



Constraints on Quantum Gravity

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Caltech & Kavli IPMU

Kavli IPMU Colloquium, 3 April 2019

What I would like to tell you today:

- Why has the unification of general relativity and quantum mechanics been difficult?
- ☆ Why is superstring theory important?
- ☆ What is the holographic principle?
- ☆ What are known and not known about quantum gravity ?

Why has the unification of general reativity and quamtum mechanics been difficult?

It is often said that, since the Einstein theory,

$$S = \int d^4x \sqrt{-g} \left(\Lambda + \frac{1}{G_N} R + \text{matter} \right)$$

is *not renormalizable*, we cannot use it to compute quantum gravity effects reliably.

This is not the whole story.

For example, the pion theory in nuclear physics,

$$S = \int d^4x \frac{(2\pi (\pi))^2}{1 + \pi (\pi)^2/F^2}$$

is also not renormalizable.

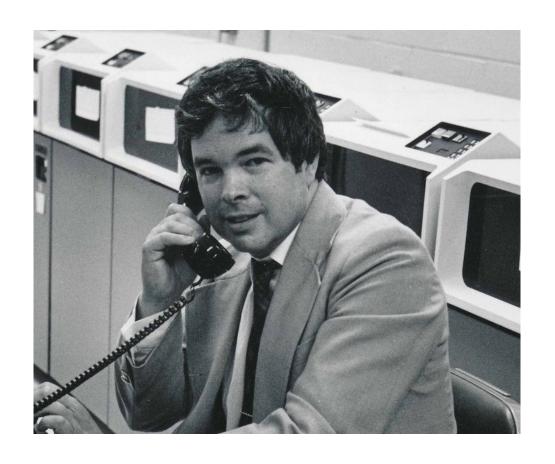
Neverthelss, we can used it to study phenomena whose energy scale is less than F [\sim 184 MeV], and compute their quantum effects reliably.

In the pion theory,

$$S = \int d^4x \frac{(2\pi (\pi))^2}{1 + \pi (\pi)^2/F^2}$$

we can expand observable quantities in powers of [energy/F] and [momenta/F].

Each term in the perturbative expansion can be calculated systematically by renomalizing finite number of parameters.



Kenneth G. Wilson (1936 - 2013)

Wilsonian View:

The pion theory is a low energy approximation to QCD. It can be derived by performing QCD functional integral while freezing the low energy pion degrees of freedom.

⇒ Effective Theory

Despite being non-renormalizable, low energy predictions including quantum effects can be made; they have been verified experimentally.

Einstein gravity is also an effective theory

As in the pion theory, the Einstein gravity can be used to make reliable predictions including quantum effects, provided energy and momenta are much less than its cutoff scale (threshold above which a more fundamental theory is required).

For example: ☆ Hawking radiation from a black hole

- ☆ Cosmic microwave background fluctions caused by quantum effects during the inflation
- ☆ Corrections to the newton potential

$$V = -\frac{G_{N}m_{1}m_{2}}{r} \left(1 + 3 \frac{G_{N}(m_{1} + m_{2})}{r} + \frac{41}{10\pi^{2}} \frac{G_{N}h}{r^{2}} + \cdots \right)$$

relativity effect

one-loop

Issues:

- [1] In relativity, energy cutoff depents on observers.Does the Wilsonian approach really work in gravity?
 - ← Black hole firewall paradox, Swampland
- [2] The pion theory can be UV-completed by QCD, which is a consistent quantum theory.
 - Is the Einstein gravity with any matter fields in low energy guaranteed to have a UV completion?
- [3] The asymptotic freedom of strong interaction was not expected in the pion theory.
 - Many interesting phenomena specific to quantum gravity, such as a fate of an evaporating black hole, top-down derivation of inflation models, and the initial singularity of the Universe, cannot be addressed by the Einstein gravity.

[2] The pion theory can be UV-completed by QCD.

Is the Einstein gravity guaranteed to have a UV completion?

Existence Theorem of Consistent Quantum Gravity

- ☆ In superstring theory, one can compute quantum effects without UV cuttoff to all order in the perturbative expansion. The AdS/CFT gives its non-pertrubative completion.
- ☆ Its low energy effective theory contains the Einstein gravity.
- By compactifying the spacetime dimensions to 4, we find:
 - several generations of quarks and leptons
 - gauge interactions including SU(3) x SU(2) x U(1)
 - Higgs mechanism

The theory contains all ingredients for the Standard Model.

[2] The pion theory can be UV-completed by QCD.

Is the Einstein gravity guaranteed to have a UV completion?

However, we will see later in this colloquium that not all low energy theories of gravity have UV-completion.

Swampland

This is contrary to what we know about theories without gravity.

Another special feature of quantum gravity:

The physical world is hierarchical. Historically, exploration of shorter distances led us to more fundamental laws of nature.

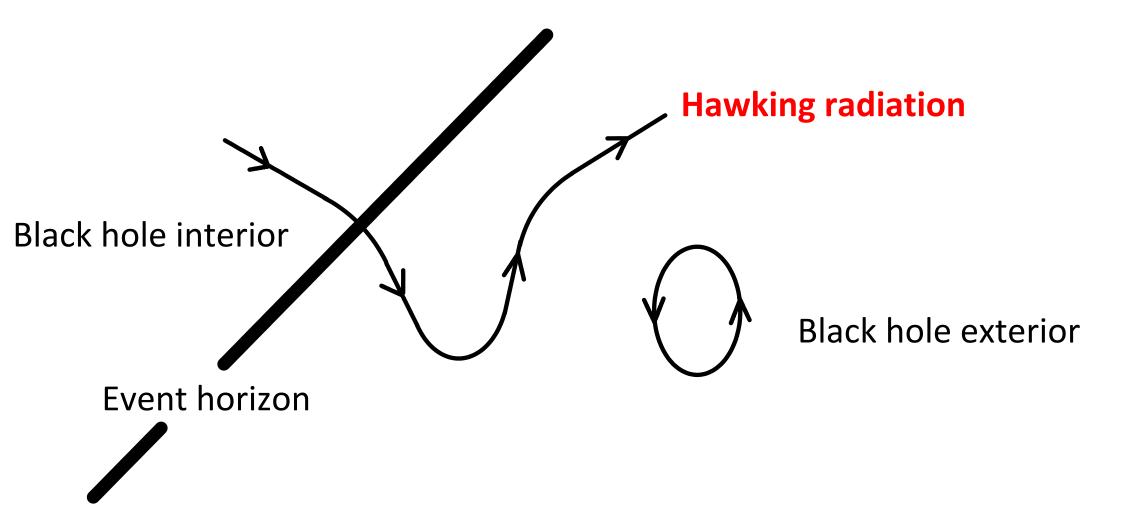
This hierarchy of scales will terminates once we complete quantum gravity.

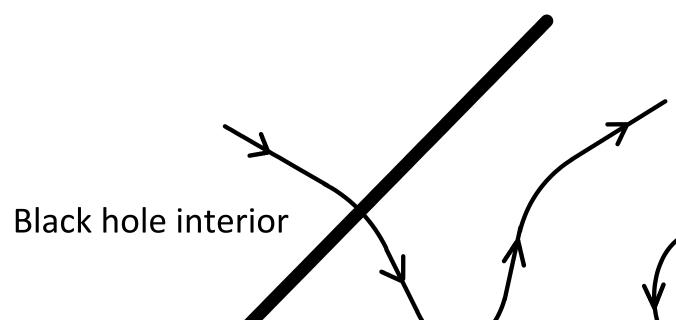
Quantum gravity will lead us to the ultimate unification of elementary particles and forces.

Black Hole Paradox and Holographic Principle



Stephen Hawking (1942 - 2018)





Hawking radiation

Black hole exterior



$$\frac{1}{T} = \frac{\partial S}{\partial E} \Rightarrow \frac{entropy}{energy}$$

$$S = \frac{1}{4GN} \left(\begin{array}{c} Area & of \\ Event & Horizon \end{array} \right)$$

$$Newton constant$$

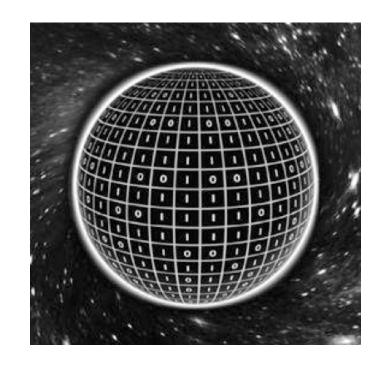
temperature

$$S = \frac{1}{4G_N} \left(\begin{array}{c} Area & of \\ Event Horizon \end{array} \right)$$
 Why is it proportional to the area?

Entropy is extensive.

Holographic Principle

Fundamental degrees of freedom for a region of spacetime are defined on the surface surrounding it.



The holographic principle is realized in string theory.



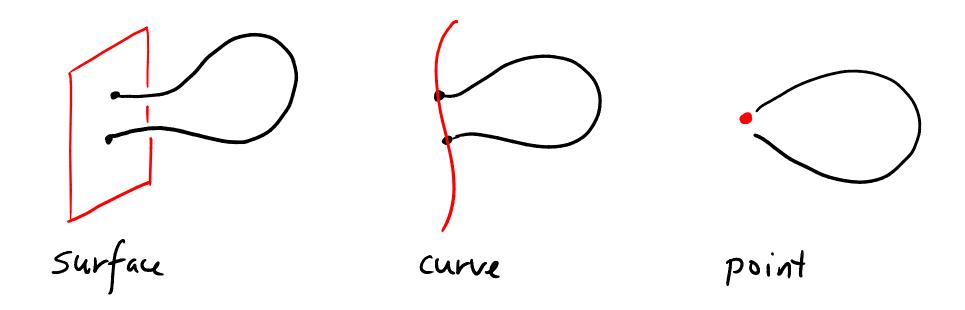
D-Branes

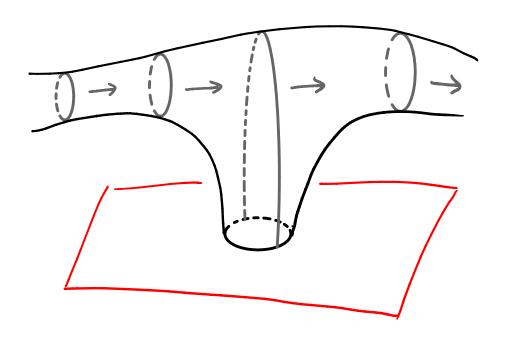
Joseph Polchinski (1954 - 2018)

There are closed strings and open strings

There are closed strings $\left(\begin{array}{c} \\ \end{array}\right)$ and open strings $\left(\begin{array}{c} \\ \end{array}\right)$

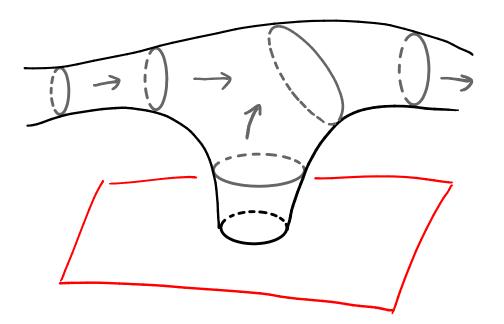
We need to specify a sub-space on which open strings can end.

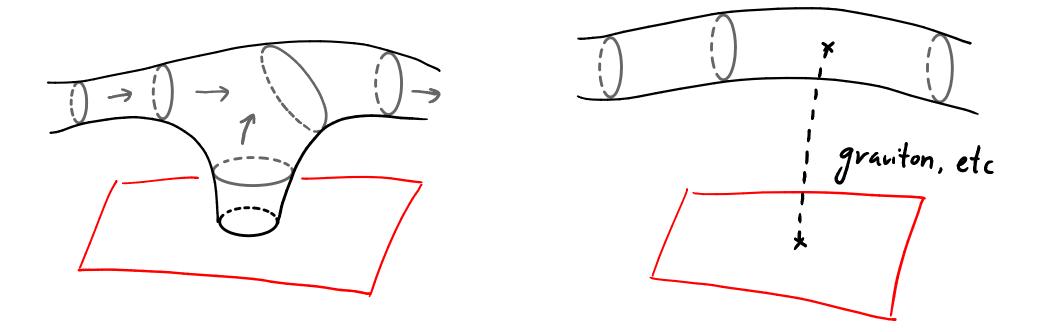




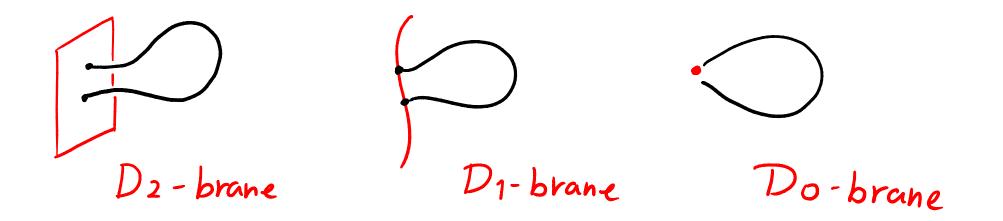
When open string end-points are located in a sub-space,

the sub-space can emit and absorb closed strings.





The graviton is also a closed string state. The fact that a sub-space can emit and absorb closed strings means that mass/energy is localized on the sub-space.

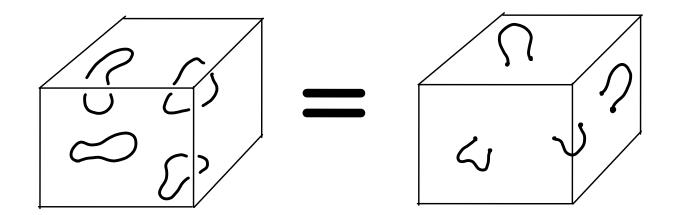


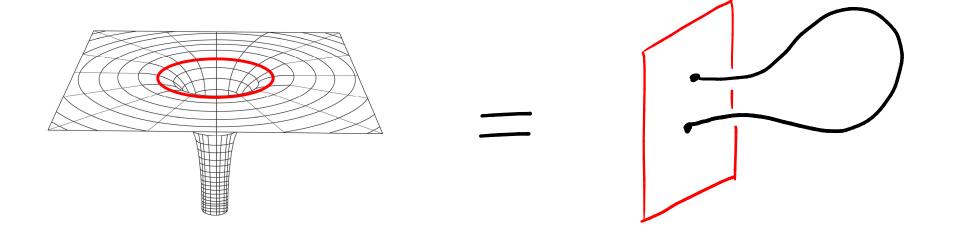
Since mass/energy is localized on the D-brane, it becomes a black hole/brane if the gravity is strong.

By quantizing open strings on the D-brane and by analyzing its Hilbert space, one can count black hole microstates.

In cases when this calculation can be done exactly,

$$S = \frac{1}{4G_N} \left(\begin{array}{c} Area & of \\ Event Horizon \end{array} \right)$$
 has been reproduced.





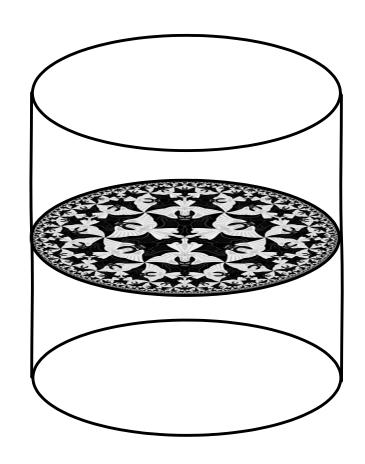
Physical phenomena on the event horizon of a black hole can be described by quantum theory of **open strings** on the corresponding D-brane.

- ⇒ Quantum theory of open strings does not contain gravity.
- ⇒ Gravitational phenomena can be described by the non-gravitational theory, localized on the horizon.

AdS/CFT correpsondence:

Gravitational theory in anti-de Sitter space (AdS) is equivalent to conformal field theory (CFT) at the boundary.

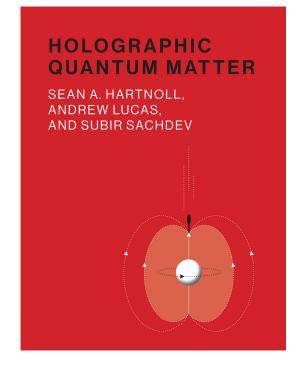


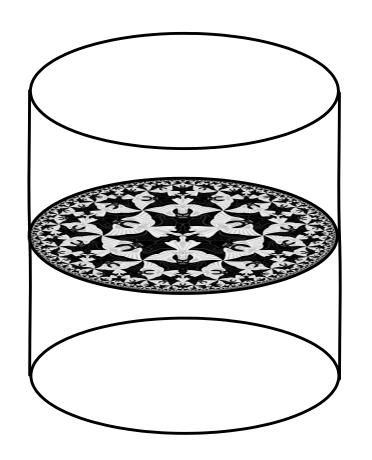


The evaporation of a black hole by the Hawking radiation can be described by a unitary time evolution in CFT.

(*In principle*, it provides a solution to the information paradox.)

The AdS/CFT correspondence defines a consistent quantum theory of gravity including non-perturbative effects.





There are important applications to condensed matter physics and hadron physics, but we will not discuss them today.

Instead, let me discuss new insights into quantum gravity provided by the holographic principle.

Quantum Entanglement and Emergence of Spacetime

Quantum Entanglement

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, Institute for Advanced Study, Princeton, New Jersey (Received March 25, 1935)

$$\mathcal{H}_{A} \otimes \mathcal{H}_{B} \text{ with } \mathcal{H}_{A} = \{10\}_{A}, 11\}_{A}, \mathcal{H}_{B} = \{10\}_{B}, 11\}_{B}\}$$

$$\begin{cases} 10 \rangle_{A} \mid 0 \rangle_{B} : \text{no entanglement} \\ 1EPR \rangle = \frac{1}{\sqrt{2}} (10)_{A} \mid 0 \rangle_{B} + |1\rangle_{A} \mid 1\rangle_{B}) : \text{maximally entangled} \end{cases}$$

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Entanglement entropy: quantifying the entanglement

$$\begin{array}{l}
|1+\rangle \in \mathcal{H}_{A} \otimes \mathcal{H}_{B}, \text{ partial trace } \mathcal{S}_{A} = th_{\mathcal{H}_{B}}(14) < |1+\rangle \\
S(14) = -th_{\mathcal{H}_{A}}(\mathcal{S}_{A} \log_{2} \mathcal{S}_{A}) \\
\downarrow^{\text{Symmetric}} = \begin{cases} 0 & (14) = 10 \\ 1 & (14) = 1EPR \end{cases}$$

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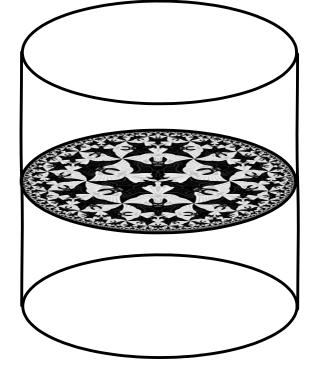
Entanglement entropy: quantifying the entanglement

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S(14) = -th_{\mathcal{H}_{A}}(P_{A} \log_{2} P_{A}) \\
= \begin{cases} 0 & (14) = 10 \rangle_{A} \log_{B} \\
1 & (14) = 1EPR \end{cases}$$

How many EPR pairs can be extracted from $|\psi\rangle$

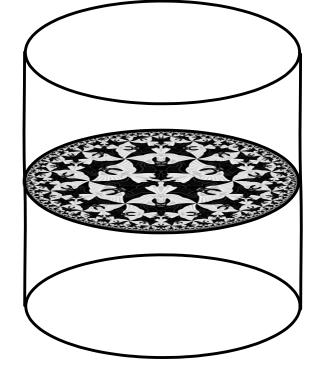
AdS/CFT correspondence:

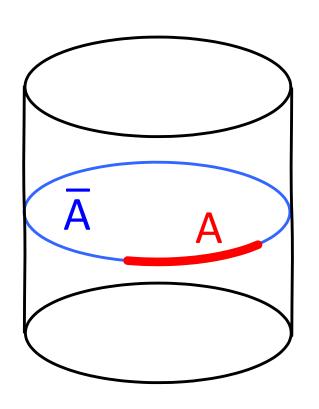
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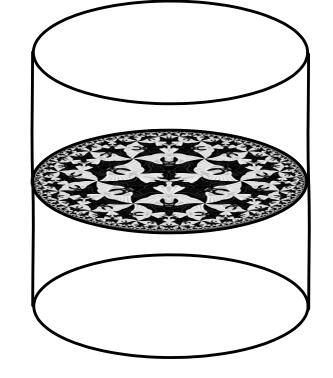


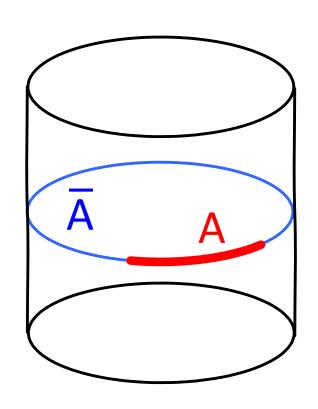


For a given choice of a sub-region A and its complement \overline{A} of the Cauchy surface, we can decompose the total Hilbert space of CFT into a direct product of Hilbert spaces associate to \overline{A} and \overline{A} .

AdS/CFT correspondence:

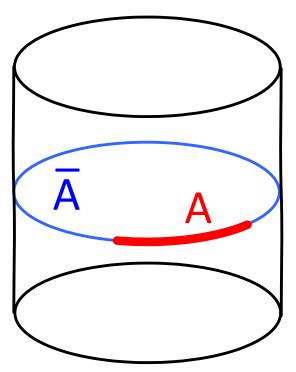
Gravitational theory in AdS is equivalent to CFT at the boundary.





For a given choice of a sub-region **A** and its completed of the Cauchy surface, we can detect the total Hilbert space of CFT into a direct product of Hilbert spaces associate to **A** and **A**.

We need to say the right set of words such as Tomita-Takesaki (富田-竹崎).



For a given choice of a sub-region A and its complement A of the Cauchy surface, we can decompose the total Hilbert space of CFT into a direct product of Hilbert spaces associate to A and A.

To measure entanglement between A and \overline{A} for $|\Psi\rangle$, $\beta(|\Psi\rangle) = t_{H_{\overline{A}}}(|\Psi\rangle\langle\Psi|) \quad \text{partial trace}$ $S(|\Psi\rangle) = -t_{H_{\overline{A}}}(|P|\log_{e}P) \quad \text{entanglement}$ entropy

Ryu-Takayanagi Formula for Entanglement Entropy

PRL 96, 181602 (2006)

PHYSICAL REVIEW LETTERS

week ending 12 MAY 2006

Holographic Derivation of Entanglement Entropy from the anti-de Sitter Space/Conformal Field Theory Correspondence

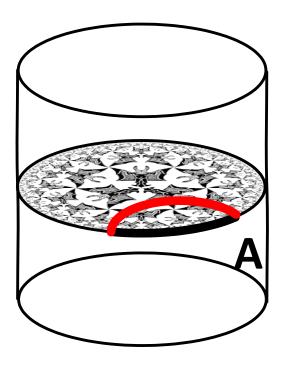
Shinsei Ryu and Tadashi Takayanagi

Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA

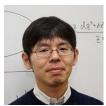
(Received 8 March 2006; published 9 May 2006)

$$g(14) = th_{\mathcal{H}_{\overline{A}}}(14) \langle \psi |$$

$$S(14>) = -th_{HA}(plug_e p)$$







Finite temparature state can be regarded as an entangled state:

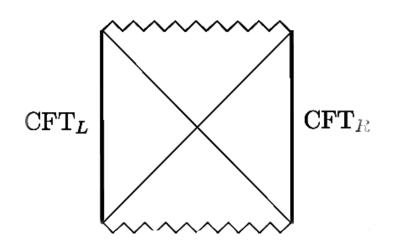
Thermo Field Double:
$$|TFD\rangle \sim \sum_{i} e^{-\frac{E_{i}}{2kT}} |i\rangle_{A} |i\rangle_{B}$$

The higher the temperature T, the more entanglement:

$$T \rightarrow 0$$
: $|TFD\rangle \sim |O\rangle_A |O\rangle_B$
 $T \rightarrow \infty$: $|TFD\rangle \sim |D| |A|i\rangle_B + \cdots$
 $E_i \ll kT$

Finite temperature state can be regarded as an entangled state.

$$\sum_{i} e^{-\frac{E_{i}}{2kT}} |i\rangle_{A} |i\rangle_{B}$$



In AdS gravity, a finite temperature state can be interpreted as an eternal black hole (with two asymptotic AdS regions).

The strength of the entanglement (i.e., the **number of EPR pairs**) is proportional to the **size of the Einstein-Rosen bridge**.

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MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

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JULY 1, 1935

PHYSICAL REVIEW

VOLUME 48

The Particle Problem in the General Theory of Relativity

A. EINSTEIN AND N. ROSEN, Institute for Advanced Study, Princeton (Received May 8, 1935)

ER = EPR?

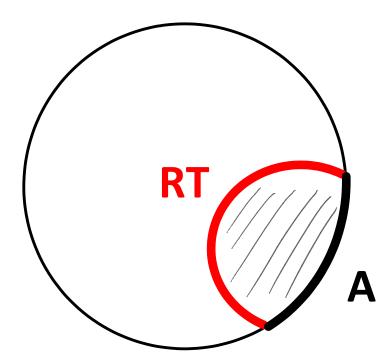
Fortschr. Phys. **61**, No. 9, 781 – 811 (2013) / **DOI** 10.1002/prop.201300020

Cool horizons for entangled black holes

Juan Maldacena^{1,*} and Leonard Susskind²

- ¹ Institute for Advanced Study, Princeton, NJ 08540, USA
- ² Stanford Institute for Theoretical Physics and Department of Physics, Stanford University, Stanford, CA 94305-4060, USA

Consider the shaded sub-region bounded by **A** on the boundary and the Ryu-Takayanagi surface **RT** subtending it.



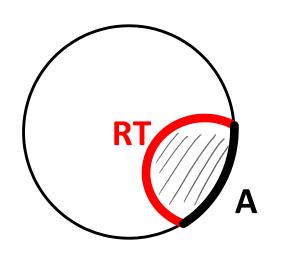
Quantum gravity operator localized in the **shaded region in AdS** can be represented by an operator acting on the sub-region **A of CFT**.

Hamilton, Kabat, Lifschytz, Lowe: hep-th/0606141

Papadodimas, Raju: 1310.6335

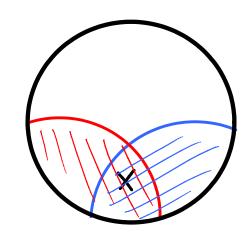
Headrick, Hubeny, Lawrence, Rangamani: 1408.6300

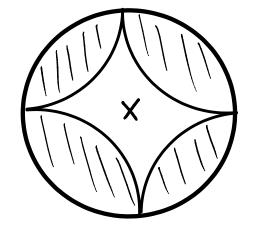
Almheiri, Dong, Harlow: 1411.7041, Dong, Harlow, Wall: 1601.05416



Quantum gravity operator localized in the **shaded region in AdS** can be represented by an operator acting on the sub-region **A of CFT**.

⇒ Reconstruction Paradox

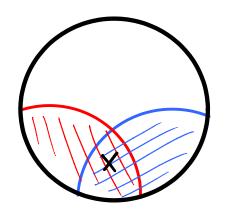


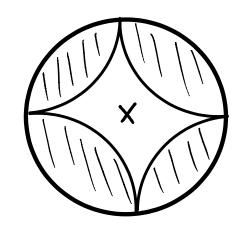


Different operators acting on different sub-spaces of CFT correspond to the same operator in AdS: uniqueness?

A local operator in AdS commutes with every local operator in CFT: contradicting its basic axioms?

Relation to Quantum Error Correcting Codes





Almheiri, Dong, Harlow: 1411.7041

Harlow: 1607.03901

Local excitations of the gravitational theory in AdS correspond to states with a special type of entanglement in CFT similar to the one used for **quantum error correcting codes**, where different sub-spaces of CFT share **quantum secret keys**.

Applications of Quantum Information to Gravitational Theory

Swampland Question

Given an effective theory of gravity, how can one judge whether it is realized as a low energy approprimation to a consistent quantum theory with ultra-violet completion, such as string theory?

Vafa: hep-th/0509212;

Vafa + HO: hep-th/0605264

Constraints on Symmetry

Symmetry has played important roles in physics:

in identifying and formulating fundamental laws of nature

and

in using these laws to understand and predict dynamics and phases of matters.

Symmetry can be deceiving:

Two seemingly different microscopic Lagrangians with **different gauge symmetries** and different matter contents **can describe the same quantum system.**

"Duality"

Equivalencen can be between full Hilbert spaces or about their low energy limits (such as in the Seiberg dualities).

Symmetry can be deceiving:

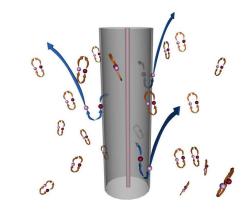
Global symmetry is well-defined and is independent of which Lagrangian description you use.

Symmetry can be deceiving:

Global symmetry is well-defined and is independent of which Lagrangian description you use.

However, it has been argued that a consistent quantum theory of gravity does not have global symmetry.

Standard argument for



No global symmetry in quantum gravity:

If there is a continuous global symmetry G, we can combine a large number of G-charge matters to make a black hole in an arbitrary large representations of G.

Let it Hawking-radiate, keeping its mass > the Planck mass.

Since the Hawking radiation is G-blind, the black hole still contains the large representation of G with the number of states exceeding the Bekenstein-Hawking entropy formula.

We have given precise formulation of the conjecture in the context of the AdS/CFT correspondence and proved them using locality of CFT. (with D. Harlow, arXiv:1810.05338 and its synopsis arXiv:1810.05337)

- (1) Quantum gravity with global symmetry is inconsistent, whether the symmetry is discrete or continuous.
- (2) Global symmetry in CFT corresponds to gauge symmetry in gravity.
- (3) Completeness: There must be physical states in every irreducible unitary representation in gauge symmetry.
 - + with some additional assumption:(4) Internal (non-spacetime) symmetry is compact.

We need to define what we mean by symmetry.

Global Symmetry

Standard definition:

For every element g of group G, there is a unitary operator U(g) on the Hilbert space such that,

$$U(g_1) U(g_2) = U(g_1 \cdot g_2),$$

 $LU(g), Hamiltonian] = 0.$

How about the projection operator onto the 42th eigenstates?

We would like to make additional locality assumptions.



Global Symmetry

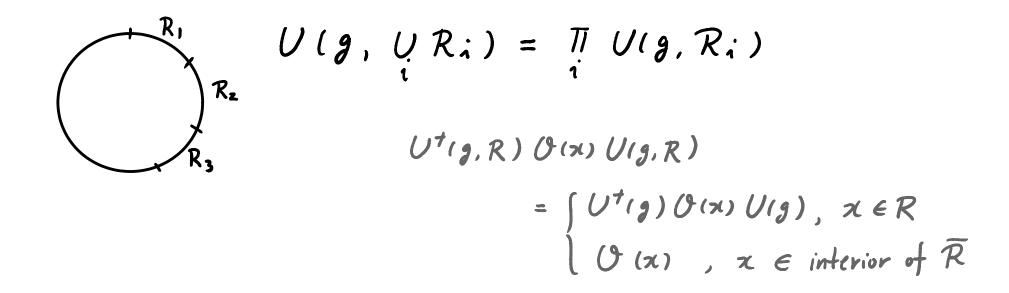
We sharpen our requirements:

- (1) Symmetry should map a local operator to a local operator.
- (2) Symmetry action should be faithful on the set of local operators.
- (3) Symmetry should commute with the energy-momentum tensor.

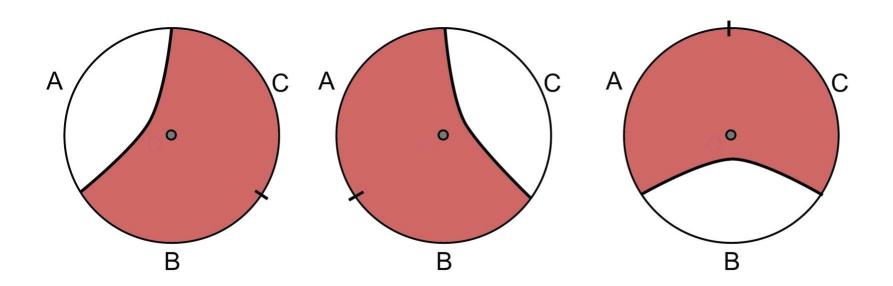
Global Symmetry

We sharpen our requirements:

- (1) Symmetry should map a local operator to a local operator.
- (2) Symmetry action should be faithful on the set of local operators.
- (3) Symmetry should commute with the energy-momentum tensor.
- (4) For a set of open disjoint subspaces of the Cauchy surface:



In the following, we will apply the entangement wedge reconstruction.

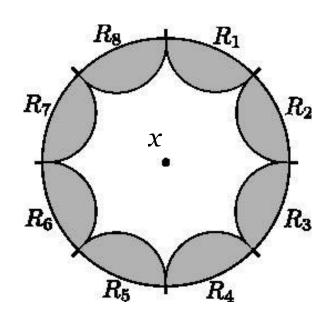


Global symmetry in AdS is inconsistent with local structure of CFT.

If a gravitational theory in AdS has global symmetry G, there must be a bulk local operator that transforms faithfully into another local operator at the same point.

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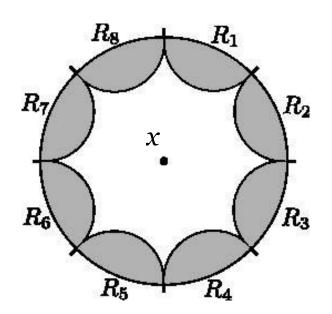
Symmetry generator,

$$U(g) = \prod_{i} U(g, R_i)$$

commute with the local operator at x in the bulk.

Contradiction

Global symmetry in AdS is inconsistent with local structure of CFT.



Symmetry generator,

$$U(g) = \prod_{i} U(g, \mathcal{R}_{i})$$

commute with the local operator at x in the bulk.

The argument works for spontaneously broken global symmetry. For example, shift symmetry of a scalar field, often invoked in inflation models, must be broken.

The argument also works for discrete spacetime symmetry such as parity and time reversal. They cannot be global symmetry. If unbroken, they must be gauge symmetries. Quantum gravity must sum over no-orientable manifolds.

The argument does not work for 2d gravity (e.g., on string worldsheet) and 3d oriented pure Einstein gravity.

Any global symmetry in low energy effective theory of quantum gravity must be approximate.

How is it broken?

Are symmetry breaking terms power suppressed by the Planck mass?

The completeness theorem states that every finite dimensional unitary representation of gauge symmetry must appear.

Can we bound the mass of the lightest charged particle?

How can we go beyond the AdS/CFT correspondence and prove quantum gravity theorems for more general spacetime?

So far, I have presented theorems I can prove.

Now, we are entering the territory of conjectures.



Weak Gravity Conjecture

In any low energy theory described by the Einstein gravity + Maxwell field + finite number of matters, if it has an UV completion as a consistent quantum theory, there must be a particle with charge Q and mass $m \ll M$ _Planck, such that:

$$m \leq \frac{|Q|}{\sqrt{G}}$$
, $G: Newton Constant$

Arkani-Hamed, Motl, Nicolis, Vafa: hep-th/0601001

$$\frac{1}{2}$$
 (m,Q) 5.t. $m \leq \frac{|Q|}{\sqrt{G}}$

Motivated by:

(1) Black Hole Physics: Extremal black holes should decay unless protected by supersymmetry.

Otherwise, charged black holes can decay to Planck-size remnants with entropies, exceeding the Bekenstein-Hawking bound.

(2) True in all known constructions from string theory.

In all cases,
$$m < \frac{Q}{\sqrt{G}} \quad (no "=") \quad unless BPS.$$

If this sharpened weak gravity conjecture is true, non-SUSY AdS supported by fluxed must be unstable.

Vafa + H.O.: 1610.1533

All known non-SUSY AdS's are marginally stable at best, and some of them are unstable in interesting ways.

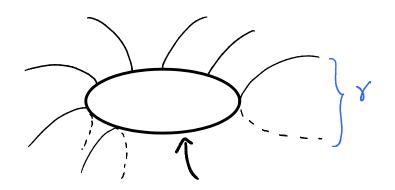
Example: AdS5 x S5 / 7 in IIB:

Supersymmetry is broken when Γ does not fit in SU(3).

★ If \(\bigcap \) has a fixed point or S5 is small, there is a tachyon violating the BF bound.

Dymarsky, Klebanov, Roiban: 0509132

★ If \(\mathbb{\gamma} \) has no fixed point and S5 is large, there is Witten's instanton, creating a bubble of nothing.



Witten (1982)

Horowitz, Orgera, Polchinski: 0709.4262

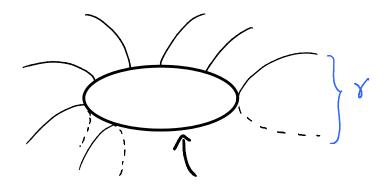
The bulk geometry terminates with S1 collapsing.

Example: AdS5 x CP3 in M Theory:

Supersymmetry is broken.

Though the fundamental group of CP3 is trivial (and thus, there is no Witten's instanton), the geometry allows a generalization of Witten's instanton where a 2-sphere collapses.

Spodyneiko + H.O.: 1703.03105



The bulk geometry terminates with S2 collapsing.



Standard Model of Particle Physics gives rise to a rich landscape of stable dS and AdS vacua in 2 and 3 dimensions upon compactification, depending on types (Majorana or Dirac) of neutrinos and their masses.

Arkani-Hamed Dubovsky, Nicolis, Villadoro: hep-th/0703067

We pointed out that the sharpened weak gravity conjecture would **rule out certain types and masses of neutrinos** if they give rise to stable non-supersymmetric AdS_3.

Vafa + H.O.: 1610.1533

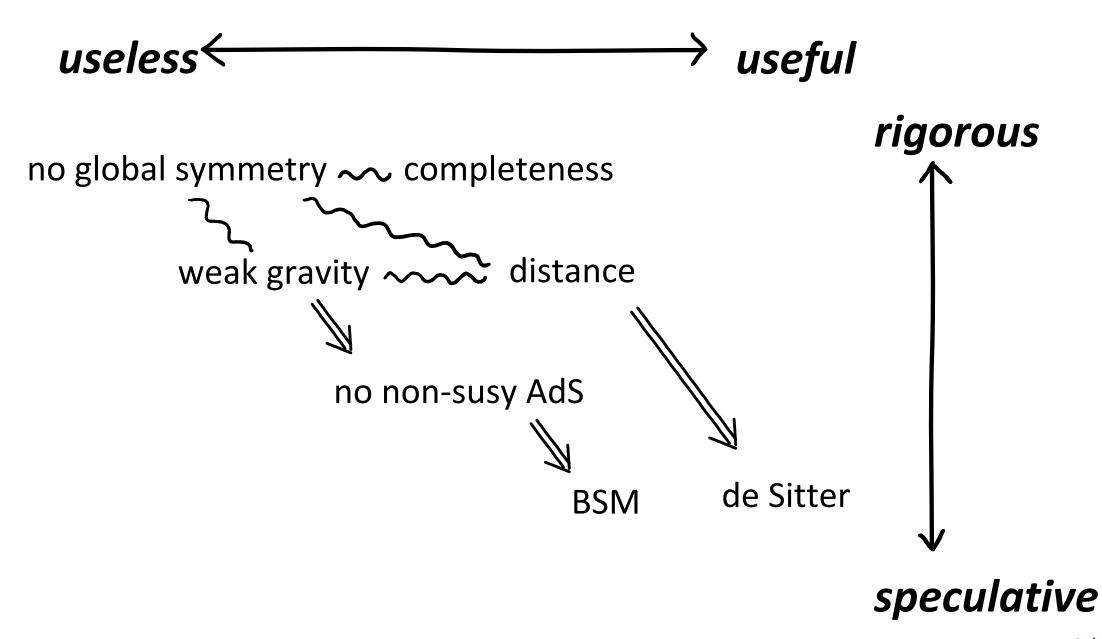
Our idea has been explored further in recent papers, leading to constraints on particle physics models beyond the Standard Model.

Ibanez, Martin-Lozano, Valenzuela: 1706.05392,1707.05811;

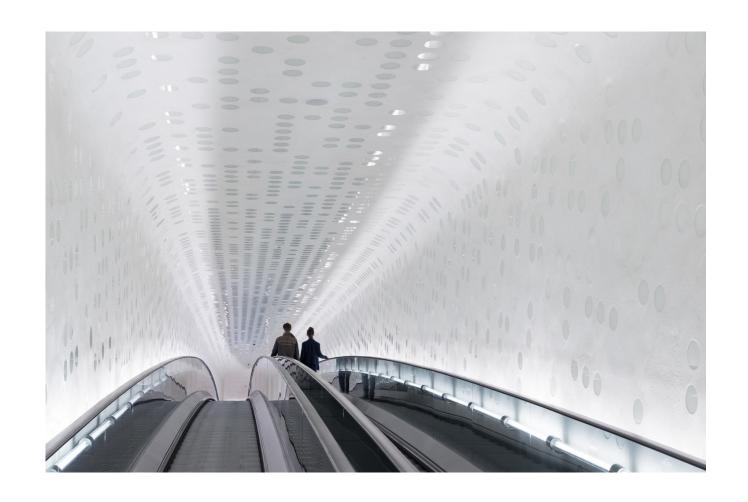
Hamada, Shiu: 1707.06326;

Gonzalo, Herraez, Ibanez: 1803.08455

Landscape of Swampland Conditions



The UV/IR connection may imply surprising IR predictions on observable phenomena from UV completion of quantum gravity.



Collaborations with quantum information may provide a key to quantum gravity.



Thank you.