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Primordial perturbations from cosmological inflation

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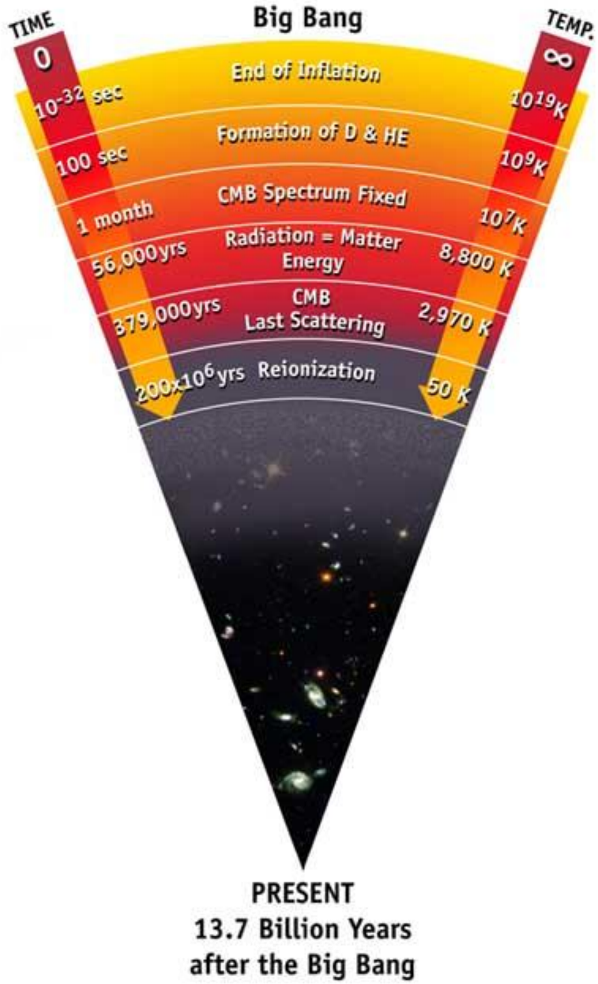
outline:

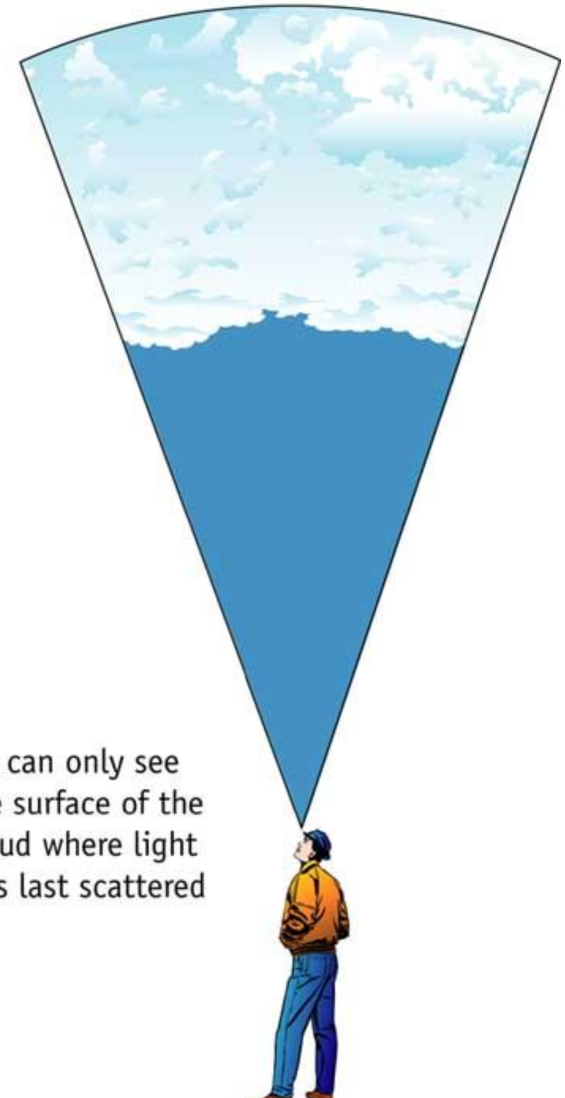
- Primordial structure
  - evidence of density inhomogeneities in the very early universe (primordial = 1 sec after big bang)
- Fluctuations from inflation
  - Vacuum fluctuations during inflation
  - Primordial density perturbations after inflation
- Distinguishing models with non-Gaussianity
  - Local non-Gaussianity
  - Bispectrum and f_{NL}
  - Higher-order statistics and scale-dependence

how far back can we look?



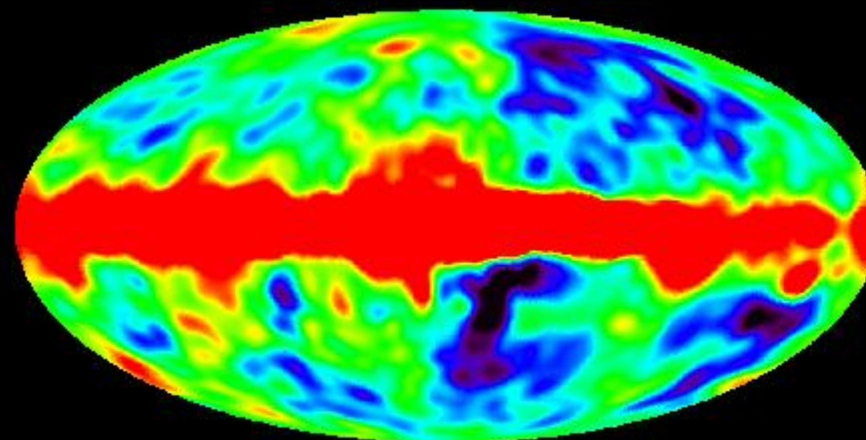
The cosmic microwave background Radiation's "surface of last scatter" is analogous to the light coming through the clouds to our eye on a cloudy day.

We can only see the surface of the cloud where light was last scattered

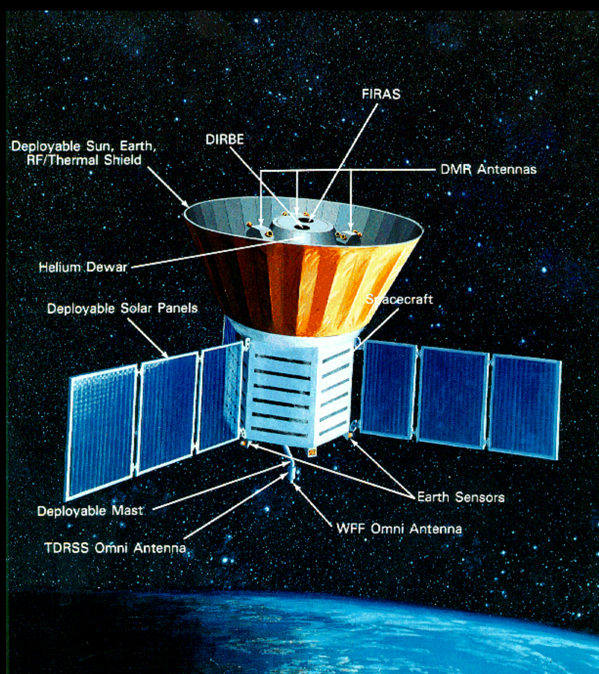


Cosmic microwave background radiation

- discovered by Penzias and Wilson 1965
- relic thermal radiation from the hot big bang



COBE satellite



2.7 K everywhere

+/- 3.3 mK redshift due local motion
(at 1 million miles per hour)

+/- 18 μK intrinsic anisotropies

an elegant, simple, well-motivated model for the origin of structure in the early universe

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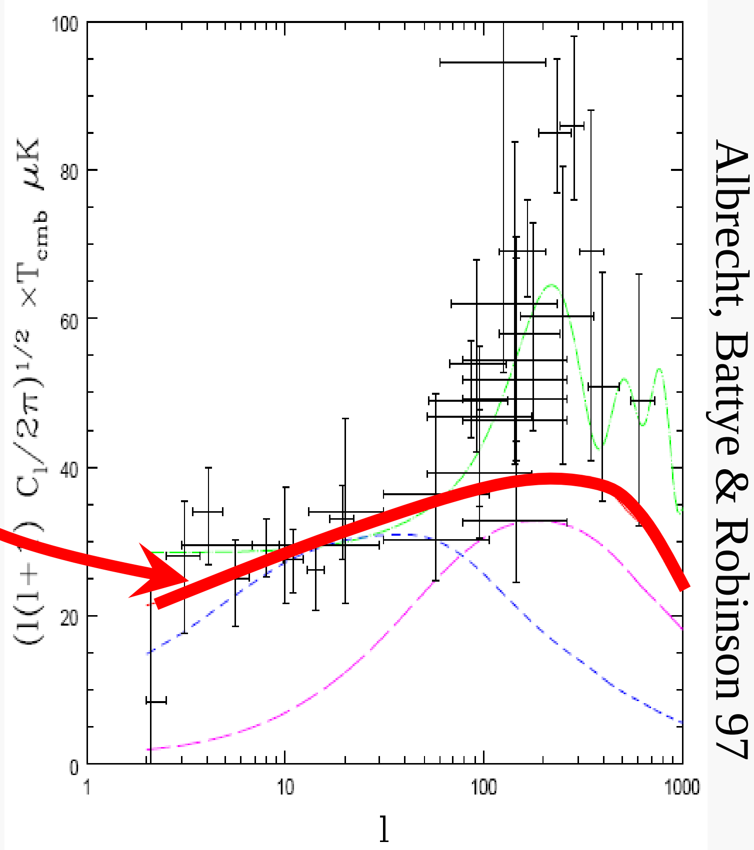
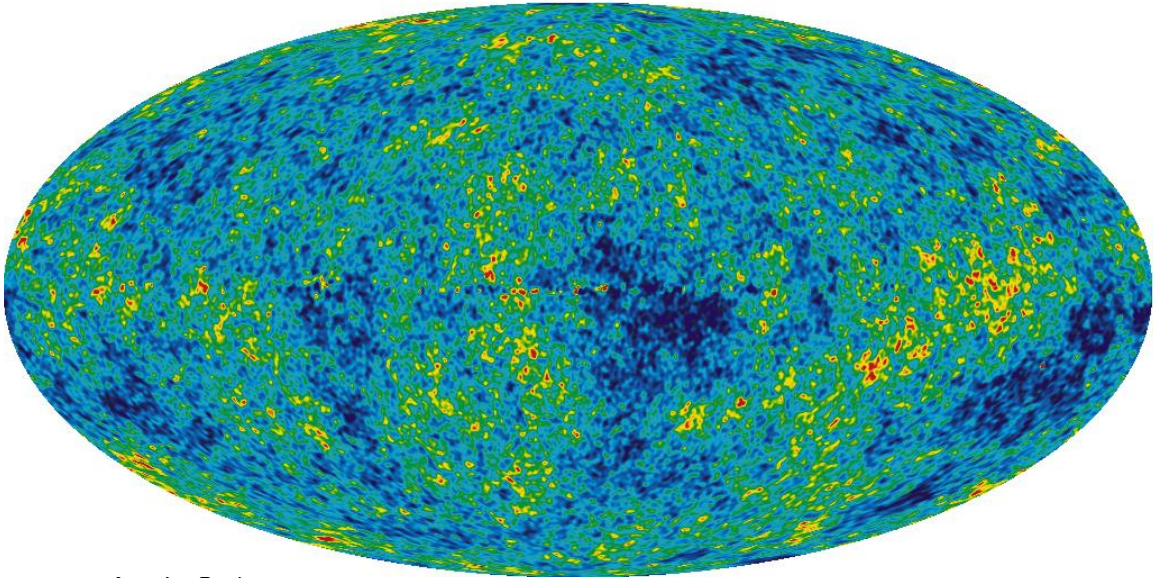
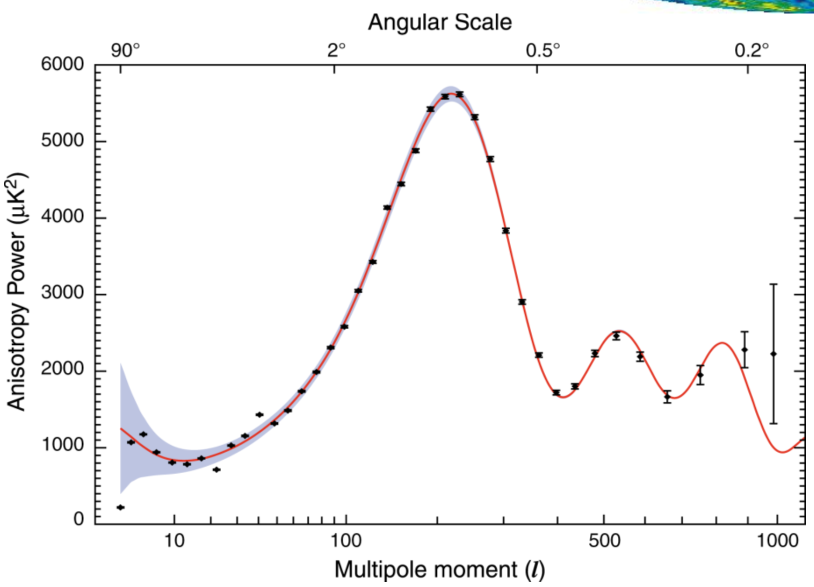
cosmic strings

formed by spontaneous symmetry breaking at GUT scale



Wilkinson Microwave Anisotropy Probe





coherent oscillations in photon-baryon plasma due to existence of primordial density perturbations

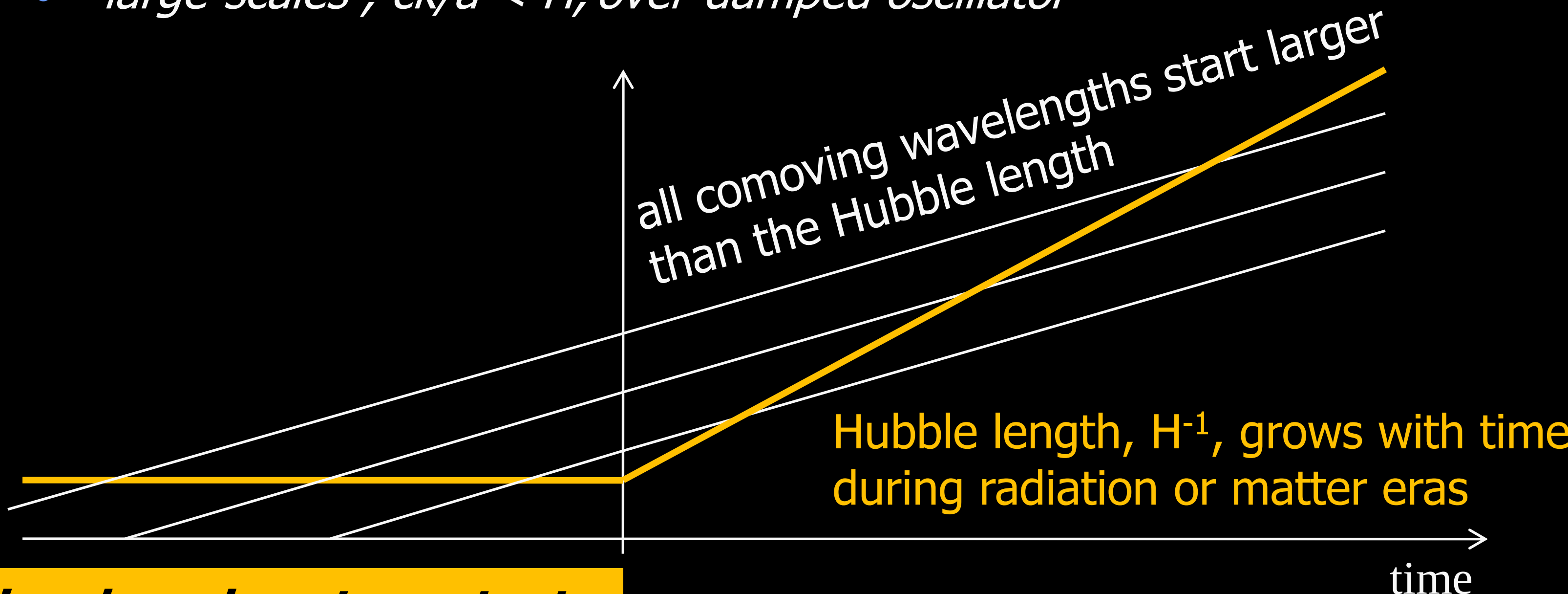
passive not active seeds for structure

Wave equation in FRW cosmology:

$$\delta\ddot{\rho} + 3H\delta\dot{\rho} + (ck/a)^2\,\delta\rho = 0$$

Characteristic timescales for waves, comoving wavemode k

- *small-scales , ck /a > H, under-damped oscillator*
- *large-scales , ck/a < H, over-damped oscillator*



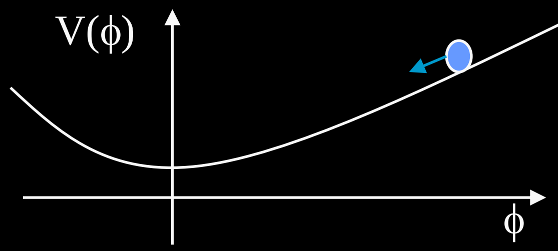
inflation, has almost constant Hubble length

Duration of inflation:

- number of "e-folds" of expansion

$$N = \ln(a) = \int H\, dt$$

- slow-roll inflation driven by a scalar field, ϕ, potential $V(\phi)$



$$N = \int \frac{H}{\dot{\phi}} d\phi = \int \frac{1}{\sqrt{\varepsilon}} \frac{d\phi}{M_{Pl}}$$

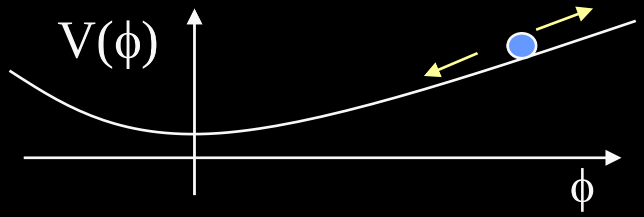
$$\text{where slow-roll parameter}: \quad \varepsilon \equiv -\frac{\dot{H}}{H^2} \approx \frac{1}{2M_{Pl}^{\;2}} \left(\frac{V'}{V} \right)^2$$

- N > 40 to solve the horizon problem

$$\text{Lyth bound}: \quad \Delta\phi > \left(40\sqrt{\varepsilon}\right) M_{Pl}$$

Vacuum fluctuations

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations (k>aH)*

$$\left\langle \delta\phi_k \delta\dot{\phi}^*_{k'} \right\rangle = i\hbar\delta\left(k-k'\right)$$

- *large-scale/overdamped perturbations in growing mode (k<aH)*

linear evolution $\Rightarrow$ *Gaussian random field*

$$\Rightarrow \quad \mathcal{P}\left(\delta\phi\right)_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k{}^2\right|}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any light scalar fields (m<3H/2) `frozen-in' on large scales

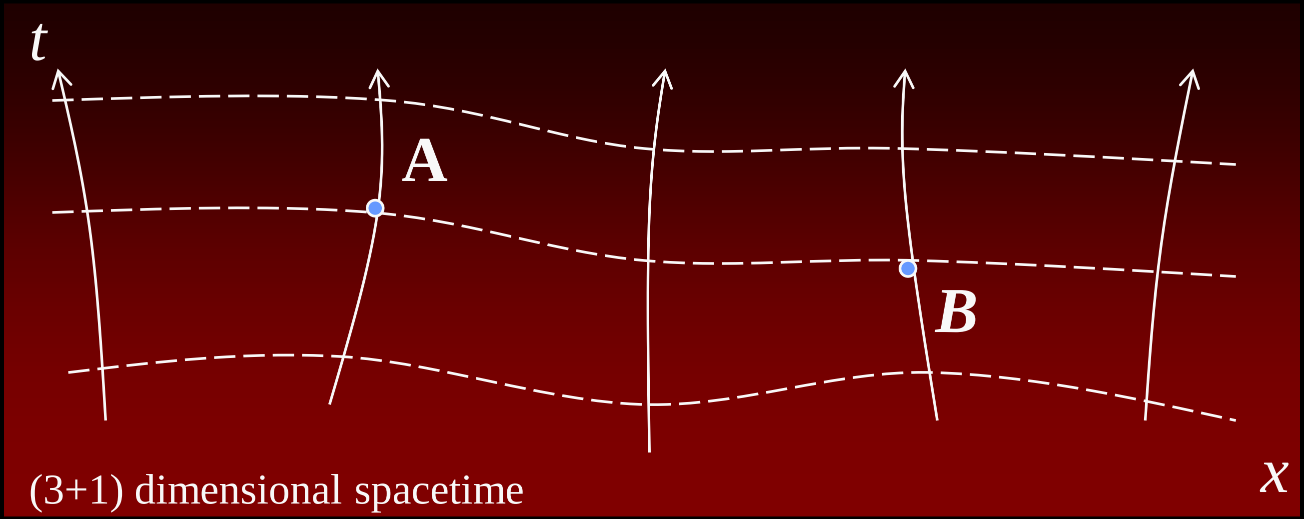
interactions = non-linearity = non-Gaussianity

suppressed during slow-roll inflation $\left\langle \delta\phi_k \delta\phi_{k'} \delta\phi_{k''} \right\rangle \approx \varepsilon \left|\delta\phi_k{}^4\right| \delta\left(k+k'+k''\right)$

defining the primordial density perturbation

gauge-dependent density perturbation, $\delta\rho$, and spatial expansion, δN

$$\rho = \rho_0(t) + \delta\rho(t,x) \qquad ds^2 = a^2(t)\, e^{2\delta N(t,x)}\, \delta_{ij}\, \delta x^i \delta x^j$$



gauge-invariant combination (Bardeen, 88)

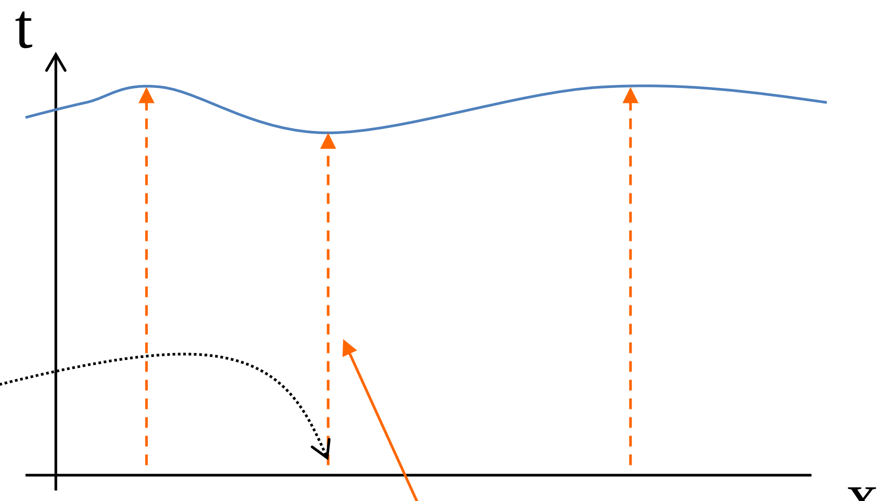
$$\zeta = \delta N - \frac{H}{\dot{\rho}}\delta\rho$$

Provides initial conditions for adiabatic perturbations on large scales, early times,

e.g., Newtonian potential in matter-dominated era: $\Phi = (3/5)\zeta$

the δN formalism for primordial perturbations

in radiation-dominated era
curvature perturbation ζ on
uniform-density hypersurface



during inflation
field perturbations $\phi(x,t_i)$ on
initial spatially-flat hypersurface

$$\boxed{N = \int_{initial}^{final} H\, dt}$$

on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N(\phi_{initial}) - \bar{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky `85; Sasaki & Stewart `96
Lyth & Rodriguez '05 – works to any order

Separate universes

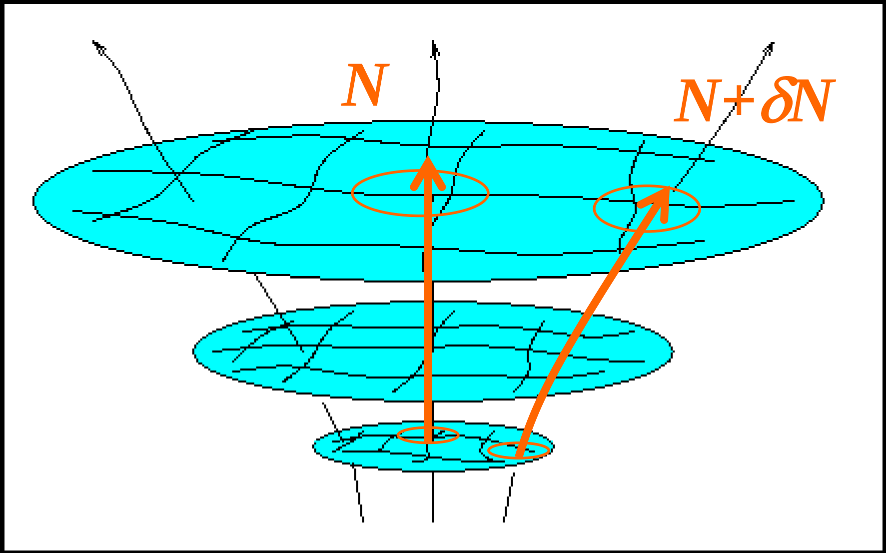
Salopek & Bond (1990)

Wands, Malik, Lyth & Liddle (2001)

- local expansion on very large scales (>> Hubble length, negligible spatial gradients) given by local Friedmann equation

$$N(x) = \int H(x)\,dt$$

$$H^2(x) = \frac{8\pi G_N}{3}\rho(x)$$



density perturbations from inflaton field

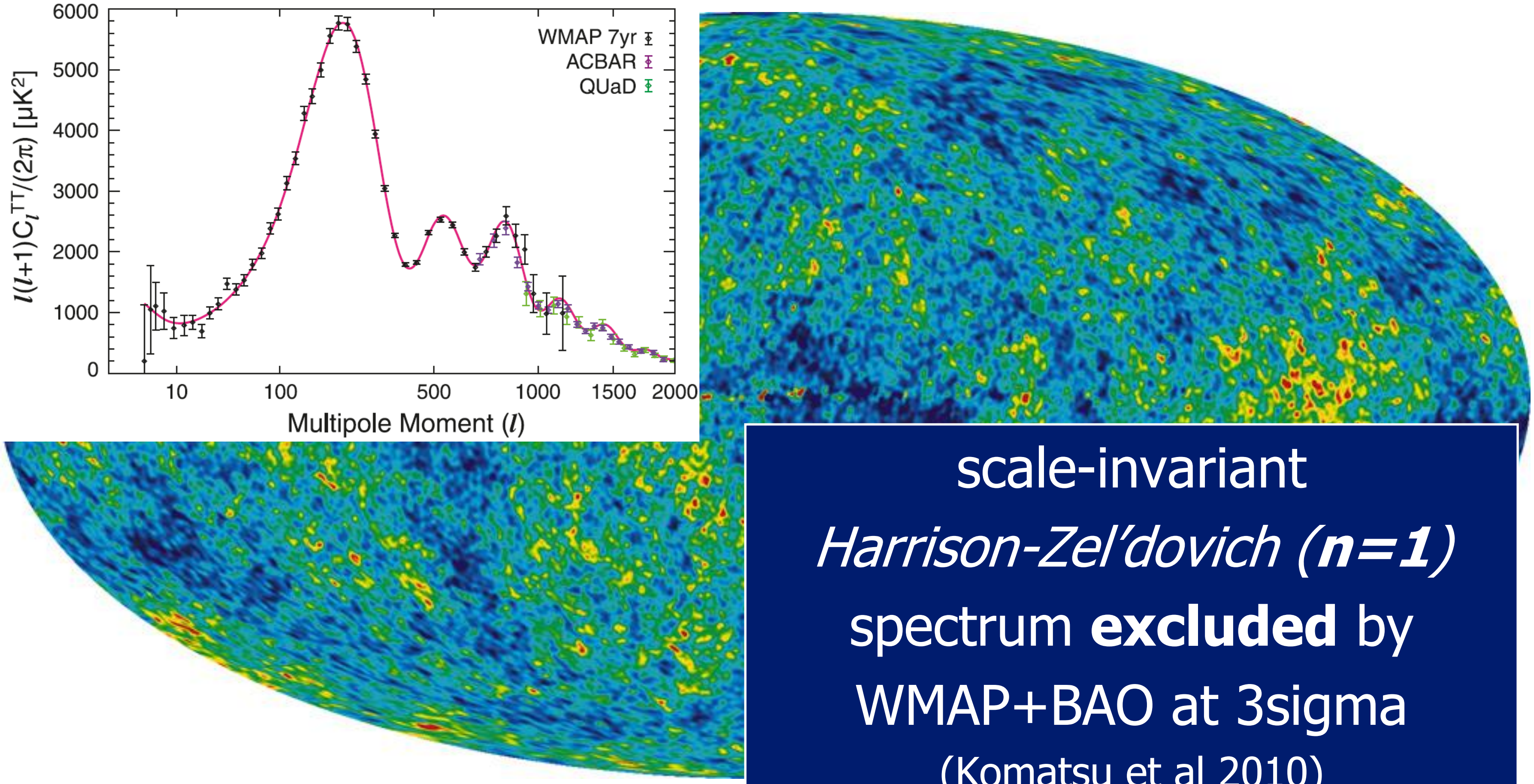
- *quantum field fluctuations on unperturbed (flat) hypersurfaces during inflation leads to scalar metric perturbation*

$$\zeta = \frac{dN}{d\phi}\delta\phi = \left(-\frac{H}{\dot{\phi}}\delta\phi \right)_{k=aH}$$

- *produce primordial density perturbations in radiation-dominated era*

$$\Rightarrow \quad \left\langle \frac{\delta T^2}{T^2} \right\rangle_{SW} \approx \frac{1}{25}\left\langle \zeta^2 \right\rangle \approx \frac{1}{25}\left(\frac{H^2}{2\pi\dot{\phi}} \right)^2_{k=aH}$$

WMAP 7 year data February 2010



scale-invariant

*Harrison-Zel'dovich (**n=1**)*

spectrum **excluded** by

WMAP+BAO at 3sigma

(Komatsu et al 2010)

Spectral tilt = slow-roll dynamics

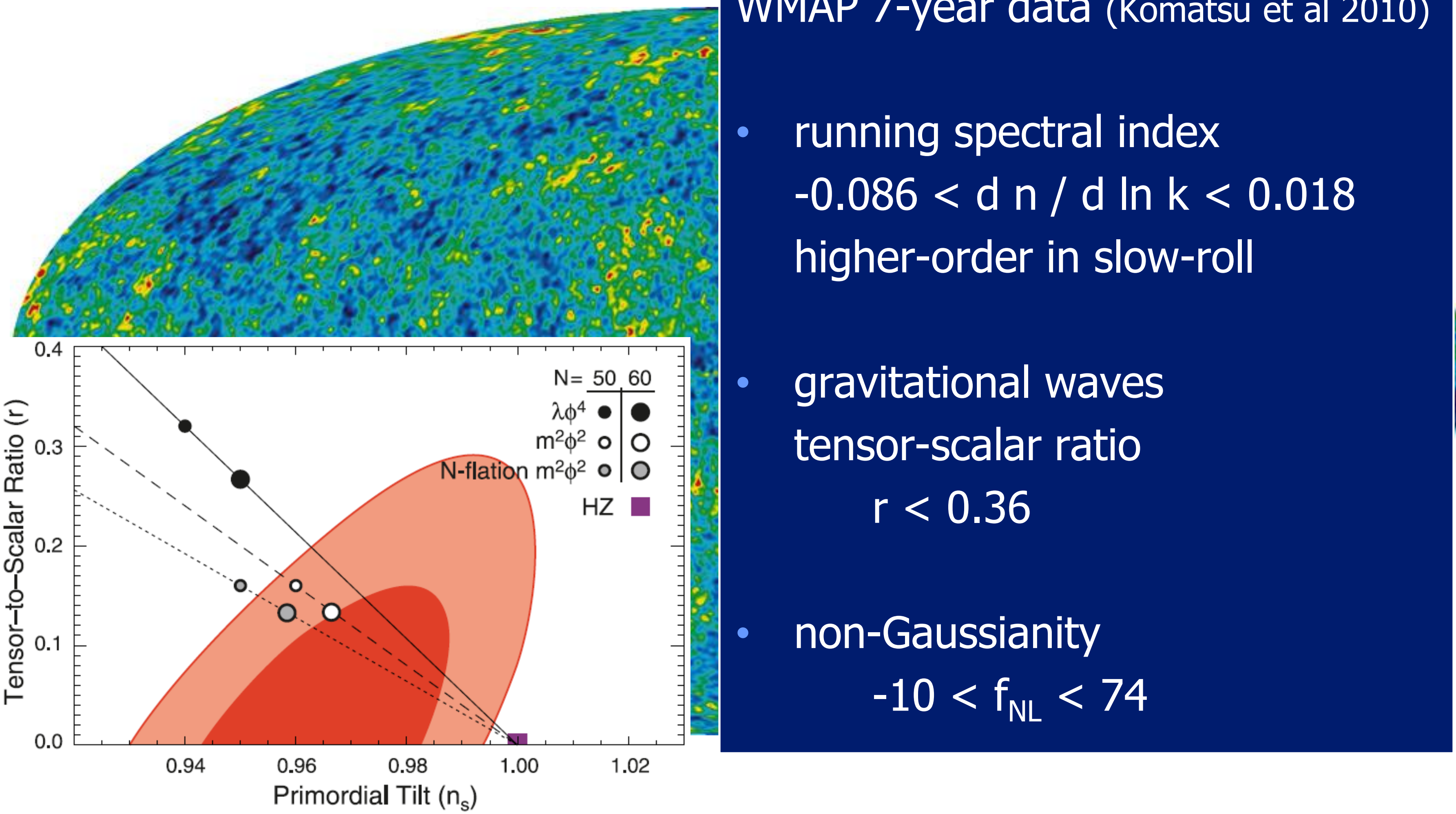
slow time-dependence during inflation
$\Rightarrow$ weak scale-dependence

$$\text{tilt}: \quad n-1 \equiv \frac{d\ln\langle\zeta^2\rangle}{d\ln k} \approx -6\varepsilon + 2\eta_\sigma$$

slow roll parameters $\quad \varepsilon = -\dot{H}/H^2 \quad , \quad \eta_\sigma = {m_\sigma}^2/3H^2$

$$\{\varepsilon, |\eta|\} << 1$$

what next?



WMAP 7-year data (Komatsu et al 2010)

- running spectral index
  $-0.086 < d\,n\,/\,d\ln k < 0.018$
  higher-order in slow-roll
- gravitational waves
  tensor-scalar ratio
  $r < 0.36$
- non-Gaussianity
  $-10 < f_{NL} < 74$

ESA Planck satellite launched!

next all-sky survey

data 2011/12?

$r \approx 0.1$?

$\mathbf{f_{NL} < 8}$

theoretical non-Gaussianity

- **possibilities limited**
  - Local-type non-Gaussianity
    - super-Hubble evolution of Gaussian random field from multi-field inflation
  - Equilateral-type non-Gaussianity
    - sub-Hubble interactions in k-inflation/DBI inflation
  - Topological defects
    - cosmic strings or textures?
- **templates required to develop optimal estimators**
  - matched filtering to extract small non-Gaussian signal

the δN formalism for primordial perturbations

in radiation-dominated era
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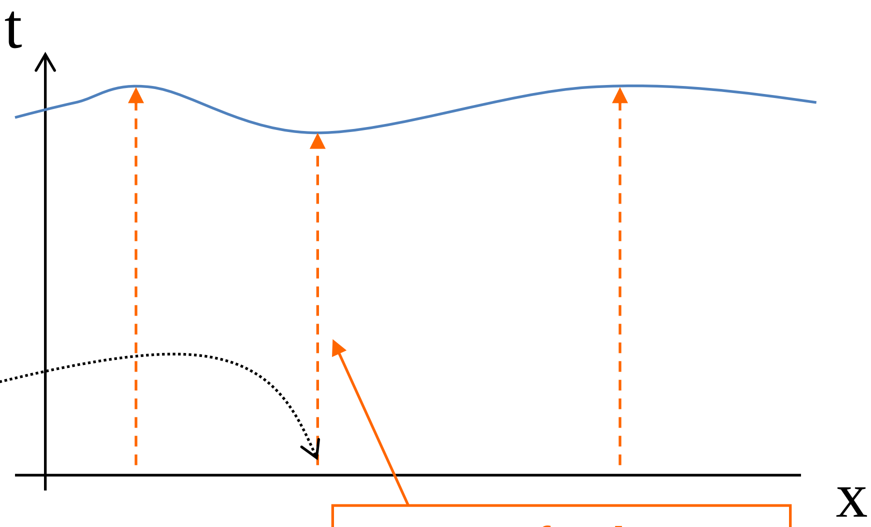


$$N = \int_{initial}^{final} H\, dt$$

on large scales, neglect spatial gradients, treat as "separate universes"

$$\zeta = N(\phi_{initial}) - \overline{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Starobinsky `85; Sasaki & Stewart `96
Lyth & Rodriguez '05 – works to any order

simplest local form of non-Gaussianity

applies to many models inflation including curvaton, modulated reheating, etc

$\zeta = \delta N(\phi)$ is local function of ***single Gaussian random field***, ϕ

$$\zeta = N'\delta\phi + \frac{1}{2}N''\delta\phi^2 + \frac{1}{6}N'''\delta\phi^3 + \ldots$$

$$\Rightarrow \quad \langle \zeta(x_1)\zeta(x_2)\rangle = N'^2 \langle \delta\phi(x_1)\delta\phi(x_2)\rangle + \ldots$$

$$\langle \zeta(x_1)\zeta(x_2)\zeta(x_3)\rangle = \frac{1}{2}N'^2 N'' \langle \delta\phi(x_1)\delta\phi(x_2)\delta\phi^2(x_3)\rangle + \ldots$$

$$= \frac{3}{5} f_{NL} \langle \zeta(x_2)\zeta(x_3)\rangle \langle \zeta(x_1)\zeta(x_3)\rangle + \ldots$$

where $$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2}$$

- odd factors of 3/5 because (Komatsu & Spergel, 2001, used) $\Phi_1 = (3/5)\zeta_1$

evidence for local non-Gaussianity?

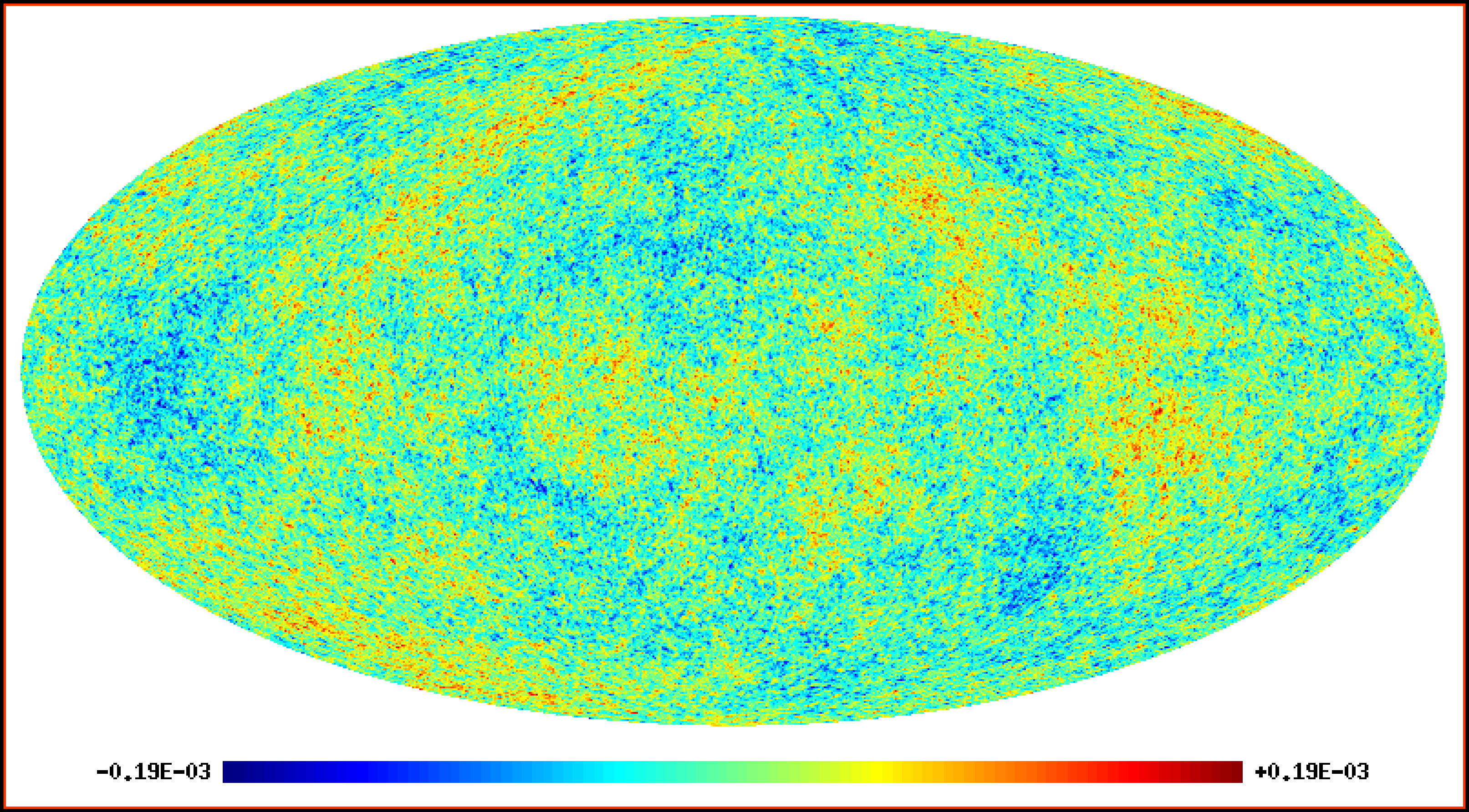
- $\Delta T/T \approx -\Phi/3$, so positive $f_{NL} \Rightarrow$ more cold spots in CMB

- various groups have attempted to measure this with the WMAP CMB data using estimators based on matched filtering (all 95% CL) :

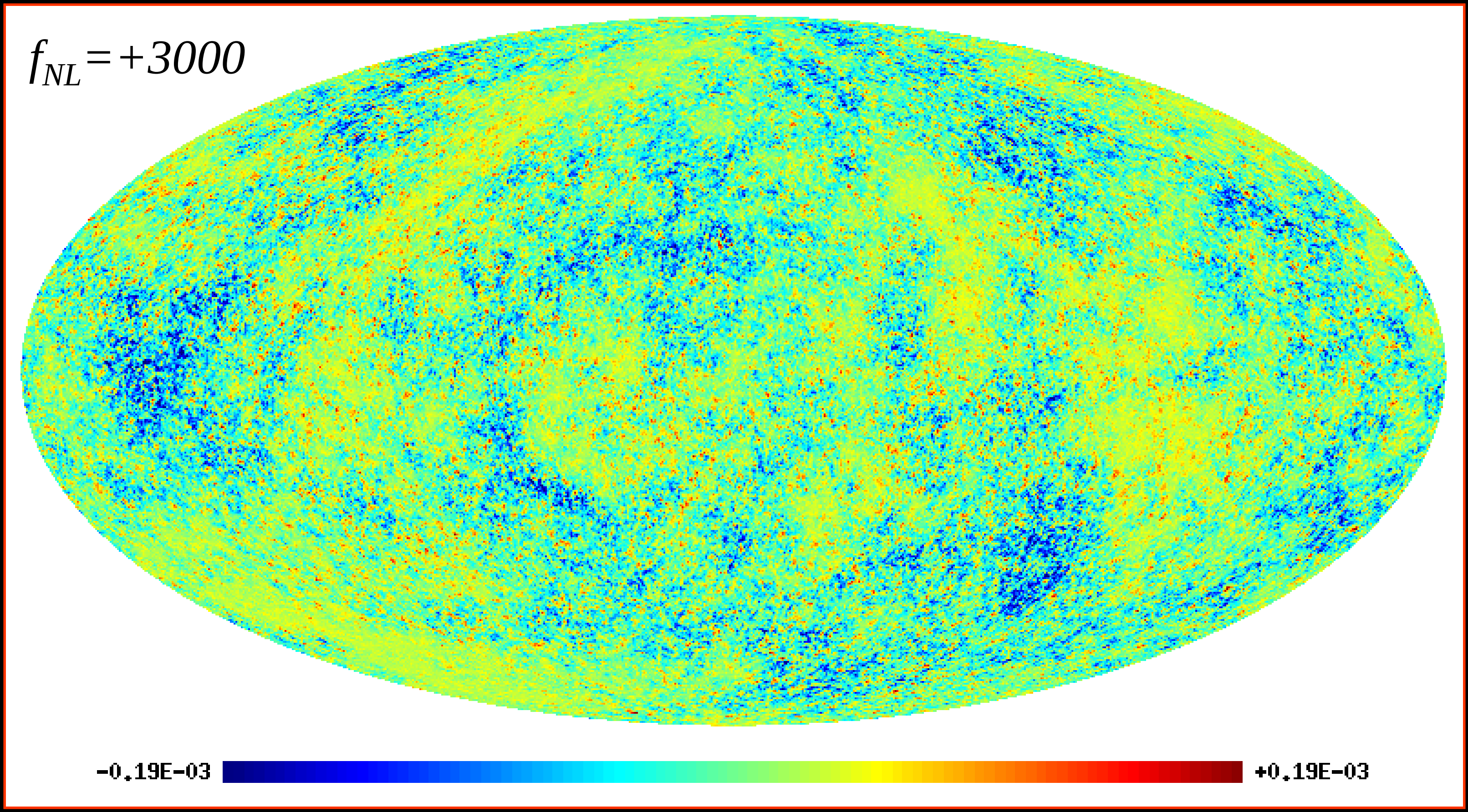
  - $27 < f_{NL} < 147$ Yadav & Wandelt WMAP3 data
  - $-9 < f_{NL} < 111$ Komatsu et al WMAP5
  - $-4 < f_{NL} < 80$ Smith et al. Optimal WMAP5
  - $-10 < f_{NL} < 74$ Komatsu et al WMAP7

- Large scale structure observations have recently given independent indications due to non-local bias on large scales (Dalal et al 2007):

  - $-29 < f_{NL} < 70$ (95% CL) Slosar et al 2008



Liguori, Matarrese and Moscardini (2003



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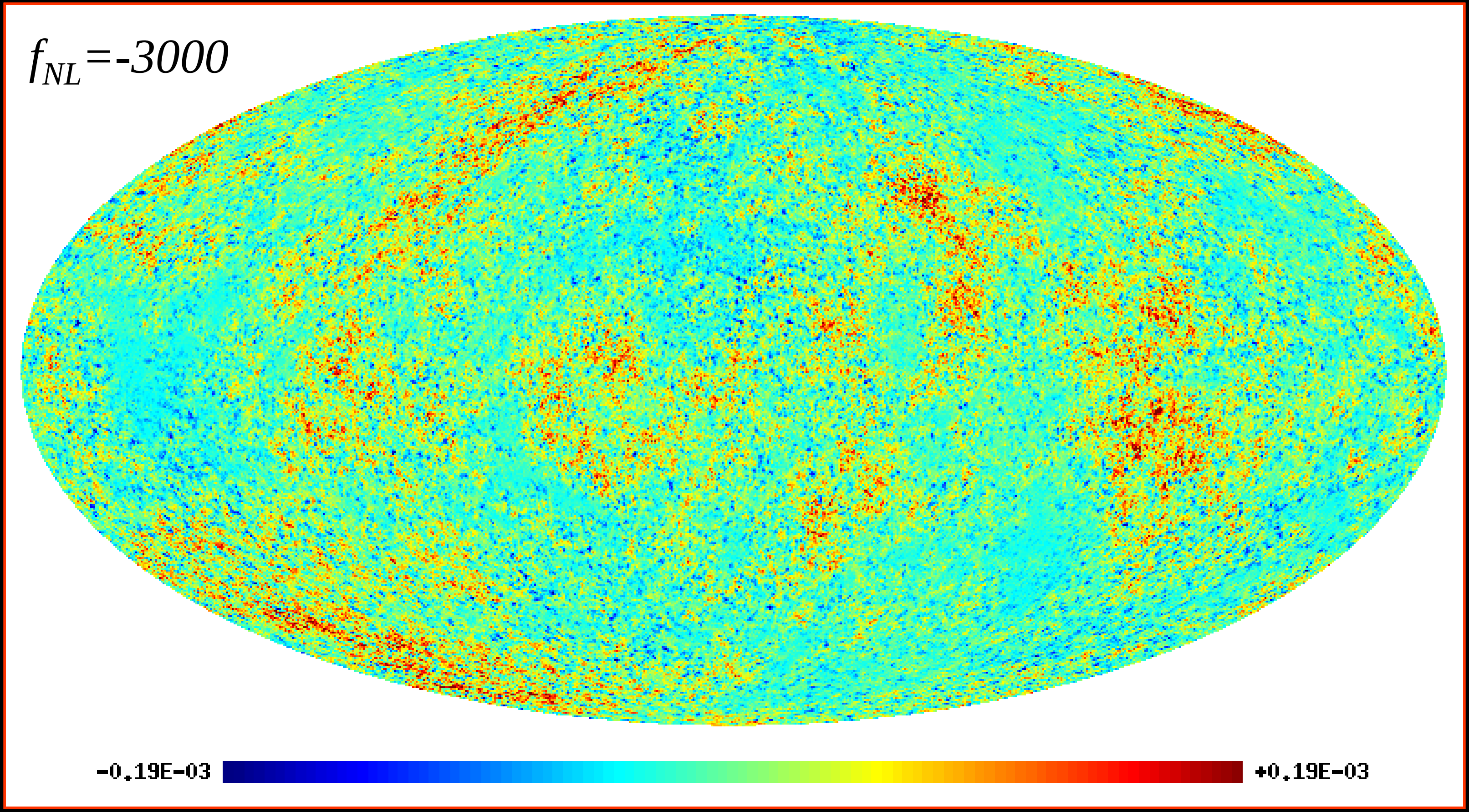

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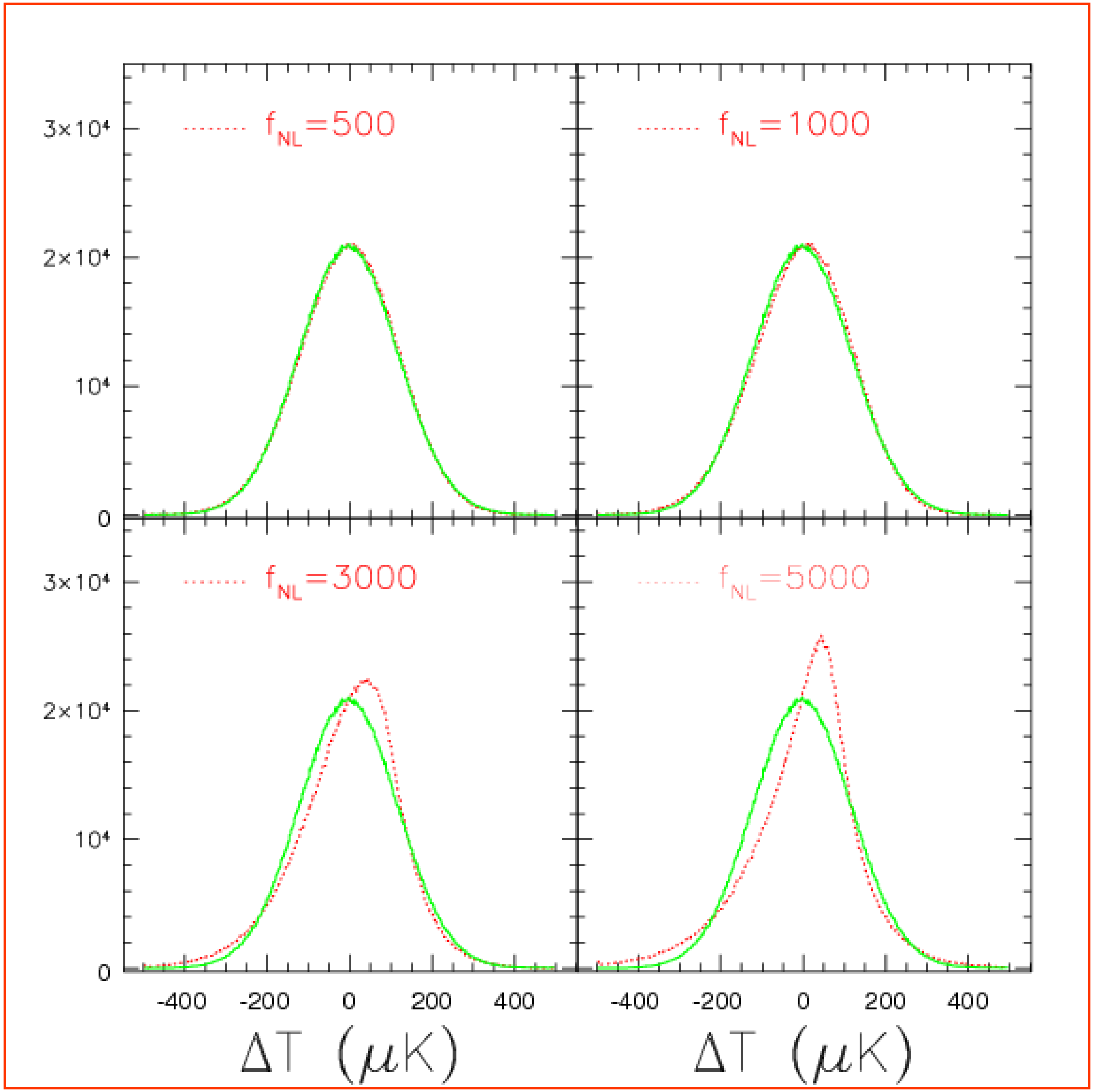


remember: f_{NL} < 100 implies Gaussian to better than 0.1%

non-Gaussianity from inflation?

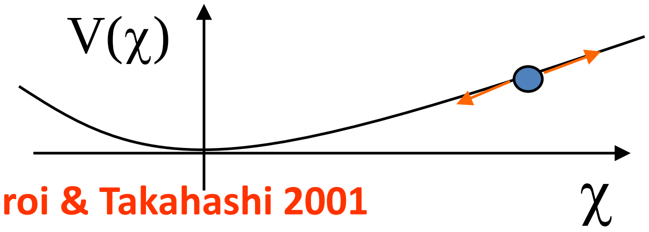
- **slow-roll single inflaton field**
  - adiabatic perturbations => zeta constant on large scales
  - can evaluate non-Gaussianity immediately after Hubble exit

$$f_{NL} = \frac{5}{6}\frac{N''}{(N')^2} = \frac{5}{6}(\eta - 2\epsilon)$$

  - **undetectable** with WMAP or Planck data

- **requires non-canonical or multi-field models**
  - self-interaction on sub-Hubble scales during inflation
    - e.g., DBI inflation in string theory models
    - this is of equilateral type, not the local form
  - local evolution on super-Hubble scales from non-inflaton fields
    - e.g., curvaton or inhomogeneous (p)reheating
    - or fast-roll during ekpyrotic/pre-big-bang collapse?

curvaton scenario:



Linde & Mukhanov 1997; Enqvist & Sloth, Lyth & Wands, Moroi & Takahashi 2001

curvaton χ = a weakly-coupled, late-decaying scalar field

- light during inflation (m<<H) hence acquires an almost scale-invariant, *Gaussian distribution of field fluctuations* on large scales
- energy density for massive field, $\rho_\chi = m^2\chi^2/2$
- spectrum of initially isocurvature density perturbations

$$\zeta_\chi \approx \frac{1}{3}\frac{\delta\rho_\chi}{\rho_\chi} \approx \frac{1}{3}\left(\frac{2\chi\delta\chi + \delta\chi^2}{\chi^2}\right)$$

- transferred to radiation when curvaton decays with some efficiency $\approx \Omega_{\chi,decay}$

$$\zeta = \Omega_{\chi,decay}\zeta_\chi$$

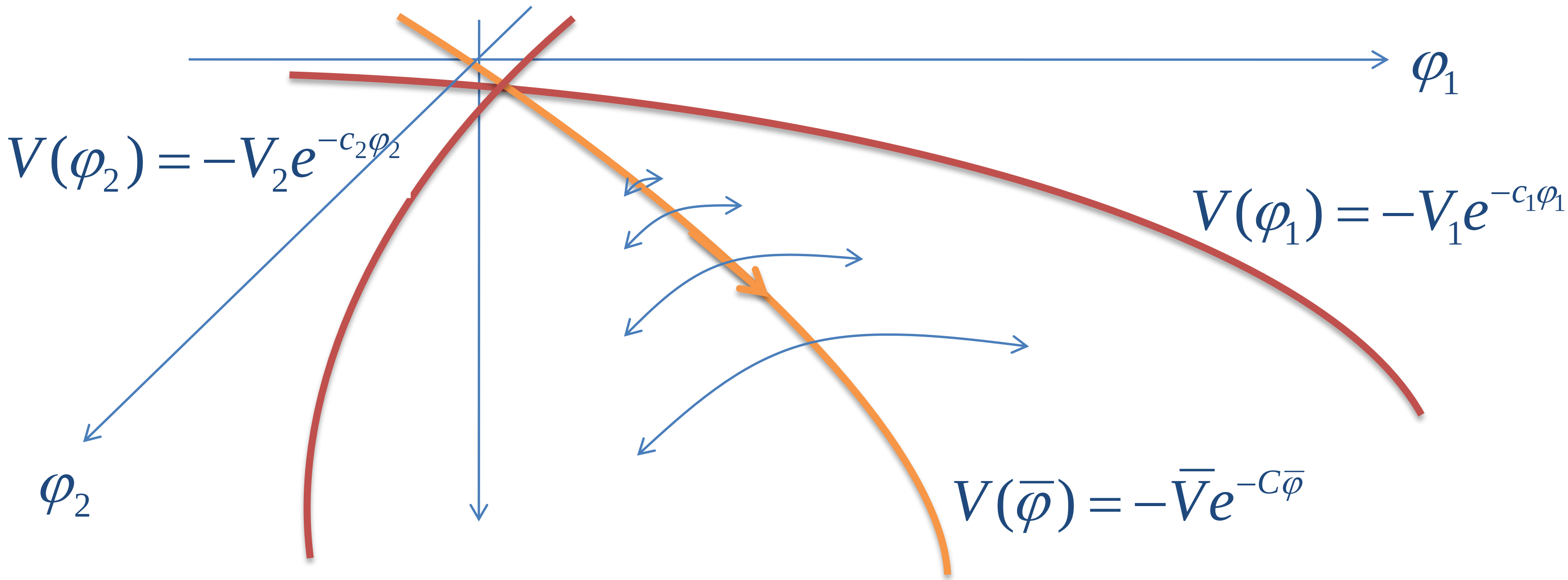
$$= \zeta_G + \frac{3}{4\Omega_{\chi,decay}}\zeta_G^{\ 2} \quad \Rightarrow \quad f_{NL} = {}^{5}\!\big/_{4\Omega_{\chi,decay}}$$

new ekpyrotic scenario:

Lehners et al; Buchbinder et al; Creminelli and Senatore 2007

Contracting universe driven by multiple scalar fields with steep strongly interacting potentials, V<0

- rapidly growing (diverging) Hubble rate during collapse
- tachyonic instability leads to rapidly growing isocurvature perturbations with scale invariant spectrum

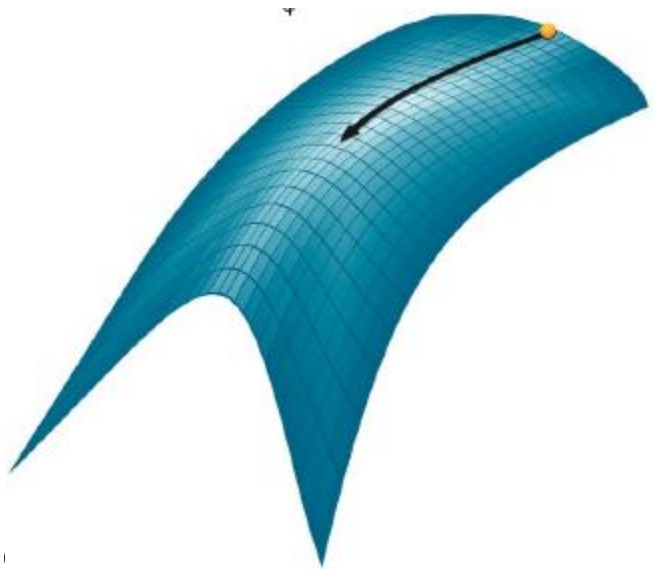


ekpyrotic non-Gaussianity

Koyama, Mizuno, Vernizzi & Wands 2007
(but see also Creminelli & Senatore, Buchbinder et al, Lehners & Steinhardt 2007)

Simplest model:

- tachyonic instability towards steepest descent (-> single field)
- converts isocurvature field perturbations to curvature/density perturbations (calculated via delta-N, but confirmed by Langlois & Vernizzi's second-order equations)



- Simple model => clear predictions:
  - small blue spectral tilt (for $c^2 >> 1$):
    - $n - 1 = 4 / c^2 > 0$
  - large and *negative* bispectrum:
    - $f_{NL} = - (5/12)\, c_i^2 < - (5/3) / (n-1)$

- Other authors consider corrections (e.g., $c_i(\varphi_i)$) and corrections to f_{NL}
  - But generally, steep potentials and fast roll
    => large non-Gaussianity

curvaton vs ekpyrotic non-Gaussianity?

Curvaton

- $f_{NL} > -5/4$
- energy density is quadratic
  - higher order statistics well described by fNL
  - even for multiple curvatons (Assadullahi, Valiviita & Wands 2008)
- unless self-interactions significant (e.g., $\lambda\phi^4$) (Enqvist et al 2009)

Ekpyrotic

- f_{NL} negative or positive?
- potentials are steep quasi-exponential
  - expect large non-linearities at all orders

Beyond f_{NL}?

- **Higher-order statistics**
  - **trispectrum $\Rightarrow$ g_{NL}** (Seery & Lidsey; Byrnes, Sasaki & Wands 2006…)
  - **delta-N gives full probability distribution function** (Sasaki, Valiviita & Wands 2007)

- **Multi-variate local non-Gaussianity**
  - **local function of more than one independent Gaussian field**
  - **adiabatic and entropy decomposition** (Langlois, Vernizzi & Wands 2008)

$$\zeta = \frac{\partial N}{\partial \sigma}\delta\sigma + \frac{\partial N}{\partial s}\delta s + \frac{1}{2}\frac{\partial^2 N}{\partial s^2}\delta s^2 + \ldots$$

    - c.f. Boubekeur & Lyth; Chambers & Rajantie (2007)
  - **e.g., mixed inflaton-curvaton model** (Bartolo & Liddle, 2002, Langlois & Vernizzi 2003, etc)
  - **scale-dependent fNL** (Byrnes, Nurmi, Tasinato & Wands 2009)

- **Non-linear isocurvature perturbations**
  - **extend δN to isocurvature modes** (Kawasaki et al; Langlois, Vernizzi & Wands 2008)
  - **limits on isocurvature density perturbations** (Hikage et al 2008)

Scale-dependent f_{NL}

$$n_{f_{NL}} = \frac{d \ln |f_{NL}|}{d \ln k}$$

Byrnes, Choi & Hall 2009
Khoury & Piazza 2009
Sefusatti, Liguori, Yadav, Jackson & Pajer 2009
Byrnes, Nurmi, Tasinato & Wands 2009

- Local single-field mode
  - scale indepednednt fNL by construction

$$\Phi(x) = \Phi_G(x) + f_{NL}\left(\Phi_G{}^2(x) - \left\langle \Phi_G{}^2 \right\rangle\right) + \ldots \quad \Rightarrow \quad n_{f_{NL}} = 0$$

- Quasi-local single-field model
  - self-interactions lead to non-Gaussian field on super-Hubble scales

$$f_{NL} = \frac{5}{6}\left(\frac{N''}{N'^2}\right)_{k=aH} \quad \Rightarrow \quad n_{f_{NL}} = \frac{d \ln \left| N''/N'^2 \right|_{k=aH}}{d \ln a}$$

- Local multiple-field models e.g., inflaton+curvaton

$$\Phi(x) = \Phi_A(x) + \Phi_B(x) + f_{NL}^{(B)}\left(\Phi_B{}^2(x) - \left\langle \Phi_B{}^2 \right\rangle\right) + \ldots \quad \Rightarrow \quad n_{f_{NL}} \neq 0$$

Adiabatic+entropy split

Gordon, Wands, Bassett & Maartens 2001
Langlois, Vernizzi & Wands 2008

$$\zeta = \frac{\partial N}{\partial \sigma}\delta\sigma + \frac{\partial N}{\partial s}\delta s + \frac{1}{2}\frac{\partial^2 N}{\partial s^2}\delta s^2 + \dots$$

- adiabatic + entropy modes both contribute to power spectrum

$$P(k) = P_\sigma(k) + P_s(k)$$

$$\text{spectral tilts}: \quad n - 1 = \frac{d\ln P}{d\ln k} - 3, \quad n_s - 1 = \frac{d\ln P_s}{d\ln k} - 3$$

- only entropy modes contribute significantly to the bispectrum

$$B(k_1,k_2,k_3) = B_s(k_1,k_2,k_3) = f_{NL}^{(s)}\left(P_s(k_1)P_s(k_2) + perms\right)$$

$$\Rightarrow \quad f_{NL} = f_{NL}^{(s)}\frac{\left(P_s(k_1)P_s(k_2) + perms\right)}{\left(P(k_1)P(k_2) + perms\right)}$$

$$\Rightarrow \quad n_{f_{NL}} = \frac{d\ln|f_{NL}|}{d\ln k} = 2\left(n_s - n\right)$$

  - adiabatic mode bispectrum suppressed (Maldacena 2002)

summary:

- ***Inflation*** *is simplest model for origin of structure compatible with current data*
- ***Spectral index*** *of density perturbations ($n \neq 1$) evidence for slow-roll dynamics during inflation*
- *Canonical* ***single-field models*** *(e.g. chaotic inflation) in* ***good agreement*** *with power spectra, but…*
- *Any* ***non-Gaussianity*** *and/or* ***non-adiabaticity*** *of primordial perturbations* ***would rule out all canonical single-field models***
- ***Still lots of models, but lots more data coming!***