac{\langle T^{0i} \rangle \delta_i^\nu}{\tau_{\text{imp}}}$$

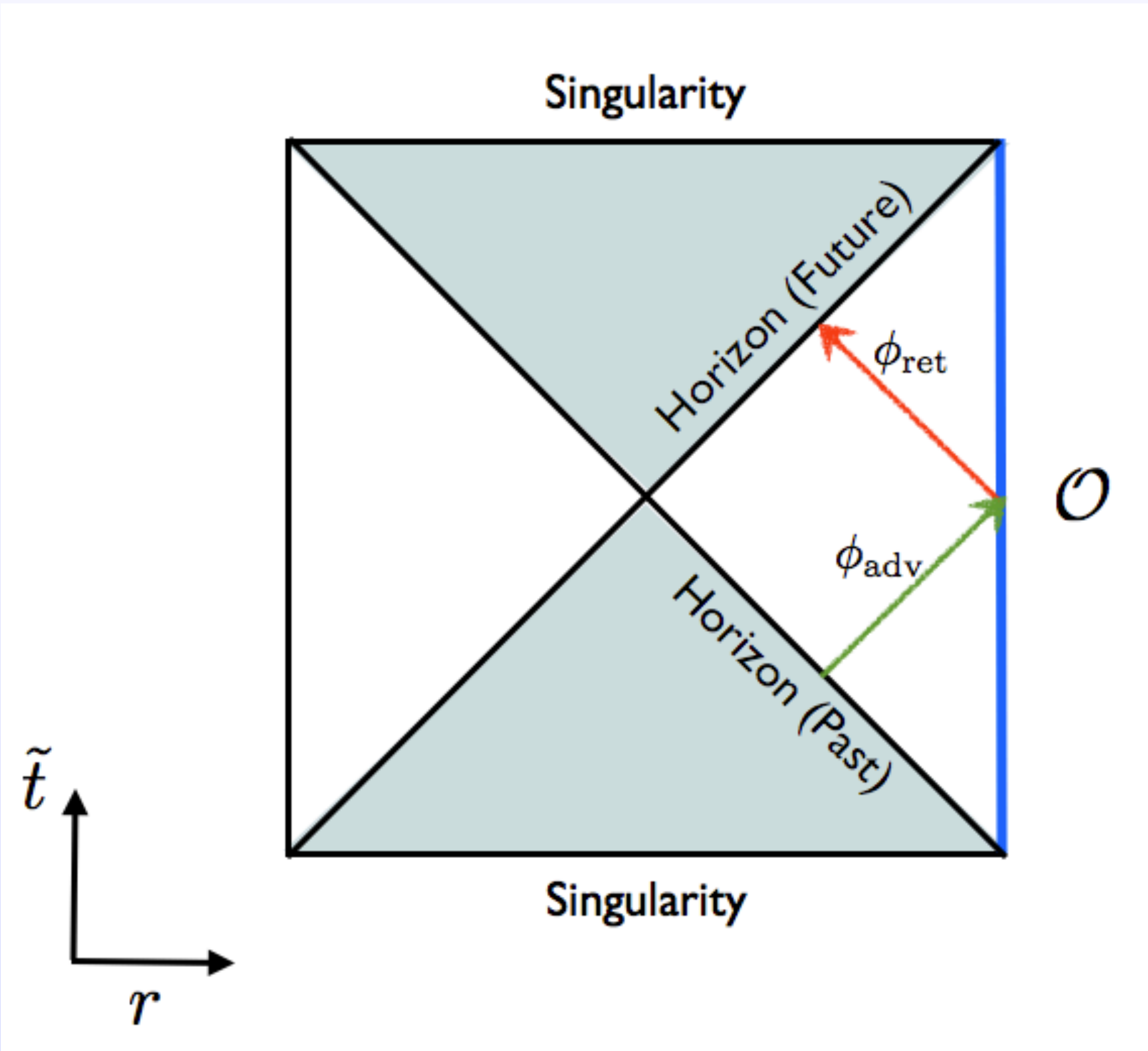
$$\nabla_\rho \langle J^\rho \rangle = 0$$

Strongly correlated matter relaxes hydrodynamically

[Policastro et.al. 2002; Minwalla et.al. 2007; Vegh, Tong, Donos, Gouteraux, Blake…]

The AdS/CFT Correspondence

$$S = \frac{1}{16\pi G_N} \int d^{d+1}x \sqrt{-g} \left(R + \frac{(d+1)d}{L^2} - \frac{1}{4} F^2 \right)$$



$$h^{\mu\nu} \leftrightarrow T^{\mu\nu}$$

$$\delta A^{\mu} \leftrightarrow J^{\mu}$$

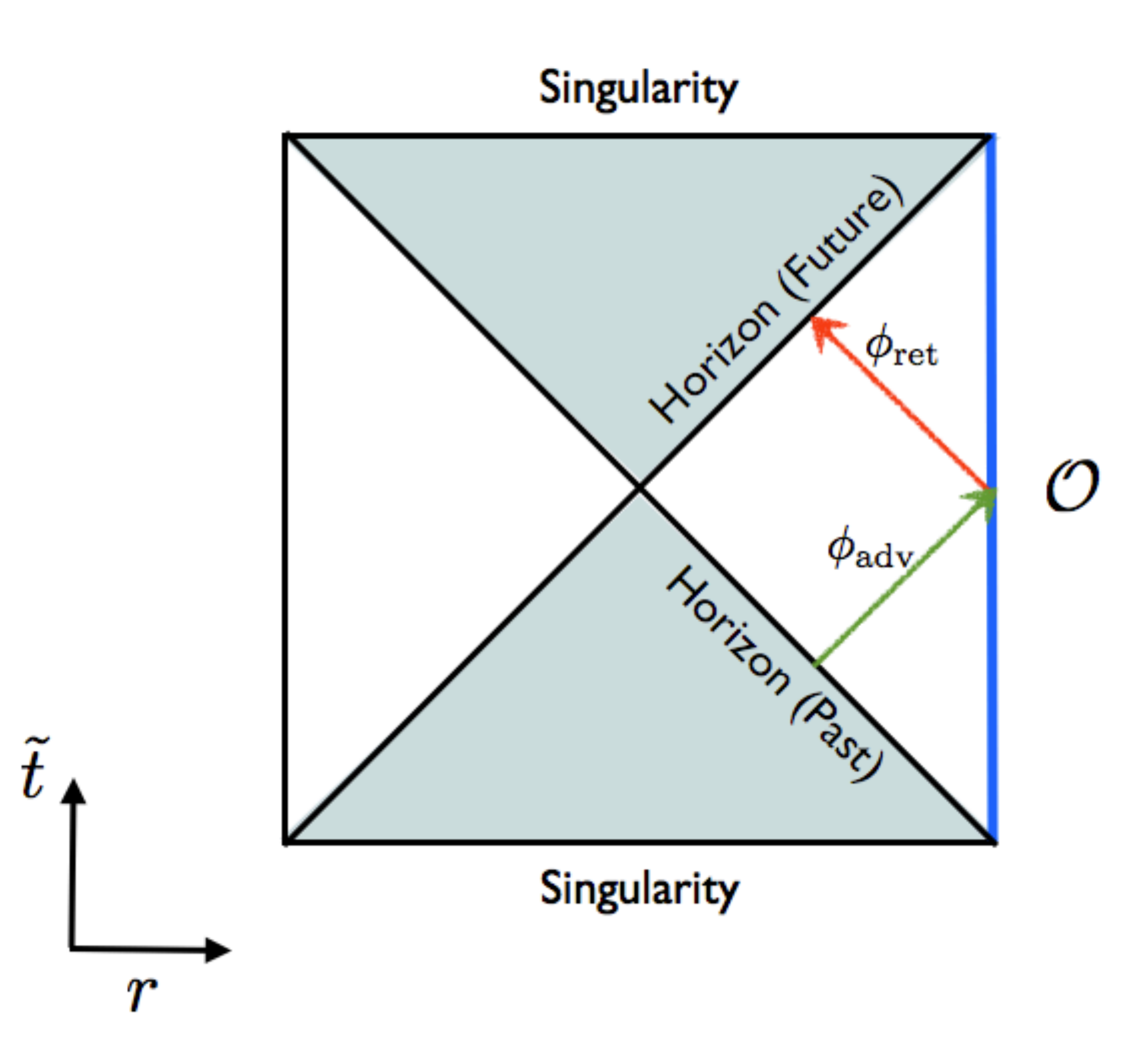
$$\Box h^{\mu\nu} = 0$$

$$\nabla_{\mu} \delta F^{\mu\nu} = 0$$

$$h^{\mu\nu} \simeq h^{\mu\nu}_{(0)} + \frac{\langle \delta T^{\mu\nu}(h_{(0)}) \rangle}{r^d}$$

$$\delta A^{\mu} \simeq \delta A^{\mu}_{(0)} + \frac{\langle \delta J^{\mu}(A_{(0)}) \rangle}{r^{d-1}}$$

Hydrodynamics from Black Holes



$$\eta = \lim_{\omega \to 0} \frac{1}{8\omega} (\delta_{ik}\delta_{jl} - \epsilon_{ik}\epsilon_{jl}) \, \mathrm{Im} \, G_R^{ij,kl}(\omega, 0)$$

$$G_R^{ij,kl} = \frac{\langle \delta T^{ij} \rangle_R}{\delta h_{(0),kl}}$$

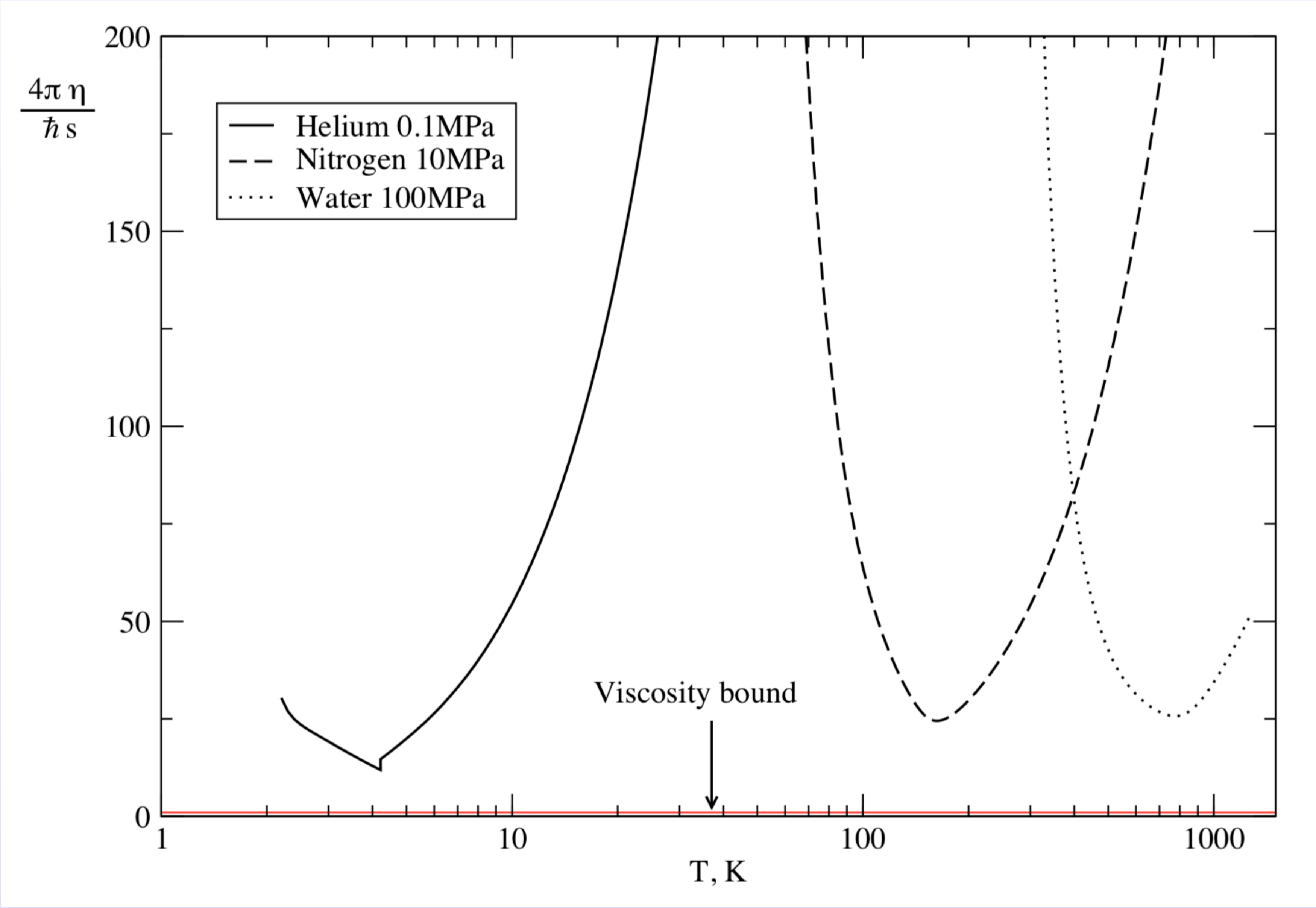
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

[Kovtun, Son, Starinets '04]

- Universal in AdS/CFT at large N and coupling
- Same in 2+1D and 3+1D

Universal Shear Viscosity

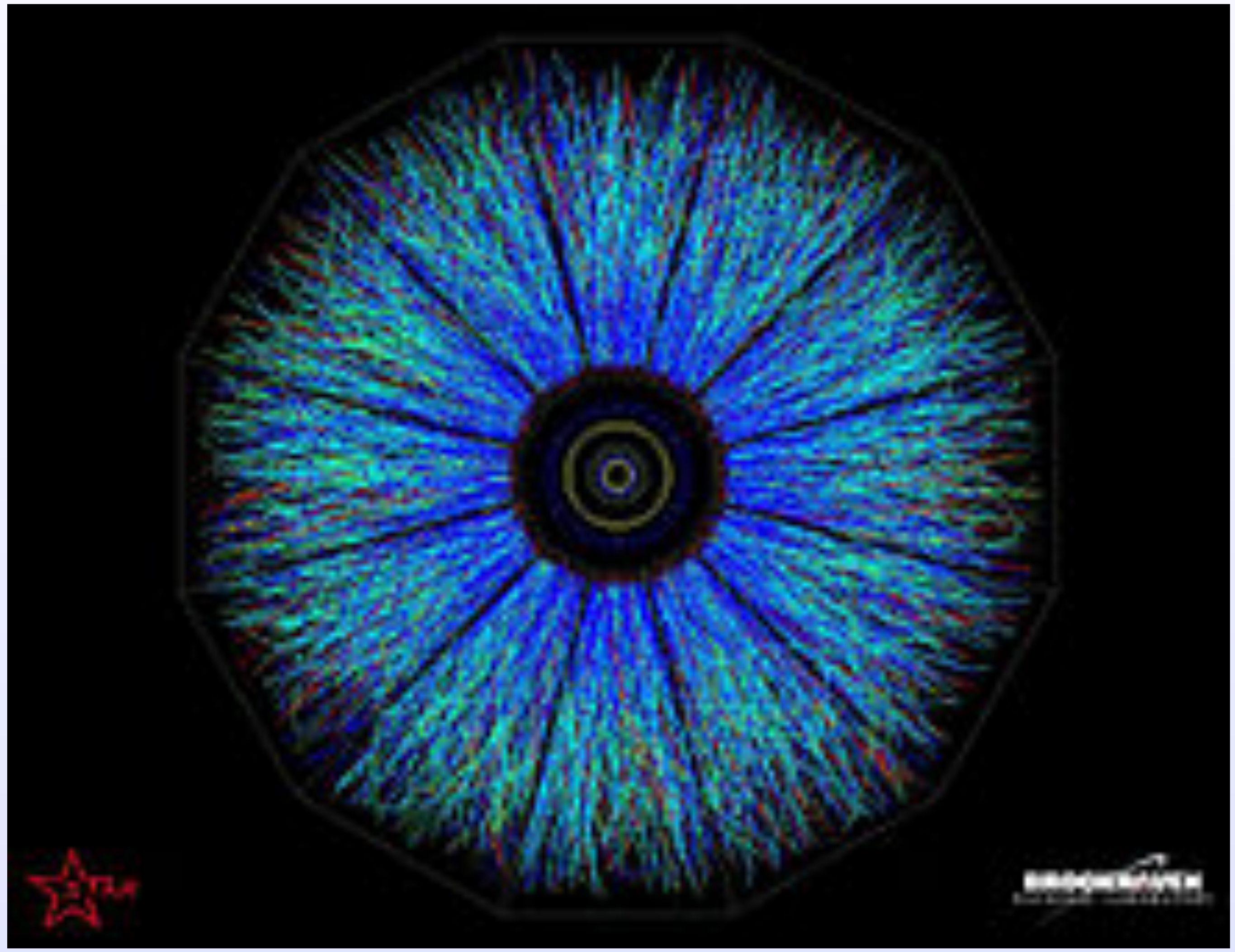
- Lower bound for interacting quantum systems



$$\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}$$

[Kovtun, Son, Starinets hep-th/0405231]

Heavy Ion Collisions



- Confirmed by RHIC & LHC: $1 < 4\pi(\eta/s)_{QGP} < 2.5$

[Heinz et.al. 1108.5323]

- Extremely viscous: $\eta \sim \frac{\hbar}{4\pi k_B} s \sim 10^{14} cp$

Pitch drop experiment

started in 1930

8 drops fell so far

but only once a drop was witnessed falling

Ig Nobel Prize in 2005

Viscosity of pitch:

$$2.3 \cdot 10^{11}\ cp$$

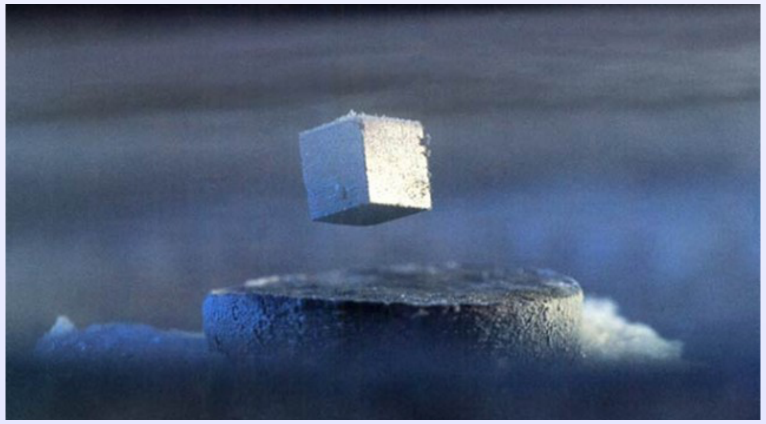
230 billion times the viscosity of water!

AdS/CFT in the lab?

AdS/CFT Prediction:

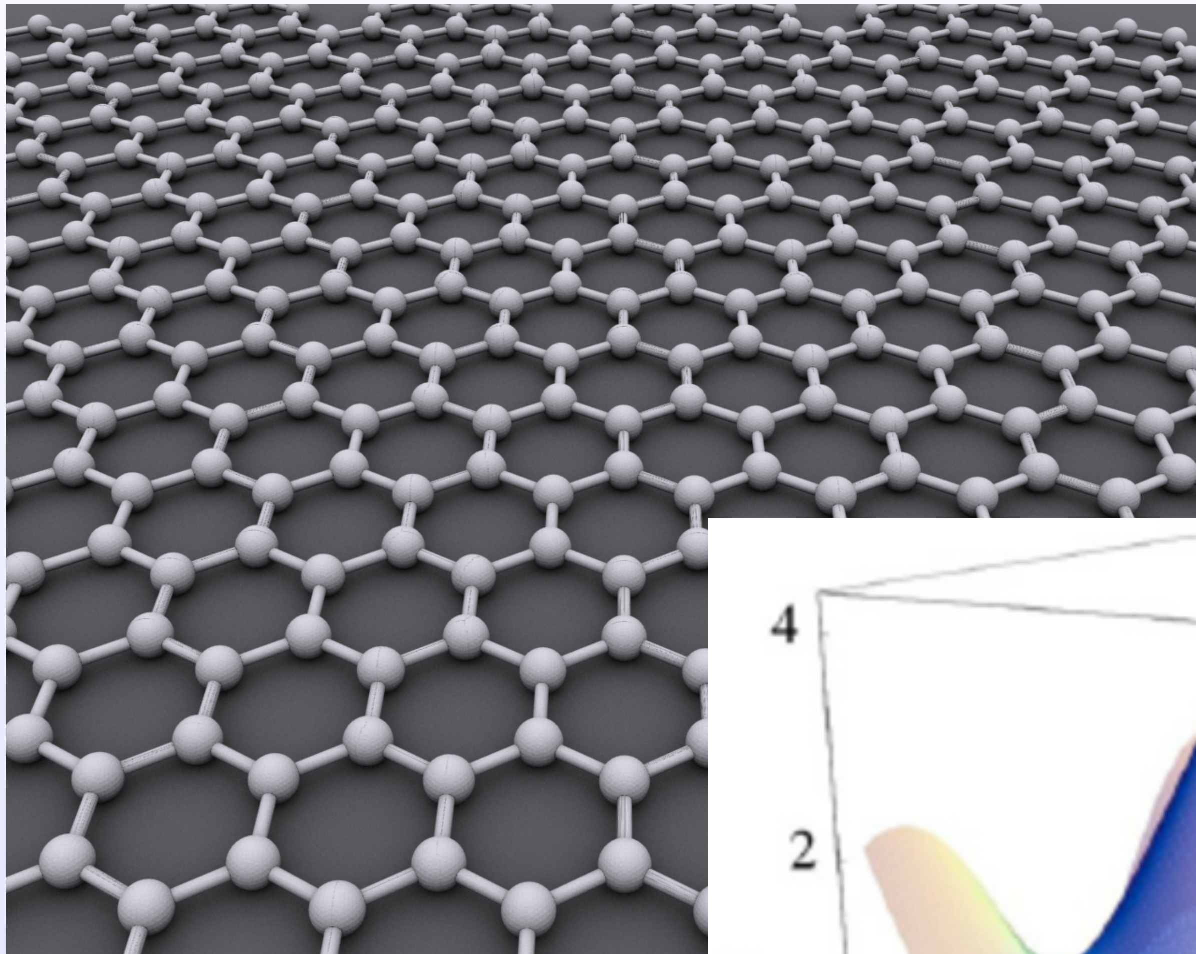
$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

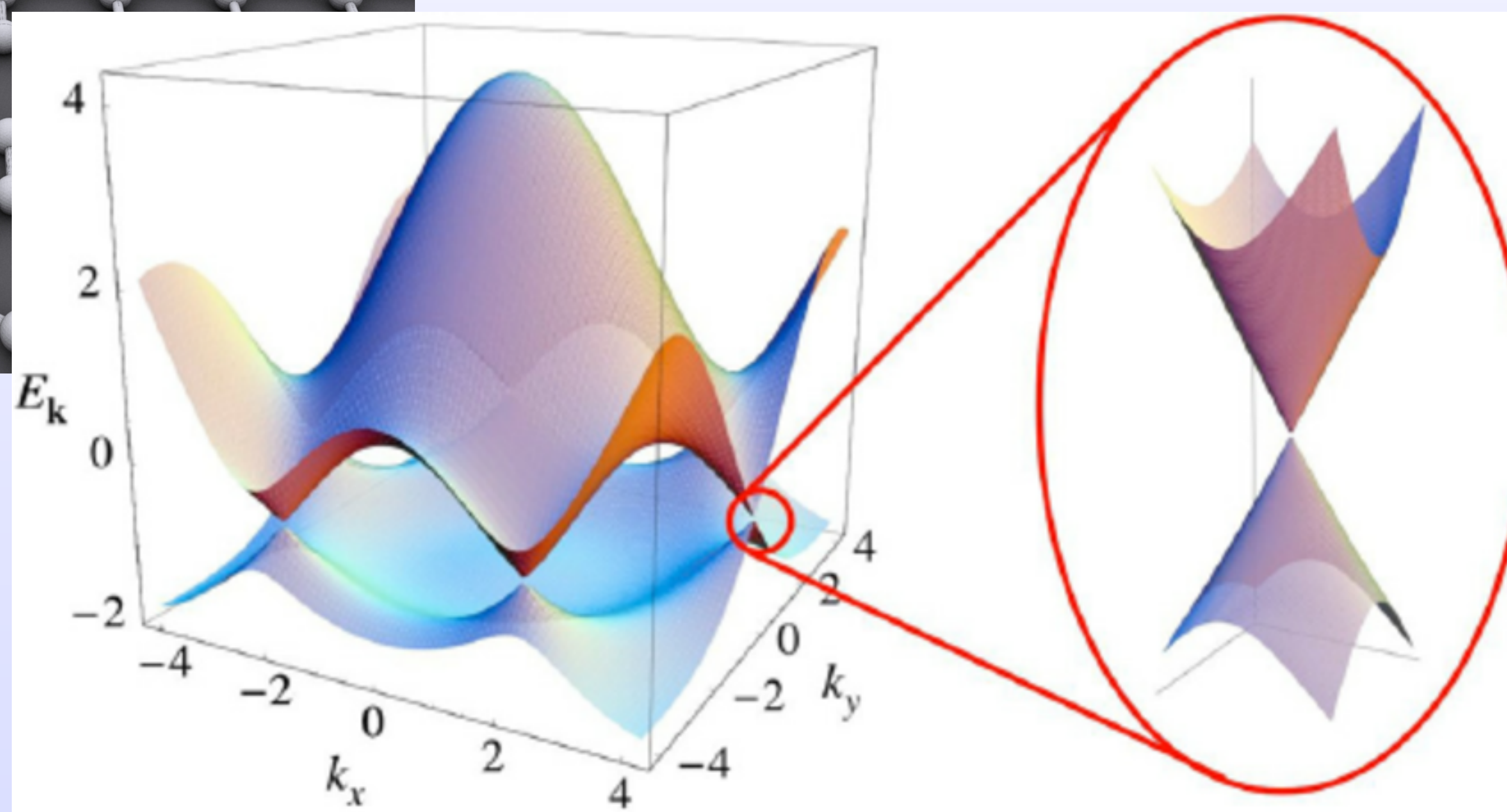
Can we measure $\frac{\eta}{s}$ in other strongly correlated electron systems?



- Which systems to look at?
- How to measure the viscosity of an electron fluid?
- How to measure entropy density?
- N.B.: AdS/CFT electronic fluids are close to ideal.

Graphene



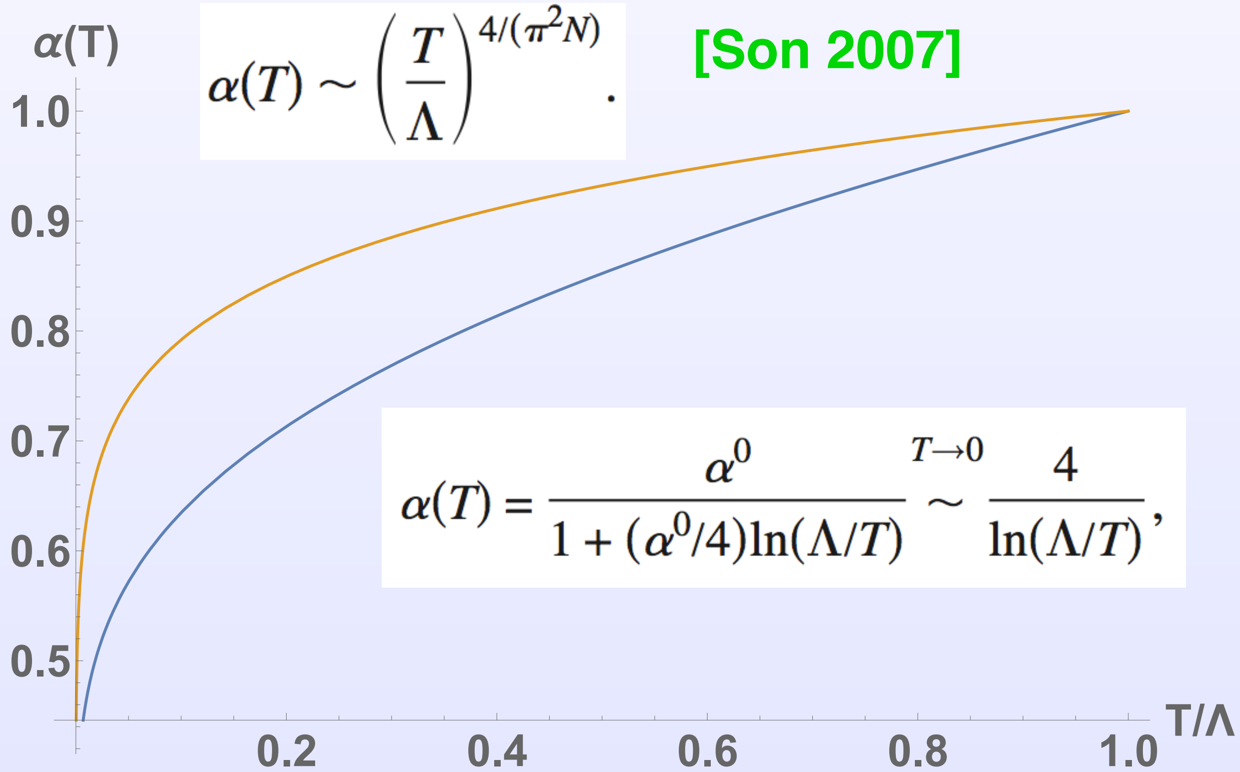


$$v_F \approx \frac{c}{300}$$

[Geim, Novozelov Nature 2005, Nobel Prize 2010]

Hydrodynamic Fluid in Graphene

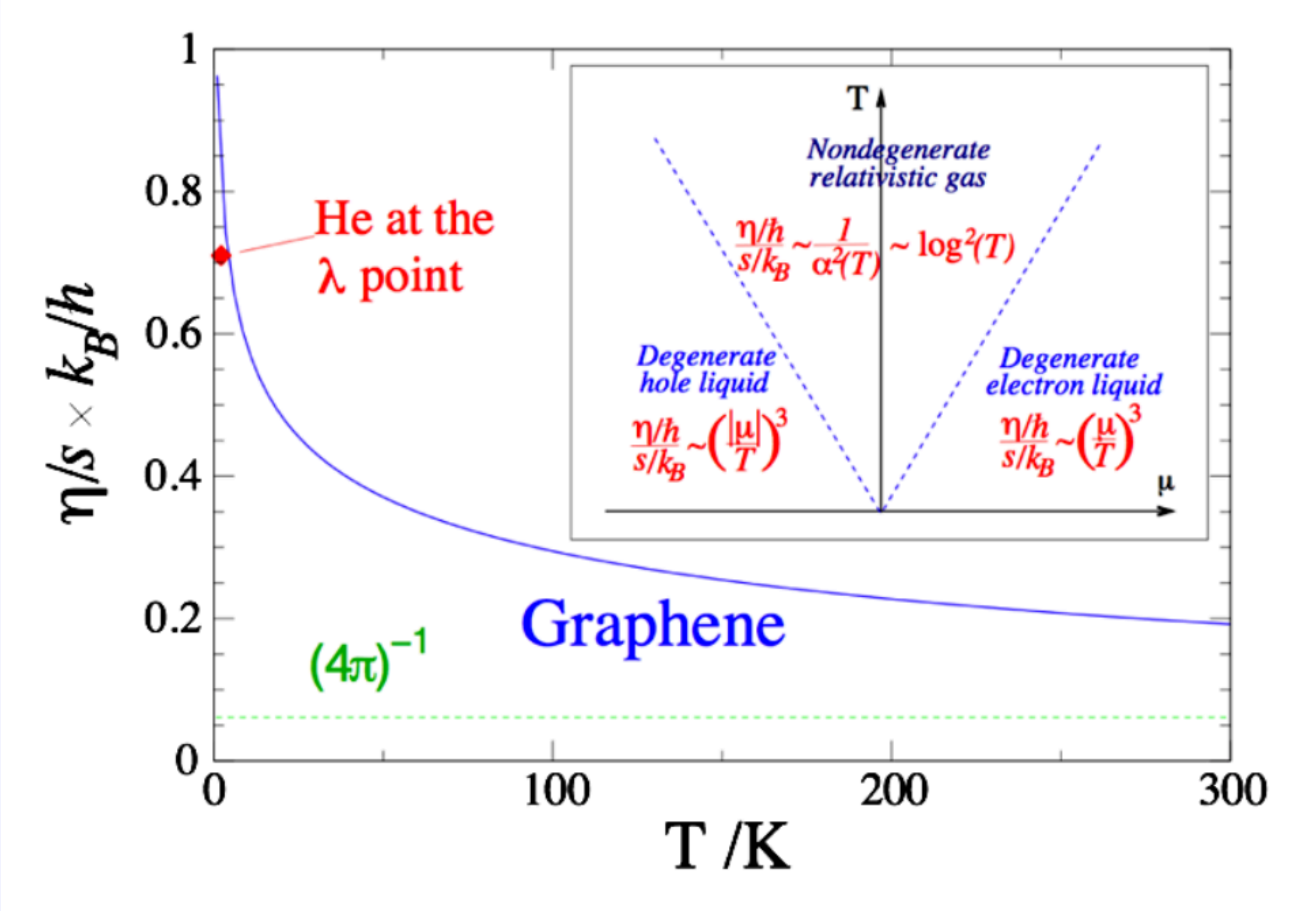
- Graphene in the nonperturbative regime



$$\alpha(T) \sim \left(\frac{T}{\Lambda}\right)^{4/(\pi^2 N)}.$$

[Son 2007]

$$\alpha(T) = \frac{\alpha^0}{1 + (\alpha^0/4)\ln(\Lambda/T)} \overset{T\to 0}{\sim} \frac{4}{\ln(\Lambda/T)},$$



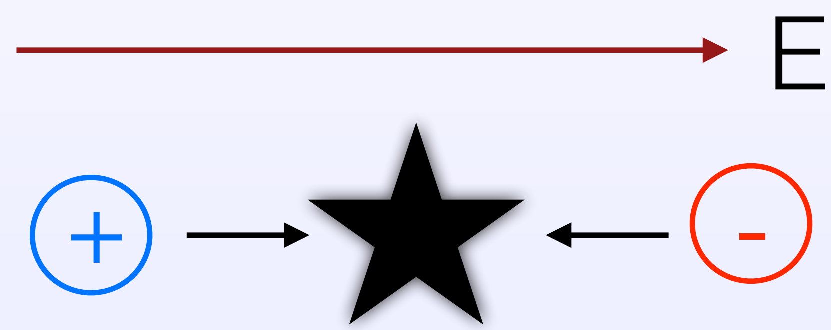
$$\eta/s = \frac{\hbar}{k_B}\frac{C_\eta \pi}{9\zeta(3)}\frac{1}{\alpha^2(T)} \simeq 0.008\,15 \times \left(\log\frac{T_\Lambda}{T}\right)^2.$$

[Fritz, Schmalian, Sachdev etal PRB, PRL 2008,09]

How to measure $\frac{\eta}{s}$?

- Transport coefficients in a P&T symmetric charged fluid:

Relevant transport coefficients:
η: Shear viscosity
σ: Quantum Critical conductivity



Bulk viscosity can be neglected
for conformal fluids or incompressible flows ($\partial_\mu v^\mu = 0$)

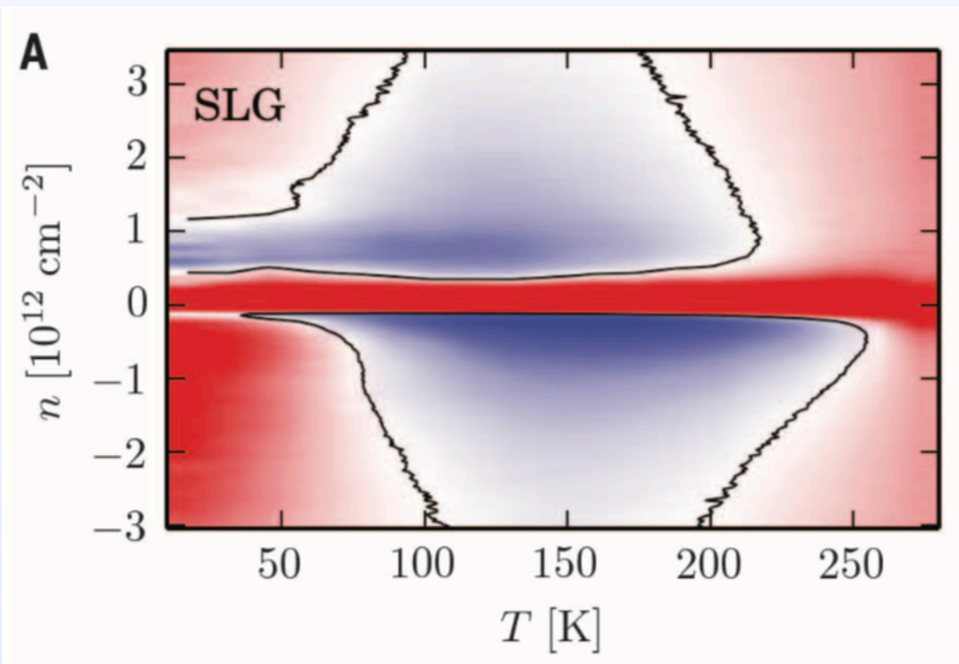
- Relation to kinematic viscosity

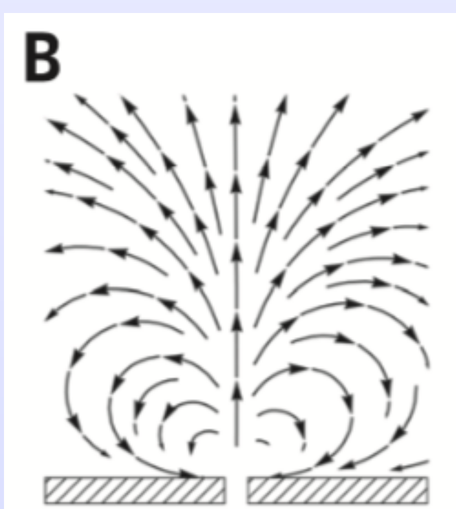
In relativistic hydro: $\nu_{\rm rel} = \frac{\eta}{\epsilon + P} = \frac{\eta}{s}\frac{s}{\epsilon+P}$

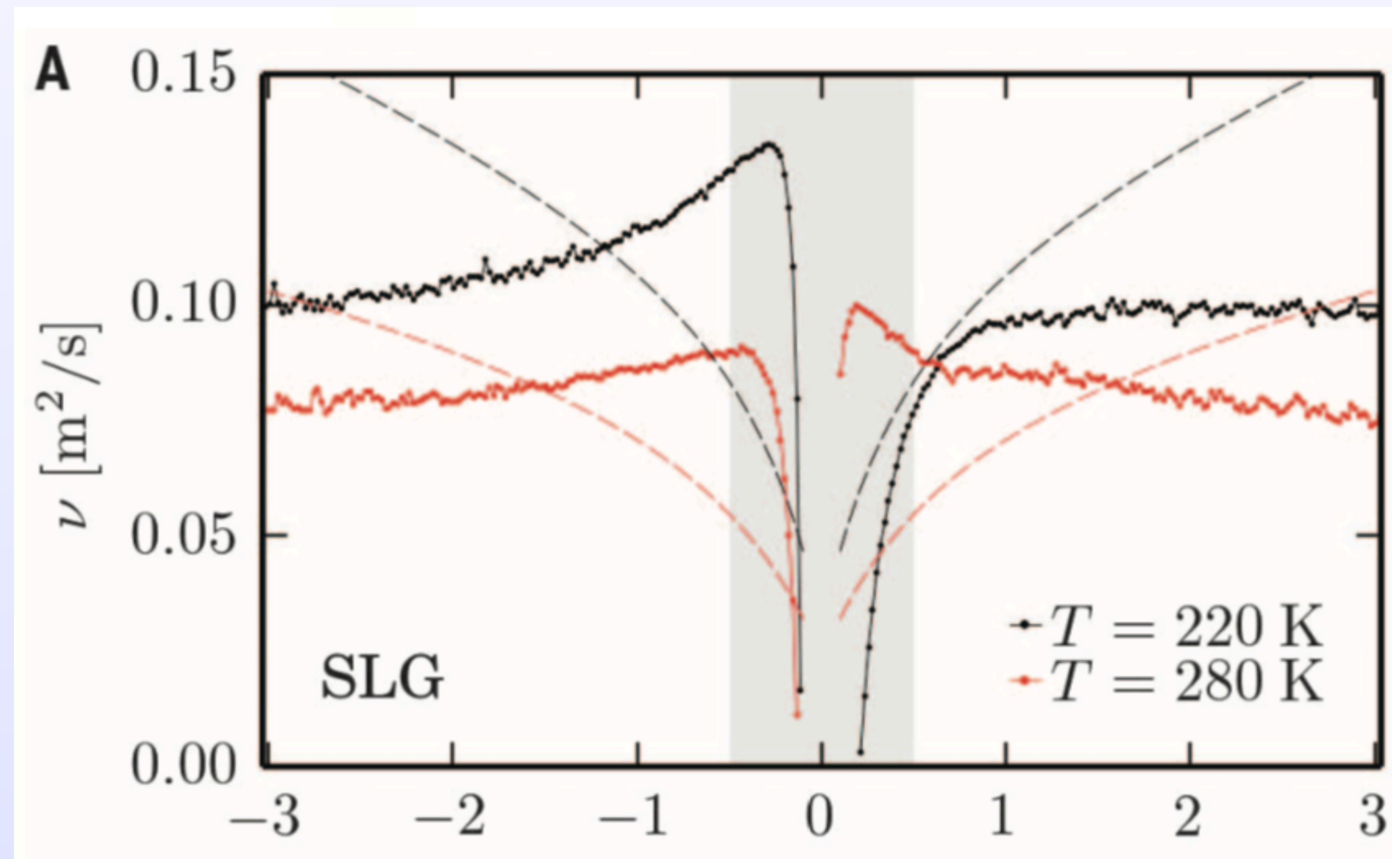
Nonrelativistic limit: $\nu = \frac{\eta}{m_{\rm eff}\, n}$

Experimental Results for Viscosity

- Nonlocal negative resistance in Doped Graphene







[Geim et.al. Science 2016]

$$\nu \sim 0.1\, m^2/s$$

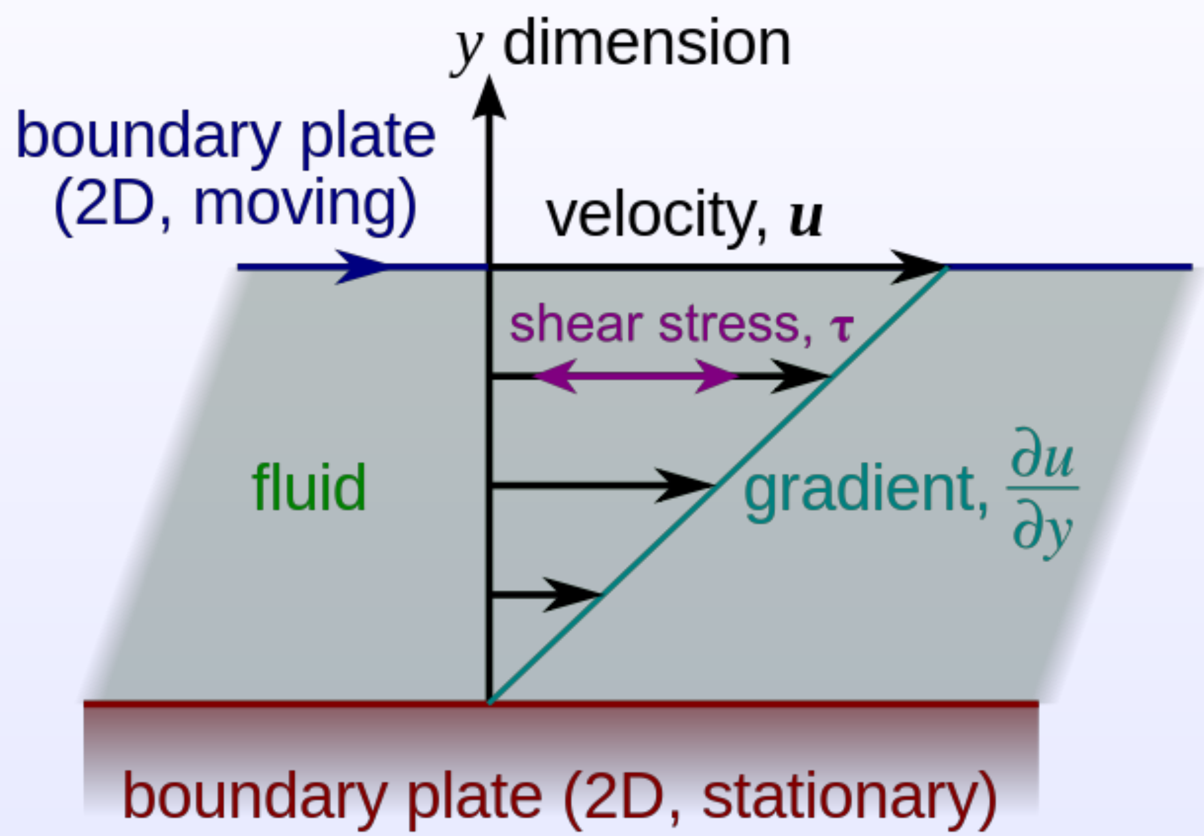
Typical Values

$$\nu \sim 10^{-6}\, m^2/s$$

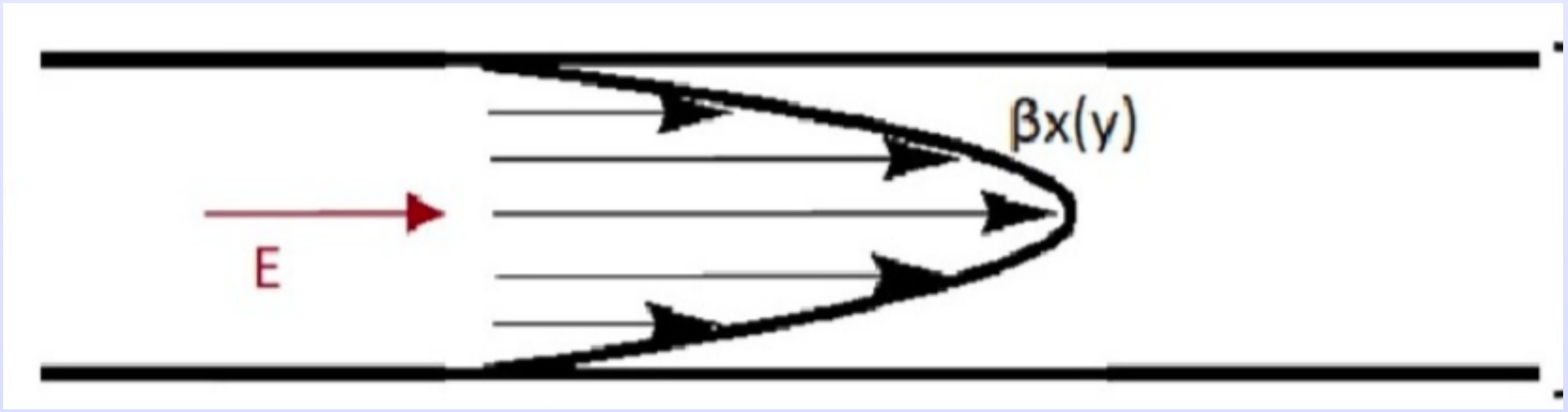
$$\nu \sim 10^{-3}\, m^2/s$$

Special Flow Geometries

Couette



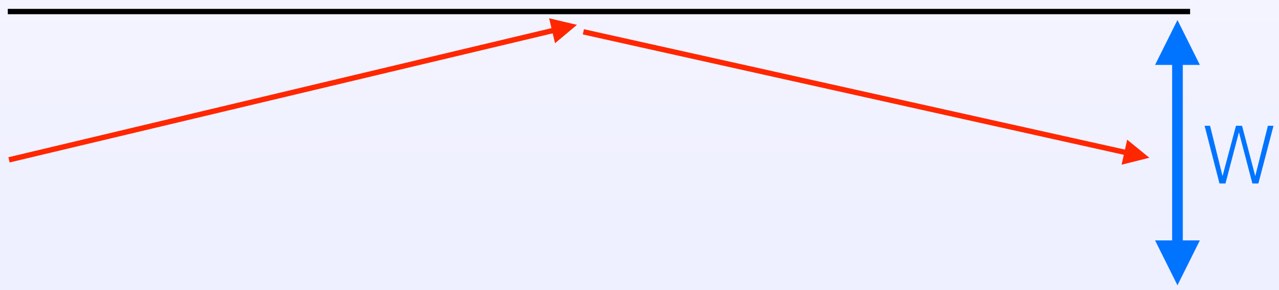
Poiseuille



Gurzhi Effect

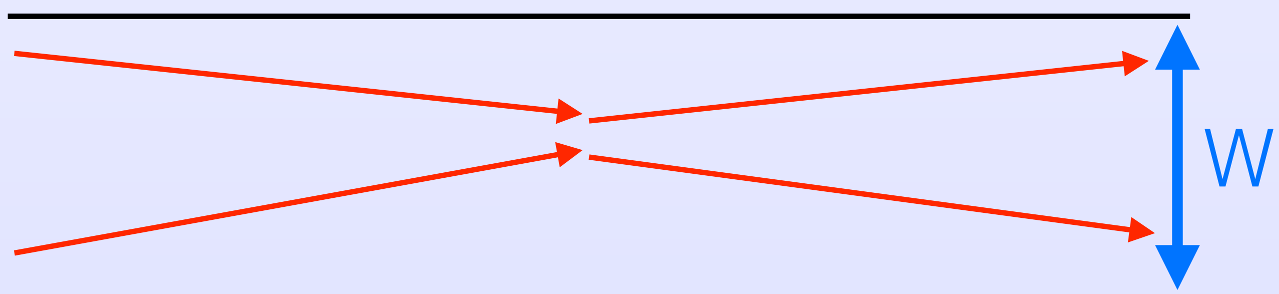
- 2D Electrons in (Al)GaAs Heterostructures

Knudsen flow regime

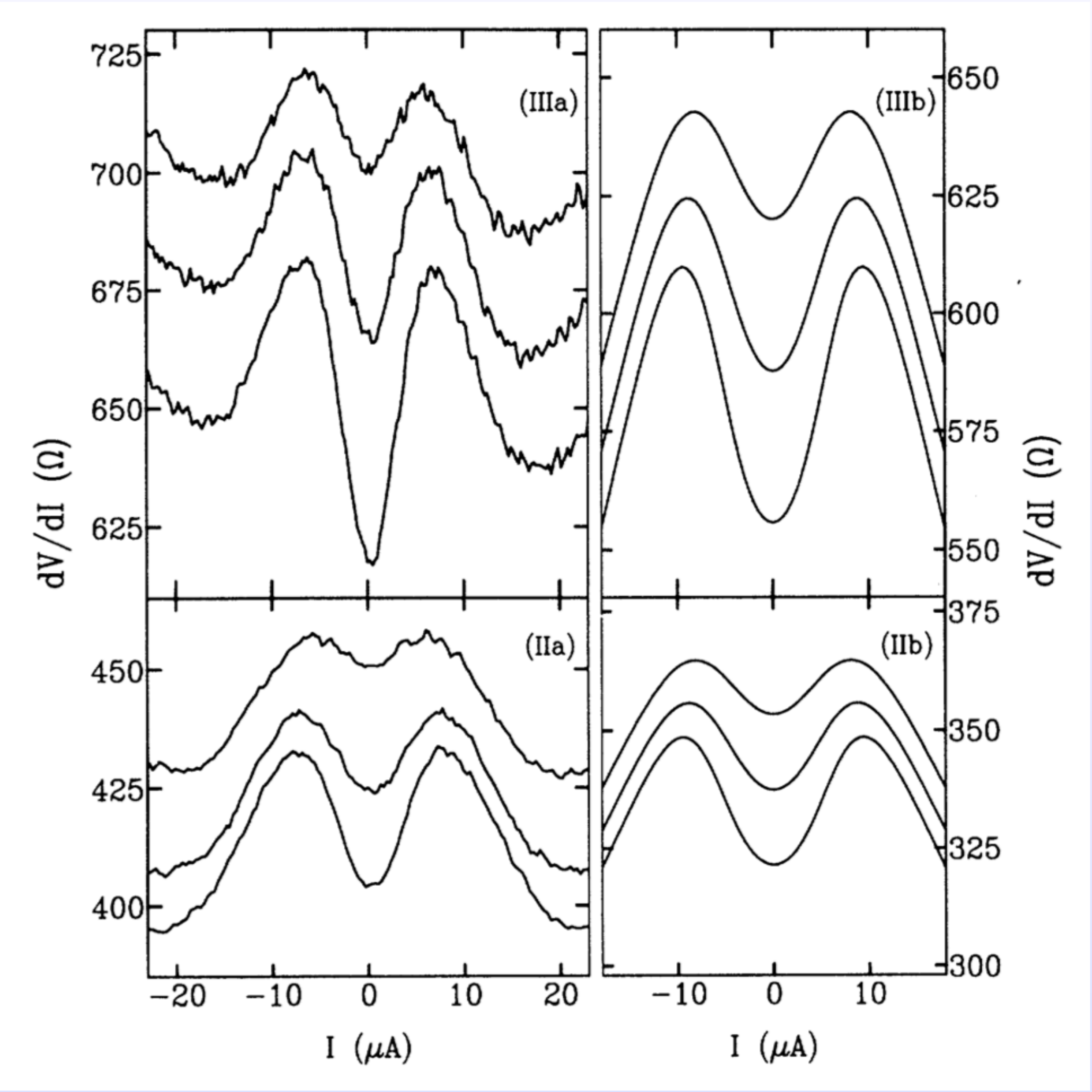


$$\ell_{ee}(T) > W$$

Poiseuille flow regime



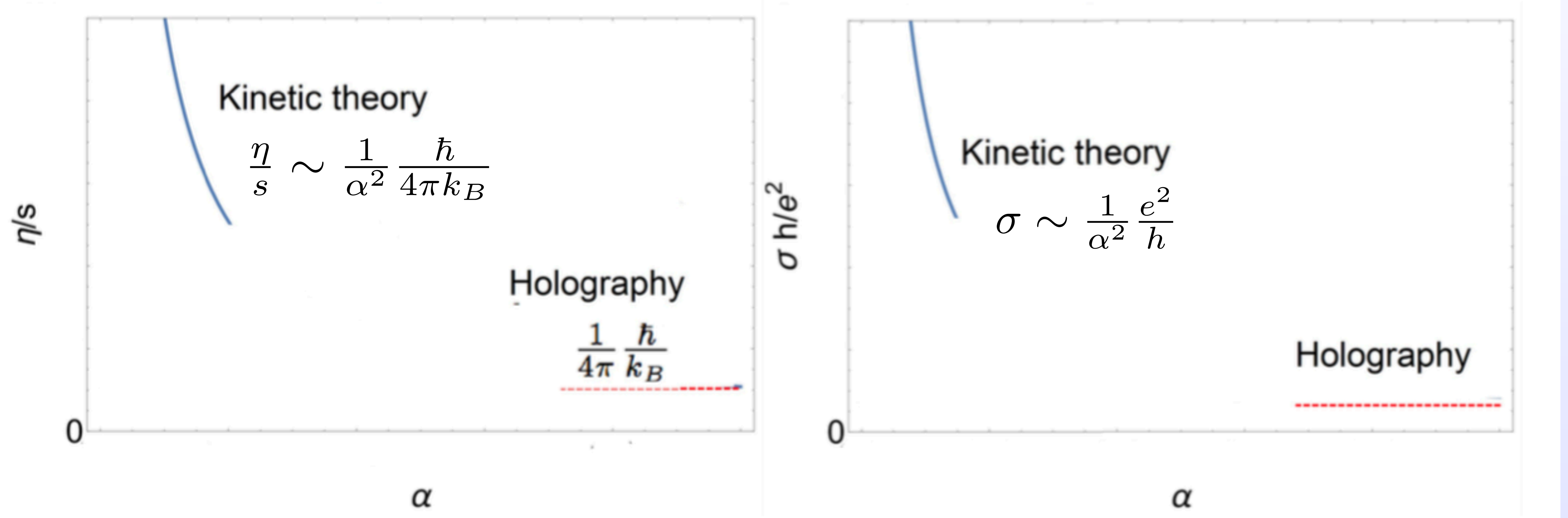
$$\ell_{ee}(T) < W$$



[Gurzhi 1968]

[Molenkamp+de Jong 1994,95]

From Weak to Strong Coupling

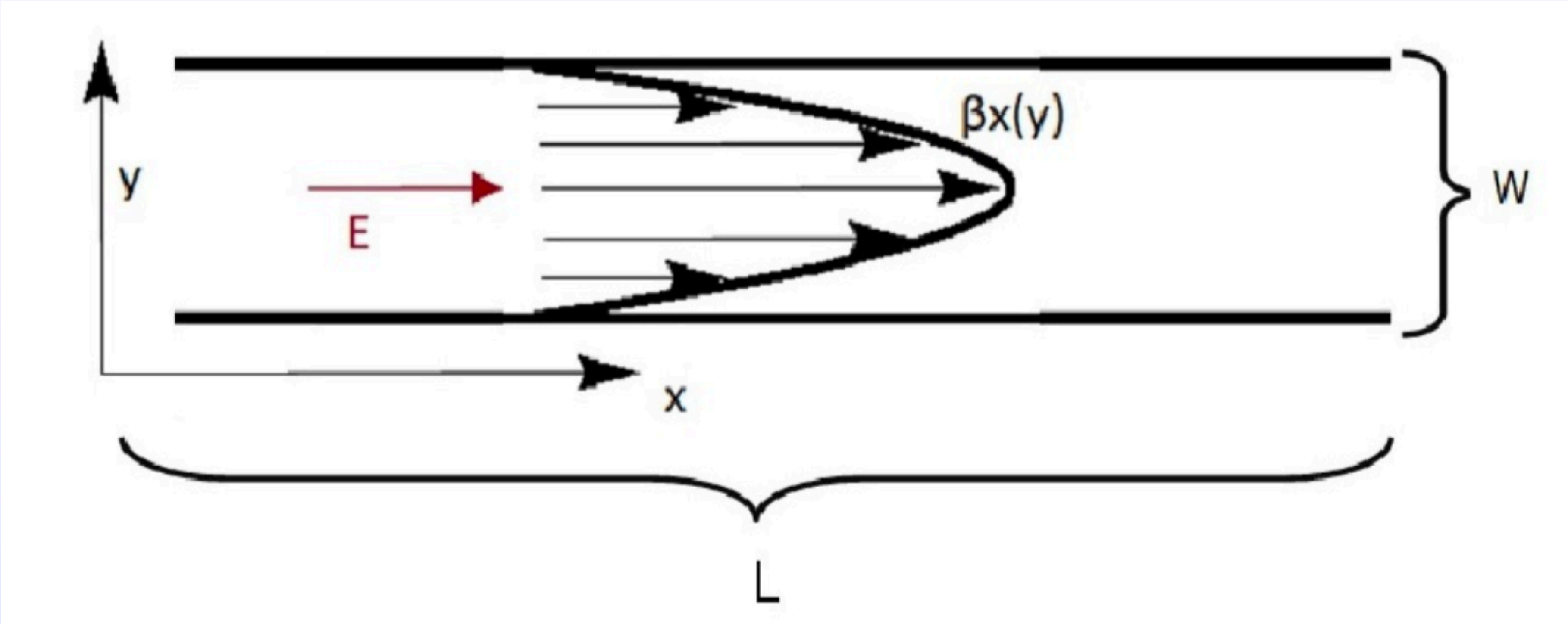


[M. Müller, J. Schmalian, and L. Fritz]
[L. Fritz, J. Schmalian, M. Müller, and S. Sachdev]

Our Setup

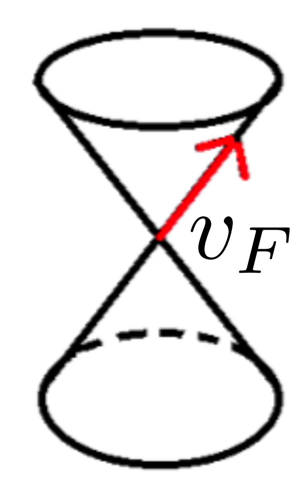
J.E., I. Matthaiakakis, R.M., D. Rodríguez-Fernández (PRB 2018)

Relativistic Hydrodynamics in a 2D Channel Setup



Zero Velocity Boundary Conditions

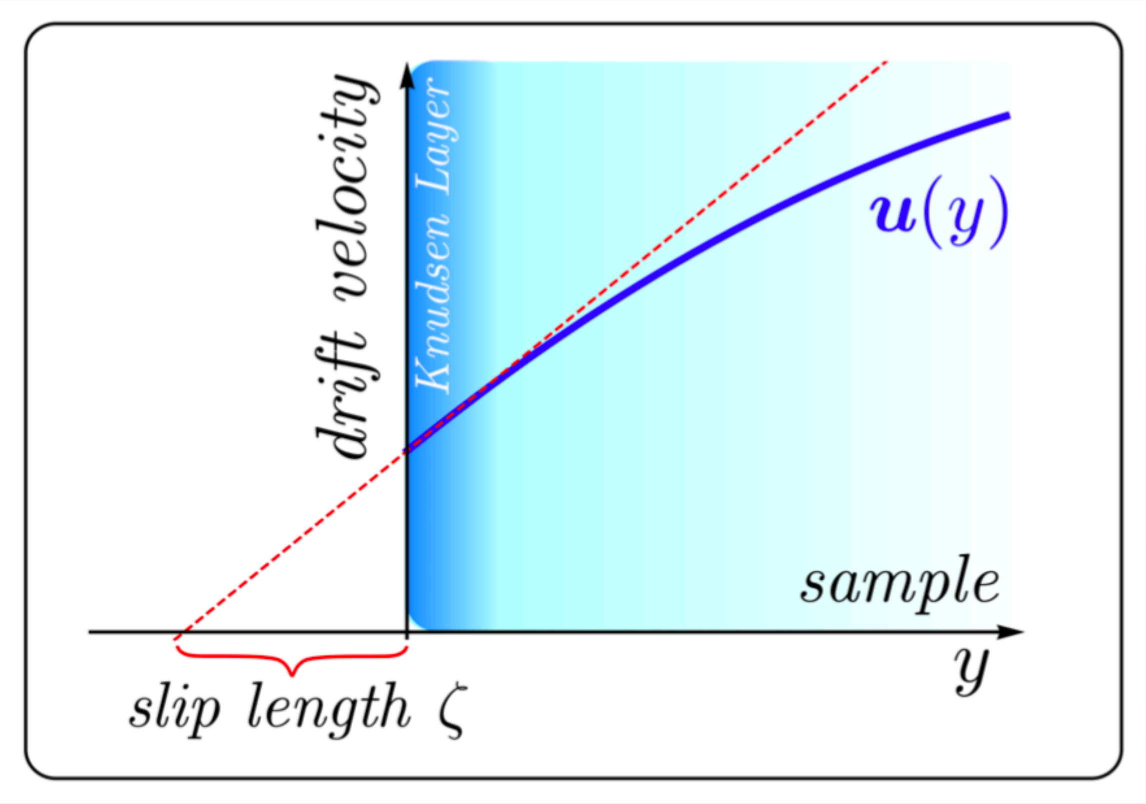
$$0 = \frac{\eta}{s}\left[\beta_x'' + \frac{2}{v_F^2}\gamma^2\beta_x\beta_x'^2\right] + \frac{1}{\gamma}\left(v_F\epsilon_r\frac{\epsilon_0\,h}{e}\frac{E_x}{\gamma}\frac{\rho}{s} - \frac{p+\varepsilon}{s\tau_{\mathrm{imp}}}\frac{\beta_x}{v_F^2}\right)$$

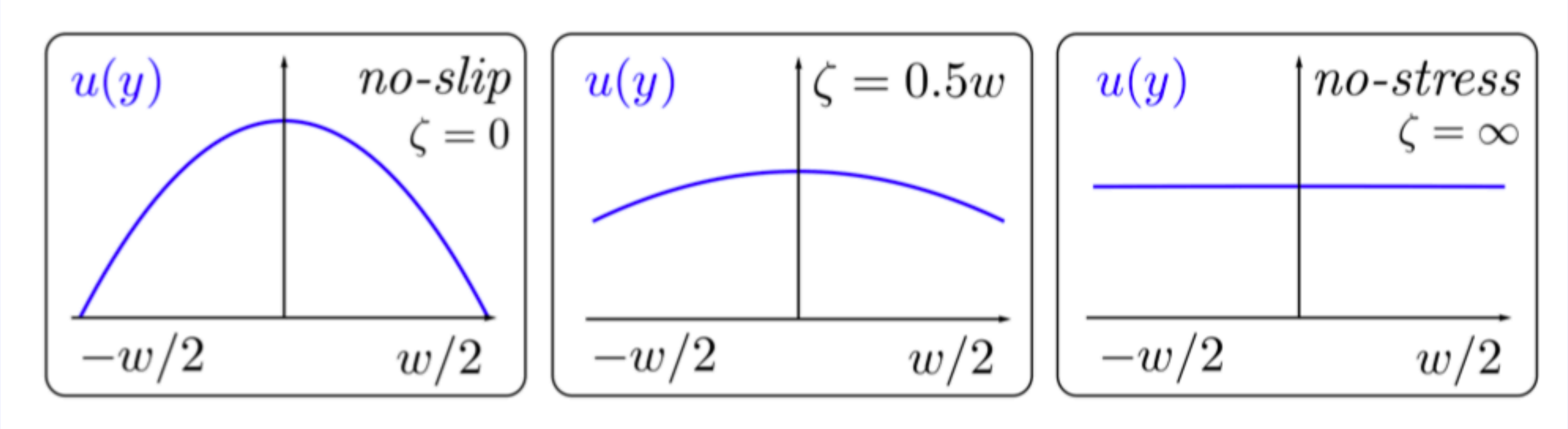


Momentum Relaxation, EOS from AdS/CFT

Boundary Conditions

$$u^t_\alpha\Big|_S = \zeta\, n_\beta \frac{\partial u^t_\alpha}{\partial x_\beta}\Big|_S$$

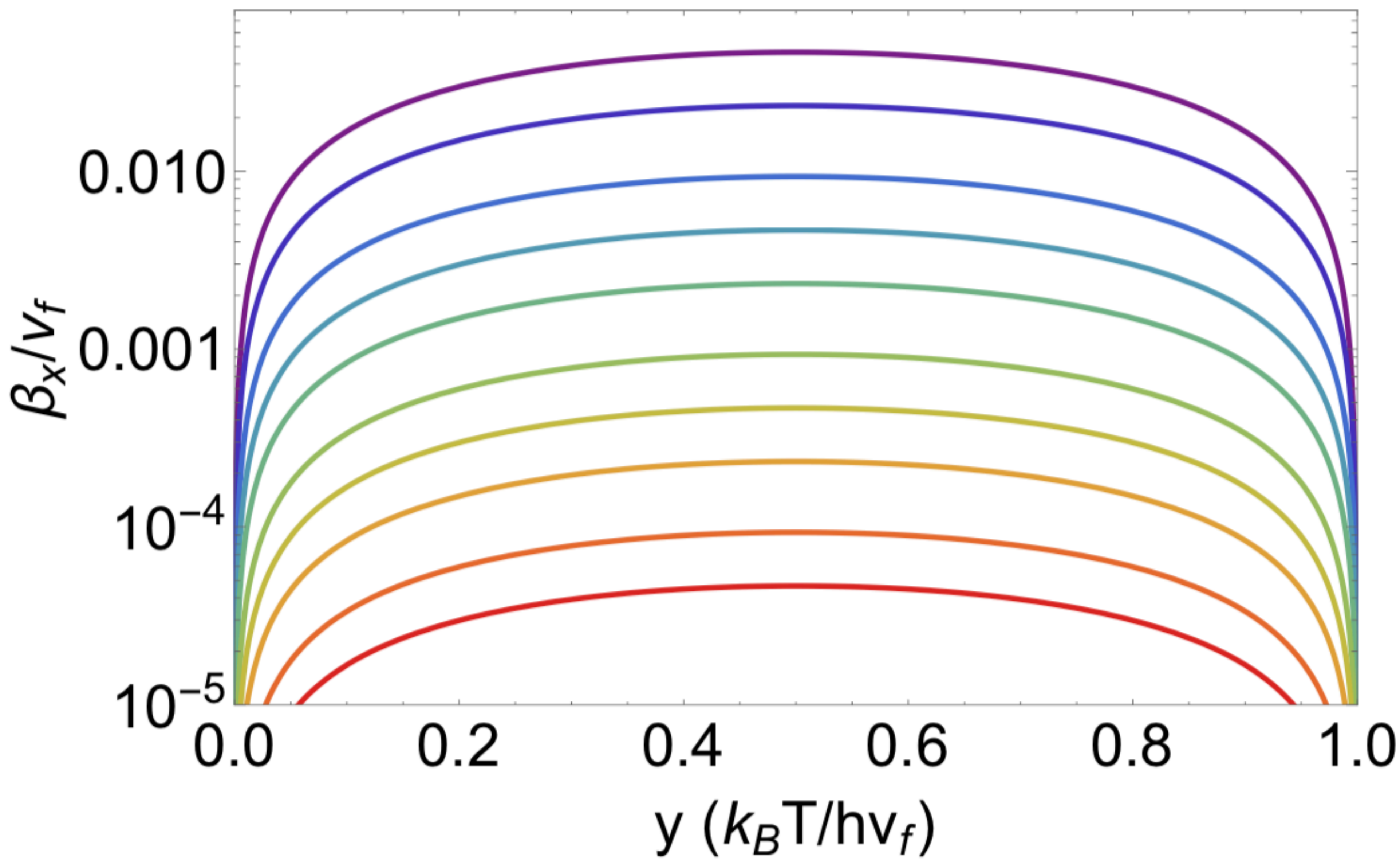




Kiselev, Schmalian PRB 2018

Relativistic Poiseuille Flows

$\mu \sim 0.1$ meV and $W = 1.5\mu m$ and $E_x \sim 10^{-3}$ V/μm



Top curve, $\eta/s = \hbar/4\pi k_B \sim 10^{-13}$ K s. Bottom curve, 10^{-10} K s

Strongly Coupled Fluids Flow Fastest

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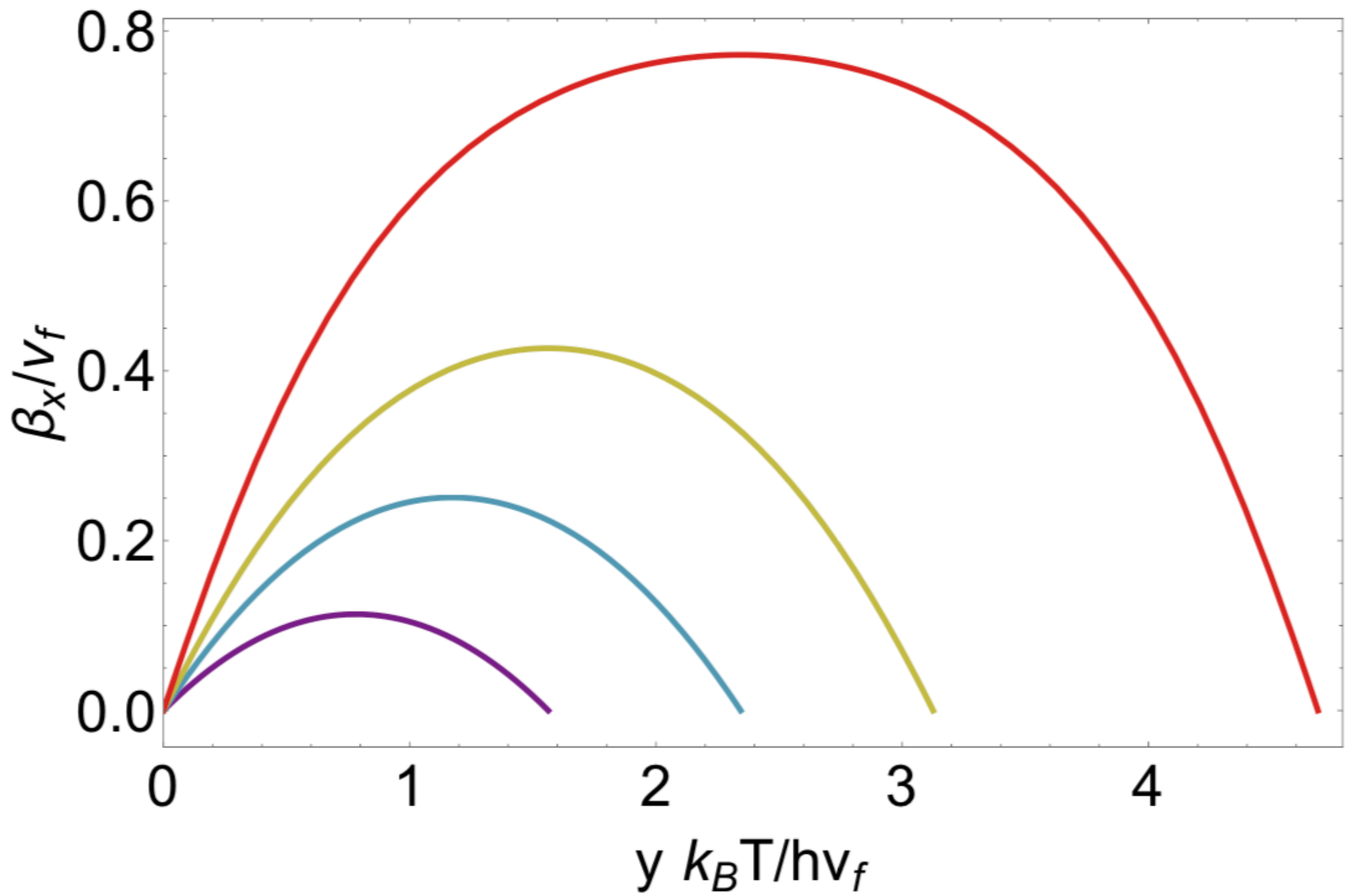
Relativistic Poiseuille Flows

$\mu \sim 0.1$ meV, $\eta/s = \hbar/4\pi k_B$ and $E_x \sim 10^{-3}$ V$/\mu m$



Top curve, $W = 5\,\mu m$. Bottom curve, $W = 1.5\,\mu m$

Strongly Coupled Fluids Easily Flow Relativistically

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Differential Wire Resistance

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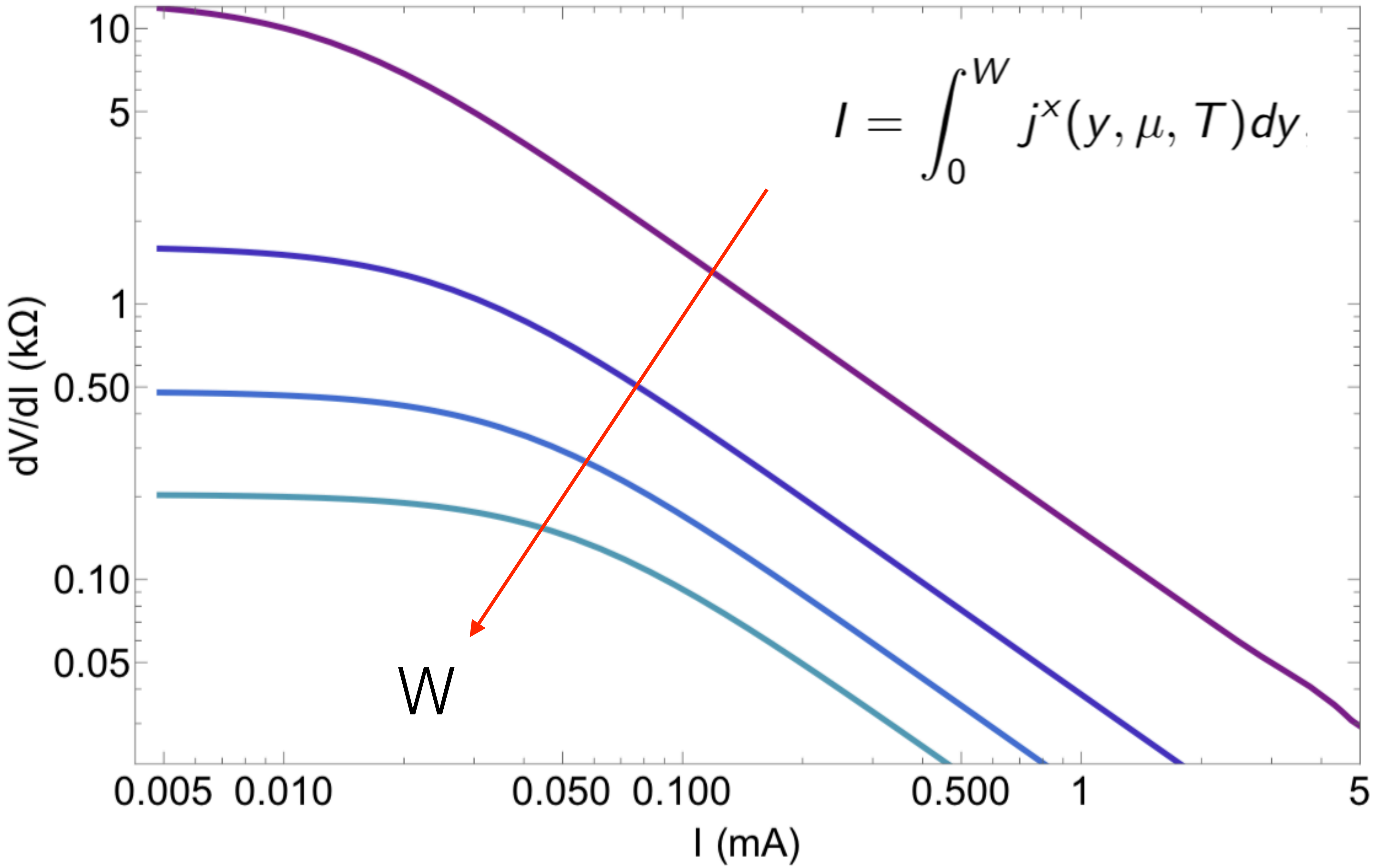
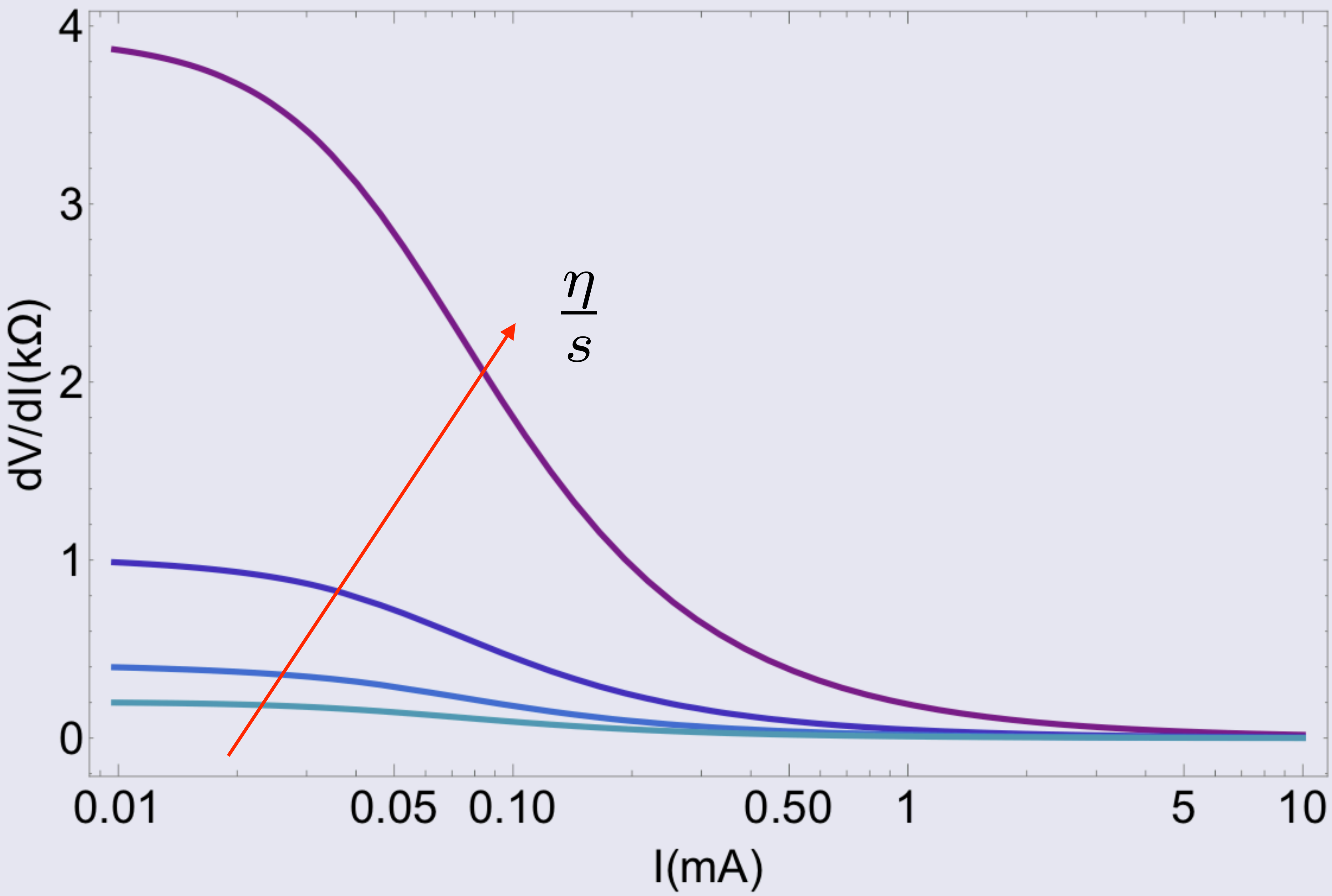
Differential resistance in the Poiseuille flow



$$I = \int_0^W j^x(y, \mu, T) dy$$

dV/dI **at different** W**.** $R(I)$ **decreases as** W **increases.**

Differential Wire Resistance



R(I) increases as η/s increases

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Conclusions/Outlook Part I

- Simulated strongly coupled holographic fluids with small $\frac{\eta}{s}$
- Applicable to Graphene, Strange Metals, Kondo Systems
- Strongly coupled fluids flow fastest through wires
- Show smallest wire resistance
- Onset of turbulence, preturbulent behavior?
- Other ways to measure eta/s? Actual experiments?