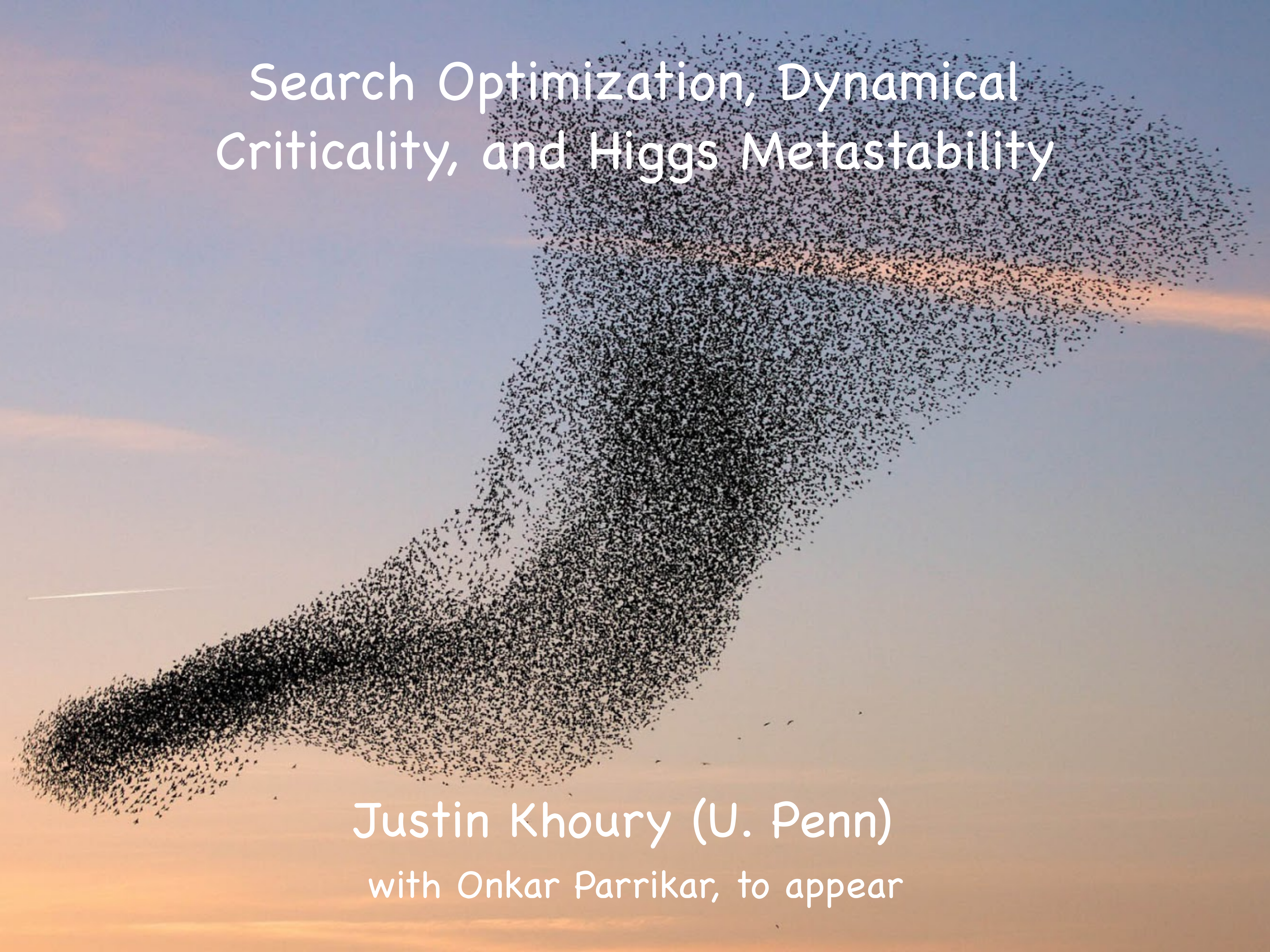


Search Optimization, Dynamical Criticality, and Higgs Metastability

Justin Khoury (U. Penn)

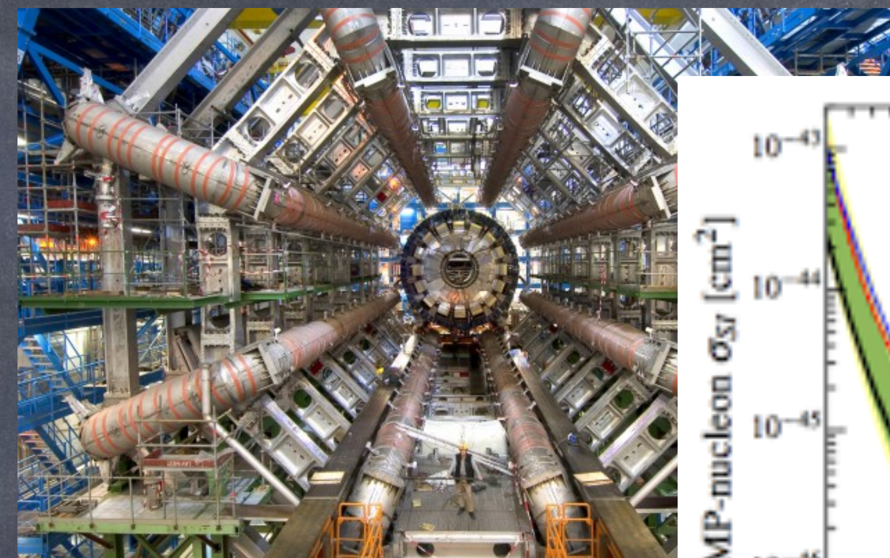
with Onkar Parrikar, to appear

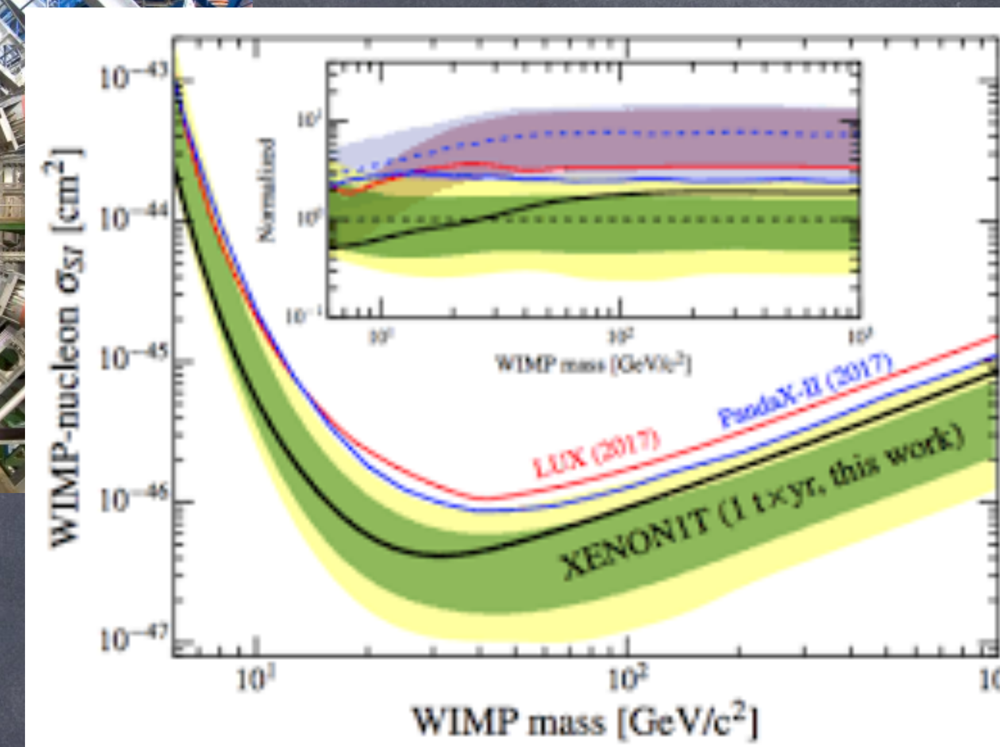
The Post-Naturalness Era?

- What stabilizes the Higgs mass (hierarchy problem)?

- No new physics at LHC

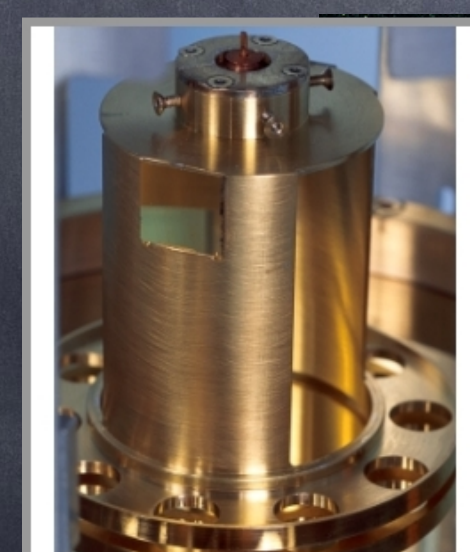
- No WIMPs detected





- What explains the vacuum energy (cosmological constant problem)?

- Constraints on deviations from GR are increasingly tight

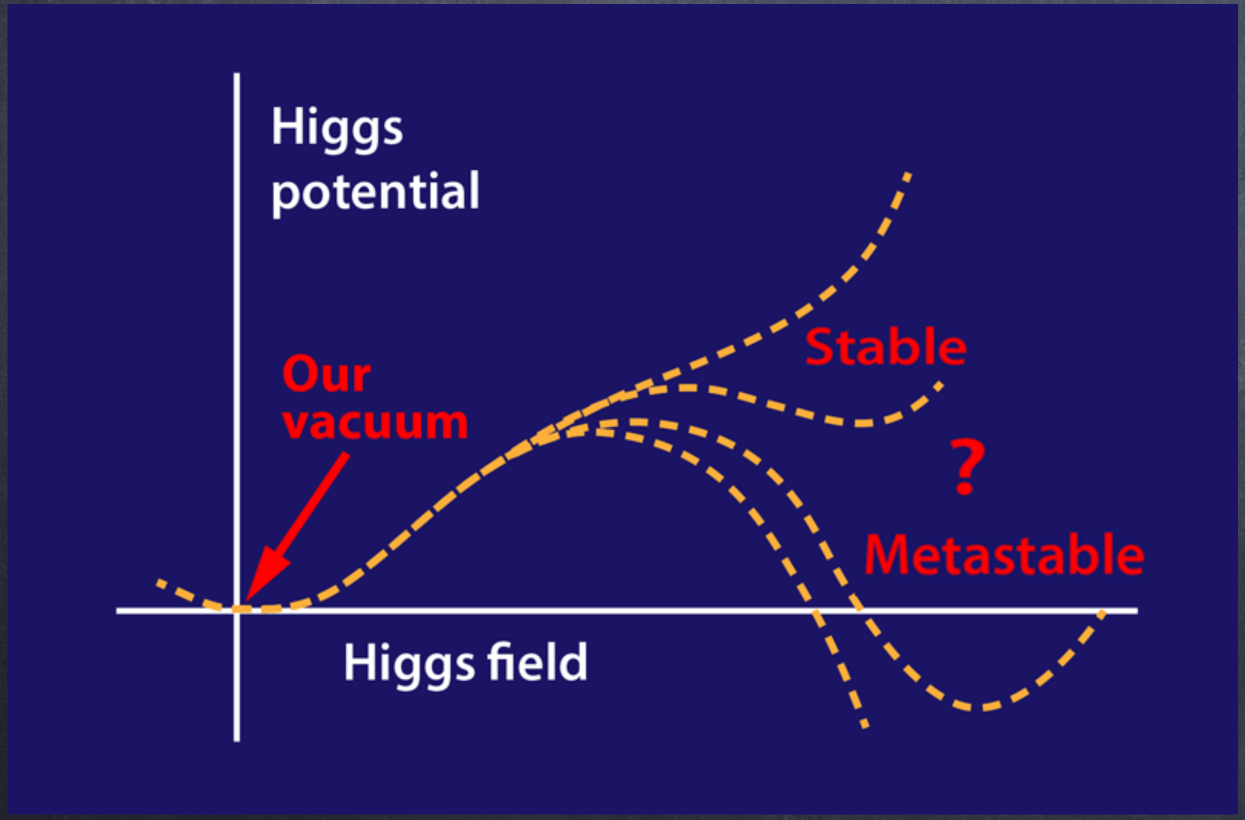


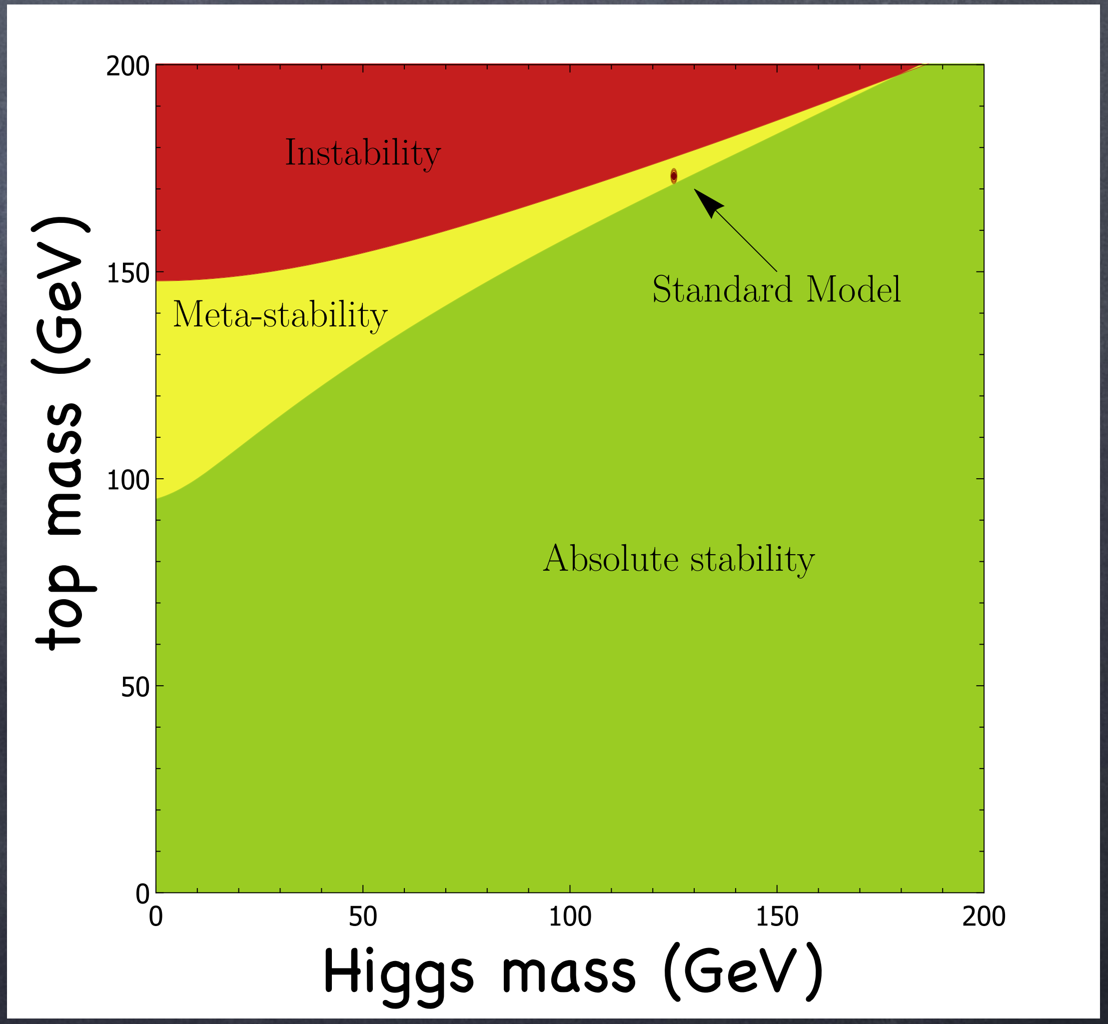
Higgs metastability

Frampton (1976); Sher (1989); Degrassi et al. (2012); Buttazzo et al. (2013); Bednyakov et al. (2015); Andreassen, Frost and Schwartz (2017)

- A disturbing consequence of a "grand desert" above the weak scale is the metastability of our vacuum

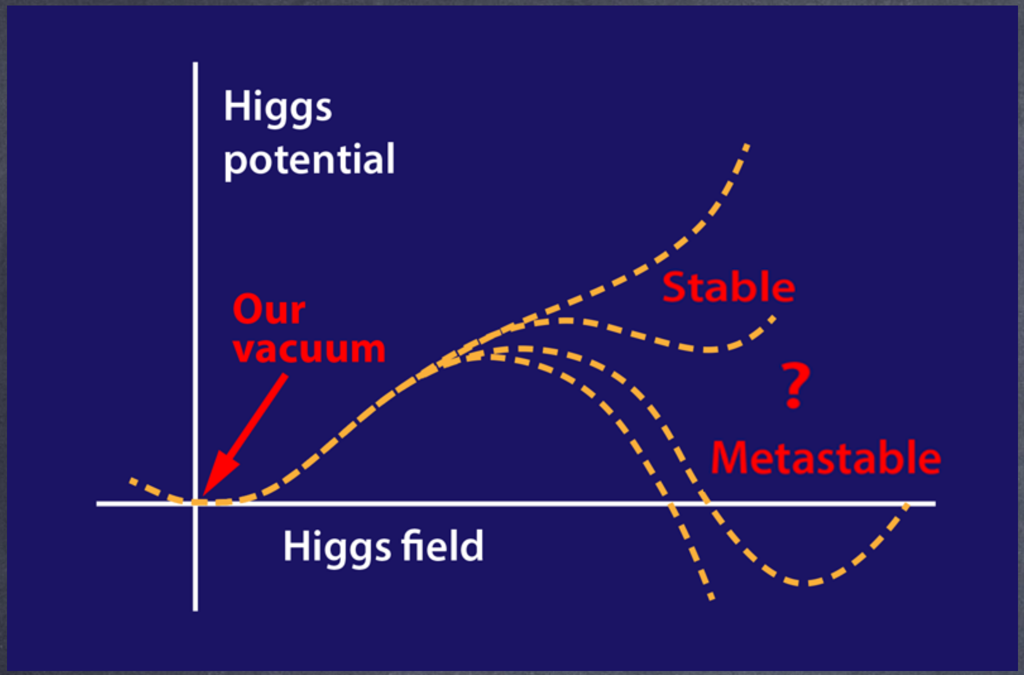
- Higgs discovery with $m_h \simeq 125 \text{ GeV}$ fixes all SM parameters, and allows computation of quantum effective potential





Andreassen, Frost and Schwartz (2017)

Higgs metastability as near criticality



Nucleation prob. within observable universe:

$$\kappa \equiv \frac{\Gamma_{\rm decay}}{H_0^4}$$

Percolation transition:

$$10^{-6} \lesssim \kappa_c \lesssim 0.24$$

Guth & Weinberg (1983)

Estimated lifetime of our vacuum:

$$\tau = 10^{526^{+409}_{-202}} \text{ years}$$

Andreassen et al. (2017)

Reassuringly long, but hinges on delicate numerical cancellation:

$$P_{\rm decay} \simeq \frac{\mu_*^4}{H_0^4} \exp\left(-\frac{8\pi^2}{3|\lambda(\mu_*)|}\right) \qquad \mu_* \simeq 3\times 10^{17}\ \text{GeV}$$

- This delicate numerical conspiracy cannot be an accident
- Difficult to conjure up an anthropic explanation

Why is our universe so precariously close to instability?

Other fine-tunings can be understood as problems of criticality.

- Weak hierarchy:

Giudice & Rattazzi (2006)

$$-M_{\mathrm{Pl}}^2 \lesssim m_h^2 \lesssim M_{\mathrm{Pl}}^2$$

$m_h^2 \simeq M_{\mathrm{Pl}}^2$
$\langle h \rangle = 0$ *

$m_h^2 \simeq 0$
$\langle h \rangle = 246\ \mathrm{GeV}$

$m_h^2 \simeq -M_{\mathrm{Pl}}^2$
$\langle h \rangle \sim M_{\mathrm{Pl}}$

* In SM, electroweak still broken at QCD scale by Higgs coupling to quark condensate.

- Cosmological constant:

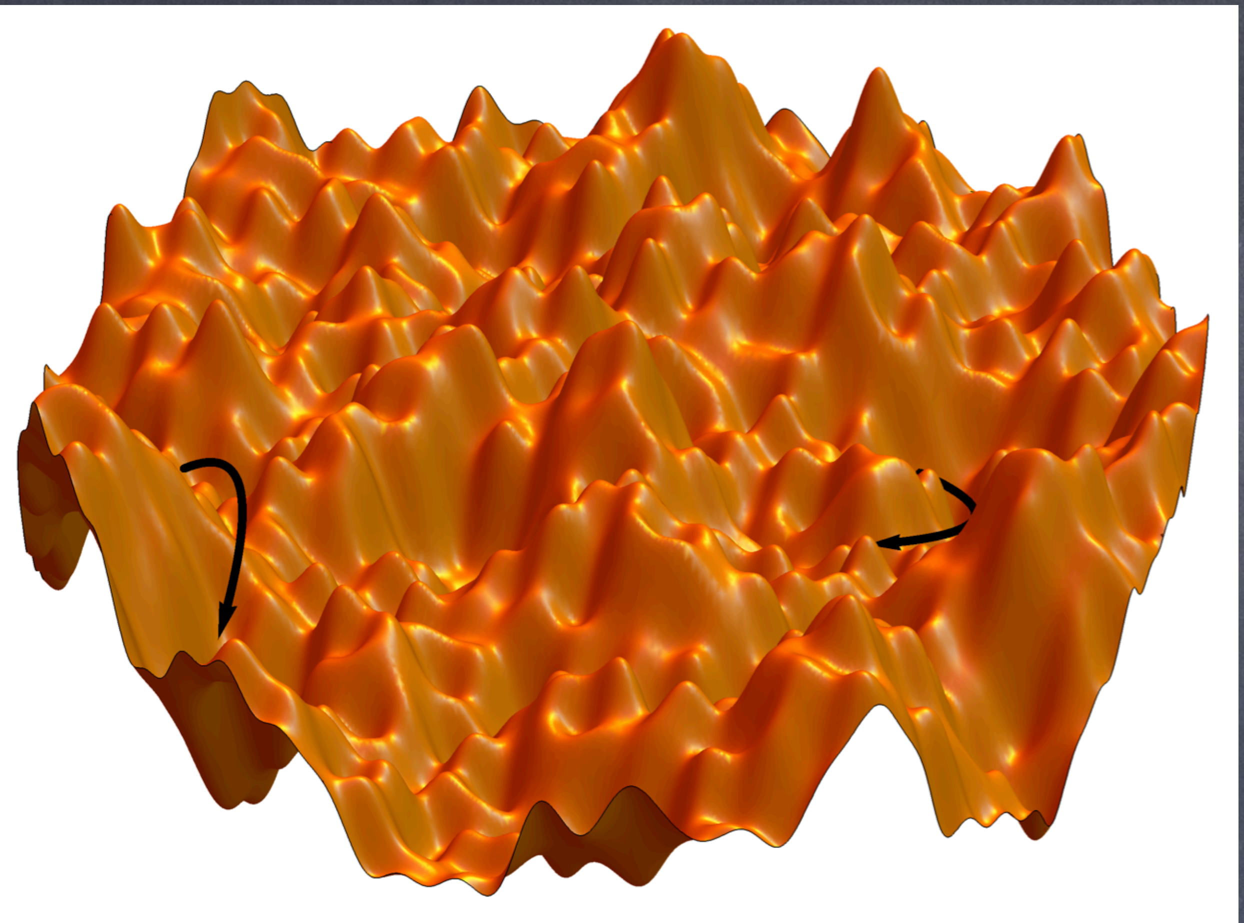
$\Lambda > 0$
stable

$\Lambda = 0$

$\Lambda < 0$
Non-linearly unstable

Landscape approach

Landscape approach



- Physical parameters vary across a vast landscape of metastable vacua
- Observed values are environmentally determined

Usual strategy:

Garriga & Vilenkin (1998)

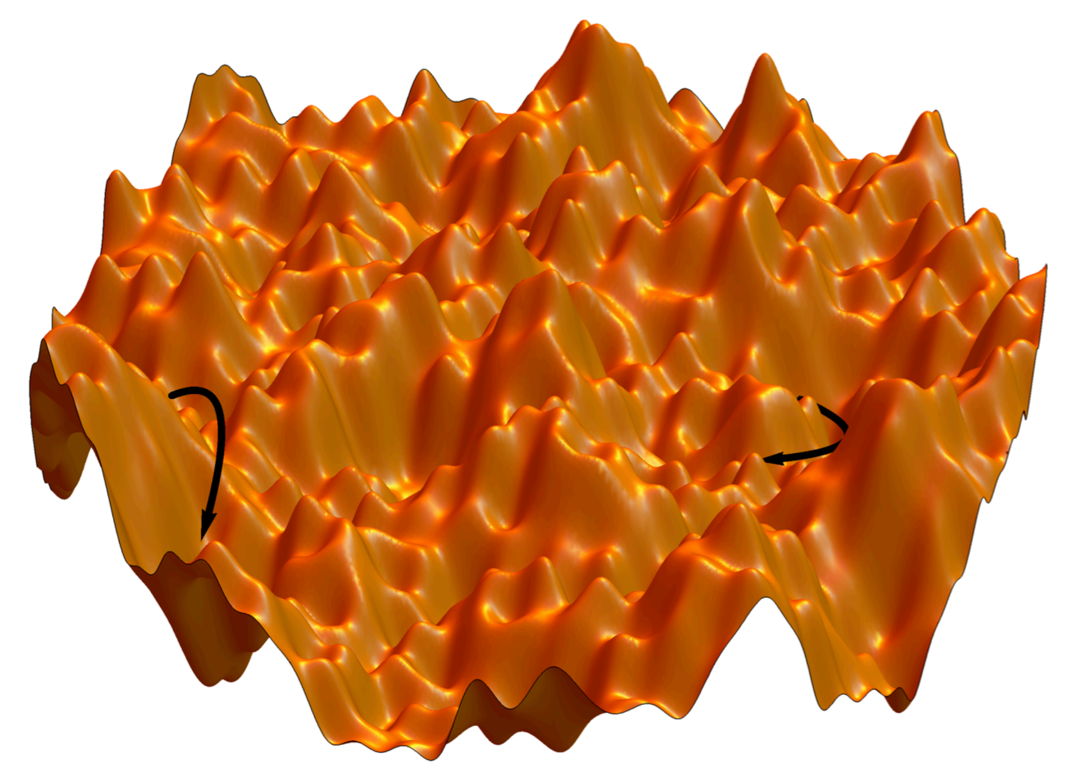
- Focus on late-time, stationary/equilibrium distribution

$f_i^\infty \equiv$ fraction of comoving volume occupied by $i^{\rm th}$ vacuum

- Hope: among all hospitable vacua, our vacuum is typical/generic

(Principle of mediocrity)

Challenges with the usual approach



- Many long-lived vacua

$\Longrightarrow$ exponentially long relaxation time

$\Longrightarrow$ frustration, aging dynamics

(glassy system)

Relatedly, finding vacua within hospitable range of Λ is NP-hard

Denef & Douglas (2007)

- Bubbles of all types are generated ∞-many times as $t \to \infty$

$\Longrightarrow$ Predictions depend on choice of time variable

$\Longrightarrow$ Predictions also depend on comoving vs physical volume

(Measure problem)

Long-standing problem

Linde & Mezhlumian (1993)
Garcia-Bellido, Linde & Linde (1994)
Garriga & Vilenkin (1998)
Garriga et al. (2006)
Vanchurin & Vilenkin (2006)
Bousso (2006)
Bousso, Freivogel & Yang (2009)...

Instead of focusing on equilibrium distributions, in this talk we will study the approach to equilibrium

Denef, Douglas, Greene & Zukowski (2018):

Suppose that the multiverse has existed for a time much shorter than the exponentially-long mixing time ($t \ll t_{\text{relax}}$).

Instead of asking:

`What hospitable vacua occur most frequently according to late-time equilibrium distribution?`

...the question becomes:

`What hospitable vacua have the right properties to be easily accessed early on?`

With Onkar Parrikar, to appear

This suggests a natural selection mechanism, which selects vacua at criticality.

Near-criticality of our universe ⟷ Non-equilibrium phase transitions in landscape dynamics

Consider a finite region of the landscape containing $N \gg 1$ vacua.

(Assume all dS vacua, and treat as closed system)

$f_i(t) \equiv$ fraction of comoving volume occupied by i^{th} vacuum

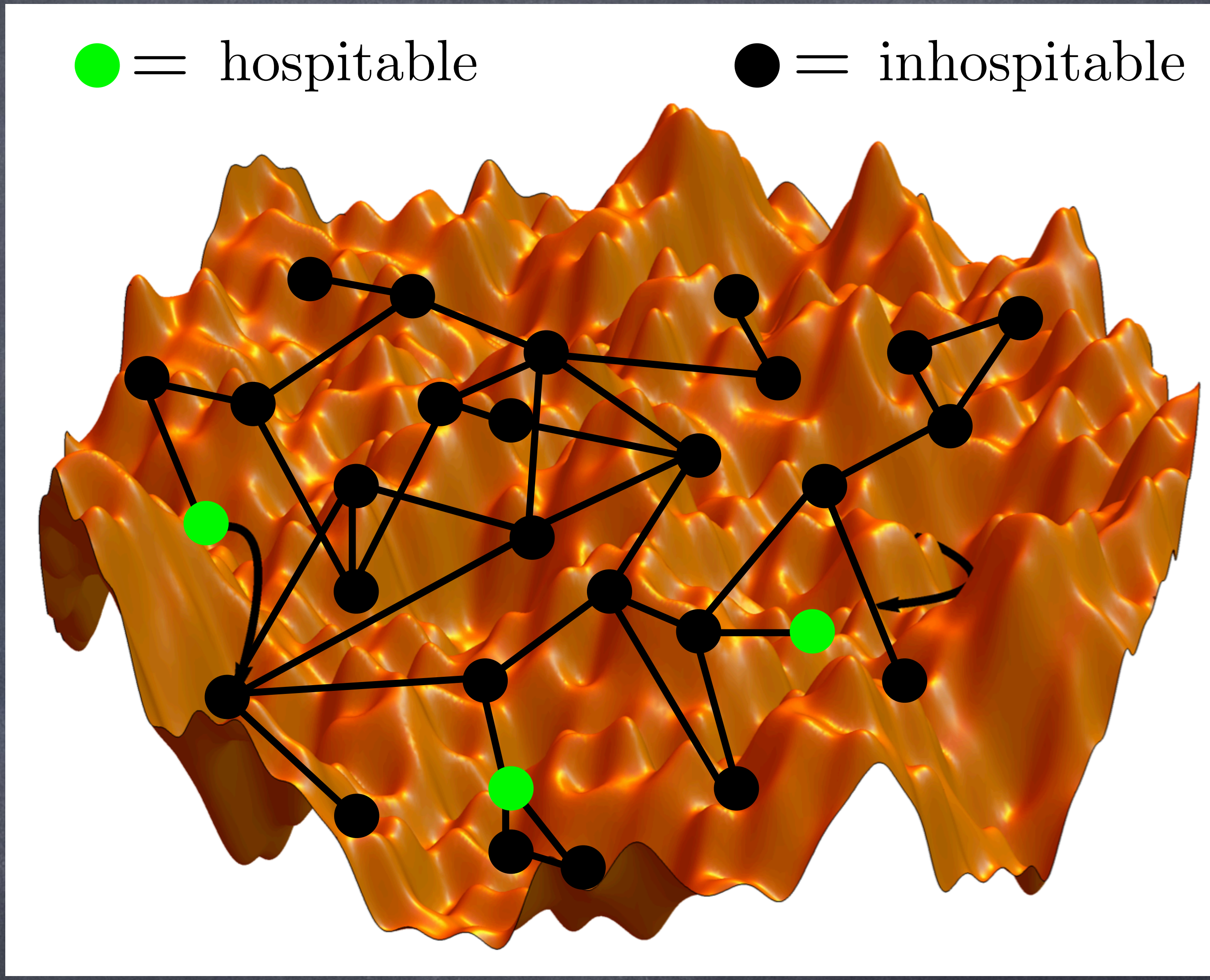
Volume is conserved: $\sum_{i=1}^{N} f_i(t) = 1$

$\kappa_{ij} \equiv$ transition rate for $j \to i$

Satisfies linear master equation for Markov process:

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = \sum_j (\kappa_{ij} f_j - \kappa_{ji} f_i)$$

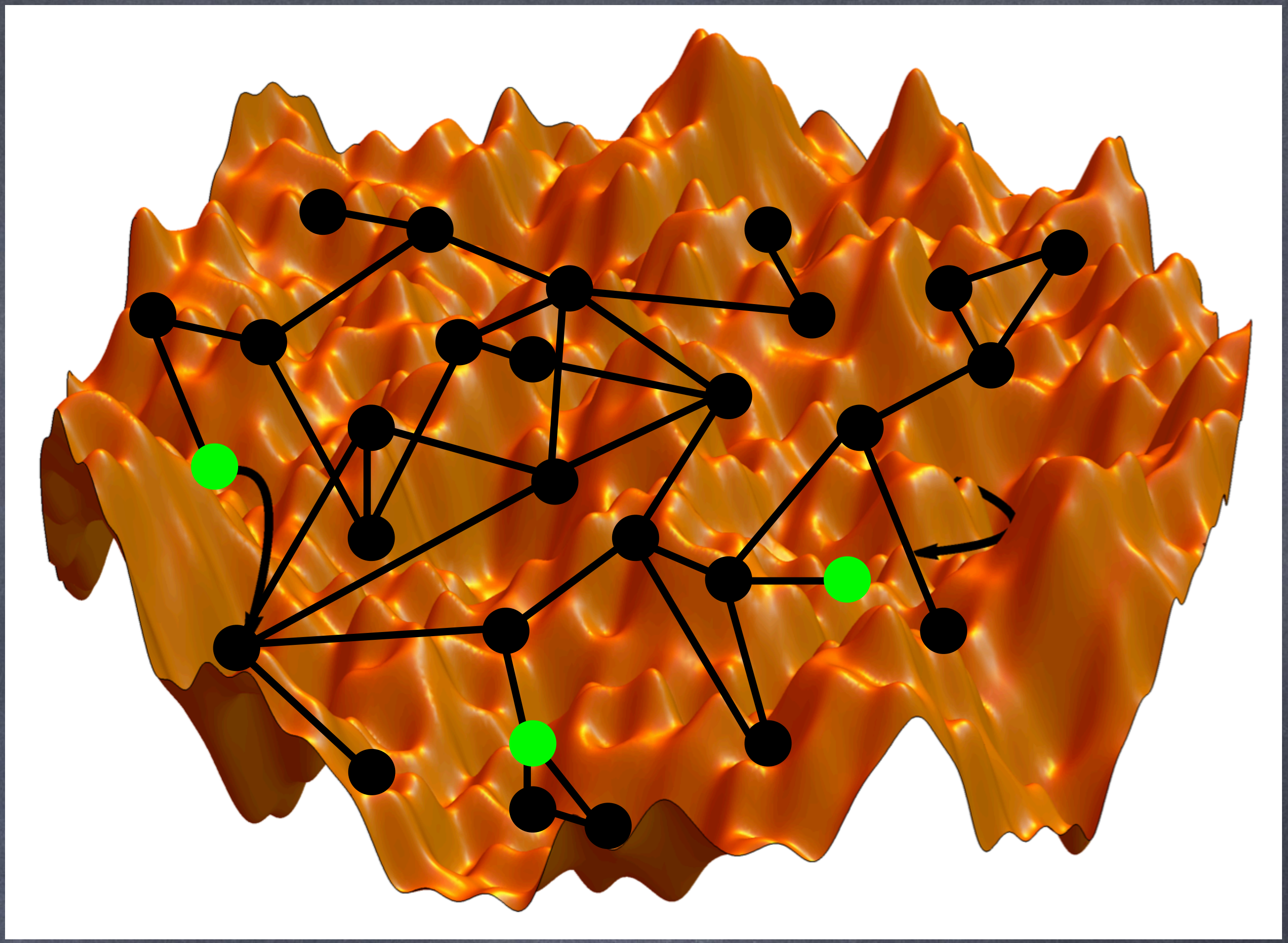
transitions into i transitions out of i



Garriga & Vilenkin (1998)

Landscape dynamics as random walk

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = \sum_j (\kappa_{ij} f_j - \kappa_{ji} f_i)$$



$\kappa_{ij} \equiv$ transition rate for $j \to i$

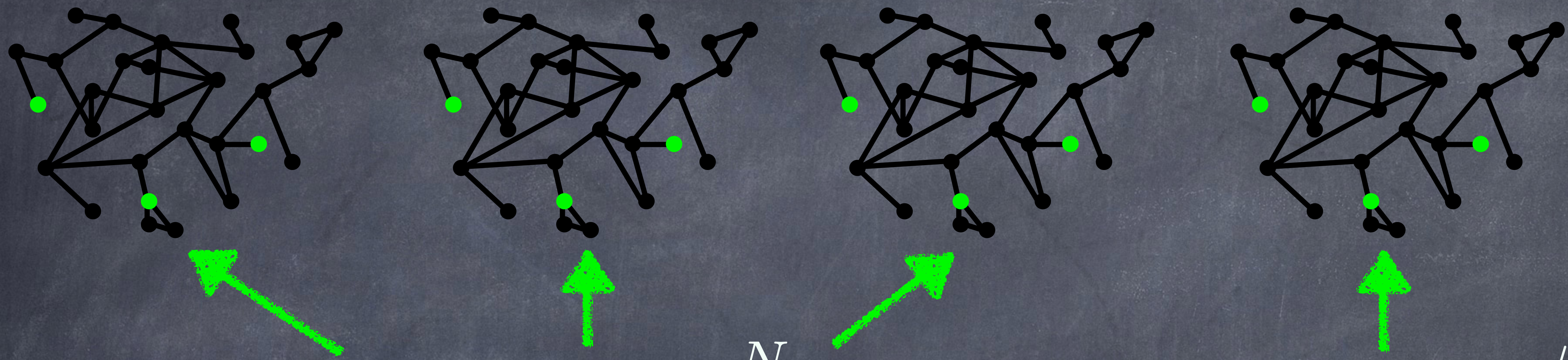
Alternative interpretation:

$f_i(t) \equiv$ probability that random walker is in i^{th} vacuum

Random walks on complex networks

- Biological networks
- Epidemiology
- Urban traffic
- World wide web
- Power grids
- ...

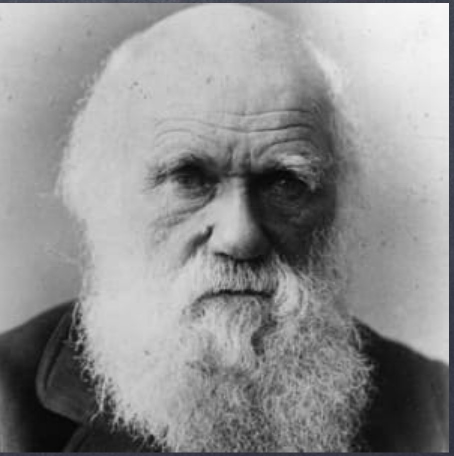
In the vastness of the landscape, imagine many replicas of the fiducial region, each with slight differences in transition rates, network topology etc.



$$t_{\text{generic}} \sim e^{N} \qquad\qquad t_{\text{optimal}} \sim N^{\#}$$

Textbook example of natural selection:

- Ensemble = gene pool; Each region = set of alleles
- “Genetic” make-up is heritable (cosmological expansion)
- Hospitable alleles compete for a finite resource (comoving volume)



Target alleles (i.e., hospitable vacua) best adapted to (i.e., easily accessed by) their environment (i.e., other vacua in the region) get naturally selected.

Naturally-selected hospitable vacua, far from being typical/mediocre, are exceptional and fine-tuned, much like complex organisms in the natural world are fine-tuned.

But they are fine-tuned for a purpose:

Hospitable vacua residing in optimal regions are exponentially more efficient at being accessed early on

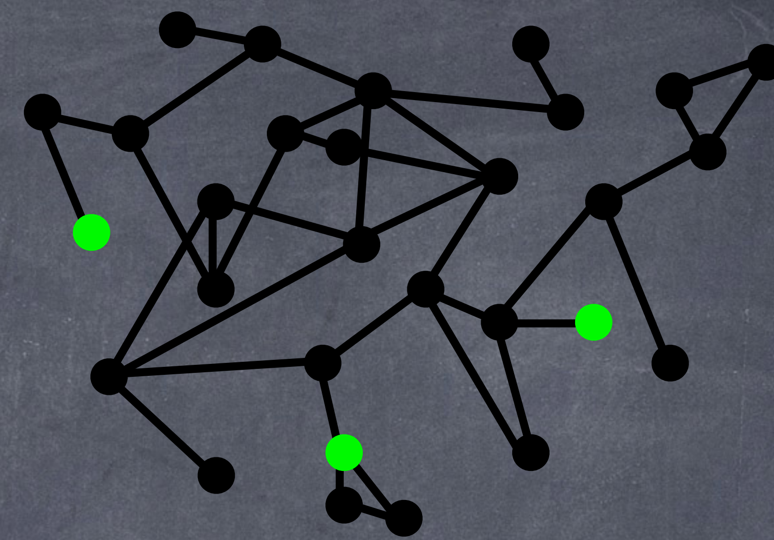
In Nature, striking relation between complexity and criticality.

(Criticality hypothesis)

Natural selection/ search optimization $\Longleftrightarrow$ Dynamical criticality

Importantly, optimality criteria (and phenomenological predictions) will be time-reparametrization invariant and independent of comoving vs physical volume.

Transition rates on the landscape



- Coleman-De Luccia: $$\kappa_{ij} = \frac{A_{ij}}{w_j}$$

adjacency matrix: $A_{ij} = A_{ji} = \left(M^4 \mathrm{e}^{-S_{\mathrm{bounce}}}\right)_{ij}$ Lee & Weinberg (1987)

weights: $w_j = H_j^3 \mathcal{N}_j^{-1} \mathrm{e}^{\frac{2\pi}{G H_j^2}}$ ← de Sitter entropy (Low-energy vacua exponentially weighted)

lapse function: $\mathrm{d}\tau_j = \mathcal{N}_j \mathrm{d}t$

proper time (→ $\mathrm{d}\tau_j$)   global time (→ $\mathrm{d}t$)

(In this talk, remain agnostic about choice of global time)

→ Random walk on weighted, undirected network   Zhang et al. (2013)

- Detailed balance: $$\frac{\kappa_{ji}}{\kappa_{ij}} = \frac{w_j}{w_i}$$

Lee & Weinberg (1987)

(Regional) equilibrium distribution

Master equation: $$\frac{\mathrm{d}f_i}{\mathrm{d}t} = \sum_j M_{ij} f_j$$

$$M_{ij} \equiv \kappa_{ij} - \delta_{ij} \sum_r \kappa_{rj}$$

Transition matrix

Perron-Frobenius theorem: M_{ij} has one vanishing eigenvalue

$$\lambda_1 = 0$$

All others are strictly negative:

$$0 > \lambda_2 \geq \ldots \geq \lambda_N$$

Zero-mode sets the stationary/equilibrium distribution:

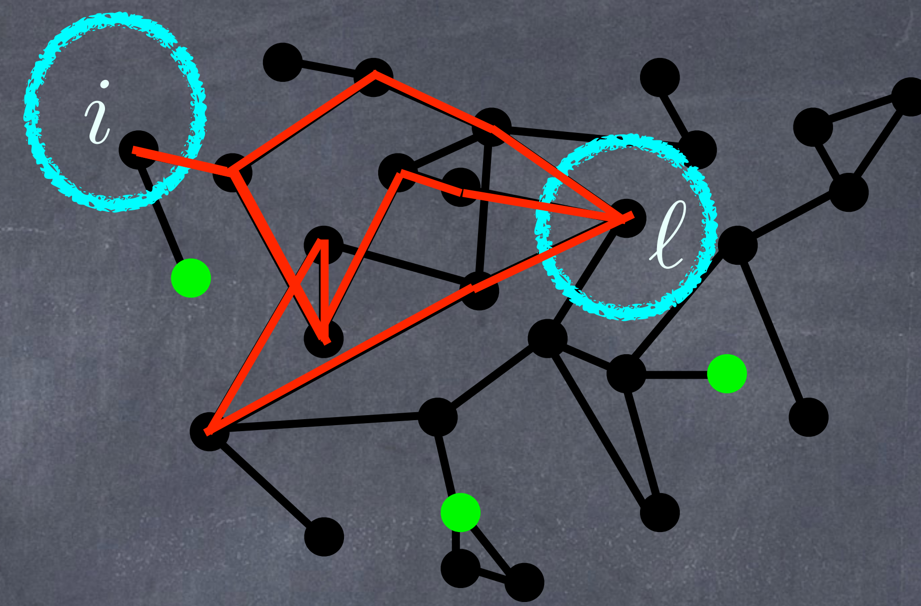
$$M \vec{f}^{\infty} = 0 \quad \Longrightarrow \quad f_i^{\infty} = \frac{w_i}{\sum_j w_j}$$

Garriga & Vilenkin (1998)

- Depends only on the weights
- Low-energy vacua exponentially favored

First-Passage Processes

How quickly is equilibrium reached?



A popular measure of search efficiency is the mean first-passage time:

$$\langle t_{i\to\ell}\rangle \equiv$$ Avg time for walker starting from node i to reach node ℓ

Global mean first-passage time (Kemeny's constant):

$$t_{\mathrm{MFPT}} \equiv \sum_k \langle t_{i\to k}\rangle f_k^{\infty}$$

Famously, independent of starting node!

Neatly expressed as spectral sum:

$$t_{\mathrm{MFPT}} = \sum_{\ell=2}^{N} \frac{1}{|\lambda_\ell|}$$

Non-zero eigenvalues of M_{ij}

Must diagonalize large $N \times N$ matrix — a daunting numerical task!

"Downward" approx'n

Schwartz-Perlov & Vilenkin (2006); Olum & Schwartz-Perlov (2007)

Recall detailed balance:

$$\frac{\kappa_{ji}}{\kappa_{ij}} = \frac{w_j}{w_i} \sim e^{-\frac{2\pi}{G}\left(\frac{1}{H_j^2} - \frac{1}{H_i^2}\right)}$$

"upward" transitions exponentially suppressed

In downward approx'n, neglect upward transitions to leading order.

By labeling vacua by increasing pot., $V_1 \leq V_2 \leq \ldots \leq V_N$, transition matrix becomes upper-triangular:

$$M = \begin{pmatrix} 0 & \cdot & \cdot & \cdot & \cdot \\ & -\kappa_2 & \cdot & \cdot & \cdot \\ & & \cdot & \cdot & \cdot \\ & 0 & & \cdot & \cdot \\ & & & & -\kappa_N \end{pmatrix}$$

Total rate out of each vacuum: $\kappa_j \equiv \sum_r \kappa_{rj}$

($\kappa_1 = 0$ in the approx'n)

But eigenvalues of upper-triangular matrix are diagonal entries:

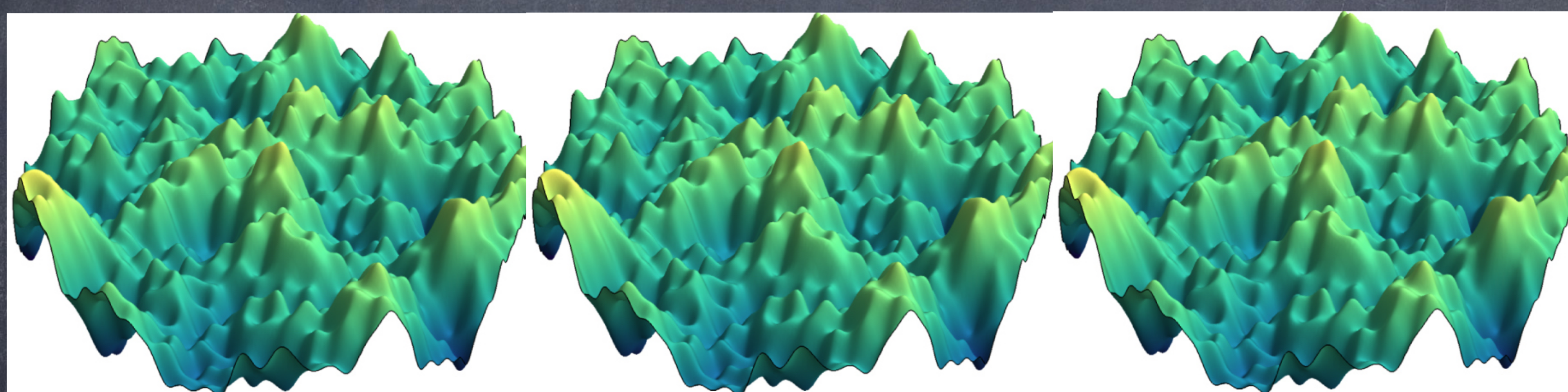
$$\Longrightarrow \quad \lambda_\ell \simeq -\kappa_\ell$$

Global mean first-passage time

$$t_{\mathrm{MFPT}} = \sum_{j=2}^{N} \frac{1}{|\lambda_j|} \simeq \sum_{j=2}^{N} \frac{1}{\kappa_j}$$

Total residency time
(diffusion in disordered media)

Ensemble of regions:



Typical regions include vacua whose only allowed transitions are upward jumps

$\Longrightarrow$ Exponentially long t_{MFPT}

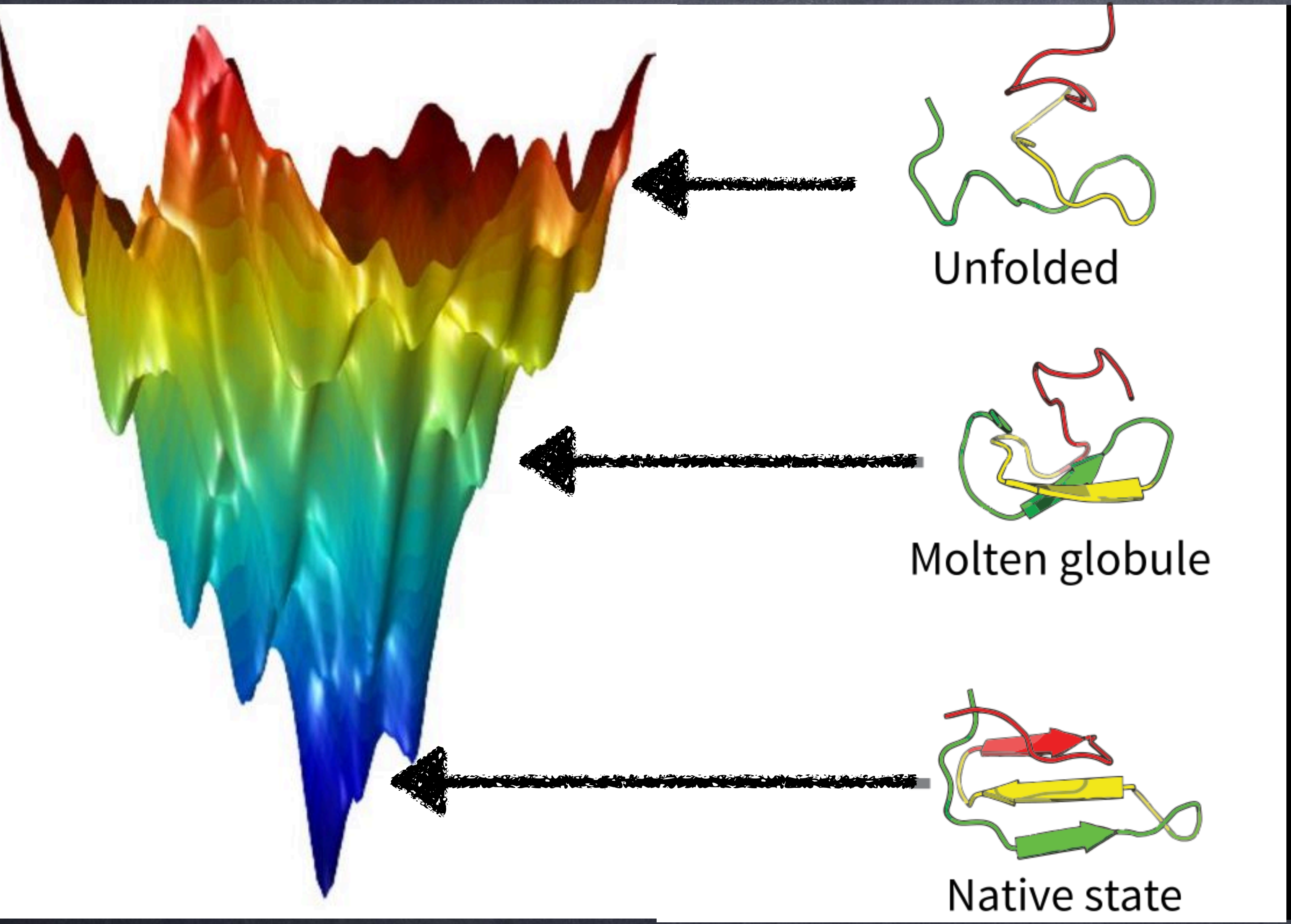
$$\Longrightarrow \quad t_{\mathrm{MFPT}} \sim e^{\frac{2\pi}{G H_{\mathrm{min}}^2}} \sim e^N$$

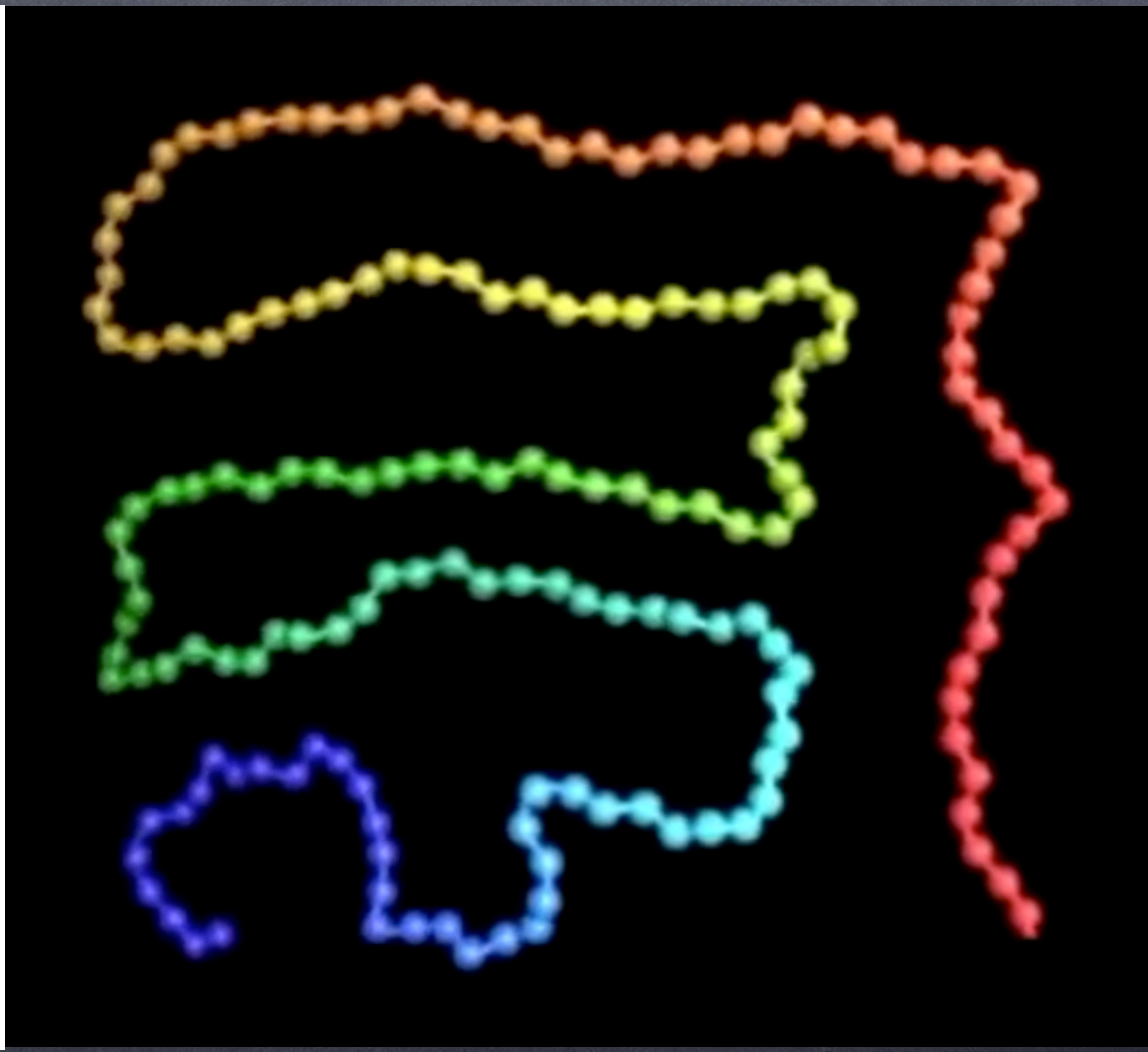
Consistent with NP-hard complexity class
Denef & Douglas (2007)

Global mean first-passage time

$$t_{\mathrm{MFPT}} = \sum_{j=2}^{N} \frac{1}{|\lambda_j|} \simeq \sum_{j=2}^{N} \frac{1}{\kappa_j}$$

Total residency time
(diffusion in disordered media)





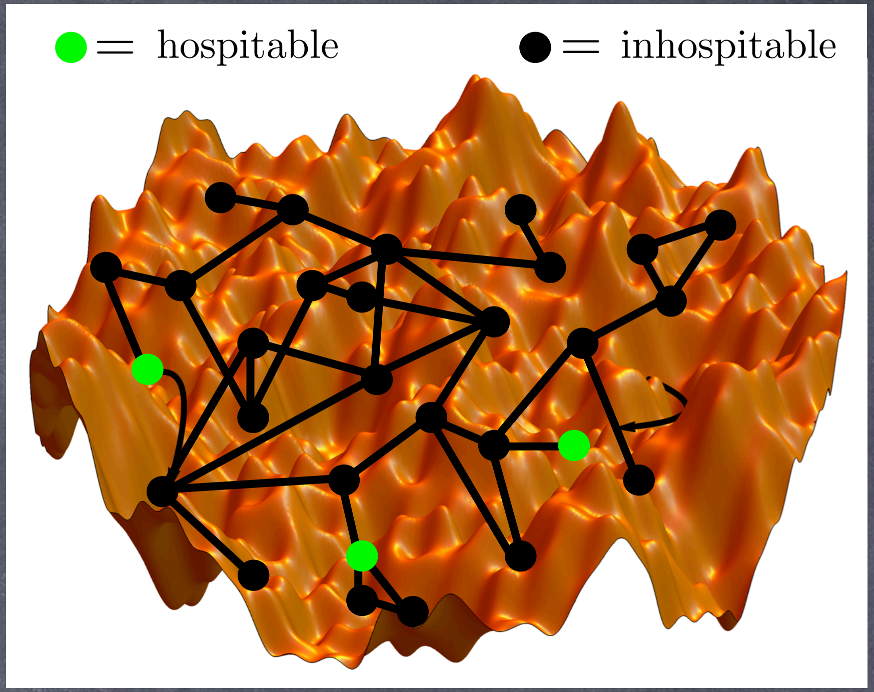
$$\Longrightarrow \qquad t_{\mathrm{MFPT}} \sim N^{\#}$$

(NP-hardness is 'worst-case'. Special cases can be polynomial.)

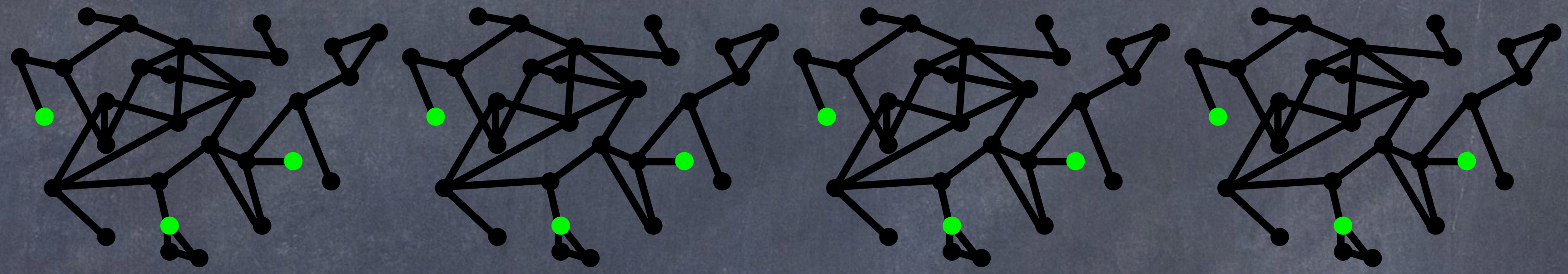
Quick recap

Landscape dynamics: random walk on weighted, undirected network

$$\frac{\mathrm{d}f_i}{\mathrm{d}t} = \sum_j \left(\kappa_{ij} f_j - \kappa_{ji} f_i\right) \quad ; \qquad \kappa_{ij} = \frac{A_{ij}}{w_j}$$



Ensemble:



- Statistically identical equilibrium distribution: $f_i^{\infty} = \dfrac{w_i}{\sum_j w_j}$
- But vastly different A_{ij}, hence different "mixing times":

Typical, glassy region:

$$t_{\mathrm{MFPT}} \sim e^N$$

Golden, funnel-like region:

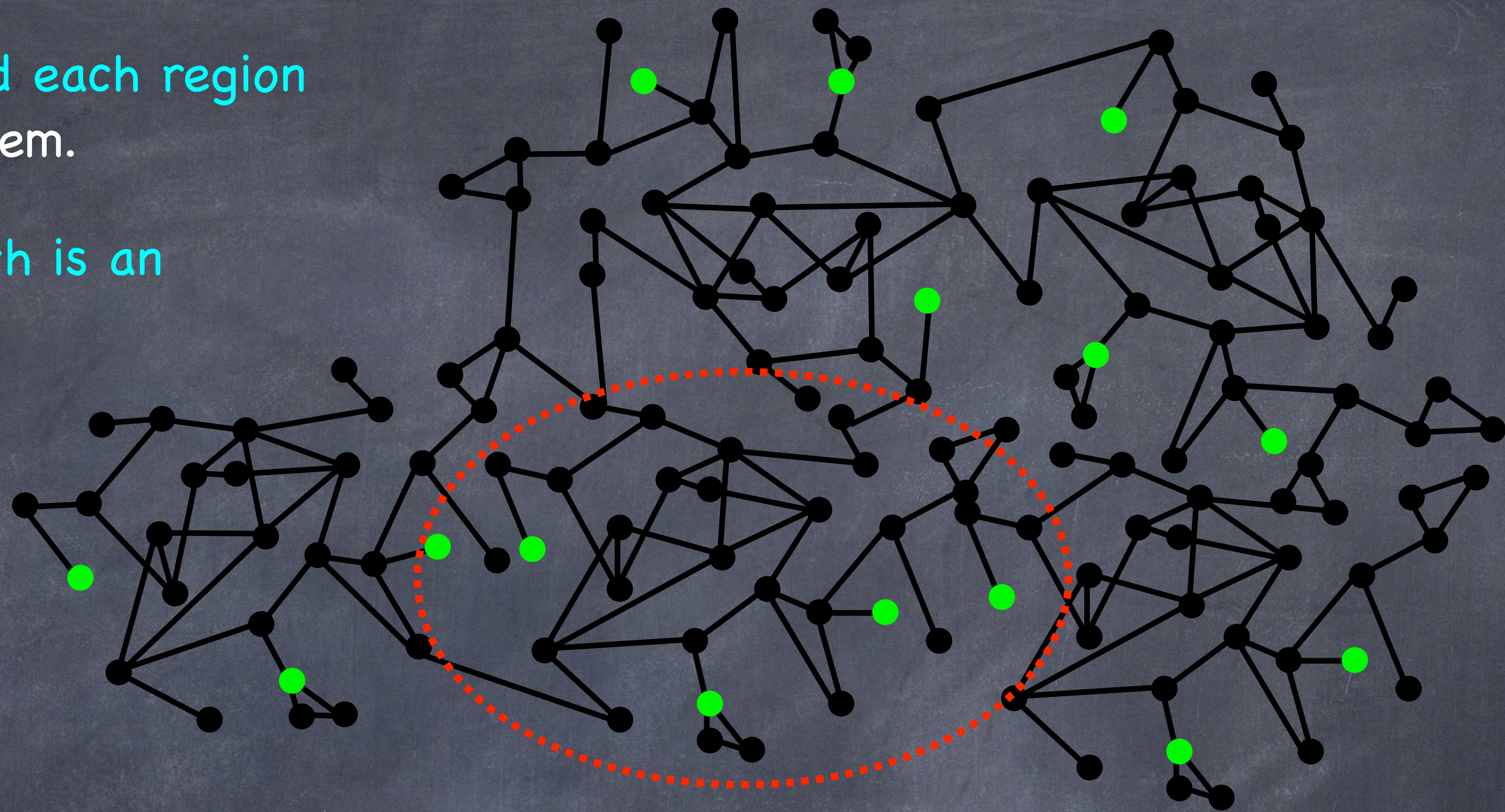
$$t_{\mathrm{MFPT}} \sim N^{\#}$$

→ Constrains topology of optimal region

→ Otherwise, favors fastest rates possible

So far treated each region as closed system.

In reality, each is an open system.



Once random walker lands in a golden region, how can we minimize the probability of escape?

In principle requires modeling environment...

Instead study a proxy requirement that depends solely on the intrinsic dynamics of the region:

Demand that walks be <u>recurrent</u> in the $N \to \infty$ limit.

Recurrent and Transient Random Walks

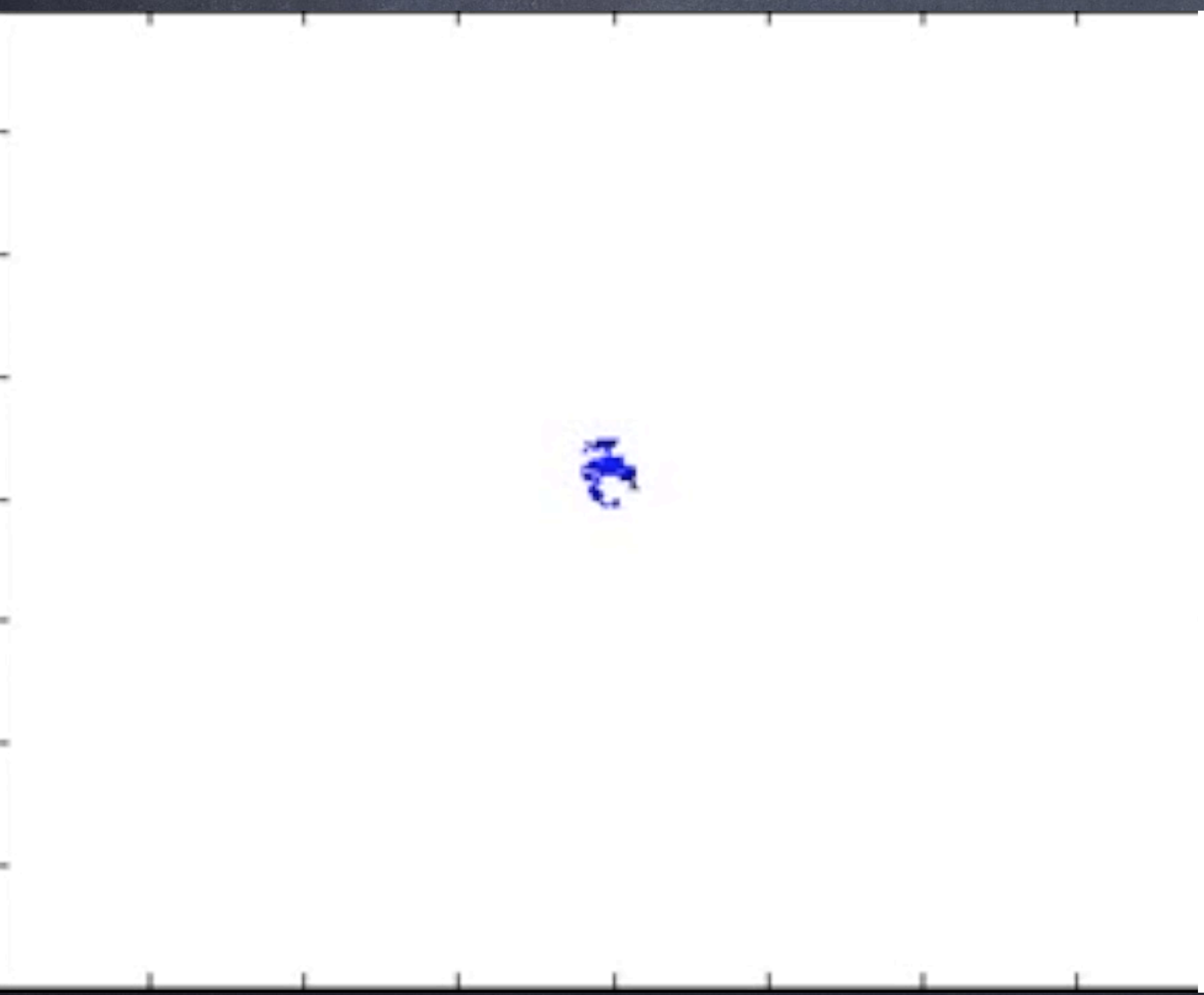
Recurrence: Random walker <u>will eventually return</u> to starting node

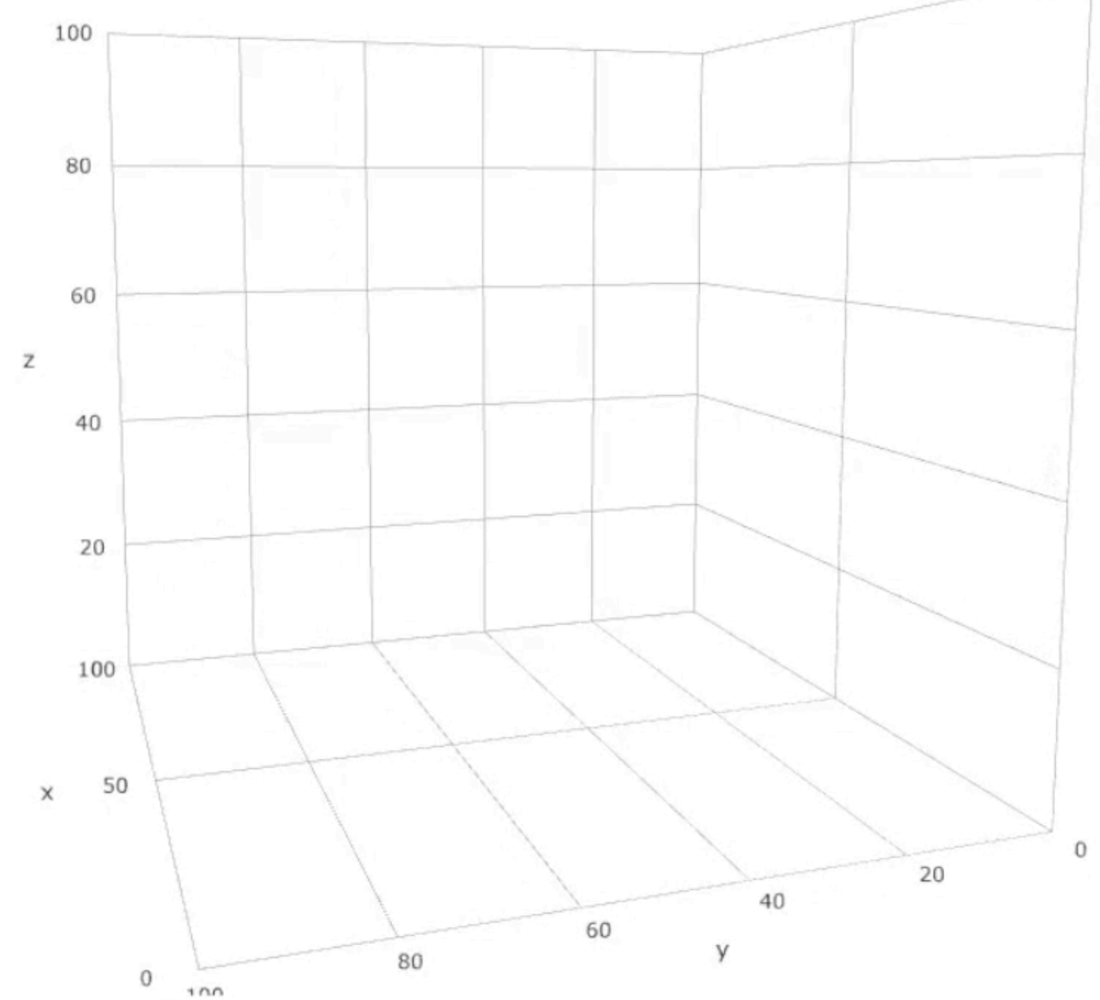
(Equivalently, random walker <u>will eventually visit</u> every node.)

Transience: Random walker may <u>never return</u> to starting node

<u>Pólya's theorem</u>

Simple random walks on $\mathbb{R}^d$ are recurrent for $d \leq 2$, and transient for $d > 2$.





"A drunk man will find his way home, but a drunk bird may get lost forever."

- Shizuo Kakutani, UCLA Colloquium

Recurrent/Transient Random Walks on Networks

First-passage probability: $F_{ki}(t) \equiv$ probability density that walker, who started at node i, visits node k for the 1st time at time t

(Note: MFPT is first moment, $\langle t_{i\to k}\rangle = \int_0^\infty \mathrm{d}t\, t F_{ki}(t)$)

In particular, $F_{ii}(t) \equiv$ first-return probability density

Escape probability: Probability that walker never returns to starting node

$$\lim_{t\to\infty} S_{ii}(t) = 1 - \int_0^\infty \mathrm{d}t\, F_{ii}(t)$$

$$\lim_{t\to\infty} S_{ii}(t) = 0 \quad \Longleftrightarrow \quad \text{Recurrence}$$

$$\lim_{t\to\infty} S_{ii}(t) = \text{finite} \quad \Longleftrightarrow \quad \text{Transience}$$

Importantly, recurrence/transience criterion is time-reparam. invariant

Escape probability: $$\lim_{t\to\infty} S_{ii}(t) \equiv \frac{1}{\mathcal{T}_i} \ ; \quad \begin{array}{lcl} \mathcal{T}_i = \infty & \Longleftrightarrow & \text{Recurrence} \\ \mathcal{T}_i < \infty & \Longleftrightarrow & \text{Transience} \end{array}$$

- Walks are always recurrent for finite N
- Non-trivial case: $N \to \infty$ (First send $N \to \infty$, then $t \to \infty$.)

Averaged over all nodes, neatly expressed as spectral sum:

$$\langle \mathcal{T} \rangle = \lim_{N\to\infty} \frac{1}{N} \sum_{\ell=2}^{N} \frac{1}{\Delta t \, |\lambda_\ell|} = \lim_{N\to\infty} \frac{t_{\mathrm{MFPT}}}{\Delta t \, N}$$

$\Delta t \equiv$ discrete time step

- Remarkably, simply related to global MFPT
- Recognized as dimensionless mean residency time.
- In downward approx'n, $\lambda_\ell \simeq -\kappa_\ell$, reduces to

$$\langle \mathcal{T} \rangle \simeq \lim_{N\to\infty} \left\langle \frac{1}{\kappa_i \Delta t} \right\rangle$$

Manifestly time-reparam. invariant

2 competing requirements

- Search efficiency $\Longleftrightarrow$ minimal t_{MFPT}
- Sweeping exploration $\Longleftrightarrow$ recurrence: $\dfrac{t_{\mathrm{MFPT}}}{\Delta t\, N} \xrightarrow[N \to \infty]{} \infty$

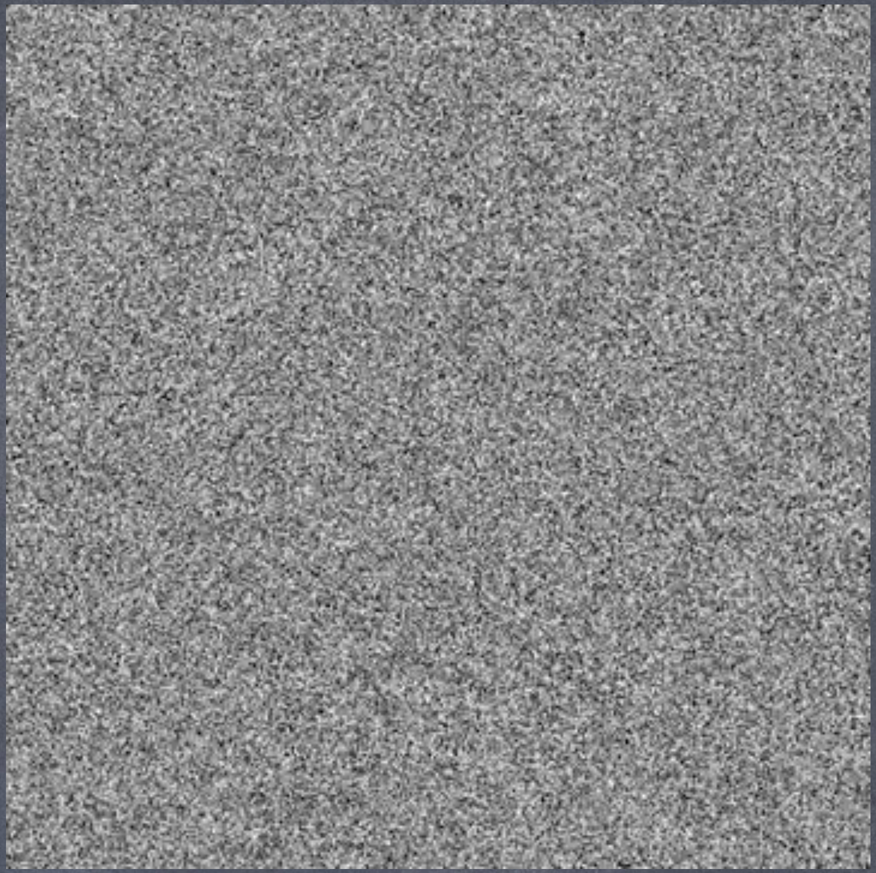
Optimal regions reach a compromise by having the shortest MFPT compatible with recurrence.

Optimality select regions at critical boundary between recurrence and transience.

Different notions of criticality

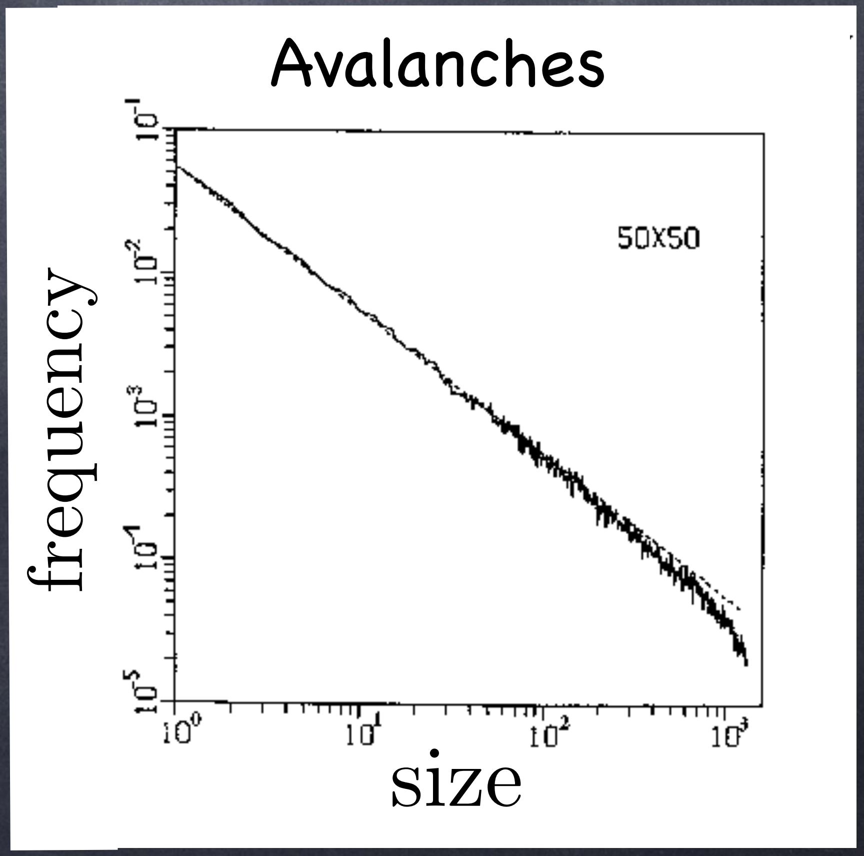
- Equilibrium phase transitions

$$(T \to T_c)$$



- Non-equilibrium (dynamical) phase transitions

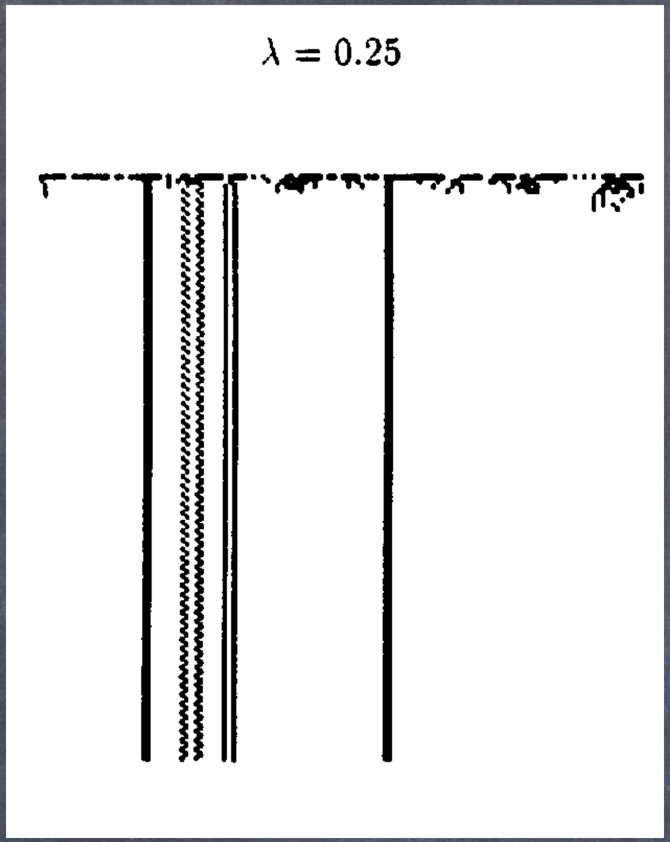
Bak, Tang & Wiesenfeld (1987)



- "Edge of chaos" dynamical phase transition: critical boundary between stable and unstable dynamics

Cellular automata:

Wolfram (1984);
Langton (1990);
Crutchfield et al. (1993)



In cellular automata, associated with optimal information processing

- information storage/transfer
- dynamic response
- universal computation

Dynamical criticality via natural selection

- Poised at dynamical criticality: compromise between robustness and adaptability

Dynamical criticality

Recurrence: $\langle \mathcal{T} \rangle \simeq \lim_{N\to\infty} \left\langle \frac{1}{\kappa_i \Delta t} \right\rangle = \infty$

- Use proper time, $\kappa_i \Delta t = \kappa_i^{\text{proper}} \Delta\tau_i$ $\sim H_i^{-1}$
- In the continuum limit,

$$\langle \mathcal{T} \rangle = \int \mathrm{d}V \, \frac{\sqrt{V}}{M_{\text{Pl}}} \, \kappa^{-1}(V) \, \mathcal{P}(V)$$

$\mathcal{P}(V) \equiv$ probability distribution

$\kappa(V) \equiv$ average proper transition rate

- Assume $\mathcal{P}(V) \simeq \text{const.}$ as $V \to 0$ Weinberg (2003)

$$\Longrightarrow \langle \mathcal{T} \rangle \simeq \int_0 \mathrm{d}V \, \frac{\sqrt{V}}{M_{\text{Pl}}} \, \kappa^{-1}(V)$$ diverges for $\kappa(V) \sim V^{\frac{3}{2}+\alpha}\,; \quad \alpha \geq 0$

Critical case ($\alpha = 0$):

$$\kappa_{\text{crit}}(V) \sim \frac{V^{3/2}}{M_{\text{Pl}}^5} \quad \Longrightarrow \quad \langle \mathcal{T} \rangle \sim \int_{V_{\min}} \frac{\mathrm{d}V}{V} \sim \ln V_{\min} \sim \ln N$$

Identical to dynamical phase transition from normal to anomalous diffusion in disordered media

Phenomenological Implications

(Do not rely on anthropics)

- Metastability of our vacuum

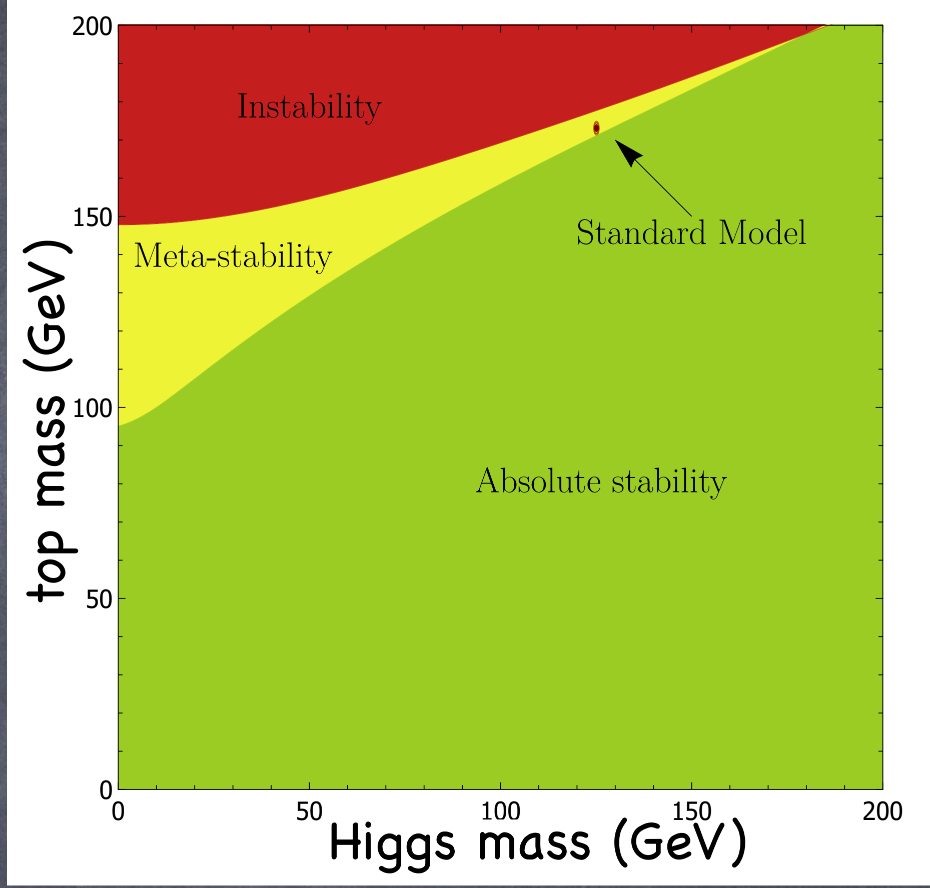
The critical decay rate implies an optimal lifetime for our vacuum of

$$\tau \sim \frac{M_{\rm Pl}^2}{H_0^3} \sim 10^{130} \text{ years}$$

SM estimate: $\tau = 10^{526^{+409}_{-202}}$ years

Agrees to within $\gtrsim 2\sigma$.

Prediction for top: $m_t^{\rm pole} \simeq 174.5$ GeV

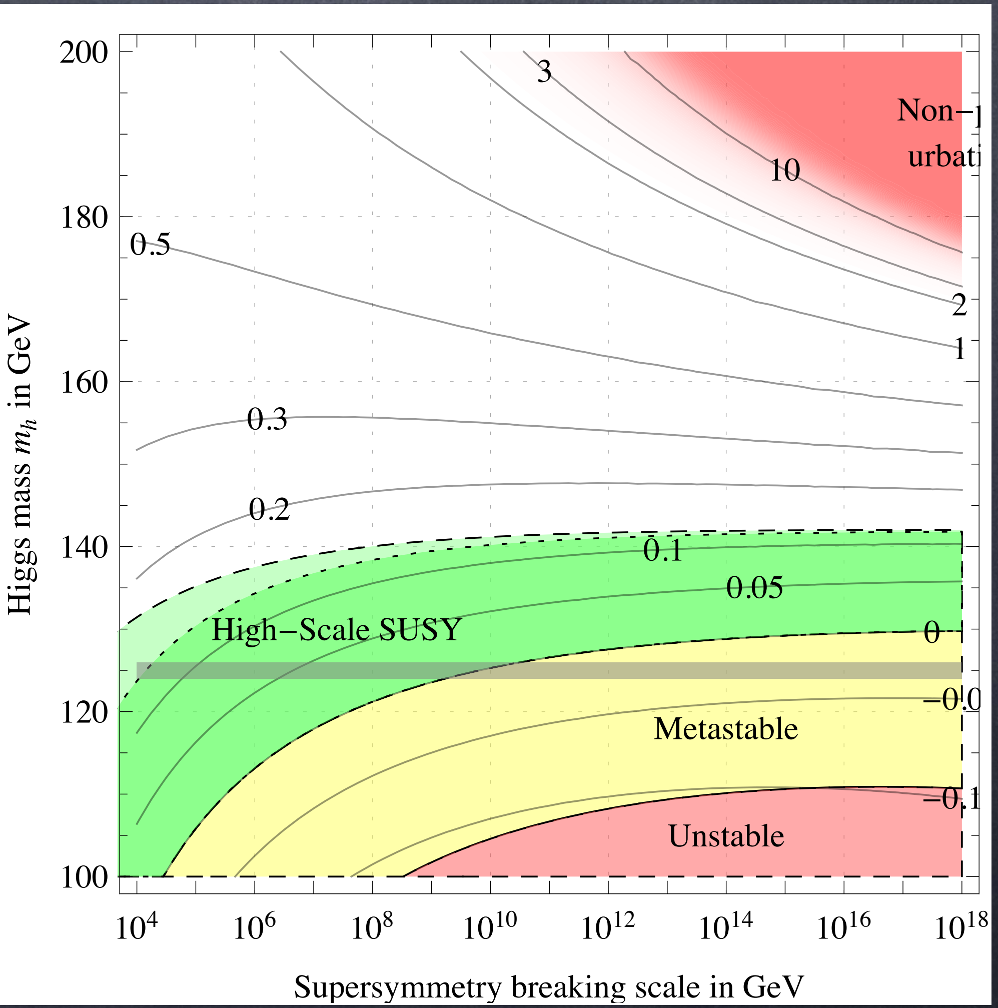


- Why no low-scale SUSY?

Suppose all SUSY partners have mass at $m_{\rm SUSY}$.

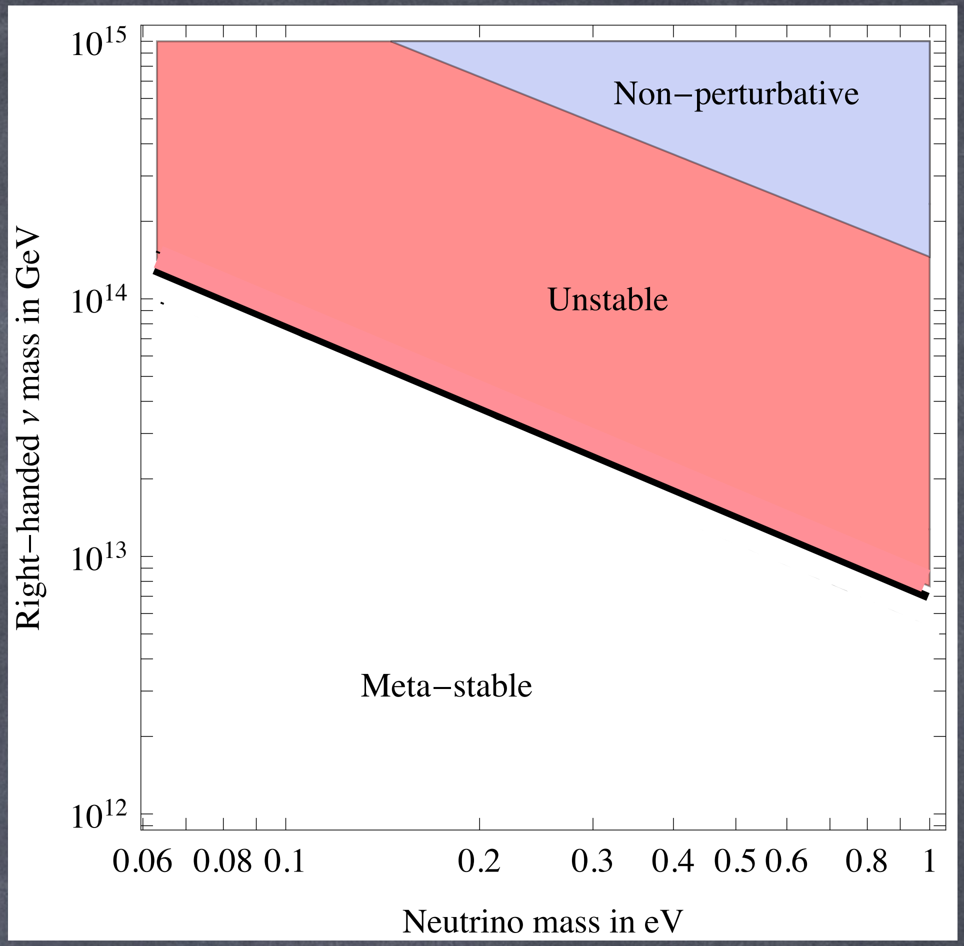
$$\Longrightarrow \quad m_{\rm SUSY} \gtrsim 10^{10} \text{ GeV}$$

High-scale SUSY favored by optimal search strategy



Giudice & Strumia (2011)

- Right-handed neutrinos



Elias-Miró et al. (2011)

Sufficiently massive RH neutrinos make EW vacuum less stable

Can bring SM lifetime closer to optimality with

$$M_N \sim 10^{13} - 10^{14}\ \mathrm{GeV}$$

- Strong CP and QCD axion

QCD axion makes EW vacuum more stable

$$f_{\mathrm{a}} \gtrsim 10^{10}\ \mathrm{GeV}$$

Hertzberg (2011)

Meanwhile,

$$\Omega_{\mathrm{a}} h^2 \simeq 0.7 \left(\frac{\theta_{\mathrm{i}}}{\pi}\right)^2 \left(\frac{f_{\mathrm{a}}}{10^{12}\ \mathrm{GeV}}\right)^{7/6}$$

- Cosmological constant (NOT time-reparam invariant)

Local equilibrium distribution: $f_i^\infty \sim \mathrm{e}^{\frac{2\pi}{G H_i^2}}$

Exponentially favors vacuum with smallest potential energy $V_{\min}$.
With N vacua, this is statistically

$$V_{\min} \sim \frac{M_{\rm Pl}^4}{N}$$

Can explain observed C.C. if our region contains $N \gtrsim 10^{120}$

Note: Could make same argument in the "global" approach to the landscape

e.g. Linde & Vanchurin (2010)

$\Longrightarrow$ favors smallest potential energy anywhere on the landscape

$\Longrightarrow$ But expect such vacuum to be nearly supersymmetric, with tiny $m_{3/2}$ and $V_{\min} \sim m_{3/2}^4$.

Instead our mechanism predicts:

$$\frac{V_{\min}}{m_{3/2}^4} \sim \frac{1}{N}$$

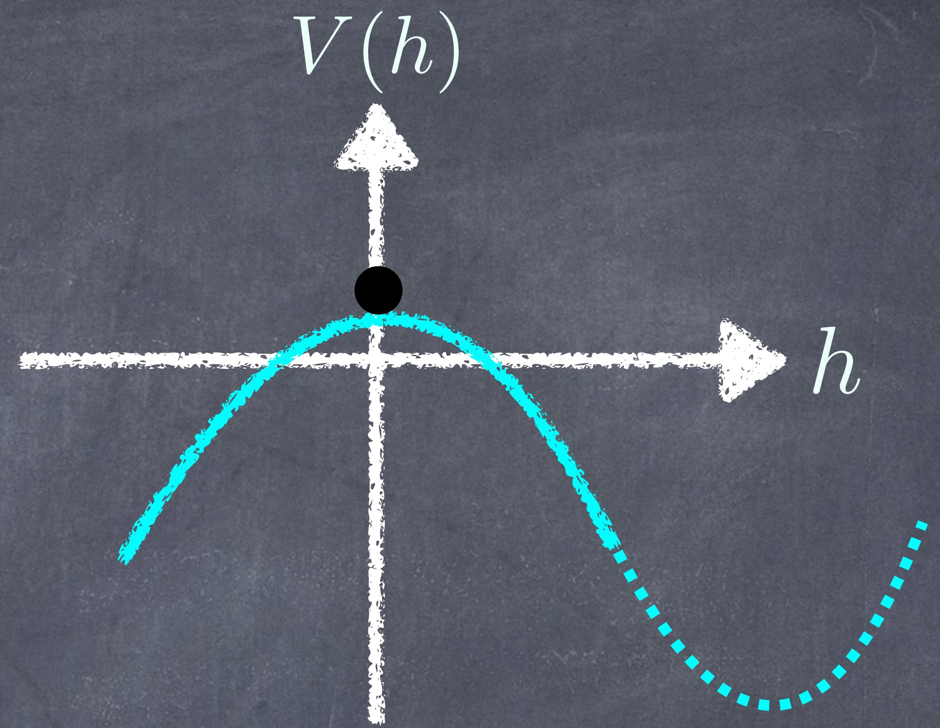
"Given the value of the CC, why is the SUSY breaking scale so large?"

Banks (2001)

AdS vacua

Tunneling into AdS vacua

$\mathrm{dS} \to \mathrm{AdS}$ transition rate can be reliably calculated   Coleman & De Luccia



Within a Hubble time, AdS region starts to collapse

Big Crunch singularity?

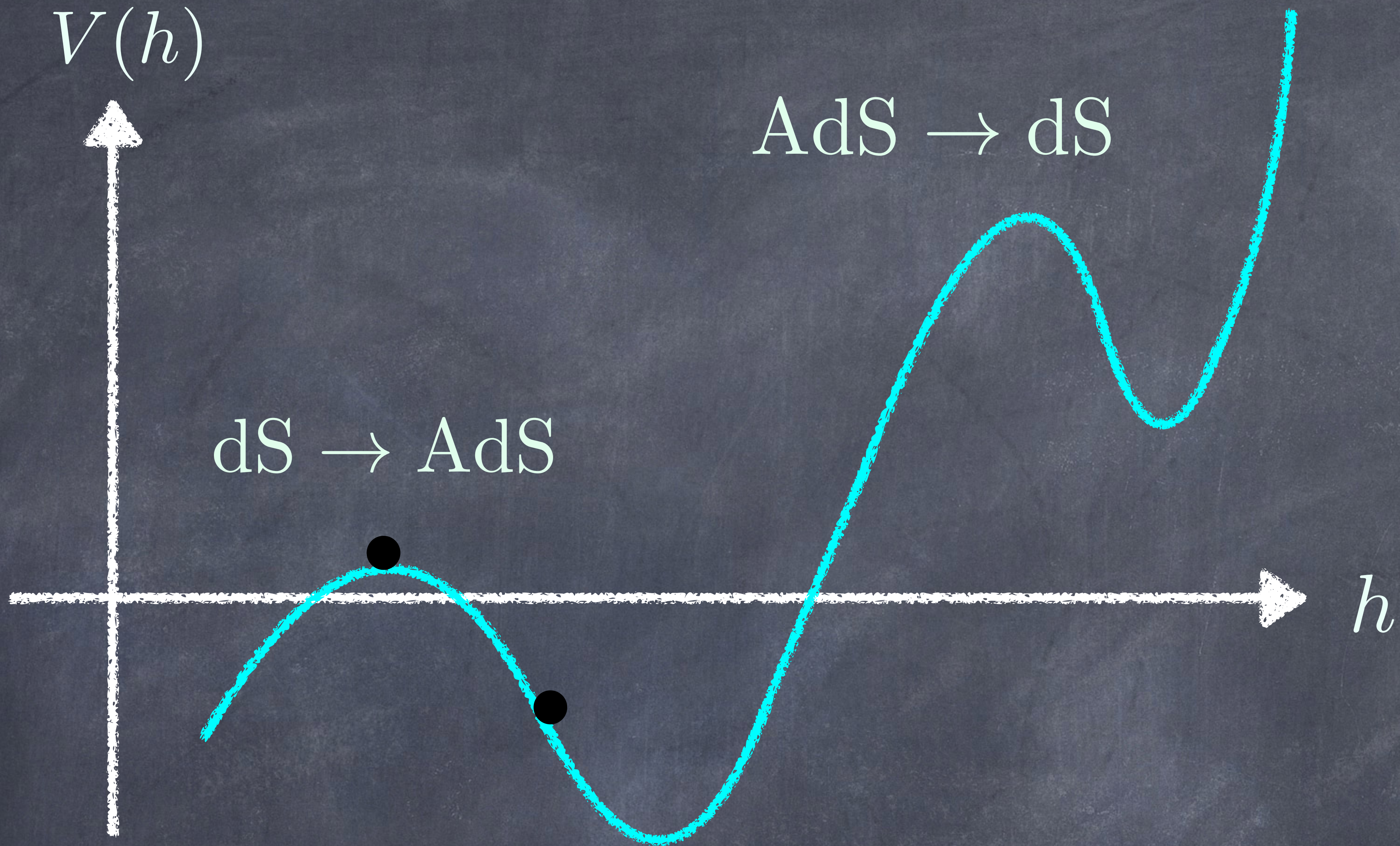
AdS vacua are terminal
$\Longrightarrow$ probability sinks

Death?

Garriga, Schwartz-Perlov, Vilenkin & Winitzki (2005)

AdS regions bounce
contraction → expansion
$\Longrightarrow \mathrm{AdS} \to \mathrm{dS}$ transitions

Piao (2004,2009); Nomura (2011)
Garriga & Vilenkin (2012,2013)



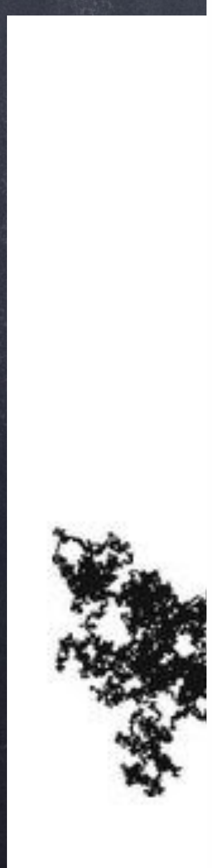
Because of high (Planckian?) energy reached at the bounce,

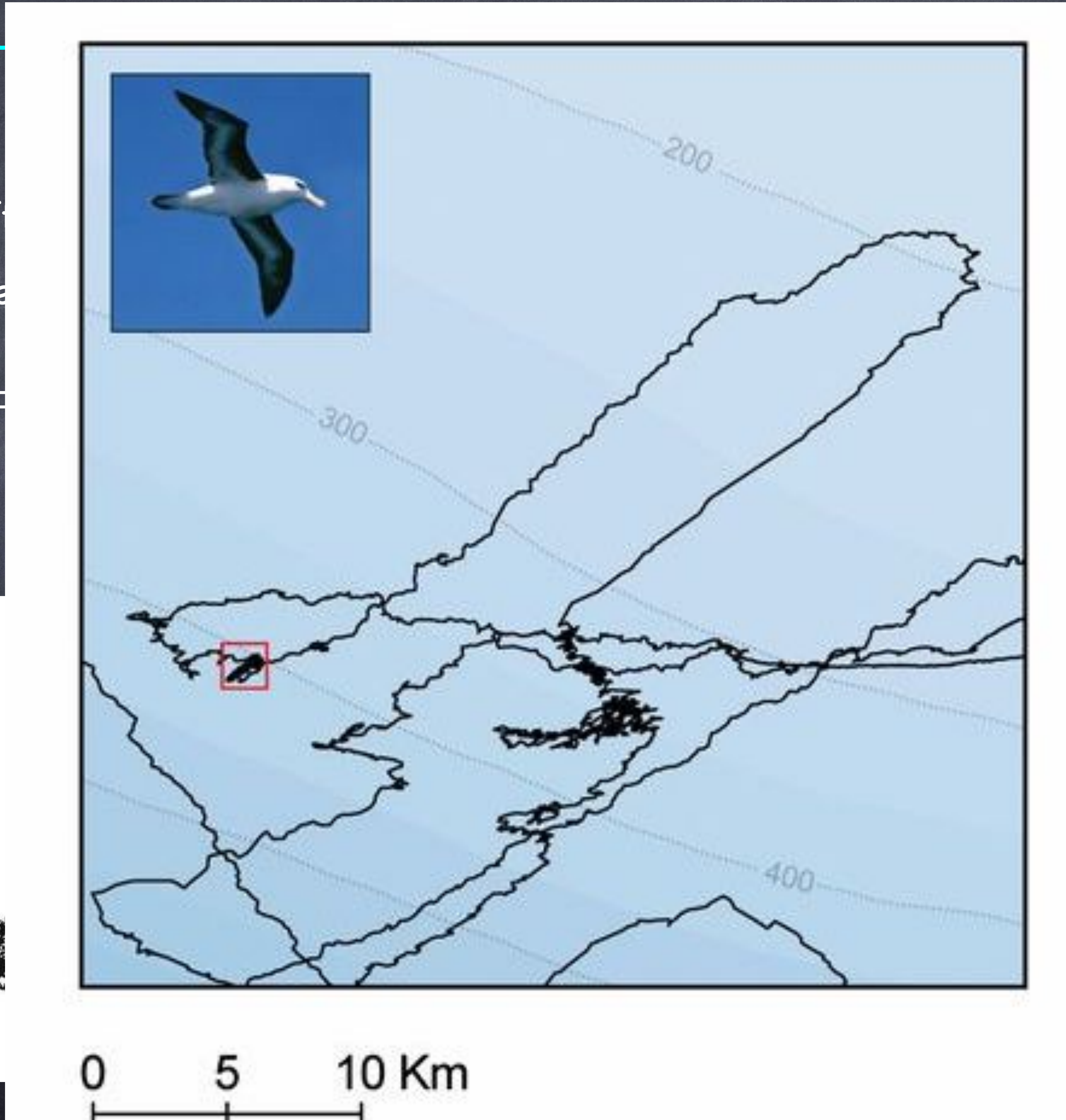
- $\mathrm{AdS} \to \mathrm{dS}$ is nearly instantaneous
- can transition (classically) to far away, high-energy vacua

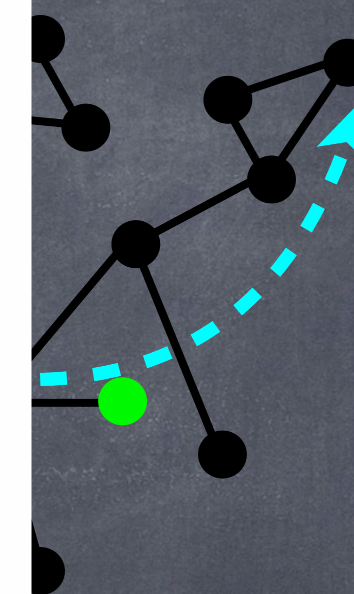
$\Longrightarrow$ AdS vacua are short-lived mediators

Suggest

- Ef
- Ne

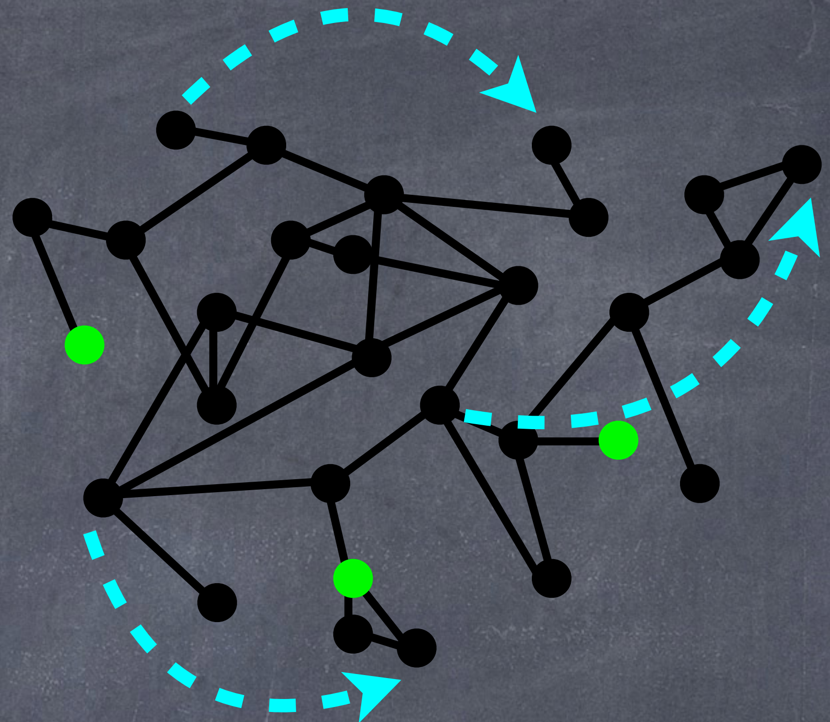






ul Lévy

Random walks with Lévy flights



$$\kappa_{ij} = \alpha\ \kappa_{ij}^{\mathrm{CDL}} + (1-\alpha)\ \kappa_{ij}^{\mathrm{AdS}} \qquad 0 \leq \alpha \leq 1$$

Local/Brownian moves

Non-local/Lévy moves

Similar to Google's PageRank matrix

$$\alpha = 0.85$$

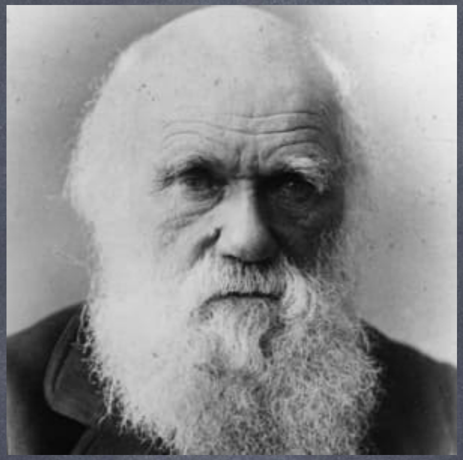
Brin & Page (1998)

Final thoughts

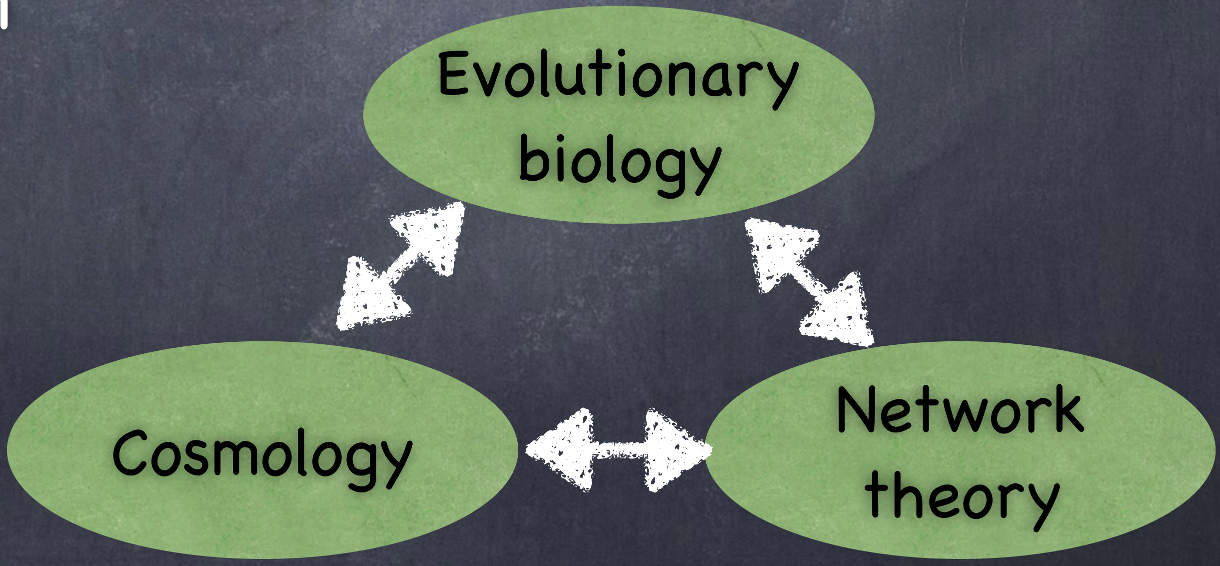
Most fine-tuning problems are problems of criticality

Generic?
- Naturalness: small parameters protected by symmetries
- Landscape (standard approach): principle of mediocrity

Natural selection: outcomes are fine-tuned and nearly critical



Search optimization on the landscape: powerful natural selection mechanism

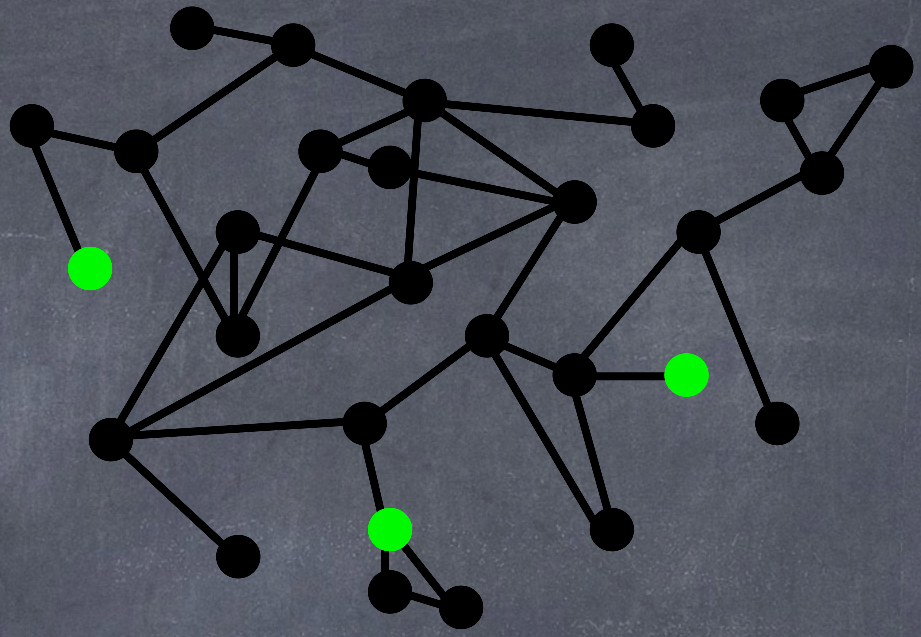


Optimal region: nearly critical vacua

$\Longrightarrow$ Higgs metastability

$\Longrightarrow$ Absence of low-scale SUSY

Tantalizing questions:



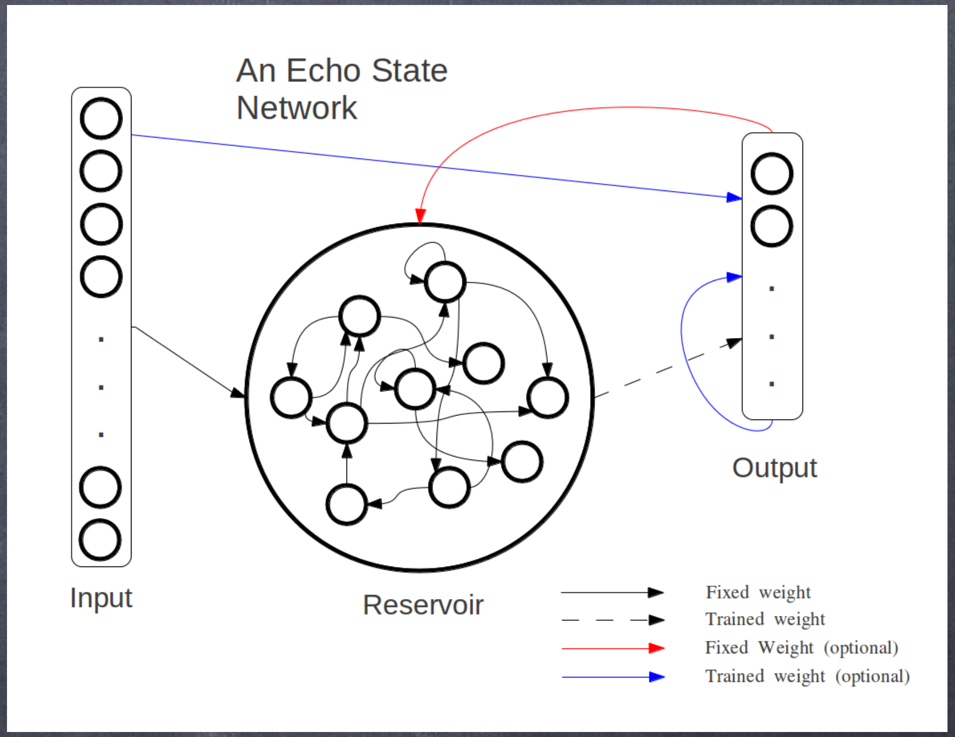
- Enhanced computational capabilities?

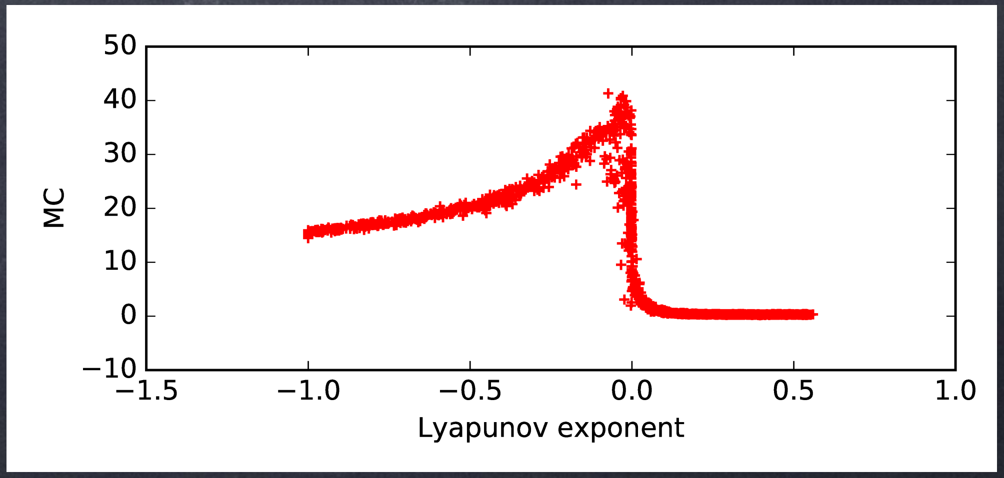
  Criticality in cellular automata, random boolean networks and neural networks is associated with optimal information processing

Non-equilibrium critical landscape dynamics

↕

Optimal information processing ?



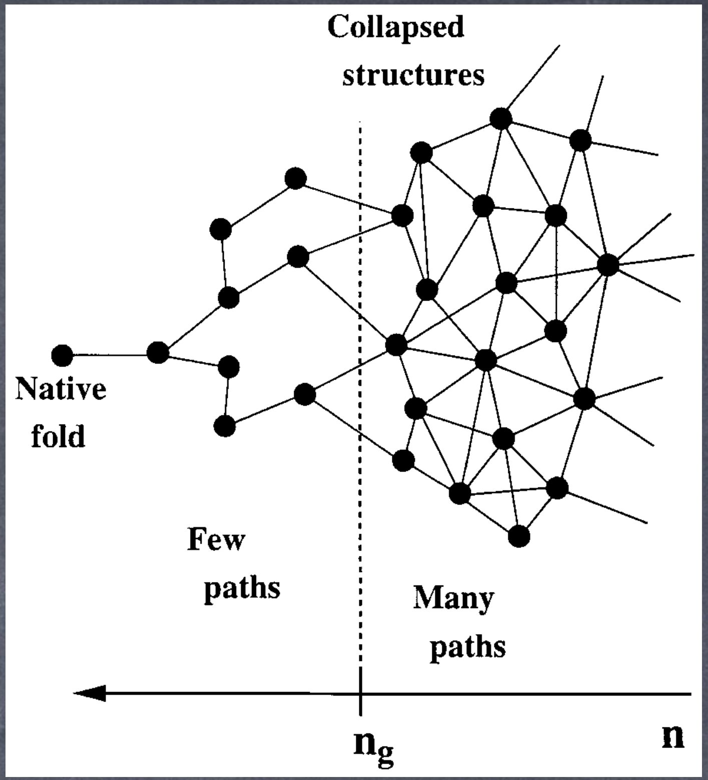


Tantalizing questions:

- Why (no more than) 3 dim'ns?

Search optimality might favor landscape regions with low effective moduli-space dimensionality, particularly near the lowest-energy vacuum.

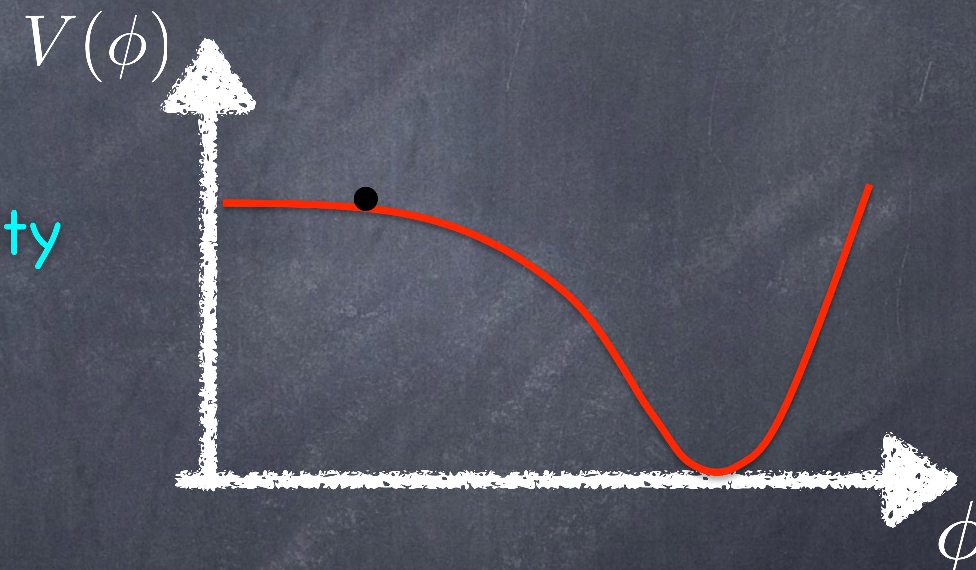
e.g. Protein landscapes



- The early universe

Slow-roll inflation also a problem of near-criticality

$$\mathrm{SO}(4,1) \longrightarrow \mathrm{ISO}(3)$$

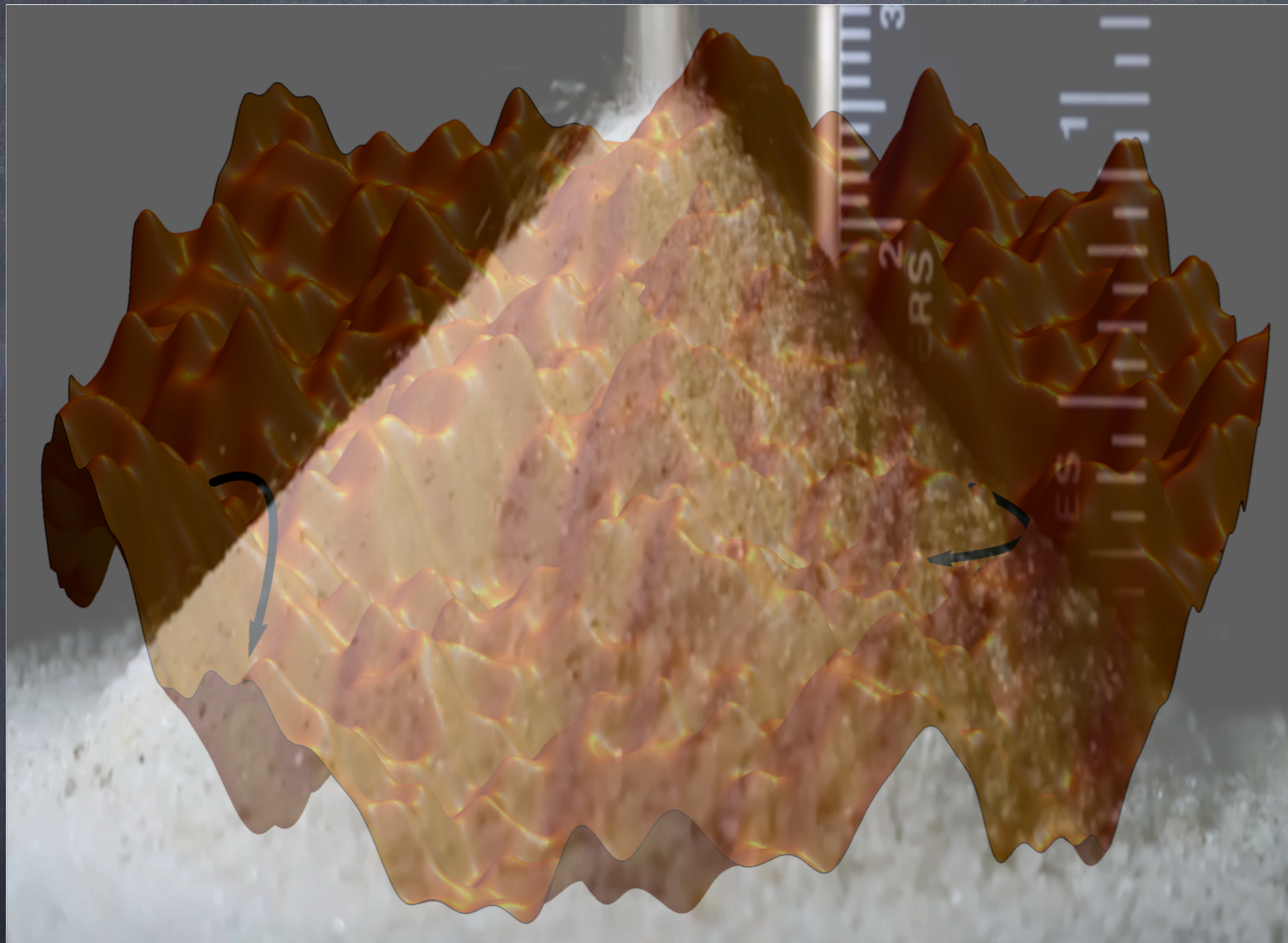


Naturally by-product in optimal/near-critical regions of the landscape?

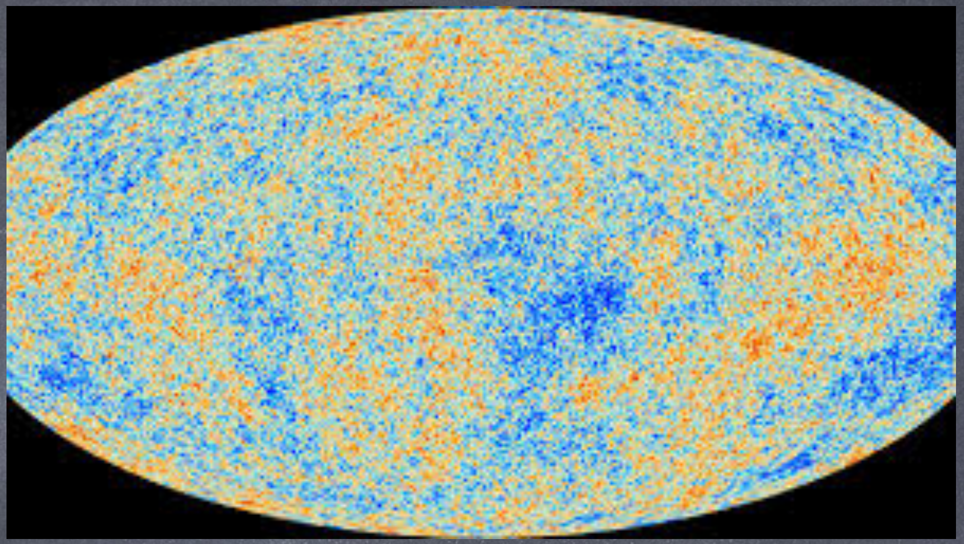
More tantalizingly: New ways of realizing inflation?

Optimal regions are open systems $\Longrightarrow$ non-equilibrium inflationary dynamics?

scale-invariant inflationary avalanches?



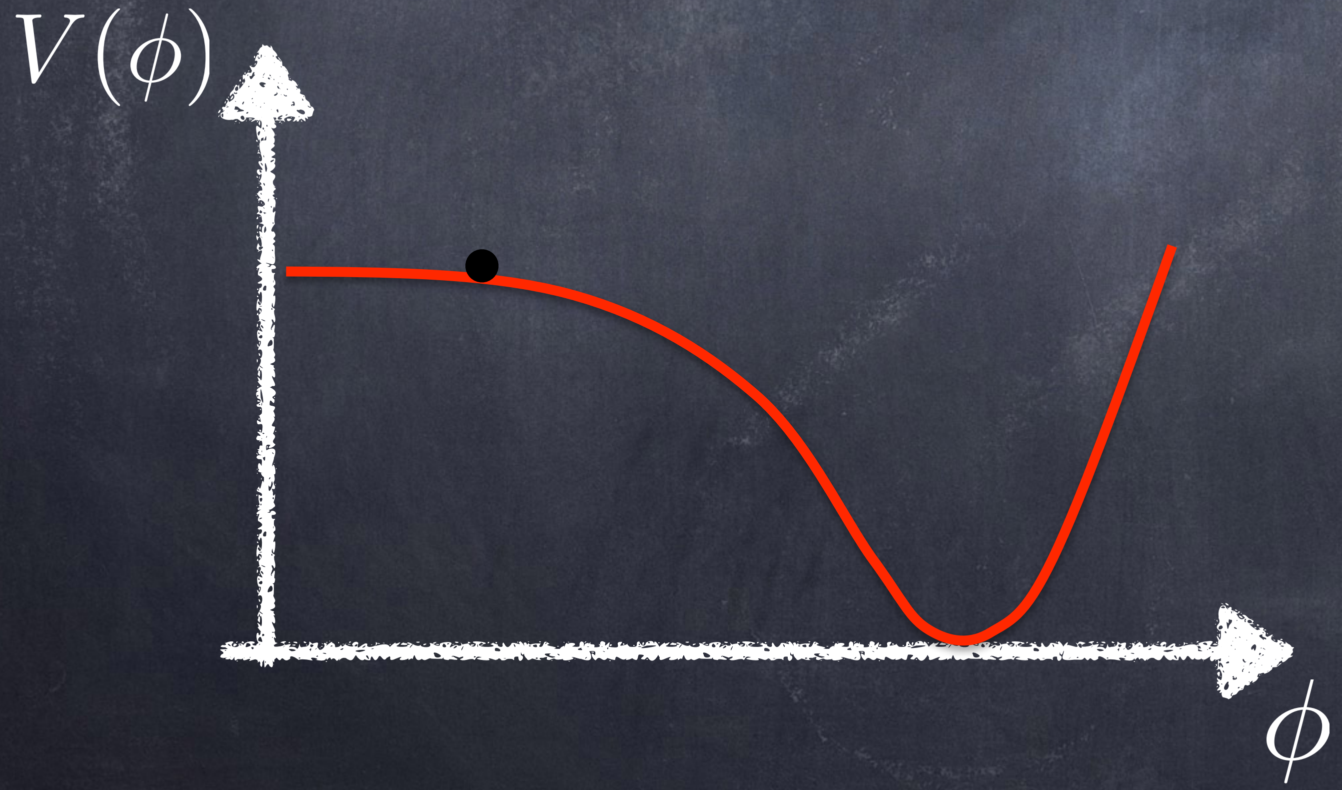
- The early universe



Scale invariant primordial spectrum suggests near-criticality in the early universe

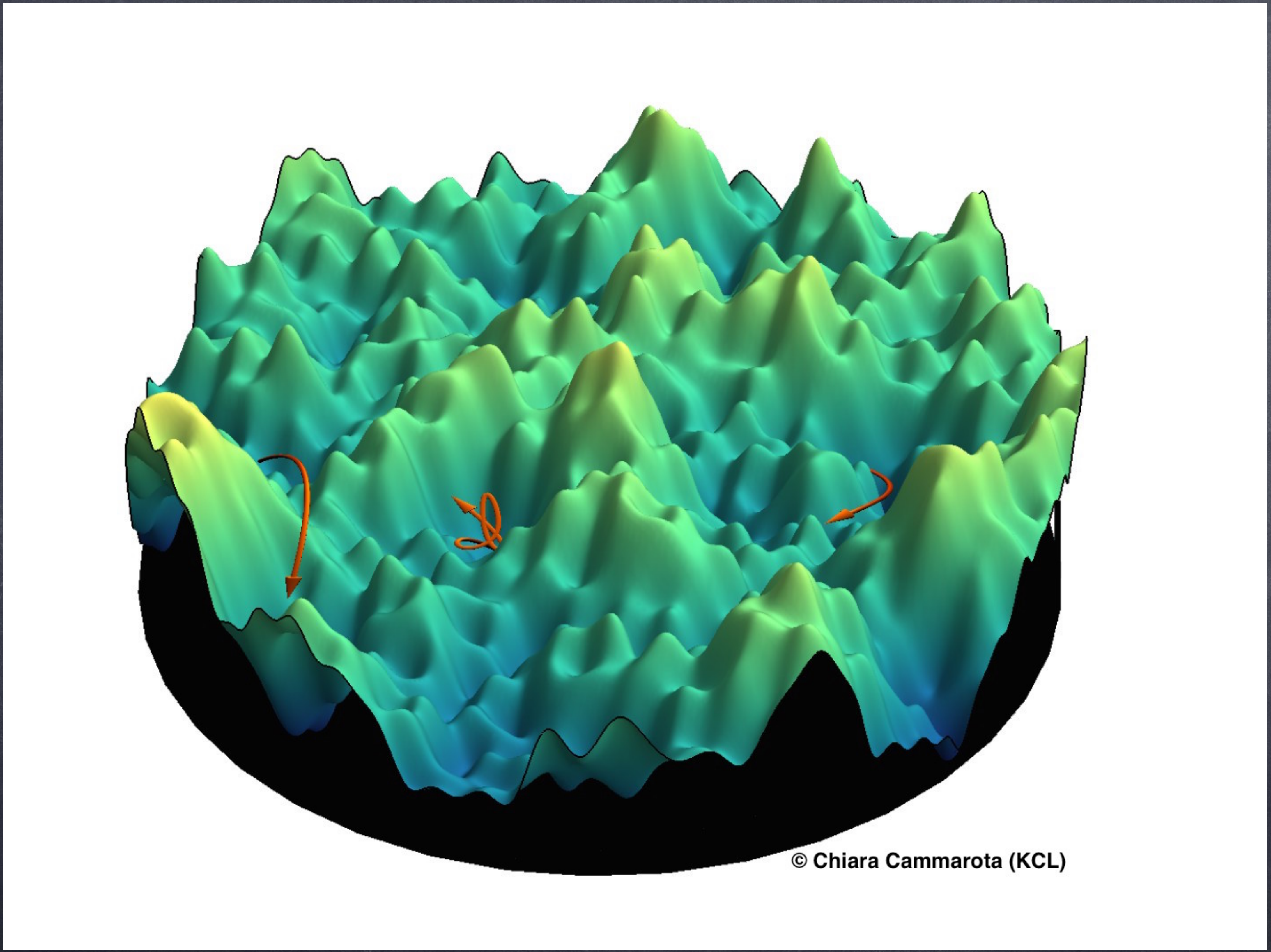
$$P(k) \sim k^{n_s - 1} \qquad n_s \simeq 0.96$$

The mechanism traditionally invoked (i.e. slow-roll inflation) relies on approximate conformal invariance



$$\mathrm{SO}(4,1) \longrightarrow \mathrm{ISO}(3)$$

Maldacena (2002);
Creminelli & Zaldarriaga (2004);
Hinterbichler, Hui & Khoury (2012,2013);
Creminelli, Norena & Simonovic (2012);
Goldberger, Hui & Nicolis (2013);
Hui, Joyce & Wong (2018)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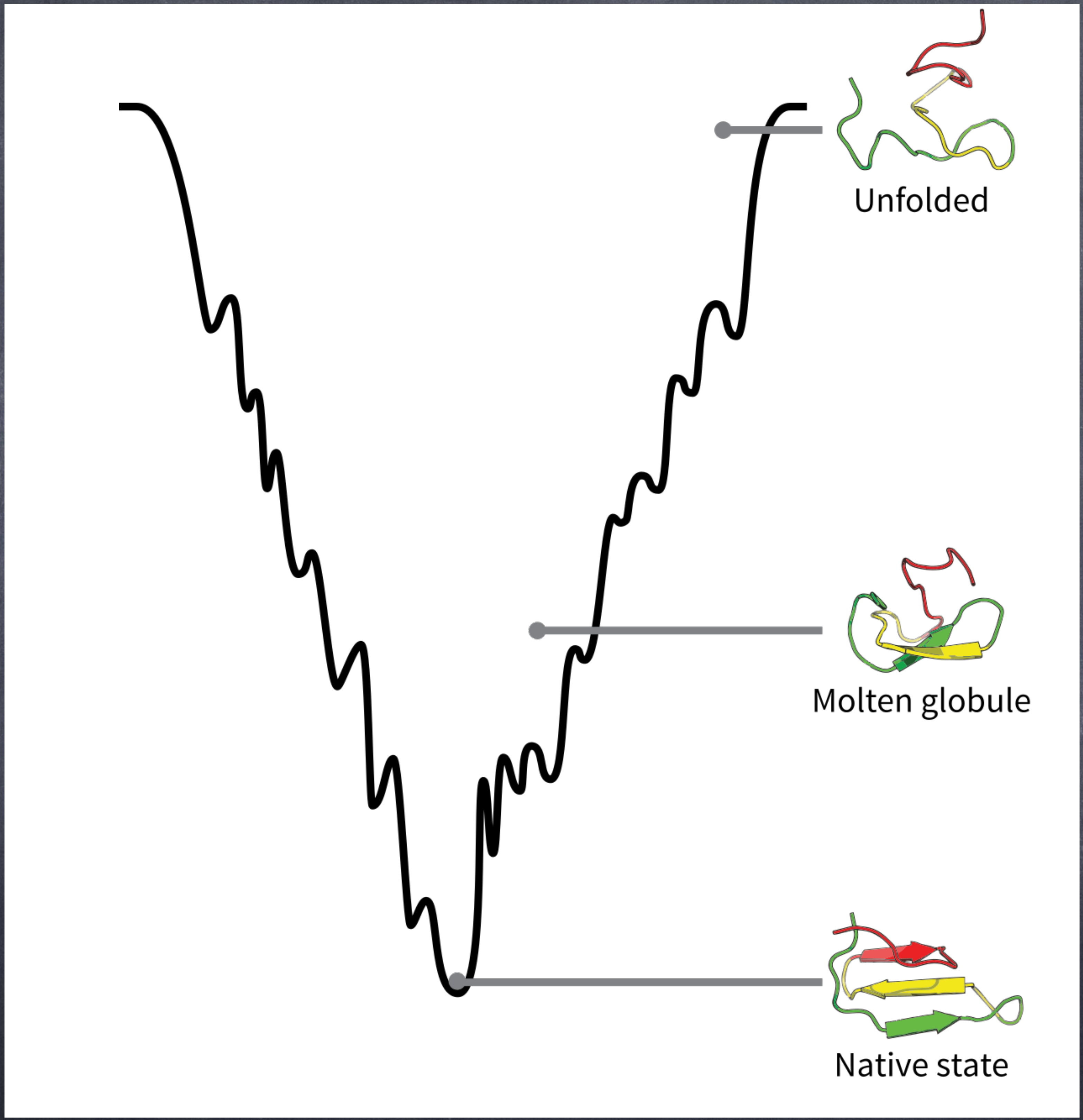
Unfolded
Molten globule
Native state