

Non-dynamical torsion from fermions and CMBR phenomenology

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based on

[P. Dona & A. Marciano, arXiv:1605.09337 \(PRD 2016\)](#)

[A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848](#)

[works in progress...](#)

Separate Universe assumption

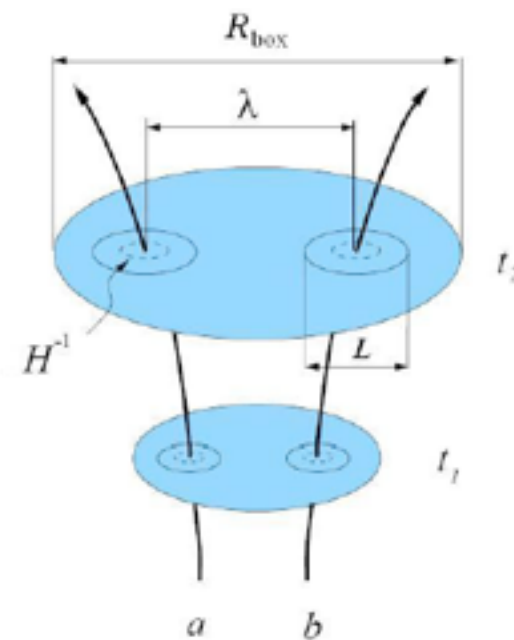
$$ds^2 = N^2 dt^2 - \gamma_{ij} (dx^i + N^i dt) (dx^j + N^j dt)$$

ADM decomposition

$$K_{ij} = -\nabla_{(j} n_{i)} = \frac{1}{2N} \left(-\partial_t \gamma_{ij} + {}^{(3)}\nabla_{(i} N_{j)} + {}^{(3)}\nabla_{(j} N_{i)} \right)$$

$$K_{ij} = -\frac{\theta}{3} \gamma_{ij} + A_{ij} \quad \theta = \frac{3}{N} \left(\frac{\dot{a}(t)}{a(t)} + \dot{\psi} \right)$$

$$N(t_1, t_2; x_j) = \frac{1}{3} \int_{\gamma(\tau)} \theta d\tau, \quad n_\mu = (N, 0)$$



Perturbations

$$\epsilon = k / (a H)$$

$$\gamma_{ij} = a(t, x_i) \tilde{\gamma}_{ij}, \quad \tilde{\gamma}_{ij} = (e^h)_{ij}, \quad a(t, x_i) = a(t) e^{\psi(t, x_i)}$$

Curvature perturbation variable

Lyth, Malik & Sasaki, JCAP 2015

$$\nabla_\mu T^{\mu\nu} = 0$$



$$\frac{d\rho(t, x_i)}{dt} + 3\tilde{H}(t, x_i)[\rho(t, x_i) + p(t, x_i)] = 0$$
$$\frac{d\rho(t)}{dt} + 3\frac{\dot{a}(t)}{a(t)}[\rho(t) + p(t)] = \dot{\psi}(t)$$

Uniform density slicing & adiabatic pressure

$$\psi(t_2, x^i) - \psi(t_1, x^i) = -\ln \left[\frac{a(t_2)}{a(t_1)} \right] - \frac{1}{3} \int_{\rho(t_1, x^i)}^{\rho(t_2, x^i)} \frac{d\rho}{\rho + p}$$



$$-\zeta(x^i) = \psi(t, x^i) + \frac{\delta\rho}{3(\rho + p)}$$

Macroscopic quantum states of matter I

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

- I Classical background fields correspond to expectation values on macroscopic (condensed) states**

$$\phi(x) := \langle \alpha | \hat{\phi} | \alpha \rangle$$

- II Matter perturbations are evaluated as the the first order expansion of the expectation values on perturbed macroscopic states**

$$\delta\phi(x) := \langle \alpha + \delta\alpha | \hat{\phi} | \alpha + \delta\alpha \rangle |_{O(\delta\alpha)}$$

Macroscopic quantum states of matter II

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

III Density matrix and infrared mode of the macroscopic state

$$\rho_{1-p}(x - x') = \int_{k,k'} e^{-i(kx - k'x')} \langle a_k^\dagger a_{k'} \rangle$$

$$\rho_{1-p}(t - t'; \vec{x} - \vec{x}') = \frac{N_0}{V} + \int_k e^{-i\vec{k} \cdot (\vec{x} - \vec{x}')} n(k)$$

II Off-diagonal long ranged order (ODLRO) and vanishing of correlations at large space-time distances

$$\lim_{||x - x'|| \rightarrow \infty} \rho_{1-p}(t - t'; \vec{x} - \vec{x}') = \langle \phi(x) \phi^\dagger(x') \rangle_0 \equiv n_0$$

Bosonic statistics and coherent states

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$\phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2E_k}} \left(a_k e^{-ikx} + a_k^\dagger e^{+ikx} \right) \quad \text{Scalar field}$$

Bosonic Hilbert space and infinite occupation numbers

$$|\alpha\rangle \equiv \prod_k |\alpha(k)\rangle = \prod_k e^{\alpha(k)a_k^\dagger - \alpha^*(k)a_k} |0\rangle = D(\alpha) |0\rangle$$

Displacement operator

$$D(\alpha)^\dagger \phi(x) D(\alpha) = \phi(x) + \phi_\alpha(x)$$

$$\langle\alpha| \mathcal{O}(\phi(x)) |\alpha\rangle = \langle 0| \mathcal{O}(\phi(x) + \phi_\alpha(x)) |0\rangle$$

Matter perturbations at linear order

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$\delta G_{\mu\nu} = \frac{8\pi G}{c^4} \langle \alpha + \delta\alpha | \widehat{T_{\mu\nu}(\phi)} | \alpha + \delta\alpha \rangle \Big|_{\mathcal{O}(\delta\alpha)}$$

Expanding perturbations in the conservation equation

$$3(\zeta + \psi) \langle \alpha | \hat{\rho} + \hat{p} | \alpha \rangle = - \langle \alpha + \delta\alpha | \hat{\rho} | \alpha + \delta\alpha \rangle \Big|_{\mathcal{O}(\delta\alpha)}$$

Example: Chaotic Inflation

$$\langle \alpha + \delta\alpha | \hat{\rho} | \alpha + \delta\alpha \rangle = \lim_{x \rightarrow y} \frac{1}{2} m^2 \langle \alpha + \delta\alpha | \hat{\phi}(x) \hat{\phi}(y) | \alpha + \delta\alpha \rangle = \frac{1}{2} m^2 [\phi_{\alpha+\delta\alpha}(x)]^2$$

Power spectrum of scalar perturbations

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

$$-\hat{\Xi} = \hat{\mathbb{1}} \psi(t, x^i) + \frac{\hat{\rho}}{3\langle \alpha | \hat{\rho} + \hat{p} | \alpha \rangle}$$

Slow-roll condition

$$\langle \alpha | \ddot{\phi} + 3H\dot{\phi} + \widehat{V'(\phi)} | \alpha \rangle = 0 \quad \longrightarrow \quad 3H\phi_\alpha \simeq -V(\phi_\alpha)$$

Power spectrum

$$\mathcal{P}_\zeta = \lim_{x \rightarrow y} \langle \alpha + \delta\alpha | \hat{\Xi}(x) \hat{\Xi}(y) | \alpha + \delta\alpha \rangle \Big|_{O(\delta\alpha^2)}$$

Fermion fields and linear perturbations

Alexander, Brandenberger, Calcagni, Hui, Nicolis, Piazza, Prokopec, Sasaki, etc...

I Pressure perturbations (non adiabatic) and conservation of curvature perturbations

$$\dot{\zeta} = -\frac{H}{\rho + p} \delta p_{\text{na}}$$

II A no-go argument:

$$\delta\phi \rightarrow \delta(\bar{\psi}\psi) = \delta\bar{\psi}\psi + \bar{\psi}\delta\psi$$

$$\psi(t) = \langle\alpha|\hat{\psi}|\alpha\rangle = \langle\alpha|R^\dagger(\varphi)R(\varphi)\hat{\psi}R^\dagger(\varphi)R(\varphi)|\alpha\rangle|_{\varphi=2\pi} = -\psi(t)$$

Fermion fields & macroscopic coherent states I

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Pauli exclusion principle and quasi particles

$$a_k^\dagger \rightarrow c_k^\dagger = a_{k\uparrow}^\dagger b_{-k\downarrow}^\dagger$$

BCS states as macroscopic coherent states

$$|\alpha\rangle \equiv e^{\int d^3k \alpha(k) c_k^\dagger - \alpha^*(k) c_k} |0\rangle = D(\alpha) |0\rangle$$

$$J_1 = \frac{1}{2} (a^\dagger b^\dagger + h.c.), \quad J_2 = -\frac{i}{2} (a^\dagger b^\dagger - h.c.), \quad J_3 = \frac{1}{2} (a^\dagger a + b^\dagger b - 1), \quad [J_i, J_j] = i\epsilon_{ij}^k J_k$$

BCS states are SU(2) coherent states

$$|\hat{n}\rangle = D(\hat{n}) |j, -j\rangle = |\xi\rangle = \exp(\xi J^+ - \bar{\xi} J^-) |j, -j\rangle$$

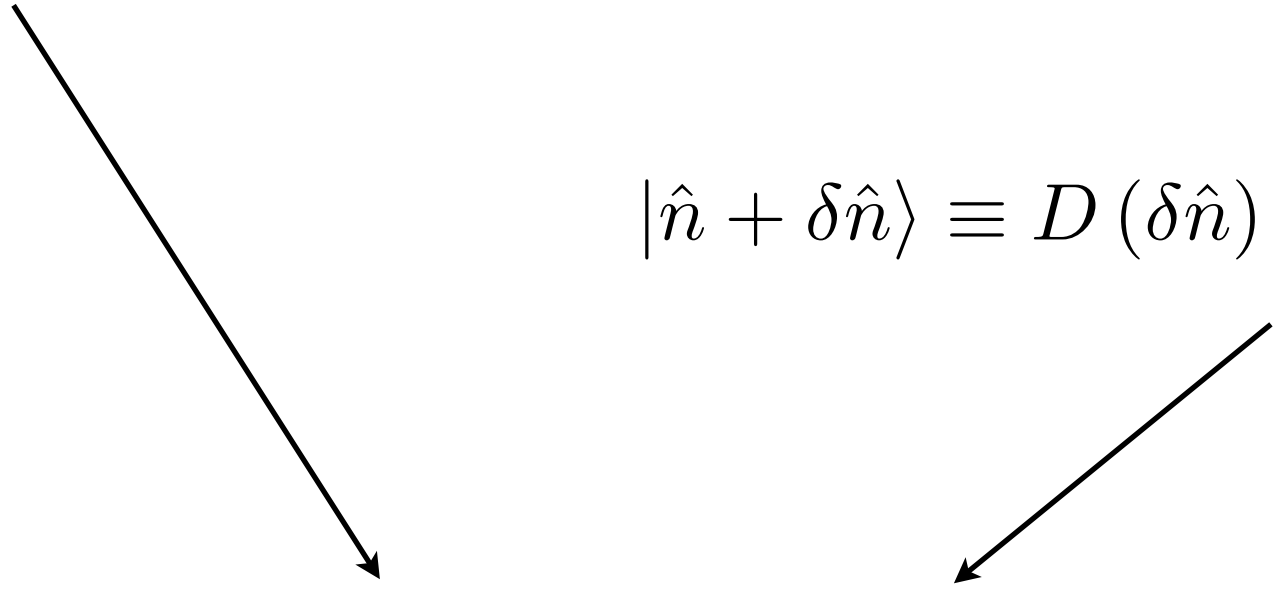
Fermion fields & macroscopic coherent states II

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Linear perturbations and SU(2) rotations

$$\langle \hat{n} | \bar{\psi} \psi | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \langle \hat{n} | \vec{J}_k | \hat{n} \rangle = \int_k \vec{\zeta}_k \cdot \hat{n}_k$$

$$|\hat{n} + \delta\hat{n}\rangle \equiv D(\delta\hat{n}) |\hat{n}\rangle = |R(\hat{z}, \delta\hat{n}) \hat{n}\rangle$$


$$\langle \hat{n} + \delta\hat{n} | \vec{J} | \hat{n} + \delta\hat{n} \rangle \approx \hat{n} + \delta\hat{n} \times \hat{n} = \hat{n} - \hat{n} \times \langle \delta\hat{n} | \vec{J} | \delta\hat{n} \rangle$$

Bogolubov transformations & non-BD states

Dona & Marciano, arXiv:1605.09337 (PRD 2016)

Adjoint action of displacement operators

$$D(\alpha)^\dagger a_k D(\alpha) = a_k + \alpha(k)$$

$$D(\alpha)^\dagger a_k^\dagger D(\alpha) = a_k^\dagger + \alpha^*(k)$$

U(1) bosonic case

$$\tilde{a} = \cos(|\xi|) a + \frac{\xi}{|\xi|} \sin(|\xi|) b^\dagger$$

$$\tilde{b}^\dagger = \cos(|\xi|) a - \frac{\xi}{|\xi|} \sin(|\xi|) b^\dagger$$

SU(2) fermionic case

The macroscopic state obtained is the Bogolubov transform of the vacuum

Gravity with non-dynamical torsion I

Rovelli & Perez, CQG 2005; Freidel & Minic, PRD 2005

$$\mathcal{S}_{\text{Holst}} = \frac{1}{2\kappa} \int_M d^4x |e| e_I^\mu e_J^\nu P^{IJ}_{KL} F_{\mu\nu}{}^{KL}(\omega)$$
$$P^{IJ}_{KL} = \delta_K^{[I} \delta_L^{J]} - \epsilon^{IJ}_{KL} / (2\gamma)$$
$$\mathcal{S}_{\text{Dirac}} = \int_M d^4x |e| \left\{ \frac{1}{2} \left[\bar{\psi} \gamma^I e_I^\mu \left(1 - \frac{i}{\alpha} \gamma_5 \right) i \nabla_\mu \psi - m \bar{\psi} \psi \right] + \text{h.c.} \right\}$$

Theory with torsion!

[Alexander, Biswas, Cai, Magueijo, Prokopec, Kibble, Poplawski, Qiu...]

Gravity with non-dynamical torsion II

S.Alexander,Y. Cai & A. Marciano PLB 2015

Theory with torsion

$$e_I^\mu C_{\mu JK} = \frac{\kappa}{4} \frac{\gamma}{\gamma^2 + 1} (\beta \epsilon_{IJKL} J^L - 2\theta \eta_{I[J} J_{K]})$$
$$J^L = \bar{\psi} \gamma^L \gamma_5 \psi$$

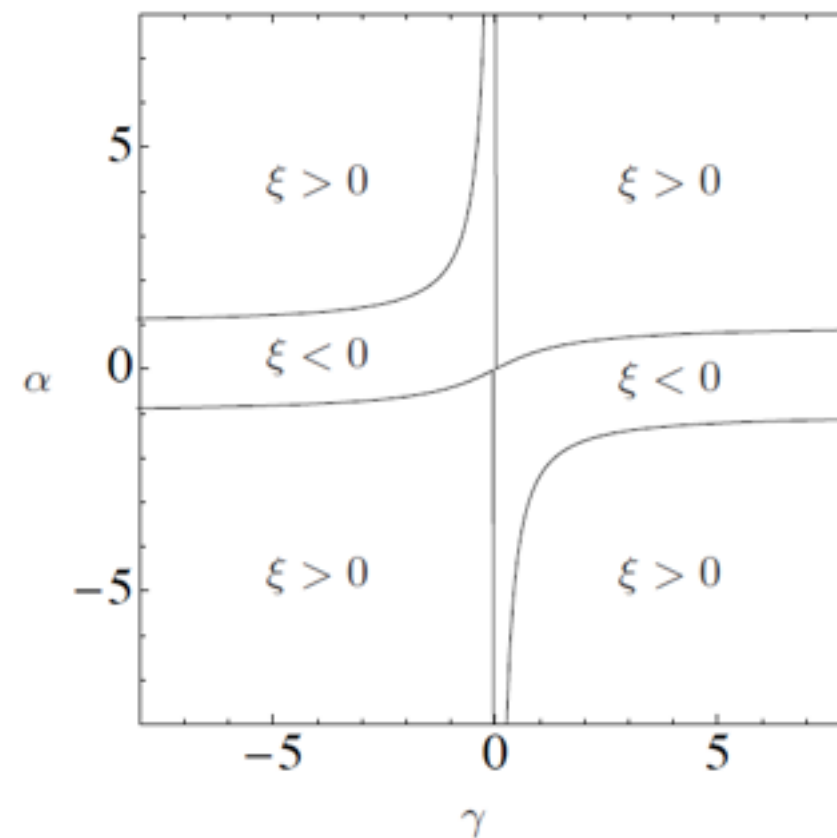
$$\mathcal{S}_{GR} = \frac{1}{2\kappa} \int_M d^4x |e| e_I^\mu e_J^\nu R_{\mu\nu}^{IJ}$$
$$\mathcal{S}_{\text{Dirac}} = \frac{1}{2} \int_M d^4x |e| \left(\bar{\psi} \gamma^I e_I^\mu i \tilde{\nabla}_\mu \psi - m \bar{\psi} \psi \right) + \text{h.c.}$$
$$\mathcal{S}_{\text{Int}} = -\xi \kappa \int_M d^4x |e| J^L J^M \eta_{LM}$$

Gravity with non-dynamical torsion III

S.Alexander, C. Bambi, A. Marciano & L. Modesto, [arXiv:1402.5880] PRD 90 (2014) 123510

A.Addazi, S.Alexander, Y. Cai & A. Marciano, arXiv: 1712.04848 (CPC 2018)

A.Addazi & A. Marciano, arXiv:1810.05513



$$\xi = \frac{3}{16} \frac{\gamma^2}{\gamma^2 + 1} \left(1 + \frac{2}{\alpha\gamma} - \frac{1}{\alpha^2} \right)$$

NJL mechanism applied to SM fermions

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

$$\int \sqrt{-g} d^4x \bar{\Psi} (\not{\partial} \gamma^\mu(x) \nabla_\mu - M) \Psi + \frac{\lambda}{2N_f} [(\bar{\Psi}\Psi)(\bar{\Psi}\Psi) + (\bar{\Psi}\not{\gamma}_5\Psi)(\bar{\Psi}\not{\gamma}_5\Psi)]$$

N_f number of fermions of the SM



$$\lambda = \xi \kappa$$

$$\int \sqrt{-g} d^4x [\bar{\Psi} \not{\partial} \gamma^\mu(x) \nabla_\mu \Psi - \frac{N_f}{2\lambda} (|\Pi|^2 + |\Sigma|^2) - \bar{\Psi} (\Sigma + \not{\gamma}_5 \Pi) \Psi]$$

Effective potential I

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

Integrating out fermionic DOF

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left\{ -\frac{1}{\lambda} (|\Pi|^2 + |\Sigma|^2) - i \ln \text{Det} \{ i I \gamma^\mu(x) \nabla_\mu - (\Sigma + i \gamma_5 \Pi) \} \right\}$$

with negligible corrections controlled by the number of fermions

$$S_{\text{eff}}[\Pi, \Sigma] + O(1/N_f)$$



$$V(\Pi, \Sigma) = \frac{1}{2\lambda} (|\Pi|^2 + |\Sigma|^2) + i \text{Tr} \ln \langle x | i \gamma^\mu(x) I \nabla_\mu - (\Sigma + i \gamma_5 \Pi) | z \rangle$$

Effective matrix potential

Effective potential II

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

Π, Σ classical slowly varying fields

Introduce $A = \Sigma + i\gamma_5\Pi$ and estimate the trace within the proper time method

$$V = \frac{1}{2\lambda}(|\Pi|^2 + |\Sigma|^2) - i\text{Tr} \ln S(x, x, A)$$
$$S(x, y; A) = \langle x | (iI\gamma^\mu \nabla_\mu - A)^{-1} | y \rangle$$

where the propagator is associated to the classical matrix equation

$$(iI\gamma^\mu(x)\nabla_\mu - A)S(x, y; A) = I \frac{1}{\sqrt{-g(x)}} \delta^4(x - y)$$

Effective potential III

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

To calculate the propagator, one performed the background expansion $A = \bar{A} + \delta A$

$$\begin{aligned}\ln \text{Det} \{iI\gamma^\mu(x)\nabla_\mu - A\} &= \text{Tr} \ln \{iI\gamma^\mu\nabla_\mu - A\} \\ &= \text{Tr} \ln \{iI\gamma^\mu(x)\nabla_\mu - A\} - \int d^4x \text{Tr} \{\delta A(x) S_F(x, x)\} \\ &\quad - \frac{1}{2} \int d^4x \int d^4y \delta A(x) S_F(x, y) \delta A(y) S_F(y, x) + \dots\end{aligned}$$

with fermion propagator

$$\sqrt{-g}(iI\gamma^\mu(x)\nabla_\mu - M)S_F(x, y) = i\delta^4(x - y)I$$

Effective potential IV

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

In the large N one can calculate the bubble diagram x into x

$$S(x, x; A) = \int \frac{d^4 q}{(2\pi)^4} \left[(I\gamma^a q_a + A) \frac{1}{q^2 - |A|^2} - \frac{1}{12} R (I\gamma^a q_a + A) \frac{1}{(q^2 - |A|^2)^2} + \frac{2}{3} R_{\mu\nu} q^\mu q^\nu (I\gamma^a q_a + A) \frac{1}{(q^2 - |A|^2)^3} - \frac{1}{8} \gamma^a [\gamma^c, \gamma^d] R_{cda\mu} q^\mu \frac{1}{(q^2 - |A|^2)^2} \right]$$

within the weakly varying curvature approximation $\dot{R} \simeq 0$

$$V(A) = \tilde{V}(A) - \frac{1}{(4\pi)^2} \frac{R}{6} \left[-|A|^2 \ln \left(1 + \frac{\Lambda^2}{|A|^2} \right) + \frac{\Lambda^2 |A|^2}{\Lambda^2 + |A|^2} \right]$$

$$\tilde{V} = V_0 + \frac{1}{2\lambda} |A|^2$$

$$- \frac{1}{4\pi^2} \left[|A|^2 \Lambda^2 + \Lambda^4 \ln \left(1 + \frac{|A|^2}{\Lambda^2} \right) - |A|^4 \ln \left(1 + \frac{\Lambda^2}{|A|^2} \right) \right]$$

Inflaton from SM fermions

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

In FLRW, impose either a custodial global symmetry or a gauge flavor symmetry



$$V(a) = V_0 + \frac{1}{2\lambda}|a|^2 - \frac{1}{4\pi^2} \left[|a|^2 \Lambda^2 + \Lambda^4 \ln \left(1 + \frac{|a|^2}{\Lambda^2} \right) - |a|^4 \ln \left(1 + \frac{\Lambda^2}{|a|^2} \right) \right] - \frac{1}{(4\pi)^2} (\dot{H} + 2H^2) \left[-|a|^2 \ln \left(1 + \frac{\Lambda^2}{|a|^2} \right) + \frac{\Lambda^2 |a|^2}{\Lambda^2 + |a|^2} \right],$$

$$\Lambda^2 = c(\xi\kappa)^{-1}$$



$$\frac{\epsilon[a]}{M_{Pl}^2} = \frac{1}{2} \left(\frac{V'[a]}{V[a]} \right)^2, \quad \frac{\eta[a]}{M_{Pl}^2} = \frac{V''[a]}{V[a]}$$

Consistency with data on CMBR

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

$$\Delta_R^2 \simeq \frac{V_0}{24\pi^2 M_{Pl}^4 \epsilon} \quad \Delta_{R, \text{exp}}^2 = 2,215 \times 10^{-9}$$

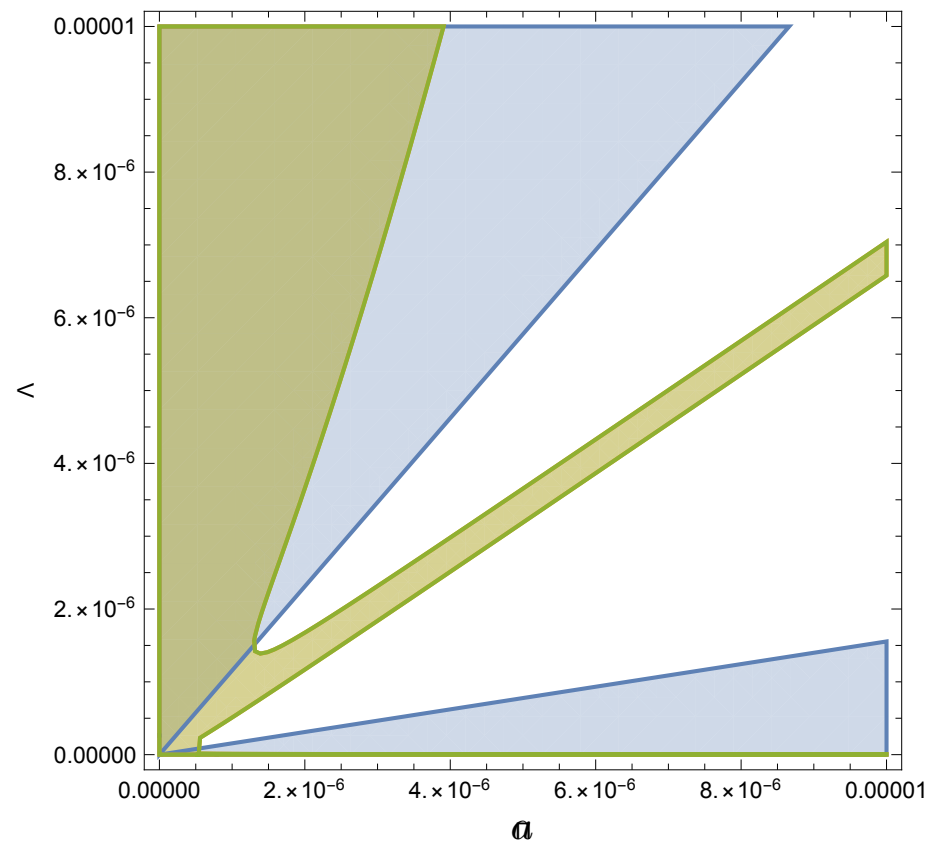
$$\begin{aligned} \epsilon &= 1/(2N/3 + 1)^{3/2} & \eta &= -1/(2N/3 + 1) \\ n_s - 1 &= -2\epsilon - \eta & n_{s, \text{exp}} &= 0.968 \pm 0.006 \end{aligned}$$

$$N = \frac{1}{M_{Pl}^2} \int_{\phi_{end}}^{\phi} d\phi \frac{V'}{V} \simeq 60 \quad \Lambda \simeq 2 \times 10^{15} \div 10^{16} \text{ GeV}$$

$$\epsilon \simeq 0.003 \quad |\eta| \simeq 0.02$$

Predictions on primordial tensor spectrum

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848



$$\lambda^{-1} = \frac{\Lambda^2}{2\pi^2}$$

$$\Lambda \simeq 2 \times 10^{15} \div 10^{16} \text{ GeV}$$



$$\mathcal{P}_T = \frac{2H_{inf}^2}{\pi^2 M_{Pl}^2} \simeq \frac{2V_0}{3\pi M_{Pl}^4} \sim 10^{-13} \div 10^{-11} \longrightarrow r = 10^{-4} \div 10^{-2}$$

Critical branches

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

There exist two critical branches for compatibility with data

I) $\lambda^{-1} = \frac{\Lambda^2}{2\pi^2}$

$$\Lambda \simeq 2 \times 10^{15} \div 10^{16} \text{ GeV} \quad V_0 = \Lambda^4$$

II) $\lambda^{-1} = \frac{\Lambda^2}{\pi^2}$

$$|a| \ll \Lambda$$

Reheating mechanism and graceful exit

A. Addazi, P. Chen & A. Marciano, arXiv:1712.04848

For the second critical branch, perturbative reheating can be easily achieved

$|a| \ll \Lambda$

Quartic term

$$V(a) = V_0 + \frac{1}{2\lambda}|a|^2 - \frac{1}{4\pi^2} \left[|a|^2 \Lambda^2 + \Lambda^4 \ln \left(1 + \frac{|a|^2}{\Lambda^2} \right) - |a|^4 \ln \left(1 + \frac{\Lambda^2}{|a|^2} \right) \right]$$
$$- \frac{1}{(4\pi)^2} (\dot{H} + 2H^2) \left[-|a|^2 \ln \left(1 + \frac{\Lambda^2}{|a|^2} \right) + \frac{\Lambda^2 |a|^2}{\Lambda^2 + |a|^2} \right],$$

Quadratic term

$\Lambda^2 = c(\xi\kappa)^{-1}$

The model converge to the form of the Coleman-Weinberg potential for $|a| \ll \Lambda$

Graceful exit mechanism from inflation, with a reliable re-heating mechanism

. A. Cerioni, F. Finelli, A. Tronconi and G. Venturi, Phys. Lett. B 2009

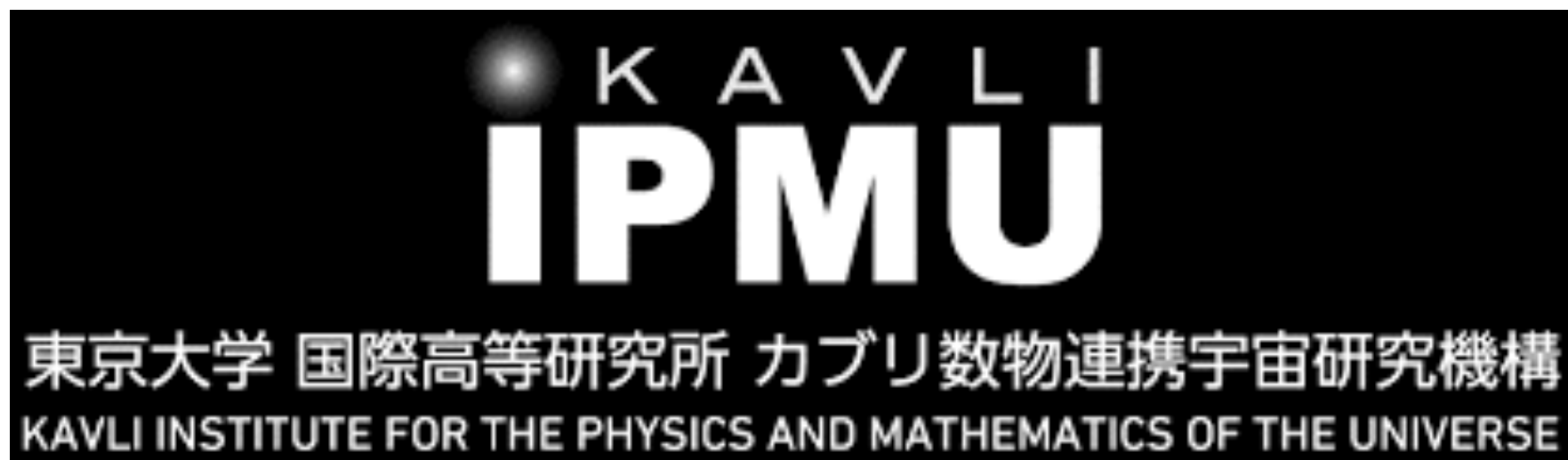
. G. Barenboim, E. J. Chun and H. M. Lee, Phys. Lett. B 2014

Conclusions

- i) Fermionic matter cosmological perturbations*
- ii) Spinorial perturbations entail non-isotropic d.o.f. not present in the scalar field perturbations*
- iii) Spinorial contributions can be recast in terms of a multi-field approach in inflationary scenarios*
- iv) Inflation can be sourced by an infrared collective mode generated by condensation of SM fermions*
- v) Constraints are satisfied, but phenomenology is reached*
 - v) A falsifiable prediction for r can be provided*

ありがとう

Thank you!



Grazie!

谢谢